

## UNIT 2: TESTING OF HYPOTHESIS:

Hypothesis: A statement related to population parameters.

↓

Two types

→ NULL: No difference statement

Ex:  $H_0: \mu = 3$ ,  $H_0: \mu_1 = \mu_2$ ,  $H_0: \sigma^2 = 2$

→ Alternate: Complementary to NULL hypothesis.

Ex:  $H_1: \mu \neq 3$ ,  $H_1: \mu_1 \neq \mu_2$ ,  $H_1: \sigma^2 \neq 2$

↓  
Two types

Single tailed (one tailed)

↓  
Tests the possibility of relationship in one direction and completely ignores the possibility of a relationship in another direction. The values differ from reference only in one direction.

Left Tailed

↓  
A hypothesis test where the rejection region is located to the extreme left of the distribution.

Right tailed.

↓  
A hypothesis test where the rejection region is located to the extreme right of the distribution.

$$H_A < \alpha$$

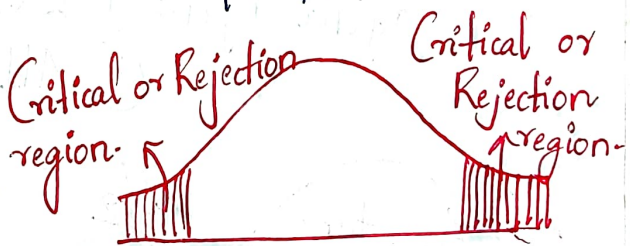
Critical region



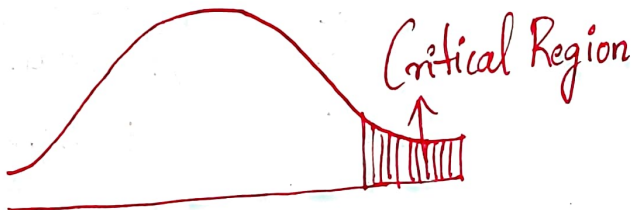
Two tailed

↓  
Tests whether a sample is greater than or less than a certain range of values. It is non-directional because the effects are tested for in both directions.

$$\begin{array}{l|l} \text{Ex: } H_0: \mu_1 = \mu_2 & H_0: \sigma^2 = 3 \\ H_1: \mu_1 \neq \mu_2 & H_1: \sigma^2 \neq 3 \end{array}$$



$$H_A > \alpha$$



## Critical Region (Rejection Region)

The set of all values of the test static that would cause us to reject the NULL hypothesis. This area is defined by Critical Value(s).



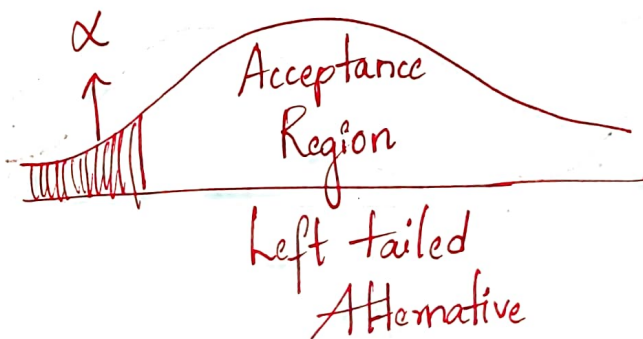
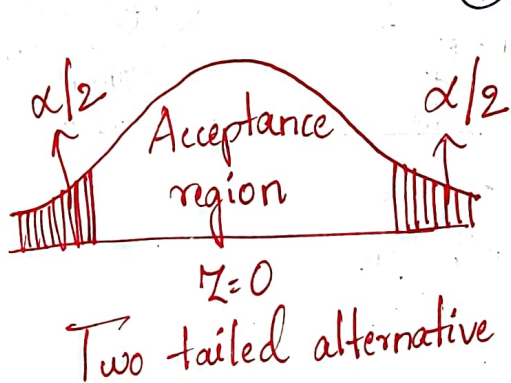
## Level of Significance:

The probability of rejecting the null hypothesis when it is true

$$\begin{aligned} P(t \in W | H_0) &= \alpha \\ P(t \in \bar{W} | H_1) &= \beta \\ W \cup \bar{W} &= S \\ W \cap \bar{W} &= \emptyset \end{aligned}$$

