- Характеристики случайных величин

$M[x]$ — матожидание случ. дискр. велич. $= \sum_{i=1}^{\infty} x_i p_i$

непрерывн велич. $= \int_{-\infty}^{\infty} x f(x) dx$

1. $M[c] = c$
2. $M[cX] = c \cdot M[x]$
3. $M[x+y] = M[x] + M[y]$
4. $M[xy] = M[x] M[y]$, но X и Y - независимые $(A = \{X = x_i\} \; B = \{Y = y_i\} \; \forall i)$

$D[x]$ — дисперсия — мат. ожидание квадрата отклонения

$= M[(X - m_x)^2] = M[X^2] - m_x^2$ при $M_x = M[x]$

дискр: $D[x] = \sum_{i=1}^{\infty} (x_i - m_x)^2 p_i$

непрерывн: $D[x] = \int_{-\infty}^{\infty} (x - m_x)^2 f(x) dx$

1. $D[c] = 0$
2. $D[cX] = c^2 D[x]$
3. $D[X+Y] = D[X] + D[Y]$, если X и Y - независимые

$M[X^m] = \alpha_m$ — m-ый момент мат. ожидание X

$M[(X - m_x)^m] = \mu_m$ — центральный момент мат. ожид. X

$D[x] = \alpha_2 - \alpha_1^2 \quad (D[x] = M[x^2] - M^2[x])$

$\varphi(z) = \sum_{k=0}^{\infty} p_k z^{x_k}$ — производящая функция

$\varphi(1) = \sum_{k=0}^{n} p_k = 1$

$\varphi'(1) = \sum_{k=0}^{\infty} x_k p_k = M[x] = \alpha_1$

$\varphi''(1) = \sum_{k=0}^{\infty} (x_k^2 - x_k) p_k$

$D[x] = \varphi''(1) + \varphi'(1) - (\varphi'(1))^2$

$P\{X = m\} = q^{m-1} p \quad (X = 1, 2, \dots)$ — геометрич. распределение (дискретное)

$\varphi(z) = \sum_{m=1}^{\infty} p_m z^{x_m} = \sum_{m=1}^{\infty} q^{m-1} p z^m = pz \sum_{m=1}^{\infty} (qz)^{m-1} = \frac{pz}{1-qz}$

$\varphi(1) = \sum_{m=1}^{\infty} q^{m-1} p = p \sum_{m=1}^{\infty} q^{m-1} = p \sum_{k=0}^{\infty} q^k = p \frac{1}{1-q} = \frac{p}{p} = 1$

$\varphi'(z) = \frac{p}{(1-qz)^2}(-q) = \frac{p}{1-qz} + \frac{pqz}{(1-qz)^2} = \frac{p - pqz + pqz}{(1-qz)^2} = \frac{p}{(1-qz)^2}$

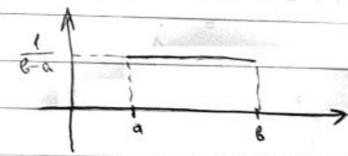
$\varphi''(z) = \frac{2pq}{(1-qz)^3}$

$D[x] = \varphi''(1) + \varphi'(1) - [\varphi'(1)]^2 = \frac{2pq}{(1-q)^3} + \frac{p}{(1-q)^2} - \frac{p^2}{(1-q)^4} =$

$= \frac{2pq}{p^3} + \frac{p}{p^2} - \frac{p^2}{p^4} = \frac{2q + p - 1}{p^2} = \frac{q}{p^2}$

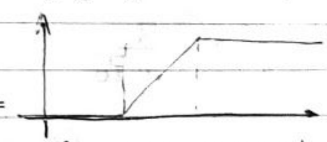
$\sigma[x] = \sqrt{\frac{q}{p^2}} = \frac{\sqrt{q}}{p}$

$f(x) = \begin{cases} \frac{1}{b-a}, & x \in [a, b] \\ 0, & x \notin [a, b] \end{cases} \qquad b > a$



$\int_{-\infty}^{\infty} f(x) dx = 1$

равномерное распределение (функция плотности)



(функция распред.)

$M[X] = \int_{-\infty}^{\infty} x f(x) dx = \int_a^b x \frac{1}{b-a} dx = \frac{1}{b-a} \frac{x^2}{2} \Big|_a^b =$

$= \frac{1}{2} \frac{b^2 - a^2}{b - a} = \frac{1}{2}(a + b)$

$M[x^2] = \int_{-\infty}^{\infty} x^2 f(x) dx = \int_a^b x^2 \frac{1}{b-a} dx = \frac{1}{3} \frac{x^3}{(b-a)} \Big|_a^b = \frac{b^3 - a^3}{3(b-a)} =$

