

main

January 24, 2026

1 Part I. Code replication

Packages and data load

```
[1]: # %pip install numpy pandas matplotlib scipy tqdm

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.optimize import minimize
from tqdm import tqdm

np.random.seed(2026)
```

```
[2]: # global parameters
ch_opt = 2.34
R2_grid = np.arange(0.1, 1.0, 0.1)
```

Model

```
[3]: def DGP1(T, J, R2, alpha=1.5):
    c = np.sqrt(R2 / (1 - R2))

    t = np.arange(1, T + 1)
    tau = t / T
    F_tau = tau**3

    X = np.random.randn(T, J)
    X[:, 0] = 1.0
    j = np.arange(1, J+1)
    theta = c * np.sqrt(2 * alpha) * j**(-(alpha + 0.5))
    mu = F_tau * (X @ theta)
    eps = np.random.randn(T)
    Y = mu + eps

    return Y, X, mu

def DGP2(T, J, R2, alpha=1.5):
```

```

c = np.sqrt(R2 / (1 - R2))

t = np.arange(1, T + 1)
tau = t / T

F_tau = np.where(tau < 0.3, 0.1,
                  np.where(tau < 0.8, 1.0, -0.5))

X = np.random.randn(T, J)
X[:, 0] = 1.0

j = np.arange(1, J+1)
theta = c * np.sqrt(2 * alpha) * j**(-(alpha + 0.5))

mu = F_tau * (X @ theta)
eps = np.random.randn(T)

Y = mu + eps
return Y, X, mu

```

[59]: `y1, _, mu1 = DGP1(200, 100, 0.5)`

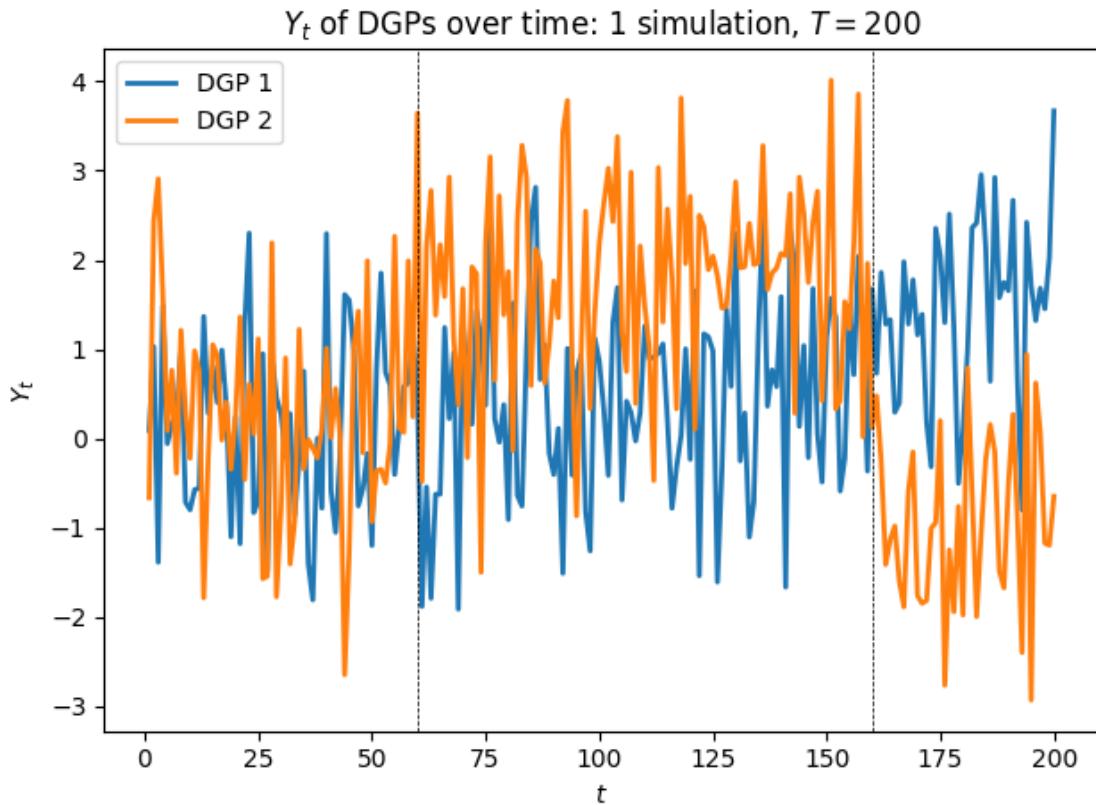
[60]: `y2, _, mu2 = DGP2(200, 100, 0.5)`

[99]: `T = 200`
`plt.plot(np.arange(1, T + 1), y1, linewidth=2, label='DGP 1')`
`plt.plot(np.arange(1, T + 1), y2, linewidth=2, label='DGP 2')`

```

plt.xlabel(r"$t$")
plt.ylabel(r"$Y_t$")
plt.axvline(0.8*T, linestyle="--", color="black", linewidth=0.5)
plt.axvline(0.3*T, linestyle="--", color="black", linewidth=0.5)
plt.title(r"$Y_t$ of DGPs over time: 1 simulation, $T=200$")
plt.legend()
plt.tight_layout()
plt.show()

```



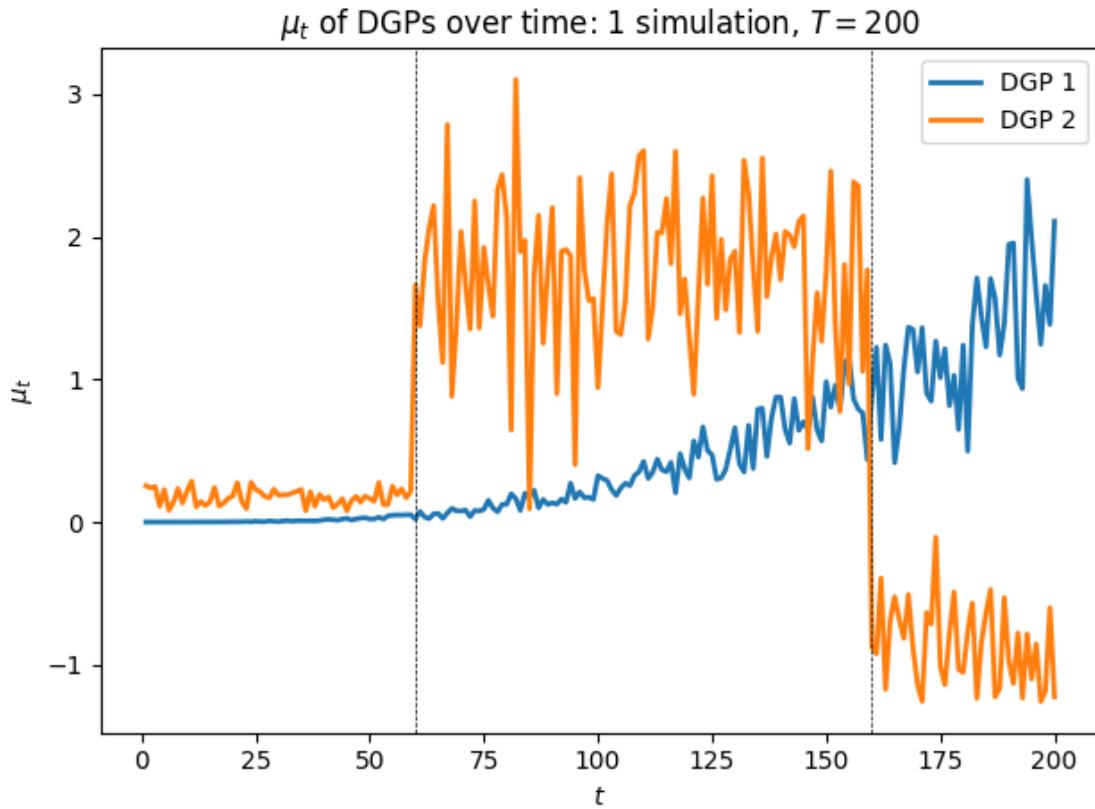
[101]: $T = 200$

```

plt.plot(np.arange(1, T + 1), mu1, linewidth=2, label='DGP 1')
plt.plot(np.arange(1, T + 1), mu2, linewidth=2, label='DGP 2')

plt.xlabel(r"$t$")
plt.ylabel(r"$\mu_t$")
plt.axvline(0.8*T, linestyle="--", color="black", linewidth=0.5)
plt.axvline(0.3*T, linestyle="--", color="black", linewidth=0.5)
plt.title(r"$\mu_t$ of DGPs over time: 1 simulation, $T=200$")
plt.legend()
plt.tight_layout()
plt.show()

