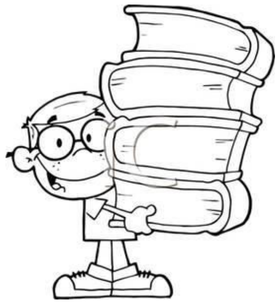
PERMUTATION & COMBINATION



Drill 1: Solutions

a. *Answer*: 30 ways

Explanation:

Entry:

Number of glass doors	=	3
Number of metal doors	=	2
Number of ways to enter	=	3 + 2
	=	5

Exit:

Number of glass doors	=	5
Number of wooden doors	=	1
Number of ways to exit	=	5 + 1
	=	6

Enter & Exit:

Number of ways to enter & exit	=	6 x 5
	=	**30 ways**

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b. *Answer:* 15 ways

Explanation:

A — 3 trains—— B — 5trains————— C

A to B:

Number of ways to travel = 3 ways

B to C:

Number of ways to travel = 5 ways

A to C

Travelling from A to C is nothing but travelling from A to B and travelling from B to C.

So, Number of ways to travel = 3*5
= **15 ways**

DRILL 2: SOLUTIONS

a. *Answer:* Combination

Explanation:
The order doesn't matter for cricket team members. Its selection and hence its **combination.**

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b. *Answer:* Permutation

Explanation:
Whenever seating arrangement comes, it is always **permutation.**

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c. *Answer:* Combination

Explanation:
All are retired judges and 4 panel judges have to be selected where the order doesn't matter hence it's a **combination.**

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d. *Answer:* Permutation

Explanation:
Whenever seating arrangement comes, it is always **permutation.**

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e. *Answer:* Combination

Explanation:
Selection. So, order doesn't matter, hence its **combination.**

Drill 3: Solutions

a) i) .Answer: A

Explanation:

In this case **repetition is allowed** because 777, 555, 333 are possible, so we can use the same prime numbers for three times.

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ii). Answer: NA

Explanation:

Here **repetition is not allowed**, because only one person can sit in a single seat.

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iii). Answer: A

Explanation:

Here **repetition is allowed** because we can post more than a single letter in a post box.

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b. Answer: 60 ways

Explanation:

In this case repetition is not allowed because its a seating arrangement.

Number of ways of filling 1^{st} chair = 5
Number of ways of filling 2^{nd} chair = 4
Number of ways of filling 3^{rd} chair = 3
Total Number of ways of filling 3 chairs = 5 x 4 x 3
= **60 ways**

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c. Answer: 1000 ways

Explanation:

In this case repetition is allowed.

So, all the three slots can have 10 possibilities.

Total number of possibilities = 10 x 10 x 10
= **1000 ways**

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d) i) .Answer: 3125 ways

Explanation:

Here its given that the repetition is allowed and hence all the places have equal and maximum possibilities of the word.

There are 5 letters in the word GREAT.

So, maximum possibility = 5

Number of ways of arrangements = $\underline{5} \times \underline{5} \times \underline{5} \times \underline{5} \times \underline{5}$

= **3125 ways**

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ii). Answer: 5! ways

Explanation:

Here its given that the repetition is not allowed.

Number of ways of filling first place = 5

Number of ways of filling second place = 4

Number of ways of filling third place = 3

Number of ways of filling fourth place = 2

Number of ways of filling fifth place = 1

Total number of ways = $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

= 120

= **5! Ways**

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iii). Answer: 48 ways

Explanation:

Here the vowels should be together.

Let us assume the two vowels E & A as a single letter.

So, now we have only 4 letters to be arranged in 4 slots.

Number of ways of filling first place = 4

Number of ways of filling second place = 3

Number of ways of filling third place = 2

Number of ways of filling fourth place = 1

Total number of ways = $\underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

= 24 ways

Vowels being A & E in this word, when arranged together can have two different arrangements as AE and EA.

