cs109a_hw3_109_submit

October 3, 2018

1 CS109A Introduction to Data Science:

1.1 Homework 3 - Forecasting Bike Sharing Usage

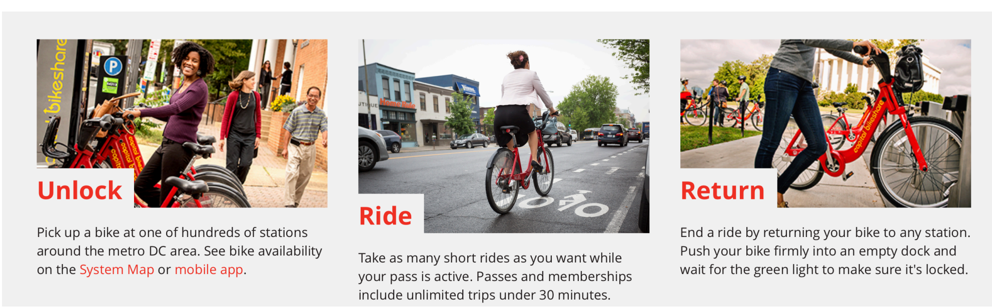
Harvard University Fall 2018 Instructors: Pavlos Protopapas, Kevin Rader

```
In [1]: #RUN THIS CELL
        import requests
        from IPython.core.display import HTML
        styles = requests.get("https://raw.githubusercontent.com/Harvard-IACS/2018-CS109A/maste
        HTML(styles)

Out[1]: <IPython.core.display.HTML object>
```

1.1.1 INSTRUCTIONS

- To submit your assignment follow the instructions given in canvas.
- Restart the kernel and run the whole notebook again before you submit.
- If you submit individually and you have worked with someone, please include the name of your [one] partner below.
- As much as possible, try and stick to the hints and functions we import at the top of the homework, as those are the ideas and tools the class supports and is aiming to teach. And if a problem specifies a particular library you're required to use that library, and possibly others from the import list.

Names of people you have worked with goes here: Avirel Epps, Erin Williams
Main Theme: Multiple Linear Regression, Subset Selection, Polynomial Regression



bike_sharing

1.1.2 Overview

You are hired by the administrators of the Capital Bikeshare program program in Washington D.C., to **help them predict the hourly demand for rental bikes** and **give them suggestions on how to increase their revenue**. Your task is to prepare a short report summarizing your findings and make recommendations.

The predicted hourly demand could be used for planning the number of bikes that need to be available in the system at any given hour of the day. It costs the program money if bike stations are full and bikes cannot be returned, or empty and there are no bikes available. You will use multiple linear regression and polynomial regression and will explore techniques for subset selection to predict bike usage. The goal is to build a regression model that can predict the total number of bike rentals in a given hour of the day, based on all available information given to you.

An example of a suggestion to increase revenue might be to offer discounts during certain times of the day either during holidays or non-holidays. Your suggestions will depend on your observations of the seasonality of ridership.

The data for this problem were collected from the Capital Bikeshare program over the course of two years (2011 and 2012).

1.1.3 Use only the libraries below:

In [2]:
```
import numpy as np
import pandas as pd
import matplotlib
import matplotlib.pyplot as plt

import statsmodels.api as sm
from statsmodels.api import OLS

from sklearn import preprocessing
from sklearn.preprocessing import PolynomialFeatures
from sklearn.metrics import r2_score
from sklearn.model_selection import train_test_split

from pandas.plotting import scatter_matrix

from pandas.core import datetools

import seaborn as sns

import itertools
import warnings
warnings.filterwarnings("ignore")

%matplotlib inline
```

```
C:\Users\erina\Anaconda3\lib\site-packages\ipykernel_launcher.py:16: FutureWarning: The pandas
  app.launch_new_instance()
```

1.2 Data Exploration & Preprocessing, Multiple Linear Regression, Subset Selection

1.2.1 Overview

The initial data set is provided in the file `data/BSS_hour_raw.csv`. You will first add features that will help with the analysis and then separate the data into training and test sets. Each row in this file represents the number of rides by registered users and casual users in a given hour of a specific date. There are 12 attributes in total describing besides the number of users the weather if it is a holiday or not etc:

- `dteday` (date in the format YYYY-MM-DD, e.g. 2011-01-01)
- `season` (1 = winter, 2 = spring, 3 = summer, 4 = fall)
- `hour` (0 for 12 midnight, 1 for 1:00am, 23 for 11:00pm)
- `weekday` (0 through 6, with 0 denoting Sunday)
- `holiday` (1 = the day is a holiday, 0 = otherwise)
- `weather`
  - 1: Clear, Few clouds, Partly cloudy, Partly cloudy
  - 2: Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist
  - 3: Light Snow, Light Rain + Thunderstorm
  - 4: Heavy Rain + Thunderstorm + Mist, Snow + Fog
- `temp` (temperature in Celsius)
- `atemp` (apparent temperature, or relative outdoor temperature, in Celsius)
- `hum` (relative humidity)
- `windspeed` (wind speed)
- `casual` (number of rides that day made by casual riders, not registered in the system)
- `registered` (number of rides that day made by registered riders)

1.2.2 General Hints

- Use pandas .describe() to see statistics for the dataset.
- When performing manipulations on column data it is useful and often more efficient to write a function and apply this function to the column as a whole without the need for iterating through the elements.
- A scatterplot matrix or correlation matrix are both good ways to see dependencies between multiple variables.
- For Question 2, a very useful pandas method is .groupby(). Make sure you aggregate the rest of the columns in a meaningful way. Print the dataframe to make sure all variables/columns are there!

1.2.3 Resources

http://pandas.pydata.org/pandas-docs/stable/generated/pandas.to_datetime.html

Question 1: Data Read-In and Cleaning

In this section, we read in the data and begin one of the most important analytic steps: verifying that the data is what it claims to be.

1.1 Load the dataset from the csv file `data/BSS_hour_raw.csv` into a pandas dataframe that you name `bikes_df`. Do any of the variables' ranges or averages seem suspect? Do the data types make sense?

1.2 Notice that the variable in column `dteday` is a pandas `object`, which is **not** useful when you want to extract the elements of the date such as the year, month, and day. Convert `dteday` into a `datetime` object to prepare it for later analysis.

1.3 Create three new columns in the dataframe: - `year` with 0 for 2011, 1 for 2012, etc. - `month` with 1 through 12, with 1 denoting January. - `counts` with the total number of bike rentals for that **hour** (this is the response variable for later).

1.2.4 Answers

1.1 Load the dataset from the csv file `data/BSS_hour_raw.csv` into a pandas dataframe that you name `bikes_df`. Do any of the variables' ranges or averages seem suspect? Do the data types make sense?

```
In [3]: bikes_df = pd.read_csv('data/BSS_hour_raw.csv', sep=",")
        display(bikes_df.head())
```

```
       dteday  season  hour  holiday  weekday  workingday  weather  temp  \
0  2011-01-01       1     0        0        6           0        1  0.24
1  2011-01-01       1     1        0        6           0        1  0.22
2  2011-01-01       1     2        0        6           0        1  0.22
3  2011-01-01       1     3        0        6           0        1  0.24
4  2011-01-01       1     4        0        6           0        1  0.24

    atemp   hum  windspeed  casual  registered
0  0.2879  0.81        0.0       3          13
1  0.2727  0.80        0.0       8          32
2  0.2727  0.80        0.0       5          27
3  0.2879  0.75        0.0       3          10
4  0.2879  0.75        0.0       0           1
```

```
In [4]: bikes_df.describe()
```

```
Out[4]:              season          hour       holiday       weekday    workingday  \
        count  17379.000000  17379.000000  17379.000000  17379.000000  17379.000000
        mean       2.501640     11.546752      0.028770      3.003683      0.682721
        std        1.106918      6.914405      0.167165      2.005771      0.465431
        min        1.000000      0.000000      0.000000      0.000000      0.000000
        25%        2.000000      6.000000      0.000000      1.000000      0.000000
        50%        3.000000     12.000000      0.000000      3.000000      1.000000
        75%        3.000000     18.000000      0.000000      5.000000      1.000000
        max        4.000000     23.000000      1.000000      6.000000      1.000000

                    weather          temp         atemp           hum     windspeed  \
        count  17379.000000  17379.000000  17379.000000  17379.000000  17379.000000
        mean       1.425283      0.496987      0.475775      0.627229      0.190098
        std        0.639357      0.192556      0.171850      0.192930      0.122340
        min        1.000000      0.020000      0.000000      0.000000      0.000000
        25%        1.000000      0.340000      0.333300      0.480000      0.104500
```

```
50%         1.000000      0.500000      0.484800      0.630000      0.194000
75%         2.000000      0.660000      0.621200      0.780000      0.253700
max         4.000000      1.000000      1.000000      1.000000      0.850700

             casual    registered
count  17379.000000  17379.000000
mean      35.676218    153.786869
std       49.305030    151.357286
min        0.000000      0.000000
25%        4.000000     34.000000
50%       17.000000    115.000000
75%       48.000000    220.000000
max      367.000000    886.000000
```

In [5]:
```
# your code here
bikes_df.loc[bikes_df['registered'] == 886.00]
```

Out[5]:
```
           dteday  season  hour  holiday  weekday  workingday  weather  temp  \
14773  2012-09-12       3    18        0        3           1        1  0.66

        atemp   hum  windspeed  casual  registered
14773  0.6212  0.44     0.2537      91         886
```

In [6]:
```
summer_df = bikes_df.loc[bikes_df['weather'] == 1]
display(summer_df.head())
```

```
       dteday  season  hour  holiday  weekday  workingday  weather  temp  \
0  2011-01-01       1     0        0        6           0        1  0.24
1  2011-01-01       1     1        0        6           0        1  0.22
2  2011-01-01       1     2        0        6           0        1  0.22
3  2011-01-01       1     3        0        6           0        1  0.24
4  2011-01-01       1     4        0        6           0        1  0.24

    atemp   hum  windspeed  casual  registered
0  0.2879  0.81        0.0       3          13
1  0.2727  0.80        0.0       8          32
2  0.2727  0.80        0.0       5          27
3  0.2879  0.75        0.0       3          10
4  0.2879  0.75        0.0       0           1
```

Answer

The majority of the data makes sense -- rides spike on holidays, there are more riders in the spring/summer/fall months than winter, and the highest volume of rides occur during the workday. However, the temperature seems to be skewed. Our summary shows an average temperature of 49^o C, which is 120^o F. In Washington, DC, average temperatures in July are 80^oF/20^oC, and in January the averages are 38^oF/4^oC. This tells us the data is skewed or some sort of scale is off. THe high temperature value is consistent across individual data points.

1.2 Notice that the variable in column `dteday` is a pandas `object`, which is not useful when you want to extract the elements of the date such as the year, month, and day. Convert `dteday` into a `datetime` object to prepare it for later analysis.

```
In [7]: bikes_df['dteday']= pd.to_datetime(bikes_df['dteday'])
        display(bikes_df.dteday.head())
```

```
0   2011-01-01
1   2011-01-01
2   2011-01-01
3   2011-01-01
4   2011-01-01
Name: dteday, dtype: datetime64[ns]
```

1.3 Create three new columns in the dataframe:

- `year` with 0 for 2011, 1 for 2012, etc.
- `month` with 1 through 12, with 1 denoting January.
- `counts` with the total number of bike rentals for that hour (this is the response variable for later).

```
In [8]: bikes_df['year']= pd.DatetimeIndex(bikes_df['dteday']).year
        bikes_df['month']= pd.DatetimeIndex(bikes_df['dteday']).month
        bikes_df['year']= bikes_df['year'].map({2012:1,2011:0})
        bikes_df['counts']=bikes_df.casual+bikes_df.registered
        display(bikes_df.head())
```

```
      dteday  season  hour  holiday  weekday  workingday  weather  temp  \
0 2011-01-01       1     0        0        6           0        1  0.24
1 2011-01-01       1     1        0        6           0        1  0.22
2 2011-01-01       1     2        0        6           0        1  0.22
3 2011-01-01       1     3        0        6           0        1  0.24
4 2011-01-01       1     4        0        6           0        1  0.24

    atemp   hum  windspeed  casual  registered  year  month  counts
0  0.2879  0.81        0.0       3          13     0      1      16
1  0.2727  0.80        0.0       8          32     0      1      40
2  0.2727  0.80        0.0       5          27     0      1      32
3  0.2879  0.75        0.0       3          10     0      1      13
4  0.2879  0.75        0.0       0           1     0      1       1
```

Question 2: Exploratory Data Analysis.

In this question, we continue validating the data, and begin hunting for patterns in ridership that shed light on who uses the service and why.

2.1 Use pandas' `scatter_matrix` command to visualize the inter-dependencies among all predictors in the dataset. Note and comment on any strongly related variables. [This will take several minutes to run. You may wish to comment it out until your final submission, or only plot a randomly-selected 10% of the rows]

2.2 Make a plot showing the *average* number of casual and registered riders during each hour of the day. `.groupby` and `.aggregate` should make this task easy. Comment on the trends you observe.

2.3 Use the variable `weather` to show how each weather category affects the relationships in question 2.2. What do you observe?

2.4 Make a new dataframe with the following subset of attributes from the previous dataset and with each entry being just **one** day:

- `dteday`, the timestamp for that day (fine to set to noon or any other time)
- `weekday`, the day of the week
- `weather`, the most severe weather that day
- `season`, the season that day falls in
- `temp`, the average temperature (normalized)
- `atemp`, the average atemp that day (normalized)
- `windspeed`, the average windspeed that day (normalized)
- `hum`, the average humidity that day (normalized)
- `casual`, the **total** number of rentals by casual users
- `registered`, the **total** number of rentals by registered users
- `counts`, the **total** number of rentals of that day

Name this dataframe `bikes_by_day`.

