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Real-time Thrust Optimization for Multiple Wingsails using Extremum Seeking

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Outline

Introduction

Oceanbird Prototype [2]

Related Work

- Research on real-time control of sail-assisted ships is still in its early days
- Focus on small scale single-sail autonomous sailboats
- Two distinct approaches: Model-based *vs* Model-free
- Extremum Seeking:
  - Common model-free approach for similar applications
  - Many extensions of the basic method

Related Work

- Lack of work on multivariate Extremum-Seeking for sailing applications
- Multivariate case is more complex:
  - Nonuniform wind flow
  - Interactions between sail
- Newton-based extremum seeking
  - User-assignable convergence

Research Objectives & Problem Formulation

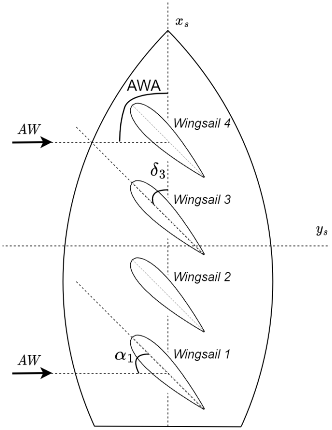
Research Objectives:

- Implement multivariate extremum seeking for wingsail thrust optimization
- Compare Gradient-based with Newton-based extremum seeking

Problem Formulation:

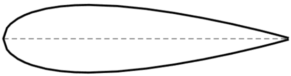
- *Given a time-variant wind angle, find the optimal sheeting angle of each wingsail that maximizes the global thrust.*
- Assumption: *The optimization criterion, the global thrust, is unknown but measurable.*

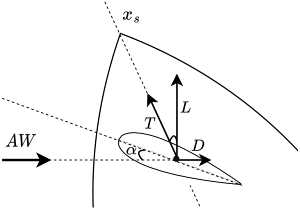
Notation



- AW : Apparent wind vector
  - AWA : apparent wind angle, measured from the ship x-axis, x_s
- α_i : Angle of attack of wingsail i
- δ_i : Sheeting angle of wingsail i

Methods - Wingsail





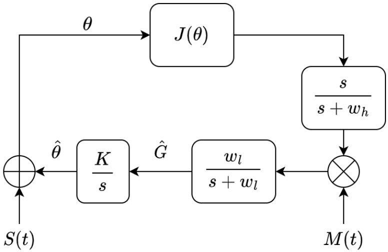
$$L_i = \frac{1}{2}\rho\ (\mathrm{AWS})^2\ c_L(\alpha_i)$$
$$D_i = \frac{1}{2}\rho\ (\mathrm{AWS})^2\ c_D(\alpha_i)$$

$$\begin{aligned} T_i &= L_i \sin(\mathrm{AWA}) - D_i \cos(\mathrm{AWA}) \\ &= \frac{1}{2}\rho\ (\mathrm{AWS})^2\ c_T(\alpha_i) \end{aligned}$$

Assumptions:

- Ship is stationary
- Stall problem is not addressed

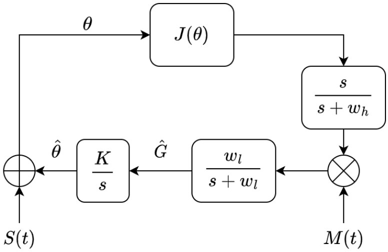
Methods - Gradient-based Extremum Seeking



$$S(t) = \begin{bmatrix} a_1 \sin(w_1 t) & \dots & a_n \sin(w_n t) \end{bmatrix}$$
$$M(t) = \begin{bmatrix} \frac{2}{a_1} \sin(w_1 t) & \dots & \frac{2}{a_n} \sin(w_n t) \end{bmatrix}$$

$$\dot{\tilde{\theta}} = -KH\tilde{\theta}, \quad \tilde{\theta} = \hat{\theta} - \theta^*$$