Number of ways(having AE together) = 24 ways

Number of ways(having EA together) = 24 ways

Total number of ways = 24 + 24

= **48 ways.**

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iv). *Answer:* 24 ways

Explanation:

Vowels and consonants should be together.

Let us assume

The vowels A & E $\Rightarrow$ single letter (EA)

The consonants G, R & T $\Rightarrow$ single letter (GRT).

So,

Number of vowels arrangement $\Rightarrow$ 2! Ways.

Number of consonants arrangement $\Rightarrow$ 3! Ways.

Total Number of arrangements $\Rightarrow$ 2*2!*3!

$\Rightarrow$ **24 ways**

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v). *Answer:* 72 ways

Explanation:

No 2 vowels are together.

$\left.\begin{array}{l}\text{If the two vowels are not}\\ \text{supposed to be together}\end{array}\right\}$ $\Rightarrow$ They should be separated by the consonants

Arrangement : *_ G_R_T_*

Now the vowels can be placed in any of these 4 places.

Number of first vowel arrangement $\Rightarrow$ 4 Ways.

Number of second vowel arrangement $\Rightarrow$ 3 Ways.

Total Number of arrangements $\Rightarrow$ 4 x 3

$\Rightarrow$ 12 ways

That G, R, T can be rearranged in 3! Ways.

So it is, 12*3! $\Rightarrow$ **72 ways**

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e) i) .Answer: 96 ways

Explanation:

Making use of the five digits 0,2,6,7 and 9

4ways x 4ways x 3ways x 2ways $\longrightarrow$ **96 ways**

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ii). *Answer:* 300 ways

Explanation:

Repetition is allowed, if not mentioned

Even numbers $\Rightarrow$ last digit can be 0, 2 & 6

I

____ ____ ____ ____
4ways x 5ways x 5ways x 3ways $\longrightarrow$ **300 ways**

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iii). Answer: 120 ways

Explanation:
Divisibilty test for 4 $\Rightarrow$ Checking last 2 digits as the multiples of 4

So, the number of possibilities :

4ways	*	5ways	*	2	0
				6	0
				7	2
				9	6
				9	2
				7	6

6 ways ==> **120 ways**

Total number of 4 digit numbers $\Rightarrow$ 4 x 5 x 6
= **120ways**

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f) i) .Answer: 144 ways

Explanation:
Since there are only 6 chairs , if 3 chairs are occupied by boys remaining 3 for girls. But there are actually 4 position for girls, from which any 3 have to be selected.

3! x 4P3 = 144 [*_B_B_B_ or B_ _ B_B etc. But anyway 4 positions are available for girls in total*].

Or, different arrangements are

BGBGBG $\Rightarrow$ 3! x 3! = 36
GBGBGB $\Rightarrow$ 3! x 3! = 36
BGGBGB $\Rightarrow$ 3! x 3! = 36
BGBGGB $\Rightarrow$ 3! x 3! = 36
Total no of ways $\Rightarrow$ **144 ways**

---

ii). *Answer:* 72 ways

Explanation:

Here no 2 boys are sitting together.

BGBGBG $\Rightarrow$ 3! x 3! = 36

GBGBGB $\Rightarrow$ 3! x 3! = 36

Total no of ways $\Rightarrow$ **72 ways**

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g. *Answer:* 12 ways

Explanation:

Number of letters = 4
Repeating letters = 2
Number of ways = 4!/2!
= **12 ways**

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h. *Answer:* 8!/2! ways

Explanation:

Number of letters = 8
Repeating letters = 2
Number of ways = **8!/2! ways**

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i. *Answer:* 5!/(3!2!) ways

Explanation:

Total number of balls = 3+2
= 5
Number of ways = **5!/(3!2!) ways**

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j. *Answer:* Question repeated.

