```
In[•]:= Needs["Notation`"];

In[•]:= Needs["MaTeX`"];

In[•]:= (*$RecursionLimit=100*)

In[•]:= (*p1 = {p10,p11,p12,p13};
        p2 = {p20,p21,p22,p23};
        q = {q0,q1,q2,q3};
        l = {l0, l1,l2, l3};
        b = {b0, b1,b2,b3};
        Q = {Q0,Q1,Q2,Q3};*)

In[•]:= (*eta = DiagonalMatrix[{1,-1,-1,-1}];*)

        (*ClearAll[p1,p2,q,l,b, Q, eta];
        vecRules = {p1 → {p10,p11,p12,p13},
          p2 → {p20,p21,p22,p23},
          q → {q0,q1,q2,q3},
          l → {l0, l1,l2, l3},
          b → {b0, b1,b2,b3},
          (*Q → {Q0,Q1,Q2,Q3},*)
          Qp → {Qp0, Qp1, Qp2, Qp3},
          eta → DiagonalMatrix[{1,-1,-1,-1}]};*)

        (*RealQ[num_] := TrueQ[Refine[Element[num, Reals]]];
        IsReal[x_] := FreeQ[RealQ[x],x∈Reals];*)

        (*scalars = {x1, x2, m1, m2, S};
        IsScalar[x_]:=MemberQ[scalars, x]||NumberQ[x]*)
```

In[ ]:=
```
ClearAll[scalars];
scalars = {x1, x2, m1, m2, S, ℏ, ρ, γ, ϵ};

$Assumptions = And @@ (Element[#, Reals] & /@ scalars);

IsScalar[expr_] := Module[{vars, dotExp},
   vars = Variables[expr];
   dotExp = Cases[vars, holdDot[_, _]];
   vars = Complement[vars, dotExp];
   vars = Join[vars, HoldForm /@ dotExp];

   (vars === {} && NumericQ[expr]) || (vars =!= {} && SubsetQ[scalars, vars])
  ];

(*IsScalar[expr_]:=Module[{vars},
   vars=Variables[expr];

   (vars==={}&&NumericQ[expr])||(vars=!={}&&SubsetQ[scalars,vars])
  ];*)
```

In[ ]:=
```
scalars
```

In[ ]:=
```
{x1, x2, m1, m2, S, ℏ, ρ, γ, ϵ}
```

Out[ ]=
```
{x1, x2, m1, m2, S, ℏ, ρ, γ, ϵ}
```

In[ ]:=
```
AddToScalars[expr_] :=
  (If[! MemberQ[scalars, HoldForm[expr]], scalars = Append[scalars, HoldForm[expr]]];
   expr);

AddDotProdToScalars[vectors_] := Module[{vecPairs},
   (*vecPairs = Subsets[vectors,{2}];*)
   vecPairs = DeleteDuplicatesBy[Tuples[vectors, 2], Sort];
   dotProdPairs = holdDot @@ # & /@ vecPairs;
   Do[AddToScalars[dotProd], {dotProd, dotProdPairs}];
  ];
```

```
In[ ]:= holdDotDef := (
    ClearAll[holdDot];
    (*holdDot[a_,b_]:=Module[{aa=a,bb=b},If[OrderedQ[{aa,bb}],a.eta.b,b.eta.a]];*)
    (*holdDot/:holdDot[a_,b_]:=If[OrderedQ[{a,b}],holdDot[a,b],holdDot[b,a]];*)
    (*holdDot/:holdDot[a_,b_]:=0/;(PossibleZeroQ[a]||PossibleZeroQ[b]);
    holdDot/:holdDot[a_,b_]:=holdDot@@Sort[{a,b}]/;!OrderedQ[{a,b}];*)
    holdDot[a_, b_] := 0 /; (PossibleZeroQ[a] || PossibleZeroQ[b]);
    holdDot[a_, b_] := holdDot @@ Sort[{a, b}] /; ! OrderedQ[{a, b}];
    holdDot[a_ + b_, x_] := holdDot[a, x] + holdDot[b, x];
    holdDot[x_, a_ + b_] := holdDot[a, x] + holdDot[b, x];
    holdDot[x_, c_?IsScalar y_] := c holdDot[x, y];
    holdDot[c_?IsScalar x_, y_] := c holdDot[x, y];
    holdDot[x_, Times[holdDot[a_, b_], y_]] := holdDot[a, b] × holdDot[x, y];
    holdDot[Times[holdDot[a_, b_] x_], y_] := holdDot[a, b] × holdDot[x, y];
    (*holdDot[c_ x_,y_]:=c holdDot[x,y]/;FreeQ[c,_List];
  holdDot[x_,c_ y_]:=c holdDot[x,y]/;FreeQ[c,_List];*)


    Notation[ParsedBoxWrapper[RowBox[{"a_", "⊙", "b_"}]] ⇔
      ParsedBoxWrapper[RowBox[{"holdDot", "[", "a_", ",", "b_", "]"}]]];
   )
```

In[ ]:=
```
holdDotDer := (
  (*ClearAll[holdDot];*)

  SetOptions[D, NonConstants → {holdDot, Wedge}];
(*D[expr_holdDot,vars__]^:=Module[{x,y,z},*)
  holdDot /: D[holdDot[a_, b_], vars__] := Module[{x = a, y = b, z},
    (*{x,y}=List@@expr;*)
    z = {vars}[[1]];
    Which[
     IsScalar[z] && x === q && y === q,
     holdDot[D[Q, {vars}[[1]]], y] + holdDot[x, D[Q, {vars}[[1]]]],
     IsScalar[z] && x === q,
     holdDot[D[Q, {vars}[[1]]], y],
     IsScalar[z] && y === q,
     holdDot[x, D[Q, {vars}[[1]]]],
     ! IsScalar[{vars}[[1]]],
     x × D[y, {vars}[[1]]] + y × D[x, {vars}[[1]]],
     True, 0]];
 )
```

In[ ]:=
```
holdDotDef
```

Notation: Future versions of the Notation package will no longer support ⇔, instead they will use ⟺. Please make this change to all your Notations.

