logarithms

Drill 1: Solutions

a. *Answer: 4*
   Explanation:

   log(x+2) + log(x-2) = log 12
   Log (x+2)(x-2)   = log 12
   (x+2)(x-2)   = 12
   $x^2 - 4$ = 12
   x^2 = 16
   x   = ± 4
   But logarithm values can only be taken for natural values
   So **x = 4**

b. *Answer: 6 + log 5*
   Explanation:

   $\log_{10}200 + \log_{10} 40 + 2\log_{10} 25 = \log_{10} (200 \times 40) + \log_{10}25^2$
   $= \log_{10} (200 \times 40 \times 25 \times 25)$
   $= \log_{10} (8 \times 10^3 \times 625)$
   $= \log_{10} (5000 \times 10^3) = \log_{10}(5 \times 10^6)$
   $= \log_{10}5 + 6\log_{10}10 =$ **6 + log 5**

c. *Answer: 13104*
   Explanation:

   2^{56}
   $\log 2^{56} = 56 \log 2$
   $= 56 \times 0.3010$
   $= 16.856$
   Number of digits = 16+1 = **17**

Drill 2 : Solutions

a. *Answer: 37*
   Explanation:

   -7,-3,1,5,9
   The 12th term is
   a = -7
   d = -3-(-7) = 4
   T$_{12}$ = a + 11d= -7 + 11(4) = -7+44=**37**

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b. *Answer: 315*
   Explanation:

   -7,-3,1,5,9
   The sum of series can be found by
   S$_n$ = n/2[2a+ (n-1) d]
   S$_{15}$ = 15/2 [2*(-7) + (15-1)*4]
   S$_{15}$ = 315

Drill 3 solutions:

a. *Answer: 6250*
   Explanation:

   2, 10, 50, 250...
   Nth term of G.P. can be found by
   t$_n$= a*r$^{(n-1)}$
   r =10/2 =50/10 = 5.
   6th term is given by,
   t$_6$ = 2*5^5
   = 2*3125
   t$_6$ = 6250

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b. *Answer: 7812*
   Explanation:

   2, 10, 50, 250, ... sum of the 6th term
   Sum of nth term
   S$_n$ = a* (r^n-1)/(r-1)
   r =10/2 =50/10 = 5.
   S$_6$ = 2* (5^6-1)/ (5-1)
   S$_6$=7812

c. *Answer: 54*

Explanation:

18, 2, 8..... ∞

$S_n = a/ (1-r)$

$r = 12/18$ (or) $8/12 = 2/3$.

$= 18/ (1-(2/3))$

$= 18/ (1/3)$

$S_n = 54$

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Drill 4 Solutions:

a. *Explanation*

Complementary angle= sum of two angles=90°

$$i.e. \theta_1 + \theta_2 = 90°$$

Supplementary angle= sum of two angles=180°

i.e. $\theta_1 + \theta_2 = 180°$

	Complementary	**Supplementary**
• 75°	90° - 75°=15°	180° - 75°=105°
• 82°	90° - 82°=8°	180° - 82°=98°
• 64°50′	90° - 64°50′=35°10′	180° - 64°50′=115°10′ [since 60′=1°]
• 21°12′	90° - 21°12′=68°48′	180° - 21°12′=158°48′
• 42°40′	90° - 42°40′=47°20′	180° - 42°40′=137°20′

b. *Explanation*

Reflex angle $=360°-\theta$

- 175°   =360°-175°   =185°
- 136°44′ =360°-136°44′ =223°16′
- 92°18′ =360°-92°18′ =267°32′
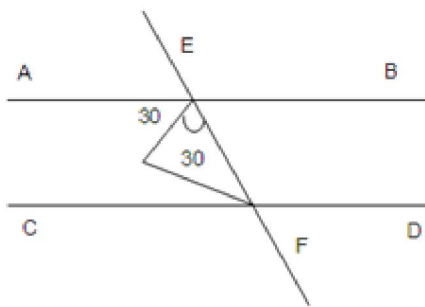- 101°01′ =360°-101°01′ =258°59′

c. *Answer: 162**

Explanation:

We know that sum of exterior angles in a polygon =360°

For 20 side polygon, each side =360°/20° =18°
Interior angle = 180°-exterior angle
                = 180°-18° =**162°**

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d. *Answer: 90°*
    Explanation:

Given that   EO is angular bisector
So,

∠ OEA should be 30° since it is an angular bisector
If ∠FEA = 60° Similarly   F =∠ 60°
If ∠F = 60° then ∠EFC is also 120°
Given that OF is angular bisector .So ∠EFO = 60°
Sum of angles in a triangle is 180°
∠OEF = 30°   ∠EFO = 60°   ∠EOF = 180° - (60+30) = **90°**

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e. *Answer: 130°*
    Explanation
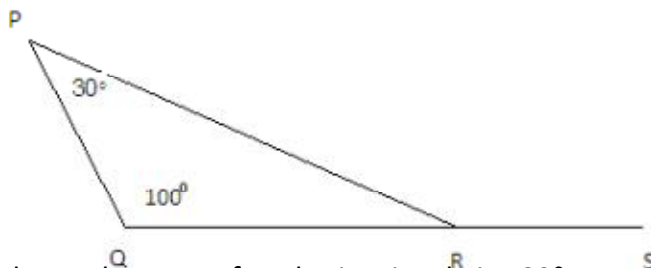
We know that sum of angles in triangle is 180°
∠P + Q + R = 180°
30 + 100 + R = 180
R = 50

QPR = 30°
PQR = 100°
PRS = 180 - PRQ
[Sum of exterior angles = 180°]
PRS = 180°-50° = **130°**

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f. *Answer: 165°*
Explanation

In the figure, PQ‖RS‖TU and WX is parallel to YZ. ∠YXS = 35°, ∠RXZ = 50°. Find ∠XYZ.
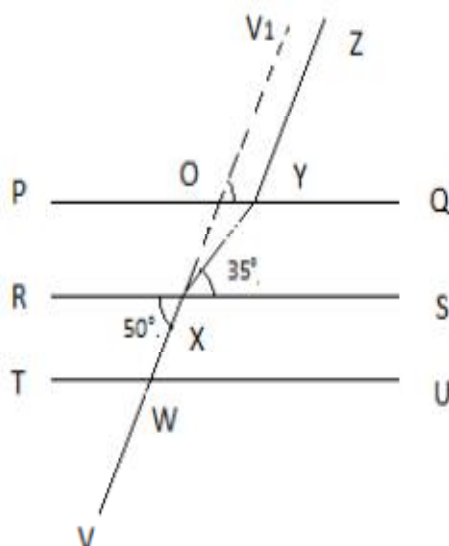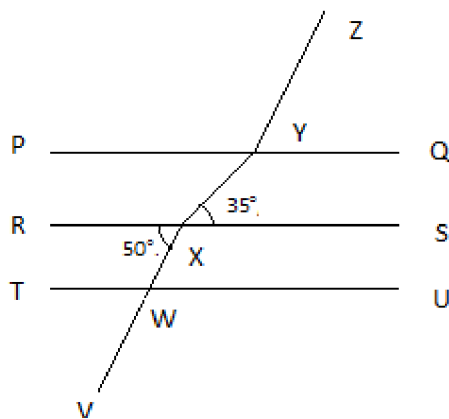
If we draw an imaginary line V₁ then angle at V₁OQ should be 50°.
Then ∠V₁OP = 180°- ∠V₁OQ = 180°-50° = 130° = ∠ZYQ
Similarly, ∠PYX = 35° Since corresponding angles are equal.
So answer should be ∠XYZ = ∠PYX + ∠ZYP
∠XYZ = 130° + 35° = 165°

Drill 5 solutions :

a. *Explanation*

   i. Given : The side of the triangle a = 4√3 cm
       Height of the equilateral triangle h = √3a/2 = (√3 x 4√3)/2 = 6cm
   ii. Area of equilateral triangle = √3/4 x a² = √3/4 x 4√3 x 4√3 = 12√3
   iii. Area of a regular hexagon = (3√3/2)a² = 3√3/2 x 4√3 x 4√3 = 72√3 cm
   iv. Perimeter of regular hexagon
        Sum of length of regular hexagon = 6a = 6 x 4√3 = 24√3