$= \frac{(b-a)(a^2 + ab + b^2)}{3(b-a)} = \frac{1}{3}(a^2 + ab + b^2)$

$D[x] = M[x^2] - M^2[x] = \frac{1}{3}(a^2 + ab + b^2) - \frac{1}{4}(a+b)^2 =$

$= \frac{1}{12}(4a^2 + 4ab + 4b^2 - 3a^2 - 6ab - 3b^2) = \frac{1}{12}(b-a)^2$

$$\sigma = \frac{1}{2\sqrt{3}}|b-a|$$

Экспоненциальное распределение:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$M[x] = \int_0^{\infty} x\lambda e^{-\lambda x}dx = \frac{1}{\lambda} \qquad \left(\int_0^{\infty} x e^{-x} dx\right)$$

$$\Downarrow$$

$$M[x^2] = \int_0^{\infty} x^2\lambda e^{-\lambda x}dx = \qquad \left(\int_0^{\infty} e^{-x}dx = 1\right)$$

$$= \left|\begin{array}{l} \lambda x = \tau \\ x = \tau/\lambda \\ dx = \frac{1}{\lambda}d\tau \end{array}\right| = \left.\begin{array}{l} \left(\Gamma(z) = \int_0^{\infty} x^{z-1}e^{-x}dx\right. \\ \left(\Gamma(z+1) = z\Gamma(z)\right. \\ \left(\Gamma(n+1) = n!\right. \end{array}\right\} \text{гамма-функция}$$

$$= \int_0^{\infty} \frac{\tau^2}{x^2}\lambda e^{-\tau}\frac{d\tau}{\lambda} =$$

$$= \frac{1}{\lambda^2}\int_0^{\infty} \tau^2 e^{-\tau}d\tau = \frac{2}{\lambda^2}$$

$$\sigma[x] = \frac{1}{\lambda}$$

Нормальное распределение

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-a)^2/2\sigma^2} \qquad N(a, \sigma)$$

$$M[x] = \int_{-\infty}^{\infty} x f(x)dx = \int_{-\infty}^{\infty} x\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-a)^2/2\sigma^2} \; |x-a=t| =$$

$$= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty}(a+t)e^{-t^2/2\sigma^2} = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} a e^{-t^2/2\sigma} + \frac{1}{\sqrt{2\pi}\sigma}\cdot$$

$$\cdot\int_{-\infty}^{\infty} t e^{-t^2/2\sigma^2}dt = \frac{a}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} e^{-t^2/2\sigma^2} = \frac{a}{\sqrt{2\pi}\sigma}\cdot\sqrt{2\pi}\sigma = a$$

$$M[x] = a$$

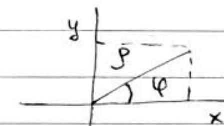
Решение интеграла:

$$\int_{-\infty}^{\infty} e^{-t^2/2\sigma^2}dt = \left|\begin{array}{l}\frac{t}{\sqrt{2}\sigma} = \tau \\ dt = \sqrt{2}\sigma d\tau\end{array}\right| = \sqrt{2}\sigma\int_{-\infty}^{\infty} e^{-\tau^2}d\tau = \sqrt{2\pi}\sigma$$

$$\boxed{\int_{-\infty}^{\infty} e^{-\tau^2}d\tau = \sqrt{\pi}}$$

$$I = \int_{-\infty}^{\infty} e^{-\tau^2}d\tau \qquad I^2 = \left(\int_{-\infty}^{\infty} e^{-\tau^2}d\tau\right)^2 = \int_{-\infty}^{\infty} e^{-x^2}dx \cdot \int_{-\infty}^{\infty} e^{-y^2}dy =$$

$$= \iint e^{-(x^2+y^2)}dxdy =$$



$$dxdy = \rho d\rho d\varphi$$

$$= \int_0^{\infty} d\rho\,\rho \int_0^{2\pi} d\varphi\, e^{-\rho^2} = 2\pi\int_0^{\infty} \rho e^{-\rho^2}d\rho =$$

$$= \frac{2\pi}{2}\cdot\int_0^{\infty} e^{-\xi}d\xi = \pi$$

$$I^2 = \pi$$

$$I = \sqrt{\pi}$$

$$D[x] = \int_{-\infty}^{\infty}(x-a)^2\frac{1}{\sqrt{2\pi}}\sigma e^{-(x-a)^2/2\sigma^2} \cdot Ax = \left|\begin{array}{l}\frac{x-a}{\sqrt{2}\sigma} = t \\ dx = \sqrt{2}\sigma dt\end{array}\right| =$$

$$= \frac{1}{\sqrt{2\pi}\sigma}2\sigma^2\sqrt{2}\sigma\cdot\int_{-\infty}^{\infty} t^2e^{-t^2}dt = \frac{2\sigma^2}{\sqrt{\pi}}\int_{-\infty}^{\infty} t^2e^{-t^2}dt$$

$$\int_{-\infty}^{\infty} t^2e^{-t^2}dt = \int_{-\infty}^{\infty} \underbrace{t}_{u}\,\underbrace{te^{-t^2}dt}_{dv} = \left|\begin{array}{l} dv = te^{-t^2}dt \\ v = \int te^{-t^2}dt = -\frac{1}{2}e^{-t^2}\end{array}\right|$$

$$= -\frac{t}{2}e^{-t^2}\Big|_{-\infty}^{\infty} + \frac{1}{2}\int_{-\infty}^{\infty} e^{-t^2}dt = \frac{\sqrt{\pi}}{2} = \ldots = \sigma^2$$

$$\left(\int u\,dv = uv - \int v\,du\right) \qquad D[x] = \sigma^2 \qquad \sigma[x] = \sigma$$

$$A = 0 \quad E = 0$$