In[ ]:=
```
vectors = {p1, p2, q, l1, l2, l3, Qp}
```

Out[ ]=
```
{p1, p2, q, l1, l2, l3, Qp}
```

In[ ]:=
```
AddDotProdToScalars[vectors]
```

In[ ]:=
```
scalars
```

Out[ ]=
```
{x1, x2, m1, m2, S, ℏ, ρ, γ, ϵ, p1 ⊙ p1, p1 ⊙ p2, p1 ⊙ q, l ⊙ p1, l1 ⊙ p1, l2 ⊙ p1, p1 ⊙ Qp,
 p2 ⊙ p2, p2 ⊙ q, l ⊙ p2, l1 ⊙ p2, l2 ⊙ p2, p2 ⊙ Qp, q ⊙ q, l ⊙ q, l1 ⊙ q, l2 ⊙ q, q ⊙ Qp,
 l ⊙ l, l ⊙ l1, l ⊙ l2, l ⊙ Qp, l1 ⊙ l1, l1 ⊙ l2, l1 ⊙ Qp, l2 ⊙ l2, l2 ⊙ Qp, Qp ⊙ Qp}
```

In[∘]:=
```
ClearAll[Wedge];

Wedge[x_, y_] := 0 /; (x === y || PossibleZeroQ[x] || PossibleZeroQ[y]);
Wedge[a_ + b_, x_] := Wedge[a, x] + Wedge[b, x];
Wedge[x_, a_ + b_] := Wedge[x, a] + Wedge[x, b];
Wedge[x_, c_?IsScalar y_] := c Wedge[x, y];
Wedge[c_?IsScalar x_, y_] := c Wedge[x, y];
Wedge[x_, y_] := -Wedge[y, x] /; OrderedQ[{y, x}];
Wedge[x_, Times[holdDot[a_, b_], y_]] := holdDot[a, b] × Wedge[x, y];
Wedge[Times[holdDot[a_, b_] x_], y_] := holdDot[a, b] × Wedge[x, y];
```

In[∘]:=
```
wedgeDer := (SetOptions[D, NonConstants → {holdDot, Wedge}];
(*D[expr_Wedge,vars__]^:=Module[{x,y,z},*)
  Wedge /: D[Wedge[a_, b_], vars__] := Module[{x = a, y = b, z},
    (*{x,y}=List@@expr;*)
    z = {vars}[[1]];
    Which[
     IsScalar[z] && x === q && y === q,
     Wedge[D[Q, {vars}[[1]]], y] + Wedge[x, D[Q, {vars}[[1]]]],
     IsScalar[z] && x === q,
     Wedge[D[Q, {vars}[[1]]], y],
     IsScalar[z] && y === q,
     Wedge[x, D[Q, {vars}[[1]]]],
     ! IsScalar[{vars}[[1]]],
     Wedge[x, D[y, {vars}[[1]]]] + Wedge[D[x, {vars}[[1]]], y],
     True, 0]];

 )
```

In[∘]:=
```
wedgeDer
```

In[∘]:=
```
D[Wedge[p2, q] / holdDot[p1, q], x2]
```

Out[∘]=

$$-\frac{\gamma\, p1 \wedge p2}{S\, p1 \odot q} - \frac{\left(\frac{\gamma\, p1 \odot p1}{S} - \frac{m1^2\, p1 \odot p2}{S}\right) p2 \wedge q}{(p1 \odot q)^2}$$

In[∘]:=
```
Wedge[p1, p2 / (p1 ⊙ p2)^2]
```

Out[∘]=

$$\frac{p1 \wedge p2}{(p1 \odot p2)^2}$$

```
In[ ]:= aWedgeDb[expr_, a_, b_] := Wedge[a, D[expr, b]];
        aWedgeDb[expr_holdDot, a_, b_] := Module[{x, y}, {x, y} = List @@ expr;
           Wedge[a, y D[x, b]] + Wedge[a, x D[y, b]]];

In[ ]:= (*ClearAll[holdDot]*)
        (*SetAttributes[holdDot,HoldAll]*)

        (*holdDot[a_,b_]:=
          Module[{aa=a,bb=b},If[OrderedQ[{aa,bb}],HoldForm[a.eta.b],HoldForm[b.eta.a]]];*)
        (*holdDot[a_,b_]:=Module[{aa=a,bb=b},If[OrderedQ[{aa,bb}],a.eta.b,b.eta.a]];
        holdDot[a_+b_, x_]:=holdDot[a,x]+holdDot[b,x];
        holdDot[x_, a_+b_]:=holdDot[a,x]+holdDot[b,x];
        holdDot[x_, c_?NumberQ y_]:= c holdDot[x,y];
        holdDot[c_?NumberQ x_, y_]:= c holdDot[x,y];
        holdDot[c_ x_,y_]:=c holdDot[x,y]/;FreeQ[c,_List];
        holdDot[x_,c_ y_]:=c holdDot[x,y]/;FreeQ[c,_List];*)

        (*UpValues[holdDot]={};
        holdDot,Dt[holdDot[x_,y_], z,___]:=Which[
          x===q&&y===q,
           holdDot[D[Q, z],y]+holdDot[x,D[Q, z]],
          x===q,
          holdDot[D[Q,z],y],
          y===q,
          holdDot[x,D[Q, z]],
          True,
           0 ]*)

        (*holdDot/:D[holdDot[x_,y_],z_,NonConstants→_]:=D[holdDot[x,y],z]*)

        (*wrapDot[expr_ ] :=expr //.{(p1.eta.p1/.vecRules)→m1^2,
           (-p1.eta.p1/.vecRules)→-m1^2,(p2.eta.p2/.vecRules)→m2^2,(-p2.eta.p2/.vecRules)→-m2^2,
           (p1.eta.p2/.vecRules)→γ,(-p1.eta.p2/.vecRules)→-γ,
           (p1.eta.q/.vecRules)→p1.eta.q,(-p1.eta.q/.vecRules)→-p1.eta.q,
           (p2.eta.q/.vecRules)→p2.eta.q,(-p2.eta.q/.vecRules)→-p2.eta.q,
           (p1.eta.l/.vecRules)→p1.eta.l,(-p1.eta.l/.vecRules)→-p1.eta.l,
           (p2.eta.l/.vecRules)→p2.eta.l,(-p2.eta.l/.vecRules)→-p2.eta.l,
           (l.eta.q/.vecRules)→l.eta.q,(-l.eta.q/.vecRules)→-l.eta.q,
           (l.eta.l/.vecRules)→l.eta.l,(-l.eta.l/.vecRules)→-l.eta.l,
           (q.eta.q/.vecRules)→q.eta.q,(-q.eta.q/.vecRules)→-q.eta.q}*)
```

```
In[∘]:= (*holdDot[a_, b_]:=
  a.eta.b//FullSimplify//.{p1.eta.p2→HoldForm[HoldPattern[γ],-p1.eta.p2→-HoldForm[γ],
       p1.eta.q→HoldForm[p1.eta.q],-p1.eta.q→-HoldForm[p1.eta.q],
       p2.eta.q→HoldForm[p2.eta.q],-p2.eta.q→-HoldForm[p2.eta.q],
       p1.eta.l→HoldForm[p1.eta.l],-p1.eta.l→-HoldForm[p1.eta.l],
       p2.eta.l→HoldForm[p2.eta.l],-p2.eta.l→-HoldForm[p2.eta.l],
       l.eta.q→HoldForm[l.eta.q],-l.eta.q→-HoldForm[l.eta.q]}*)

In[∘]:= (*wrapDot[expr_, ] :=
 expr //.{p1.eta.p2→HoldForm[γ],-p1.eta.p2→-HoldForm[γ],p1.eta.q→HoldForm[p1.eta.q],
   -p1.eta.q→-HoldForm[p1.eta.q],p2.eta.q→HoldForm[p2.eta.q],
   -p2.eta.q→-HoldForm[p2.eta.q],p1.eta.l→HoldForm[p1.eta.l],
   -p1.eta.l→-HoldForm[p1.eta.l],p2.eta.l→HoldForm[p2.eta.l],-p2.eta.l→
    -HoldForm[p2.eta.l],l.eta.q→HoldForm[l.eta.q],-l.eta.q→-HoldForm[l.eta.q],
   l.eta.l→HoldForm[l.eta.l],-l.eta.l→-HoldForm[l.eta.l],
   q.eta.q→HoldForm[q.eta.q],-q.eta.q→-HoldForm[q.eta.q]}*)

In[∘]:= (*holdDot[expr_] :=Simplify[expr ,{p1.eta.p2→HoldForm[γ],p1.eta.q→HoldForm[p1.eta.q],
   p2.eta.q→HoldForm[p2.eta.q],p1.eta.l→HoldForm[p1.eta.l],
   p2.eta.l→HoldForm[p2.eta.l],l.eta.q→HoldForm[l.eta.q]}]*)

In[∘]:= holdDotDef
```

