

CRT

$$x \equiv 2 \pmod{3}$$

$$x \equiv 4 \pmod{5}$$

$$x \equiv 5 \pmod{7}$$

$$a_1 = 2$$

$$a_2 = 4$$

$$a_3 = 5$$

$$m_1 = 3$$

$$m_2 = 5$$

$$m_3 = 7$$

$$M = m_1 m_2 m_3 = 3 \times 5 \times 7 = 105$$

$$M_1 = \frac{M}{m_1} = \frac{105}{3} = 35$$

$$M_2 = \frac{M}{m_2} = \frac{105}{5} = 21$$

$$M_3 = \frac{M}{m_3} = \frac{105}{7} = 15$$

Soln,

$$x \equiv a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3 \pmod{M}$$

$$M_1 y_1 \equiv 1 \pmod{m_1}$$

$$35 y_1 \equiv 1 \pmod{3}$$

$$2 y_1 \equiv 1 \pmod{3}$$

$$4 y_1 \equiv 2 \pmod{3}$$

$$y_1 \equiv 2 \pmod{3}$$

$$\boxed{y_1 = 2}$$

$$M_2 y_2 \equiv 1 \pmod{m_2}$$

$$21 y_2 \equiv 1 \pmod{5}$$

$$1 y_2 \equiv 1 \pmod{5}$$

$$\boxed{y_2 = 1}$$

$$M_3 y_3 \equiv 1 \pmod{m_3}$$

$$15 y_3 \equiv 1 \pmod{7}$$

$$1 y_3 \equiv 1 \pmod{7}$$

$$\boxed{y_3 = 1}$$

Put in eqⁿ 1

$$x = (2 \times 35 \times 2 + 4 \times 21 \times 1 + 5 \times 15 \times 1) \pmod{105}$$

$$= 140 + 84 + 75 \pmod{105}$$

$$= 299 \pmod{105}$$

$$x = 89 \pmod{105}$$

Also $x = 89 + 105k$ is also solⁿ where $k = 0, 1, \dots$

We can check up answers

$$89 \equiv 2 \pmod{3}$$

$$89 - 2 = 87$$

$$= 87/3 = 29$$

$$89 \equiv 4 \pmod{5}$$

$$= \frac{89-4}{5} = 17$$

$$2 \equiv \left(\frac{89-2}{3} \right) \pmod{5}$$

$$15 \equiv \frac{201}{2} \pmod{5}$$

$$21 \equiv \frac{201}{5} \pmod{5}$$

So divisible

$$89 \equiv 5 \pmod{7}$$

$$\frac{89-5}{7} = \frac{84}{7} = 12$$

$$(1 \pmod{7}) \equiv 1 \pmod{5}$$

$$(2 \pmod{7}) \equiv 2 \pmod{5}$$

$$(3 \pmod{7}) \equiv 3 \pmod{5}$$

$$(4 \pmod{7}) \equiv 4 \pmod{5}$$

$$(5 \pmod{7}) \equiv 5 \pmod{5}$$

$$(6 \pmod{7}) \equiv 6 \pmod{5}$$