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k. *Answer:* 4^5 ways

Explanation:

Consider each letter

Number of ways of posting first letter $\Rightarrow$ 4 ways
Number of ways of posting second letter $\Rightarrow$ 4 ways
Number of ways of posting third letter $\Rightarrow$ 4 ways
Number of ways of posting fourth letter $\Rightarrow$ 4 ways

Total number of ways of posting four letters ⇒ 4 x 4 x 4 x 4 x 4

= **4^5 ways**

Drill 4: Solutions

a. *Answer:* 4! Ways

Explanation:

Permutation of n people ⇒ (n-1)! Ways

Therefore, There are 5 people = **4! Ways**

And yes! The arrangement is different in clockwise and anti – clockwise duration.

b. *Answer:* 6! /2 ways

Explanation:

No! The anti – clockwise and clockwise arrangement are not different. [*Necklace*]

Therefore, number of arrangement = (n-1)! / 2 = **6! /2 ways**

c. *Answer:* 5! /2 ways

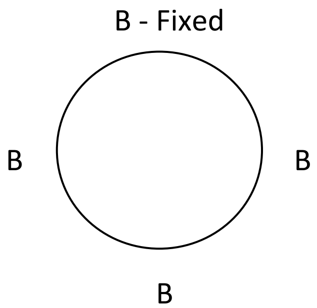
Explanation:

No! The anti – clockwise and clockwise arrangements are not different. [*Garland*]

Number of arrangement = (n-1)! /2 = **5! /2**

d. *Answer:* 3! x 4! ways

Explanation:



Number of Boys arrangement ⇒ 3! Ways.

Number of Girls arrangement ⇒ 4! Ways.

Total number of ways = **3! x 4! ways**

Drill 5: Solutions

a. *Answer:* Yes

b. *Answer:* 1568

Explanation:

Selecting 6 batsmen out of 8	=	$8C_6$ **= 28**
Selecting 5 bowlers out of 8	=	$8C_5$ **= 56**
Therefore, for selecting a team	=	batsmen and bowlers
	=	28 x 56
	=	**1568**

c) i) .Answer: $5C_2 \times 3C_2$

Explanation:

Selecting 2 men out of 5	=	$5C_2$
Selecting 2 women out of 3	=	$3C_2$
Therefore, number of ways	=	$\mathbf{5C_2 \times 3C_2}$

ii). *Answer:* $(5C_2 \times 3C_2) + (5C_3 \times 3C_1) + 5C_4$

Explanation:

At least 2 men:

Case 1:

Selecting 2 men and 2 women = $5C_2 \times 3C_2$

Case 2:

Selecting 3 men and 1 women = $5C_3 \times 3C_1$

Case 3:

Selecting 4 men = $5C_4$

Therefore, number of ways = $\mathbf{(5C_2 \times 3C_2) + (5C_3 \times 3C_1) + 5C_4}$

iii). *Answer:* $5C_4 + (5C_3 \times 3C_1) + (5C_2 \times 3C_2)$

Explanation:

No more than 2 women:

Case 1:

Selecting 4 women = $5C_4$

Case 2:

Selecting 3 men and 1 women = $5C_3 \times 3C_1$

Case 3:

Selecting 2 men and 2 women = $5C_2 \times 3C_2$

Therefore, number of ways = $\mathbf{5C_4 + (5C_3 \times 3C_1) + (5C_2 \times 3C_2)}$

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iv). *Answer:* $7C_3$

Explanation:

When one particular person is always selected, we need not consider that person during selection of committee members.
Initially there were 8 members in total [5 men and 3 women].
Now, there are only 7 members out of which we have to select 3
No. of ways = $\mathbf{7C_3}$

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v). *Answer:* $7C_4$

Explanation:

If one particular member should never be selected we need not consider that member during selection. So out of remaining 7 members
We would select 4 = $\mathbf{7C_4}$

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d) i) .Answer: $6\ C_3$

Explanation:

Select 3 out of 6 = $\mathbf{6\ C_3}$

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ii). *Answer:* 31 or 2^6-1

Explanation:

One friend = $6C_1$
Two friends = $6C_2$
Three friends = $6C_3$

Total no. of ways = $6C_1 + 6C_2 + \ldots\ldots\ldots\ldots + 6C_6$
= 31

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iii). Answer: 57 or 2^6-1-6

Explanation:
At least 2 friends:
Select 2 friends = $6C_2$
Select 3 friends = $6C_3$
Therefore, no. of ways = $6C_2 + 6C_3 + \ldots\ldots\ldots\ldots + 6C_6$
= **57**

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e. Answer: 29 ways

Explanation:
When there are 5 similar orange, if I have to select 1, there is only one way.
For selecting 2 oranges, there is only 1 way (because selecting any 2 oranges is always same) i.e. selecting 1, 2 is same as selecting 3, 4.
Selecting similar oranges:
0 orange = 1 way
1 orange = 1 way
2 orange = 1 way
3 orange = 1 way
4 orange = 1 way
5 orange = 1 way
Therefore for 5 oranges = 6 ways.
Similarly for 4 grape = 5 ways
But for selecting one toffee, we would select
0 orange and 1 grape
Or
0 grapes and 1 orange
Hence total combination = 6 * 5
= 30
But there is a chance of selecting 0 orange and o grape which should not be included.
Ans: 30 -1=**29 ways.**

DRILL 6: SOLUTIONS

a. *Answer: i) 16 ii)216 iii)51*52*

Explanation:

i. Number of outcomes per toss of a coin = 2.
Total outcome of 4 tosses = 2 & 2 & 2 & 2
= 2^4
= **16**
If 4 coins are tossed at once, will the no. of outcomes be the same? Yes
Because, whatever maybe the case the outcomes will always be the same.
[But output may vary]

ii. Number of outcomes per roll of a die = 6
Total no. of outcomes for rolling it thrice = 6 & 6 & 6
= 6^3
= **216**.

iii. With replacement of 2 cards
The total no. of cards = 52
With replacement = **52 * 52**
Without replacement of 2 cards = **52 * 51**

b. *Answer: i) 23/65 ii) 5/12 iii) 3/8 iv) 11/16 v) 26C2 / 52C2 + 4C2 / 52C2 – 2C2 / 52C2*

Explanation:

i. Total no. of alphabets = 26
Total no. of vowels = 5
Both of them are supposed to be vowels.
No. of ways of selecting 2 vowels = $5C_2$
No. of ways of selecting 2 alphabets = $26C_2$
Probability = $5C_2 / 26C_2$
= 2 / 65
At least one of them is a vowel.
Total Probability = selecting 1 vowel + selecting 2 vowels
Selecting one vowel = $5C_1$ & $21C_1 / 26C_2$
Selecting two vowels = $5C_2 / 26C_2$
Total probability = $5C_1 * 21C_1 / 26C_2$ or $5C_2 / 26C_2$
= 21 / 65 + 2 / 65
= **23 / 65.**

ii. Two dice are rolled, the sum of the values on prime number.
(1, 1), (1, 2), (1, 4), (1, 6),
(2, 1), (2, 3), (2, 5)
(3, 2), (3, 4)
(4, 1), (4, 3)
(5, 2), (5, 6)
(6, 1), (6, 5)
Total probability on throwing 2 dice = 36

Total events on sum of values as prime number = 15
Probability = 15 / 36
= **5 / 12**

iii. Total events on tossing 3 coins = 2 * 2 * 2
= 8
Exactly 2 of them tails means
So, probability is **3 / 8**

iv. Total events when 4 coins are tossed = 2 * 2 * 2 * 2
= 16
Probability of getting 2 or more tails = 11
Probability = **11 / 16**

v. Both of them are spades
From 13 cards [spades] 2 cards selected mean $13C_2$
Total event is from 52 cards selection 2 means $52C_2$
Probability = $13C_2 / 52C_2$
= **3 / 51**
Both of them are red or both of them are kings.
If 2 cards are selected from red = $26C_2$
If 2 cards are selected from kings = $4C_2$
Selecting 2 cards from 52 cards = $52C_2$
Probability = $\mathbf{26C_2 / 52C_2 + 4C_2 / 52C_2 - 2C_2 / 52C_2}$
[2C2 / 52C2 subtracted because king card is counted twice]

