

```

In[*]:= Needs["Notation`"];

In[*]:= Needs["MaTeX`"];

In[*]:= (*$RecursionLimit=100*)

In[*]:= (*p1 = {p10,p11,p12,p13};
p2 = {p20,p21,p22,p23};
q = {q0,q1,q2,q3};
l = {l0, l1,l2, l3};
b = {b0, b1,b2,b3};
Q = {Q0,Q1,Q2,Q3};*)

In[*]:= (*eta = DiagonalMatrix[{1,-1,-1,-1}];*)

In[*]:= (*ClearAll[p1,p2,q,l,b, Q, eta];
vecRules = {p1 -> {p10,p11,p12,p13},
p2 -> {p20,p21,p22,p23},
q -> {q0,q1,q2,q3},
l -> {l0, l1,l2, l3},
b -> {b0, b1,b2,b3},
(*Q -> {Q0,Q1,Q2,Q3},*)
Qp -> {Qp0, Qp1, Qp2, Qp3},
eta -> DiagonalMatrix[{1,-1,-1,-1}]};*)

In[*]:= (*RealQ[num_] := TrueQ[Refine[Element[num, Reals]]];
IsReal[x_] := FreeQ[RealQ[x],x∈Reals];*)

In[*]:= (*scalars = {x1, x2, m1, m2, S};
IsScalar[x_] := MemberQ[scalars, x]||NumberQ[x]*)

```

```

In[*]:= ClearAll[scalars];
scalars = {x1, x2, m1, m2, S,  $\hbar$ ,  $\rho$ ,  $\gamma$ ,  $\epsilon$ };

$Assumptions = And @@ (Element[#, Reals] & /@ scalars);

IsScalar[expr_] := Module[{vars, dotExp},
  vars = Variables[expr];
  dotExp = Cases[vars, holdDot[_], _];
  vars = Complement[vars, dotExp];
  vars = Join[vars, HoldForm /@ dotExp];

  (vars === {} && NumericQ[expr]) || (vars != {} && SubsetQ[scalars, vars])
];

(*IsScalar[expr_] := Module[{vars},
  vars = Variables[expr];

  (vars === {} && NumericQ[expr]) || (vars != {} && SubsetQ[scalars, vars])
];*)

```

```

In[*]:= scalars

```

```

Out[*]=
{x1, x2, m1, m2, S,  $\hbar$ ,  $\rho$ ,  $\gamma$ ,  $\epsilon$ }

```

```

In[*]:= AddToScalars[expr_] :=
  (If[! MemberQ[scalars, HoldForm[expr]], scalars = Append[scalars, HoldForm[expr]]];
  expr);

AddDotProdToScalars[vectors_] := Module[{vecPairs},
  (*vecPairs = Subsets[vectors, {2}];*)
  vecPairs = DeleteDuplicatesBy[Tuples[vectors, 2], Sort];
  dotProdPairs = holdDot @@ # & /@ vecPairs;
  Do[AddToScalars[dotProd], {dotProd, dotProdPairs}];
];

```

```

In[*]:= holdDotDef := (
  ClearAll[holdDot];
  (*holdDot[a_,b_]:=Module[{aa=a,bb=b},If[OrderedQ[{aa,bb}],a.eta.b,b.eta.a]];*)
  (*holdDot/:holdDot[a_,b_]:=If[OrderedQ[{a,b}],holdDot[a,b],holdDot[b,a]];*)
  (*holdDot/:holdDot[a_,b_]:=0/(PossibleZeroQ[a]||PossibleZeroQ[b]);
  holdDot/:holdDot[a_,b_]:=holdDot@@Sort[{a,b}]/;!OrderedQ[{a,b}];*)
  holdDot[a_, b_] := 0 /; (PossibleZeroQ[a] || PossibleZeroQ[b]);
  holdDot[a_, b_] := holdDot @@ Sort[{a, b}] /; !OrderedQ[{a, b}];
  holdDot[a_ + b_, x_] := holdDot[a, x] + holdDot[b, x];
  holdDot[x_, a_ + b_] := holdDot[a, x] + holdDot[b, x];
  holdDot[x_, c_?IsScalar y_] := c holdDot[x, y];
  holdDot[c_?IsScalar x_, y_] := c holdDot[x, y];
  holdDot[x_, Times[holdDot[a_, b_], y_]] := holdDot[a, b] x holdDot[x, y];
  holdDot[Times[holdDot[a_, b_], x_], y_] := holdDot[a, b] x holdDot[x, y];
  (*holdDot[c_ x_, y_] := c holdDot[x, y]; FreeQ[c, _List];
  holdDot[x_, c_ y_] := c holdDot[x, y]; FreeQ[c, _List];*)

```

```

  Notation[ParsedBoxWrapper[RowBox[{"a_", "⊙", "b_"}]] ⇔
    ParsedBoxWrapper[RowBox[{"holdDot", "[", "a_", ",", "b_", "]"}]]];
)

```

```

In[*]:= holdDotDer := (
  (*ClearAll[holdDot];*)

  SetOptions[D, NonConstants → {holdDot, Wedge}];
  (*D[expr_holdDot, vars_]^:=Module[{x,y,z},*)
  holdDot/: D[holdDot[a_, b_], vars_] := Module[{x = a, y = b, z},
    (*{x,y}=List@@expr;*)
    z = {vars}[[1]];
    Which[
      IsScalar[z] && x === q && y === q,
      holdDot[D[Q, {vars}[[1]], y] + holdDot[x, D[Q, {vars}[[1]]],
      IsScalar[z] && x === q,
      holdDot[D[Q, {vars}[[1]], y],
      IsScalar[z] && y === q,
      holdDot[x, D[Q, {vars}[[1]]],
      ! IsScalar[{vars}[[1]],
      x x D[y, {vars}[[1]] + y x D[x, {vars}[[1]],
      True, 0];
    ]
  )

