NUMBERS

DRILL 1: SOLUTIONS

a. *Answer:*

Explanation:

i) 6543 **(Rational)**
ii) √9 = 3 **(Rational)**
iii) 0.13563487623… (**Irrational** number which follows NTNR)
iv) 0.57324354354354… (**Rationa**l number which follows NTR)
v) √11 (**Irrational** number which follows NTNR)

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b. *Answer:*

Explanation:

i) Real, Rational, Whole, Integer and Natural ➔ Ans: **1**(Or any natural number)
ii) A number that is Real and Irrational ➔ Ans: **√3** (=1.732… follows NTNR]
iii) Real, Rational and Non integer ➔ Ans: **1/3**

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c. *Answer:*

Explanation:

In the given diagram, b and f are not possible.
Since all whole numbers belongs to rational number.
Similarly, a and e are not possible since all integers belong to rational and all whole numbers belongs to integers.

-0.263 **(c)** (follows NRT)
245 **(g)**
-67 **(d)**
4783 **(g)**
0.32424… **(c)**
0 **(g)**
Π (-)
√5 (-) } ⇒ both are irrational numbers which are not represented in the diagram.

DRILL 2:

a. *Answer:*

Explanation:

Number	Divisible by
15	3, 5
27	3, 9
36	2, 3, 4, 6, 9
268	2, 4
4518	2, 3, 6, 9
3619	11
15000	2, 3, 4, 5, 6, 8

Students have to circle these numbers based on the divisibility rule learned.

b. *Answer: 5*

Explanation:

The only number which divides the given number 87,195 is **5**

c. *Answer:*

Explanation:

i) Rule for 9 : sum of digits should be divisible by 9.
Ans : **1**
ii) Rule for 33 : The number should be divisible by 11 and 3; checking for 3 will give the answer.
Ans : **2**
iii) Rule for 11 : The difference between sums of alternate numbers should be 0 or 11.
Ans : **4**
iv) Rule for 8 : Last three digits should be divisible by 8.
Ans : **2**

DRILL 3:

a. *Answer: 4*

Explanation:

Last digit of 4^{55}
Power cycle for 4 is 4, 6

- 4 to the power of any odd value is 4
- 4 to the power of any even value is 6

Here the power is 55, so the answer is **4**

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b. *Answer: 9*

Explanation:

Last digit of 135647^{34}
Power cycle pattern for 7 is 7,9,3,1
Here the power is 34
(34/4) gives the remainder 2 (since power cycle of 7 repeats for every 4 values, so we are dividing the power by 4) so now 7^{34} is similar to 7^2
Hence the last digit is **9**.

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c. *Answer: 2*

Explanation:

Find the remainder when 3^{75} is divided by 5.
Power cycle of 3 is 3,9,7,1
Power cycle value when divided by 5, the reminder also follows a pattern;
3/5 = R(3),
9/5 = R(4),
7/5 = R(2),
1/5 = R(1).
$3^{75/4} = 3^{R(3)}$
$= 2$
According to the pattern given above, the reminder is **2**.

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d. *Answer: 7*

Explanation:

To find the rightmost non-zero integer in the expression $1430^{343} + 1470^{367}$
1430^{367} have more number of zeroes than 1430^{343}.
So the rightmost non-zero will be from 1430^{343} (the value should be 3^{343})
Power cycle pattern for 3 is 3,9,7,1.

$3^{343/4}=3^3$ (since the pattern is repeated for every 4 terms)
3^3 means the unit place digit is **7**.
So the last digit of the given number is **7**.

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e. *Answer: 3*

Explanation:

To find the remainder when 7^{203} is divided by 4
The power cycle of 7 is 7,9,3,1
When these values are divided by 4, the remainder will also be repeated for every 4 terms.
i.e. 7/4 =R(3),
9/4 =R(1),
3/4 =R(3),
1/4 =R(1).
The remainder pattern is 3,1,3,1.
So we can conclude that any odd power value divided by 4 the remainder should be 3 and any even power value is divided by 4 the remainder should be 1.
Here in 7^{203}, the power is odd, so the remainder is **3**.