## Critical Value or Significant Value

↓  
depends upon: ① Level of significance  
② Alternate Hypothesis



## Types of Error:

Type I  
 ↓  
 probability of rejecting a good lot (producer's risk) denoted by  $\alpha$

Type II  
 ↓  
 probability of accepting a bad lot (consumer's risk) denoted by  $\beta$

## Level of Significance

	1%	5%	10%
Two-tailed	$Z_{\alpha/2} = 2.58$	$Z_{\alpha/2} = 1.96$	$Z_{\alpha/2} = 1.65$
Right-tailed	$Z_{\alpha} = 2.33$	$Z_{\alpha} = 1.65$	$Z_{\alpha} = 1.29$
left-tailed	$Z_{\alpha} = -2.33$	$Z_{\alpha} = -1.65$	$Z_{\alpha} = -1.29$

power of the test =  $1 - \beta$

For left-tailed:

Calculated value < table value → reject  $H_0$   
 Calculated value > table value → accept  $H_0$

For 2-tailed & right-tailed  
 Cal Value < table value → accept  $H_0$   
 Cal Value > table value → reject  $H_0$

## Test for single proportion:

$$Z = \frac{x - np}{\sqrt{npq}} \sim N(0,1)$$

$n$  → no. of times coin is tossed

$p$  → prob of success

$q$  → " " failure

COIN Toss PROBLEMS



## Test for single proportion

Cont'd:

$$Z = \frac{p - P}{\sqrt{PQ/n}} \sim N(0,1)$$

$p \rightarrow$  sample proportion

$P \rightarrow$  population proportion

$Q \rightarrow 1 - P$   $p = x/n$

$n \rightarrow$  sample size

## Confidence interval for true proportion:

$$p - z_{\alpha/2} \sqrt{pq/n} < P < p + z_{\alpha/2} \sqrt{pq/n}$$

NOTE: When level of significance not given, take default 5% (0.05 LOS)

Test for single proportion means: One sample and a claim about a proportion of the sample is tested.

## Test for difference of proportions:

$$\textcircled{1} Z = \frac{p_1 - p_2}{\sqrt{PQ(1/n_1 + 1/n_2)}} \quad \begin{matrix} p_1 = \text{I sample proportion} \\ p_2 = \text{II " " " "} \\ Q = 1 - P \end{matrix}$$

$$P = \frac{X_1 + X_2}{n_1 + n_2} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} \quad \begin{matrix} n_1 \rightarrow \text{I sample size} \\ n_2 \rightarrow \text{II " " " "} \end{matrix}$$

$$\textcircled{2} Z = \frac{P_1 - P_2}{\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}} \sim N(0,1)$$

$\rightarrow$  Used when two different populations are discussed.  
Men, Women / City A, City B.

$P_1 \rightarrow$  I population proportion

$P_2 \rightarrow$  II " " "

$Q_1 \rightarrow 1 - P_1$ ,  $Q_2 \rightarrow 1 - P_2$

$n_1 \rightarrow$  I sample size

$n_2 \rightarrow$  II " " "

③ Confidence interval:

$$(p_1 - p_2) \pm z_{\alpha/2} \sqrt{pq(1/n_1 + 1/n_2)}$$

Both ① & ② are used when two different categories of populations are given.

populations are of

(1) used when the number of proportion is known.

(2) is " " " percentage " " " " .

## TEST FOR SINGLE MEAN:

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$\bar{x} \rightarrow$  Sample mean

$\mu \rightarrow$  population mean

$\sigma \rightarrow$  S.D of pop<sup>n</sup>. If not given, use  
S.D of sample (s).

$n \rightarrow$  ~~no~~ sample size.