```

```
In[*]:= holdDotDef
```

**Notation:** Future versions of the Notation package will no longer support  $\Leftrightarrow$ , instead they will use  $\Leftrightarrow$ . Please make this change to all your Notations.

```
In[*]:= vectors = {p1, p2, q, l, l1, l2, Qp}
```

```
Out[*]:= {p1, p2, q, l, l1, l2, Qp}
```

```
In[*]:= AddDotProdToScalars[vectors]
```

```
In[*]:= scalars
```

```
Out[*]:= {x1, x2, m1, m2, S, h, ρ, γ, ε, p1 ⊙ p1, p1 ⊙ p2, p1 ⊙ q, l ⊙ p1, l1 ⊙ p1, l2 ⊙ p1, p1 ⊙ Qp,
p2 ⊙ p2, p2 ⊙ q, l ⊙ p2, l1 ⊙ p2, l2 ⊙ p2, p2 ⊙ Qp, q ⊙ q, l ⊙ q, l1 ⊙ q, l2 ⊙ q, q ⊙ Qp,
l ⊙ l, l ⊙ l1, l ⊙ l2, l ⊙ Qp, l1 ⊙ l1, l1 ⊙ l2, l1 ⊙ Qp, l2 ⊙ l2, l2 ⊙ Qp, Qp ⊙ Qp}
```

```
In[*]:= ClearAll[Wedge];
```

```
Wedge[x_, y_] := 0 /; (x === y || PossibleZeroQ[x] || PossibleZeroQ[y]);
Wedge[a_ + b_, x_] := Wedge[a, x] + Wedge[b, x];
Wedge[x_, a_ + b_] := Wedge[x, a] + Wedge[x, b];
Wedge[x_, c_?IsScalar y_] := c Wedge[x, y];
Wedge[c_?IsScalar x_, y_] := c Wedge[x, y];
Wedge[x_, y_] := -Wedge[y, x] /; OrderedQ[{y, x}];
Wedge[x_, Times[holdDot[a_, b_], y_]] := holdDot[a, b] * Wedge[x, y];
Wedge[Times[holdDot[a_, b_] x_, y_] := holdDot[a, b] * Wedge[x, y];
```

```
In[*]:= wedgeDer := (SetOptions[D, NonConstants -> {holdDot, Wedge}];
```

```
(*D[expr_Wedge, vars_]^:=Module[{x,y,z},*)
Wedge /: D[Wedge[a_, b_], vars_] := Module[{x = a, y = b, z},
(*{x,y}=List@@@expr;*)
z = {vars}[[1]];
Which[
IsScalar[z] && x === q && y === q,
Wedge[D[Q, {vars}[[1]], y] + Wedge[x, D[Q, {vars}[[1]]],
IsScalar[z] && x === q,
Wedge[D[Q, {vars}[[1]], y],
IsScalar[z] && y === q,
Wedge[x, D[Q, {vars}[[1]]],
! IsScalar[{vars}[[1]],
Wedge[x, D[y, {vars}[[1]]] + Wedge[D[x, {vars}[[1]], y],
True, 0];
)
```

```

In[*]:= aWedgeDb[expr_, a_, b_] := Wedge[a, D[expr, b]];
aWedgeDb[expr_holdDot, a_, b_] := Module[{x, y}, {x, y} = List @@ expr;
Wedge[a, y D[x, b]] + Wedge[a, x D[y, b]]];

In[*]:= (*ClearAll[holdDot]*)
(*SetAttributes[holdDot, HoldAll]*)

In[*]:= (*holdDot[a_, b_] :=
Module[{aa=a, bb=b}, If[OrderedQ[{aa, bb}], HoldForm[a.eta.b], HoldForm[b.eta.a]]];*)
(*holdDot[a_, b_] := Module[{aa=a, bb=b}, If[OrderedQ[{aa, bb}], a.eta.b, b.eta.a];
holdDot[a_+b_, x_] := holdDot[a, x] + holdDot[b, x];
holdDot[x_, a_+b_] := holdDot[a, x] + holdDot[b, x];
holdDot[x_, c_?NumberQ y_] := c holdDot[x, y];
holdDot[c_?NumberQ x_, y_] := c holdDot[x, y];
holdDot[c_ x_, y_] := c holdDot[x, y]; FreeQ[c, _List];
holdDot[x_, c_ y_] := c holdDot[x, y]; FreeQ[c, _List];*)

In[*]:= (*UpValues[holdDot]={};
holdDot, Dt[holdDot[x_, y_], z, ___] := Which[
x===q && y===q,
holdDot[D[Q, z], y] + holdDot[x, D[Q, z]],
x===q,
holdDot[D[Q, z], y],
y===q,
holdDot[x, D[Q, z]],
True,
0 ]*)

(*holdDot/:D[holdDot[x_, y_], z_, NonConstants->_] := D[holdDot[x, y], z]*)

In[*]:= (*wrapDot[expr_] := expr //. {(p1.eta.p1/.vecRules)->m1^2, (-p1.eta.p1/.vecRules)->-m1^2,
(p2.eta.p2/.vecRules)->m2^2, (-p2.eta.p2/.vecRules)->-m2^2,
(p1.eta.p2/.vecRules)->y, (-p1.eta.p2/.vecRules)->-y,
(p1.eta.q/.vecRules)->p1.eta.q, (-p1.eta.q/.vecRules)->-p1.eta.q,
(p2.eta.q/.vecRules)->p2.eta.q, (-p2.eta.q/.vecRules)->-p2.eta.q,
(p1.eta.l/.vecRules)->p1.eta.l, (-p1.eta.l/.vecRules)->-p1.eta.l,
(p2.eta.l/.vecRules)->p2.eta.l, (-p2.eta.l/.vecRules)->-p2.eta.l,
(l.eta.q/.vecRules)->l.eta.q, (-l.eta.q/.vecRules)->-l.eta.q,
(l.eta.l/.vecRules)->l.eta.l, (-l.eta.l/.vecRules)->-l.eta.l,
(q.eta.q/.vecRules)->q.eta.q, (-q.eta.q/.vecRules)->-q.eta.q}*)