Methods - Gradient-based Extremum Seeking



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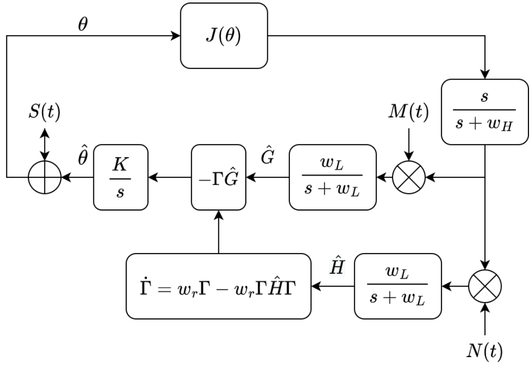
$$\dot{\tilde{\theta}} = -K{\color{red}H}\tilde{\theta}, \quad \tilde{\theta} = \hat{\theta} - \theta^*$$

Unknown convergence rate of $\tilde{\theta}$ to 0

Methods - Newton-based Extremum Seeking [1]



$$N_{i,i}(t) = \frac{16}{a_i}\left(\sin(w_i t)^2 - \frac{1}{2}\right)^2$$

$$N_{i,j}(t) = \frac{4}{a_i a_j}\sin(w_i t)\sin(w_j t)$$

Methods - Newton-based Extremum Seeking

Hessian is estimated as $\hat{H} = N(t)y$. To avoid numerical singularities, its inverse is estimated using the filter:

$$\dot{\Gamma} = w_r \Gamma - w_r \Gamma \hat{H} \Gamma, \; \Gamma^* = \hat{H}^{-1}$$

Linearization around the equilibrium point, assuming $\hat{H} \approx H$:

$$\dot{\tilde{\theta}} = -K\tilde{\theta}$$

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User-assignable convergence rate of $\tilde{\theta}$ to 0

Methods - Thrust Optimization

- Optimization criterion: global thrust

$$J = \sum_{1}^{n} T_i$$

- Inputs: sheeting angles

$$\theta = \left[\begin{array}{ccc} \delta_1 & ... & \delta_n \end{array}\right]$$

- Tuning parameters:
  - $\{a_i, f_i\}$: Perturbation signal
  - $\{w_l, w_h\}$: LP/HP Filters cut-off frequency
  - K : integrator gain
  - w_r : Hessian filter convergence rate

Methods - Extremum Seeking Tuning

Perturbation frequency, f_i:

- Physical system properties:
  - Frequency of c_T measurements, $f_s = 10$ Hz
  - Wingsail aerodynamics, $\tau = 0.1$s
  - Sail rotating rate ≈ 9 deg/s
- Extremum seeking algorithm constraints:
  - $\frac{1}{w_i} >> \tau$
  - $w_i \neq \beta w_j, \quad \beta \in \{1, 2, 3\}, \quad \forall i, j$ distinct
  - $w_i \neq \frac{1}{2}(w_j \pm w_k), \quad \forall i, j, k$ distinct
  - $w_i \neq (w_j + w_k \pm w_l), \quad \forall i, j, k, l$ distinct
- Performance maximization
  - Fastest sail should be the most disturbed
  - $\hat{H} \approx H$ over $\Pi = LCM\left\{\frac{1}{f_i}\right\}$

Methods - Extremum Seeking Tuning

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- Performance maximization
  - Fastest sail should be the most disturbed
  - $\hat{H} \approx H$ over $\Pi = LCM \left\{ \frac{1}{f_i} \right\}$

Perturbation frequency upper bound: $\bar{f}_i = 0.2$ Hz

Methods - Extremum Seeking Tuning

Perturbation amplitude, a_i:

- Large enough to be distinguished from equal-frequency AWA variations
- Small enough to avoid loss of performance

Low-Pass cut-off frequency, w_l:

- $w_l << \min\{\underbrace{|w_i - w_j|}_{\hat{G}}, \underbrace{w_i, |w_i - w_j - w_k|, |2w_i - w_j|}_{\hat{H}}\},$

  $\forall i \neq j \neq k$

Results - Test setup

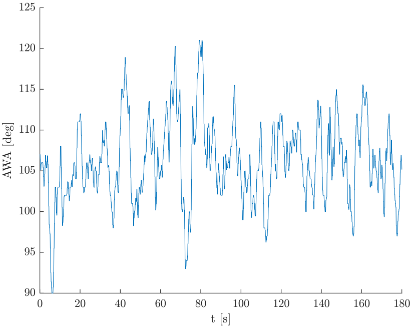
2 wingsails:

