PERMUTATION, COMBINATION AND PROBABILITY

Concepts

- In general, if a_1 can be done in p_1 ways, a_2 can be done in p_2 ways,... and a_n can be done in p_n ways, then the events a_1, a_2, a_3 ...and a_n can be done in $(p_1 \times p_2 \dots p_n)$ ways.
- Number of ways of arranging **n items** in **r places** is given by $^nP_r = n! / (n - r)!$
- The total number of permutations of **n** different things taken all at a time = **n!**
- The total number of arrangements of **n** different things taken **r** at a time, in which a particular thing always occurs = $r \times {}^{n-1}P_{r-1}$
- The total number of permutations of **n** different things taken **r** at a time in which a particular thing never occurs = $^{n-1}P_r$
- The total number of permutation of **n** dissimilar things taken **r** at a time with repetitions = n^r
- The number of permutations of **n** things taken all at a time when **p** of them are alike and of one kind, **q** of them are alike and of second kind, all other being different, is $n! / (p! \times q!)$
- Number of ways of selecting **r** items out of n items is given by $^nC_r = {}^nP_r / r! = n! / r! (n - r)!$
- Number of combinations of **n** different things taken **r** at a time in which **p** particular things will always occur is $^{(n-p)}C_{r-p}$
- Number of combinations of **n** different things taken **r** at a time in which **p** particular things will never occur is $^{(n-p)}C_r$
- $^nC_0 + {}^nC_1 + {}^nC_2 + \dots {}^nC_n = 2^n$
- The number of ways in which **(m + n)** things can be divided into two groups containing **m & n** things respectively is $(m + n)! / (m! \times n!)$
- If E & F are two mutually exclusive events, then the probability that either event E or event F will occur in a single trial is given by **P(E or F) = P(E) + P(F)**
- If the event are not mutually exclusive, then **P(E or F) = P(E) + P(F)– P(E & F together)**
- If the events E & F are independent then **P(E & F) = P(E) × P(F)**
- If **p** is the probability that an event will occur and **q = (1 − p)** is the probability of the non-occurrence of the event, then we say that the **odds in favour** of the event occurring are **p : q** and the **odds against** its occurring are **q : p.**

Drill

1. A shopping mall has 3 distinct glass doors and 2 distinct metal doors for entry and has 5 distinct glass doors and a wooden door for exit.
   In how many ways can you enter the mall?
   In how many ways can you leave the mall?
   In total, how many ways can you enter and leave the mall?
2. If there are three trains from A to B and 5 trains from B to C, in how many ways can one travel from A to C by train (assume there are no direct trains from A to C)? _____
3. The number of ways a cricket team of 11 can be selected from a 16 – member squad. P/C
4. The number of ways 7 dignitaries can seat themselves in seven chairs kept on the stage. P/C
5. The number of ways a panel of 4 judges can be formed from 6 retired judges. P/C
6. The number of ways 5 friends can occupy 7 empty chairs in a theatre. P/C

7. The number of ways Raghav can invite 3 out of 7 friends to his house for a party. P/C
8. For the following questions, identify whether repetition is allowed (A) or not allowed (NA).
   (i) How many 3 - digit numbers can be formed using single digit prime numbers? A/NA
   (ii) In how many ways can 15 friends sit in 20 chairs in a movie theatre? A/NA
   (iii) In how many ways can 4 letters be posted in 6 post boxes? A / NA
9. In how many ways can 5 friends be seated in three chairs?
10. What is the maximum number of attempts required to open a 3–slot number lock where each slot can have any digit between 0 and 9?
11. How many 5 letter words (with or without meaning) can be formed using the letters of the word 'GREAT'?
    (i) If repetition of alphabets is allowed=_____.
    (ii) Without repetition of alphabets=_____.
    (iii) Such that all the vowels are together=_____.
    (iv) Such that the vowels are together and the consonants are together=_____.
    (v) No two vowels are together=_____.
12. Making use of the five digits 0, 2, 6, 7, and 9
    a. How many 4-digit numbers can you make without repetition of digits?
    b. How many 4-digit even numbers can you form?
    c. How many 4-digit numbers divisible by four can you form?
13. In how many ways can three boys and three girls sit in six chairs?
14. How many 4-letter words can be formed using all the letters of the word 'TEST'?
15. How many words can be formed using all the letters of the word 'ELEPHANT'?
16. In how many ways can 3 red balls and 2 blue balls be arranged in a straight line?
17. In how many ways can 5 letters be posted in 4 post-boxes?
18. You go to a jewellery shop to buy a beaded necklace and the necklace displayed on the side is shown to you. After some time, the sales man rotates the displayed jewellery by a certain angle and claims it to be different jewellery. Will you accept it?
19. In how many ways can 5 friends sit around a table?
20. How many necklaces can be formed with 7 different beads?
21. How many garlands can be formed with 6 different coloured roses?
22. In how many ways can 4 boys and 4 girls sit around a table, if no two boys should sit together?
23. From a squad of 8 members, in how many ways can you pick a team of 5 members?
    Is selecting 3 members, who will not be a part of the team, the same as selecting 5 members who will be a part of the team?
24. In how many ways can you form a cricket team of 11(comprising of 6 batsmen and 5 bowlers) from a squad of 16 (8 batsmen and 8 bowlers)?
25. In how many ways can you form a committee of 4 from 5 men and 3 women?
    (i) If the committee comprises of 2 men and 2 women?
    (ii) The committee has at least 2 men?