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f. *Answer: 48*

Explanation:

Find the highest power of 12 in 100!
To find number of 12 in 100! We should know number of 4's and 3's.
i.e. 12 can be prime factorized into 4 and 3.
Again 4 can be also prime factorized into $2^{2.}$
Now we are going to find number of 2's and 3's in 100!

3	100
3	33
3	11
3	3
	1

2	100
2	50
2	25
2	12
2	6
2	3
	1

Number of 3's is 33+11+3+1=48.
Number of 2's is 50+25+12+6+3+1=97.
So number of 4's =48.
We can form 12 with the pair of 3's and 4's only.
So the number of 12's in 100 is **48**.

DRILL 4:

a. *Answer: 45*

Explanation:

3600 can be factorized as : $2^4*3^2*5^2$
Hence number of factor will be : (4+1)*(2+1)*(2+1)
5*3*3=**45**

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b. *Answer: 10*

Explanation:

HCF of 20 and 30
Factors of 20 -> 1, 2, 4, 5, 10, 20
Factors of 30 -> 1, 2, 3, 5, 6, 10, 15, 30.
Common factors are 1, 2, 5 and 10.
Highest common factor is **10**

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c. *Answer: 2*

Explanation:

Factors of 12 -> 1, 2, 3, 4, 6, 12
Factors of 14 -> 1, 2, 7
Common factors are ->1 and 2
Highest common factor is **2**

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d. *Answer: 35*

Explanation:

Multiples of 5 -> 5, 10, 15, 20, 25, 30, 35, 40...
Multiples of 7 -> 7, 14, 21, 28, 35, 42...
Least common multiple (LCM) is **35**.

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e. *Answer: 30*

Explanation:

Multiples of 6 -> 6, 12, 18, 24, 30, 36...
Multiples of 10 -> 10, 20, 30, 40...
Common multiples are 30, 60…
Least common multiple is **30**.

[Take the highest number i.e. 10 and check the next few multiples to find a common multiple]

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f. *Answer: 350*

Explanation:

HCF(50,70)

Can be found using L-division;

2	50 , 70
5	25 , 35
	5 , 7

HCF = 2*5 = 10

LCM = 2*5*5*7 = **350**

[LCM can be formed by taking the multiples of greatest number i.e 70]

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g. *Answer: 15 seconds*

Explanation:

This question is the application of LCM

LCM of 3 and 5 = **15 seconds.**

Hence both the lights will flash together for every **15 seconds.**

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h. *Answer:112*

Explanation:

The ratio between circumference of front wheel and back wheel is 4:7

We know,

Distance= circumference * revolution

Circumference is inversely proportional to revolution

Hence ratio of revolution is 7:4

The difference in ratio is 3 but it is given that there is a difference of 12 revolutions.

Hence (7:4)*4 = 28:16

Hence distance covered = 7* 16=**112.**

DRILL 5:

a) *Answer: 6*

Explanation:

Find the greatest number that will exactly divide 24, 36 and 42?
This question is the application of HCF [Type 1].
i.e HCF (24,36,42)= **6**.

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b) *Answer: 6*

Explanation:

Find the greatest number that will divide 27, 38, 47 leaving remainders of 3, 2 and 5 respectively?
Type 2: HCF [(27-3), (38-2), (47-5)]
HCF (24, 36, 42) = **6**

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c) *Answer: 36*

Explanation:

Find the greatest number which when it divides 74,110 and 182 will leave the same remainder in each case?
Type 3: HCF [(110-74), (182-110)]
HCF (36, 72) = **36**

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d) *Answer: 120*

Explanation:

Find the least number which is exactly divisible by 12, 8 and 10?
Type 4: LCM (12, 8, 10) =**120**

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e) *Answer: 7*

Explanation:

Find the least number which when divided by 12, 8 and 10 leaves the same remainder 7 in each case?
Type 5: LCM (12, 8, 10) +7
[But here since the remainder is less than all the three numbers, so the least number we require will be 7 itself]
Ans is **7**.

f) *Answer: 118*

Explanation:

Find the least number which when divided by 12, 8 and 10 leaves the remainders 10, 6, 8 respectively?
Type 6: Since we have a difference of 2 in each case, we would first determine the
LCM of given numbers and subtracts 2 from them
LCM (12, 8, 10) - 2 = 120-2
= 118.