```



Methodology: Estimator

```
[68]: def check_true_dgp(alpha=1.5, eps=1e-3):
    j = np.arange(1, 101)
    # c values implied by R^2 in [0.1, 0.9]
    cs = [0.33, 0.68, 1.00, 1.53, 3.00]

    for c in cs:
        theta = c * np.sqrt(2 * alpha) * j**(-(alpha + 0.5))
        # find first index where theta_j <= eps since theta is monotonically
        ↪decreasing
        fail_idx = np.where(theta <= eps)[0]

        if len(fail_idx) == 0:
            print(f"c = {c:.2f}: _j remains above tolerance={eps:.0e} for all"
      ↪j   ↪{j[-1]}")
        else:
            j_star = j[fail_idx[0]]
            print(f"c = {c:.2f}: _j falls below tolerance={eps:.0e} at j ="
      ↪{j_star}")
    plt.plot(j, theta, label=f"$c_{\{\theta\}} = {c:.2f}$")
```

```

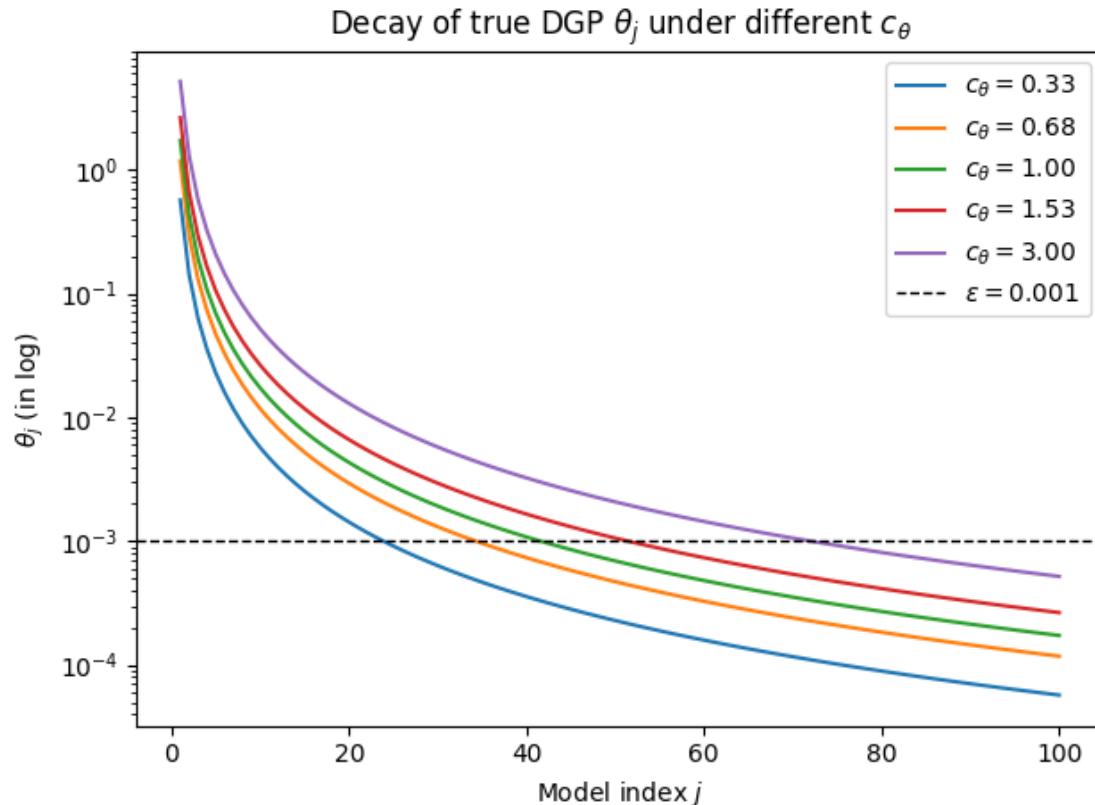
plt.axhline(eps, color="black", linestyle="--", linewidth=1, label=f"\varepsilon = {eps}")
plt.xlabel(r"Model index $j$")
plt.ylabel(r"$\theta_j$ (in log)")
plt.title(r"Decay of true DGP $\theta_j$ under different $c_\theta$")
plt.yscale("log")
plt.legend()
plt.tight_layout()
plt.show()
check_true_dgp()

```

```

c = 0.33: _j falls below tolerance=1e-03 at j = 24
c = 0.68: _j falls below tolerance=1e-03 at j = 35
c = 1.00: _j falls below tolerance=1e-03 at j = 42
c = 1.53: _j falls below tolerance=1e-03 at j = 52
c = 3.00: _j falls below tolerance=1e-03 at j = 73

```



```
[9]: def ls_oracle_mse(mu_hat_mat, mu_true):
    M = mu_hat_mat.shape[1]
```

```

def obj(w):
    mu_w = mu_hat_mat @ w
    return np.mean((mu_w - mu_true)**2)

cons = (
    {'type': 'eq', 'fun': lambda w: np.sum(w) - 1},
    {'type': 'ineq', 'fun': lambda w: w}
)

w0 = np.full(M, 1.0 / M)
res = minimize(obj, w0, constraints=cons, method='SLSQP')
mu_oracle = mu_hat_mat @ res.x

return np.mean((mu_oracle - mu_true)**2)

```

```

[10]: def kernel_vec(t, T, h):
    s = np.arange(1, T + 1) # Epanechnikov
    u = (s - t) / (T * h)
    k = 0.75 * (1 - u**2)
    k[np.abs(u) > 1] = 0.0
    return k

def local_jackknife(Y, X, h):
    T = len(Y); mu_tilde = np.empty(T)
    for i in range(T):
        t = i+1
        K = kernel_vec(t, T, h)
        K[i] = 0.0
        W = np.diag(K)
        beta = np.linalg.solve(X.T @ W @ X, X.T @ W @ Y)
        mu_tilde[i] = X[i, :] @ beta
    return mu_tilde

def local_estimator(Y, X, h):
    T = len(Y); mu_hat = np.empty(T)
    for i in range(T):
        t = i+1
        K = kernel_vec(t, T, h)
        # K[i] = 0.0
        W = np.diag(K)
        beta = np.linalg.solve(X.T @ W @ X, X.T @ W @ Y)
        mu_hat[i] = X[i, :] @ beta
    return mu_hat

def obj_cv(w, mu_mat, Y, K):
    mu_w = mu_mat @ w
    resid = Y - mu_w

```

```

    return np.sum(resid**2 * K)

def tvjma(Y, X_list, h):
    T, M = len(Y), len(X_list)
    # Local estimator
    mu_hat_mat = np.empty((T, M))
    for m, X in enumerate(X_list):
        mu_hat_mat[:, m] = local_estimator(Y, X, h)
    # Jackknife Prediction
    mu_tilde_mat = np.empty((T, M))
    for m, X in enumerate(X_list):
        mu_tilde_mat[:, m] = local_jackknife(Y, X, h)

    # Optimization for time varying weights with kernel fct & obj_cv (with mu_tilde)
    w_hat = np.zeros((T, M)); mu_hat = np.empty(T)
    for i in range(T):
        t = i+1
        K = kernel_vec(t, T, h)
        # K[i] = 0.0
        cons = ({'type':'eq','fun':lambda w: np.sum(w)-1},
                {'type':'ineq','fun':lambda w: w})
        w0 = np.full(M, 1./M) # initial value = uniform
        res = minimize(obj_cv, w0, args=(mu_tilde_mat, Y, K),
                       method='SLSQP', constraints=cons,
                       options={'ftol':1e-9})
        w_hat[i] = res.x
        mu_hat[i] = mu_hat_mat[i] @ w_hat[i]
    return mu_hat, mu_hat_mat, mu_tilde_mat, w_hat

def build_X_list(X):
    T, J = X.shape
    X_list = []

    for m in range(1, J + 1):
        X_list.append(X[:, :m])

    return X_list

def monte_carlo(T, R2, dgp_num, ch, n_rep=1000):
    MSE_inf_all = [] # will store lowest MSE across candidate models per replication
    MSE_tvjma_all = [] # will store MSE of TVJMA per replication
    J = int(np.round(3 * T**(1/3)))
    h = ch * T**(-0.2)

```

```

for r in (range(n_rep)):
    if dgp_num == 1:
        Y, X, mu_true = DGP1(T, 100, R2)
    elif dgp_num == 2:
        Y, X, mu_true = DGP2(T, 100, R2)
    else:
        raise ValueError("dgp must be 'DGP1' or 'DGP2'")

    X = X[:, :J]
    X_list = build_X_list(X)
    mu_tvma, mu_hat_mat, mu_tilde_mat, w_hat = tvjma(Y, X_list, h)

    M = len(X_list)
    mu_hat_ls = np.empty((T, M))

    for m, Xm in enumerate(X_list):
        beta_ls = np.linalg.lstsq(Xm, Y, rcond=None)[0]
        mu_hat_ls[:, m] = Xm @ beta_ls

    mse_oracle = ls_oracle_mse(mu_hat_ls, mu_true)
    MSE_inf_all.append(mse_oracle)

    # TVJMA risk
    mse_tvjma = np.mean((mu_tvma - mu_true)**2)
    MSE_tvjma_all.append(mse_tvjma)

# Convert to numpy arrays
MSE_inf_all = np.array(MSE_inf_all)
MSE_tvjma_all = np.array(MSE_tvjma_all)

normalized_risk = MSE_tvjma_all.mean() / MSE_inf_all.mean()

return normalized_risk, MSE_tvjma_all/MSE_inf_all

```

```

[11]: T = 50
rel_mse = np.zeros(len(R2_grid))
mse = []
mse_tvjma = []

for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse[i], mse = monte_carlo(T, R2, dgp_num = 1, ch= 2.34, n_rep=100)
    mse_tvjma.append(mse)

plt.plot(R2_grid, rel_mse, linewidth=2)
plt.axhline(1.0, linestyle="--", color="black")
plt.xlabel(r"$R^2$")
plt.ylabel("Relative MSE")

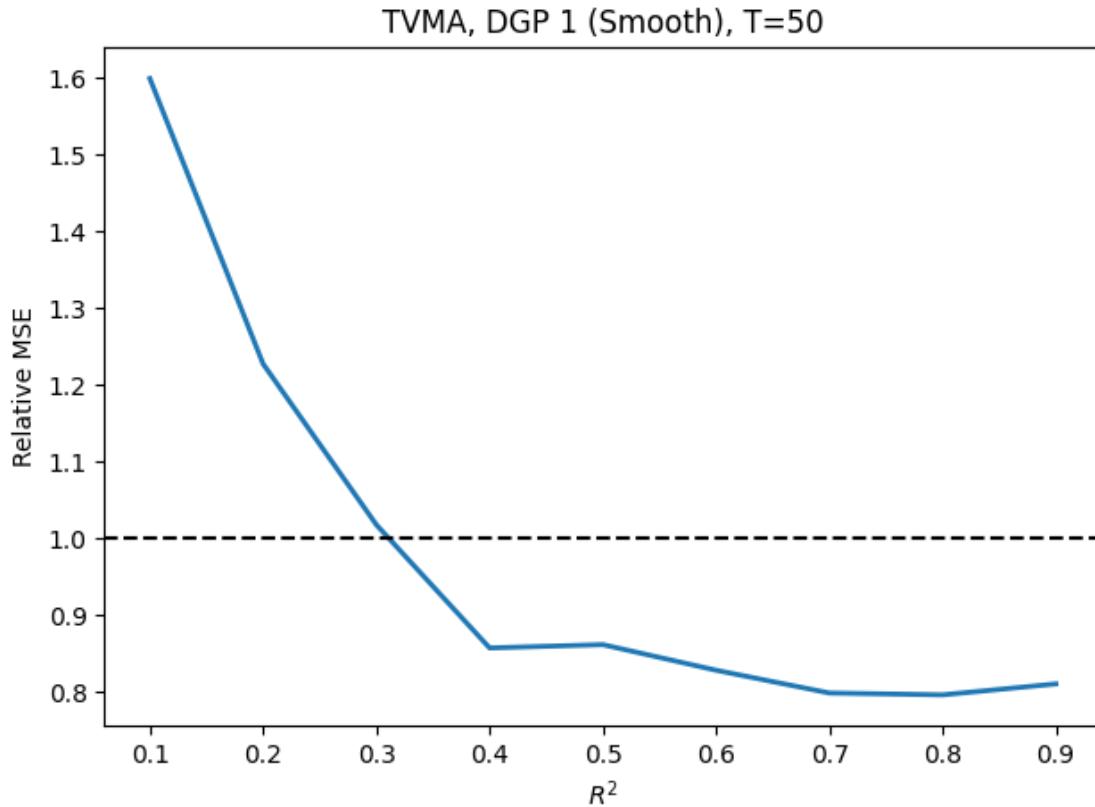
```

```

plt.title(f"TVMA, DGP 1 (Smooth), T={T}")
plt.tight_layout()
plt.show()