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```
In[∘]:= α1 = (γ x2 - m2^2 x1) / S;
        α2 = (γ x1 - m1^2 x2) / S;
        Q = α1 p1 + α2 p2 + Qp;
        q == Q;

In[∘]:= holdDotDer

In[∘]:= wedgeDer

In[∘]:= D[holdDot[p1, q] / holdDot[p2, q], x1]
Out[∘]=
```

$$-\frac{p1\odot q\left(-\frac{m2^2\, p1\odot p2}{S}+\frac{\gamma\, p2\odot p2}{S}\right)}{(p2\odot q)^2}+\frac{-\frac{m2^2\, p1\odot p1}{S}+\frac{\gamma\, p1\odot p2}{S}}{p2\odot q}$$

```
In[∘]:= onShellP[expr_] := expr //. {p1⊙q → 0, p2⊙q → 0, p1⊙p2 → γ, p1⊙p1 → m1^2, p2⊙p2 → m2^2}
        (*onShellP[expr_]:=expr//.{HoldForm[p1.eta.q]→0, HoldForm[p2.eta.q]→0}*)
```

In[ ]:=
```
ClearAll[simplifiedForm];
simplifiedForm[expr_] := expr /. {(coef1_. ϵ + coef2_. l1dot_ + rest_.)^(n_.) :>
    (coef2)^n * (ϵ * (coef1 / coef2) / Abs[coef1 / coef2] + l1dot + Distribute[rest / coef2])^n /;
     l1dot == l1⊙p1 || l1dot == l1⊙p2 || l1dot == l2⊙p1 || l1dot == l2⊙p2}
```

In[ ]:=
```
cancelFactors[expr_] :=
  expr //. {dot1 : d1_^(n_.) * (c1_. ϵ + dot2 : d2_ + d3___)^(m_) /; d1 == d2 + d3 ->
    If[Abs[n] < Abs[m], (c1 ϵ + d1)^(m + n), d1^(m + n)]}
```

In[ ]:=
```
supressQml[expr_] :=
   expr /. {-l1⊙l1 + 2 l1⊙l2 + 2 l1⊙q - l2⊙l2 + 2 l2⊙q - q⊙q -> -qml1ml2,
    l1⊙l1 + 2 l1⊙l2 - 2 l1⊙q + l2⊙l2 - 2 l2⊙q + q⊙q -> qml1ml2};
```

In[ ]:=
```
modifyNum[expr_] :=
  expr //. {(dot1_ /; MemberQ[{l1⊙p1, l1⊙p2, l2⊙p1, l2⊙p2}, dot1])^(n_.) *
      (c1_ ϵ + dot2 : d1_ + d2___)^(m_.) /; dot1 == d1 :>
    Expand[Hold[dot1 + d2] - d2]^n * (c1 ϵ + d1 + d2)^m}
```

In[ ]:=
```
factorRho[exp_] :=
  exp /. {(expr_Plus)^(n_) :> Factor[expr]^n /; ContainsAll[Variables[expr], {ϵ, l1⊙p1}] ||
      ContainsAll[Variables[expr], {ϵ, l1⊙p2}]}
```

In[ ]:=
```
a = D[p1⊙q, x1]
b = D[p1⊙q, x2]
```

Out[ ]=

$$-\frac{m2^2\, p1\odot p1}{S}+\frac{\gamma\, p1\odot p2}{S}$$

Out[ ]=

$$\frac{\gamma\, p1\odot p1}{S}-\frac{m1^2\, p1\odot p2}{S}$$

$p_1 \wedge q$ term for first two diagrams

Two Loop

In[ ]:=
```
numPhotons = 3;
numScalars = 4;
numVertices = 6;
numLoops = 2;
photons = Permutations[{1, 2, 3}]
```

Out[ ]=
```
{{1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1}, {3, 1, 2}, {3, 2, 1}}
```

In[ ]:=
```
verticesSet = {};
Do[AppendTo[verticesSet, {v1, v2}], {v1, photons}, {v2, photons}]
```

In[ ]:=
```
numDiagrams = Length[verticesSet]
```

Out[ ]=
```
36
```

In[ ]:=
```
l3 = q - l1 - l2;
P1[0] = p1;
P2[0] = p2;
P1[4] = p1 + ℏ * (l1 + l2 + l3);
P2[4] = p2 - ℏ * (l1 + l2 + l3);
```

In[ ]:=
```
{v1, v2} = verticesSet[[16]]
```

Out[ ]=
```
{{2, 1, 3}, {2, 3, 1}}
```

In[ ]:=
```
vertexFactors = {};
diagrams = {};
overallFactor = (-ⅈ)^(numVertices+numPhotons) * ⅈ^numScalars * ℏ^(4*numLoops-numVertices/2-2*numPhotons);

Do[
 {v1, v2} = verticesSet[[diag]];