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b. *Explanation*

Length of room l = 60 m
Breadth of room b = 50m
Height of room h = 20m
i. Floor area of each room
   lb = 50 x 60 = **3000m²**
ii. Volume of each room = l x b x h = 50 x 60 x 20 = **60000m³**
iii. Tiles required to make the walls =
     2(bh + lh) (Since walls are only on 4 sides, lb need not be calculated)
     2(1200) + 2(1000)
     =2400 + 2000 = 4400
     No of tiles = 4400/ (3x2) = 4400/6 = **733 (1/3)**

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c. *Explanation*

   Diameter = 14 cm h = 6cm r = 7cm
   Volume of cylinder = πr²h
   =πx7x7x6 = **924 cm³**
   Volume of cone = 1/3πr²h
   =1/3 x 22/7 x 7 x 7 x6 = **308 cm³**
   Total surface area of cylinder = 2πr² + 2πrh
   = 2πr(r+h)
   = 2 x π x 7(7+6)
   = **472 cm²**

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d. *Answer: 440 m²*
   Explanation
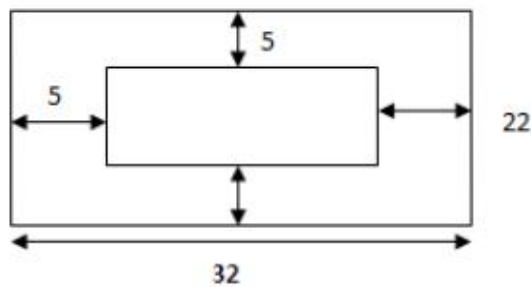
Area of garden = Total area – Area of rectangular plot
= ((22+10)x(12+10) – (22x12))
= [32 x 22] – [22 x 12]
= 20 x 22 = **440m²**

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e. *Answer: 256cm²*
   Explanation
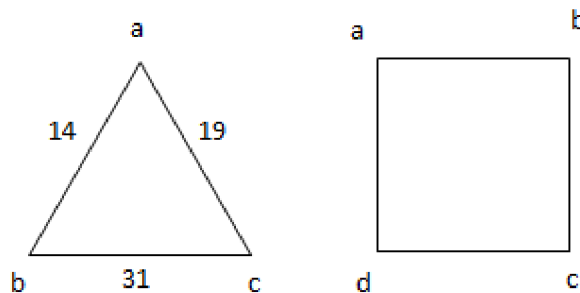
Perimeter of triangle = 14+19+31 = 64 cm³
Now, perimeter of triangle = perimeter of square (Since the rope is made as a square)
∴ Perimeter of square = 4a =64
A=64/4=16 cms
Area of square = a^2 =16^2 =**256 cm²**

Drill 6 Solutions:

a. *Explanation*

   X^2-6X+5=0
(i) Using formula [-b±√(b²-4ac)]/2a
   [6±√(36-20)]/2 = (6±4)/2
   = (6+4)/2 = 5
   (6-4)/2=1
   The roots are **1,5**

(ii) x^2+5x-84=0
   x^2+12x-7x-84=0
   x(x+12)-7(x+12)=0
   (x-7)(x+12)=0
   x=7 x=-12 The roots are **7,-12**

(iii) Formula $[-b \pm \sqrt{(b^2-4ac)}]/2a$

$\qquad [-13 \pm \sqrt{(169+1740)}]/10$

$\qquad \mathbf{(-13 \pm \sqrt{1909})/10}$

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b. *Answer: $x^2+8x-33=0$*
Explanation

Roots of the equation are 3,-11
(x-3)(x+11)
$X^2+11x-3x-33=0$

$$\boxed{x^2+8x-33=0}$$

Googly Questions:

1. Wrong

Log(4x+3) – log(3x-7) = log12
Actually the answer given is **wrong** because the formula should be

$$\boxed{\text{Log a – log b = log a/b}}$$

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2. Correct

The surface area = $6a^2$ which will be equal to the surface area generated because of the cuts made on the cube. Hence there will be a 100% increase in the surface area.