```

100% | 9/9 [01:17<00:00, 8.65s/it]



```
[12]: # np.savetxt("partI_rel_mse_t50_dgp1.csv", rel_mse, delimiter=",")
```

2 Part II. Code extension

When $T=200$, the optimal bandwidth is ...

```

[13]: # DGP 1 and T=200
T= 200
rel_mse_1s_T200 = np.zeros(len(R2_grid))
rel_mse_1m_T200 = np.zeros(len(R2_grid))
rel_mse_1l_T200 = np.zeros(len(R2_grid))
rel_mse_1opt_T200 = np.zeros(len(R2_grid))
mse_temp=[]
mse_1s_T200 = []
mse_1m_T200 = []

```

```
mse_11_T200 = []
mse_1opt_T200 = []
```

```
[14]: h_test=0.3
ch_test=h_test/T**(-0.2)
print(f"${c_h$ for simulation = {np.round(ch_test, 3)}}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_1s_T200[i], mse_temp = monte_carlo(T, R2, dgp_num=1, ch=ch_test, n_rep=100)
    mse_1s_T200.append(mse_temp)
```

```
$c_h$ for simulation = 0.866
100%|      | 9/9 [11:11<00:00, 74.61s/it]
```

```
[15]: h_test=0.5
ch_test2=h_test/T**(-0.2)
print(f"${c_h$ for simulation = {np.round(ch_test, 3)}}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_1m_T200[i], mse_temp = monte_carlo(T, R2, dgp_num=1, ch=ch_test2, n_rep=100)
    mse_1m_T200.append(mse_temp)
```

```
$c_h$ for simulation = 0.866
100%|      | 9/9 [11:10<00:00, 74.51s/it]
```

```
[16]: h_test=0.9
ch_test3=h_test/T**(-0.2)
print(f"${c_h$ for simulation = {np.round(ch_test, 3)}}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_11_T200[i], mse_temp = monte_carlo(T, R2, dgp_num=1, ch=ch_test3, n_rep=100)
    mse_11_T200.append(mse_temp)
```

```
$c_h$ for simulation = 0.866
100%|      | 9/9 [11:09<00:00, 74.43s/it]
```

```
[17]: print(f"${c_h$ for simulation = {np.round(ch_opt, 3)}}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_1opt_T200[i], mse_temp = monte_carlo(T, R2, dgp_num=1, ch=ch_opt, n_rep=100)
    mse_1opt_T200.append(mse_temp)
```

```
$c_h$ for simulation = 2.34
100%|      | 9/9 [11:06<00:00, 74.04s/it]
```

```
[18]: np.savetxt("rel_mse_1s_T200.csv", rel_mse_1s_T200, delimiter=",")
np.savetxt("rel_mse_1m_T200.csv", rel_mse_1m_T200, delimiter=",")
```

```

np.savetxt("rel_mse_11_T200.csv", rel_mse_11_T200, delimiter=",")
np.savetxt("rel_mse_1opt_T200.csv", rel_mse_1opt_T200, delimiter=",")
np.savetxt("mse_1s_T200.csv", mse_1s_T200, delimiter=",")
np.savetxt("mse_1m_T200.csv", mse_1m_T200, delimiter=",")
np.savetxt("mse_11_T200.csv", mse_11_T200, delimiter=",")
np.savetxt("mse_1opt_T200.csv", mse_1opt_T200, delimiter=",")

```

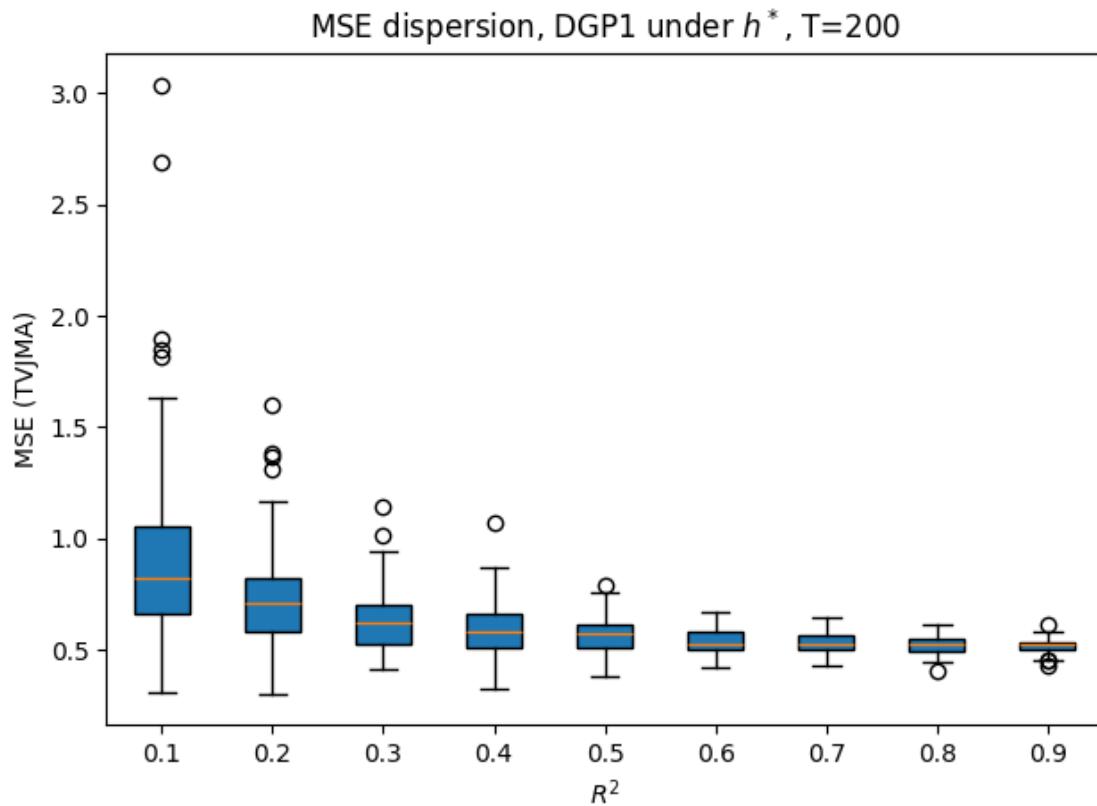
```

[77]: plt.boxplot(mse_1opt_T200,
                  labels=[f"r2:{r2:.1f}" for r2 in R2_grid],
                  showfliers=True, patch_artist=True)
plt.xlabel(r"$R^2$")
plt.ylabel("MSE (TVJMA)")
plt.title(f"MSE dispersion, DGP1 under $h^*$, T={T}")
plt.tight_layout()
plt.show()

