Mechanism Design in Social Networks

Presentation 1

Current strategies for Auctions

1. Classical Auction - Winner pays his bid, but information is passed to only a few of the possible bidders who are in contact with the seller i.e it is not incentive compatible. (Local Optimisation)

2. Network Auction(VCG mechanism) - Aim to make auctions more widespread and increase information flow among bidders so that maximum bid can be raised.

Terms Related to Network Auctions

1. Truthfulness - Reporting a bid equal to what the bidder actually thinks the product is worth (Not more nor less).
2. Feasible mechanism - A bidder cannot perform an action until he has received information about the auction. (Action set will be a null set.)
3. Efficient Allocation - Maximum value of sum of bids for available products is the bid(s) that win(s).

Definition 3. An allocation π is *efficient* if for all $\mathbf{a}' \in A$,

$$\pi \in \arg\max_{\pi' \in \Pi} \sum_{i \in N_{-s}, a'_i \neq null} \pi'_i(\mathbf{a}')v'_i$$

4. Utility in a network auction is defined as $u_i(a_i, \mathbf{a}', (\pi, p)) = \pi_i(\mathbf{a}')v_i - p_i(\mathbf{a}')$.

5. Individual Rationality - Utility is non negative if she truthfully reports her bid irrespective of who she tells or doesn't tell.

Definition 4. A mechanism (π, p) is *individually rational* (IR) if $u_i(a_i, ((v_i, r_i'), \mathbf{a}'_{-i}), (\pi, p)) \geq 0$ for all $i \in N_{-s}$, all $r_i' \in \mathcal{P}(r_i)$, and all $\mathbf{a}'_{-i} \in A_{-i}^{(v_i, r_i')}$.

6. Incentive Compatibility - Reporting her value truthfully and pssing on the information to all he neighbours is a dominant strategy.

7. Weakly budget balanced - Seller will never be in loss.

Definition 6. A mechanism $\mathcal{M} = (\pi, p)$ is *weakly budget balanced* if for all $\mathbf{a}' \in A$, $Rev^{\mathcal{M}}(\mathbf{a}') \geq 0$.

VCG Model

Basic Outline

Buyer with highest bid wins and pays the price of the second highest bid to the seller. Other buyers who are in the diffusion critical node of the winner will receive a reward.

Diffusion Critical Node - If all the paths of information flow to j have to pass through i then i is a diffusion critical node for the buyer j.

Reward - Pay received by the seller if buyer i participated in the network auction - Pay received by the seller if buyer i did not participate in the network auction.

Payment method in VCG

Payment for each buyer i is defined as

$$p_i^{vcg}(\mathbf{a}') = W(\mathbf{a}'_{-d_i}) - (W(\mathbf{a}') - \pi_i^*(\mathbf{a}')v_i') \qquad (1)$$

where $W(\mathbf{a}'_{-d_i}) = \sum_{j \in -d_i, a_j' \neq null} \pi_j^*(\mathbf{a}'_{-d_i})v_j'$, $W(\mathbf{a}') = \sum_{j \in N_{-s}, a_j' \neq null} \pi_j^*(\mathbf{a}')v_j'$, and π^* is an efficient allocation.

(Essentially social decrease in welfare in absence of participation of i.)

Desirable Properties Of VCG (with Proof)

1. Individual Rationality :

Buyers Utility under VCG is given by

(π^*, p^{vcg}) is $u_i(a_i, \mathbf{a}', (\pi^*, p^{vcg})) = \pi_i^*(\mathbf{a}')v_i - p_i^{vcg}(\mathbf{a}') = W(\mathbf{a}') + \pi_i^*(\mathbf{a}')(v_i - v_i') - W(\mathbf{a}'_{-d_i})$.

As W(a’) >= $W(a'_{-di})$, the utility is non negative when i plays its truthful value, hence it is individually rational.

2. Incentive Compatibility :

Will be proved by the following 2 points

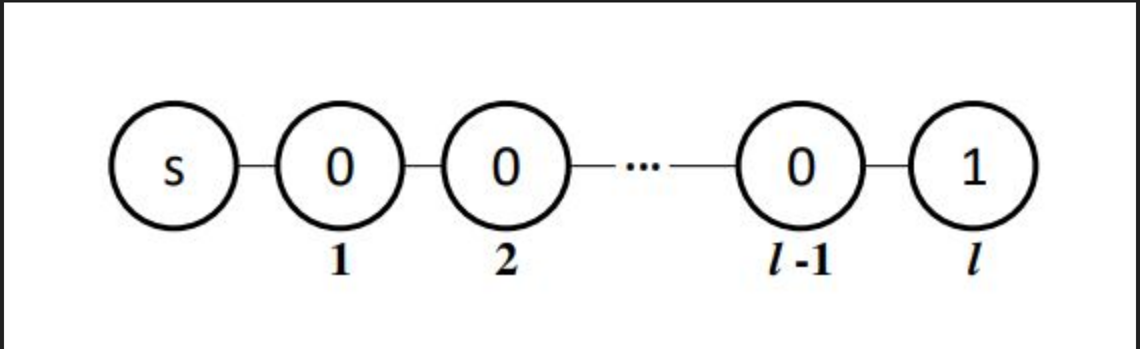
A. For each buyer i, by fixing r'_i , utility if i is maximised when i reports its true value - i’s payment does not depend on i’s valuation. I’s utility which turns out to be