GOOGLY QUESTIONS:

1. *Answer:* **Wrong**

   Explanation:

   Since the question given is to find the minimum number of rooms and not the number of participants in each room.

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2. *Answer:* **Wrong**

   Explanation:

   a. 23 - rational
   b. 25.6 - rational
   c. 26.2464646…… - rational (NRT)
   d. 24.34542319... - Irrational
   e. Pi - Irrational (NRNT)

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3. *Answer:* **Wrong**

   Explanation:

   In prime factorized format, base numbers should be prime numbers only. But here 10 is in the base and it can be decomposed further to 2*5.

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4. *Answer:* **Correct**

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5. *Answer:* **Wrong**

   Explanation:

The largest power of 10 in 100! Has to be encountered by counting the number of 5's and 2's in 100!

Because 10 is a composite number and cannot be done directly.

100/5 = 20

20/5 = 4

24

5's are 24 times in 100!

2	100
2	50
2	25
2	12
2	6
2	3
	1

Number of 2's in 100! is 50+25+12+6+3+1=97.

But 10 can be formed only from a pair of 5 and 2.

So 5's are limited and we have only 24.

CONCEPT REVIEW QUESTIONS:

1. *Answer: 121*

Explanation:

We know that factors are also divisors, which means when the given number is divided by the factor, it is completely divisible i.e., Remainder is 0.

Given number, $(a*4^3*6^6*3^{11})$ when divided by 11^2 and 3^3 should give us the remainder 0.

$$\frac{(a*4^3*6^6*13^{11})}{11^2*3^3}$$ Here $6^6 = 3^6*2^6$

3^3 in denominator can be cancelled by the 3^6 in the numerator.

Now the denominator is 11^2 which is not present anywhere in the numerator, so that we can get the remainder to be 0.

Therefore 11^2 should be the smallest value of 'a' so that the values can cancel each other.

Hence a = 11^2 = **121**

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2. *Answer: 645*

Explanation:

$323^2 - 322^2$

The above question is of the form $a^2 - b^2$, where a = 323, b = 322

$(a^2 - b^2)$ = (a+b)*(a-b)
= (323+322)*(323-322)
= (645)*(1)
= **645**

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3. *Answer:192ways*

Explanation:

The divisibility test for 4 is that the last 2 digits should be divisible by 4.

_ _ _ 1 2
1 6
2 4
3 2
3 6
5 2
5 6
6 4

The last two digits can be filled in any of above written 8 ways.
Since the numbers we use should not repeat, the first place can be filled with any of the '4' digits apart from what is used in the last two digits.
Similarly the second place can be filled with the other '3' digits and the remaining two digits can be used in the third place.
Therefore, the number of different 5 digit numbers = 4*3*2*(8ways)
= **192 ways**

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4. *Answer: 6 numbers*

Explanation:

20*21*22*23*24 ➔ Till this part of calculation we will be getting only one zero, but when we multiply 25, which has two 5's in it, our answer will result in 3 zeroes.
Hence we have to multiply **6 numbers.**

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5. *Answer: 1 or 5*

Explanation:

'X' is a prime number greater than 5, the first prime number will be '7' which when divide by 6, will give us a remainder '1' and the next prime number is '11' which when divided by 6, will give us a remainder '5'.
Similarly, 13/6 = Remainder (1)
17/6 = Remainder (5)
Hence the remainder is either **1 or 5.**

6. *Answer:* 881

Explanation:

Let 'X' be the given number and it is given that
X divided by 9 ➔ Quotient – A, Remainder – 8
A divided by 11 ➔ Quotient – B, Remainder – 9
B divided by 13 ➔ Quotient – C, Remainder – 8
X divided by 1287 ➔ Quotient – y, Remainder – ??
We know that,
Number or Dividend = (Divisor * Quotient) + Remainder