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3. Wrong.

Ravi jumps from the height of 21 and he doesn't bounce back to the same height of 21. Instead, he will bounce $2/3^{rd}$ of 21 feet. So the answer is wrong because the sum of terms in GP = a/1-r
We can't start the a with 42 because he is not bouncing back the same 21

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4. Correct.

Decagon→10 sides

Value of external angle = 360/10 = 36°

External angle + internal angle = 180° $\Rightarrow$ so internal angle = **144°**

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5. **Wrong.**

$X^2+9x+20$

$X^2+ 4x +5x+20=0$

$X(x+4) +5(x+4) =0$

$(x+5)(x+4)=0$

X+5=0   x+4=0

X=-5   x=-4

So the roots are not 5,4 instead it is **(-5,-4)**

Concept Review Questions:

1. *Answer: option(a)*

   Explanation:

   log 3+log 2

   We know the identity $(\log_a m + \log_a n = \log_a (m*n))$

   So the answer is log (3*2) =**log6**

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2. *Answer: option(d)*

   Explanation:

   log 3x-log 6=log 9

   We know the identity $(\log_a m - \log_a n = \log_a (m/n))$

   m=3x, n=6, m/n=9

   3x/6=9, 3x=54

   $\therefore$**x=18**

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3. *Answer: option(b)*

   Explanation:

   We know that $\log_a b = 1/\log_b a$

   $1/\log_2 50 + 1/\log_5 50 + 1/\log_5 50$ can be written as

   $\log_{50}2 + \log_{50}5 + \log_{50}5$

   $=\log_{50} (2x5x5) = \log_{50}50 =$**1**

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4. *Answer: option(d)*

   Explanation:

$\log_m p = p \log m$
Same way $\log 9^2 = 2\log 9$
$(2 \log 9)/ (\log 9)$ **= 2**

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5. *Answer: option(b)*
   Explanation:

   125^{50}
   Taking log we get = $\log (125)^{50}$
   125 can be written as 5^3
   $\log (5^3)^{50} = \log 5^{150}$
   $= 150\log 5$ ($\log 5 = 0.69897$)
   $= 150(0.69897)$
   $= 104.84 \cong$ **105**

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6. *Answer: option(b)*
   Explanation:

   Let the number be x
   Actual number = 26x
   Processed number = 62x
   So the difference in the number = 900
   62x-26x = 900
   36x = 900
   x=900/36 =25
   So the number (actual) =26x = 26 x 25 = **650**

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7. *Answer: option(b)*
   Explanation:

   No of apples purchased for Rs. 360➔?
   Going from the options
   (a) 20
   360/20 = 18 Rs/apples
   If the cost is increased by 3 = 18 + 3 =21
   360/21 = 17.14 (Wrong option)
   (b) 24
   360/24 = 15Rs/apple
   If increased by Rs.3 it will become Rs 18/apple
   So the no. of apples = 360/18 = 20 which is 4 less than the actual one and satisfies the condition
   ∴**Karan purchased 24 apples**

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8. *Answer: option(b)*

Explanation:

In this problem the 10th and 11th terms are the middle terms. So considering the concept of average

(Which is the middle one), we get
Sum/n = 210/20 =21/2
Here 21 represent the sum and 2 represents the 10th and 11th terms. $\therefore$ **The sum is 21**

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9. *Answer: option(a)*
   Explanation:

   Assume there are 3 numbers in AP .They are a-d, a, a+d. The middle term is average $\therefore$ Sum/n = 36/3 = 12
   So a =12
   If difference is 1; the terms are 11, 12, 13 but the sum of squares= 121 + 144 + 169 =434 $\pm$440
   So taking the difference is 2.The terms are 10, 12 and 14
   The sum of squares = 100+144+196 =440
   $\therefore$ 10, 12, 14 satisfies the condition .So the larger number is **14**

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10. *Answer: option(c)*
    Explanation:

    We know that sum of terms of infinitely long decreasing GP = a/1-r
    128 + (64+64) + (32+32) + (16+16) + (8+8) + (4+4) + (2+2) + (1+1)....
    128+ 128 + 64 + 32 + 16 + 8 + 4 + 2
    We can apply the formula a/(1-r)
    = 128 + 128/(1-1/2) = 128 + 256 =**384**

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11. *Answer: option(b)*
   Explanation:

   For the arithmetic mean we can tell sum of the (1st term + last term) divided by 2
   = (-4+40)/2 = **18**
   Since the difference between the terms is the same we can use the above method.