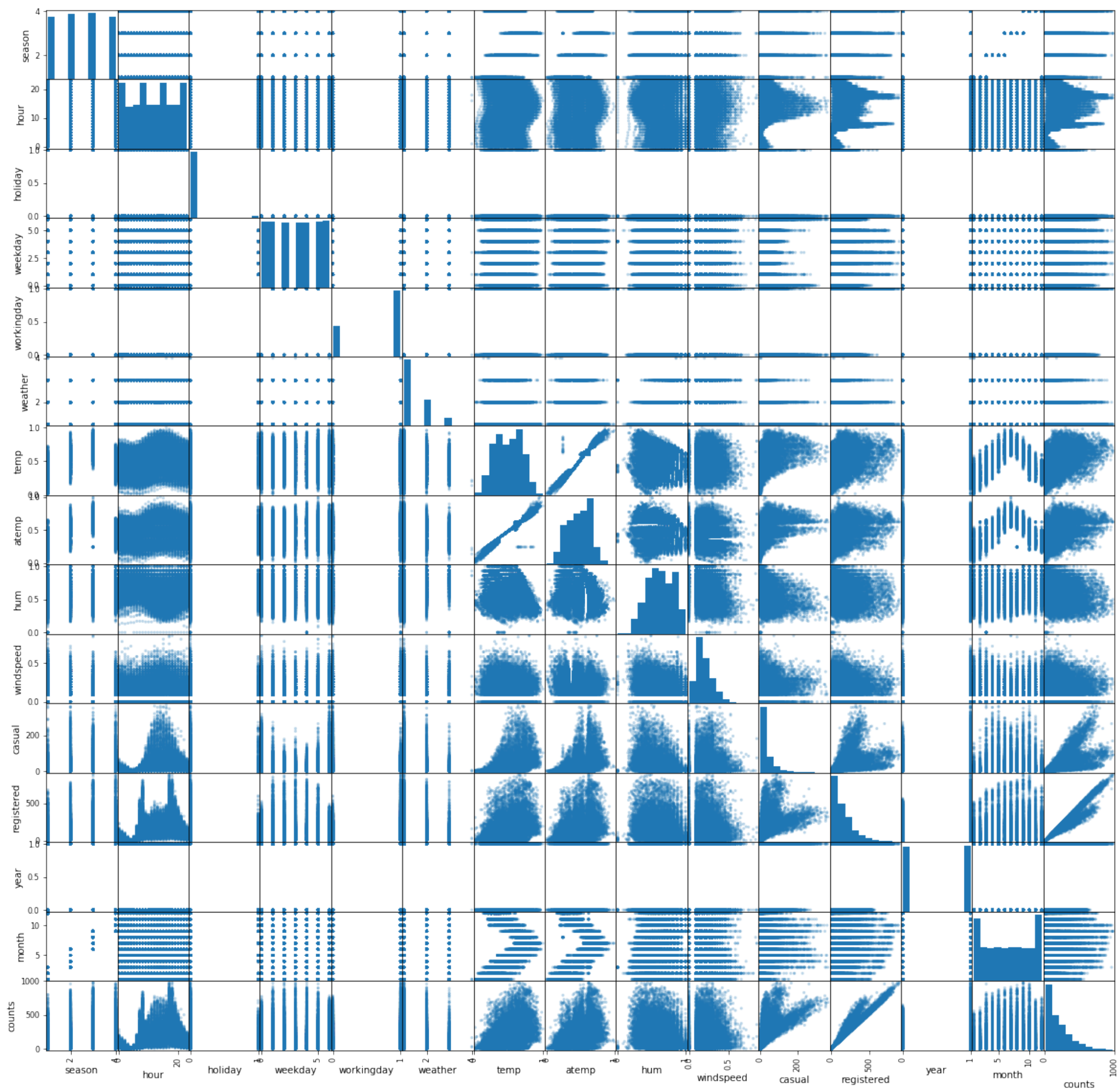
Make a plot showing the *distribution* of the number of casual and registered riders on each day of the week.

2.5 Use `bikes_by_day` to visualize how the distribution of **total number of rides** per day (casual and registered riders combined) varies with the **season**. Do you see any **outliers**? Here we use the pyplot's boxplot function definition of an outlier as any value 1.5 times the IQR above the 75th percentile or 1.5 times the IQR below the 25th percentiles. If you see any outliers, identify those dates and investigate if they are a chance occurence, an error in the data collection, or a significant event (an online search of those date(s) might help).

1.2.5 Answers

2.1 Use pandas' `scatter_matrix` command to visualize the inter-dependencies among all predictors in the dataset. Note and comment on any strongly related variables. [This will take several minutes to run. You may wish to comment it out until your final submission, or only plot a randomly-selected 10% of the rows]

```
In [9]: scatter_matrix(bikes_df, alpha = 0.3, figsize=(20,20));
```



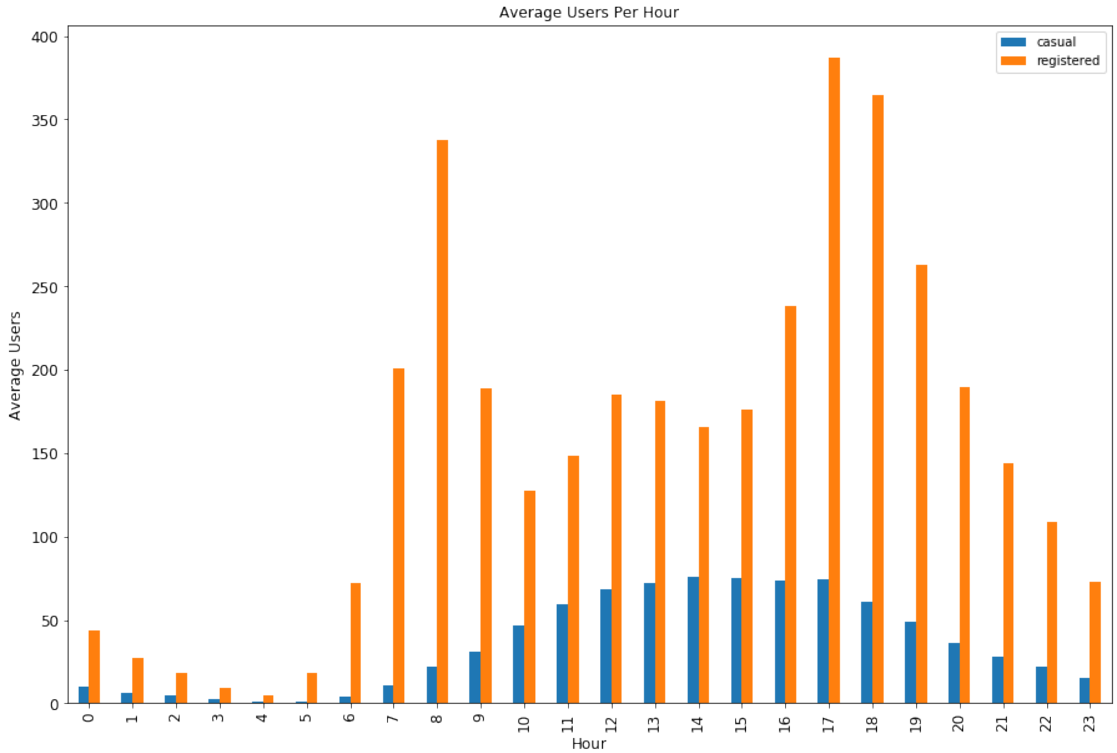
Answer
We don't see any surprising relationships. Most notably, counts increases during daytime hours and is higher during late spring, summer, and early fall months. Additionally, counts is closely realted to the weather type, with significantly higher values when the weather is rated as "1" or "2".

2.2 Make a plot showing the *average* number of casual and registered riders during each hour of the day. `.groupby` and `.aggregate` should make this task easy. Comment on the trends you observe.

```
In [10]: hour_casual_mean = bikes_df.groupby('hour').agg({'casual':'mean'})
         hour_registered_mean= bikes_df.groupby('hour').agg({'registered':'mean'})
```

```
In [11]: avg_hour_frames = [hour_casual_mean,hour_registered_mean]
         avg_hour_df = pd.concat(avg_hour_frames, axis = 1)
```

```
In [12]: user_plot = avg_hour_df.plot(kind='bar', title ="Average Users Per Hour", figsize=(15
         user_plot.set_xlabel("Hour", fontsize=12)
         user_plot.set_ylabel("Average Users", fontsize=12)
         plt.show()
```



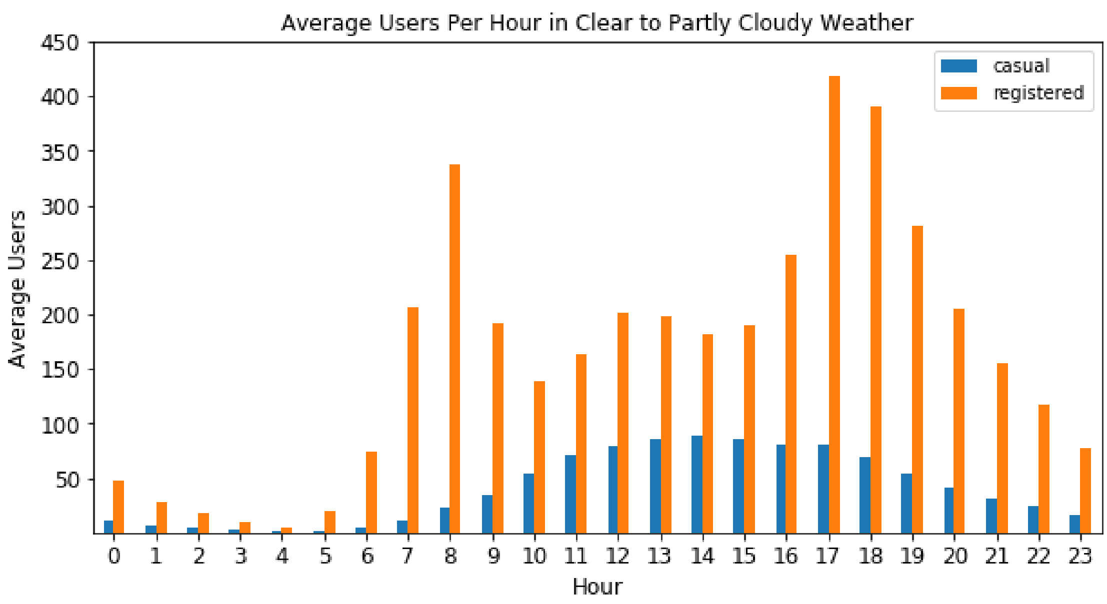
Answer
We can easily observe that rides for registered users spike from 0700-0900 and again from 1600-1900, which generally aligns with rush hour. This makes sense, since registered users likely rely on the bike share as a means of commuting. Casual users spike from 1200 to 1800, indicating that they are likely tourists who are using the bike share to see DC or get to an attraction.

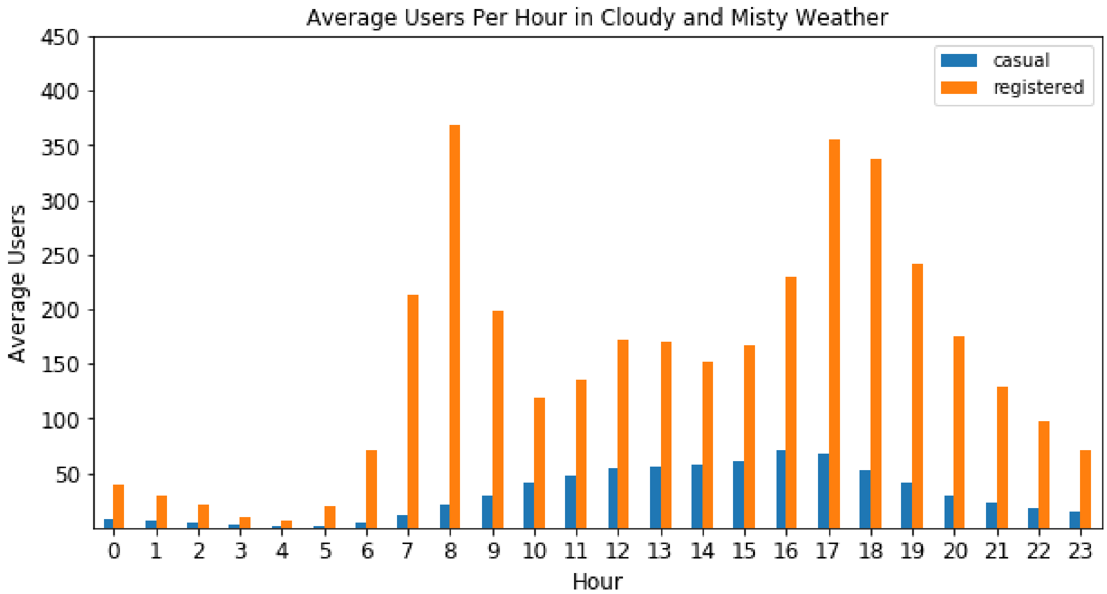
2.3 Use the variable `weather` to show how each weather category affects the relationships in question 2.2. What do you observe?