$$(\pi^*, p^{vcg}) \text{ is } u_i(a_i, \mathbf{a}', (\pi^*, p^{vcg})) = \pi_i^*(\mathbf{a}')v_i - p_i^{vcg}(\mathbf{a}') = W(\mathbf{a}') + \pi_i^*(\mathbf{a}')(v_i - v_i') - W(\mathbf{a}'_{-d_i}).$$

Is maximised by reporting true value v_i

B. From the above property the middle term diffuses to 0 and W(a’) is maximised when the information is diffused to all the neighbors of buyer i , hence VCG is incentive compatible

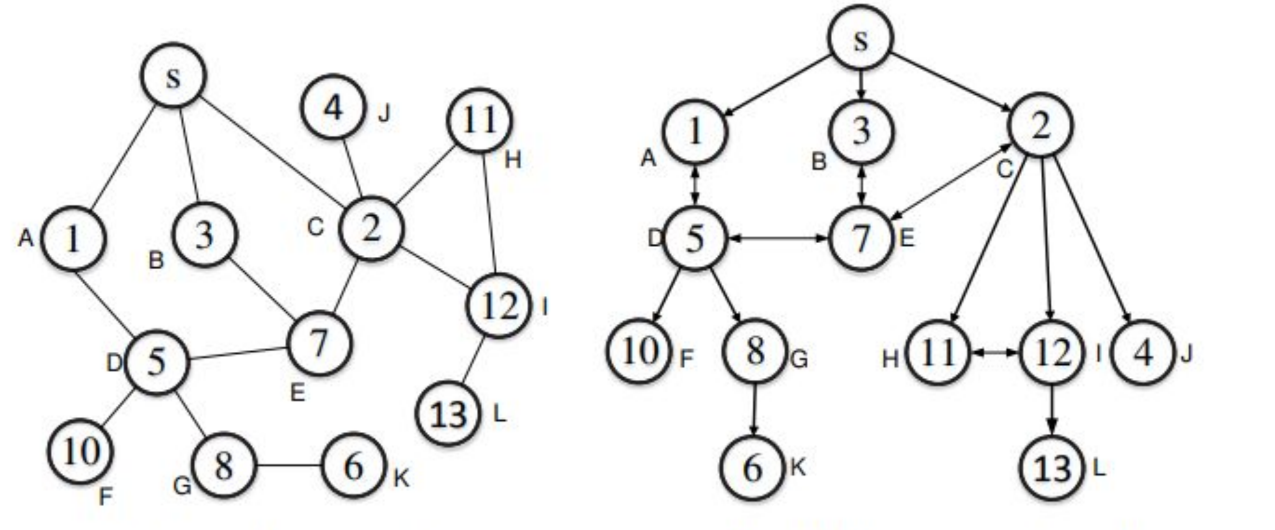
VCG Mechanism is not weakly budget balanced



In this case applying VCG, the revenue of the seller will be -(l-1) and the seller would prefer to have a classical auction where his revenue would've been 0.

Information Diffusion Mechanism

Diffusion critical Sequence - Ordered set of diffusion critical nodes.(Unique diffusion critical sequence for every auction)



(a) social network (b) diffusion network

C_m denotes the diffusion critical sequence of buyer m. If m is the winner then allocation in IDM is defined as If

$$\pi_i^{idm}(\mathbf{a}') = \begin{cases} 1 & \text{if } i \in C_m \setminus \{m\} \text{ and } v_i' = v^*_{-d_{i+1}}, \\ 1 & \text{if } i = m, \\ 0 & \text{otherwise.} \end{cases}$$

If there are multiple i's with allocation, then winner is i with the least index in C_m

Say w belonging to C_m wins the auction then the payments to the seller are defined as

$$p_i^{idm}(\mathbf{a}') = \begin{cases} v^*_{-d_i} - v^*_{-d_{i+1}} & \text{if } i \in C_w \setminus \{w\}, \\ v^*_{-d_i} & \text{if } i = w, \\ 0 & \text{otherwise.} \end{cases}$$

Types Of Buyers in IDM

1. *The winner*: w.
2. *On-path buyers*: all w's diffusion critical nodes, i.e., $C_w \setminus \{w\}$.
3. *Unlucky buyers*: if $w \neq m$, then all d_{w+1} are unlucky buyers, otherwise, all $d_m \setminus \{m\}$ are unlucky buyers.
4. *Normal buyers*: all buyers who are not classified in any of the other three status.

