UNIT-I

TIME SERIES

Def:- A set of ordered observation of a Quantitative variable taken at successive points in time is known as 'time series.'

In other words, arrangement on stastical data in chronological order, i.e., in accordance with occurence of time is known as Time Series.

Time in terms of years, months, days, hours (or) minutes.

According to Ya-lun chou "a time series, maybe defined as a collection of readings belonging to different time perids of some economic variable (or) composite of variables."

Mathematically a time series is defined by the function relationship $y(t)=f(t)$ where

$y(t) \rightarrow$ value of the phenomena under consideration at time 't'.

For example, the population $(y(t))$ of a country (or) a place in different years(t).

② No. of births & deaths $[y(t)]$ in different months(t).

③ The sale $[y(t)]$ of a departmental stores in different weeks(t)

④ The temperature [$y(t)$] in different ways

⑤ Share value [$y(t)$] changes in hours (t).

Note:- The values of a phenomena (or) variable at time $t_1, t_2, t_3, ---, t_n$ are $y_1, y_2, ---, y_n$ respectively. Then the series

$$t : t_1 \quad t_2 \quad t_3 \quad \cdots \quad t_n$$

$$y_t : y_1 \quad y_2 \quad y_3 \quad --- \quad y_n$$

Constitute a time-series.

Components of Time-Series:-

Various components of time-series canbe classified into the following four categories.

(a) Secular trend (or) Long term Movement

(b) Periodic changes. (or) short term Fluctuations

(i) Seasonal variation

(ii) Cyclic Variation

(c) Random (or) Irregular Movements.

Trend:-

By secular trend (or) simple trend we mean the general tendency of the data to increase (or) decrease during a long period of time. This is true of most of series of Business and Economic statistics. For example, an upward tendency would be seen in data pertaining to population, agricultural production, currency in circulation etc, while a downward tendency will be noticed in data of deaths and births, epidemics etc, as a result of advancement in Medical sciences, better medical facilities, literacy & higher standard of living.

Periodic changes:-

It would be observed that in many social & economic phenomena, apart from the growth factor in a time-series there are forces at work which prevent the smooth flow of the series in a particular direction and tend to repeat themselves over a period of time. These forces donot act continuously but operate in a regular spasmodic manner. The resultant effect of such forces maybe classified as:

1. Seasonal Variations and
2. Cyclic Variations

1. Seasonal Variations:-

These variations in a time series are due to rhythmic forces which operate in a regular & periodic manner over a span of lessthan a year i.e, during a period of 12 months and have the same (or) almost same pattern year after year. Thus seasonal variations in a timeseries willbe there if the data are recorded quarterly (every three months), monthly, weekly, daily, hourly and so on.

The seasonal variations maybe attributed to the following two causes.

(i) Those resulting from natural forces.

(ii) Those resulting from man-made conventions.

② Cyclic Variations:-

The oscillatory movements in a time series with period of oscillation morethan one year are termed cyclic fluctuations. One complete period is called a 'cycle'. The cyclic movements in a time series are generally attributed to the so-called 'Business Cycle', which may also be referred to as the 'four phase cycle' composed of prosperity(period of boom), recession, depression and recovery, and normally lasts from seven to eleven years. The Upswings and downswings in business depend upon the cumulative nature of the econo mic forces (affecting the equilibrium of demand and supply) and the interaction between them. Most of the economic and commercial series, e.g., series relating to prices, production and wages, etc., are affected by business cycles. Cyclic fluctuations, through more (or) less regular, arenot periodic.

Irregular (or Random) Component:-

Apart from the regular variations, almost all the series contain another factor called the random (or) irregular (or) residual fluctuations, which are not accounted for by secular trend and seasonal and cyclic variations. These fluctuations are purely random, erratic, unforeseen, unpredictable and are due to numerous, non-recurring and irregular circumstances which are beyond the control of human hand but at the same time are a part of our system such as earthquakes, wars, floods, famines, epidemics, revolutions, etc.

9th Jan 20

Analysis of Time Series:-

The main problems in the analysis of time series are:

(i) To identify the forces (or) components at work, the net effect of whose interaction is exhibited by the movement of a time series, and

(ii) To isolate, study, analyse and measure them independently ie, by holding other things constant.

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Mathematical models for Time Series:-

The following are the two models commonly used for the decomposition of a time series into its components.

i) **Decomposition by Additive Hypothesis (or Additive Model):-**

According to the additive model, a timeseries canbe expressed as

$$y_t = T_t + S_t + C_t + R_t$$

where y_t is the time-series value at time t, T_t represents the trend value, S_t, C_t and R_t represent the seasonal cyclic and random fluctuations at time t. Obviously, the term S_t willnot appear in a series of annual data. The additive model implies that seasonal forces (in different years), cyclical forces (in different cycles) and irregular forces (in different long term period) operate with equal absolute effect irrespective of the trend value. As such (C_t and S_t) will have +ve (or) -ve values, according as whether we are in an above normal (or) below normal phase of the cycle (& year) & the total of +ve & -ve value for any cycle (& any year) willbe zero.