## Confidence Interval:

$$(\bar{x} - \mu) \pm z_{\alpha/2} \left( \frac{s}{\sqrt{n}} \right)$$

## TEST FOR DIFFERENCE OF MEANS

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\alpha_1 \rightarrow$  1st Sample mean

$$a_2 \rightarrow 2^{\text{nd}} \quad \text{"} \quad \text{"} \quad \text{"}$$

$S_1 \rightarrow 1^{\text{st}}$  " S.D

$c \rightarrow 2^{\text{nd}}.$  " "

$S_2 \rightarrow 2^{nd}$  size

$n_1 \rightarrow$  1st, 2nd, 3rd, 4th, 5th, 6th, 7th, 8th, 9th, 10th, 11th, 12th, 13th, 14th, 15th, 16th, 17th, 18th, 19th, 20th, 21st, 22nd, 23rd, 24th, 25th, 26th, 27th, 28th, 29th, 30th, 31st, 32nd, 33rd, 34th, 35th, 36th, 37th, 38th, 39th, 40th, 41st, 42nd, 43rd, 44th, 45th, 46th, 47th, 48th, 49th, 50th, 51st, 52nd, 53rd, 54th, 55th, 56th, 57th, 58th, 59th, 60th, 61st, 62nd, 63rd, 64th, 65th, 66th, 67th, 68th, 69th, 70th, 71st, 72nd, 73rd, 74th, 75th, 76th, 77th, 78th, 79th, 80th, 81st, 82nd, 83rd, 84th, 85th, 86th, 87th, 88th, 89th, 90th, 91st, 92nd, 93rd, 94th, 95th, 96th, 97th, 98th, 99th, 100th, 101st, 102nd, 103rd, 104th, 105th, 106th, 107th, 108th, 109th, 110th, 111th, 112th, 113th, 114th, 115th, 116th, 117th, 118th, 119th, 120th, 121st, 122nd, 123rd, 124th, 125th, 126th, 127th, 128th, 129th, 130th, 131st, 132nd, 133rd, 134th, 135th, 136th, 137th, 138th, 139th, 140th, 141st, 142nd, 143rd, 144th, 145th, 146th, 147th, 148th, 149th, 150th, 151st, 152nd, 153rd, 154th, 155th, 156th, 157th, 158th, 159th, 160th, 161st, 162nd, 163rd, 164th, 165th, 166th, 167th, 168th, 169th, 170th, 171st, 172nd, 173rd, 174th, 175th, 176th, 177th, 178th, 179th, 180th, 181st, 182nd, 183rd, 184th, 185th, 186th, 187th, 188th, 189th, 190th, 191st, 192nd, 193rd, 194th, 195th, 196th, 197th, 198th, 199th, 200th, 201st, 202nd, 203rd, 204th, 205th, 206th, 207th, 208th, 209th, 210th, 211st, 212nd, 213rd, 214th, 215th, 216th, 217th, 218th, 219th, 220th, 221st, 222nd, 223rd, 224th, 225th, 226th, 227th, 228th, 229th, 230th, 231st, 232nd, 233rd, 234th, 235th, 236th, 237th, 238th, 239th, 240th, 241st, 242nd, 243rd, 244th, 245th, 246th, 247th, 248th, 249th, 250th, 251st, 252nd, 253rd, 254th, 255th, 256th, 257th, 258th, 259th, 260th, 261st, 262nd, 263rd, 264th, 265th, 266th, 267th, 268th, 269th, 270th, 271st, 272nd, 273rd, 274th, 275th, 276th, 277th, 278th, 279th, 280th, 281st, 282nd, 283rd, 284th, 285th, 286th, 287th, 288th, 289th, 290th, 291st, 292nd, 293rd, 294th, 295th, 296th, 297th, 298th, 299th, 300th, 301st, 302nd, 303rd, 304th, 305th, 306th, 307th, 308th, 309th, 310th, 311st, 312nd, 313rd, 314th, 315th, 316th, 317th, 318th, 319th, 320th, 321st, 322nd, 323rd, 324th, 325th, 326th, 