MATRIX CHAIN MULTIPLICATION:

18/2/21
Thursday.

* Dynamic prog used & Tabulation.
* Consider a matrix

A_1 matrix → 2X3
A_2 matrix → 3X3
→ Non-symmetric matrices.

A_1 matrix → 2X2
A_2 matrix → 2X2
→ Multiplication of symmetric matrices.

* 1st we check if both matrices are multiplicable or not.

Case 1:
A_1 → 2X3
A_2 → 3X3
→ multiplicable

Case 2:
A_1 → 2X1
A_2 → 5X3
→ Not multiplicable.

* Cost of multiplication for:

A_1 → 2X3
A_2 → 3X3

A_1: C_1 C_2 C_3; r_1, r_2

A_2: C_1 C_2 C_3; r_1, r_2, r_3

Output Matrix 'O' → O_{2X3} = [ ]

To get a single element → 3 multiplications.

To get 2nd element → 3 multiplications.

To get 2x3 Matrix, for each element we perform 3 multiplications.

No. of elements in 2x3 Matrix: 6.

" " Multiplications per element → 3

Total Cost = 6x3

= 18

Example:

A_1 (2x2) $\begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$ A_2 (2x3) $\begin{bmatrix} 4 & 5 & 1 \\ 6 & 7 & 1 \end{bmatrix}$

2x3x2 = 12 Multiplications.

Consider multiplication of 4 Matrices:

A x B x C x D

↓ ↓ ↓ ↓

2x8 2x3 3x1 1x4

① ((A·B)·C)·D

② (A·B)(C·D)

③ A·(B·(C·D))

→ Diff Soln.

Out of all the soln's, the soln that takes min cost → benefit soln.

As cost ↑ → Time Complexity ↑.

① Find cost for each soln

② Arrange the costs in a table

③ Pick the cost with least Value.

Consider 3 Matrices:

$$\underset{2\times2}{A} \times \underset{2\times3}{B} \times \underset{3\times4}{C}$$

① $(A \cdot B) \cdot C$

② $A \cdot (B \cdot C)$

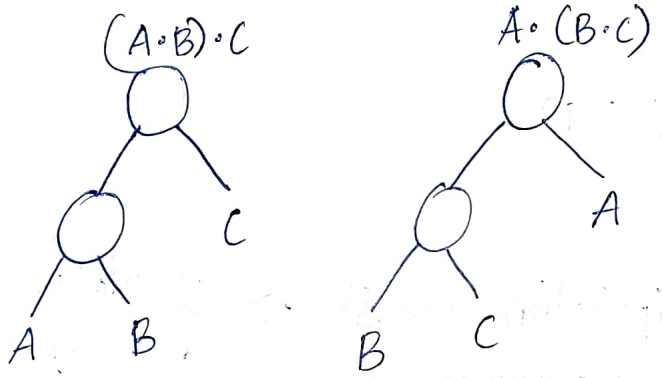
For 3 Matrices → 2 Multiplications

" 4 " → 3 "

" 2 " → 1 "

⇒ **For N Matrices → N-1 Multiplications.**

Representing the Sol^{n}s for 3 Matrix Multiplication as a tree:



⇒ 2 Trees.

$\underbrace{(A \cdot B)}_{2\times3\ \times2} \cdot C$

→ 12 Multiplications.

$(A \cdot B)$ → Output Matrix is 2×3.

Now $O \cdot C$ → ~~2×4~~ 2×3 3×4

2×4×3

= 24 Multiplications

Total → 12+24 = 36 "

Now:

$A \cdot (B \cdot C)$

$2 \times 3 \quad 3 \times 4$

$2 \times 4 \times 3 = 24$ Multiplications

Output of $(B \cdot C) = 2 \times 4$.

$(A \cdot O) \rightarrow (2 \times 2) \quad (2 \times 4)$

$2 \times 4 \times 2 = 16$ Multiplications.

Total $\rightarrow$ 40 Multiplications.

Cost of $(A \cdot B) \cdot C \leftrightarrow$ Cost of $(A \cdot (B \cdot C))$

Consider $A \times B \times C \times D$

$n = 4$

No. of Multiplications $\rightarrow 3$. $\Rightarrow$ 3 nodes in tree representation

No. of Tree Representations $T(n) = \frac{2nC_n}{n+1}$

$\Rightarrow$ For Multiplication of 4 Matrices:-

$T(3) = \frac{2 \cdot 3C_2}{3+1}$ $\quad T(3) = \frac{2^{3C_2}}{3+1}$

$\Rightarrow T(3) = \frac{2^{\frac{3!}{2!(3-2)!}}}{4}$

A (2x2) B (2x3) C (3x1) D (1x4)

M	A	B	C	D
A	0	12	10	18
B		0	6	14
C			0	12
D				0

How to fill the table?

Diagonals: fill with zero

1st Row

2nd Col → mul A, B

3rd Col → min (mul A, B, C)

4th Col → min (mul A, B, C, D)

2nd Row

3rd Col → mul B, C

4th Col → min (mul B, C, D)

3rd ROW

4th Col → mul C, D

going along diagonals:

① 1st row, 2nd Col:

multiply A, B → A·B (2x2 2x3)

2x2x3 = 12

② 2nd row, 3rd Col:

multiply B, C → B·C (2x3 3x1)

2x3x1 = 6

③ 3rd row, 4th Col:

multiply C, D → C·D (3x1 1x4)

3x1x4 = 12

④ 1st row, 3rd Col

multiply A, B, C (2x2 2x3 3x1)

A·(B·C) (A·B)·C

M[A,A]+M[B,C]

+ M[A·(B·C)]

= A·(B·C) (2x2) (2x3 3x1)

2x2x1

= 4

A · (B · C)

= M[A,A] + M[B,C] + M[A,B,C]

= 0 + 6 + 4

= 10

(A · B) · C

2×2 2×3 3×1

2×3×1

= 6

(A · B) · C

= M[A,B] + M[C,C] + M[(A · B) · C]

= 12 + 0 + 6

= 18

Min (A · (B · C), (A · B) · C)

Min (10, 18)

= 10

(5) 2nd row, 4th Col:

B · (C · D)

2×3 3×1 1×4

2×3×4 = 24

B · (C · D) = M[B,B] + M[C,D] + M[B · (C · D)]

= 0 + 12 + 24

= 36

(B · C) · D

2×3 3×1 1×4

2×1×4 = 8

$(B \cdot C) \cdot D$

$= M[B,C] + M[D,D] + M[(B \cdot C) \cdot D]$

$= 6 + 0 + 8$

$= 14$

$\text{Min}[B \cdot (C \cdot D), (B \cdot C) \cdot D]$

$= \text{Min}[36, 14]$

$= 14$

⑥ 1st Row, 4th Col:

$(((A \cdot B) \cdot C) \cdot D)$ =

~~$M[A,B]$~~ $+ M[A,B,C] + M[D,D] + M[A,B,C,D]$

$10 + 0 + (2 \times 1 \times 4)$

$10 + 8$

18

$(A \cdot B)(C \cdot D)$

$= M[A,B] + M[C,D] + (2 \times 3 \times 4)$

$= 12 + 12 + 48$

$= 72$

$(A \cdot (B \cdot (C \cdot D)))$

$= M[A,A] + M[B,C,D] + m(A,B,C,D)$

$= 0 + 14 + (2 \times 2 \times 4)$

$= 14 + 16$

$= 30$

$\text{Min}(18, 72, 30)$

$= 18$