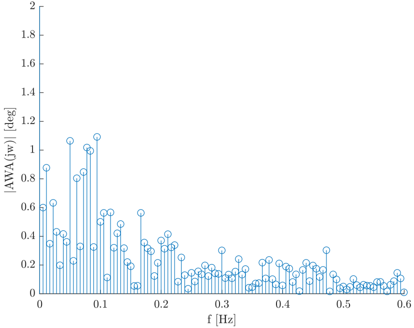
- Simulated AWA
- Real AWA measurement datasets
- Filtered AWA measurement datasets

4 wingsails:

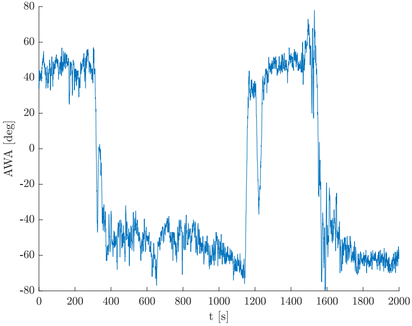
- Simulated AWA
- Filtered AWA measurement datasets

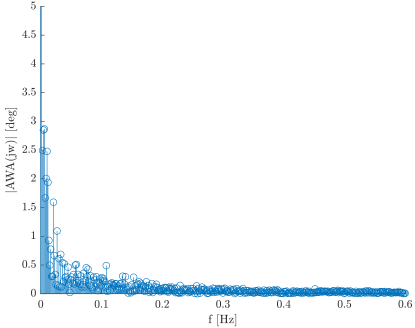
Results - Measurement datasets



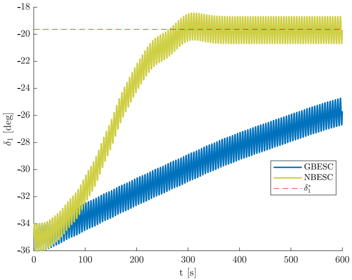


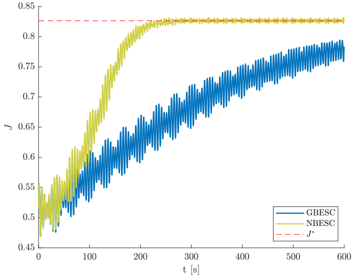
Results - Measurement datasets

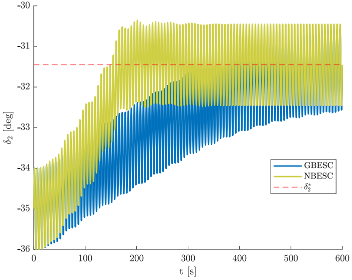


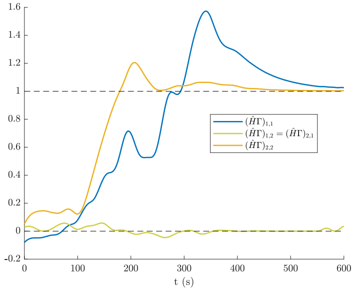


Results - 2D | Simulated AWA

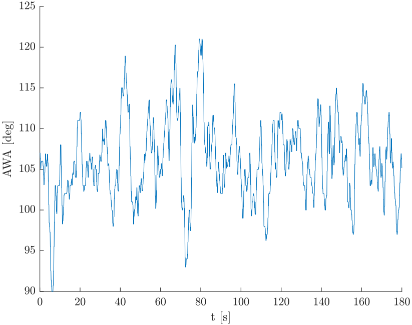








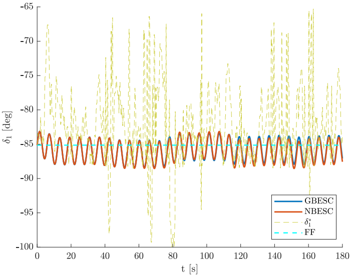
Results - 2D | Measured AWA at constant heading