```

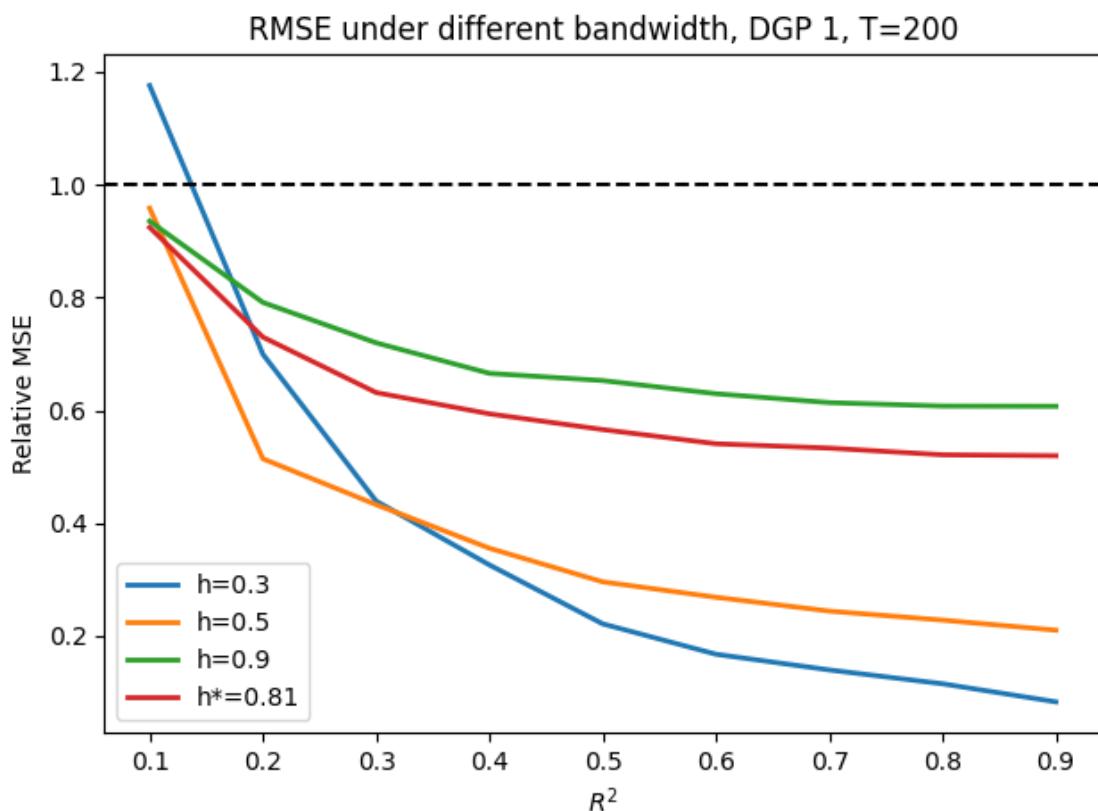
/tmp/ipykernel_76/144169825.py:1: MatplotlibDeprecationWarning: The 'labels' parameter of boxplot() has been renamed 'tick_labels' since Matplotlib 3.9; support for the old name will be dropped in 3.11.

```
plt.boxplot(mse_1opt_T200,
```



```
[76]: plt.plot(R2_grid, rel_mse_1s_T200, linewidth=2, label='h=0.3')
plt.plot(R2_grid, rel_mse_1m_T200, linewidth=2, label='h=0.5')
plt.plot(R2_grid, rel_mse_1l_T200, linewidth=2, label='h=0.9')
plt.plot(R2_grid, rel_mse_1opt_T200, linewidth=2, label='h*=0.81')

plt.axhline(1.0, linestyle="--", color="black")
plt.xlabel(r"$R^2$")
plt.ylabel("Relative MSE")
plt.title(f"RMSE under different bandwidth, DGP 1, T={T}")
plt.legend()
plt.tight_layout()
plt.show()
```



```
[21]: # data = [mu_1s, mu_1m, mu_1l, mu_1opt]
# labels = ['h=0.3', 'h=0.5', 'h=0.9', 'h*=0.81']

# plt.figure()
# plt.boxplot(data, labels=labels, showfliers=True)
# plt.ylabel("Mu at time T=26")
# plt.title(f"TVMA, DGP 1 (Smooth Structural Changes), T={T}")
```

```
# plt.tight_layout()
# plt.show()
```

When $T=50$, the optimal bandwidth is...

[22]: # DGP 1 and $T=50$

```
T= 50
rel_mse_1s_T50 = np.zeros(len(R2_grid))
rel_mse_1m_T50 = np.zeros(len(R2_grid))
rel_mse_1l_T50 = np.zeros(len(R2_grid))
rel_mse_1opt_T50 = np.zeros(len(R2_grid))

mse_1s_T50 = []
mse_1m_T50 = []
mse_1l_T50 = []
mse_1opt_T50 = []
```

[23]:

```
h_test=T**(-0.2) * ch_test
print(f"${h$ for simulation = {h_test} with ch = {np.round(ch_test, 3)}$}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_1s_T50[i], mse_temp = monte_carlo(T, R2, dgp_num=1, ch=ch_test, n_rep=100)
    mse_1s_T50.append(mse_temp)
```

\$h\$ for simulation = 0.3958523732318683 with ch = 0.866

100% | 9/9 [01:23<00:00, 9.31s/it]

[24]:

```
h_test=T**(-0.2) * ch_test2
print(f"${h$ for simulation = {h_test} with ch = {np.round(ch_test2, 3)}$}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_1m_T50[i], mse_temp = monte_carlo(T, R2, dgp_num=1, ch=ch_test2, n_rep=100)
    mse_1m_T50.append(mse_temp)
```

\$h\$ for simulation = 0.6597539553864472 with ch = 1.443

100% | 9/9 [01:20<00:00, 8.97s/it]

[25]:

```
h_test=T**(-0.2) * ch_test3
print(f"${h$ for simulation = {h_test} with ch = {np.round(ch_test3, 3)}$}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_1l_T50[i], mse_temp = monte_carlo(T, R2, dgp_num=1, ch=ch_test3, n_rep=100)
    mse_1l_T50.append(mse_temp)
```

\$h\$ for simulation = 1.1875571196956047 with ch = 2.597

100% | 9/9 [01:19<00:00, 8.80s/it]

```
[26]: print(f"${c_h$ for simulation = {np.round(ch_opt, 3)}}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_1opt_T50[i], mse_temp = monte_carlo(T, R2, dgp_num=1, ch=ch_opt, n_rep=100)
    mse_1opt_T50.append(mse_temp)
```

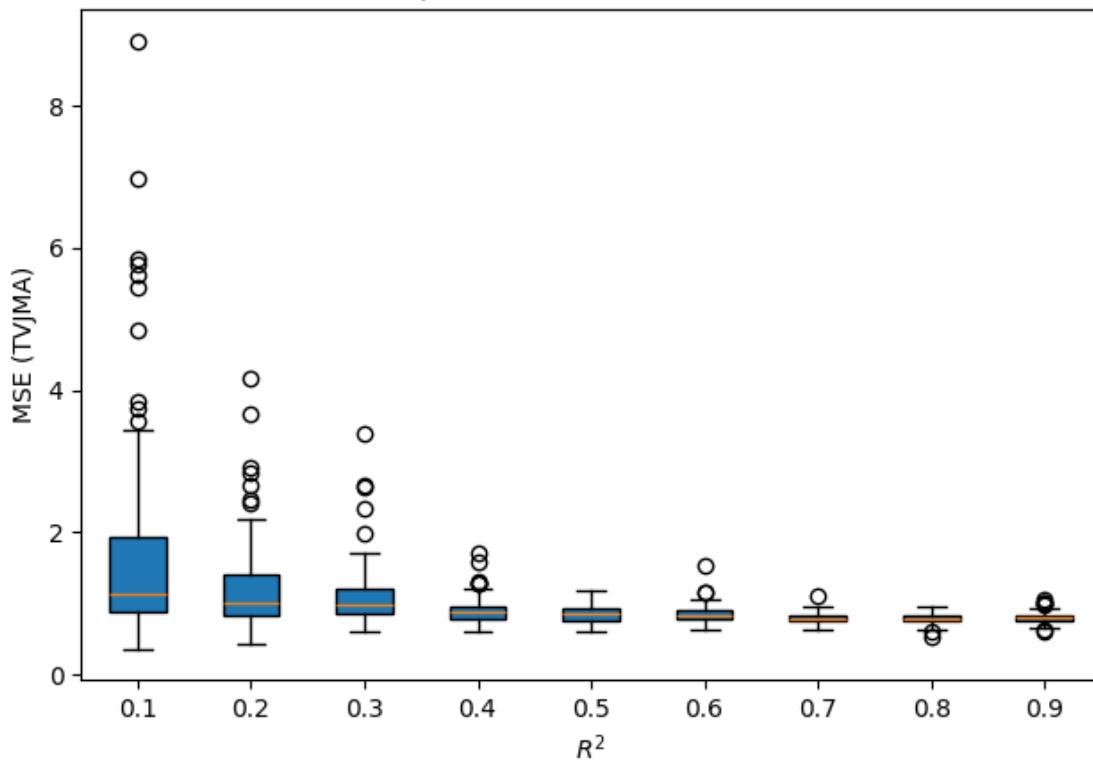
\$c_h\$ for simulation = 2.34
100% | 9/9 [01:20<00:00, 8.97s/it]

```
[27]: np.savetxt("rel_mse_1s_T50.csv", rel_mse_1s_T50, delimiter=",")
np.savetxt("rel_mse_1m_T50.csv", rel_mse_1m_T50, delimiter=",")
np.savetxt("rel_mse_11_T50.csv", rel_mse_11_T50, delimiter=",")
np.savetxt("rel_mse_1opt_T50.csv", rel_mse_1opt_T50, delimiter=",")
np.savetxt("mse_1s_T50.csv", mse_1s_T50, delimiter=",")
np.savetxt("mse_1m_T50.csv", mse_1m_T50, delimiter=",")
np.savetxt("mse_11_T50.csv", mse_11_T50, delimiter=",")
np.savetxt("mse_1opt_T50.csv", mse_1opt_T50, delimiter=",")
```

```
[85]: T=50
plt.boxplot(mse_1opt_T50,
            labels=[f"{r2:.1f}" for r2 in R2_grid],
            showfliers=True, patch_artist=True)
plt.xlabel(r"$R^2$")
plt.ylabel("MSE (TVJMA)")
plt.title(f"MSE dispersion, DGP2 under $h^*$, T={T}")
plt.tight_layout()
plt.show()
```

/tmp/ipykernel_76/1048645959.py:2: MatplotlibDeprecationWarning: The 'labels' parameter of boxplot() has been renamed 'tick_labels' since Matplotlib 3.9; support for the old name will be dropped in 3.11.
plt.boxplot(mse_1opt_T50,