In practice, most of these arrays relating to economicdata confirm to multiplicative model.

iii) **Mixed Model:-**

In addition to additive and multiplication models, the components in a timeseries maybe combined in a large Number of other ways. Some of the mixed models resulting from different combinations of additive & Multiplicative models are given below.

① $y_t = T_t C_t + S_t \cdot R_t$

② $y_t = T_t + S_t C_t R_t$

③ $y_t = T_t + S_t + C_t \cdot R_t$

Note:

① If Trend component (T_t) is known, then using multiplicative model it canbe isolated from the given time series to give

$$S_t \times C_t \times R_t = \frac{y_t}{T_t} = \frac{\text{original value}}{\text{Trend value}}$$

② For the annual data, for which the seasonal component (S_t) isnot there, we have

$$y_t = T_t \times C_t \times R_t$$

$$\Rightarrow \boxed{C_t \times R_t = \frac{y_t}{T_t}}$$

Uses of Time Series:

The Time Series analysis is of greater importance not only to businessman (or) an economist but also to people working in various disciplines in natural, social and physical sciences. some of its uses are enumerated below:

① It enables us to study the past behaviour of the phenomenon under consideration, i.e., to determine the type and nature of the variations in the data.

② The segregation and study of the various components is of paramount importance to a businessman in the planning of future operations and in the formulation of executive and policy decisions.

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② the segregation and study of the various components is of paramount importance to a businessman in the planning of future operations and in the formulation of executive and policy decisions.

③ It helps to compare the actual current performance of accomplishments (on the basis of the past performances) & analyse the causes of such variations, if any.

④ It enables us to predict (or) estimate (or) forecast the behaviour of the phenomenon in the future which is very essential for business planning.

⑤ It helps us to compare the changes in the values of different phenomenon at different times (or) places, etc.

Measurement of Trend:

Trend canbe studied (or) measured by the following methods:

① Graphic Method
② Method of semi-averages
③ Method of Curve-fitting by principle of least squares.
④ Method of moving averages

Method of Curve-fitting by principle of Least Squares:

Let y_t be the value of the variable corresponding to time 't'. Then

(i) A straight line: $y_t = a + bt$

(ii) Second degree parabola: $y_t = a + bt + ct^2$

(iii) k^{th} degree polynomial : $y_t = a_0 + a_1 t + a_2 t^2 + \dots + a_k t^k$

(iv) Exponential curve : $y_t = ab^t$

(v) Second degree curve fitted to logrithm : $y_t = ab^t c^{t^2}$

(vi) Growth curves : (a) $y_t = a + bc^t$ [Modified exponential curve]
(b) $y_t = a(b^{c^t})$ [Gompertz curve]

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(c) $y_t = \frac{k}{1+e^{(a+bt)}}$ [Logistic curve]

Fitting of St. line by least squares Method:

Let the straight line between given time series values (y_t) and time 't' be given by the equation

$$y_t = a + bt \quad \text{--- (1)}$$

Taking summation on b.s

$$\Sigma y_t = \Sigma a + \Sigma bt$$

$$\Sigma y_t = na + b\Sigma t \quad \text{--- (2)}$$

Multiplying eq.(2) with t on b.s

$$\Sigma y_t \cdot t = nat + b\Sigma t^2$$

$$\Sigma y_t \cdot t = a\Sigma t + b\Sigma t^2 \quad \text{--- (3)}$$

$\therefore$ Eq. (2) & (3) are the Normal equations for estimating 'a' & 'b'.

(1) In a certain Industry the production of a certain commodity (in thousand units) during the years 2004–14 is given in the following table. (1) graph the data (2) Obtain least square line fitting the data & construct graph of trend line

Year	Production (y_t)	$x = t - 2009$	$x \cdot y_t$	x^2	$y = 95.0363 + (3.9463)x$
2004	66.6	-5	-333	25	75.3048
2005	84.9	-4	-339.6	16	79.2511
2006	88.6	-3	-265.8	9	83.1974
2007	78.0	-2	-156	4	87.1437
2008	96.8	-1	-96.8	1	91.09
2009	105.2	0	0	0	95.0363

				1	98.9826
2010	93.2	1	93.2	4	102.9289
2011	111.6	2	223.2	9	106.8752
2012	88.3	3	264.9	16	110.8215
2013	117.0	4	468	25	114.7678
2014	115.2	5	576		
	$\Sigma y_t = 1045.4$	$\Sigma x = 0$	$\Sigma x.y_t = 434.1$	$\Sigma x^2 = 110$	

Here n=11, that is odd.