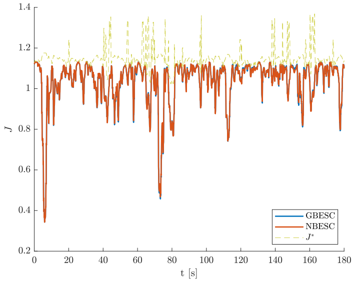


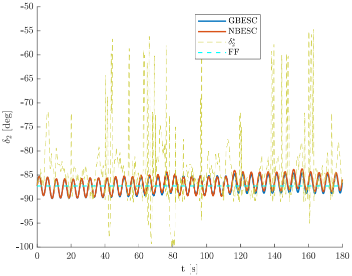
Tuning:

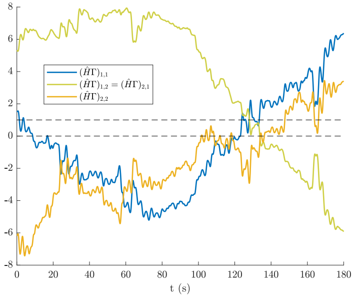
- Output LPF, $w_l >> \max\{w_i\}$
- $a_i = 2^\circ, \forall i$
- $f_i = \begin{bmatrix} 0.17 & 0.2 \end{bmatrix}$ Hz
- $K_g = -K_n \Gamma(0)$
- $\Gamma(0) = \gamma_0 \mathbf{I}$, $\gamma_0 \in \mathbb{R}$
- K_n, w_r **small**

Results - 2D | Measured AWA at constant heading





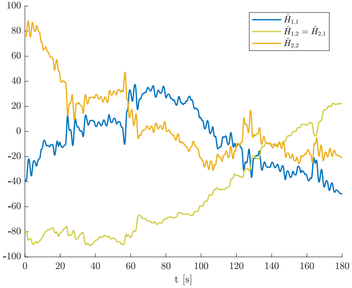


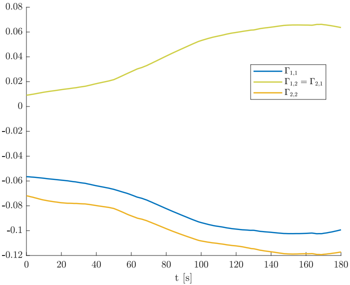


Results - 2D | Measured AWA at constant heading

Recall:

- $\hat{H} \approx H$ in average over Π, but H is not constant over Π
- $\Gamma^* = \hat{H}^{-1}$ is only locally stable



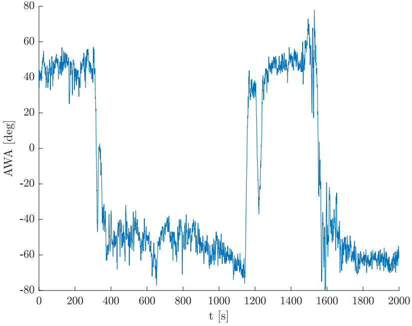


Results - 2D | Measured AWA at constant heading

Conclusions:

- Low integrator gain K to avoid instability
- Newton-based extremum seeking is less robust
  - $\hat{H}$ is not guaranteed to stay in a small neighborhood of H
- Extremum seeking trade-off between disturbance rejection and convergence rate

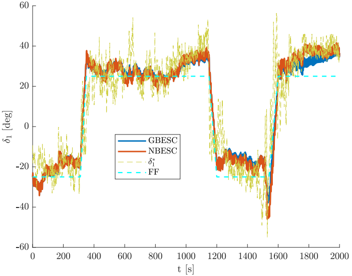
Results - 2D | Measured AWA while tacking

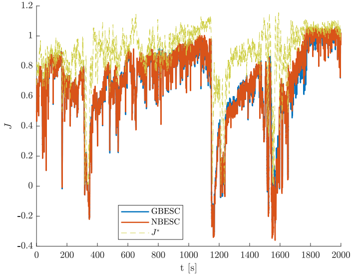


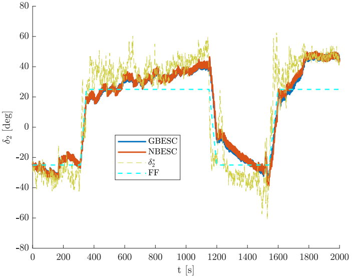
Tuning:

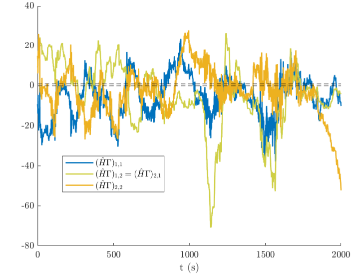
- Output LPF, $w_l >> \max\{w_i\}$
- $a_i = 2^\circ, \forall i$
- $f_i = \left[\begin{array}{cc} 0.2 & 0.17 \end{array}\right]$ Hz
- $K_g = -K_n \Gamma(0)$
- $\Gamma(0) = \gamma_0 \mathbf{I}, \gamma_0 \in \mathbb{R}$
- K_n, w_r **small**
- Piecewise linear FF

Results - 2D | Measured AWA while tacking









Results - 2D | Measured AWA while tacking

KPIs

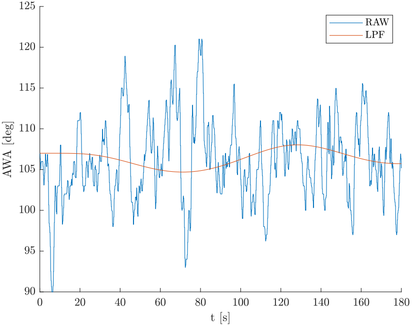
	cT	MAE δ_1 [deg]	MAE δ_2 [deg]
GBESC	13542	7.5504	9.9924
NBESC	13562	7.3912	9.9294
Optimal	17759	-	-

Results - 2D | Measured AWA while tacking

Conclusions:

- Newton-based extremum seeking is not robust
- Gradient-based extremum seeking is moderately accurate
- “Rough” FF is enough

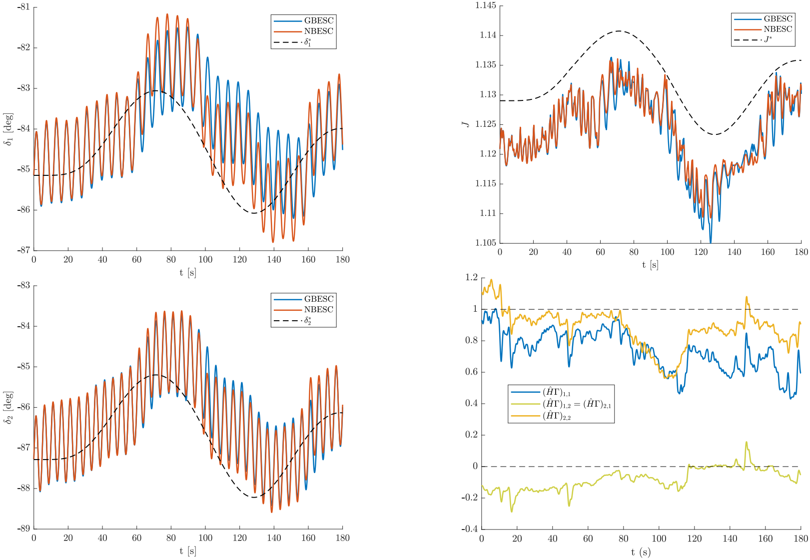
Results - 2D | Filtered AWA at constant heading



Tuning:

- $a_i = 1°, \forall i$
- $f_i = \begin{bmatrix} 0.17 & 0.2 \end{bmatrix}$ Hz
- $K_g = -K_n \Gamma(0)$
- w_r **large**
- $\Gamma(0) = \gamma_0 \mathbf{I}$, $\gamma_0 \in \mathbb{R}$

Results - 2D | Filtered AWA at constant heading

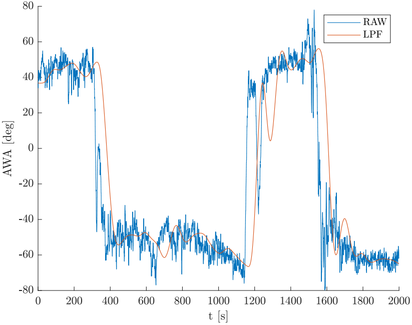


Results - 2D | Filtered AWA at constant heading

KPIs

	cT	MAE δ_1 [deg]	MAE δ_2 [deg]
GBESC	2021	0.9214	0.8379
NBESC	2025	0.8265	0.7958
Optimal	2039	-	-