MSE dispersion, DGP2 under h^* , T=50



[86]: T=50

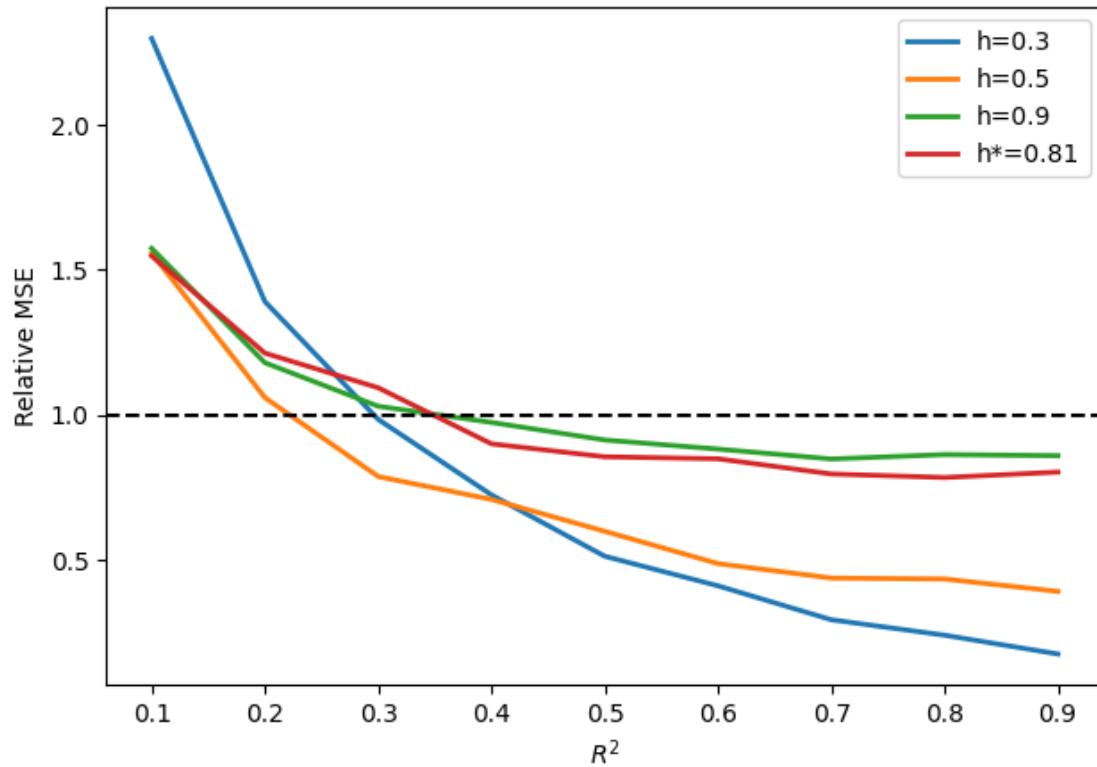
```

plt.plot(R2_grid, rel_mse_1s_T50, linewidth=2, label='h=0.3')
plt.plot(R2_grid, rel_mse_1m_T50, linewidth=2, label='h=0.5')
plt.plot(R2_grid, rel_mse_1l_T50, linewidth=2, label='h=0.9')
plt.plot(R2_grid, rel_mse_1opt_T50, linewidth=2, label='h*=0.81')

plt.axhline(1.0, linestyle="--", color="black")
plt.xlabel(r"$R^2$")
plt.ylabel("Relative MSE")
plt.title(f"RMSE under different bandwidth, DGP 2, T={T}")
plt.legend()
plt.tight_layout()
plt.show()

```

RMSE under different bandwidth, DGP 2, T=50



The optimal bandwidth is subject to the size of samples, T!

The bandwidth choice is also sensitive to the different DGP

When T=50,

```
[30]: # DGP 2 and T=50
T= 50
rel_mse_2s_T50 = np.zeros(len(R2_grid))
rel_mse_2m_T50 = np.zeros(len(R2_grid))
rel_mse_2l_T50 = np.zeros(len(R2_grid))
rel_mse_2opt_T50 = np.zeros(len(R2_grid))

mse_2s_T50 = []
mse_2m_T50 = []
mse_2l_T50 = []
mse_2opt_T50 = []
```

```
[31]: h_test=T**(-0.2) * ch_test
print(f"$h$ for simulation = {h_test} with ch = {np.round(ch_test, 3)}")
for i, R2 in enumerate(tqdm(R2_grid)):
```

```

    rel_mse_2s_T50[i], mse_temp = monte_carlo(T, R2, dgp_num=2, ch=ch_test, u
    ↵n_rep=100)
    mse_2s_T50.append(mse_temp)

```

\$h\$ for simulation = 0.3958523732318683 with ch = 0.866

100%| 9/9 [01:25<00:00, 9.52s/it]

```
[32]: h_test=T**(-0.2) * ch_test2
print(f"$h$ for simulation = {h_test} with ch = {np.round(ch_test2, 3)}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_2m_T50[i], mse_temp = monte_carlo(T, R2, dgp_num=2, ch=ch_test2, u
    ↵n_rep=100)
    mse_2m_T50.append(mse_temp)
```

\$h\$ for simulation = 0.6597539553864472 with ch = 1.443

100%| 9/9 [01:21<00:00, 9.03s/it]

```
[33]: h_test=T**(-0.2) * ch_test3
print(f"$h$ for simulation = {h_test} with ch = {np.round(ch_test3, 3)}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_21_T50[i], mse_temp = monte_carlo(T, R2, dgp_num=2, ch=ch_test3, u
    ↵n_rep=100)
    mse_21_T50.append(mse_temp)
```

\$h\$ for simulation = 1.1875571196956047 with ch = 2.597

100%| 9/9 [01:20<00:00, 8.98s/it]

```
[34]: print(f"${c_h}$ for simulation = {np.round(ch_opt, 3)}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_2opt_T50[i], mse_temp = monte_carlo(T, R2, dgp_num=2, ch=ch_opt, u
    ↵n_rep=100)
    mse_2opt_T50.append(mse_temp)
```

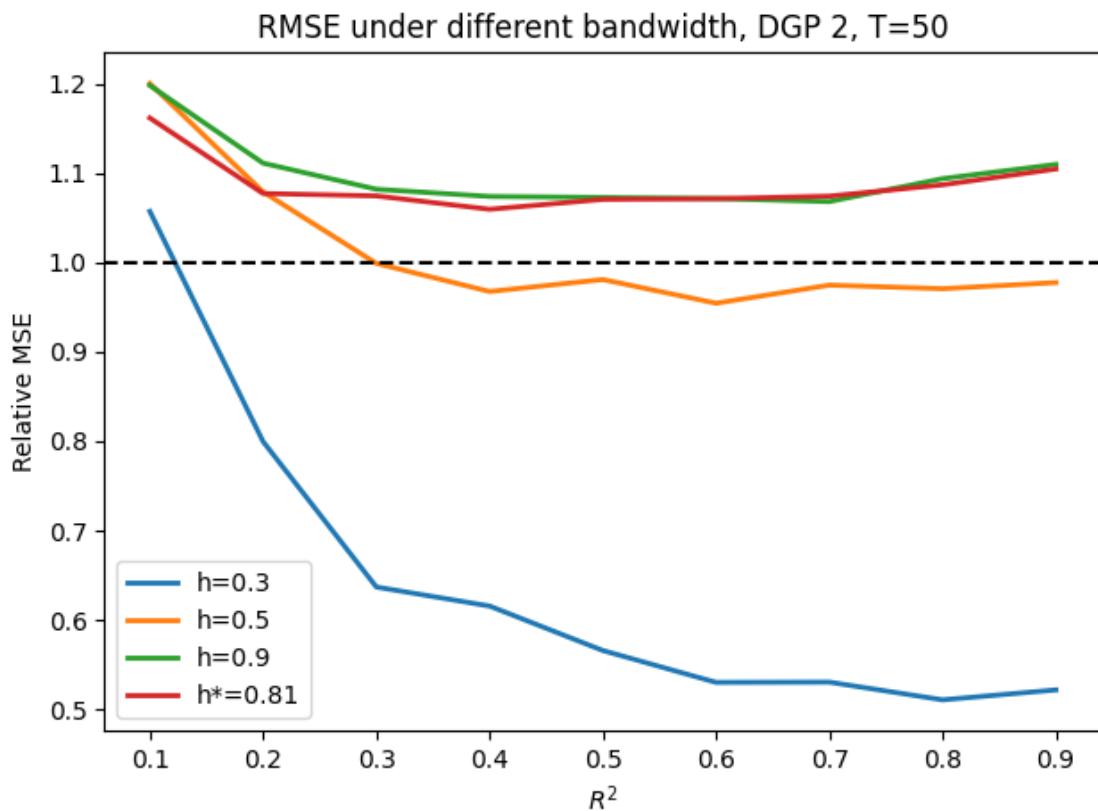
\$c_h\$ for simulation = 2.34

100%| 9/9 [01:19<00:00, 8.80s/it]

```
[82]: np.savetxt("rel_mse_2s_T50.csv", rel_mse_2s_T50, delimiter=",")
np.savetxt("rel_mse_2m_T50.csv", rel_mse_2m_T50, delimiter=",")
np.savetxt("rel_mse_21_T50.csv", rel_mse_21_T50, delimiter=",")
np.savetxt("rel_mse_2opt_T50.csv", rel_mse_2opt_T50, delimiter=",")
np.savetxt("mse_2s_T50.csv", mse_2s_T50, delimiter=",")
np.savetxt("mse_2m_T50.csv", mse_2m_T50, delimiter=",")
np.savetxt("mse_21_T50.csv", mse_21_T50, delimiter=",")
np.savetxt("mse_2opt_T50.csv", mse_2opt_T50, delimiter=",")
```

```
[87]: plt.plot(R2_grid, rel_mse_2s_T50, linewidth=2, label='h=0.3')
plt.plot(R2_grid, rel_mse_2m_T50, linewidth=2, label='h=0.5')
plt.plot(R2_grid, rel_mse_2l_T50, linewidth=2, label='h=0.9')
plt.plot(R2_grid, rel_mse_2opt_T50, linewidth=2, label='h*=0.81')

plt.axhline(1.0, linestyle="--", color="black")
plt.xlabel(r"$R^2$")
plt.ylabel("Relative MSE")
plt.title(f"RMSE under different bandwidth, DGP 2, T={T}")
plt.legend()
plt.tight_layout()
plt.show()
```