```

```
In[*]:= (*holdDot[a_, b_] := a.eta.b//
FullSimplify[{p1.eta.p2→HoldForm[HoldPattern[y], -p1.eta.p2→-HoldForm[y],
p1.eta.q→HoldForm[p1.eta.q], -p1.eta.q→-HoldForm[p1.eta.q],
p2.eta.q→HoldForm[p2.eta.q], -p2.eta.q→-HoldForm[p2.eta.q],
p1.eta.l→HoldForm[p1.eta.l], -p1.eta.l→-HoldForm[p1.eta.l],
p2.eta.l→HoldForm[p2.eta.l], -p2.eta.l→-HoldForm[p2.eta.l],
l.eta.q→HoldForm[l.eta.q], -l.eta.q→-HoldForm[l.eta.q]}]*)
```

```
In[*]:= (*wrapDot[expr_, ] := expr // {p1.eta.p2→HoldForm[y], -p1.eta.p2→-HoldForm[y],
p1.eta.q→HoldForm[p1.eta.q], -p1.eta.q→-HoldForm[p1.eta.q],
p2.eta.q→HoldForm[p2.eta.q], -p2.eta.q→-HoldForm[p2.eta.q],
p1.eta.l→HoldForm[p1.eta.l], -p1.eta.l→-HoldForm[p1.eta.l],
p2.eta.l→HoldForm[p2.eta.l], -p2.eta.l→-HoldForm[p2.eta.l],
l.eta.q→HoldForm[l.eta.q], -l.eta.q→-HoldForm[l.eta.q],
l.eta.l→HoldForm[l.eta.l], -l.eta.l→-HoldForm[l.eta.l],
q.eta.q→HoldForm[q.eta.q], -q.eta.q→-HoldForm[q.eta.q]}*)
```

```
In[*]:= (*holdDot[expr_] :=
Simplify[expr, {p1.eta.p2→HoldForm[y], p1.eta.q→HoldForm[p1.eta.q],
p2.eta.q→HoldForm[p2.eta.q], p1.eta.l→HoldForm[p1.eta.l],
p2.eta.l→HoldForm[p2.eta.l], l.eta.q→HoldForm[l.eta.q]}]*)
```

```
In[*]:= holdDotDef
```

**Notation:** Future versions of the Notation package will no longer support  $\Leftrightarrow$ , instead they will use  $\iff$ . Please make this change to all your Notations.

```
In[*]:=  $\alpha_1 = (\gamma x_2 - m^2 x_1) / S;$ 
 $\alpha_2 = (\gamma x_1 - m^2 x_2) / S;$ 
 $Q = \alpha_1 p_1 + \alpha_2 p_2 + Q_p;$ 
 $q == Q;$ 
```

```
In[*]:= holdDotDer
```

```
In[*]:= wedgeDer
```

```
In[*]:= D[holdDot[p1, q]/holdDot[p2, q], x1]
```

```
Out[*]=
```

