LOGARITHM, PROGRESSIONS, GEOMETRY AND QUADRATIC EQUATION

Concepts

Logarithms

1. $\log_a a = 1$
2. $\log_a 1 = 0$
3. $\log_a(m \times n) = \log_a m + \log_a n$
4. $\log_a \frac{m}{n} = \log_a m - \log_a n$
5. $\log_a m^p = p \times \log_a m$
6. $\log_a m = \frac{\log_a m}{\log_a a}$
7. $a^{\log b} = b^{\log a}$
8. $\log_a b = \frac{1}{\log_b a}$

Mensuration

Figure	Name	Perimeter/ Circumference	Area
(a)	square	$4a$	a^2
(b)	rectangle	$2l + 2w$ or $2(l+w)$	lw
(c)	parallelogram	$2a + 2b$ or $2(a+b)$	bh
(d)	triangle	$a + b + c$	$\frac{1}{2}bh$
(e)	trapezoid	$a + b_1 + c + b_2$	$\frac{1}{2}(b_1+b_2)h$
(f)	regular polygon	ns n = number of sides	$\frac{1}{2}ap$ p = perimeter a = apothem
(g)	circle	πd or $2\pi r$	πr^2

Arithmetic progression

n^{th} term of an A.P., $T_n = a + (n - 1) \times d$

Sum of the terms of an A.P., $S_n = \frac{n}{2} \times (2a + (n-1) \times d)$

Geometric Progression

n^{th} term of a G.P., $T_n = a \times r^{(n-1)}$

Sum of first n terms of a G.P., $S_n = \frac{a(r^n - 1)}{(r-1)}$

Sum of terms of an infinitely long decreasing G.P., $S_n = \frac{a}{(1-r)}$

Quadratic Equation

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If α and β are the roots of the quadratic equation $ax2 + bx + c = 0$,

- Sum of the roots , $\alpha + \beta = -b/a$
- Product of the roots, $\alpha\beta = c/a$

If the sum and the product of the roots of the equation are known, then the quadratic equation can be formed as

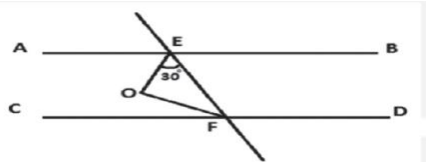
$$x^2 - x(\alpha + \beta) + \alpha\beta = 0$$

Drill

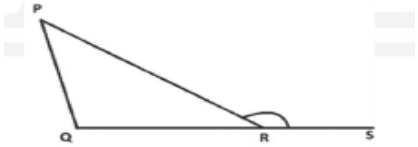
1. Simplify: $\log (x + 2) + \log (x - 2) = \log 12$
2. Evaluate $\log_{10}200 + \log_{10}40 + 2\log_{10}25$
3. If $\log 2 = 0.3010$, find the number of digits in 2^{56}
4. Find the 12^{th} term of series -7, -3, 1, 5, 9, …
5. Find the sum of the first 15 terms of the series -7,-3, 1, 5, 9,….
6. Find the 6^{th} term of the series 2, 10, 50, 250, …
7. Find the sum of first 6 terms of the series 2, 10, 50, 250, …
8. Find the sum of terms of the series 18, 12, 8, ……∞
9. Find the complementary angle (CA) and supplementary angle(SA) for the following:

	CA	SA		CA	SA
75°			21° 12′		
82°			42° 40′		
64° 50′					

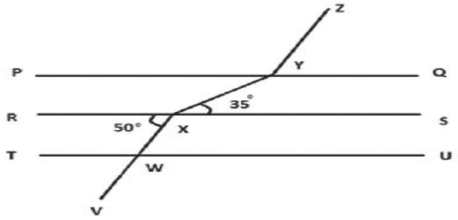
10. Find the reflex angle for the following:
    175° = _____. 136° 44′ = _____.
    92° 18′ = _____. 101° 01′ = _____.
11. Find the respective measures of the interior angle and the exterior angle of a regular polygon of twenty sides.
12. In the figure, AB ‖ CD, a transversal line which cuts the parallel lines at E and F respectively. EO is an angular bisector and ĐOEF = 30°. OF is the angular bisector of ĐCFE. Find ĐEOF and ĐEFO.



13. In the figure, QR is produced to S and ĐQPR=30° and ĐPQR=100°. Find ĐPRS



14. In the figure, PQ‖RS‖TU and WX is parallel to YZ. ĐYXS=35°, ĐRXW=50°. Find ĐXYZ



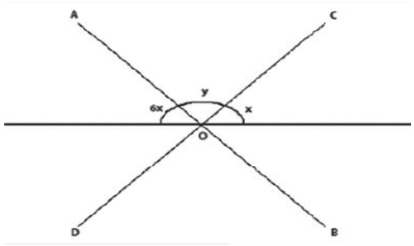
15. Let the length of the side be 4√3 cm.
    i. What is the height of an equilateral triangle?
    ii. What is the area of an equilateral triangle?
    iii. What is the area of a regular hexagon?
    iv. What is the perimeter of a regular hexagon?
16. Mr. Frankenstein wants to build a house with 7 rooms each having equal area. The length of each room is 60 m, breadth is 50 m and height is 20 m. For every unit of area, he requires 120 bricks. Tiles with the dimensions 3 m x 2 m are fixed only on the walls.

i. What is the floor area in each room?
ii. What is the volume of each room?
iii. How many tiles are required to make the walls of a particular room?