∴ We shift the origin to the middle time period i.e., 2009

Let $x = t - 2009$.

let the least square line of y_t on x then

$y_t = a + bx$

The Normal equations for estimating 'a' & 'b' are

$$\Sigma y_t = na + b\Sigma x \quad — ②$$

$$\Sigma x y_t = a\Sigma x + b\Sigma x^2 \quad — ③$$

From ②,

$1045.4 = na + b.0$

$\Rightarrow a = \frac{1045.4}{11} = 95.036$ $\boxed{a = 95.0363}$

③ $\Rightarrow 434.1 = 95.036(0) + 110b$

$\Rightarrow \boxed{b = 3.9463}$

Note: Estimate of production of commodity during the years 2015 & 2016 are

For the FY 2015,

$$y_t = 95.0363 + 6(3.9463)$$
$$= 118.7141$$

For the FY 2016,

$$y_t = 95.0363 + 7(3.9463)$$
$$= 122.6604$$

② Fit a St-line to the following data relating to the sales of a leading department store assuming that same rate of change continues what would be predicted earnings for the year 2016?

Year	2007	2008	2009	2010	2011	2012	2013	2014
Sales (Crores)	76	80	130	144	138	120	174	190

Find the trend values & also draw the graph?

Soln Here $n=8$, that is even

Hence we shift the origin to the A.M of two middle years

i.e, 2010 & 2011

we define $x = \dfrac{t - \left[\frac{2010+2011}{2}\right]}{\frac{1}{2}\text{interval}} = (t-2010.5)2 = 2t-4021$

Year	Production (y_t)	$x=2t-4021$	xy_t	x^2	$y=131.5+(7.33)x$
2007	76	-7	-532	49	80.19
2008	80	-5	-400	25	94.85
2009	130	-3	-390	9	109.51
2010	144	-1	-144	1	124.17
2011	138	1	138	1	138.83
2012	120	3	360	9	153.49
2013	174	5	870	25	168.15
2014	190	7	1330	49	182.81
	$\Sigma y_t=1052$	$\Sigma x=0$	$\Sigma x \cdot y_t=1232$	$\Sigma x^2=168$	

let the least square st. line of y_t on x then

$y_t = a + bx$

The Normal eqn's for estimating 'a' & 'b' are

$\Sigma y_t = na + b\Sigma x \longrightarrow 1052 = 8a + b.0 \Rightarrow a = 131.5$

$\Sigma x y_t = a\Sigma x + b\Sigma x^2 \Rightarrow 1232 = 8.0 + b(168) \Rightarrow b = 7.33$

predicted earnings for the year 2016 is,

$y = 131.5 + (7.33)x$

Here $x = 2t - 4021$

$= 4032 - 4021 = 11$

$y = 131.5 + (7.33)11 = 212.13$

③ Fitting of second degree (parabolic) trend:

let the second degree parabolic trend curve be

$$y_t = a + bt + ct^2$$

Proceeding similarly as in the case of a straight line, the Normal equations for estimating a, b & c are given by

$$\Sigma y_t = na + b\Sigma t + c\Sigma t^2$$

$$\Sigma y_t . t = a\Sigma t + b\Sigma t^2 + c\Sigma t^3$$

$$\Sigma y_t . t^2 = a\Sigma t^2 + b\Sigma t^3 + c\Sigma t^4$$

The following figures are the production data of a certain factory manufacturing air-conditioners:

Year:	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Production: (1000 units)	17	20	19	26	24	40	35	55	51	74	79

① Fit the second degree parabolic trend curve to the above data & obtain trend values.

year(t)	Production (y_t)	$x = t - 2005$	x^2	x^3	x^4	yx	yx^2	$y_t = 34 + 6.28x + 0.6x^2$
2000	17	-5	25	-125	625	-85	425	17.6
2001	20	-4	16	-64	256	-80	320	18.48
2002	19	-3	9	-27	81	-57	171	20.56
2003	26	-2	4	-8	16	-52	104	23.84
2004	24	-1	1	-1	1	-24	24	28.32
2005	40	0	0	0	0	0	0	34
2006	35	1	1	1	1	35	36	40.88
2007	55	2	4	8	16	110	220	48.96
2008	51	3	9	27	81	153	459	58.24
2009	74	4	16	64	256	296	1184	68.72
2010	79	5	25	125	625	395	1975	80.4

$\Sigma y_t = 440$ $\quad \Sigma x = 0$ $\quad \Sigma x^2 = 110$ $\quad \Sigma x^3 = 0$ $\quad \Sigma x^4 = 1958$ $\quad \Sigma yx^2 = 4917$