Results - 2D | Filtered AWA while tacking



Tuning:

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- $K_g = -K_n\Gamma(0)$
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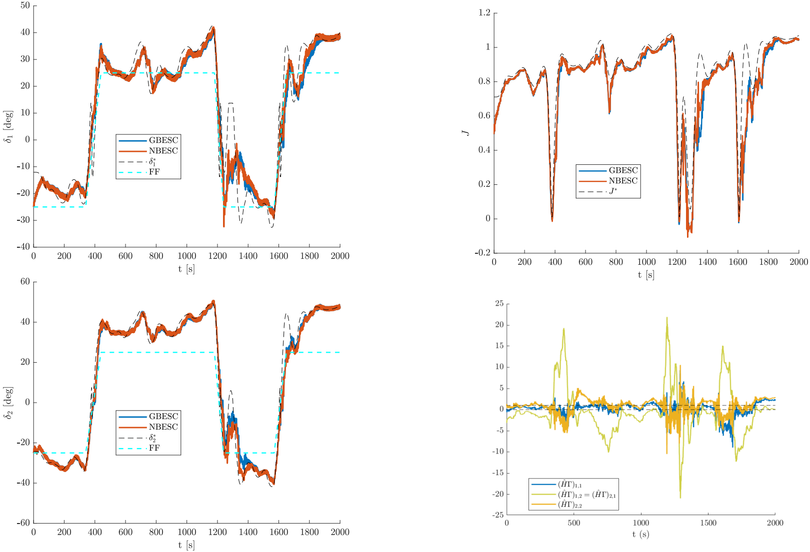
- K_n, w_r **small**
- Piecewise Linear FF

Results - 2D | Filtered AWA while tacking



Results - 2D | Filtered AWA while tacking

KPIs

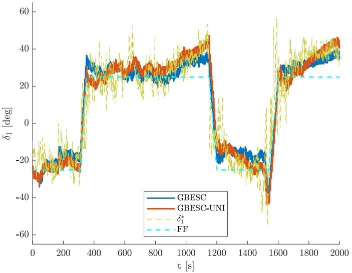
	cT	MAE δ_1 [deg]	MAE δ_2 [deg]
GBESC	15891	4.9480	3.4136
NBESC	16000	4.6152	3.2871
Optimal	17323	-	-

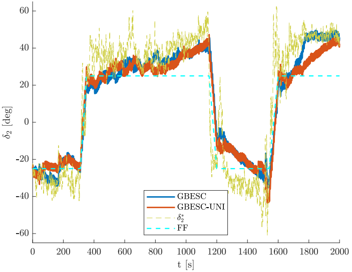
Results - 2D | Filtered AWA

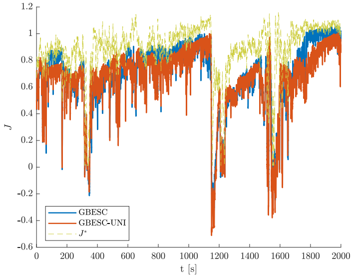
Conclusions:

- Extremum seeking performs better for less disturbed AWA
- Newton-based extremum seeking does not improve performance significantly
  - Particularly poor performance when maneuvering
- Gradient-based extremum seeking is accurate