(iii) The committee has no more than 2 women?
(iv) One particular member always has to be selected?
(v) One particular member should never be selected?

26. From 6 friends, in how many ways can you invite
    (i) 3 friends to your house for a party?
    (ii) 1 or more friends to your house for a party?
    (iii) At least 2 friends to your house for a party?

27. In how many ways can you give one or more toffees to your friend from the 5 similar orange and 4 similar grape flavoured toffees?

28. If a coin is tossed 4 times, how many different outcomes are possible?
    (i) Number of outcomes per toss of a coin =_____.
    (ii) Total number of outcomes for 4 tosses =_____.
    (iii) If 4 coins are tossed at once, will the number of outcomes be the same? Yes/No

29. If a die is rolled 3 times, how many different outcomes are possible?

30. If 2 cards are drawn at random from a pack of cards, how many different outcomes are possible?
    (i) With replacement _____
    (ii) Without replacement _____

31. If two different letters are selected at random from the English alphabet, what is the probability that
    (i) Both of them are vowels?
    (ii) At least one of them is a vowel?

32. If 2 dice are rolled, what is the probability that the sum of the values on them is a prime number?

33. If 3 coins are tossed simultaneously, what is the probability that exactly 2 of them will show tails?

34. If 4 coins are tossed simultaneously, what is the probability of getting 2 or more tails?

35. If two cards are drawn at random from a pack of cards, what is the probability that
    (i) Both of them are spades?
    (ii) Both of them are red or both of them are kings?

36. Problems on odds in favour / odds against
    (i) Two fair coins are tossed. What are the odds in favour of getting heads on both the coins?
    (ii) If 3 dice are rolled simultaneously, what are the odds against getting a prime number on each of the 3 faces?

Concept review questions

1. How many ten letter words can be formed with all the letters of the word 'ENGAGEMENT'?

   a. 10!   b. 10! / (3! x 2! x 2!) c. 10! / (3! x 2!) d. None

2. How many 5 - digit numbers divisible by 4 can be formed using the digits 5, 6, 7, 8, and 9 such that there is no repetition of digits?

   a.30   b. 21   c. 24   d. 18

3. Find the sum of all 4 - digit numbers formed by taking all the digits 2, 4, 5, and 7.

   a. 118899   b. 119988   c. 19998   d. 19988

4. A conference is attended by 25 participants. If each participant shakes hand with every other participant, what will be the resultant number of handshakes?

   a. 300   b. 25   c. 276   d. 325

5. How many diagonals does an octagon have?

   a. 56   b. 20   c. 28   d. 24

6. How many parallelograms are formed by a set of 6 parallel lines intersecting another set of 4 parallel lines?

   a. 24   b. 90   c. 15   d. None

7. If all the possible words using the letters of the word 'DRAW' are formed without repetition and arranged in alphabetical order, what will be the position of the word 'WARD'?

   a. 23   b. 24   c. 19   d. 20

8. In how many ways can 6 tennis players be divided into 3 teams of 2 each?

   a. 20   b. 45   c. 90   d. None

9. What is the probability that a clerk while randomly placing 5 letters (each intended for a particular recipient) in 5 addressed envelopes will place exactly one of those letters in a wrong envelope?

   a.5!   b. 1/5!   c. 0   d. None

10. If two different numbers are randomly selected from the first 10 natural numbers, what is the probability that the sum of the selected numbers will be 11?
    a. 1/55   b. 1/11   c. 1/9   d. 1/45

11. From a bag with 2 white, 3 black and 5 red marbles, 3 marbles are randomly selected. What is the probability that all the 3 selected marbles are red in colour?

    a. 1/2   b. 1/120   c. 1/12   d. None

12. A football player was practicing penalty shots. If the probability of scoring a goal for each attempt is 80%, what is the probability that he will score a goal in each of his 3 attempts?

    a. 80%   b. 64%   c. 0%   d. 51%

13. If the letters of the word 'TRAP' are jumbled at random, what is the probability that the position in which the vowel appears will remain unchanged?

    a. 6/23   b. 1/24   c. 1/6   d. 1/4

14. A man plays a game of dice in a casino. The man has to pay Rs. 100 for every roll of the dice. If a multiple of 3 turns up, the man doubles his money; else, he forfeits it. In the long run, what is his expected gain or loss per roll of the dice?

    a. Rs. 33 1/3 gain   b. No gain or loss

    c. Rs. 33 1/3 loss   d. Rs. 100 loss

15. There are two boxes with numerous balls in them. The first box has 13 red and 17 white balls. The second box has 12 red and 8 green balls. If one of those boxes is selected at random and a ball is selected at random from the selected box, what is the probability that the selected ball will be white?

    a. 17/30   b. 17/50   c. 17/60   d. 29/60