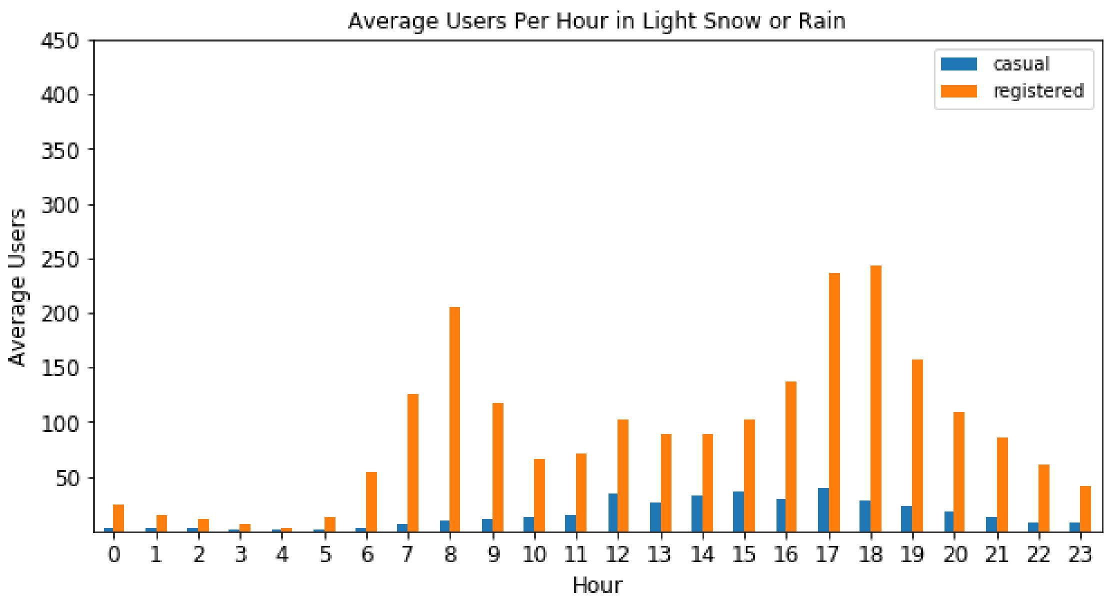
```
In [13]: # sort into data frames for each weather type
         weather1_df = bikes_df.loc[bikes_df['weather'] == 1]
         weather2_df = bikes_df.loc[bikes_df['weather'] == 2]
         weather3_df = bikes_df.loc[bikes_df['weather'] == 3]
         weather4_df = bikes_df.loc[bikes_df['weather'] == 4]
```

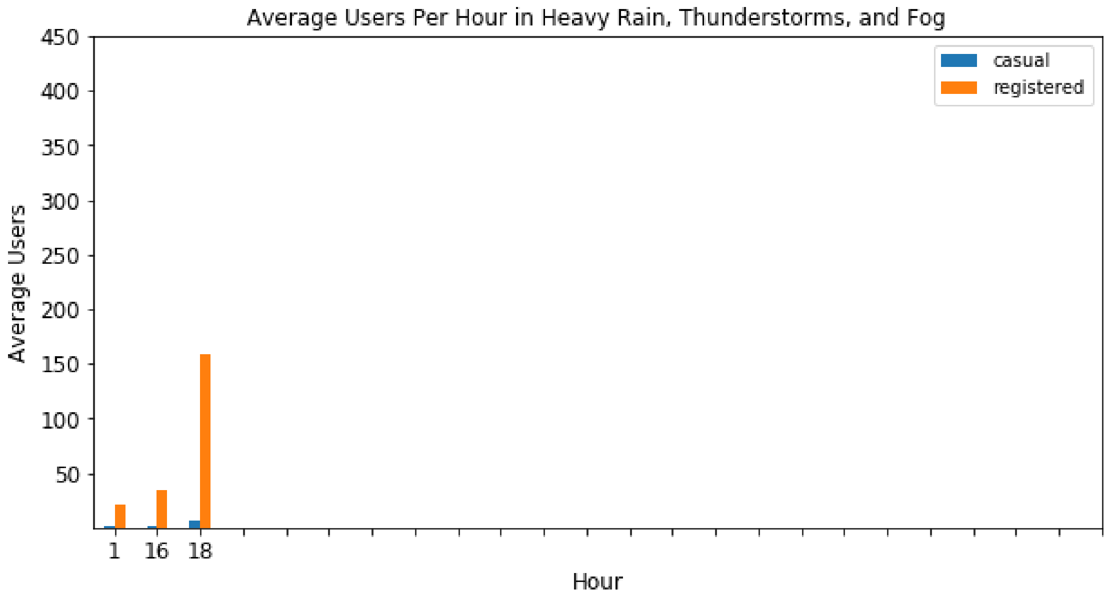
```
In [14]: mean_weather1 = weather1_df.groupby(['hour']).agg({'casual':'mean', 'registered':'mea
         mean_weather2 = weather2_df.groupby(['hour']).agg({'casual':'mean', 'registered':'mea
         mean_weather3 = weather3_df.groupby(['hour']).agg({'casual':'mean', 'registered':'mea
         mean_weather4 = weather4_df.groupby(['hour']).agg({'casual':'mean', 'registered':'mea
```

In [15]:
```
user_plot1 = mean_weather1.plot.bar(rot = 0, title ="Average Users Per Hour in Clear t
user_plot1.set_xlabel("Hour", fontsize=12)
user_plot1.set_ylabel("Average Users", fontsize=12)
user_plot1.set_yticks([50,100,150,200,250,300,350,400,450])
plt.show()

user_plot2 = mean_weather2.plot.bar(rot = 0, title ="Average Users Per Hour in Cloudy
user_plot2.set_xlabel("Hour", fontsize=12)
user_plot2.set_ylabel("Average Users", fontsize=12)
user_plot2.set_yticks([50,100,150,200,250,300,350,400,450])
plt.show()

user_plot3 = mean_weather3.plot.bar(rot = 0, title ="Average Users Per Hour in Light S
user_plot3.set_xlabel("Hour", fontsize=12)
user_plot3.set_ylabel("Average Users", fontsize=12)
user_plot3.set_yticks([50,100,150,200,250,300,350,400,450])
plt.show()

user_plot4 = mean_weather4.plot.bar(rot = 0, title ="Average Users Per Hour in Heavy R
user_plot4.set_xlabel("Hour", fontsize=12)
user_plot4.set_ylabel("Average Users", fontsize=12)
user_plot4.set_yticks([50,100,150,200,250,300,350,400,450])
user_plot4.set_xticks([0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23])
plt.show()
```




Average Users Per Hour in Cloudy and Misty Weather
casual
registered
Average Users
Hour
450
400
350
300
250
200
150
100
50
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23


Average Users Per Hour in Light Snow or Rain
casual
registered
Average Users
Hour
450
400
350
300
250
200
150
100
50
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23



Answer We can see that while the trend line generally holds, the averages begin to fall as weather gets bad (decreasing from 1 to 2, and from 2 to 3), and riders almost completely drop off during the worst weather conditions.

2.4 Make a new dataframe with the following subset of attributes from the previous dataset and with each entry being just one day:

- `dteday`, the timestamp for that day (fine to set to noon or any other time)
- `weekday`, the day of the week
- `weather`, the most severe weather that day
- `season`, the season that day falls in
- `temp`, the average temperature (normalized)
- `atemp`, the average atemp that day (normalized)
- `windspeed`, the average windspeed that day (normalized)
- `hum`, the average humidity that day (normalized)
- `casual`, the **total** number of rentals by casual users
- `registered`, the **total** number of rentals by registered users
- `counts`, the **total** number of rentals of that day

Name this dataframe `bikes_by_day`.

Make a plot showing the *distribution* of the number of casual and registered riders on each day of the week.

```
In [16]: bikes_by_day = pd.DataFrame(bikes_df.groupby('dteday').agg({
             'dteday': np.min,
             'temp': np.mean,
             'atemp': np.mean,
```

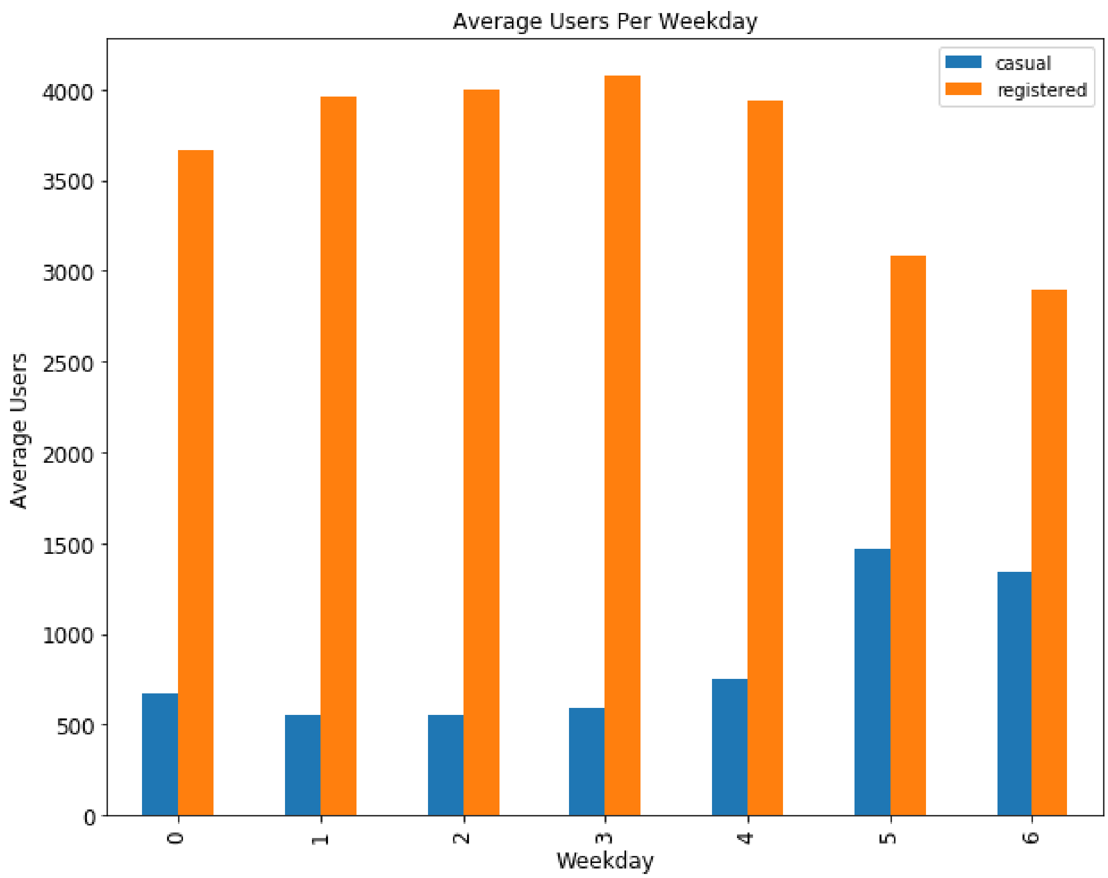
```
        'windspeed': np.mean,
        'hum': np.mean,
        'casual': np.sum,
        'registered': np.sum,
        'counts': np.sum
    }))
    bikes_by_day2 = pd.DataFrame()
    bikes_by_day2['dteday'] = pd.to_datetime(bikes_df['dteday'])
    bikes_by_day2['weekday'] = pd.DatetimeIndex(bikes_df['dteday']).weekday #0 = monday,
    bikes_by_day2['season'] = pd.DatetimeIndex(bikes_df['dteday']).month
    bikes_by_day2['season'] = ((bikes_by_day2.season%12 + 3)//3)-1 # 0 = winter, 1 = sprin
    bikes_by_day2 = bikes_by_day2.drop_duplicates(["dteday"])
    bikes_by_day2.set_index('dteday')
    bikes_by_day = pd.merge(bikes_by_day, bikes_by_day2, on=['dteday'])
    display(bikes_by_day.head())
```

```
      dteday      temp     atemp  windspeed       hum  casual  registered  \
0 2011-01-01  0.344167  0.363625   0.160446  0.805833     331         654
1 2011-01-02  0.363478  0.353739   0.248539  0.696087     131         670
2 2011-01-03  0.196364  0.189405   0.248309  0.437273     120        1229
3 2011-01-04  0.200000  0.212122   0.160296  0.590435     108        1454
4 2011-01-05  0.226957  0.229270   0.186900  0.436957      82        1518

   counts  weekday  season
0     985        5       0
1     801        6       0
2    1349        0       0
3    1562        1       0
4    1600        2       0
```

```
In [17]: casual_mean = bikes_by_day.groupby('weekday').agg({'casual':'mean'})
         registered_mean= bikes_by_day.groupby('weekday').agg({'registered':'mean'})
```

```
In [18]: avg_week_frames = [casual_mean,registered_mean]
         avg_week_df = pd.concat(avg_week_frames, axis = 1)
```

```
In [19]: user_plot = avg_week_df.plot(kind='bar', title ="Average Users Per Weekday", figsize=
         user_plot.set_xlabel("Weekday", fontsize=12)
         user_plot.set_ylabel("Average Users", fontsize=12)
         plt.show()
```



Answer
Registered users are highest during weekdays. Registered users likely use the bikes to commute to work and run errands, so this makes sense. Casual users increase on weekends. Generally speaking, more tourists visit DC on the weekends, so this also makes sense.

2.5 Use `bikes_by_day` to visualize how the distribution of **total number of rides** per day (casual and registered riders combined) varies with the **season**. Do you see any **outliers**? Here we use the pyplot's boxplot function definition of an outlier as any value 1.5 times the IQR above the 75th percentile or 1.5 times the IQR below the 25th percentiles. If you see any outliers, identify those dates and investigate if they are a chance occurence, an error in the data collection, or a significant event (an online search of those date(s) might help).