327th, 328th, 329th, 330th, 331st, 332nd, 333rd, 334th, 335th, 336th, 337th, 338th, 339th, 340th, 341st, 342nd, 343rd, 344th, 345th, 346th, 347th, 348th, 349th, 350th, 351st, 352nd, 353rd, 354th, 355th, 356th, 357th, 358th, 359th, 360th, 361st, 362nd, 363rd, 364th, 365th, 366th, 367th, 368th, 369th, 370th, 371st, 372nd, 373rd, 374th, 375th, 376th, 377th, 378th, 379th, 380th, 381st, 382nd, 383rd, 384th, 385th, 386th, 387th, 388th, 389th, 390th, 391st, 392nd, 393rd, 394th, 395th, 396th, 397th, 398th, 399th, 400th, 401st, 402nd, 403rd, 404th, 405th, 406th, 407th, 408th, 409th, 410th, 411st, 412nd, 413rd, 414th, 415th, 416th, 417th, 418th, 419th, 420th, 421st, 422nd, 423rd, 424th, 425th, 426th, 427th, 428th, 429th, 430th, 431st, 432nd, 433rd, 434th, 435th, 436th, 437th, 438th, 439th, 440th, 441st, 442nd, 443rd, 444th, 445th, 446th, 447th, 448th, 449th, 450th, 451st, 452nd, 453rd, 454th, 455th, 456th, 457th, 458th, 459th, 460th, 461st, 462nd, 463rd, 464th, 465th, 466th, 467th, 468th, 469th, 470th, 471st, 472nd, 473rd, 474th, 475th, 476th, 477th, 478th, 479th, 480th, 481st, 482nd, 483rd, 484th, 485th, 486th, 487th, 488th, 489th, 490th, 491st, 492nd, 493rd, 494th, 495th, 496th, 497th, 498th, 499th, 500th, 501st, 502nd, 503rd, 504th, 505th, 506th, 507th, 508th, 509th, 510th, 511st, 512nd, 513rd, 514th, 515th, 516th, 517th, 518th, 519th, 520th, 521st, 522nd, 523rd, 524th, 525th, 526th, 527th, 528th, 529th, 530th, 531st, 532nd, 533rd, 534th, 535th, 536th, 537th, 538th, 539th, 540th, 541st, 542nd, 543rd, 544th, 545th, 546th, 547th, 548th, 549th, 550th, 551st, 552nd, 553rd, 554th, 555th, 556th, 557th, 558th, 559th, 560th, 561st, 562nd, 563rd, 564th, 565th, 566th, 567th, 568th, 569th, 570th, 571st, 572nd, 573rd, 574th, 575th, 576th, 577th, 578th, 579th, 580th, 581st, 582nd, 583rd, 584th, 585th, 586th, 587th, 588th, 589th, 590th, 591st, 592nd, 593rd, 594th, 595th, 596th, 597th, 598th, 599th, 600th, 601st, 602nd, 603rd, 604th, 605th, 606th, 607th, 608th, 609th, 610th, 611st, 612nd, 613rd, 614th, 615th, 616th, 617th, 618th, 619th, 620th, 621st, 622nd, 623rd, 624th, 625th, 626th, 627th, 628th, 629th, 630th, 631st, 632nd, 633rd, 634th, 635th, 636th, 637th, 638th, 639th, 640th, 641st, 642nd, 643rd, 644th, 645th, 646th, 647th, 648th, 649th, 650th, 651st, 652nd, 653rd, 654th, 655th, 656th, 657th, 658th, 659th, 660th, 661st, 662nd, 663rd, 664th, 665th, 666th, 667th, 668th, 669th, 670th, 671st, 672nd, 673rd, 674th, 675th, 676th, 677th, 678th, 679th, 680th, 681st, 682nd, 683rd, 684th, 685th, 686th, 687th, 688th, 689th, 690th, 691st, 692nd, 693rd, 694th, 695th, 696th, 697th, 698th, 699th