$$-\frac{p_1 \odot q \left( -\frac{m^2 p_1 \odot p_2}{S} + \frac{\gamma p_2 \odot p_2}{S} \right)}{(p_2 \odot q)^2} + \frac{-\frac{m^2 p_1 \odot p_1}{S} + \frac{\gamma p_1 \odot p_2}{S}}{p_2 \odot q}$$

```
In[*]:= onShellP[expr_] :=
expr //. {p1 ⊙ q → 0, p2 ⊙ q → 0, p1 ⊙ p2 → γ, p1 ⊙ p1 → m1^2, p2 ⊙ p2 → m2^2}
(*onShellP[expr_] := expr //. {HoldForm[p1.eta.q]→0, HoldForm[p2.eta.q]→0}*)
```

```
In[*]:= (*simplifiedForm[expr_]:=expr/.{(-m1^2 m2^2/s + y^2/s) -> 1, (m1^2 m2^2/s - y^2/s) -> -1}/.
      {(coef1_ ε+coef2_ l⊗p1)^(n_) -> (coef2)^n*(ε*(coef1/coef2)/Abs[coef1/coef2]+l⊗p1)^n,
      (coef1_ ε+coef2_ l⊗p2)^(n_) -> (coef2)^n*(ε*(coef1/coef2)/Abs[coef1/coef2]+l⊗p2)^n}*)
```

```
In[*]:= (-m1^2 m2^2/(W*S) + y^2/(W*S))/.(T1:-m1^2 m2^2/c_.)+(T2:y^2/c_.)-> S/c
```

```
Out[*]:=
      -m1^2 m2^2/(S W) + y^2/(S W)
```

```
In[*]:= simplifiedForm[expr_] :=
      expr /. {(-m1^2 m2^2/s + y^2/s) -> 1, (m1^2 m2^2/s - y^2/s) -> -1, -m1^2 m2^2 + y^2 -> S, m1^2 m2^2 - y^2 -> -S} //.
      {(coef1_. ε+coef2_. l1dot_+rest_.)^(n_.) -> (coef2)^n*
      (ε*(coef1/coef2)/Abs[coef1/coef2]+l1dot+Distribute[rest/coef2])^n /;
      l1dot == l⊗p1 || l1dot == l⊗p2 || l1dot == l1⊗p1 || l1dot == l1⊗p2 ||
      l1dot == l2⊗p1 || l1dot == l2⊗p2}
```

```
In[*]:= cancelFactors[expr_] :=
      expr //. {dot1:d1_^(n_.)*(c1_. ε+ dot2:d2_+d3____)^(m_) /; d1 == d2+d3 ->
      If[Abs[n] < Abs[m], (c1 ε+ d1)^(m+n), d1^(m+n)]}
```

```
In[*]:= factorRho[exp_] :=
      exp /. {(expr_Plus)^(n_) -> Factor[expr]^n /; ContainsAll[Variables[expr], {ε, l⊗p1}] ||
      ContainsAll[Variables[expr], {ε, l⊗p2}]}
```

```
In[*]:= simplifyTerm[expr_] := expr //. {(num_a_)/(Power[den_, -n_]) ->
      num/(den^(-n-1)) /; ContainsAll[Variables[den], {ε, a}] /; a == l⊗p1 || a == l⊗p2}
```

```
In[*]:= simplifyTerm[expr_] := expr //.
      {(num_Power[a_, m_:1])/(Power[den_, -n_:1]) -> num Power[a, m-1]/(den^(-n-1)) /;
      ContainsAll[Variables[den], {ε, a}] /; a == l⊗p1 || a == l⊗p2}
```

```
In[*]:= simplifyTerm[expr_] := {lst = {};
```

```
      Do[AppendTo[lst, expr[[i]] //. {(num_Power[a_, m_:1])/(Power[den_, -n_:1]) ->
      num Power[a, m-1]/(den^(-n-1)) /; ContainsAll[Variables[den], {ε, a}] /;
      a == l⊗p1 || a == l⊗p2}], {i, 1, Length[expr]}];
```

```
      Total[lst];
```

```
In[*]:= cancelFactors[expr_] :=
      expr //. {dot1:d1_^(n_.)*(c1_. ε+ dot2:d2_+d3____)^(m_) /; d1 == d2+d3 ->
      If[Abs[n] < Abs[m], (c1 ε+ d1)^(m+n), d1^(m+n)]}
```

```

In[*]:= wedgeToDot[expr_] :=
  expr /. {HoldPattern[Wedge[l1, c_]] :-> l ⊗ c, HoldPattern[Wedge[c_, l1]] :-> -l ⊗ c}

In[*]:= extractLog[expr_, expnt_] := Coefficient[FactorRho[
  simplifyTerm[Expand[simplifiedForm[onShellP[expr]]]] /. {l -> ρ l, ε -> ρ ε}], ρ, expnt]

In[*]:= delx := (D[#, {x1, 1}] - D[#, {x2, 1}]) &;

In[*]:= L[1] = 1 / ((p1 ⊗ l + i ε) (p2 ⊗ l - i ε));
L[2] = 1 / ((p1 ⊗ l + i ε) (p2 ⊗ (q - l) - i ε));
L[3] = 1 / ((p1 ⊗ (q - l) + i ε) (p2 ⊗ (q - l) - i ε));
L[4] = 1 / ((p1 ⊗ (q - l) + i ε) (p2 ⊗ l - i ε));

In[*]:= Cf[1] = (1/2) ⎛ 2 γ + ħ (p2 - p1) ⊗ l -  $\frac{\hbar * \gamma l \otimes l}{(p1 \otimes l + i \epsilon)}$  +  $\frac{\hbar * \gamma l \otimes l}{(p2 \otimes l - i \epsilon)}$  ⎞;

Cf[2] = (1/2) ⎛ 2 γ + ħ (p2 + p1) ⊗ l - 2 ħ * p1 ⊗ q -  $\frac{\hbar * \gamma l \otimes l}{(p1 \otimes l + i \epsilon)}$  +  $\frac{\hbar * \gamma (q - l) \otimes (q - l)}{(p2 \otimes (q - l) - i \epsilon)}$  ⎞;

Cf[3] = (1/2) ⎛ 2 γ + ħ (p2 - p1) ⊗ (2 q - l) -  $\frac{\hbar * \gamma (q - l) \otimes (q - l)}{(p1 \otimes (q - l) + i \epsilon)}$  +  $\frac{\hbar * \gamma (q - l) \otimes (q - l)}{(p2 \otimes (q - l) - i \epsilon)}$  ⎞;

Cf[4] = (1/2) ⎛ 2 γ - ħ (p2 + p1) ⊗ l + 2 ħ * p2 ⊗ q -  $\frac{\hbar * \gamma (q - l) \otimes (q - l)}{(p1 \otimes (q - l) + i \epsilon)}$  +  $\frac{\hbar * \gamma l \otimes l}{(p2 \otimes l - i \epsilon)}$  ⎞;

In[*]:= A0[1] = i  $\frac{(2 p1 + \hbar (q + \rho * l)) \otimes (2 p2 - \hbar (q + \rho * l))}{(q - \rho * l) \otimes (q - \rho * l)}$ ;

A0[2] = i  $\frac{(2 p1 + \hbar (q + \rho * l)) \otimes (2 p2 - \hbar (q - \rho * l))}{(q - \rho * l) \otimes (q - \rho * l)}$ ;

A0[3] = i  $\frac{(2 p1 + \hbar (q - \rho * l)) \otimes (2 p2 - \hbar (q - \rho * l))}{(q - \rho * l) \otimes (q - \rho * l)}$ ;

A0[4] = i  $\frac{(2 p1 + \hbar (q - \rho * l)) \otimes (2 p2 - \hbar (q + \rho * l))}{(q - \rho * l) \otimes (q - \rho * l)}$ ;

In[*]:= Do[Do[Cfhbar[i][n] = Coefficient[Normal[Series[Cf[i], {ħ, 0, n}]]], ħ, n];, {i, 1, 4},
  {n, 0, 2}];

In[*]:= Do[Do[A0hbar[i][n] = Coefficient[Normal[Series[A0[i], {ħ, 0, n}]]], ħ, n];, {i, 1, 4},
  {n, 0, 1}];

In[*]:= Do[Do[Do[A0hbarl[i][n, m] = Coefficient[Normal[Series[A0hbar[i][n], {ρ, 0, m}]]], ρ, m],
  {i, 1, 4}];, {m, 0, 3}], {n, 0, 1}];

```



In[\*]:= D[p1 ⊗ q, x1]

Out[\*]:=

$$-\frac{m^2 p_1 \otimes p_1}{S} + \frac{\gamma p_1 \otimes p_2}{S}$$

In[\*]:= a1 = aWedgeDb[A0[1] /. {ρ → 0}, l1, p2] /.