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12. *Answer: option(d)*
    Explanation:

So for the numbers to be a multiple of both 5 and 3, the number should start from 60 and end at 4500 within the given range of 55 to 4505

The first number which is multiple of both 5 and 3 is 15 and 4500 is the 300th term.

Since we are starting from 60, we have to eliminate 15, 30, 45 behind (3 terms)

So the no. of terms = 300-3 = **297**

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13. *Answer: option(b)*
    Explanation:

Arithmetic mean of 2 numbers a, b or a+b/2 and geometric mean = $\sqrt{ab}$

Going from the options

a=4/5, b =1/5

a+b = 1

(a+b)/2 =1/2 as given

GM = $\sqrt{ab}$ = $\sqrt{(4/5 \times 1/5)}$ = $\sqrt{(4/25)}$ = 2/5

∴ **4/5, 1/5 satisfies the condition**

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14. *Answer: option(a)*
    Explanation:

Given that flower triples for every 10 minutes

So minutes   flowers

          0   3^1

10   3^2

20   3^3

So we need to find 59049 in terms of powers of 3 instead we can go for power of 9 and we can double the power 9 = 3^2. Since the unit digit is 9 it will be enough if we check for the odd powers of 9 (power cycles)
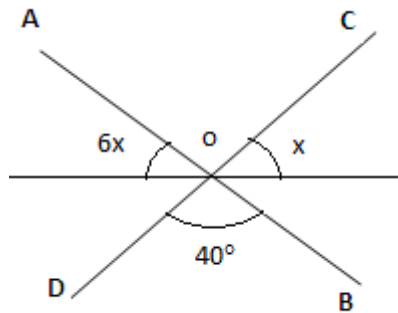
9^3= 729

9^5=59049

∴ 9^5= 3^{10}

∴ For 90 minutes the power will be 3^{10}

90 minutes = **1 hour** + **30 minutes**

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15. *Answer: option(b)*
    Explanation:

AOC = 40° (Vertically opposite angle)

6x + 40 + x =180
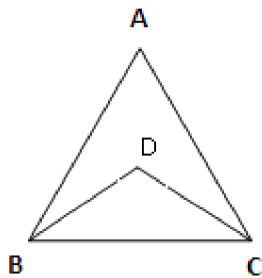
7x = 140 →**x= 20°**

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16. *Answer: option(d)*

  Explanation:

The total interior angle of an equilateral triangle = 180

Bisector forms an angle 60°/2 = 30°

∴ DBC + BCD = 30 + 30 = 60

From triangle BDC, total angle = 180

DBC + BCD + BDC =180°

BDC = 180 -60 =**120°**

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17. *Answer: option(a)*

  Explanation:

We know that area of trapezium

=1/2 x sum of parallel sides x height

Parallel sides = 11m and 25m

Height = 12m

= ½ *(11+25) x 72

=1/2 x 36 x 12 = **216 m²**

18. *Answer: option(d)*
    Explanation:

    Given c =110 from this information we are not able to conclude any answer .So it is **cannot be determined**

19. *Answer: option(a)*
    Explanation:

    From the options
     If Abhilash has 10 coins
     Then with Ravi = 10 +2 =12
     Coins with Raghav = 10-2 =8
     8, 10, 12
     Product = 8x10x12 = 960
     ∴**10** satisfies condition

20. *Answer: option(a)*
    Explanation:

    The roots of the equation
    $x^2+10x+24=0$ are 6 and 4.The reciprocal & roots are 1/6 and ¼
    Sum of the roots = 5/12
    Product of the roots =1/24
    In the generalised equations $ax^2+bx+c=0$, the co-efficient of x will have the sum of the roots and constant term c will have product of roots
    $x^2+5/12x+1/24 =0$
    $24x^2+10x+1=0$