In [20]:
```
# your code here
display(bikes_by_day.head())
```

	dteday	temp	atemp	windspeed	hum	casual	registered	\
0	2011-01-01	0.344167	0.363625	0.160446	0.805833	331	654	
1	2011-01-02	0.363478	0.353739	0.248539	0.696087	131	670	
2	2011-01-03	0.196364	0.189405	0.248309	0.437273	120	1229	
3	2011-01-04	0.200000	0.212122	0.160296	0.590435	108	1454	
4	2011-01-05	0.226957	0.229270	0.186900	0.436957	82	1518	

```
   counts  weekday  season
0     985        5       0
1     801        6       0
2    1349        0       0
3    1562        1       0
4    1600        2       0
```

```
In [21]: bikes_by_day_winter = bikes_by_day.loc[bikes_by_day['season'] == 0]
         bikes_by_day_spring = bikes_by_day.loc[bikes_by_day['season'] == 1]
         bikes_by_day_summer = bikes_by_day.loc[bikes_by_day['season'] == 2]
         bikes_by_day_fall = bikes_by_day.loc[bikes_by_day['season'] == 3]
```

```
In [22]: display(bikes_by_day_winter.head())
```

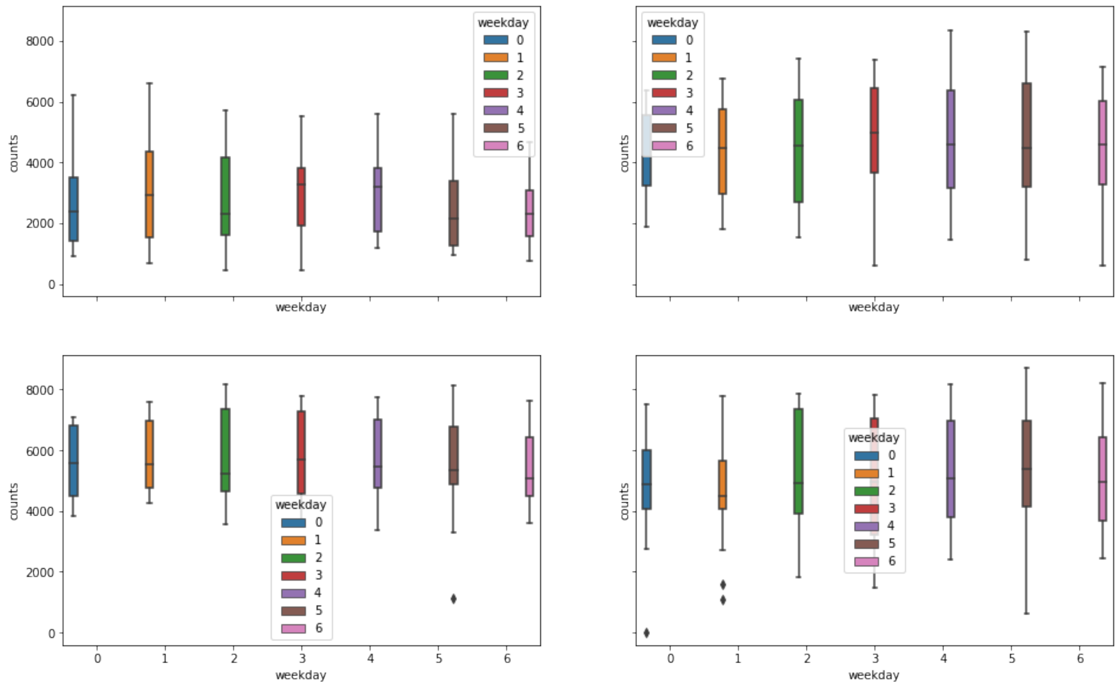
```
      dteday      temp     atemp  windspeed       hum  casual  registered  \
0 2011-01-01  0.344167  0.363625   0.160446  0.805833     331         654
1 2011-01-02  0.363478  0.353739   0.248539  0.696087     131         670
2 2011-01-03  0.196364  0.189405   0.248309  0.437273     120        1229
3 2011-01-04  0.200000  0.212122   0.160296  0.590435     108        1454
4 2011-01-05  0.226957  0.229270   0.186900  0.436957      82        1518

   counts  weekday  season
0     985        5       0
1     801        6       0
2    1349        0       0
3    1562        1       0
4    1600        2       0
```

```
In [23]: test = bikes_by_day_winter.loc[bikes_by_day_winter['weekday']== 1]
         display(test.head())
```

```
       dteday      temp     atemp  windspeed       hum  casual  registered  \
3  2011-01-04  0.200000  0.212122   0.160296  0.590435     108        1454
10 2011-01-11  0.169091  0.191464   0.122132  0.686364      43        1220
17 2011-01-18  0.216667  0.232333   0.146775  0.861667       9         674
24 2011-01-25  0.223478  0.234526   0.129796  0.616957     186        1799
31 2011-02-01  0.192174  0.234530   0.053213  0.829565      47        1313

    counts  weekday  season
3     1562        1       0
10    1263        1       0
17     683        1       0
24    1985        1       0
31    1360        1       0
```

```
In [24]: fig, axes = plt.subplots(nrows=2, ncols=2,sharex=True, sharey=True, figsize=(16, 10))
         sns.boxplot(x='weekday', hue="weekday", y='counts', data=bikes_by_day_winter,  ax=axes
```

```
sns.boxplot(x='weekday', hue="weekday", y='counts', data=bikes_by_day_spring, ax=axes
sns.boxplot(x='weekday', hue="weekday", y='counts', data=bikes_by_day_summer, ax=axes
sns.boxplot(x='weekday', hue="weekday", y='counts', data=bikes_by_day_fall, ax=axes[1
```



Check the outliers that are visible on these plots

```
In [25]: def find_quartiles(data):
             q1 = np.percentile(data, 25)
             q3 = np.percentile(data, 75)
             return q1, q3

         def find_outlier(data, q1, q3):
             outliers = []
             lower = q1 - 1.5 * (q3 - q1)
             upper = q3 + 1.5 * (q3 - q1)
             for i in data:
                 if i <= lower or i >= upper:
                     outliers.append(i)
                 else:
                     pass
             return outliers
```

```
In [26]: summer_outliers = bikes_by_day_summer.loc[bikes_by_day_summer['weekday']==5]
         summer_outliers2= summer_outliers['counts']
         summer_quartiles = find_quartiles(summer_outliers2)
         find_summer_outliers = find_outlier(summer_outliers2,summer_quartiles[0],summer_quart
         find_summer_outliers
```

Out[26]: [1115]

```
In [27]: display(summer_outliers.loc[summer_outliers['counts']==1115])
```

```
        dteday  temp     atemp  windspeed   hum  casual  registered  counts  \
238 2011-08-27  0.68  0.635556   0.375617  0.85     226         889    1115

     weekday  season
238        5       2
```

```
In [28]: fall_outliers_su = bikes_by_day_fall.loc[bikes_by_day_fall['weekday']==0]
         fall_quartiles_su = find_quartiles(fall_outliers_su['counts'])
         find_fall_su_outliers = find_outlier(fall_outliers_su['counts'],fall_quartiles_su[0],f
         find_fall_su_outliers
```

Out[28]: [22]

```
In [29]: display(fall_outliers_su.loc[fall_outliers_su['counts']==22])
```

```
        dteday  temp   atemp  windspeed   hum  casual  registered  counts  \
667 2012-10-29  0.44  0.4394     0.3582  0.88       2          20      22

     weekday  season
667        0       3
```

```
In [30]: fall_outliers_m = bikes_by_day_fall.loc[bikes_by_day_fall['weekday']==1]
         fall_quartiles_m = find_quartiles(fall_outliers_m['counts'])
         find_fall_m_outliers = find_outlier(fall_outliers_m['counts'],fall_quartiles_m[0],fal
         find_fall_m_outliers
```

Out[30]: [1607, 1096]

```
In [31]: display(fall_outliers_m.loc[fall_outliers_m['counts']==1607])
         display(fall_outliers_m.loc[fall_outliers_m['counts']==1096])
```

```
        dteday      temp     atemp  windspeed     hum  casual  registered  \
325 2011-11-22  0.416667  0.421696   0.118792  0.9625      69        1538

     counts  weekday  season
325    1607        1       3


        dteday      temp     atemp  windspeed       hum  casual  registered  \
668 2012-10-30  0.318182  0.309909   0.213009  0.825455      87        1009

     counts  weekday  season
668    1096        1       3
```

```
In [32]: bikes_df.loc[bikes_df['dteday']=='2011-08-27']
```

```
Out[32]:          dteday  season  hour  holiday  weekday  workingday  weather  temp  \
         5618  2011-08-27       3     0        0        6           0        1  0.70
         5619  2011-08-27       3     1        0        6           0        1  0.70
         5620  2011-08-27       3     2        0        6           0        1  0.70
         5621  2011-08-27       3     3        0        6           0        2  0.70
         5622  2011-08-27       3     4        0        6           0        2  0.70
         5623  2011-08-27       3     5        0        6           0        2  0.70
         5624  2011-08-27       3     6        0        6           0        2  0.70
         5625  2011-08-27       3     7        0        6           0        2  0.70
         5626  2011-08-27       3     8        0        6           0        2  0.70
         5627  2011-08-27       3     9        0        6           0        2  0.70
         5628  2011-08-27       3    10        0        6           0        2  0.70
         5629  2011-08-27       3    11        0        6           0        3  0.66
         5630  2011-08-27       3    12        0        6           0        3  0.66
         5631  2011-08-27       3    13        0        6           0        3  0.66
         5632  2011-08-27       3    14        0        6           0        3  0.64
         5633  2011-08-27       3    15        0        6           0        3  0.64
         5634  2011-08-27       3    16        0        6           0        3  0.64
         5635  2011-08-27       3    17        0        6           0        3  0.64

                atemp   hum  windspeed  casual  registered  year  month  counts
         5618  0.6667  0.84     0.1045      33         112     0      8     145
         5619  0.6667  0.84     0.1642      13          51     0      8      64
         5620  0.6667  0.84     0.1940      18          59     0      8      77
         5621  0.6667  0.84     0.2239       8          22     0      8      30
         5622  0.6667  0.84     0.2239       1           3     0      8       4
         5623  0.6667  0.84     0.2985       1          11     0      8      12
         5624  0.6667  0.84     0.2985       3          15     0      8      18
         5625  0.6667  0.84     0.3582       2          26     0      8      28
         5626  0.6667  0.84     0.2537      14          62     0      8      76
         5627  0.6667  0.84     0.4179      28         128     0      8     156
         5628  0.6667  0.79     0.4627      51         154     0      8     205
         5629  0.5909  0.89     0.4179      15          73     0      8      88
         5630  0.6061  0.83     0.4925      11          65     0      8      76
         5631  0.6061  0.83     0.3881      10          33     0      8      43
         5632  0.5758  0.89     0.5522       4          19     0      8      23
         5633  0.5758  0.89     0.5522       2          28     0      8      30
         5634  0.5758  0.89     0.5224      10          14     0      8      24
         5635  0.5758  0.89     0.8358       2          14     0      8      16
```

Answer

We see outliers on the following days:
Friday, 2011-08-27: Weather value was a "3" from 1100 onward.
Monday, 2011-11-22: Weather value was a "3" for the entire day.
Sunday, 2012-10-29 and Monday, 2012-10-30: Weather value was a "3" for both days.
These weather values may have caused extremely low useage on these days.

Question 3: Prepare the data for Regression

In order to build and evaluate our regression models, a little data cleaning is needed. In this problem, we will explicitly create binary variables to represent the categorical predictors, set up the train-test split in a careful way, remove ancillary variables, and do a little data exploration that will be useful to consider in the regression models later.

3.1 Using `bikes_df`, with hourly data about rentals, convert the categorical attributes ('season', 'month', 'weekday', 'weather') into multiple binary attributes using **one-hot encoding**.

3.2 Split the updated `bikes_df` dataset in a train and test part. Do this in a 'stratified' fashion, ensuring that all months are equally represented in each set. Explain your choice for a splitting algorithm.

3.3 Although we asked you to create your train and test set, but for consistency and easy checking, we ask that for the rest of this problem set you use the train and test set provided in the he files `data/BSS_train.csv` and `data/BSS_test.csv`. Read these two files into dataframes `BSS_train` and `BSS_test`, respectively. Remove the `dteday` column from both the train and the test dataset (its format cannot be used for analysis). Also, remove any predictors that would make predicting the `count` trivial. Note we gave more meaningful names to the one-hot encoded variables.