 Do[
  P1[i] = P1[i - 1] + ℏ * ToExpression["l" <> ToString[v1[[i]]]] // Simplify;
  P2[i] = P2[i - 1] - ℏ * ToExpression["l" <> ToString[v2[[i]]]] // Simplify;
  , {i, 1, 3}];

 vertexFactor = 1;
 Do[{i, j} = Extract[Position[verticesSet[[diag]], vertex], {{1, 2}, {2, 2}}];
  vertexFactor *= Simplify[(P1[i - 1] + P1[i]) ⊙ (P2[j - 1] + P2[j])], {vertex, 1, 3}];

 diagram = overallFactor * vertexFactor / (l1 ⊙ l1 * l2 ⊙ l2 * l3 ⊙ l3 *
      Factor[(P1[1] ⊙ P1[1] - m1^2 + ⅈ ℏ ϵ) * (P1[2] ⊙ P1[2] - m1^2 + ⅈ ℏ ϵ) * (P2[1] ⊙ P2[1] - m2^2 + ⅈ ℏ ϵ) *
        (P2[2] ⊙ P2[2] - m2^2 + ⅈ ℏ ϵ) /. {p1 ⊙ p1 → m1^2, p2 ⊙ p2 → m2^2}]);

 AppendTo[vertexFactors, vertexFactor];
 AppendTo[diagrams, diagram]
 , {diag, 1, numDiagrams}]
```

In[ ]:=
```
diagrams[[1]]
```

Out[ ]=

$$-\Big(\Big(i\left(-\hbar^2\, l1\odot l1-2\,\hbar\, l1\odot p1+2\,\hbar\, l1\odot p2+4\, p1\odot p2\right)\left(-4\,\hbar^2\, l1\odot l1-4\,\hbar^2\, l1\odot l2-4\,\hbar\, l1\odot p1+4\,\hbar\, l1\odot p2-\hbar^2\, l2\odot l2-2\,\hbar\, l2\odot p1+2\,\hbar\, l2\odot p2+4\, p1\odot p2\right)\left(-\hbar^2\left(l1\odot l1+2\, l1\odot l2+l2\odot l2\right)-2\,\hbar\,(l1\odot p1+l2\odot p1)+2\,\hbar\,(l1\odot p2+l2\odot p2)-2\,\hbar^2\,(l1\odot q+l2\odot q)+4\, p1\odot p2-2\,\hbar\, p1\odot q+2\,\hbar\, p2\odot q-\hbar^2\, q\odot q\right)\Big)\Big/\Big(\hbar^5\, l1\odot l1\left(\epsilon-i\,\hbar\, l1\odot l1-2\, i\, l1\odot p1\right)\left(\epsilon-i\,\hbar\, l1\odot l1+2\, i\, l1\odot p2\right) l2\odot l2\left(\epsilon-i\,\hbar\, l1\odot l1-2\, i\,\hbar\, l1\odot l2-2\, i\, l1\odot p1-i\,\hbar\, l2\odot l2-2\, i\, l2\odot p1\right)\left(\epsilon-i\,\hbar\, l1\odot l1-2\, i\,\hbar\, l1\odot l2+2\, i\, l1\odot p2-i\,\hbar\, l2\odot l2+2\, i\, l2\odot p2\right)\left(l1\odot l1+2\, l1\odot l2-2\, l1\odot q+l2\odot l2-2\, l2\odot q+q\odot q\right)\Big)\Big)$$

In[ ]:=
```
verticesSet[[3]]
```

Out[ ]=

{{1, 2, 3}, {2, 1, 3}}

In[∘]:=
```
Do[hbarExpansion = simplifiedForm[Normal[Series[ℏ^3 diagrams[[diag]], {ℏ, 0, 0}]]];
 Do[diagramsHbar[diag, hbarOrder] = Coefficient[hbarExpansion, ℏ, hbarOrder],
  {hbarOrder, -2, 0}];
 ClearAll[hbarExpansion]
 , {diag, 1, numDiagrams}]
```

In[∘]:=
```
Normal[Series[DiracDelta'[2 p1⊙q + ℏ q⊙q] DiracDelta[2 p2⊙q - ℏ q⊙q], {ℏ, 0, 2}]]
```

Out[∘]=

$$\frac{1}{2}\,\text{DiracDelta}[p2\odot q]\,\text{DiracDelta}'[2\,p1\odot q]+\hbar\Big(-q\odot q\,\text{DiracDelta}'[2\,p1\odot q]\,\text{DiracDelta}'[2\,p2\odot q]+\frac{1}{2}\,\text{DiracDelta}[p2\odot q]\,q\odot q\,\text{DiracDelta}''[2\,p1\odot q]\Big)+\frac{1}{4}\hbar^2\Big(-4\,(q\odot q)^2\,\text{DiracDelta}'[2\,p2\odot q]\,\text{DiracDelta}''[2\,p1\odot q]+2\,(q\odot q)^2\,\text{DiracDelta}'[2\,p1\odot q]\,\text{DiracDelta}''[2\,p2\odot q]+\text{DiracDelta}[p2\odot q]\,(q\odot q)^2\,\text{DiracDelta}^{(3)}[2\,p1\odot q]\Big)$$

In[∘]:=
intermediateTerms[expr_] :=

$$\text{expr}\;/\!/.\;\Big\{\Big(-\frac{m1^2\,m2^2}{s}+\frac{\gamma^2}{s}\Big)\to 1,\;\Big(\frac{m1^2\,m2^2}{s}-\frac{\gamma^2}{s}\Big)\to -1,\;\Big(-\frac{m2^2\,p1\odot p1}{s}+\frac{\gamma\,p1\odot p2}{s}\Big)\to 1,$$
$$\Big(\frac{m2^2\,p1\odot p1}{s}-\frac{\gamma\,p1\odot p2}{s}\Big)\to -1,\;\Big(\frac{\gamma\,p1\odot p2}{s}-\frac{m1^2\,p2\odot p2}{s}\Big)\to 1,$$
$$\Big(-\frac{\gamma\,p1\odot p2}{s}+\frac{m1^2\,p2\odot p2}{s}\Big)\to -1,\;\Big(\frac{\gamma\,p1\odot p1}{s}-\frac{m1^2\,p1\odot p2}{s}\Big)\to 0,$$
$$\Big(-\frac{\gamma\,p1\odot p1}{s}+\frac{m1^2\,p1\odot p2}{s}\Big)\to 0,\;\Big(-\frac{m2^2\,p1\odot p2}{s}+\frac{\gamma\,p2\odot p2}{s}\Big)\to 0,\;\Big(\frac{m2^2\,p1\odot p2}{s}-\frac{\gamma\,p2\odot p2}{s}\Big)\to 0\Big\}$$

In[∘]:=
```
MaTeX[
 "\\begin{aligned}R_1=p_1\\wedge\\partial_{p_1}\\,A_0-\\frac{1}{2}(\\partial_{x_1}-\\
   partial_{x_2})(q^2\\,p_1\\wedge\\partial_{p_1}\\,A_{-1})+\\frac{1}{8}(\\partial_{x_1}
   ^2+\\partial_{x_2}^2-2\\,\\partial_{x_1}\\partial_{x_2})(q^4\\,p_1\\wedge\\
   partial_{p_1}\\,A_{-2})\\end{aligned}"]
```

Out[∘]=

$$R_1=p_1\wedge\partial_{p_1}\,A_0-\frac{1}{2}(\partial_{x_1}-\partial_{x_2})(q^2\,p_1\wedge\partial_{p_1}\,A_{-1})+\frac{1}{8}(\partial_{x_1}^2+\partial_{x_2}^2-2\,\partial_{x_1}\partial_{x_2})(q^4\,p_1\wedge\partial_{p_1}$$

In[ ]:=
```
ClearAll[R11, R12, R13];
Do[R11 = aWedgeDb[diagramsHbar[diag, 0], p1, p1];
  workspace = aWedgeDb[diagramsHbar[diag, -1], p1, p1];
  R12 = -1/2 (D[q ⊙ q * workspace, x1] - D[q ⊙ q * workspace, x2]);
  workspace = aWedgeDb[diagramsHbar[diag, -2], p1, p1];
  R13 = 1/8 (D[(q ⊙ q)^2 * workspace, {x1, 2}] +
       D[(q ⊙ q)^2 * workspace, {x2, 2}] - 2 D[D[(q ⊙ q)^2 * workspace, {x2, 1}], {x1, 1}]);