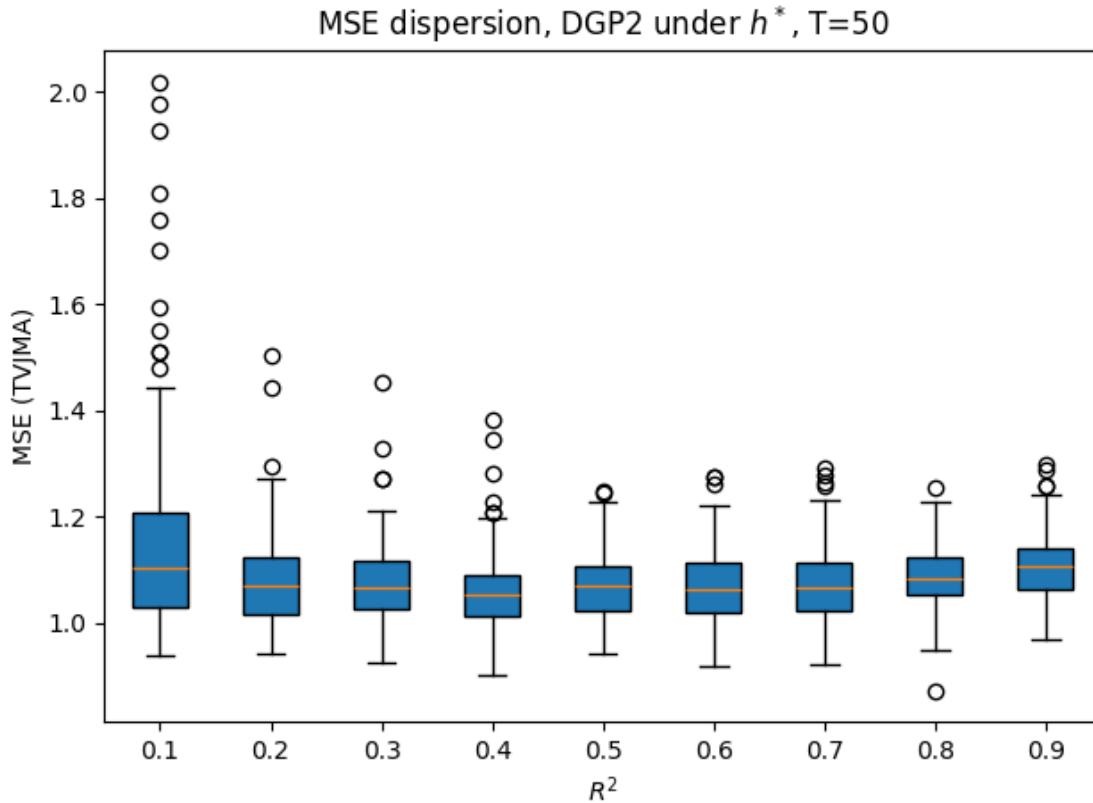


```
[88]: plt.boxplot(mse_2opt_T50,
                 labels=[f"r2:{.1f}" for r2 in R2_grid],
                 showfliers=True, patch_artist=True)
plt.xlabel(r"$R^2$")
plt.ylabel("MSE (TVJMA)")
plt.title(f"MSE dispersion, DGP2 under $h^*$, T={T}")
plt.tight_layout()
```

```
plt.show()
```

```
/tmp/ipykernel_76/536797711.py:1: MatplotlibDeprecationWarning: The 'labels'  
parameter of boxplot() has been renamed 'tick_labels' since Matplotlib 3.9;  
support for the old name will be dropped in 3.11.
```

```
plt.boxplot(mse_2opt_T50,
```



When T=200,

```
[38]: # DGP 2 and T=200  
T= 200  
rel_mse_2s_T200 = np.zeros(len(R2_grid))  
rel_mse_2m_T200 = np.zeros(len(R2_grid))  
rel_mse_2l_T200 = np.zeros(len(R2_grid))  
rel_mse_2opt_T200 = np.zeros(len(R2_grid))  
  
mse_2s_T200 = []  
mse_2m_T200 = []  
mse_2l_T200 = []  
mse_2opt_T200 = []
```

```
[39]: h_test=T**(-0.2) * ch_test
print(f"$h$ for simulation = {h_test} with ch = {np.round(ch_test, 3)}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_2s_T200[i], mse_temp = monte_carlo(T, R2, dgp_num=2, ch=h_test, n_rep=100)
    mse_2s_T200.append(mse_temp)
```

\$h\$ for simulation = 0.3 with ch = 0.866
100% | 9/9 [11:31<00:00, 76.80s/it]

```
[40]: h_test=T**(-0.2) * ch_test2
print(f"$h$ for simulation = {h_test} with ch = {np.round(ch_test2, 3)}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_2m_T200[i], mse_temp = monte_carlo(T, R2, dgp_num=2, ch=h_test2, n_rep=100)
    mse_2m_T200.append(mse_temp)
```

\$h\$ for simulation = 0.5 with ch = 1.443
100% | 9/9 [11:53<00:00, 79.27s/it]

```
[41]: h_test=T**(-0.2) * ch_test3
print(f"$h$ for simulation = {h_test} with ch = {np.round(ch_test3, 3)}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_2l_T200[i], mse_temp = monte_carlo(T, R2, dgp_num=2, ch=h_test3, n_rep=100)
    mse_2l_T200.append(mse_temp)
```

\$h\$ for simulation = 0.8999999999999999 with ch = 2.597
100% | 9/9 [11:56<00:00, 79.65s/it]

```
[42]: print(f"${c_h}$ for simulation = {np.round(ch_opt, 3)}")
for i, R2 in enumerate(tqdm(R2_grid)):
    rel_mse_2opt_T200[i], mse_temp = monte_carlo(T, R2, dgp_num=2, ch=h_opt, n_rep=100)
    mse_2opt_T200.append(mse_temp)
```

\$c_h\$ for simulation = 2.34
100% | 9/9 [11:51<00:00, 79.02s/it]

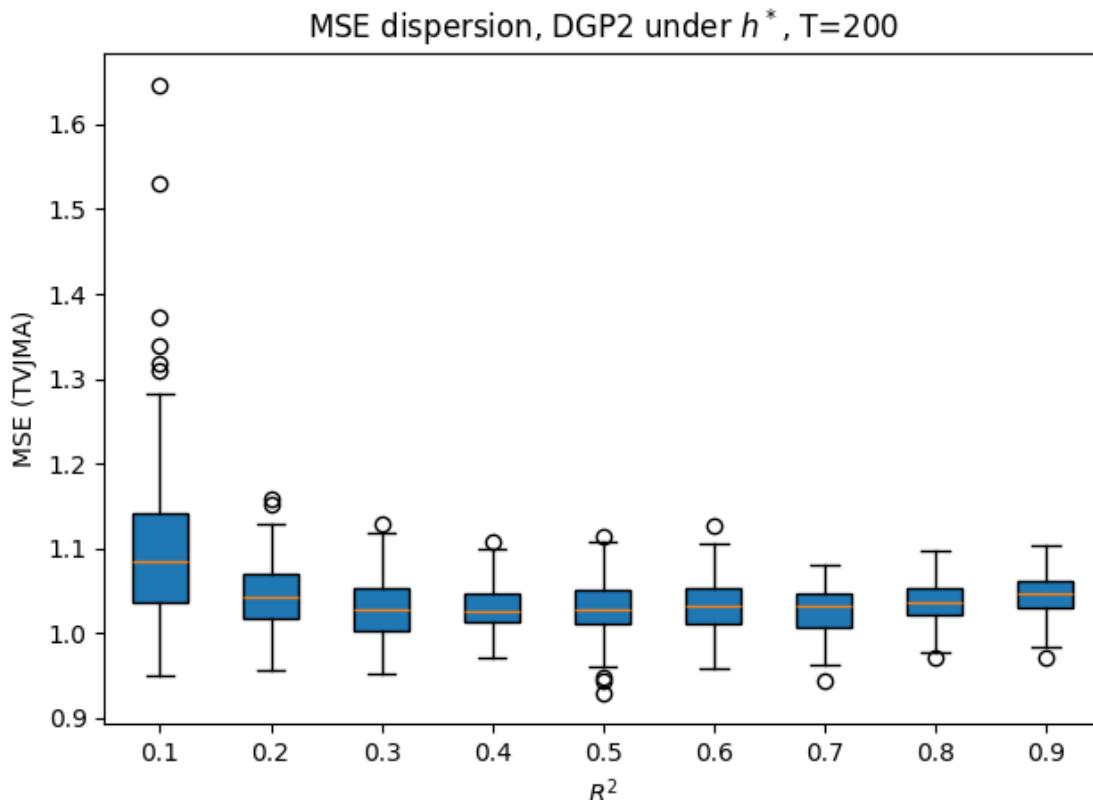
```
[43]: np.savetxt("rel_mse_2s_T200.csv", rel_mse_2s_T200, delimiter=",")
np.savetxt("rel_mse_2m_T200.csv", rel_mse_2m_T200, delimiter=",")
np.savetxt("rel_mse_2l_T200.csv", rel_mse_2l_T200, delimiter=",")
np.savetxt("rel_mse_2opt_T200.csv", rel_mse_2opt_T200, delimiter=",")
np.savetxt("mse_2s_T200.csv", mse_2s_T200, delimiter=",")
np.savetxt("mse_2m_T200.csv", mse_2m_T200, delimiter=",")
np.savetxt("mse_2l_T200.csv", mse_2l_T200, delimiter=",")
```

```
np.savetxt("mse_2opt_T200.csv", mse_2opt_T200, delimiter=",")
```

```
[89]: T=200
plt.boxplot(mse_2opt_T200,
            labels=[f"r2:{.1f}" for r2 in R2_grid],
            showfliers=True, patch_artist=True)
plt.xlabel(r"$R^2$")
plt.ylabel("MSE (TVJMA)")
plt.title(f"MSE dispersion, DGP2 under $h^*$, T={T}")
plt.tight_layout()
plt.show()
```