Answers

3.1 Using `bikes_df`, with hourly data about rentals, convert the categorical attributes ('season', 'month', 'weekday', 'weather') into multiple binary attributes using one-hot encoding.

```
In [33]: bikes_df_orig = bikes_df.copy()
```

```
In [34]: #Months weekdays and weather (better way to do it)
         bikes_df = pd.get_dummies(bikes_df_orig, columns=['season','month', 'weekday', 'weath
         bikes_df['month']= pd.DatetimeIndex(bikes_df['dteday']).month
```

```
In [35]: display(bikes_df.head())
         display(bikes_df.info())
```

```
      dteday  hour  holiday  workingday  temp   atemp   hum  windspeed  \
0 2011-01-01     0        0           0  0.24  0.2879  0.81        0.0
1 2011-01-01     1        0           0  0.22  0.2727  0.80        0.0
2 2011-01-01     2        0           0  0.22  0.2727  0.80        0.0
3 2011-01-01     3        0           0  0.24  0.2879  0.75        0.0
4 2011-01-01     4        0           0  0.24  0.2879  0.75        0.0

   casual  registered  ...    weekday_1  weekday_2  weekday_3  weekday_4  \
0       3          13  ...            0          0          0          0
1       8          32  ...            0          0          0          0
2       5          27  ...            0          0          0          0
3       3          10  ...            0          0          0          0
4       0           1  ...            0          0          0          0

   weekday_5  weekday_6  weather_2  weather_3  weather_4  month
0          0          1          0          0          0      1
```

```
1           0           1           0           0           0       1
2           0           1           0           0           0       1
3           0           1           0           0           0       1
4           0           1           0           0           0       1

[5 rows x 36 columns]


<class 'pandas.core.frame.DataFrame'>
RangeIndex: 17379 entries, 0 to 17378
Data columns (total 36 columns):
dteday        17379 non-null datetime64[ns]
hour          17379 non-null int64
holiday       17379 non-null int64
workingday    17379 non-null int64
temp          17379 non-null float64
atemp         17379 non-null float64
hum           17379 non-null float64
windspeed     17379 non-null float64
casual        17379 non-null int64
registered    17379 non-null int64
year          17379 non-null int64
counts        17379 non-null int64
season_2      17379 non-null uint8
season_3      17379 non-null uint8
season_4      17379 non-null uint8
month_2       17379 non-null uint8
month_3       17379 non-null uint8
month_4       17379 non-null uint8
month_5       17379 non-null uint8
month_6       17379 non-null uint8
month_7       17379 non-null uint8
month_8       17379 non-null uint8
month_9       17379 non-null uint8
month_10      17379 non-null uint8
month_11      17379 non-null uint8
month_12      17379 non-null uint8
weekday_1     17379 non-null uint8
weekday_2     17379 non-null uint8
weekday_3     17379 non-null uint8
weekday_4     17379 non-null uint8
weekday_5     17379 non-null uint8
weekday_6     17379 non-null uint8
weather_2     17379 non-null uint8
weather_3     17379 non-null uint8
weather_4     17379 non-null uint8
month         17379 non-null int64
dtypes: datetime64[ns](1), float64(4), int64(8), uint8(23)
```

```
memory usage: 2.1 MB
```

```
None
```

3.2 Split the updated `bikes_df` dataset in a train and test part. Do this in a 'stratified' fashion, ensuring that all months are equally represented in each set. Explain your choice for a splitting algorithm.

```
In [36]: try:
             # Doesn't work: a value is missing
             train_data, test_data = train_test_split(bikes_df, test_size = 0.1,
                                                      stratify=bikes_df['month'])
         except:
             # Count the missing lines and drop them
             missing_rows = np.isnan(bikes_df['month'])
             print("Uh oh, {} lines missing data! Dropping them".format(np.sum(missing_rows)))
             bikes_df = bikes_df.dropna(subset=['month'])
             train_data, test_data = train_test_split(bikes_df, test_size = 0.1,
                                                      stratify=bikes_df['month'])
```

```
In [37]: train_data.shape, test_data.shape
```

```
Out[37]: ((15641, 36), (1738, 36))
```

Answer

When we split the data, we are stratifying by month. This ensure we have an accurate representation of data for each month (generally, 1/12th of the data). If we don't stratify, we risk getting excessive amounts of data from a particular month, or having a particular month underrepresented in our data. This also impacts the data we get for a particular season. We've also chosen to drop data points where the month isn't entered. While these missing values could possibly be filled as we look at our data more closely, it would be time consuming and we don't lose a significant amount of data by dropping.

3.3 Although we asked you to create your train and test set, but for consistency and easy checking, we ask that for the rest of this problem set you use the train and test set provided in the he files `data/BSS_train.csv` and `data/BSS_test.csv`. Read these two files into dataframes `BSS_train` and `BSS_test`, respectively. Remove the `dteday` column from both the train and the test dataset (its format cannot be used for analysis). Also, remove any predictors that would make predicting the `count` trivial. Note we gave more meaningful names to the one-hot encoded variables.

```
In [38]: BSS_train = pd.read_csv('data/BSS_train.csv', sep=",")
         display(BSS_train.head())

         BSS_test = pd.read_csv('data/BSS_test.csv', sep=",")
         display(BSS_test.head())
```

```
   Unnamed: 0      dteday  hour  holiday  year  workingday  temp   atemp  \
0           0  2011-01-01     0        0     0           0  0.24  0.2879
1           1  2011-01-01     1        0     0           0  0.22  0.2727
2           2  2011-01-01     2        0     0           0  0.22  0.2727
3           3  2011-01-01     3        0     0           0  0.24  0.2879
4           4  2011-01-01     4        0     0           0  0.24  0.2879

    hum  windspeed  ...    Dec  Mon  Tue  Wed  Thu  Fri  Sat  Cloudy  Snow  \
0  0.81        0.0  ...      0    0    0    0    0    0    1       0     0
1  0.80        0.0  ...      0    0    0    0    0    0    1       0     0
2  0.80        0.0  ...      0    0    0    0    0    0    1       0     0
3  0.75        0.0  ...      0    0    0    0    0    0    1       0     0
4  0.75        0.0  ...      0    0    0    0    0    0    1       0     0

   Storm
0      0
1      0
2      0
3      0
4      0

[5 rows x 36 columns]

   Unnamed: 0      dteday  hour  holiday  year  workingday  temp   atemp  \
0           6  2011-01-01     6        0     0           0  0.22  0.2727
1           9  2011-01-01     9        0     0           0  0.32  0.3485
2          20  2011-01-01    20        0     0           0  0.40  0.4091
3          33  2011-01-02    10        0     0           0  0.36  0.3485
4          35  2011-01-02    12        0     0           0  0.36  0.3333

    hum  windspeed  ...    Dec  Mon  Tue  Wed  Thu  Fri  Sat  Cloudy  Snow  \
0  0.80     0.0000  ...      0    0    0    0    0    0    1       0     0
1  0.76     0.0000  ...      0    0    0    0    0    0    1       0     0
2  0.87     0.2537  ...      0    0    0    0    0    0    1       1     0
3  0.81     0.2239  ...      0    0    0    0    0    0    0       1     0
4  0.66     0.2985  ...      0    0    0    0    0    0    0       1     0

   Storm
0      0
1      0
2      0
3      0
4      0

[5 rows x 36 columns]
```

```
In [39]: BSS_train = BSS_train.drop(['dteday', 'casual', 'registered'], axis=1) #drop irreleva
```

```
BSS_test = BSS_test.drop(['dteday', 'casual', 'registered'], axis=1) #drop irrelevant
```

In [40]:
```
display(BSS_train.head(), BSS_test.head(), BSS_test.info())
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 3476 entries, 0 to 3475
Data columns (total 33 columns):
Unnamed: 0    3476 non-null int64
hour          3476 non-null int64
holiday       3476 non-null int64
year          3476 non-null int64
workingday    3476 non-null int64
temp          3476 non-null float64
atemp         3476 non-null float64
hum           3476 non-null float64
windspeed     3476 non-null float64
counts        3476 non-null int64
spring        3476 non-null int64
summer        3476 non-null int64
fall          3476 non-null int64
Feb           3476 non-null int64
Mar           3476 non-null int64
Apr           3476 non-null int64
May           3476 non-null int64
Jun           3476 non-null int64
Jul           3476 non-null int64
Aug           3476 non-null int64
Sept          3476 non-null int64
Oct           3476 non-null int64
Nov           3476 non-null int64
Dec           3476 non-null int64
Mon           3476 non-null int64
Tue           3476 non-null int64
Wed           3476 non-null int64
Thu           3476 non-null int64
Fri           3476 non-null int64
Sat           3476 non-null int64
Cloudy        3476 non-null int64
Snow          3476 non-null int64
Storm         3476 non-null int64
dtypes: float64(4), int64(29)
memory usage: 896.2 KB
```

```
   Unnamed: 0  hour  holiday  year  workingday  temp   atemp   hum  windspeed  \
0           0     0        0     0           0  0.24  0.2879  0.81        0.0
1           1     1        0     0           0  0.22  0.2727  0.80        0.0
2           2     2        0     0           0  0.22  0.2727  0.80        0.0
```

```
3          3    3        0     0           0  0.24  0.2879  0.75        0.0
4          4    4        0     0           0  0.24  0.2879  0.75        0.0

   counts  ...    Dec  Mon  Tue  Wed  Thu  Fri  Sat  Cloudy  Snow  Storm
0      16  ...      0    0    0    0    0    0    1       0     0      0
1      40  ...      0    0    0    0    0    0    1       0     0      0
2      32  ...      0    0    0    0    0    0    1       0     0      0
3      13  ...      0    0    0    0    0    0    1       0     0      0
4       1  ...      0    0    0    0    0    0    1       0     0      0

[5 rows x 33 columns]

   Unnamed: 0  hour  holiday  year  workingday  temp   atemp   hum  windspeed  \
0           6     6        0     0           0  0.22  0.2727  0.80     0.0000
1           9     9        0     0           0  0.32  0.3485  0.76     0.0000
2          20    20        0     0           0  0.40  0.4091  0.87     0.2537
3          33    10        0     0           0  0.36  0.3485  0.81     0.2239
4          35    12        0     0           0  0.36  0.3333  0.66     0.2985

   counts  ...    Dec  Mon  Tue  Wed  Thu  Fri  Sat  Cloudy  Snow  Storm
0       2  ...      0    0    0    0    0    0    1       0     0      0
1      14  ...      0    0    0    0    0    0    1       0     0      0
2      36  ...      0    0    0    0    0    0    1       1     0      0
3      53  ...      0    0    0    0    0    0    0       1     0      0
4      93  ...      0    0    0    0    0    0    0       1     0      0

[5 rows x 33 columns]

None
```

Question 4: Multiple Linear Regression

4.1 Use statsmodels to fit a multiple linear regression model to the training set using all the predictors (no interactions or polynomial terms) to predict `counts`, and report its R^2 score on the train and test sets.

4.2 Examine the estimated coefficients and report which ones are statistically significant at a significance level of 5% (p-value < 0.05). You should see some strange values, such as `July` producing 93 fewer rentals, all else equal, than January.

4.3 To diagnose the model, make two plots: first a histogram of the residuals, and second a plot of the residuals of the fitted model $e = y - \hat{y}$ as a function of the predicted value $\hat{y}$. Draw a horizontal line denoting the zero residual value on the Y-axis. What do the plots reveal about the OLS assumptions (linearity, constant variance, and normality)?

4.4 Perhaps we can do better via a model with polynomial terms. Build a dataset `X_train_poly` from `X_train` with added x^2 terms for `temp`, `hour`, and `humidity`. Are these polynomial terms important? How does predicted ridership change as each of `temp`, `hour`, and `humidity` increase?

4.5 The strange coefficients from 4.2 could also come from *multicolinearity*, where one or more predictors capture the same information as existing predictors. Why can multicolinearity lead to erroneous coefficient values? Create a temporary dataset `X_train_drop` that drops the following 'redundant' predictors from `X_train`: `workingday` `atemp` `spring` `summer` and `fall`. Fit a multiple linear regression model to `X_train_drop`. Are the estimates more sensible in this model?

1.2.6 Answers

4.1 Use statsmodels to fit a multiple linear regression model to the training set using all the predictors (no interactions or polynomial terms) to predict `counts`, and report its R^2 score on the train and test sets.

```
In [41]: y_train = BSS_train['counts']
         y_test = BSS_test['counts']

         def build_model_design(df):
             design_mat = df[['hour','holiday','year','workingday','temp', 'atemp', 'hum', 'wi
             #Add a constant
             design_mat = sm.add_constant(design_mat)
             return design_mat

         X_train = build_model_design(BSS_train)
         X_test = build_model_design(BSS_test)
         display(X_train.head(), X_test.head(), y_train.head(), y_test.head())
```

```
   const  hour  holiday  year  workingday  temp   atemp   hum  windspeed  \
0    1.0     0        0     0           0  0.24  0.2879  0.81        0.0
1    1.0     1        0     0           0  0.22  0.2727  0.80        0.0
2    1.0     2        0     0           0  0.22  0.2727  0.80        0.0
3    1.0     3        0     0           0  0.24  0.2879  0.75        0.0
4    1.0     4        0     0           0  0.24  0.2879  0.75        0.0

   spring  ...   Dec  Mon  Tue  Wed  Thu  Fri  Sat  Cloudy  Snow  Storm
0       0  ...     0    0    0    0    0    0    1       0     0      0
1       0  ...     0    0    0    0    0    0    1       0     0      0
2       0  ...     0    0    0    0    0    0    1       0     0      0
3       0  ...     0    0    0    0    0    0    1       0     0      0
4       0  ...     0    0    0    0    0    0    1       0     0      0

[5 rows x 32 columns]


   const  hour  holiday  year  workingday  temp   atemp   hum  windspeed  \
0    1.0     6        0     0           0  0.22  0.2727  0.80     0.0000
1    1.0     9        0     0           0  0.32  0.3485  0.76     0.0000
2    1.0    20        0     0           0  0.40  0.4091  0.87     0.2537
3    1.0    10        0     0           0  0.36  0.3485  0.81     0.2239
4    1.0    12        0     0           0  0.36  0.3333  0.66     0.2985

   spring  ...   Dec  Mon  Tue  Wed  Thu  Fri  Sat  Cloudy  Snow  Storm
0       0  ...     0    0    0    0    0    0    1       0     0      0
```

```
1       0  ...      0     0     0     0     0     0     1       0      0       0
2       0  ...      0     0     0     0     0     0     1       1      0       0
3       0  ...      0     0     0     0     0     0     0       1      0       0
4       0  ...      0     0     0     0     0     0     0       1      0       0

[5 rows x 32 columns]


0    16
1    40
2    32
3    13
4     1
Name: counts, dtype: int64


0     2
1    14
2    36
3    53
4    93
Name: counts, dtype: int64
```