{HoldPattern[Wedge[l1, c\_]] ⇒ l ⊗ c, HoldPattern[Wedge[c\_, l1]] ⇒ -l ⊗ c};

a2 = aWedgeDb[aWedgeDb[A0[1] /. {ρ → 0}, l1, p1] /.

{HoldPattern[Wedge[l1, c\_]] ⇒ l ⊗ c, HoldPattern[Wedge[c\_, l1]] ⇒ -l ⊗ c}, p1, p1];

a3 = aWedgeDb[aWedgeDb[A0[1] /. {ρ → 0}, l1, p2] /.

{HoldPattern[Wedge[l1, c\_]] ⇒ l ⊗ c, HoldPattern[Wedge[c\_, l1]] ⇒ -l ⊗ c}, p1, p1];

b1 = D[Wedge[p1, q] \* (aWedgeDb[A0[1] /. {ρ → 0}, l1, p1] /.

{HoldPattern[Wedge[l1, c\_]] ⇒ l ⊗ c, HoldPattern[Wedge[c\_, l1]] ⇒ -l ⊗ c}, x1];

b2 = D[Wedge[p1, q] \* (aWedgeDb[A0[1] /. {ρ → 0}, l1, p2] /.

{HoldPattern[Wedge[l1, c\_]] ⇒ l ⊗ c, HoldPattern[Wedge[c\_, l1]] ⇒ -l ⊗ c}, x1];

a1 = simplifiedForm@onShellP@a1;

a2 = simplifiedForm@onShellP@a2;

a3 = simplifiedForm@onShellP@a3;

b1 = simplifiedForm@onShellP@b1;

b2 = simplifiedForm@onShellP@b2;

In[\*]:= sum = 0;

Do[sum += L[i] \* Cf[i], {i, {1, 2, 3, 4}}];

In[\*]:= ClearAll[a1, a2, a3, a4, a5, a6]

```
In[*]:= a1 = Coefficient[extractLog[
  sum*(aWedgeDb[A0[1] /. {ρ → 0}, p1, p1] - D[Wedge[p1, q]*(A0[1] /. {ρ → 0}), x1]), -2], ħ, 1]
```

```
Out[*]=
```

$$\begin{aligned} & \frac{4 i \gamma l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) q \odot q} - \frac{4 i \gamma^3 l \odot q p1 \wedge p2}{S(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) q \odot q} + \\ & \frac{4 i \gamma l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} - \frac{4 i \gamma^3 l \odot q p1 \wedge p2}{S(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} - \\ & \frac{4 i \gamma l \odot q p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} + \frac{4 i \gamma^3 l \odot q p1 \wedge p2}{S(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} - \\ & \frac{4 i \gamma l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) q \odot q} + \frac{4 i \gamma^3 l \odot q p1 \wedge p2}{S(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) q \odot q} \end{aligned}$$

```
In[*]:= a2 = Coefficient[
  extractLog[(aWedgeDb[sum /. {γ → p1 ⊙ p2}, p1, p1] - Wedge[p1, q]*D[sum, x1])*
  (A0[1] /. {ρ → 0}), -2], ħ, 1]
```

```
Out[*]=
```

$$\begin{aligned} & \frac{8 i \gamma^2 l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2) q \odot q} + \frac{4 i \gamma^2 l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 q \odot q} - \\ & \frac{4 i \gamma^2 l \odot q l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 q \odot q} - \frac{8 i \gamma^2 l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2) q \odot q} + \\ & \frac{4 i \gamma l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) q \odot q} + \frac{4 i \gamma l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} - \\ & \frac{4 i \gamma l \odot q p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} - \frac{4 i \gamma l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) q \odot q} + \\ & \frac{4 i m^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) q \odot q} - \frac{4 i m^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q} - \\ & \frac{4 i \gamma^3 p1 \wedge q}{S(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q} + \frac{4 i \gamma^3 p1 \wedge q}{S(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q} \end{aligned}$$

```
In[*]:= a3 = Coefficient[
  extractLog[-(aWedgeDb[sum /. {γ → p1 ⊗ p2}, p1, p1] - Wedge[p1, q] * D[sum, x1]) *
    (wedgeToDot[aWedgeDb[A0[1] /. ρ → 0, l1, q]]), -2], ħ, 1]
```

```
Out[*]=
```

$$\begin{aligned}
& -\frac{8i\gamma^2 l \otimes q l \wedge p1}{(-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2) q \otimes q} - \frac{4i\gamma^2 l \otimes q l \wedge p1}{(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2 q \otimes q} + \\
& \frac{4i\gamma^2 l \otimes q l \wedge p1}{(i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2 q \otimes q} + \frac{8i\gamma^2 l \otimes q l \wedge p1}{(-i\epsilon + l \otimes p1)^3 (i\epsilon + l \otimes p2) q \otimes q} - \\
& \frac{4i\gamma l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2) q \otimes q} - \frac{4i\gamma l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1) (i\epsilon + l \otimes p2)^2 q \otimes q} + \\
& \frac{4i\gamma l \otimes q p1 \wedge p2}{(i\epsilon + l \otimes p1) (i\epsilon + l \otimes p2)^2 q \otimes q} + \frac{4i\gamma l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2) q \otimes q}
\end{aligned}$$

```
In[*]:= a4 = Coefficient[
  extractLog[-sum * (aWedgeDb[wedgeToDot[aWedgeDb[A0[1] /. ρ → 0, l1, p1]], p1, p1] -
    D[Wedge[p1, q] * wedgeToDot[aWedgeDb[A0[1] /. ρ → 0, l1, p1]], x1]), -2], ħ, 1]
```

```
Out[*]=
```

$$-\frac{2i\gamma^2 p1 \wedge p2}{S(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} + \frac{2i\gamma^2 p1 \wedge p2}{S(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)}$$

```
In[*]:= a2 + a3 + a4
```

```
Out[*]=
```