$\Sigma yx = 691$

$11a + b(0) + 110c = 440 \Rightarrow 11a + 110c = 440$ — (1)

$a(0) + b(110) + c(0) = 691 \Rightarrow 110b = 691$

$b = \frac{691}{110} = 6.2818$ — (2)

$110a + b(0) + 1958c = 4917 \Rightarrow 110a + 1958c = 4917$

$\Rightarrow 10a + 178c = 447$ — (3)

Solving (1) & (3)

$110a + 1958c = 4917$

$\ominus\ 110a + 1100c = 4400$

$858c = 517$

$\boxed{b = 6.2818}$

we get $\boxed{c = 0.6025,\ a \simeq 34}$

Trend values

$y_t = a + bx + cx^2$

$= 34 + 6.2818x + 0.6025x^2$

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Fitting of exponential curver

let $y_t = a.b^t$ —①

Taking log on both sides

$\log y_t = \log a + t\log b$

i.e., $Y = A + tB$ —② where $Y = \log y_t$, $A = \log a$, $B = \log b$

Taking summation on b.s

$\Sigma Y = nA + B\Sigma t$ —③

$\Sigma Y.t = A\Sigma t + B\Sigma t^2$ — ④

By solving equations ③ & ④, we estimate the values of A & B.

where $a = \text{Antilog}(A)$
$b = \text{Antilog}(B)$

④ Fit an exponential curve $y = ab^x$ to the following data by the method of least squares of population figures in India. Estimate the population of 1991, 2001 & 2011.

Sol: let the exponential curve $y = ab^x$

$\log y = \log a + x\log b$

$Y = A + BX$

Then the normal eqn. are

$\Sigma v = nA + B\Sigma u$

$\Sigma uv = a\Sigma u + B\Sigma u^2$

The Arithmetic for fitting the linear trend to the given data can be reduced to a great extent if we shif the origin in x to 1941 and change the square by defining a new variable u as follows. where $u = \frac{x - 1941}{10}$

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Census (year) (x)	Population (in crores) (Y)	$u = \frac{x-1941}{10}$	$v = \log y$	u^2	Σuv
1911	25.0	-3	1.3979	9	-4.1937
1921	25.1	-2	1.3996	4	-2.7992
1931	27.9	-1	1.4456	1	-1.4456
1941	31.9	0	1.5037	0	0
1951	36.1	1	1.5575	1	1.5575
1961	43.9	2	1.6424	4	3.2848
1971	54.7	3	1.7379	9	5.2137
	$\Sigma Y = 244.6$	$\Sigma u = 0$	$\Sigma v = 10.6846$	$\Sigma u^2 = 28$	$1.6175 = \Sigma uv$

$10.6846 = 7A + B(0) \Rightarrow A = 1.52637 \longrightarrow a = \text{Antilog } A = 33.6$

$1.6175 = 1.52637(0) + B(28) \rightarrow B = 0.0577 \rightarrow b = \text{Antilog } B = 1.142$

Estimating population in 1991, 2001, 2011

$t = 1991 \Rightarrow x = 5 \rightarrow (33.6)(1.142)^6 = 65.2634$ cr

$t = 2001 \rightarrow x = 6 \Rightarrow (33.6)(1.142)^6 = 74.53$ cr

$t = 2011 \Rightarrow x = 7 \rightarrow (33.6)(1.142)^7 = 85.11$ cr

Moving Average Method:-

It consists in measurement of trend by smoothing out the fluctuations of data by means of a moving average of extent m is a series of successive averages (A.M) of m terms at a time, starting with 1^{st}, 2^{nd}, 3^{rd} term etc. Thus, the first average is the mean of the 1^{st} m terms; the 2^{nd} is the mean of the m terms from 2^{nd} to $(m+1)$th term, the 3^{rd} is the mean of the mean of the m terms from 3^{rd} to $(m+2)$th term, and so on..

If 'm' is odd i.e., $m = 2k+1$ (say) moving average is placed against the mid-value of the time interval it covers i.e., against $t = k+1$ and if m is even i.e., $m = 2k$ (say), it is placed between the two middle values of the time interval it covers.

① A study of demand (d_i) for the past 12 years ($t = 1, 2, \dots 12$) has indicated the following

$$d_i = \begin{cases} 100, & t = 1,2,3,4,5 \\ 20, & t = 6 \\ 100, & t = 7,8,9,10,11,12 \end{cases}$$

Compute a five year moving average?