Some results

Lemma 2. *For all* $\mathbf{a}' \in A$*, all* $i \in N_{-s}$*,* $p_i^{idm}(\mathbf{a}')$ *is independent of* v_i'*.*

Lemma 3. *Given the buyers' action profile* $\mathbf{a}'$*, if* $i \notin C_w$ *under* $r_i' = r_i$*, then* $i \notin C_w$ *under any* $r_i' \neq r_i$*.*

Utilities of Different Players and Individual Rationality

Utility of unlucky buyers and normal buyers is 0 always.

Utility of on path buyers is -ve of their payment i.e $V^*_{-d(i+1)} - V^*_{d-i}$ which is always non negative.

Utility of winner is $V_i - V^*_{d-i}$ which is again non negative

Hence IDM is individually rational.

Incentive Compatibility

1. Unlucky buyers have a utility of 0 irrespective of what they report.
2. Normal buyers will have a utility of 0 as long as she is outside C_w, for her to change C_w she'd have to bid more than the winner but then her utility is V_i-V'_m,where V'_m is the original highest bid and hence utility becomes less than 0.
3. Utility of on path buyers is given by $V^*_{-d(i+1)}$- V^*_{d-i} which will remain the same if she bids anything less than $V^*_{-d(i+1)}$. If she bids higher than or equal to $V^*_{-d(i+1)}$, then she becomes the winner and her utility is V_i - V^*_{d-i}, as V_i is less than $V^*_{-d(i+1)}$, it is best for an on path buyer to report his true value.
4. For a winner the utility will remain same for anything she bids above true value and it degenerates to 0 if she bids less and becomes an unlucky buyer.

Proof that every buyer is never in loss if he passes on information to all her neighbours.

1. Unlucky or normal buyers will always have a utility of 0.
2. An on path buyer will retain her utility if she still passes on information in the diffusion critical sequence but if she doesn’t she might degenerate to the winner(case discussed in previous slide) or degenerate to a normal buyer where her utility is 0.
3. A winners utility will remain unaltered if he wins when he passes information to all his neighbours.

Therefore, we have $u_i(a_i, ((v_i, r_i'), \mathbf{a}'_{-i}), (\pi^{idm}, p^{idm})) \geq u_i(a_i, ((v_i', r_i'), \mathbf{a}'_{-i}), (\pi^{idm}, p^{idm}))$ for all i and all v_i'.

fected. Therefore, $u_i(a_i, ((v_i, r_i), \mathbf{a}'_{-i}), (\pi^{idm}, p^{idm})) \geq u_i(a_i, ((v_i, r_i'), \mathbf{a}''_{-i}), (\pi^{idm}, p^{idm}))$ for all i and all r_i'.

Putting together the above analysis, we get that $u_i(a_i, ((v_i, r_i), \mathbf{a}'_{-i}), (\pi^{idm}, p^{idm})) \geq u_i(a_i, ((v_i', r_i'), \mathbf{a}''_{-i}), (\pi^{idm}, p^{idm}))$ for all i, all v_i' and all r_i', i.e., IDM is incentive compatible. $\square$

Weakly Budget Balanced

All the terms in the summation of the payments cancel out except the first term(non negative) and hence we can see that the IDM mechanism is weakly budget balanced.

Proof. According to the payment policy, the seller's revenue in IDM is $\sum_{i \in C_w \setminus \{w\}} (v^*_{-d_i} - v^*_{-d_{i+1}}) + v^*_{-d_w} = v^*_{-d_1}$, where buyer 1 is the first buyer in C_w. Since $v^*_{-d_1}$ is non-negative, then IDM is weakly budget balanced.

VCG Vs IDM

Revenue of IDM is always greater than or equal to revenue of VCG.

$$\begin{aligned} Rev^{VCG} &= \sum_{i \in C_m} (W_{-d_i}(\mathbf{a'}_{-d_i}) - W_{N_{-s} \setminus \{i\}}(\mathbf{a'})) \\ &= \sum_{i \in C_m \setminus \{m\}} (v^*_{-d_i} - v^*_{N_{-s}}) + (v^*_{-d_m} - 0) \\ &= \sum_{i \in C_m \setminus \{1\}} (v^*_{-d_i} - v^*_{N_{-s}}) + v^*_{-d_1} \\ &\leq v^*_{-d_1} = Rev^{IDM} \end{aligned}$$

THANK YOU