$$\begin{aligned}
& -\frac{2i\gamma^2 p1 \wedge p2}{S(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} + \frac{2i\gamma^2 p1 \wedge p2}{S(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} + \\
& \frac{4im^2\gamma^2 p1 \wedge q}{S(-i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2) q \otimes q} - \frac{4im^2\gamma^2 p1 \wedge q}{S(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2) q \otimes q} - \\
& \frac{4i\gamma^3 p1 \wedge q}{S(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2) q \otimes q} + \frac{4i\gamma^3 p1 \wedge q}{S(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2) q \otimes q}
\end{aligned}$$

```
In[*]:= sum = 0;
```

```
Do[sum += L[i] * Cf[i], {i, {1, 2, 3, 4}}];
```

```
In[*]:= b1 = extractLog[D[sum * q ⊗ q * aWedgeDb[A0[1] /. ρ → 0, p1, p1], x1] -
  D[sum * q ⊗ q * aWedgeDb[A0[1] /. ρ → 0, p1, p1], x2], -2];
```

```
In[*]:= b2 = extractLog[D[(A0[1] /. ρ → 0) * q ⊗ q * aWedgeDb[sum, p1, p1], x1] -
  D[(A0[1] /. ρ → 0) * q ⊗ q * aWedgeDb[sum, p1, p1], x2], -2];
```

```
In[*]:= b3 = -extractLog[
  D[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]] * q ⊗ q * aWedgeDb[sum, p1, p1], x1] -
  D[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]] *
    q ⊗ q * aWedgeDb[sum, p1, p1], x2], -2];
```

```

In[*]:= b4 = -extractLog[
  D[sum * q ⊙ q * aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], p1, p1], x1] -
  D[sum * q ⊙ q * aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], p1, p1],
  x2], -2];

In[*]:= b5 =  $\frac{1}{2}$  extractLog[
  D[wedgeToDot[aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], l1, q]] *
  q ⊙ q * aWedgeDb[sum, p1, p1], x1] -
  D[wedgeToDot[aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], l1, q]] *
  q ⊙ q * aWedgeDb[sum, p1, p1], x2], -2];

In[*]:= c1 = extractLog[D[D[sum * Wedge[p1, q] * (A0[1] /. ρ → 0), x1] -
  D[sum * Wedge[p1, q] * (A0[1] /. ρ → 0), x2], x1], -2];

In[*]:= c2 = -extractLog[
  D[D[sum * Wedge[p1, q] * q ⊙ q * wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], x1] - D[sum *
  Wedge[p1, q] * q ⊙ q * wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], x2], x1], -2];

In[*]:= c3 =
 $\frac{1}{2}$  extractLog[D[D[sum * Wedge[p1, q] * q ⊙ q * wedgeToDot[aWedgeDb[wedgeToDot[aWedgeDb[
  (A0[1] /. ρ → 0), l1, q]], l1, q]], x1] - D[sum * Wedge[p1, q] * q ⊙ q * wedgeToDot[
  aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], l1, q]], x2], x1], -2];

In[*]:= Coefficient[Expand[b1 + b2 + b3 + b4 + b5], ħ, 0]

In[*]:= Coefficient[Expand[c1 + c2 + c3], ħ, 0] - Coefficient[Expand[b1 + b2 + b3 + b4 + b5], ħ, 0]

In[*]:= wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]]

Out[*]=

$$\frac{i(-2\hbar l \odot p_1 + 2\hbar l \odot p_2 - 2\hbar^2 l \odot q)}{q \odot q} - \frac{2i l \odot q(4p_1 \odot p_2 - 2\hbar p_1 \odot q + 2\hbar p_2 \odot q - \hbar^2 q \odot q)}{(q \odot q)^2}$$


```

In[ ]:= Coefficient[Expand[b3],  $\hbar$ , 0]

Out[ ]:=

$$\begin{aligned}
& -\frac{16 i \gamma^2 l \odot q l \wedge p_1}{(-i \epsilon + l \odot p_1)^3 (-i \epsilon + l \odot p_2) q \odot q} - \frac{8 i \gamma^2 l \odot q l \wedge p_1}{(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)^2 q \odot q} + \\
& \frac{8 i \gamma^2 l \odot q l \wedge p_1}{(i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)^2 q \odot q} + \frac{16 i \gamma^2 l \odot q l \wedge p_1}{(-i \epsilon + l \odot p_1)^3 (i \epsilon + l \odot p_2) q \odot q} - \\
& \frac{8 i m_1^2 \gamma^2 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} - \frac{8 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} + \\
& \frac{8 i m_1^2 \gamma^2 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} + \frac{8 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} + \\
& \frac{8 i m_2^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) q \odot q} + \frac{8 i \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) q \odot q} - \\
& \frac{8 i m_2^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} - \frac{8 i \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q}
\end{aligned}$$

In[ ]:= extractLog[simplifyTerm[simplifiedForm[onShellP[Expand[sum]]], -2]

Out[ ]:=

$$\begin{aligned}
& -\frac{\gamma}{(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2)} + \frac{\gamma}{(i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2)} + \\
& \frac{\gamma}{(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)} - \frac{\gamma}{(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)} + \frac{\gamma \hbar l \odot q}{(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2)} + \\
& \frac{\gamma \hbar l \odot q}{(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2} - \frac{\gamma \hbar l \odot q}{(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2} - \frac{\gamma \hbar l \odot q}{(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)}
\end{aligned}$$

```

In[ ]:= extractLog[Total[Select[
  List @@ simplifyTerm[simplifiedForm[Expand[onShellP[aWedgeDb[sum, p1, p1]]]],
  ContainsAll[List @@ Numerator[#, {p1 ∧ q} &]]] -
  simplifyTerm[Expand[simplifiedForm[onShellP[p1 ∧ q D[sum, x1]]]], -2]