Sol:

t	d_i	5-yearly moving total	5-yearly moving average
1	100	—	—
2	100	—	—
3	100	500	100
4	100	420	84
5	100	420	84
6	20	420	84
7	100	420	84
8	100	420	84
9	100	500	100
10	100	500	100
11	100	—	—
12	100	—	—

② Calculate 5-yearly and 7-yearly moving averages of the data given below to obtain trend values and give their graphic representation

Year	1	2	3	4	5	6	7	8	9	10
Value	220	208	156	210	218	240	230	220	228	244

Year	11	12	13	14	15	16	17	18	19	20
Value	260	254	244	236	260	280	270	260	254	270

Year	21	22	23	24	25	26	27	28	29	30
Value	292	284	276	270	290	310	300	296	286	312

Year	Annual Figures	5-Yearly Moving Totals	5-Yearly Moving avg.	7-yearly Moving totals	7-yearly Moving avg.
1	220				
2	208				
3	156	1012	202.4		
4	210	1032	206.4	1482	211.71
5	218	1054	210.8	1482	211.71
6	240	1118	223.6	1502	214.57
7	230	1136	227.2	1590	227.14
8	220	1162	232.4	1640	234.29
9	228	1182	236.4	1676	239.43
10	244	1186 +20 = 1206	241.2	1680	240
11	260	1230	246	1686	240.86
12	254	1238	247.6	1726	246.57
13	244	1254	250.8	1778	254
14	236	1274	254.8	1804	257.71
15	260	1290	258	1804	257.71

16	280	1306	261.2	1804	257.71
17	270	1324	264.8	1830	261.43
18	260	1334	266.8	1886	269.43
19	254	1346	269.2	1910	272.86
20	270	1360	272	1906	272.29
21	292	1376	275.2	1906	272.29
22	284	1392	278.4	1936	276.57
23	276	1412	282.4	1992	284.57
24	270	1430	286	2022	288.86
25	290	1446	289.2	2026	289.43
26	310	1466	293.2	2028	289.71
27	300	1482	296.4	2064	294.86
28	296	1504	300.8		
29	286				
30	312				

③ Workout the four yearly moving average for the following data

Year ①	Tonnage of goods carried ②	Column of differences ③	4-yearly moving total (Not centred) ④	2 period moving Total (centered) ⑤	4 year moving total Trend values (centred) ⑥ = ⑤ ÷ 8
2000	2204	–			
2001	2500	2424-2204 =220	9744	19708	19708/8 = 2463.5
2002	2360	2634-2500 =134	9964	20062	2507.75
2003	2680	544	10098	20740	2592.5
2004	2424	418	10642	21702	2712.75
2005	2634	748	11060	22868	2858.5
2006	2904	318	11808 12126	23934	2921.75
2007	3098	344	12470	24596	3074.5
2008	3172	74	12544	25014	3126.75
2009	2952	–	–		
2010	3248	–	–		
2011	3172				

First 2 columns of the table are year & tonnage of goods carried, which form a given time series.

The total of first 4 observations is

2204 + 2500 + 2360 + 2680 = 9744

which is placed in between the 1st & 4th years. i.e, it is written b/w 2nd & 3rd years i.e, 2001 & 2002

Next we construct the 3rd column of differences b/w the items carried in 2000 & 2004

= 2424 − 2204

= 220

which is placed b/w the years 2001 & 02. In this way, we obtain the subsequent differences.

The remaining moving totals of column④ canbe obtained in the same way.

In column⑤, 2-period moving totals centred are calculated from the corresponding moving totals in column 4 and are placed in b/w these columns.

In column⑥, we findout the average of centred moving total of column⑤ by dividing with 8. By this method, every centred moving average has been placed infront of some year which is consistent with the given values.

Measurement of Seasonal Variations

It has already been pointed out that one of the types of fluctuations found in time series data is the seasonal component. Many economic and business series have distinct seasonal patterns that are pronounced enough to predict future behaviour of the series. The objectives for studying seasonal patterns in a time series in a time series are necessitated by the following reasons:

(i) To isolate the seasonal variations i.e., to determine the effect of seasonal swings on the value of the given phenomenon, and

(ii) To eliminate them, i.e., to determine the value of the phenomenon if there were no seasonal ups and downs in the series. This is known as de-seasonalising the given data and is necessary for the study of cyclic variations.

The determination of seasonal effects is of paramount importance in planning (i) business efficiency or

(ii) a production programme

For example, the head of a departmental store would be interested to study the variations in the demands of different articles for different months inorder to plan his future stocks to cater to the public demands due to seasonal swings. Moreover, the isolation and elimination of seasonal factor from the data is necessary to study the effect of cycles. Obviously, for the study of seasonal variations the data must be given for 'parts' of year, viz, monthly (or) quarterly, weekly, daily (or) hourly.

<u>Method of Simple Averages</u>: Different Methods for measuring seasonal variations are discussed below. This is the simplest method of measuring seasonal variations in a time series and involves the following steps:

(i) Arrange the data by years and months (or quarters if quarterly data are given).