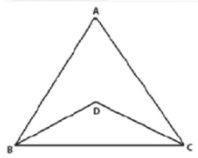
17. Let the diameter be 14 cm and height be 6 cm.
    i. What is the volume of the cylinder?
    ii. What is the volume of the cone?
    iii. What is the total surface area of the cylinder?
18. A rectangular plot of dimension 12 m x 22 m is surrounded by a garden of width 5 m. What is the area of the garden?
19. A triangle is made from a rope. The sides of triangle are 19 cm, 14 cm and 31 cm. What will be the area of the square made from the rope?
20. Solve the following quadratic equations
    i. $x^2 - 6x + 5 = 0$ ii. $x^2 + 5x - 84 = 0$ iii. $5x^2 + 13x - 87 = 0$
21. If the roots of an equation are 3 and -11, form the quadratic equation.

Concept review questions

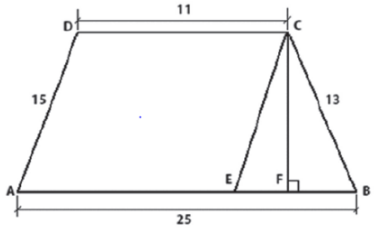
1. What is the value of log 2 + log 3?
   a. log 6 b. 6 c. log 3/2 d. log 2/3
2. Solve value for x: log 3x - log 6 = log 9.
   a. 6 b. 8 c. 12 d. 18
3. The value of $\frac{1}{\log_2 50} + \frac{1}{\log_5 50} + \frac{1}{\log_2 50}$ is
   a. 0 b. 1 c. 10 d. 50
4. $\frac{\log 9^2}{\log 9}$ is equal to
   a. 0 b. 1 c. ½ d. None
5. How many digits are there in 125^{50} given that log 5 = 0.69897?
   a. 103 b. 105 c. 107 d. 108
6. Mahendar had to multiply a number by 26 but he multiplied it by 62 and as a result, the product is increased by 900. What is the actual product?
   a. 750 b. 650 c. 850 d. 950
7. Karan purchased some apples for Rs. 360. If the cost of an apple is increased by Rs. 3, then he would get 4 apples less for the same amount. How many apples did Karan purchase?
   a. 20 b. 24 c. 30 d. 36
8. The sum of the first twenty terms of an arithmetic progression is 210. Find the sum of its tenth and eleventh terms.
   a. 42 b. 21 c. 28 d. 56
9. The sum of 3 numbers in an arithmetic progression is 36. The sum of their squares is 440. Find the largest among the three numbers.
   a. 14 b. 12 c. 16 d. 18
10. A ball is dropped from a height of 128 m. It bounces back rising to a height of 64 m. Each time it further touches the floor, it rises to the height of half the height it fell from before the previous bounce. Find the total distance travelled by the ball.
    a. 512 b. 256 c. 384 d. None
11. The arithmetic mean of -4, 0, 4, 8, 12,……., 40 is
    a. 16 b. 18 c. 28 d. 38
12. How many numbers are there in between 55 and 4505, which are multiples of 5 and divisible by 3?
    a. 287 b. 295 c. 296 d. 297
13. The A.M and G.M of two numbers are 1/2 and 2/5 respectively. Find the numbers.
    a. 1/5, 2/5 b. 4/5, 1/5 c. 1/5, 3/5 d. 1/5, 1/5
14. In a pond, flowers triple in every 10 minutes. If there are 3 flowers in the pond initially, then in what time the number of flowers become 59049?
    a. 1 hr 30 min b. 1 hr 40 min c. 1 hr d. 2 hrs
15. In the figure, AB and CD are intersecting at O and ÐBOD = 40°. Find x.



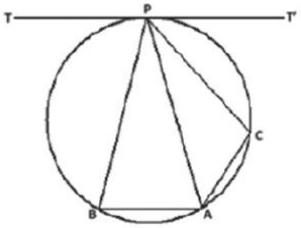
    a. 40° b. 20° c. 140° d. 120°
16. In the figure, ABC is an equilateral triangle. The bisectors of ÐABC and ÐACB meet at D. Find ÐBDC.



    a. 90° b. 60° c. 30° d. 120°
17. Find the area of a trapezium whose parallel sides are 11 m and 25 m long and the non parallel sides are 15 m and 13 m long respectively. The perpendicular distance between the two parallel sides is 12 m.



    a. 216 sq m b. 66 sq m c. 150 sq m d. 78 sq m
18. Given ÐC = 110°; Find ÐAPT.



    a. 70° b. 110° c. 40° d. Cannot determined
19. If Ravi has 2 coins more than Abilash and Abilash has 2 coins more than Raghav. If the product of the number of coins with them is 960, find the number of coins with Abilash.
    a. 10 b. 12 c. 8 d. None
20. Find the equation whose roots are the reciprocal of the roots of the equation $x^2 + 10x + 24 = 0$
    a. $24x^2 + 10x + 1 = 0$ b. $x^2 + 24x + 10 = 0$
    c. $10x^2 + 1x + 24 = 0$ d. None