c. *Answer: i) 1:3 ii) 7:1*

Explanation:

i. 2 coins are tossed, total events = 4
Probability of getting heads on both the coins-
HH
HT
TH
TT
Favorable event = 1
Probability = 1 / 4
Odds in favor = Probability of getting Heads on both: Probability of not getting head
Probability of getting head = 1 / 4
Probability of not getting head = 1 – (probability of getting head)
= 1 – 1 / 4
= 3 / 4
Odd in favor = (1/4): (3/4)
= **1: 3**

ii. Three dice are rolled -prime no. on each faces.

Probability of a die getting a prime number = 3 / 6
= 1 / 2 [2, 3, 5]

Probability of 3 dice getting a prime number = (1 / 2) * (1 / 2) * (1 / 2)
= 1 / 8

Probability of not getting a prime number = 1 – (1 /8)
= 7 / 8

Probability of odd favour = 1 /8 : 7/8
= 1:7

Odd against = **7:1**

Googly Questions : Solutions

1. *Answer:* Wrong

Explanation:

Number of arrangements of 6 friends in circular table is (n-1)! = (6-1)! = 5!
Hence the arrangement can be made in 120 ways.

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2. *Answer:* Correct

Explanation:

The probability of getting an even number = 3 / 6 (2, 4, 6)
The Probability of getting prime number = 3 / 6 (2, 3, 5)
The value '2' is counted twice. So one time should be removed,
Probability = (3 / 6) + (3 / 6) – (1 / 6)
= 5 / 6

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3. *Answer:* Correct

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4. *Answer:* Wrong

Explanation:

Number of ways of dividing 9 friends into group of 4 & 5 = 9! / (4! * 5!)
After dividing 4 & 5 persons, we should arrange them in a circular fashion
i. e. 3! And 4!

Total ways = 9! 3! / 5!

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5. *Answer:* wrong

   Explanation:

4w	4w	3w	2w
2 or 3 or 7 or 9			0 or 3

Number of ways: 4 * 4 * 3 * 2 = **96 ways**

Concept Review Question

1. *Answer:* 10! / 3! 2! 2!

   Explanation:

   The_word "ENGAGEMENT "has 10 letters which can be arranged in 10! Ways.
   The letter 'E' repeats 3 times
   The letter 'N' repeats 2 times
   The letter 'G' repeats 2 times
   Therefore, the number of 10 letter words that can be formed are = **10! / 3! 2! 2!**

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2. *Answer: 24 ways*

   Explanation:

   The divisibility rule for 4 is, the last two digits should be divisible by 4.
   The given digits are 5, 6, 7, 8 and 9.
   The combination of the last two digits which satisfies the divisibility of 4 are

   __ __ __ 5 6
   __ __ __ 6 8
   __ __ __ 7 8
   9 6

   So last two digits can be filled in 4 ways.
   The remaining 3 places can be filled by remaining 3 digits because repetition is not allowed.

3 * 2 * 1 * 4 ways = **24 ways**

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3. *Answer:* *119988*

   Explanation:

   The given digits are 2, 4, 5 and 7.
   We need to find the sum of all four digit numbers formed by 2, 4, 5,7.
   The last position can be

   __ __ __ 2
   __ __ __ 4
   __ __ __ 5
   __ __ __ 7
   18 (Sum)

   The sum is 18 this can be arranged in other 3 places in 3! Ways.
   Therefore, 18 * 3! = 108 will be the maximum sum in each place.