In [42]:
```python
model1 = sm.OLS(
    BSS_train.counts,
    sm.add_constant(BSS_train[['hour','holiday','year','workingday','temp', 'atemp',
).fit()
model1.summary()

prediction = model1.predict(X_train)
model1.summary()
```

Out[42]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                 counts   R-squared:                       0.407
Model:                            OLS   Adj. R-squared:                  0.405
Method:                 Least Squares   F-statistic:                     316.8
Date:                Wed, 03 Oct 2018   Prob (F-statistic):               0.00
Time:                        23:09:25   Log-Likelihood:                -88306.
No. Observations:               13903   AIC:                         1.767e+05
Df Residuals:                   13872   BIC:                         1.769e+05
Df Model:                          30
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
```

```
const          -21.0830      8.641     -2.440      0.015     -38.020      -4.146
hour             7.2214      0.184     39.144      0.000       6.860       7.583
holiday        -18.0958      6.597     -2.743      0.006     -31.027      -5.165
year            76.3519      2.380     32.084      0.000      71.687      81.017
workingday      11.3178      2.751      4.114      0.000       5.926      16.710
temp           333.2482     44.162      7.546      0.000     246.684     419.812
atemp           74.6312     46.207      1.615      0.106     -15.940     165.202
hum           -205.4959      7.801    -26.343      0.000    -220.786    -190.205
windspeed       22.5168     10.753      2.094      0.036       1.439      43.595
spring          43.1541      7.417      5.818      0.000      28.615      57.693
summer          29.5426      8.773      3.367      0.001      12.346      46.739
fall            68.5953      7.492      9.156      0.000      53.911      83.280
Feb             -7.6430      5.966     -1.281      0.200     -19.336       4.050
Mar            -11.6737      6.665     -1.752      0.080     -24.737       1.390
Apr            -41.5244      9.878     -4.204      0.000     -60.886     -22.163
May            -33.2927     10.543     -3.158      0.002     -53.958     -12.628
Jun            -65.8039     10.716     -6.141      0.000     -86.809     -44.799
Jul            -93.4805     12.086     -7.734      0.000    -117.171     -69.789
Aug            -59.2081     11.832     -5.004      0.000     -82.401     -36.015
Sept           -16.0517     10.575     -1.518      0.129     -36.780       4.676
Oct            -16.1602      9.865     -1.638      0.101     -35.497       3.177
Nov            -25.8732      9.527     -2.716      0.007     -44.547      -7.199
Dec            -10.2043      7.614     -1.340      0.180     -25.128       4.719
Mon             -2.6601      2.978     -0.893      0.372      -8.498       3.177
Tue             -6.1425      3.208     -1.915      0.056     -12.430       0.145
Wed              2.2964      3.183      0.721      0.471      -3.943       8.536
Thu             -3.1611      3.185     -0.993      0.321      -9.404       3.082
Fri              2.8892      3.186      0.907      0.364      -3.355       9.133
Sat             14.9459      4.382      3.411      0.001       6.357      23.535
Cloudy           6.7868      2.900      2.341      0.019       1.103      12.470
Snow           -28.2859      4.819     -5.870      0.000     -37.731     -18.841
Storm           42.3569     98.377      0.431      0.667    -150.475     235.189
==============================================================================
Omnibus:                     2831.359   Durbin-Watson:                   0.755
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             5657.789
Skew:                           1.224   Prob(JB):                         0.00
Kurtosis:                       4.943   Cond. No.                     1.17e+16
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly speci
[2] The smallest eigenvalue is 1.87e-26. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
"""
```

In [43]:
```
print("The r2 on the train data is:")
print(r2_score(BSS_train.counts, model1.predict(X_train)))
```

```
print("The r2 on the test data is:")
print(r2_score(BSS_test.counts, model1.predict(X_test)))
```

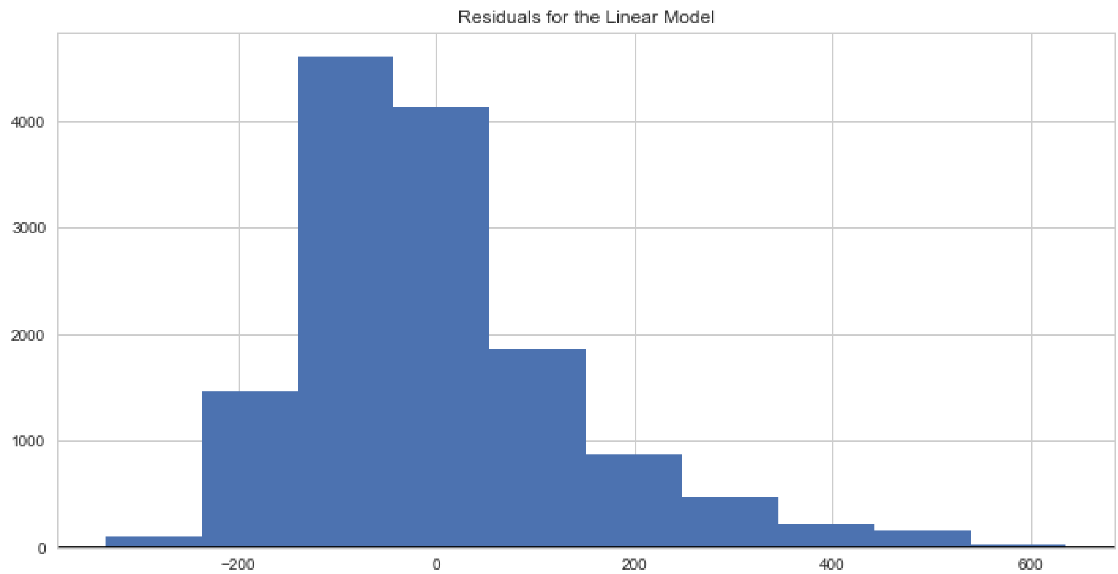
```
The r2 on the train data is:
0.4065387827969087
The r2 on the test data is:
0.40638554757102263
```

4.2 Examine the estimated coefficients and report which ones are statistically significant at a significance level of 5% (p-value < 0.05). You should see some strange values, such as `July` producing 93 fewer rentals, all else equal, than January. Based on (p value < 0.05), the following coefficients are significant:
-hour
-holiday
-year
-working day
-temp
-humidity
-windspeed
-seasons: spring, summer, and fall
-months: april through august, inclusive, and november
-days of the week: saturday
-weather conditions: snow, clouds

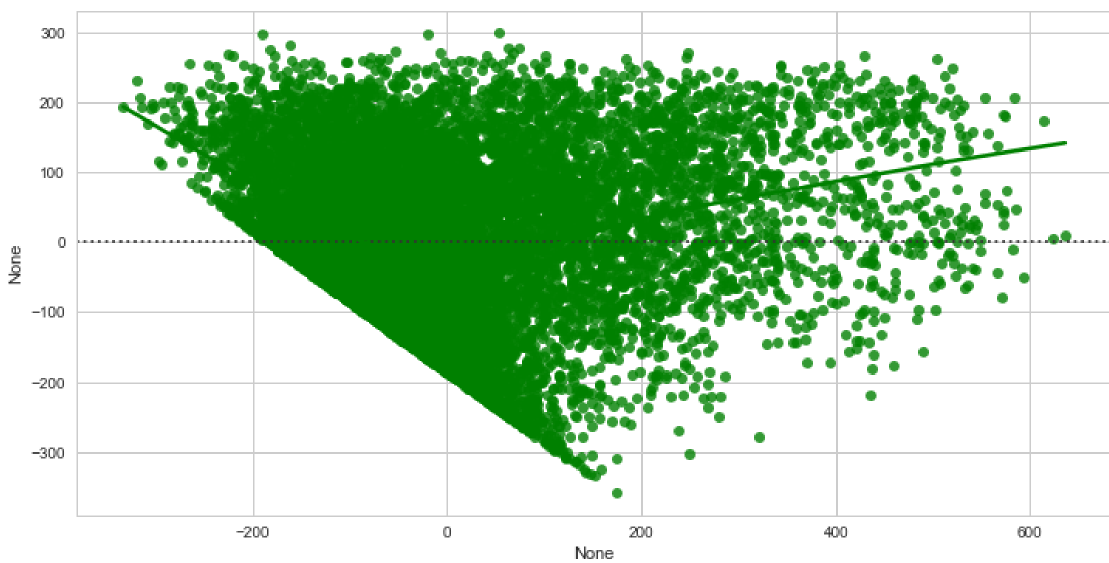
4.3 To diagnose the model, make two plots: first a histogram of the residuals, and second a plot of the residuals of the fitted model $e = y - \hat{y}$ as a function of the predicted value $\hat{y}$. Draw a horizontal line denoting the zero residual value on the Y-axis. What do the plots reveal about the OLS assumptions (linearity, constant variance, and normality)?

In [44]:
```
sns.set(style="whitegrid")
matplotlib.rcParams['figure.figsize']=(12,6)

residual = y_train - prediction
plt.hist(residual, label="Residual")
plt.axhline(0, color='k')
plt.title("Residuals for the Linear Model")
```

Out[44]:
```
Text(0.5,1,'Residuals for the Linear Model')
```



In [45]: `sns.residplot(residual, prediction, lowess=True, color="g");`



The assumptions of linearity, normality, and constance variance are all violated. A polynomial regression to the degree of 2 would probably better fit this data, since there seems to be a curve.

4.4 Perhaps we can do better via a model with polynomial terms. Build a dataset `X_train_poly` from `X_train` with added x^2 terms for `temp`, `hour`, and `humidity`. Are these polynomial terms important? How does predicted ridership change as each of `temp`, `hour`, and `humidity` increase?

```
In [46]: def build_poly_model(df):
             poly_design = df[['hour','holiday','year','workingday','temp', 'atemp', 'hum', 'w
             poly_design['temp2'] = df['temp']**2
             poly_design['hour2'] = df['hour']**2
             poly_design['hum2'] = df['hum']**2

             #reindex so variables are in a pretty order
             poly_design = poly_design[['hour','hour2','holiday','year','workingday','temp', '

             poly_design = sm.add_constant(poly_design)

             return poly_design

         X_train_poly = build_poly_model(BSS_train)
         X_test_poly = build_poly_model(BSS_test)


         poly_model = sm.OLS(BSS_train.counts, X_train_poly).fit()
         poly_model.summary()
```

```
Out[46]: <class 'statsmodels.iolib.summary.Summary'>
         """
                                     OLS Regression Results
         ==============================================================================
         Dep. Variable:                 counts   R-squared:                       0.501
         Model:                            OLS   Adj. R-squared:                  0.500
         Method:                 Least Squares   F-statistic:                     421.8
         Date:                Wed, 03 Oct 2018   Prob (F-statistic):               0.00
         Time:                        23:10:31   Log-Likelihood:                -87102.
         No. Observations:               13903   AIC:                         1.743e+05
         Df Residuals:                   13869   BIC:                         1.745e+05
         Df Model:                          33
         Covariance Type:            nonrobust
         ==============================================================================
                          coef    std err          t      P>|t|      [0.025      0.975]
         ------------------------------------------------------------------------------
         const       -185.2131     14.016    -13.214      0.000    -212.687    -157.739
         hour          39.5786      0.662     59.777      0.000      38.281      40.876
         hour2         -1.3570      0.027    -50.567      0.000      -1.410      -1.304
         holiday      -13.0061      6.056     -2.148      0.032     -24.877      -1.135
         year          81.0305      2.199     36.854      0.000      76.721      85.340
         workingday    13.2894      2.524      5.265      0.000       8.342      18.237
         temp         132.7247     58.298      2.277      0.023      18.452     246.997
         temp2        109.4437     36.470      3.001      0.003      37.958     180.930
         atemp         67.4957     43.532      1.550      0.121     -17.833     152.824
         hum           11.8636     36.114      0.329      0.743     -58.925      82.652
         hum2        -108.7057     28.944     -3.756      0.000    -165.440     -51.971
         windspeed     -6.9100      9.920     -0.697      0.486     -26.354      12.534
```