  R11 = onShellP[R11] // intermediateTerms;
  R12 = onShellP[R12] // intermediateTerms;
  R13 = onShellP[R13] // intermediateTerms;

  ClearAll[workspace];

  R1[diag] = supressQml@(R11 + R12 + R13);
  R1[diag] = cancelFactors@R1[diag];
  R1[diag] = cancelFactors@ReleaseHold@Expand@modifyNum@R1[diag];
  R1[diag] = cancelFactors@ReleaseHold@Expand@modifyNum@R1[diag];

  ClearAll[R11, R12, R13];

  , {diag, 1, numDiagrams}];
```

In[ ]:=
```
(*Calculated previously. Total sum
 for each diagram without doing simplification*)
ClearAll[a];
Do[(*a[diag]=Expand[R11[diag]+R12[diag]+R13[diag]+R21[diag]+R22[diag]+R23[diag]];*)
 a[diag] = Expand[R1[diag]],
 {diag, 1, numDiagrams}]
```

In[ ]:=
```
(*Sum of the number of terms of all diagrams*)
len = 0;
Do[len += Length[List @@ a[diag]], {diag, 1, numDiagrams}]
len
```

Out[ ]=
```
27 318
```

In[ ]:=
```
totalSum = 0;

Do[totalSum += a[diag], {diag, 1, numDiagrams}]
```

In[ ]:=
```
Length[List @@ totalSum]
```
Out[ ]=

12 526

In[ ]:=
```
MaTeX["\\begin{aligned}R_2=-\\partial_{x_1}(p_1\\wedge
q\\,A_0)+\\frac{1}{2}(\\partial_{x_1}^2-\\partial_{x_1}\\partial_{x_2})(q^2\\,p_1\\
wedge
q\\,A_{-1})-\\frac{1}{8}(\\partial_{x_1}^3+\\partial_{x_1}\\partial_{x_2}^2-2\\,\\
partial_{x_1}^2\\partial_{x_2})(q^4\\,p_1\\wedge q\\,A_{-2})\\end{aligned}"]
```
Out[ ]=

$$R_2 = -\partial_{x_1}(p_1 \wedge q\, A_0) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})(q^2\, p_1 \wedge q\, A_{-1}) - \frac{1}{8}(\partial_{x_1}^3 + \partial_{x_1}\partial_{x_2}^2 - 2\,\partial_{x_1}^2\partial_{x_2})(q^4\, p_1 \wedge q\, A$$

In[ ]:=
```
D[q ⊙ q / (p2 ⊙ l + p2 ⊙ q), {x1, 1}, {x2, 2}] // intermediateTerms
```
Out[ ]=

$$-\frac{4\,\gamma}{S\,(l \odot p2 + p2 \odot q)^2} + \frac{2\left(-\frac{2\,m2^2\,p1\odot q}{S} + \frac{2\,\gamma\,p2\odot q}{S}\right)}{(l \odot p2 + p2 \odot q)^3}$$

In[ ]:=
```
D[q ⊙ q / (p2 ⊙ l + p2 ⊙ q), {x2, 2}, {x1, 1}] // intermediateTerms
```
Out[ ]=

$$-\frac{4\,\gamma}{S\,(l \odot p2 + p2 \odot q)^2} + \frac{2\left(-\frac{2\,m2^2\,p1\odot q}{S} + \frac{2\,\gamma\,p2\odot q}{S}\right)}{(l \odot p2 + p2 \odot q)^3}$$

In[ ]:=
```
ClearAll[R21, R22, R23];
Do[R21 = -D[Wedge[p1, q] * diagramsHbar[diag, 0], x1];
  workspace1 = D[q ⊙ q * Wedge[p1, q] * diagramsHbar[diag, -1], {x1, 1}];
  R22 = 1/2 (D[workspace1, {x1, 1}] - D[workspace1, {x2, 1}]);
  workspace1 = D[(q ⊙ q)^2 * Wedge[p1, q] * diagramsHbar[diag, -2], {x1, 1}];
  workspace2 = D[workspace1, {x2, 1}];
  workspace1 = D[workspace1, {x1, 1}];
  workspace1 = workspace1 - workspace2;
  workspace2 = D[workspace1, {x2, 1}];
  workspace1 = D[workspace1, {x1, 1}];
  workspace1 = workspace1 - workspace2;
  R23 = -1/8 * workspace1;

  R21 = onShellP[R21] // intermediateTerms;
  R22 = onShellP[R22] // intermediateTerms;
  R23 = onShellP[R23] // intermediateTerms;

  ClearAll[workspace1, workspace2];

  R2[diag] = supressQml@(R21 + R22 + R23);
  R2[diag] = cancelFactors@R2[diag];
  R2[diag] = cancelFactors@ReleaseHold@Expand@modifyNum@R2[diag];
  R2[diag] = cancelFactors@ReleaseHold@Expand@modifyNum@R2[diag];

  ClearAll[R21, R22, R23];

  , {diag, 1, numDiagrams}];
```

In[ ]:=
```
ClearAll[R];
Do[R[diag] = R1[diag] + R2[diag];
  , {diag, 1, numDiagrams}];