(ii) Compute the average $\bar{x}_i$ ($i=1,2,..12$) for the ith month for all the years. (ith month, $i=1,2,...12$ represents Jan, Feb, Mar, ---. Dec) respectively

(iii) Compute the average $\bar{x}$ of the monthly averages i.e., $\bar{x} = \frac{1}{12}\sum_{i=1}^{12}\bar{x}_i$

(iv) Seasonal indices for different months are obtained by expressing monthly averages as percentage of $\bar{x}$. thus

Seasonal Index for ith month $= \frac{\bar{x}_i}{\bar{x}} \times 100$; $i=1,2,--,12$

<u>Remarks</u> (1) If instead of monthly averages, we use monthly totals for all the years, the result remains the same.

(2) Total of seasonal indices is $12 \times 100 = 1200$ for monthly data and $4 \times 100 = 400$ for quarterly data.

Use the method of Monthly averag[illegible] indices for the following data of production of a commodity for the years 2012, 2013, 2014.

Month ①	Production in lakhs of Tonnes 2012 ②	2013 ③	2014 ④	Total ⑤	Monthly Avg ⑤÷3	Seasonal Index
Jan	12	15	16	43	14.3333	$\frac{14.33}{13.6695} \times 100 = 104.896$
Feb	11	14	15	40	13.3333	97.56
Mar	10	13	14	37	12.3333	90.24
April	14	16	16	46	15.3333	112.25
May	15	16	15	46	15.3333	112.25
June	15	15	17	47	15.6667	114.669
July	16	17	16	49	16.3333	119.548
August	13	12	13	38	12.6667	92.711
Sep	11	13	10	34	11.3333	82.952
Oct	10	12	10	32	10.6667	78.073
Nov	12	13	11	36	12	87.832
Dec	15	14	15	44	14.6667	107.35
Total				492	163.95	1200.321
				41	13.6625	100.027

Step①:- Arrange the data by years & months.

Step②:- compute the averages $\overline{x}_i$ $(i = 1, 2, 3, \ldots, 12)$ [Respective month]

Step③:- Compute the average $\overline{x}$ of the monthly averages

i.e., $\overline{x} = \frac{1}{12} \sum_{i=1}^{12} \overline{x}_i$

Step④:- Seasonal indices for i^{th} month $= \frac{\overline{x}_i}{\overline{x}} \times 100$, where $i = 1, 2, \ldots$

Merits and Demerits:-

This Method is based on the basic assumption that data donot contain any trend and cyclic components and consists in eliminating irregular components by averaging the monthly (or quarterly) values over different years since most of the economic time series have trends, these assumptions arenot in general true and as such this method, though simple isnot of much practical utility.

Ratio to Trend Method:-

This Method is an improvement over the simple averages method and is based on the assumption that seasonal variation for any given month is constant factor of the trend. The measurement of seasonal variation by this method consists of the following steps.

(i) Compute the trend values by the principle of least squares by fitting an appropriate mathematical curve (straight line, 2nd degree parabolic curve (or) exponential curve, etc).

(ii) Express the original data as the percentage of the trend values. Assuming the Multiplicative model, these percentages will therefore contain the seasonal, cyclic and irregular components

(iii) The cyclic and irregular components are then wiped out by averaging the percentages for different months (quarters) if the data are monthly (quarterly), thus leaving us with indices of seasonal variations. Either Arithmetic mean (or) median can used for averaging, but median is preferred to A.M since the latter gives undue weightage to extreme values which arenot primarily due to seasonal swings. If there are few abnormal values, modified mean (which consists of calculating A.M after dropping out the extreme (or) abnormal values) may be used with advantage.

(iv) Finally, these indices, obtained in step (iii) are adjusted to a total of 1200 for monthly data (or) 400 for quarterly data by multiplying them throughout by a constant 'k' given by

$$k = \frac{1200}{\text{Total of the indices}} \quad \text{and} \quad k = \frac{400}{\text{Total of the indices}}$$

Merits and Demerits

Since this method attempts at ironing out the cyclical (or) irregular components by the process of averaging, the purpose willbe accomplished only if the cyclical variations are known to be absent (or) they arenot so pronounced even if present. On the other hand, if the series exhibits pronounced cyclical swings, the trend values obtained by the least square method can never follow the actual data as closely as 12-month moving average and as such the seasonal indices obtained by 'ratio to trend' method are liable to be more biased than those obtained by 'ratio to moving average' discussed later.

The obvious advantage of this method over the moving average method lies in the fact that 'ratio to trend' canbe obtained for each month for which the data are available & as such, unlike the 'ratio to moving average' method, there is no loss of data.