```
spring        43.7116      6.805      6.424      0.000      30.374      57.049
summer        33.9087      8.066      4.204      0.000      18.098      49.720
fall          72.1937      6.878     10.497      0.000      58.712      85.675
Feb            1.6487      5.538      0.298      0.766      -9.207      12.504
Mar            9.5583      6.304      1.516      0.129      -2.798      21.914
Apr          -10.7152      9.238     -1.160      0.246     -28.824       7.393
May           -2.7388      9.789     -0.280      0.780     -21.926      16.449
Jun          -23.0368      9.922     -2.322      0.020     -42.485      -3.588
Jul          -53.5230     11.163     -4.795      0.000     -75.405     -31.642
Aug          -23.6944     10.965     -2.161      0.031     -45.188      -2.201
Sept          10.9055      9.887      1.103      0.270      -8.475      30.286
Oct            2.8452      9.262      0.307      0.759     -15.309      20.999
Nov          -16.5926      8.897     -1.865      0.062     -34.032       0.846
Dec           -6.9106      7.078     -0.976      0.329     -20.784       6.963
Mon           -2.4620      2.731     -0.901      0.367      -7.816       2.892
Tue           -3.8629      2.943     -1.313      0.189      -9.631       1.905
Wed            2.1275      2.920      0.729      0.466      -3.596       7.851
Thu           -0.2540      2.922     -0.087      0.931      -5.981       5.473
Fri            4.7347      2.923      1.620      0.105      -0.995      10.465
Sat           16.7983      4.021      4.178      0.000       8.917      24.679
Cloudy        -8.4327      2.680     -3.146      0.002     -13.687      -3.179
Snow         -47.3269      4.583    -10.327      0.000     -56.310     -38.344
Storm         35.5800     90.246      0.394      0.693    -141.315     212.475
==============================================================================
Omnibus:                     2946.543   Durbin-Watson:                   0.890
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6094.033
Skew:                           1.253   Prob(JB):                         0.00
Kurtosis:                       5.059   Cond. No.                     1.35e+16
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
[2] The smallest eigenvalue is 4.56e-24. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
"""
```

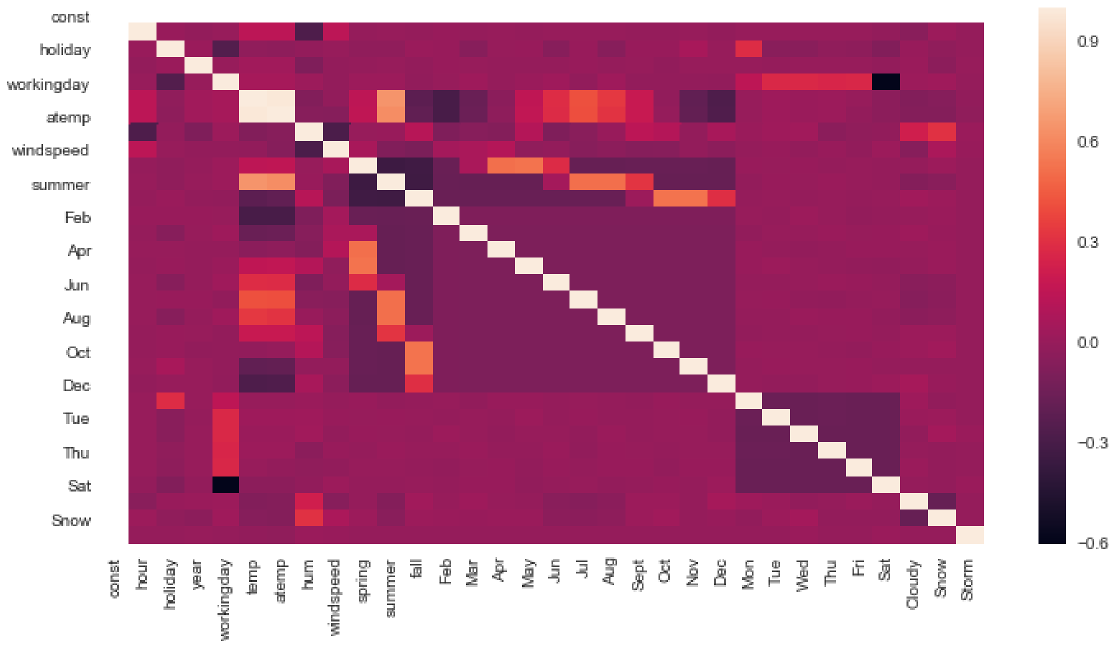
Answer

By adding these polynomial terms, our R-squared value increases from 0.406 to 0.501. We also see that x^2 terms for `temp`, `hour`, and 'humidity' all have significant p values, indicating that these polynomials play a significant role in our model. 'temp'^{2} causes an increase in ridership; whereas 'hum'^{2} and 'hour'^{2} cause decreases.

4.5 The strange coefficients from 4.2 could also come from *multicolinearity*, where one or more predictors capture the same information as existing predictors. Why can multicolinearity lead to erroneous coefficient values? Create a temporary dataset `X_train_drop` that drops the following 'redundant' predictors from `X_train`: `workingday` `atemp` `spring` `summer` and `fall`. Fit a multiple linear regression model to `X_train_drop`. Are the estimates more sensible in this model?

```
In [47]: X_train.corr()
         sns.heatmap(X_train.corr())
```

```
Out[47]: <matplotlib.axes._subplots.AxesSubplot at 0x1add3024c50>
```




```
In [48]: X_train_drop = X_train.drop(['workingday', 'atemp', 'spring', 'summer', 'fall'], axis=
         display(X_train_drop.head())

         model3 = sm.OLS(
             y_train,
             X_train_drop).fit()
         model3.summary()
```

	const	hour	holiday	year	temp	hum	windspeed	Feb	Mar	Apr	...	\
0	1.0	0	0	0	0.24	0.81	0.0	0	0	0	...	
1	1.0	1	0	0	0.22	0.80	0.0	0	0	0	...	
2	1.0	2	0	0	0.22	0.80	0.0	0	0	0	...	
3	1.0	3	0	0	0.24	0.75	0.0	0	0	0	...	
4	1.0	4	0	0	0.24	0.75	0.0	0	0	0	...	

	Dec	Mon	Tue	Wed	Thu	Fri	Sat	Cloudy	Snow	Storm
0	0	0	0	0	0	0	1	0	0	0
1	0	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	0	1	0	0	0
3	0	0	0	0	0	0	1	0	0	0
4	0	0	0	0	0	0	1	0	0	0

```
[5 rows x 27 columns]
```

```
Out[48]: <class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                 counts   R-squared:                       0.402
Model:                            OLS   Adj. R-squared:                  0.401
Method:                 Least Squares   F-statistic:                     358.3
Date:                Wed, 03 Oct 2018   Prob (F-statistic):               0.00
Time:                        23:10:32   Log-Likelihood:                -88363.
No. Observations:               13903   AIC:                         1.768e+05
Df Residuals:                   13876   BIC:                         1.770e+05
Df Model:                          26
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        -20.0627      8.541     -2.349      0.019     -36.805      -3.321
hour           7.2378      0.185     39.095      0.000       6.875       7.601
holiday      -35.8906      7.395     -4.854      0.000     -50.385     -21.396
year          76.3039      2.389     31.945      0.000      71.622      80.986
temp         406.2359     13.279     30.593      0.000     380.208     432.264
hum         -201.5103      7.800    -25.835      0.000    -216.799    -186.221
windspeed     11.9668     10.448      1.145      0.252      -8.512      32.446
Feb           -7.6897      5.986     -1.285      0.199     -19.422       4.043
Mar            2.8889      6.158      0.469      0.639      -9.182      14.960
Apr            1.0237      6.594      0.155      0.877     -11.902      13.950
May            7.2426      7.613      0.951      0.341      -7.680      22.165
Jun          -30.6611      8.346     -3.674      0.000     -47.020     -14.302
Jul          -67.7620      9.062     -7.477      0.000     -85.525     -49.999
Aug          -34.2712      8.628     -3.972      0.000     -51.183     -17.359
Sept          20.6406      7.882      2.619      0.009       5.191      36.090
Oct           50.7025      6.823      7.431      0.000      37.329      64.076
Nov           42.3211      6.111      6.926      0.000      30.344      54.299
Dec           34.2134      5.952      5.748      0.000      22.546      45.881
Mon            9.2907      4.570      2.033      0.042       0.333      18.248
Tue            4.7929      4.442      1.079      0.281      -3.914      13.500
Wed           13.2143      4.417      2.992      0.003       4.557      21.871
Thu            8.0051      4.445      1.801      0.072      -0.708      16.718
Fri           13.0474      4.429      2.946      0.003       4.367      21.728
Sat           14.1461      4.397      3.217      0.001       5.528      22.764
Cloudy         6.7192      2.909      2.310      0.021       1.018      12.421
Snow         -29.1668      4.828     -6.041      0.000     -38.631     -19.703
Storm         40.3125     98.759      0.408      0.683    -153.267     233.893
==============================================================================
```

```
Omnibus:                     2850.389   Durbin-Watson:                   0.749
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             5702.134
Skew:                           1.231   Prob(JB):                         0.00
Kurtosis:                       4.944   Cond. No.                     1.13e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
[2] The condition number is large, 1.13e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
"""
```

Answer

Multicollinearity increase the variance of the coefficient estimates and make the estimates very sensitive to minor changes in the model. The result is that the coefficient estimates are unstable and difficult to interpret. It can cause extreme values, and changes in signs. By removing these terms, we are able to get a better model.

Question 5: Subset Selection

Perhaps we can automate finding a good set of predictors. This question focuses on forward stepwise selection, where predictors are added to the model one by one.

5.1 Implement forward step-wise selection to select a minimal subset of predictors that are related to the response variable. Run your code on the richest dataset, `X_train_poly`, and determine which predictors are selected.

We require that you implement the method **from scratch**. You may use the Bayesian Information Criterion (BIC) to choose the best subset size.

Note: Implementing from scratch means you are not allowed to use a solution provided by a Python library, such as sklearn or use a solution you found on the internet. You have to write all of the code on your own. However you MAY use the `model.bic` attribute implemented in statsmodels.

5.2 Does forward selection eliminate one or more of the colinear predictors we dropped in Question 4.5 (`workingday atemp spring summer` and `fall`)? If any of the five predictors are not dropped, explain why.

5.3 Fit the linear regression model using the identified subset of predictors to the training set. How do the train and test R^2 scores for this fitted step-wise model compare with the train and test R^2 scores from the polynomial model fitted in Question 4.4?

1.2.7 Answers

5.1 Implement forward step-wise selection to select a minimal subset of predictors that are related to the response variable. Run your code on the richest dataset, `X_train_poly`, and determine which predictors are selected.

We require that you implement the method from scratch. You may use the Bayesian Information Criterion (BIC) to choose the best subset size.

Note: Implementing from scratch means you are not allowed to use a solution provided by a Python library, such as sklearn or use a solution you found on the internet. You have to write

all of the code on your own. However you MAY use the `model.bic` attribute implemented in statsmodels.

```
In [49]: x = X_train_poly.iloc[:,:-1]
         y = BSS_train['counts']
         x.shape, y.shape