```

Out[ ]:=

$$\begin{aligned}
& \frac{\gamma \hbar l \odot l p1 \wedge q}{2(-i\epsilon + l \odot p1)^2(-i\epsilon + l \odot p2)^2} + \frac{\gamma \hbar l \odot l p1 \wedge q}{(-i\epsilon + l \odot p1)^3(-i\epsilon + l \odot p2)} - \\
& \frac{\hbar p1 \wedge q}{2(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} + \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} + \frac{\gamma \hbar l \odot l p1 \wedge q}{2(-i\epsilon + l \odot p1)^2(i\epsilon + l \odot p2)^2} - \\
& \frac{\gamma \hbar l \odot l p1 \wedge q}{(-i\epsilon + l \odot p1)^3(i\epsilon + l \odot p2)} + \frac{\hbar p1 \wedge q}{2(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} - \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} - \\
& \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} - \frac{\hbar p1 \wedge q}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} + \frac{\gamma^2 \hbar p1 \wedge q}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)}
\end{aligned}$$

```

In[ ]:= ClearAll[sum];
sum = 0;
Do[sum += L[i] * Cf[i] /. {γ → p1 ⊙ p2}, {i, {1, 2, 3, 4}}];

```

In[\*]:= **extractLog**[delx[{aWedgeDb[sum /. {γ → p1 ⊗ p2}, p1, p1] - Wedge[p1, q] \* D[sum, x1]]], -3]

Out[\*]=

$$\begin{aligned}
& -\frac{2\gamma l \wedge p1}{(-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)} - \frac{\gamma l \wedge p1}{(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2} + \\
& \frac{\gamma l \wedge p1}{(i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2} + \frac{2\gamma l \wedge p1}{(-i\epsilon + l \otimes p1)^3 (i\epsilon + l \otimes p2)} + \frac{6\gamma \hbar l \otimes q l \wedge p1}{(-i\epsilon + l \otimes p1)^4 (-i\epsilon + l \otimes p2)} - \\
& \frac{2\gamma \hbar l \otimes q l \wedge p1}{(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^3} + \frac{2\gamma \hbar l \otimes q l \wedge p1}{(i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^3} + \frac{4\gamma \hbar l \otimes q l \wedge p1}{(-i\epsilon + l \otimes p1)^3 (i\epsilon + l \otimes p2)^2} - \\
& \frac{6\gamma \hbar l \otimes q l \wedge p1}{(-i\epsilon + l \otimes p1)^4 (i\epsilon + l \otimes p2)} - \frac{p1 \wedge p2}{(-i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)} - \frac{p1 \wedge p2}{(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} + \\
& \frac{p1 \wedge p2}{(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} + \frac{p1 \wedge p2}{(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)} + \frac{2\hbar l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)} - \\
& \frac{2\hbar l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^3} + \frac{2\hbar l \otimes q p1 \wedge p2}{(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^3} + \frac{2\hbar l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2} - \\
& \frac{2\hbar l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)^3 (i\epsilon + l \otimes p2)} + \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)} + \\
& \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} + \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} + \frac{m^2 \gamma \hbar p1 \wedge q}{S(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} - \\
& \frac{\gamma^2 \hbar p1 \wedge q}{S(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} - \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)}
\end{aligned}$$

In[\*]:= **extractLog**[simplifiedForm[onShellP[delx[sum]]], -5]

Out[\*]=

0

In[\*]:= **simplifyTerm**[Expand[extractLog[  
simplifiedForm[onShellP[aWedgeDb[sum, p1, p1] - Wedge[p1, q] \* D[sum, x1]]], -2]]]

Out[\*]=

$$\begin{aligned}
& -\frac{\gamma l \wedge p1}{(-i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)} + \frac{\gamma l \wedge p1}{(i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)} + \\
& \frac{2\gamma \hbar l \otimes q l \wedge p1}{(-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)} - \frac{p1 \wedge p2}{(-i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} + \frac{p1 \wedge p2}{(i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} + \\
& \frac{\hbar l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)^2} + \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)}
\end{aligned}$$

```
In[*]:= b1 = simplifyTerm[Expand[extractLog[
  simplifiedForm[onShellP[aWedgeDb[sum12, p1, p1] - Wedge[p1, q] * D[sum12, x1]]], -3]]]
```

```
Out[*]=
```

$$\frac{\gamma \hbar q \odot q \wedge p_1}{2 (i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)^2} + \frac{\hbar q \odot q p_1 \wedge p_2}{2 (i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2}$$

```
In[*]:= b2 = simplifyTerm[Expand[extractLog[
  simplifiedForm[onShellP[aWedgeDb[sum14, p1, p1] - Wedge[p1, q] * D[sum14, x1]]], -3]]]
```

```
Out[*]=
```

$$-\frac{\gamma \hbar q \odot q \wedge p_1}{(-i \epsilon + l \odot p_1)^3 (-i \epsilon + l \odot p_2)} - \frac{\hbar q \odot q p_1 \wedge p_2}{2 (-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2)}$$

```
In[*]:= ClearAll[b3];
b3 = extractLog[simplifiedForm[onShellP[sum]], -3]
```

```
Out[*]=
```

$$-\frac{\gamma \hbar q \odot q}{2 (-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2)} - \frac{\gamma \hbar q \odot q}{2 (-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2} +$$

$$\frac{\gamma \hbar q \odot q}{2 (i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2} + \frac{\gamma \hbar q \odot q}{2 (-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)}$$

```
In[*]:= Coefficient[simplifyTerm[
  Expand[onShellP[(aWedgeDb[wedgeToDot[aWedgeDb[A0[1] /. {ρ → 0}, l1, q]], p1, p1] -
    D[p1 ∧ q * wedgeToDot[aWedgeDb[A0[1] /. {ρ → 0}, l1, q]], x1]) * b3]]], h, 1]
```

```
Out[*]=
```

$$\frac{4 i \gamma l \odot q p_1 \wedge p_2}{(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} - \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} +$$

$$\frac{4 i \gamma l \odot q p_1 \wedge p_2}{(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} - \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} -$$

$$\frac{4 i \gamma l \odot q p_1 \wedge p_2}{(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} + \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} -$$

$$\frac{4 i \gamma l \odot q p_1 \wedge p_2}{(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} +$$