Q) Using Ratio to Trend Method, determine the quarterly seasonal indices for the adjoining data

Year	I Qrt.	II Qrt	III Qrt	IV Qrt
2015	30	40	36	34
2016	34	52	50	44
2017	40	58	54	48
2018	54	76	68	62
2019	80	92	86	82

First of all we will determine the Trend values for the quarterly averages by fitting a linear trend by the method of least squares.

Let the st.line trend equation be

$y = a + bx$ [origin : 1997, x unit : 1 year, y unit : Quarterly Avg. (Million Rs.)]

Note: For the above problem, origin is 2017.

x

y is quarterly averages (in million rupees)

The Normal equations are

$\Sigma y = na + b\Sigma x$ — (2)

$\Sigma xy = a\Sigma x + b\Sigma x^2$ — (3)

Year (t) (1)	Total of quarterly values (Million Rs.) (2)	Average of quarterly values (y)	$x = t - \text{origin} = t - 2017$	xy	x^2	Trend values $y = a + bx = 56 + 12x$
2015	140	35	-2	-70	4	32
2016	180	45	-1	-45	1	44
2017	200	50	0	0	0	56
2018	260	65	1	65	1	68
2019	340	85	2	170	4	80
Σ		$\Sigma y = 280$	$\Sigma x = 0$	$\Sigma xy = 120$	$\Sigma x^2 = 10$	

$280 = 5a + b.0$

$\Rightarrow \cancel{5}a = \cancel{280}\ 56$

$\boxed{a = 56}$

$120 = a.0 + b.10$

$\boxed{b = 12}$

From the trend equation we observe that the yearl increment in trend values

$\therefore$ The Quarterly increment is $b/4 = \frac{12}{4} = 3$

The positive value of b implies that we have increasing trend. Next we determine the quarterly trend values as follows:

For the year 2015, the average quarterly trend value is 32. which is infact the trend value for the middle quarter i.e, half of the 2nd quarter and half of the 3rd quarter of 2015. Since the quarterly increment is 3. We obtain trend values for the 2nd & 3rd quarter of year 2015 as 32-1.5 and 32+1.5 i.e, 30.5 & 33.5 respectively and consecutively The trend value for 1st quarter is 30.5-3=27.5 and 4th quarter is 33.5+3=36.5

Similarly, you can get the trend values for other years as given in the following figure

For finding trend eliminated values of the respective quarters i.e, each year respective quarter values by each year respective trend values multipled by 100.

Year	Trend values I Qt	II Qrt.	III Qrt.	IV Qrt.	Trend Eliminated Values I Qrt.	II Qrt	III Qrt	IV Qrt.
2015	27.5	30.5	33.5	36.5	109.1	131.1	107.5	93.1
2016	39.5	42.5	45.5	48.5	86.1	122.4	109.9	90.7
2017	51.5	54.5	57.5	60.5	77.7	106.4	93.9	70.3
2018	63.5	66.5	69.5	72.5	85	114.3	97.8	85.5
2019	75.5	78.5	81.5	84.5	106	117.1	105.5	97
		Total			463.9	591.3	514.6	436.6
Average (AM)	(Seasonal Indices)				92.78	118.26	102.92	87.32
Adjusted	Seasonal Indices				92.4831	117.8816	102.5907	87.0406

The indices obtained above are adjusted to a total of 400 (since the sum of indices = 92.78 + 118.26 + 102.92 + 87.32 = 401.28) which is greater than 400) by multiplying each of them by a constant factor k called correction factor given by

$$K = \frac{400}{\text{sum of indices}} = \frac{400}{401.28} = 0.9968$$

Ratio to Moving Average Method:-

Moving Average eliminates periodic movements if the extent (period of moving average) is equal to the period of the oscillatory movements sought to be eliminated. Thus for a monthly data, a 12 month moving average should completely eliminate the seasonal movements if they are of constant pattern and intensity. The method of getting seasonal indices by moving average involves the following steps:

(i) Calculate the centred 12-month moving average of the data. These moving average values will give estimates of the combined effects of trend and cyclic variations

(ii) Express the original data (except for 6 months in the beginning and 6 months at the end) as percentages of the centred moving average values. Using multiplicative model, these percentages would then represent the seasonal and irregular components.

iii) The preliminary seasonal indices are now obtained by eliminating the irregular(or) random component by averaging these percentages. As discussed in ratio to trend method, step (iii), either arithmatic mean (or) median (preferably median) canbe used for averaging.

iv) The sum of these indices = S (say) will not, in general, be 1200 (or 400) for monthly (or quarterly) data. Finally, an adjustment is done to make the sum of the indices 1200 (or 400) by multiplying throughout by a constant factor = 1200/S (or) 400/S i.e., by expressing the preliminary seasonal indices as the percentage of their A.M. The resultant gives the desired indices of seasonal variations.