Results - 2D | Multivariate vs univariate





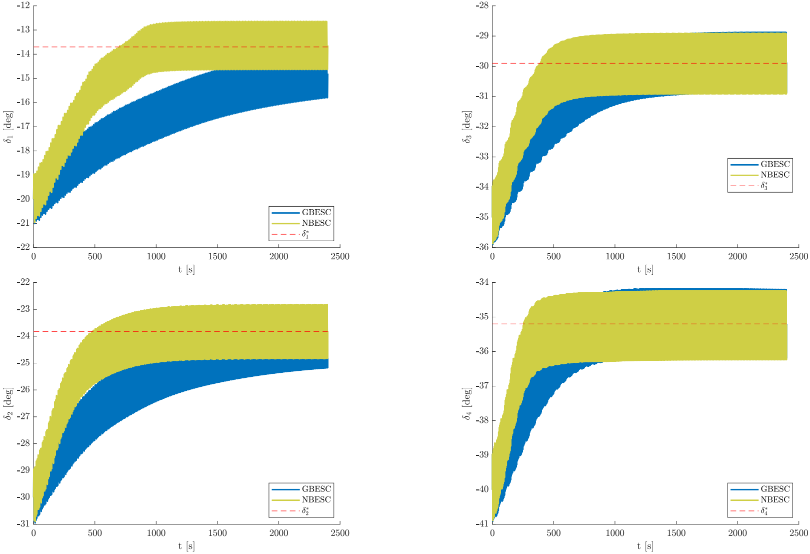


Results - 2D | Multivariate vs univariate

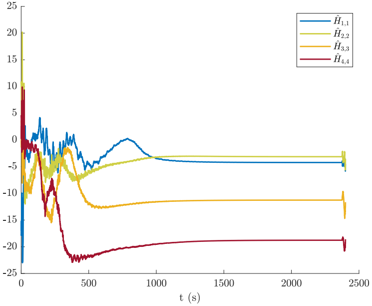
KPIs

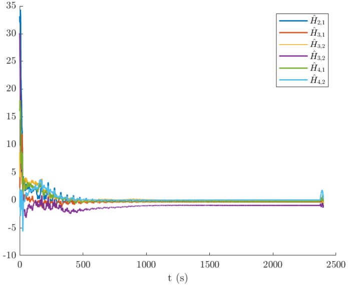
	cT	MAE δ_1 [deg]	MAE δ_2 [deg]
GBESC	13542	7.5504	9.9924
GBESC-UNI	13086	7.2880	11.1778
Optimal	17759	-	-

Results - 4D | Simulated AWA

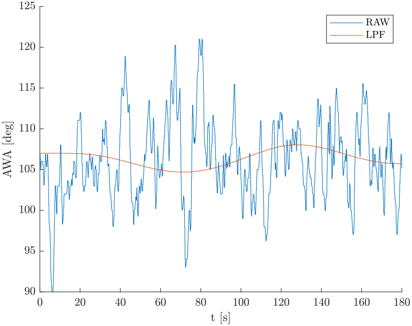


Results - 4D | Simulated AWA





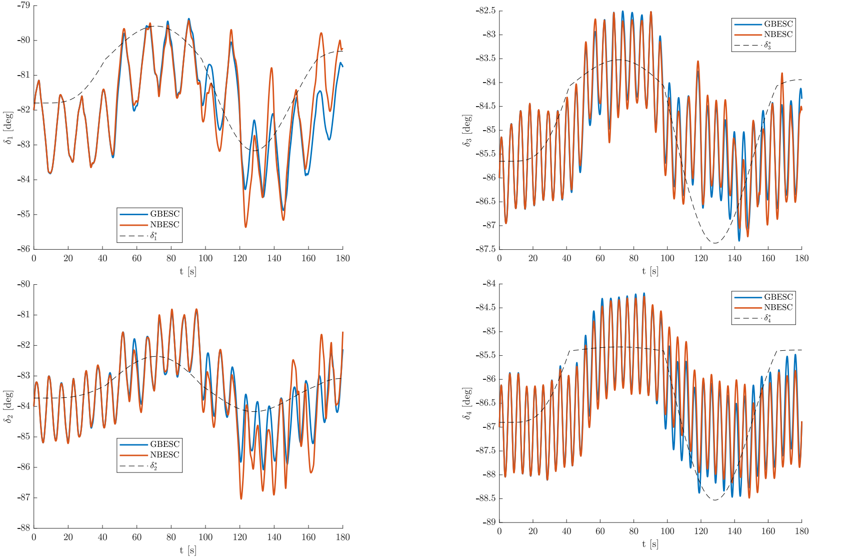
Results - 4D | Filtered AWA at constant heading