ClearAll[R1, R2];
```

In[ ]:=
```
Do[R[diag] = cancelFactors@ReleaseHold@Expand@modifyNum@R[diag];
 , {diag, 1, numDiagrams}]
```

In[ ]:=
```
R[1][[1]]
```

Out[ ]=

$$\frac{2\,i\,\gamma\;\mathrm{l1}\wedge\mathrm{p1}}{\mathrm{qml1ml2}\;\mathrm{l1}\odot\mathrm{l1}\left(i\,\epsilon+\mathrm{l1}\odot\mathrm{p1}\right)^{2}\left(-i\,\epsilon+\mathrm{l1}\odot\mathrm{p2}\right)\mathrm{l2}\odot\mathrm{l2}}$$

In[ ]:=
```
len = 0;
Do[len += Length[List @@ R[diag]], {diag, 1, numDiagrams}];
len
```
Out[ ]=

43 112

In[ ]:=
```
sumDiagrams = 0;
Do[sumDiagrams += R[diag], {diag, 1, numDiagrams}]
```
In[ ]:=
```
Length[List @@ sumDiagrams]
```
Out[ ]=

14 862

In[ ]:=
```
a = sumDiagrams;
```
In[ ]:=
```
a = cancelFactors@ReleaseHold@Expand@modifyNum@a;
```
In[ ]:=
```
lenA = Length[List @@ a]
```
Out[ ]=

14 862

```
(*/.{(coef1_. ϵ+coef2_.l1⊙m_+rest_.)^(n_.):→(ρ*coef1*ϵ+coef2*l1⊙m+rest)^(n)}*)
```

In[ ]:=
```
factorRho[a[[14 000]] /.
  (coef1_. ϵ + coef2_. l1 ⊙ m_ + rest_)^(n_.) → (coef1 * ϵ + ρ * coef2 * l1 ⊙ m + rest)^(n)]
```
Out[ ]=

$$-\frac{32\, i\, m2^2\, \gamma^4\, l1\odot l2\; l1\odot p1\; q\odot q\; p1\wedge q}{qml1ml2^3\, S^2\, l1\odot l1\, (i\,\epsilon + l1\odot p2)\, l2\odot l2\, (i\,\epsilon + \rho\, l1\odot p1 + l2\odot p1)^2\, (i\,\epsilon + \rho\, l1\odot p2 + l2\odot p2)}$$

In[ ]:=
$$\text{Series}\left[\frac{1}{(i\,\epsilon + \rho\, l1\odot p1 + l2\odot p1)^2\,(i\,\epsilon + \rho\, l1\odot p2 + l2\odot p2)},\ \{\rho, 0, 1\}\right]$$

Out[ ]=

$$\frac{1}{(i\,\epsilon + l2\odot p1)^2\,(i\,\epsilon + l2\odot p2)} + \left(\frac{l1\odot p2}{(i\,\epsilon + l2\odot p1)^2\,(\epsilon - i\, l2\odot p2)^2} - \frac{2\, l1\odot p1}{(i\,\epsilon + l2\odot p1)^3\,(i\,\epsilon + l2\odot p2)}\right)\rho + O[\rho]^2$$

In[ ]:=
```
b = {};
Do[
  AppendTo[b, Expand[Normal[a[[i]] /. {(coef1_. ϵ + coef2_. l1 ⊙ vec1_ + rest1_)^(n_.) → Series[
        (coef1 * ϵ + ρ * coef2 * l1 ⊙ vec1 + rest1)^(n), {ρ, 0, 1}]}]]], {i, 1, Length[a]}];
b = Total[b];
```
In[ ]:=
```
Length[b]
```
Out[ ]=

31 651

In[∘]:=
```
b = List @@ b;
Do[b[[i]] = cancelFactors[simplifiedForm[b[[i]]]], {i, 1, Length[b]}];
```

In[∘]:=
```
b = Total[b];
Length[b]
```

Out[∘]=
```
26 680
```

```
c = Coefficient[factorRho[b /. {ρ → 1} /. {(coef1_. ϵ + coef2_. l1 ⊙ vec1_)^(n_.) →
      (ρ * coef1 * ϵ + coef2 * l1 ⊙ vec1)^(n)} /. {l1 → ρ l1}], ρ, -4];
```

In[∘]:=
```
Length[c]
```

Out[∘]=
```
2038
```

In[∘]:=
```
c[[1000]]
```

Out[∘]=

$$-\frac{6\, i\, \gamma^2\, \mathrm{l1}\odot \mathrm{q}\ \mathrm{p1}\wedge \mathrm{p2}}{\mathrm{l1}\odot\mathrm{l1}\left(-i\,\epsilon+\mathrm{l1}\odot\mathrm{p1}\right)^2\left(-i\,\epsilon+\mathrm{l1}\odot\mathrm{p2}\right)\mathrm{l2}\odot\mathrm{l2}\left(-i\,\epsilon+\mathrm{l2}\odot\mathrm{p1}\right)^2\left(i\,\epsilon+\mathrm{l2}\odot\mathrm{p2}\right)}$$

In[∘]:=
```
Partition[{1, 2, 4, 2, 0, 3, 2, 7}, UpTo[3]]
```

Out[∘]=
```
{{1, 2, 4}, {2, 0, 3}, {2, 7}}
```

In[∘]:=
```
RandomSample[{1, 2, 4, 2, 0, 3, 2, 7}]
```

Out[∘]=
```
{4, 2, 1, 3, 7, 2, 2, 0}
```

```
a = List @@ a;

Do[b = RandomSample[b];
  partition = Partition[a, UpTo[Sqrt[lenA]]];
  lenPartition = Length[partition];


  , {numIteration, 1, 100}];
```

In[∘]:=
```
b = Select[List @@ a,
   ContainsAny[List @@ Numerator[#], {l1 ⊙ p1, l1 ⊙ p2, l2 ⊙ p1, l2 ⊙ p2}] &];
c = Complement[List @@ a, b];
```

In[∘]:=
```
Length@b
Length@c
```

Out[∘]=
```
2464
```

Out[∘]=
```
12 398
```

In[∘]:=
```
d = Total@b;
```

In[∘]:=
```
d = cancelFactors@ReleaseHold@Expand@modifyNum@d;
```

In[∘]:=
```
Length[List @@ d]
```

Out[∘]=
```
3180
```

Out[∘]=

$$\frac{12\, i\, \gamma\, l1{\wedge}p1}{qml1ml2\, l1{\odot}l1\, (i\,\epsilon + l1{\odot}p1)^2\, l2{\odot}l2\, (-i\,\epsilon + l2{\odot}p1)} + \frac{12\, i\, \gamma\, l1{\wedge}p1}{qml1ml2\, l1{\odot}l1\, (\cdots 1 \cdots)^2\, l2{\odot}l2\, (i\,\epsilon + l2{\odot}p1)} + \cdots 4846 \cdots$$

Full expression not available (original memory size: 5.6 MB)

In[∘]:=
```
Length[Select[List @@ d,
  ContainsAny[List @@ Numerator[#], {l1⊙p1, l1⊙p2, l2⊙p1, l2⊙p2}] &]]
```

Out[∘]=
```
2464
```

In[∘]:=
```
factorRho[exp_] :=
 exp /. {(expr_Plus)^(n_) :> Factor[expr]^n /; ContainsAll[Variables[expr], {ϵ}] &&
     ContainsAll[Variables[expr], {l1⊙p1, l1⊙p2, l2⊙p1, l2⊙p2}]}
```

In[∘]:=
```
cancelFactors@ReleaseHold@
  Expand@modifyNum[-((32 ⅈ m2^2 γ^4 l1⊙l2 l2⊙p1 l2⊙p2 q⊙q p1∧q) / (qml1ml2^3 S^2 l1⊙l1
       (ⅈ ϵ + l1⊙p1) (-ⅈ ϵ + l1⊙p2) l2⊙l2 (ⅈ ϵ + l1⊙p1 + l2⊙p1)^2 (-ⅈ ϵ + l1⊙p2 + l2⊙p2)))]