<u>Merits and Demerits:</u>

Of all the methods of measuring seasonal variations, the 'Ratio to the moving average method' is the most satisfactory, flexible and widely used method, for estimating the seasonal fluctuations in a time series because it irons out both trend and cyclical components from the indices of seasonal variations.

<u>Demerits:</u>

There is a loss of some trend values in the beginning and at the end, accordingly seasonal indices for 1st 6 months (2 quarters) of the first year and last 6 months (2 quarters) of the last year cannot be determined.

Link Relative Method:

This method also known as Pearson's method is based on averaging the link relatives. Link relative is the value of one season expressed as a percentage of the value of the preceding season. Here, the word 'season' refers to time period; it would mean month for monthly data, quarter for quarterly data, etc. thus, for monthly data:

$$\boxed{\text{Link relative for any month} = \frac{\text{Current month's value}}{\text{Previous month's value}} \times 100}$$

The steps involved in this method maybe summed up as follows:

(i) Convert the original data into link relatives (L.R) by the above formula.

(ii) As in the case of 'ratio to trend' method, average the link relatives for each month (quarter) of the data are monthly (quarterly). Mean (or) median maybe used for averaging.

(iii) Convert the average (Mean or Median) link relatives into Chain relatives on the base of the first season.

Chain relative (C.R) for any season is obtained on multiplying the link relative of that season by the chain relative of the preceding season and dividing by 100 i.e

$$C.R \text{ for any season} = \frac{(\text{L.R. of that season}) \times (\text{C.R of preceding season})}{100}$$

C.R for first season is taken as 100.

Thus for monthly data, the chain relative for first season (month) i.e, for January, is taken to be 100.

$$C.R \text{ for February} = \frac{\text{L.R of Feb} \times \text{C.R of Jan}}{100} = \text{L.R. of Feb}$$

$(\because \text{C.R. of Jan} = 100)$

$$C.R \text{ for March} = \frac{\text{L.R of March} \times \text{C.R for Feb}}{100}$$

$\vdots$

$$C.R \text{ for December} = \frac{\text{L.R of Dec} \times \text{C.R. for Nov}}{100}$$

Now, by taking this December value as a base, a new chain relative for January canbe obtained as:

$$\text{New C.R. for January} = \frac{\text{L.R. of Jan(New year)} \times \text{C.R of Dec(Old year)}}{100}$$

Usually, this willnot be 100 due to trend and so we have to adjust the chain relatives for trend.

(iv) This adjustment is done by subtracting a 'correction factor' from each chain relative.

$$d = \frac{1}{12}[\text{Second(New) C.R. for January} - 100]$$

then, assuming linear trend, the correction factor for Feb, March, ---, December is $d, 2d, ---, 11d$ respectively.

(V) Finally, adjust the corrected chain relatives to total 1200 by expressing the corrected chain relatives as percentages of their Arithmetic Mean. The resultant gives the adjusted monthly indices of seasonal variation

Merits and Demerits:

1. The link relatives averaged together contain both the trend and cyclic components. Although the trend is subsequently eliminated by applying correction, the method is effective only if the growth is of constant amount (or) constant rate, i.e. if the assumption of linear trend is valid.

2. This method is effective only if the growth is of constant amount (or) constant rate

3. This method utilises data more completely than Moving average Method. as there is loss of only one link relative

Q) Complete the seasonal indices by the 'Link Relatives' Method for the adjoining data relating to the average quarterly prices (Rs per kg) of a commodity for five years

Quarter \ year	1996	1997	1998	1999	2000
I	30	35	31	31	34
II	26	28	29	31	36
III	22	22	28	25	26
IV	36	36	32	35	33

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Sol: Computation of Seasonal Indices by using Link Relatives Method:

Year	Quarter-I	Quarter-II	Quarter-III	Quarter-IV
1996	-	86.7	84.6	163.6
1997	97.2	80	78.6	163.6
1998	86.1	93.5	96.6	114.3
1999	96.9	100	80.6	140
2000	97.1	105.9	72.2	126.9
L.R Total	377.3	466.1	412.6	708.4
Avg. of LR	94.3	93.2	82.5	~~141.7~~
Chain relative	100	93.2	76.9	108.97
Adjusted C.R				
Seasonal Indices.				

Step①: Link relative of any quarter = $\frac{\text{current quarter value}}{\text{previous quarter value}} \times 100$

Chain relative for any quarter = $\frac{(\text{L.R of that quarter}) \times (\text{C.R of previous quarter})}{100}$

New second C.R for 1st quarter is $\frac{\text{L.R of 1st quarter} \times \text{C.R of 4th quarter}}{100}$

$$= \frac{94.3 \times 108.97}{100} = 102.76$$

Correction factor(d) = $\frac{\text{New C.R of 1st quarter} - 100}{4}$

$$= \frac{102.76 - 100}{4} = 0.69$$