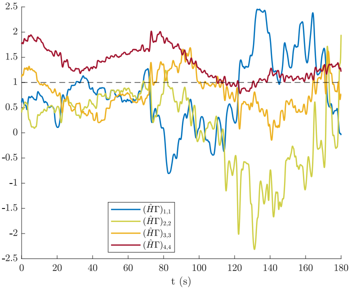
Tuning:

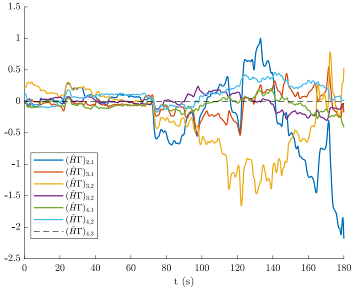
- $a_i = 1^\circ, \forall i$
- $f_i = \begin{bmatrix} 0.08 & 0.14 & 0.17 & 0.2 \end{bmatrix}$ Hz
- $K_g = -K_n \Gamma(0)$
- $\Gamma(0) = \gamma_0 \mathbf{I}$, $\gamma_0 \in \mathbb{R}$
- w_r **large**

Results - 4D | Filtered AWA at constant heading



Results - 4D | Filtered AWA at constant heading



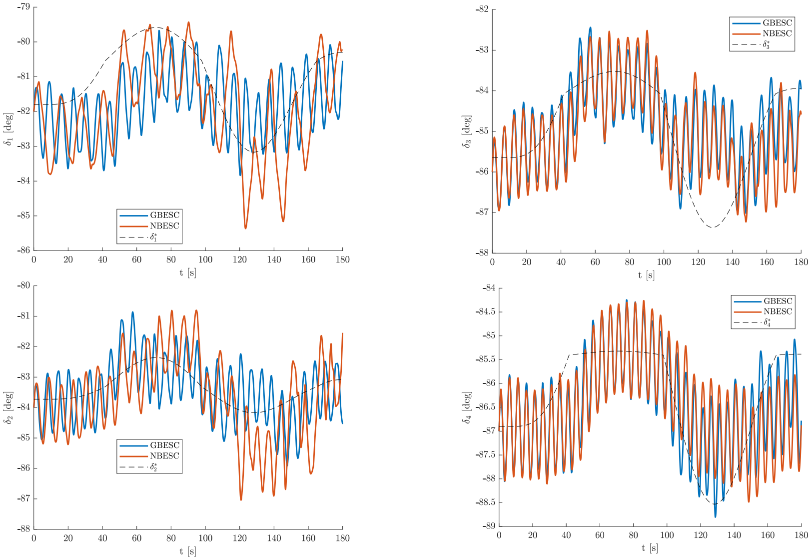


Results - 4D | Filtered AWA

Conclusions:

- Newton-based extremum seeking does not improve performance
  - Frequency selection is too restrictive
- Poor tracking due to low f_i upper bound

Results - 4D | Filtered AWA at constant heading



Results - 4D | Filtered AWA

Conclusions:

- Gradient-based higher f_i allow for higher gains
- Frequency selection is too conservative

Conclusions

- Gradient-based extremum seeking is accurate to track optima or average value of optima depending on AWA variation
- Newton-based extremum seeking is not robust to high-frequency AWA variations, and thus is not able to improve performance
- Conservative frequency selection led to couplings between output variations due to AWA and δ.
  - Main improvement is in frequency increase
- Multivariate control performed better than univariate one

Future work

- Finish tuning and simulating 4D configuration for measurement datasets
- Simulate with ship dynamics and kinematics
  - Rolling period
  - Drag increase
  - Waves disturbances
- More informative FF
- Wind disturbance rejection
- Adaptive gain
- Other Extremum Seeking derivations
  - Sliding mode control

References

[1] Azad Ghaffari, Miroslav Krstić, and Dragan Nešić. "Multivariable Newton-based extremum seeking". In: *Automatica* 48.8 (2012), pp. 1759–1767.

[2] Wallenius Marine. *From drawing board to Oceanbird*. 2021. URL: www.walleniusmarine.com/blog/ship-design-newbulding/from-drawing-board-to-oceanbird/ (visited on 07/12/2022).