(*cancelFactors[expr_]:=
 expr//.{(c1_. dota1_+c2_. dotb1_.)^(n_.)*(c3_. ϵ+c4_. dota2_+c5_. dotb2_.)^(m_)/;
    c1===c4&&c2===c5&&dota1==dota2&&dotb1==dotb2:>(c3 ϵ+c1 dota1+c2 dotb1)^(m+n)}*)
```

In[∘]:=

$$QL[x\_,\ y\_] := Log\left[\frac{x{\odot}y + \sqrt{((x{\odot}y)\hat{}2 - x{\odot}x\, y{\odot}y)}}{x{\odot}y - \sqrt{((x{\odot}y)\hat{}2 - x{\odot}x\, y{\odot}y)}}\right] \frac{x{\odot}x\, y{\odot}y}{((x{\odot}y)\hat{}2 - x{\odot}x\, y{\odot}y)^{3/2}} \left(-l{\odot}y + \frac{l{\odot}x}{k{\odot}x}\, k{\odot}y\right)$$

In[∘]:=
```
LogSoftOrder2 = Coefficient[
   Normal@Series[(QL[p1, p2] - QL[p1, -p2 + ρ q] + QL[-p1 - ρ q, p2] - QL[-p1 - ρ q, -p2 + ρ q]) /.
      {(p1 + ρ q)⊙(p1 + ρ q) → m1^2, (p2 - ρ q)⊙(p2 - ρ q) → m2^2,
       p1⊙p1 → m1^2, p2⊙p2 → m2^2}, {ρ, 0, 2}], ρ, 2];
```

In[∘]:= **LogSoftOrder2**

Out[∘]=

$$
\mathrm{m1}^2\,\mathrm{m2}^2\left(-\frac{2\,\mathrm{p1}\odot\mathrm{q}\left(\frac{\frac{\mathrm{k}\odot\mathrm{q}\;\mathrm{l}\odot\mathrm{p1}}{\mathrm{k}\odot\mathrm{p1}}-\mathrm{l}\odot\mathrm{q}}{\left(-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2\right)^{3/2}}+\frac{3\left(-\frac{\mathrm{k}\odot\mathrm{p2}\;\mathrm{l}\odot\mathrm{p1}}{\mathrm{k}\odot\mathrm{p1}}+\mathrm{l}\odot\mathrm{p2}\right)\mathrm{p1}\odot\mathrm{p2}\;\mathrm{p1}\odot\mathrm{q}}{\left(-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2\right)^{5/2}}\right)}{\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}}-\right.
$$

$$
\left(\left(-\frac{\mathrm{k}\odot\mathrm{p2}\;\mathrm{l}\odot\mathrm{p1}}{\mathrm{k}\odot\mathrm{p1}}+\mathrm{l}\odot\mathrm{p2}\right)\left(\mathrm{m1}^4\,\mathrm{m2}^4\,\mathrm{p1}\odot\mathrm{p2}\,(\mathrm{p1}\odot\mathrm{q})^2-\mathrm{m1}^2\,\mathrm{m2}^2\,(\mathrm{p1}\odot\mathrm{p2})^3\,(\mathrm{p1}\odot\mathrm{q})^2-\right.\right.
$$

$$
\left.\left.\mathrm{m1}^2\,\mathrm{m2}^2\,(\mathrm{p1}\odot\mathrm{p2})^2\,\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}\,(\mathrm{p1}\odot\mathrm{q})^2\right)\right)\Big/
$$

$$
\left(\left(\mathrm{m1}^2\,\mathrm{m2}^2-(\mathrm{p1}\odot\mathrm{p2})^2\right)^2\left(-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2\right)^{3/2}\left(-\mathrm{p1}\odot\mathrm{p2}+\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}\right)\right.
$$

$$
\left.\left(\mathrm{p1}\odot\mathrm{p2}+\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}\right)^2\right)-
$$

$$
\left(\frac{3\left(\frac{\mathrm{k}\odot\mathrm{q}\;\mathrm{l}\odot\mathrm{p1}}{\mathrm{k}\odot\mathrm{p1}}-\mathrm{l}\odot\mathrm{q}\right)\mathrm{p1}\odot\mathrm{p2}\;\mathrm{p1}\odot\mathrm{q}}{\left(-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2\right)^{5/2}}+\frac{3\left(-\frac{\mathrm{k}\odot\mathrm{p2}\;\mathrm{l}\odot\mathrm{p1}}{\mathrm{k}\odot\mathrm{p1}}+\mathrm{l}\odot\mathrm{p2}\right)\left(\mathrm{m1}^2\,\mathrm{m2}^2+4\,(\mathrm{p1}\odot\mathrm{p2})^2\right)(\mathrm{p1}\odot\mathrm{q})^2}{2\left(\mathrm{m1}^2\,\mathrm{m2}^2-(\mathrm{p1}\odot\mathrm{p2})^2\right)^2\left(-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2\right)^{3/2}}\right)
$$

$$
\left.\mathrm{Log}\left[\frac{\mathrm{p1}\odot\mathrm{p2}-\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}}{\mathrm{p1}\odot\mathrm{p2}+\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}}\right]\right)+
$$

$$
\mathrm{m1}^2\,\mathrm{m2}^2\left(-\frac{2\,\mathrm{p2}\odot\mathrm{q}\left(\frac{\mathrm{k}\odot\mathrm{p2}\left(-\mathrm{k}\odot\mathrm{q}\;\mathrm{l}\odot\mathrm{p1}+\mathrm{k}\odot\mathrm{p1}\;\mathrm{l}\odot\mathrm{q}\right)}{(\mathrm{k}\odot\mathrm{p1})^2\left(-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2\right)^{3/2}}-\frac{3\left(\frac{\mathrm{k}\odot\mathrm{p2}\;\mathrm{l}\odot\mathrm{p1}}{\mathrm{k}\odot\mathrm{p1}}-\mathrm{l}\odot\mathrm{p2}\right)\mathrm{p1}\odot\mathrm{p2}\;\mathrm{p2}\odot\mathrm{q}}{\left(-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2\right)^{5/2}}\right)}{\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}}+\right.
$$

$$