$X = 9A + 8$ --------- (1)
$A = 11B + 9$ -------- (2)
$B = 13C + 8$ --------- (3)
$X = 1287y +?$

Substituting (3) in (2) and (2) in (1), we get

$A = 11(13C + 8) + 9$
$= 143C + 88 + 9$
$= 143C + 97$
$X = 9(143C + 97) + 8$
$= 1287C + 873 + 8$
$= 1287C + 881$

When X is divided by 1287, the remainder will be **881**

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7. *Answer: 2, 6*

Explanation:

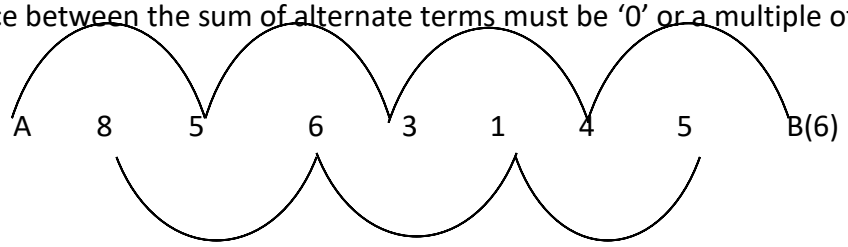
88 can be written as 8*11, where 8 and 11 are co-primes.

Divisibility rule for 8:

Last three numbers should be divisible by 8. Hence taking "45B" substituting the values from options, we can see, only '6' satisfies the condition, so last three numbers has to be "456".

Divisibility rule for 11:

The difference between the sum of alternate terms must be '0' or a multiple of 11.



$(18 + A) – 20 = 0$
$A = 2$
Answer: **2, 6**

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8. *Answer: 59*

Explanation:

Similar to 6th problem

8 → 3

11 → 7

(Ignore)

88 → ??

(8*7) = 56

56 + 3 = 59

Remainder = **59**

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9. *Answer: 5*

Explanation:

Using power cycle concept,

$(3)^{22}$ = 9 (unit digit)

$(5)^{46}$ = 5 (5 raised to any power will have 5 in the unit's digit)

$(1)^{43}$ = 1 (unit digit)

9*5*1 = 45

Hence the unit's digit will be **5**.

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10. *Answer: 18 litres*

Explanation:

Keyword: Maximum (H.C.F.)

Prime factorizing:

2	126
7	63
3	9
	3

2	54
3	27
3	9
3	3
	1

2	144
2	72
2	36
2	18
3	9
3	3
	1

126 = **2*3*3***7

54 = **2*3*3***3

144 = **2*3*3***2*2*2

H.C.F. = 2*3*3

= **18 litres**

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11. *Answer: 622*

Explanation:

Keyword: Least number (L.C.M.)

The number divided by		Remainder
7	→	6
8	→	6
11	→	6

L.C.M. of (7, 8, 11) + 6 will be the answer.
(The least number can be 6, but since it is not given in the options, we consider the next least number)
L.C.M. of 7,8,11 = 616 [Product of 7,8,11 will be the L.C.M. because they are co-primes]
Answer: LCM + 6 = **622**

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12. *Answer: 94*

Explanation:

Keyword: Least Number (L.C.M.)

Number divided by		Remainder
12	→	10
24	→	22
32	→	30

In the above concepts, we can see there is a negative remainder '2' which means we have to find the L.C.M. of the numbers and subtract '2' from it.

2	12, 24, 32
2	6, 12, 16
2	3, 6, 8
3	3, 3, 4
	1, 1, 4

L.C.M = 2*2*2*3*4
= 96
L.C.M. – 2 = 96 – 2
= **94**

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13. *Answer: 12.30pm*

Explanation:

Bells ring together, which means we have to find the L.C.M. of the given intervals.
L.C.M = 2*3*5*7
= 210mins
After 9am, bells will ring together after 210 minutes (3hours and 30 minutes).
9am + (3hr and 30mins) = **12.30pm** (option is wrong)

14. *Answer: 12, 1/315*

Explanation:

LCM of fraction = (LCM of numerator)/ (HCF of denominator)
HCF of fraction = (HCF of numerator) / (LCM of denominator)
Here,
LCM of numerator = 12
HCF of denominator = 1
Therefore,
LCM of the fraction = 12
HCF of numerator = 1
LCM of denominator = 5*7*4
= 315
Therefore,
HCF of the fractions = 1/315
Answer: **12, 1/315**

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15. *Answer: 2*

Explanation:

Keyword: Only 3 factors
3 is an odd number, which means that it is enough to check for the perfect square numbers in two-digit numbers.
The perfect square numbers are 16, 25, 36, 49, 64 and 81 out of which only 25 and 49 have 3 factors.
Hence answer is **2**.
Note: the square of a prime number will always have only 3 factors
For eg: 121→ 1, 11, 121

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16. *Answer:3*

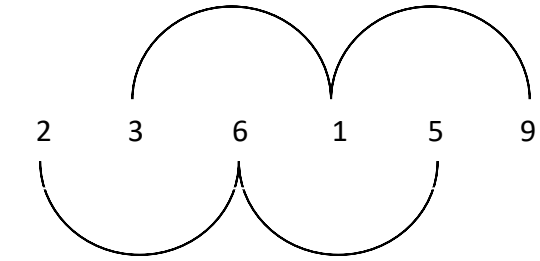
Explanation:

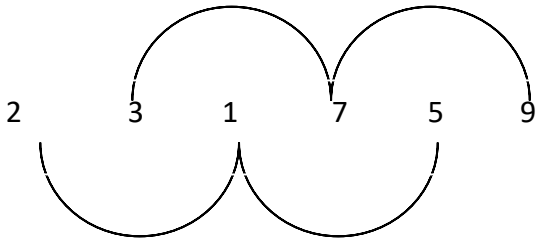
By checking the basic values,
$10^1 - 7 = 3$ [Divisible by 3]
$10^2 - 7 = 93$ [Divisible by 3]
$10^3 - 7 = 993$ [Divisible by 3]
.
.
.
$10^{25} - 7$ will also be divisible by 3
Answer: **3**

17. *Answer: 61 or 17*

Explanation:

We know, that for divisibility test for 11, it is difference between the sum of the alternate terms must be '0' or a multiple of 11.
Substituting values from the options,





3+1+9 = 13,
2+6+5 = 13.
13 – 13 = 0

3+7+9 = 19,
2+1+5 = 8
19 – 8 = 11
Answer: **61 or 17**

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18. *Answer: 0*

Explanation:

$6561 \rightarrow 9^4$
The divisibility rule for '9' is that the digital root has to be 9.
The digital root is satisfying as it is 9 for the given number, which means the values in the denominator will definitely cancel out with the value in the numerator, without leaving any remainder.
Hence answer is **0**.

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19. *Answer: 1*

Explanation:

Let the number be 'N' which when divided by D leaves a remainder 13.
Let the quotient be n.
Given,
N = nD + 13
3N = 3(nD+13)
= 3nD + **39**
But the remainder here is 2.
Which means the number has to be (39-2) = 37.
Number of possibilities = 1

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20. *Answer: more than one of the above*

Explanation:

$a^n - b^n$ is divisible by (a+b), when n is **even.**
Case 1:
In $7^{6n} - 6^{6n}$, a = 7 and b = 6.
a+b = 7+6
= 13
a+b = 13
$7^{6n} - 6^{6n} = 49^{3n} - 36^{3n}$, here n can be **odd or even**, which means $a^n - b^n$ is divisible by (a - b)
Case 2:
In $49^{3n} - 36^{3n}$, a = 49 and b = 36.
a-b = 49-36
a-b = 13
$49^{3n} - 36^{3n} = 343^{2n} - 216^{2n}$, here n has to be **even**, which means $a^n - b^n$ is divisible by (a - b) and (a+b)
In $343^{2n} - 216^{2n}$, a = 343 and b = 216.
a-b = 343-216
a-b = 127 and a+b = 559
Answer: More than one of the above.