/tmp/ipykernel_76/149930678.py:2: MatplotlibDeprecationWarning: The 'labels' parameter of boxplot() has been renamed 'tick_labels' since Matplotlib 3.9; support for the old name will be dropped in 3.11.

```
plt.boxplot(mse_2opt_T200,
```



```
[96]: def plot_mu_paths(T, R2, dgp_num, ch_list, labels, seed=2026):
    np.random.seed(seed)

    # Generate one dataset
```

```

if dgp_num == 1:
    Y, X, mu_true = DGP1(T, 100, R2)
    dgp_name = "DGP 1"
else:
    Y, X, mu_true = DGP2(T, 100, R2)
    dgp_name = "DGP 2"

J = int(np.round(3 * T**(1/3)))
X = X[:, :J]
X_list = build_X_list(X)

tgrid = np.arange(1, T + 1)

plt.figure(figsize=(10, 5))
plt.plot(tgrid, mu_true, color="black", linewidth=2.5, label="True  $\mu_t$ ")

# Loop over bandwidths
for ch, lab in zip(ch_list, labels):
    h = ch * T**(-0.2)
    mu_hat, _, _, _ = tvjma(Y, X_list, h)
    plt.plot(tgrid, mu_hat, linewidth=1.5, label=lab)

# Structural break lines for DGP 2
if dgp_num == 2:
    plt.axvline(0.3 * T, color="gray", linestyle="--", linewidth=1)
    plt.axvline(0.8 * T, color="gray", linestyle="--", linewidth=1)

plt.xlabel(r" $t$ ")
plt.ylabel(r" $\mu_t$ ")
plt.title(f"TVJMA mean paths - {dgp_name}, $T={T}$, $R^2={R2}$")
plt.legend()
plt.tight_layout()
# plt.savefig("mu_paths_DGP1_T200_R2_05.png")
plt.show()

```

[90]:

```

T = 200
R2 = 0.5

```

```

ch_list = [
    ch_test,      # corresponds to h 0.3
    ch_test2,     # corresponds to h 0.5
    ch_test3,     # corresponds to h 0.9
    ch_opt        # optimal bandwidth
]

labels = [

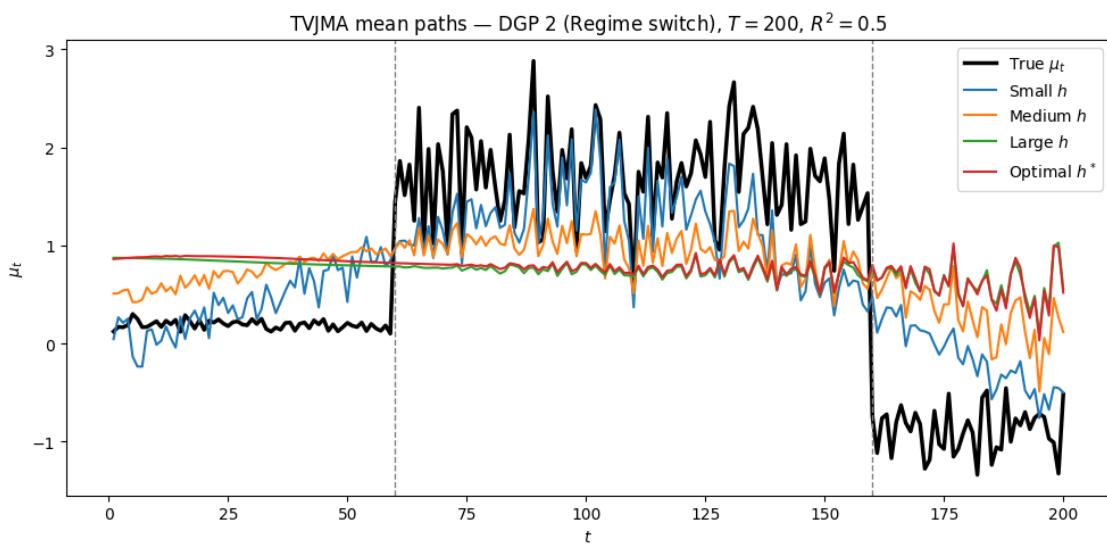
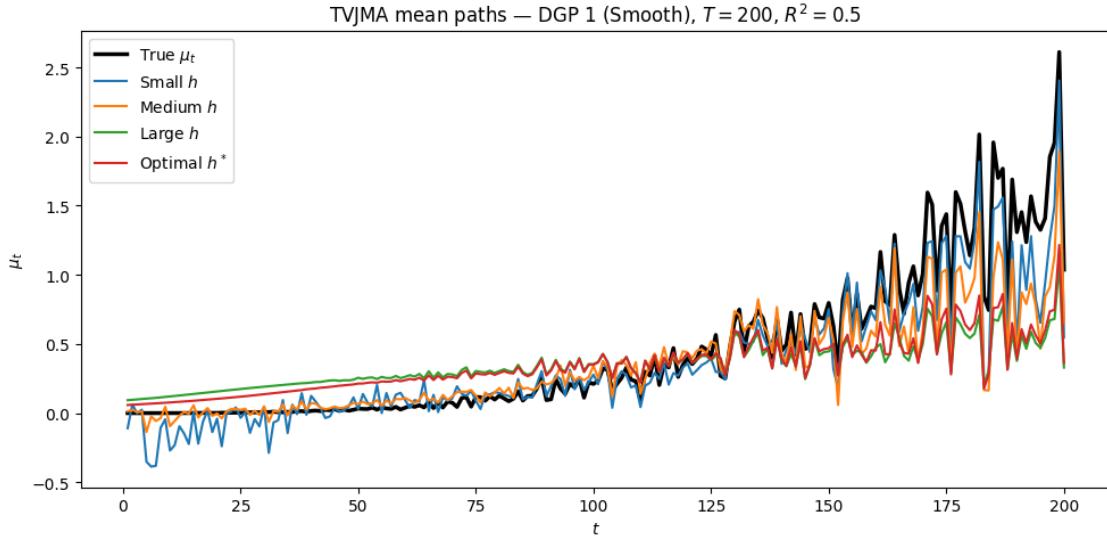
```

```

    "Small $h$",
    "Medium $h$",
    "Large $h$",
    "Optimal $h^*$"
]

plot_mu_paths(T, R2, dgp_num=1, ch_list=ch_list, labels=labels)
plot_mu_paths(T, R2, dgp_num=2, ch_list=ch_list, labels=labels)

```



```
[117]: def plot_avg_weights(T, R2, dgp_num, ch_list, labels, seed=2026):
    np.random.seed(seed)

    if dgp_num == 1:
        Y, X, _ = DGP1(T, 100, R2)
        dgp_name = "DGP 1"
    else:
        Y, X, _ = DGP2(T, 100, R2)
        dgp_name = "DGP 2"

    M = int(np.round(3 * T**(1/3)))

    X = X[:, :M]
    X_list = build_X_list(X)

    plt.figure(figsize=(8, 5))

    for ch, lab in zip(ch_list, labels):
        h = ch * T**(-0.2)
        _, _, w_hat = tvjma(Y, X_list, h)
        avg_w = w_hat.mean(axis=0)

        x = np.arange(1, M + 1)
        plt.plot(x, avg_w, linewidth=2, label=lab)

    plt.xlabel("Model index $j$", fontsize=12)
    plt.ylabel("Average weight", fontsize=12)

    plt.xticks(np.arange(1, M + 1))

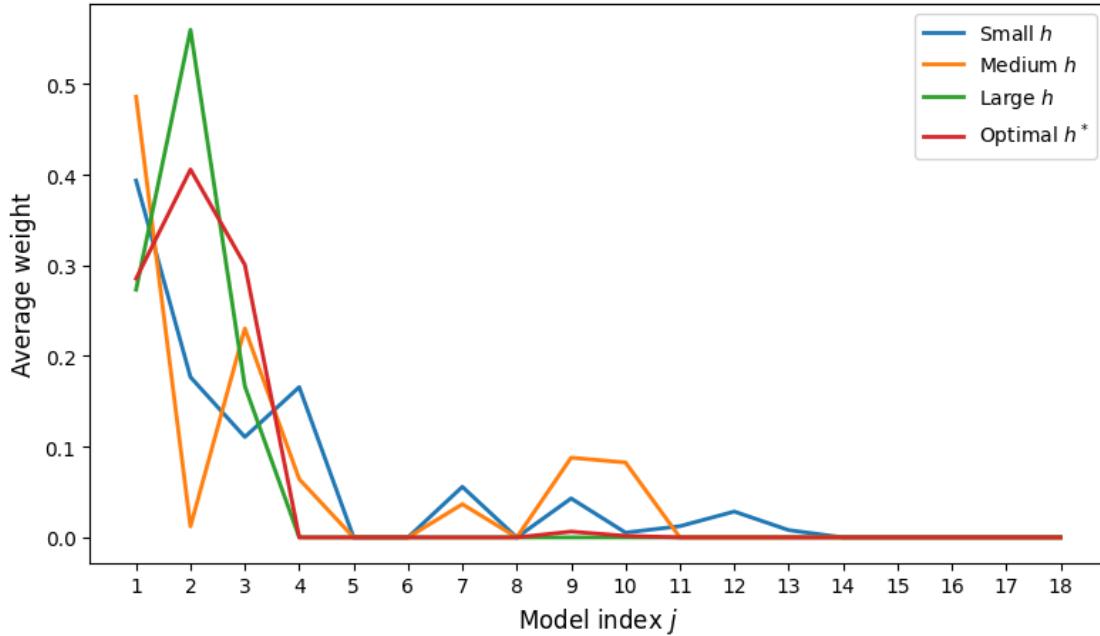
    plt.title(f"Average TVJMA Weights, {dgp_name}, $T={T}$, $R^2={R2}$",  

             fontsize=13)
    plt.legend()
    plt.tight_layout()
    plt.show()
```