\left(\left(\frac{\mathrm{k}\odot\mathrm{p2}\;\mathrm{l}\odot\mathrm{p1}}{\mathrm{k}\odot\mathrm{p1}}-\mathrm{l}\odot\mathrm{p2}\right)\left(\mathrm{m1}^4\,\mathrm{m2}^4\,\mathrm{p1}\odot\mathrm{p2}\,(\mathrm{p2}\odot\mathrm{q})^2-\mathrm{m1}^2\,\mathrm{m2}^2\,(\mathrm{p1}\odot\mathrm{p2})^3\,(\mathrm{p2}\odot\mathrm{q})^2-\right.\right.
$$

$$
\left.\left.\mathrm{m1}^2\,\mathrm{m2}^2\,(\mathrm{p1}\odot\mathrm{p2})^2\,\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}\,(\mathrm{p2}\odot\mathrm{q})^2\right)\right)\Big/
$$

$$
\left(\left(\mathrm{m1}^2\,\mathrm{m2}^2-(\mathrm{p1}\odot\mathrm{p2})^2\right)^2\left(-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2\right)^{3/2}\left(-\mathrm{p1}\odot\mathrm{p2}+\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}\right)\right.
$$

$$
\left.\left(\mathrm{p1}\odot\mathrm{p2}+\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}\right)^2\right)+\left(-\frac{\mathrm{k}\odot\mathrm{p2}\left(-(\mathrm{k}\odot\mathrm{q})^2\,\mathrm{l}\odot\mathrm{p1}+\mathrm{k}\odot\mathrm{p1}\;\mathrm{k}\odot\mathrm{q}\;\mathrm{l}\odot\mathrm{q}\right)}{(\mathrm{k}\odot\mathrm{p1})^3\left(-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2\right)^{3/2}}-\right.
$$

$$\frac{3\,k\odot p2\,\Big(-k\odot q\;l\odot p1+k\odot p1\;l\odot q\Big)\,p1\odot p2\;p2\odot q}{(k\odot p1)^2\,\Big(-m1^2\,m2^2+(p1\odot p2)^2\Big)^{5/2}}+$$

$$\frac{3\,\Big(\frac{k\odot p2\;l\odot p1}{k\odot p1}-l\odot p2\Big)\Big(m1^2\,m2^2+4\,(p1\odot p2)^2\Big)\,(p2\odot q)^2}{2\,\Big(m1^2\,m2^2-(p1\odot p2)^2\Big)^2\,\Big(-m1^2\,m2^2+(p1\odot p2)^2\Big)^{3/2}}\Bigg)$$

$$\mathrm{Log}\Bigg[\frac{p1\odot p2-\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}}{p1\odot p2+\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}}\Bigg]\Bigg)+m1^2\,m2^2$$

$$\Bigg(\frac{2\,(p1\odot q-p2\odot q)\Bigg(\frac{-l\odot q+\frac{k\odot p1\;k\odot q\;l\odot p1+k\odot p2\;k\odot q\;l\odot p1-k\odot p1\;k\odot p2\;l\odot q}{(k\odot p1)^2}}{\Big(-m1^2\,m2^2+(p1\odot p2)^2\Big)^{3/2}}-\frac{3\,\Big(-\frac{k\odot p2\;l\odot p1}{k\odot p1}+l\odot p2\Big)\,p1\odot p2\,(-p1\odot q+p2\odot q)}{\Big(-m1^2\,m2^2+(p1\odot p2)^2\Big)^{5/2}}\Bigg)}{\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}}-$$

$$\Bigg(\Bigg(-\frac{k\odot p2\;l\odot p1}{k\odot p1}+l\odot p2\Bigg)\Big(m1^2\,m2^2\,p1\odot p2\,\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}\,(p1\odot q)^2-$$

$$2\,m1^2\,m2^2\,p1\odot p2\,\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}\;p1\odot q\;p2\odot q+$$

$$m1^2\,m2^2\,p1\odot p2\,\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}\,(p2\odot q)^2-2\,m1^4\,m2^4\,\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}\;q\odot q+$$

$$2\,m1^2\,m2^2\,(p1\odot p2)^2\,\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}\;q\odot q\Big)\Bigg)\Big/$$

$$\Big(\Big(m1^2\,m2^2-(p1\odot p2)^2\Big)^2\,\Big(-m1^2\,m2^2+(p1\odot p2)^2\Big)^{3/2}\,\Big(-p1\odot p2+\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}\Big)$$

$$\Big(p1\odot p2+\sqrt{-m1^2\,m2^2+(p1\odot p2)^2}\Big)\Big)-\Bigg(\frac{(k\odot p1+k\odot p2)\,\Big(-(k\odot q)^2\;l\odot p1+k\odot p1\;k\odot q\;l\odot q\Big)}{(k\odot p1)^3\,\Big(-m1^2\,m2^2+(p1\odot p2)^2\Big)^{3/2}}-$$

$$\frac{3\,\Big(-l\odot q+\frac{k\odot p1\;k\odot q\;l\odot p1+k\odot p2\;k\odot q\;l\odot p1-k\odot p1\;k\odot p2\;l\odot q}{(k\odot p1)^2}\Big)\,p1\odot p2\,(-p1\odot q+p2\odot q)}{\Big(-m1^2\,m2^2+(p1\odot p2)^2\Big)^{5/2}}+$$

$$\frac{\Big(-\frac{k\odot p2\;l\odot p1}{k\odot p1}+l\odot p2\Big)\Bigg(\frac{15\,(p1\odot p2)^2\,(-p1\odot q+p2\odot q)^2}{\Big(-m1^2\,m2^2+(p1\odot p2)^2\Big)^2}-\frac{3\,\Big((-p1\odot q+p2\odot q)^2-2\,p1\odot p2\;q\odot q\Big)}{-m1^2\,m2^2+(p1\odot p2)^2}\Bigg)}{2\,\Big(-m1^2\,m2^2+(p1\odot p2)^2\Big)^{3/2}}\Bigg)$$

$$\mathrm{Log}\left[\frac{\mathrm{p1}\odot\mathrm{p2}+\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}}{\mathrm{p1}\odot\mathrm{p2}-\sqrt{-\mathrm{m1}^2\,\mathrm{m2}^2+(\mathrm{p1}\odot\mathrm{p2})^2}}\right]\Bigg)$$