$$n_2 \rightarrow 2^{100}$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$\sigma \rightarrow$  population S.D

Confidence interval formula:

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

# CONFIDENCE INTERVAL PROBLEM FOR TRUE PROPORTIONS.

Note points:

- \* Confidence interval problems are always **Two-tailed.**
- \* Confidence interval is the same as confidence limits.  
↳ Both have upper and lower bounds.

## PROBLEMS:

- ① In a random sample of 160 workers exposed to a certain amount of radiation, 24 experienced some ill effects. Construct a 99% confidence interval for the corresponding true proportion.

Soln: Given that:

$$n = 160$$

$$x = 24$$

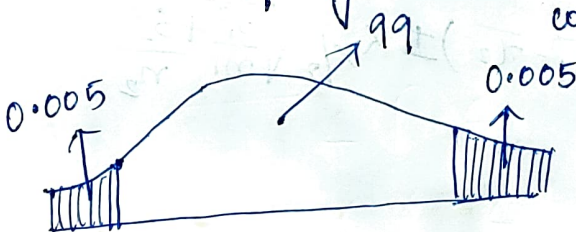
$$p = x/n = \frac{24}{160} = 0.15$$

$$q = 1 - p = 1 - 0.15 = 0.85$$

Given confidence region  $\rightarrow 99$

Critical region  $\rightarrow 100 - 99 = 1$

$\Rightarrow \alpha = 1\%$  level of significance. Since it is two-tailed, we consider  $\alpha/2 = \frac{0.01}{2} = 0.005$ .





$\Rightarrow 100 - 0.005$   
 $\Rightarrow 99.995$

Why are we doing this? It is two-tailed. The fraction part is considered as 0.005  $\rightarrow$  we take the absolute value disregarding the sign. Knowing the value for any one of the extreme ends is sufficient.

~~We consider the fraction part~~

$1 - 0.005$  i.e.  $0.995$   
 $= 0.995$

Hence, we redefine the confidence level.

Search this value in the table (Normal Distribution table).

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.04										
0.1										
0.2										
...										
2.5									0.995	

$\Rightarrow Z_{\alpha/2} = 2.58$

$$p - Z_{\alpha/2} \sqrt{pq/n} < P < p + Z_{\alpha/2} \sqrt{pq/n}$$

$$0.07117 < P < 0.22228$$

(2) A random sample of 500 apples was taken from a large consignment and 60 were found to be bad. Obtain 98% confidence limits.

Soln: Given:

$n = 500$

$X = 60$

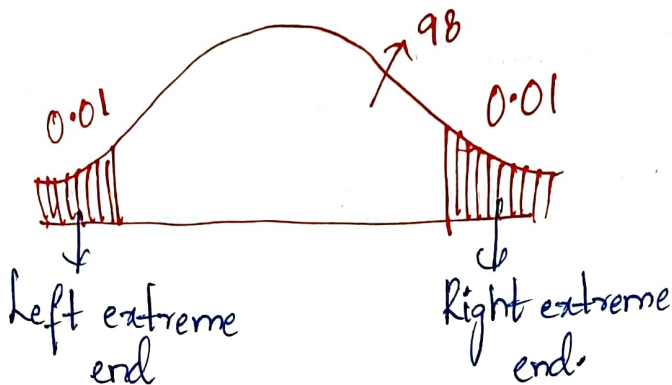
$p = 0.12$

$q = 1 - p = 0.88$

Given confidence region  $\rightarrow 98$

Critical region  $\rightarrow 100 - 98$   
 $= 2$

Two-tailed  $\rightarrow Z_{\alpha/2} \Rightarrow \alpha = 0.02$   
 $\frac{\alpha}{2} = \frac{0.02}{2} = 0.01$



It is sufficient that we find the Z-score for one extreme end. So we redefine the confidence area as follows:

~~100 - 0.02~~  
 ~~$= 99.99$~~

$1 - 0.01$   
 $= 0.99$

~~Fraction part~~  $\rightarrow$   $\boxed{0.99}$  in  
Search it in the table.

$\Rightarrow$

	0.01	0.02	$\boxed{0.03}$	0.04	0.05	0.06	0.07
0.0							
0.1							
0.2							
...							
...							
$\boxed{2.3}$			$\rightarrow 0.9901$				

$$\Rightarrow p - Z_{\alpha/2} \sqrt{pq/n} < P < p + Z_{\alpha/2} \sqrt{pq/n}$$
$$9 < P < 16$$



Suppose we are asked for, 96% Confidence interval

Confidence region  $\rightarrow 96$

Critical "  $\rightarrow 100 - 96$   
 $= 4$

$$4\% \text{ LOS} \Rightarrow Z_{\alpha/2} = \frac{0.04}{2} = 0.02$$

New confidence region  $\rightarrow$  ~~100 - 0.02~~  $1 - 0.02$   
 $=$  ~~99.98~~  $= 0.98$