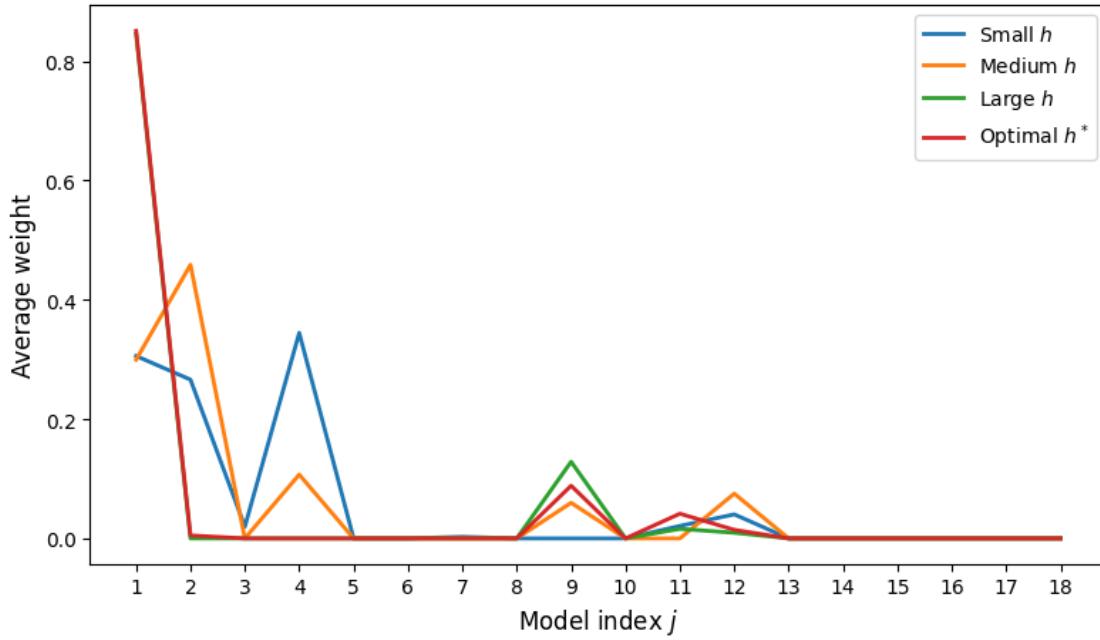
```
[118]: T = 200
R2 = 0.5

plot_avg_weights(T, R2, dgp_num=1, ch_list=ch_list, labels=labels)
plot_avg_weights(T, R2, dgp_num=2, ch_list=ch_list, labels=labels)
```

Average TVJMA Weights, DGP 1, $T = 200, R^2 = 0.5$



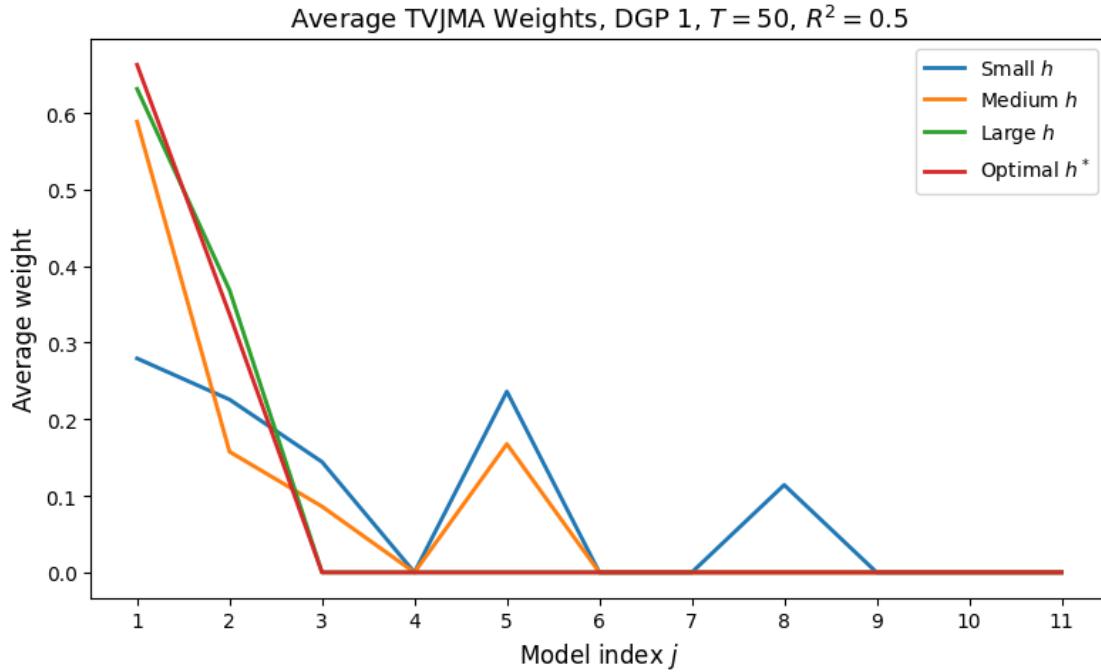
Average TVJMA Weights, DGP 2, $T = 200, R^2 = 0.5$

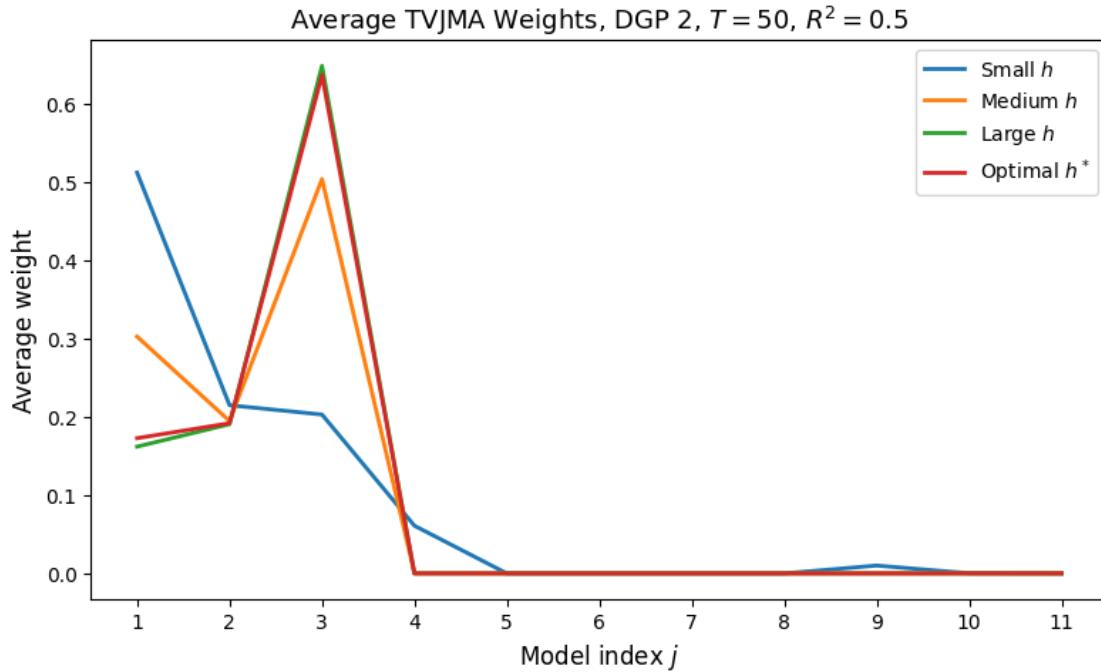


2.0.1 Appendix

```
[120]: T = 50  
R2 = 0.5
```

```
plot_avg_weights(T, R2, dgp_num=1, ch_list=ch_list, labels=labels)  
plot_avg_weights(T, R2, dgp_num=2, ch_list=ch_list, labels=labels)
```





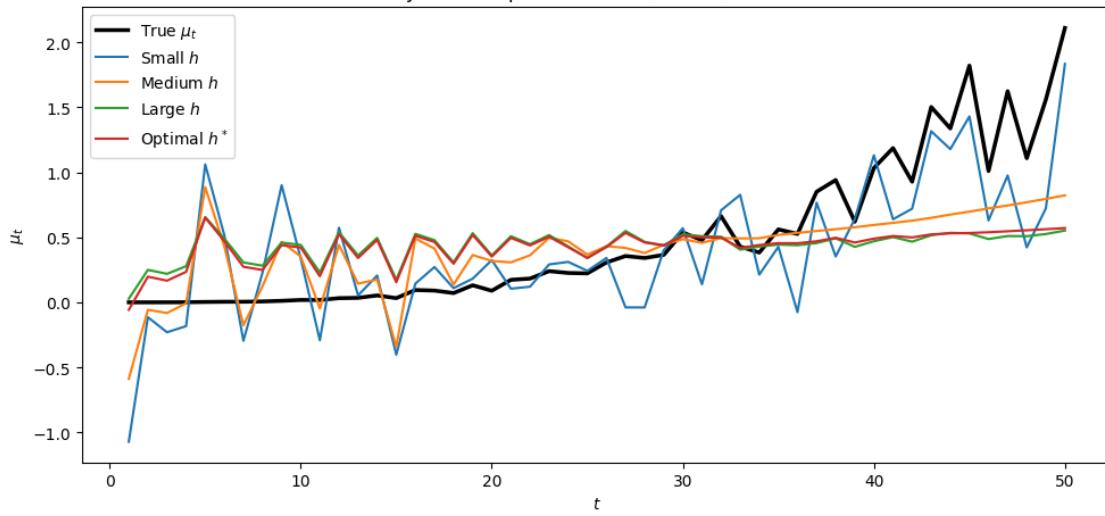
```
[97]: T = 50
R2 = 0.5

ch_list = [
    ch_test,      # corresponds to h 0.3
    ch_test2,     # corresponds to h 0.5
    ch_test3,     # corresponds to h 0.9
    ch_opt        # optimal bandwidth
]

labels = [
    "Small $h$",
    "Medium $h$",
    "Large $h$",
    "Optimal $h^*$"
]

plot_mu_paths(T, R2, dgp_num=1, ch_list=ch_list, labels=labels)
plot_mu_paths(T, R2, dgp_num=2, ch_list=ch_list, labels=labels)
```

TVJMA mean paths — DGP 1, $T = 50$, $R^2 = 0.5$



TVJMA mean paths — DGP 2, $T = 50$, $R^2 = 0.5$