         predictors = x.columns

         display(x, y, predictors)
```

```
       const  hour  hour2  holiday  year  workingday  temp   temp2   atemp  \
0        1.0     0      0        0     0           0  0.24  0.0576  0.2879
1        1.0     1      1        0     0           0  0.22  0.0484  0.2727
2        1.0     2      4        0     0           0  0.22  0.0484  0.2727
3        1.0     3      9        0     0           0  0.24  0.0576  0.2879
4        1.0     4     16        0     0           0  0.24  0.0576  0.2879
5        1.0     5     25        0     0           0  0.24  0.0576  0.2576
6        1.0     7     49        0     0           0  0.20  0.0400  0.2576
7        1.0     8     64        0     0           0  0.24  0.0576  0.2879
8        1.0    10    100        0     0           0  0.38  0.1444  0.3939
9        1.0    11    121        0     0           0  0.36  0.1296  0.3333
10       1.0    12    144        0     0           0  0.42  0.1764  0.4242
11       1.0    13    169        0     0           0  0.46  0.2116  0.4545
12       1.0    14    196        0     0           0  0.46  0.2116  0.4545
13       1.0    15    225        0     0           0  0.44  0.1936  0.4394
14       1.0    16    256        0     0           0  0.42  0.1764  0.4242
15       1.0    17    289        0     0           0  0.44  0.1936  0.4394
16       1.0    18    324        0     0           0  0.42  0.1764  0.4242
17       1.0    19    361        0     0           0  0.42  0.1764  0.4242
18       1.0    21    441        0     0           0  0.40  0.1600  0.4091
19       1.0    22    484        0     0           0  0.40  0.1600  0.4091
20       1.0    23    529        0     0           0  0.46  0.2116  0.4545
21       1.0     0      0        0     0           0  0.46  0.2116  0.4545
22       1.0     1      1        0     0           0  0.44  0.1936  0.4394
23       1.0     2      4        0     0           0  0.42  0.1764  0.4242
24       1.0     3      9        0     0           0  0.46  0.2116  0.4545
25       1.0     4     16        0     0           0  0.46  0.2116  0.4545
26       1.0     6     36        0     0           0  0.42  0.1764  0.4242
27       1.0     7     49        0     0           0  0.40  0.1600  0.4091
28       1.0     8     64        0     0           0  0.40  0.1600  0.4091
29       1.0     9     81        0     0           0  0.38  0.1444  0.3939
...      ...   ...    ...      ...   ...         ...   ...     ...     ...
13873    1.0     9     81        0     1           0  0.24  0.0576  0.2121
13874    1.0    11    121        0     1           0  0.26  0.0676  0.2273
13875    1.0    12    144        0     1           0  0.28  0.0784  0.2273
13876    1.0    13    169        0     1           0  0.30  0.0900  0.2576
13877    1.0    14    196        0     1           0  0.30  0.0900  0.2727
```

13878	1.0	15	225	0	1	0	0.28	0.0784	0.2576
13879	1.0	16	256	0	1	0	0.28	0.0784	0.2424
13880	1.0	17	289	0	1	0	0.26	0.0676	0.2273
13881	1.0	18	324	0	1	0	0.24	0.0576	0.2121
13882	1.0	21	441	0	1	0	0.20	0.0400	0.2121
13883	1.0	22	484	0	1	0	0.20	0.0400	0.1970
13884	1.0	23	529	0	1	0	0.20	0.0400	0.1970
13885	1.0	1	1	0	1	1	0.18	0.0324	0.1818
13886	1.0	2	4	0	1	1	0.16	0.0256	0.1667
13887	1.0	3	9	0	1	1	0.16	0.0256	0.1818
13888	1.0	4	16	0	1	1	0.14	0.0196	0.1667
13889	1.0	7	49	0	1	1	0.16	0.0256	0.1818
13890	1.0	8	64	0	1	1	0.14	0.0196	0.1515
13891	1.0	10	100	0	1	1	0.20	0.0400	0.2121
13892	1.0	11	121	0	1	1	0.22	0.0484	0.2273
13893	1.0	12	144	0	1	1	0.24	0.0576	0.2273
13894	1.0	13	169	0	1	1	0.26	0.0676	0.2576
13895	1.0	14	196	0	1	1	0.28	0.0784	0.2727
13896	1.0	15	225	0	1	1	0.28	0.0784	0.2879
13897	1.0	16	256	0	1	1	0.26	0.0676	0.2576
13898	1.0	17	289	0	1	1	0.26	0.0676	0.2879
13899	1.0	18	324	0	1	1	0.26	0.0676	0.2727
13900	1.0	20	400	0	1	1	0.26	0.0676	0.2576
13901	1.0	22	484	0	1	1	0.26	0.0676	0.2727
13902	1.0	23	529	0	1	1	0.26	0.0676	0.2727

	hum	...	Nov	Dec	Mon	Tue	Wed	Thu	Fri	Sat	Cloudy	Snow
0	0.81	...	0	0	0	0	0	0	0	1	0	0
1	0.80	...	0	0	0	0	0	0	0	1	0	0
2	0.80	...	0	0	0	0	0	0	0	1	0	0
3	0.75	...	0	0	0	0	0	0	0	1	0	0
4	0.75	...	0	0	0	0	0	0	0	1	0	0
5	0.75	...	0	0	0	0	0	0	0	1	1	0
6	0.86	...	0	0	0	0	0	0	0	1	0	0
7	0.75	...	0	0	0	0	0	0	0	1	0	0
8	0.76	...	0	0	0	0	0	0	0	1	0	0
9	0.81	...	0	0	0	0	0	0	0	1	0	0
10	0.77	...	0	0	0	0	0	0	0	1	0	0
11	0.72	...	0	0	0	0	0	0	0	1	1	0
12	0.72	...	0	0	0	0	0	0	0	1	1	0
13	0.77	...	0	0	0	0	0	0	0	1	1	0
14	0.82	...	0	0	0	0	0	0	0	1	1	0
15	0.82	...	0	0	0	0	0	0	0	1	1	0
16	0.88	...	0	0	0	0	0	0	0	1	0	1
17	0.88	...	0	0	0	0	0	0	0	1	0	1
18	0.87	...	0	0	0	0	0	0	0	1	1	0
19	0.94	...	0	0	0	0	0	0	0	1	1	0
20	0.88	...	0	0	0	0	0	0	0	1	1	0

21	0.88	...	0	0	0	0	0	0	0	0	1	0
22	0.94	...	0	0	0	0	0	0	0	0	1	0
23	1.00	...	0	0	0	0	0	0	0	0	1	0
24	0.94	...	0	0	0	0	0	0	0	0	1	0
25	0.94	...	0	0	0	0	0	0	0	0	1	0
26	0.77	...	0	0	0	0	0	0	0	0	0	1
27	0.76	...	0	0	0	0	0	0	0	0	1	0
28	0.71	...	0	0	0	0	0	0	0	0	0	1
29	0.76	...	0	0	0	0	0	0	0	0	1	0
...	...	...	...	...	...	...	...	...	...	...	...	...
13873	0.52	...	0	1	0	0	0	0	0	0	0	0
13874	0.41	...	0	1	0	0	0	0	0	0	0	0
13875	0.36	...	0	1	0	0	0	0	0	0	0	0
13876	0.36	...	0	1	0	0	0	0	0	0	0	0
13877	0.36	...	0	1	0	0	0	0	0	0	0	0
13878	0.38	...	0	1	0	0	0	0	0	0	0	0
13879	0.38	...	0	1	0	0	0	0	0	0	0	0
13880	0.41	...	0	1	0	0	0	0	0	0	0	0
13881	0.44	...	0	1	0	0	0	0	0	0	1	0
13882	0.51	...	0	1	0	0	0	0	0	0	0	0
13883	0.55	...	0	1	0	0	0	0	0	0	0	0
13884	0.51	...	0	1	0	0	0	0	0	0	0	0
13885	0.55	...	0	1	1	0	0	0	0	0	0	0
13886	0.59	...	0	1	1	0	0	0	0	0	0	0
13887	0.59	...	0	1	1	0	0	0	0	0	0	0
13888	0.69	...	0	1	1	0	0	0	0	0	0	0
13889	0.64	...	0	1	1	0	0	0	0	0	0	0
13890	0.69	...	0	1	1	0	0	0	0	0	0	0
13891	0.69	...	0	1	1	0	0	0	0	0	1	0
13892	0.60	...	0	1	1	0	0	0	0	0	1	0
13893	0.56	...	0	1	1	0	0	0	0	0	1	0
13894	0.44	...	0	1	1	0	0	0	0	0	1	0
13895	0.45	...	0	1	1	0	0	0	0	0	1	0
13896	0.45	...	0	1	1	0	0	0	0	0	1	0
13897	0.48	...	0	1	1	0	0	0	0	0	1	0
13898	0.48	...	0	1	1	0	0	0	0	0	1	0
13899	0.48	...	0	1	1	0	0	0	0	0	1	0
13900	0.60	...	0	1	1	0	0	0	0	0	1	0
13901	0.56	...	0	1	1	0	0	0	0	0	0	0
13902	0.65	...	0	1	1	0	0	0	0	0	0	0

[13903 rows x 34 columns]

0	16
1	40
2	32
3	13

4	1
5	1
6	3
7	8
8	36
9	56
10	84
11	94
12	106
13	110
14	93
15	67
16	35
17	37
18	34
19	28
20	39
21	17
22	17
23	9
24	6
25	3
26	2
27	1
28	8
29	20
	...
13873	74
13874	136
13875	144
13876	169
13877	160
13878	138
13879	133
13880	123
13881	125
13882	47
13883	36
13884	49
13885	19
13886	11
13887	1
13888	3
13889	85
13890	196
13891	120
13892	157
13893	224

```
13894    203
13895    247
13896    315
13897    214
13898    164
13899    122
13900     89
13901     61
13902     49
Name: counts, Length: 13903, dtype: int64
```

```
Index(['const', 'hour', 'hour2', 'holiday', 'year', 'workingday', 'temp',
       'temp2', 'atemp', 'hum', 'hum2', 'windspeed', 'spring', 'summer',
       'fall', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct',
       'Nov', 'Dec', 'Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat', 'Cloudy',
       'Snow'],
      dtype='object')
```

```
In [50]: for k in range (1, len(predictors)):
             predictors = x.columns

             best_r_squared = -np.inf
             best_model_data = None

             # Enumerate subsets of the given size
             subsets_of_size_k = itertools.combinations(predictors, len(predictors))
```

Let's create a loop that just finds the best predictor

```
In [51]: def lowest_bic(est_predictors, more_predictors):
             current_preds = []
             running_bic = []
             current_bic = []

             for i, predictor in enumerate(more_predictors):
                 design_mat = est_predictors + [predictor]
                 curr_fitted_model = OLS(endog = y_train, exog = X_train_poly[design_mat], has
                 current_bic.append(curr_fitted_model.bic)

             lowest_bic = min(current_bic)
             return (lowest_bic, more_predictors[current_bic.index(lowest_bic)])
```

Ok, now let's put this in another loop that itteratively adds the next predictor and stores the bic of that "next" best stepqise model so that we can compare them. Break the loop when the BIC starts to go up.

```
In [52]: all_predictors = X_train_poly.drop(columns=['const']).columns
         all_predictors = list(all_predictors)
```

```
predictors_list = []
test_predictors = all_predictors.copy()
best_bic = None
while True:
    new_bic, last_best_predictor = lowest_bic(predictors_list, test_predictors)

    if best_bic == None:
        best_bic = new_bic
    elif best_bic < new_bic:
        break
    elif best_bic > new_bic:
        best_bic = new_bic
    predictors_list = predictors_list + [last_best_predictor]
    test_predictors.remove(last_best_predictor)
    print(new_bic, last_best_predictor)
```

```
181370.05948206075 temp
179717.78889599905 hour
178357.82706914432 hum2
176722.95912070328 hour2
175981.5677215192 year
175316.11289253167 hum
174975.6742919337 fall
174848.4445545477 Snow
174780.8933556406 Jul
174739.97889847725 temp2
174681.61599833728 spring
174618.93471527623 Sept
174606.3563833533 Oct
174593.35851715243 holiday
174583.79233540065 summer
174579.33274081693 May
174576.98736332514 atemp
174576.9208246853 Mar
```

In [53]: `display(predictors_list)`

```
['temp',
 'hour',
 'hum2',
 'hour2',
 'year',
 'hum',
 'fall',
 'Snow',
 'Jul',
```

```
 'temp2',
 'spring',
 'Sept',
 'Oct',
 'holiday',
 'summer',
 'May',
 'atemp',
 'Mar']
```

```
In [54]: drop_predictors = []
         for predictor in all_predictors:
             if predictor not in predictors_list:
                 drop_predictors.append(predictor)
             else:
                 pass
```

```
In [55]: drop_predictors
```

```
Out[55]: ['workingday',
          'windspeed',
          'Feb',
          'Apr',
          'Jun',
          'Aug',
          'Nov',
          'Dec',
          'Mon',
          'Tue',
          'Wed',
          'Thu',
          'Fri',
          'Sat',
          'Cloudy',
          'Storm']
```

5.2 Does forward selection eliminate one or more of the colinear predictors we dropped in Question 4.5 (`workingday` `atemp` `spring` `summer` and `fall`)? If any of the five predictors are not dropped, explain why. The predictors `atemp`, `spring`, `summer`, and `fall` were not removed. This is because removing it does not cause a significant decrease in the BIC score for the model.

5.3 Fit the linear regression model using the identified subset of predictors to the training set. How do the train and test R^2 scores for this fitted step-wise model compare with the train and test R^2 scores from the polynomial model fitted in Question 4.4?

```
In [56]: X_train_poly_drop = X_train_poly.drop(drop_predictors, axis=1)
         display(X_train_poly_drop.head())
```

```
model4 = sm.OLS(
    y_train,
    X_train_poly_drop).fit()
model4.summary()
```

```
   const  hour  hour2  holiday  year  temp   temp2   atemp   hum    hum2  \
0    1.0     0      0        0     0  0.24  0.0576  0.2879  0.81  0.6561
1    1.0     1      1        0     0  0.22  0.0484  0.2727  0.80  0.6400
2    1.0     2      4        0     0  0.22  0.0484  0.2727  0.80  0.6400
3    1.0     3      9        0     0  0.24  0.0576  0.2879  0.75  0.5625
4    1.0     4     16        0     0  0.24  0.0576  0.2879  0.75  0.5625

   spring  summer  fall  Mar  May  Jul  Sept  Oct  Snow
0       0       0     0    0    0    0     0    0     0
1       0       0     0    0    0    0     0    0     0
2       0       0     0    0    0    0     0    0     0
3       0       0     0    0    0    0     0    0     0
4       0       0     0    0    0    0     0    0     0
```

Out[56]:

```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                 counts   R-squared:                       0.499
Model:                            OLS   Adj. R-squared:                  0.499
Method:                 Least Squares   F-statistic:                     769.0
Date:                Wed, 03 Oct 2018   Prob (F-statistic):               0.00
Time:                        23:10:37   Log-Likelihood:                -87125.
No. Observations:               13903   AIC:                         1.743e+05
Df Residuals:                   13884   BIC:                         1.744e+05
Df Model:                          18
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       -166.4134     13.365    -12.451      0.000    -192.611    -140.216
hour          39.4373      0.649     60.800      0.000      38.166      40.709
hour2         -1.3502      0.026    -51.075      0.000      -1.402      -1.298
holiday      -26.2386      6.450     -4.068      0.000     -38.881     -13.596
year          81.1137      2.191     37.023      0.000      76.819      85.408
temp          99.3996     55.038      1.806      0.071      -8.482     207.281
temp2        112.9786     34.946      3.233      0.001      44.480     181.478
atemp         82.4241     41.700      1.977      0.048       0.686     164.162
hum           -6.7628     35.640     -0.190      0.850     -76.622      63.096
hum2         -99.3923     28.593     -3.476      0.001    -155.439     -43.346
spring        35.6581      4.190      8.509      0.000      27.444      43.872
```

```
summer        21.3772      5.511      3.879      0.000      10.574      32.180
fall          62.5140      3.894     16.053      0.000      54.881      70.147
Mar           15.9047      4.257      3.736      0.000       7.561      24.248
May           13.1080      4.739      2.766      0.006       3.819      22.397
Jul          -30.3681      4.946     -6.139      0.000     -40.064     -20.672
Sept          30.8493      4.568      6.753      0.000      21.895      39.803
Oct           18.2902      4.811      3.802      0.000       8.860      27.720
Snow         -43.4153      4.328    -10.032      0.000     -51.898     -34.933
==============================================================================
Omnibus:                     2961.647   Durbin-Watson:                   0.887
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6161.407
Skew:                           1.256   Prob(JB):                         0.00
Kurtosis:                       5.080   Cond. No.                     1.58e+04
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly spec
[2] The condition number is large, 1.58e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
"""
```

Answer

The R^2 value decreases slightly, from 0.501 to 0.499, when compared to the model in 4.4.

2 Written Report to the Administrators [20 pts]

Question 6

Write a short repost stating some of your findings on how the administrators can increase the bike share system's revenue. You might want to include suggestions such as what model to use to predict ridership, what additional services to provide, or when to give discounts, etc. Include your report as a pdf file in canvas. The report should not be longer than one page (300 words) and should include a maximum of 5 figures.

Answers 6 See PDF