~~fraction part~~  $\rightarrow$  0.98

$\downarrow$   
Search in table

$$Z_{\alpha/2} = 2.05$$

# SM Model Question Paper

## UNIT - 2

3a) Given:  $H_0: P_1 = 0.85$  [Two-tailed]  
 $H_1: P_1 \neq 0.85$

$$X = 18$$

$$n = 30$$

$$\alpha = 0.05$$

$$p = \frac{18}{30}$$

$$= 0.6$$

$$P = 0.85$$

$$Q = 1 - 0.85$$

$$= 0.15$$

Formula to be used:  $\frac{p - P}{\sqrt{PQ/n}}$

$$Z = \frac{0.6 - 0.85}{\sqrt{(0.85 \times 0.15) / 30}}$$

$$= \frac{0.6 - 0.85}{\sqrt{0.06519}}$$

$$= \frac{-0.25}{0.06519}$$

$$= \frac{-0.25}{0.06519}$$

$$= \frac{-0.25}{0.06519}$$

$$= \frac{-0.25}{0.06519}$$

$$= \frac{-0.25}{0.06519}$$

$$= \frac{-0.25}{0.06519}$$

$$= \frac{-0.25}{0.06519}$$

$Z$  at 5% LOS for 2-tailed is 1.96  
 Cal Val > Tab Val.

Reject  $H_0$

Inference: The survival rate cannot be taken as 85%.

3) b) Given:

$$n = 1000$$

$$x = 540$$

$$\hat{p} = \frac{x}{n}$$

$$\hat{p} = 0.54$$

$$P = 0.5$$

$$Q = 1 - 0.5$$

$$= 0.5$$

$$H_0: p_1 = 0.5 \left[ \begin{array}{l} 0.5 \text{ because} \\ p_2 \neq 0.5 \text{ [equally populated]} \end{array} \right]$$

$$\alpha = 0.01$$

Formula to be used  $\Rightarrow Z = \frac{\hat{p} - P}{\sqrt{PQ/n}}$

$$= \frac{0.54 - 0.5}{\sqrt{(0.5 \times 0.5)/1000}}$$

$$Z = 2.531$$

$Z$  at 1% LOS for 2-tailed  $\rightarrow 2.58$

Cal Val  $<$  Tab Val

Accept  $H_0$

Inference: Rice & Wheat eaters are equally populated.



4) a) Given:

### POPULATION PERCENTAGES.

$$n_1 \rightarrow 1200 \quad P_1 \rightarrow 0.3$$

$$n_2 \rightarrow 900 \quad P_2 \rightarrow 0.25$$

$$Q_1 \rightarrow 1 - P_1$$

$$= 1 - 0.3$$

$$= 0.7$$

$$Q_2 \rightarrow 1 - P_2$$

$$= 1 - 0.25$$

$$= 0.75$$

$H_0$ : Differently likely  
hidden

$H_1$ : Not likely to be  
hidden.

$\alpha = 5\%$  (default)

Formula to be used:  $P_1 - P_2$

$$\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}$$

$$Z = \frac{0.3 - 0.25}{\sqrt{\frac{0.3 \times 0.7}{1200} + \frac{0.25 \times 0.75}{900}}}$$

$$Z = 2.55$$

Z at 5% LOS for two-tailed  $\rightarrow 1.96$

Cal Val  $>$  Tab Val

Reject  $H_0$

Inference: Not likely to be hidden.

4) b) Given:  
 $H_0: \mu_1 \rightarrow \text{Australians height} = \text{Englishmen}$   
 $H_1: \mu_1 \rightarrow \text{'' '' '' } > \text{'' '' ''}$

↓  
Test for difference of Means problem:

$$\bar{x} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$\bar{x}_1 = 67.85 \quad \bar{x}_2 = 68.55$$

$$S_1 = 2.56 \quad S_2 = 2.52$$

$$n_1 = 6400 \quad n_2 = 1600$$

$$Z = \frac{68.55 - 67.85}{\sqrt{\frac{(2.52)^2}{1600} + \frac{(2.56)^2}{6400}}}$$

$$Z = 9.9$$

$Z$  at 5% Level of Significance for right tailed is 1.65.

Cal Val  $\geq$  Tab Val

Reject  $H_0$ .

Inference: Australians are on avg taller than Englishmen.