   ___ ___ ___ ___
   ___ ___ ___ ___
   ___ ___ ___ ___
   ___ ___ ___ ___
   11 11 10

   108 108 108 108
   11 9 9 8 8

   ∴ The sum will be **119988**

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4. *Answer:* *300*

   Explanation:

   A hand shake requires two persons.
   There are 25 persons from which 2 persons can be selected at a time.
   So the maximum no of ways selecting 2 persons from 25 persons is $25C_2$.
   For 1^{st} person, he would have made 24 handshakes with others.
   24+23+22+.....+1 = (24*25)/2
   (i.e.) 25C2 = **300**

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5. *Answer:* *20*

   Explanation:

An octagon has 8 vertices.
The number of line segments that can be formed from 8 points are,
8C2 = 28 which include diagonals and sides.
8C2 = no of diagonals + no of sides
28 = D + 8
D = **20**

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6. *Answer: 90*

   Explanation:

   There are 6 parallel lines and 4 parallel lines (intersecting)
   To form a parallelogram we require a pair of horizontal and a pair of vertical lines.
   So the no of parallelograms formed = 6C2 * 4C2
   = (6*5)/2 * (4*3)/2
   = **90**

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7. *Answer: WARD*

   Explanation:

   Arranging the word 'DRAW' in alphabetical order, we get ADRW
   The no of words starting with letter A will be
   ADRW
   ARWD
   A_ _ _
   A _ _ _
   The no of words starting with letter D will be
   DARW = 3! = 6
   The no of words starting with letter R will be
   RADW = 3! = 6
   The 19^{th} word will be WADR
   The 20^{th} word will be **WARD**

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8. *Answer: 15*

   Explanation:

   No of tennis players = 6
   6 players divided into 3 teams of 2 each
   No of ways arranging 6 players = 6!
   There are 2 players in 3 teams
   Therefore, 6! / (2!*2!*2!)
   Now since repetition is not allowed, it will be 6! / (2!*2!*2!*3!) = **15**

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9. *Answer: 0*

Explanation:

We should place exactly one letter in a wrong envelope.
If 1 letter is placed in a wrong envelope, the other letter goes to a wrong recipient.
So, it impossible to put exactly one letter in the wrong envelope.
Hence probability is **0**.
1 2 3 4 5 – letters
3 2 **1** 4 5 – envelope
X X

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10. *Answer: 1/9*

Explanation:

The sum of selected numbers should be
11 = (1, 10) (2, 9) (3, 8) (4, 7) (5, 6)
The ways of selecting 2 numbers out of 10 natural numbers = 10C2
Therefore, probability that the sum of selected numbers be 11 = 5/ (10C2)
= 5/45
= **1/9**

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11. *Answer: 1/12*

Explanation:

Total number of marbles = 10
Selecting 3 marbles from 10 = 10C3
There are 5 marbles that are red out of which three should be selected = 5C3
Therefore, probability that the 3 selected marbles will be red = 5C3/10C3
= 10/120
= **1/12**

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12. *Answer: 0.512*

Explanation:

Probability of scoring a goal in each attempt = 80%
80% of 100 = 80
80% of 80 = 64
80% of 64 = 51.2
Therefore,
51.2% = **0.512**

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13. *Answer: ¼*

Explanation:

Position of the vowel should remain unchanged.
So A is kept constant.
So ways of arranging 'TRAP' = 4!
Keeping A constant = 3!
∴ Probability of vowel remains unchanged = 3!/4!
= ¼

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14. *Answer: Rs 33.33 loss*

Explanation:

The man pays Rs 100 for each roll.
So after a long run, he rolls the dice for 6 times and pays Rs 600 (dice 6 faces)
He gets double when the dice has multiples of 3 on the top face.
There are two possibilities; 3 and 6.
So he gets Rs 400.
600 – 400 = 200 loss
He rolls for 6 times→ 200/6 = **Rs 33.33 loss**

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15. *Answer: 17/60*

Explanation:

The probability of selecting a bag from 2 bags = ½
Total no of balls = 13+17
= 30
The no of white balls = 17
Therefore, probability of getting a white ball = ½ * 17/30
= **17/60**