$$\frac{4 i m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) q \odot q} - \frac{4 i m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} -$$

$$\frac{4 i \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^3 p_1 \wedge q}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q}$$



```
In[*]:= b3 = Total[Select[List @@ b3, ContainsAll[List @@ Numerator[##], {l ∧ p1}] &]]
```

```
Out[*]=
```

$$\begin{aligned}
& -\frac{\gamma l \wedge p1}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \frac{\gamma l \wedge p1}{(i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \\
& \frac{\gamma l \wedge p1}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} - \frac{\gamma l \wedge p1}{(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} + \frac{2 \gamma \hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} + \\
& \frac{\gamma \hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot q l \wedge p1}{(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} - \frac{2 \gamma \hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2)}
\end{aligned}$$

```
In[*]:= Do[If[ContainsAll[List @@ Numerator[b3[[i]], {l ⊙ q, l ∧ p1}], b3[[i]] = K4[b3[[i]]], {i, 1, Length[b3]]]
```

```
In[*]:= simplifyTerm[Expand[b3 /. {HoldPattern[Wedge[l, c_]] =>
```

$$\frac{1}{S} \left( \text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m^2 p1] + \text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m^2 p2] \right)]]$$

```
Out[*]=
```

$$\begin{aligned}
& \frac{\gamma^2 p1 \wedge p2}{S(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} - \frac{\gamma^2 p1 \wedge p2}{S(i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} - \frac{\gamma^2 p1 \wedge p2}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} + \\
& \frac{\gamma^2 p1 \wedge p2}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} - \frac{2 m^2 \gamma \hbar l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)^3} + \frac{\gamma^2 \hbar l \odot q p1 \wedge p2}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} + \\
& \frac{2 \gamma^2 \hbar l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} - \frac{m^2 \gamma \hbar l \odot q p1 \wedge p2}{S(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} + \frac{2 \gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2} + \frac{m^2 \gamma \hbar p1 \wedge q}{2 S(i\epsilon + l \odot p1)^2} - \\
& \frac{m^2 \gamma \hbar l \odot p2 p1 \wedge q}{S(-i\epsilon + l \odot p1)^3} - \frac{\gamma \hbar l \odot l p1 \wedge q}{S(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} - \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} + \\
& \frac{m^2 \gamma \hbar p1 \wedge q}{2 S(i\epsilon + l \odot p2)^2} + \frac{\gamma \hbar l \odot l p1 \wedge q}{2 S(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} - \frac{\gamma^2 \hbar p1 \wedge q}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)}
\end{aligned}$$

```
In[*]:= extractLog[simplifyTerm[Expand[simplifiedForm[onShellP[sum]]], -2]
```

```
Out[*]=
```

$$\begin{aligned}
& -\frac{\gamma}{(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} + \frac{\gamma}{(i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} + \\
& \frac{\gamma}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} - \frac{\gamma}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} + \frac{\gamma \hbar l \odot q}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \\
& \frac{\gamma \hbar l \odot q}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot q}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot q}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)}
\end{aligned}$$

In[\*]:= **simplifiedForm**[onShellP[sum12]]

Out[\*]=

$$\frac{2\gamma - \frac{\gamma \hbar l \otimes l}{i\epsilon + l \otimes p1} + \frac{\gamma \hbar l \otimes l}{-i\epsilon + l \otimes p2} + \hbar(-l \otimes p1 + l \otimes p2)}{2(i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} - \frac{2\gamma - \frac{\gamma \hbar l \otimes l}{i\epsilon + l \otimes p1} + \hbar(l \otimes p1 + l \otimes p2) - \frac{\gamma \hbar(l \otimes l - 2l \otimes q + q \otimes q)}{i\epsilon + l \otimes p2}}{2(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)}$$

In[\*]:= **a** =

**extractLog**[**simplifiedForm**[onShellP[aWedgeDb[sum, p1, p1] - Wedge[p1, q] \* D[sum, x1]]],  
-2] /. {**HoldPattern**[Wedge[l, c\_]]  $\rightarrow$   
 $\frac{1}{S} \left( \text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m^2 p1] + \text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m^2 p2] \right)$ };

In[\*]:= **simplifyTerm**[

**Expand**[**Total**[**Select**[**List**@@a, **ContainsAll**[**List**@@**Numerator**[#], {p1  $\wedge$  p2, l  $\otimes$  q}]  
(\***&&ContainsNone**[**List**@@**Numerator**[#], {l  $\otimes$  q}]\*)(\***&&**  
**ContainsAll**[**List**@@**Denominator**[#], {S<sup>2</sup>}]\*) &]] /. {**HoldPattern**[Wedge[l, c\_]]  $\rightarrow$   
 $\frac{1}{S} \left( \text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m^2 p1] + \text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m^2 p2] \right)$ }]

Out[\*]=

$$\begin{aligned} & \frac{\hbar l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)} - \frac{2\gamma^2 \hbar l \otimes q p1 \wedge p2}{S(-i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)} + \\ & \frac{\hbar l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} - \frac{\gamma^2 \hbar l \otimes q p1 \wedge p2}{S(-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} - \\ & \frac{\hbar l \otimes q p1 \wedge p2}{(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} + \frac{\gamma^2 \hbar l \otimes q p1 \wedge p2}{S(i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)^2} - \frac{\hbar l \otimes q p1 \wedge p2}{(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)} + \\ & \frac{m^2 \gamma \hbar l \otimes q p1 \wedge p2}{S(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)} + \frac{2\gamma^2 \hbar l \otimes q p1 \wedge p2}{S(-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)} - \frac{m^2 \gamma \hbar l \otimes q p1 \wedge p2}{S(i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)} \end{aligned}$$

In[\*]:= **K4**[**expr\_**] := **simplifyTerm**[**Expand**[ $\frac{1}{2S} (\text{expr} /. \{l \otimes q l \wedge p1 \rightarrow -p1 \wedge q\})$

$(m^2 l \otimes p1 l \otimes p1 + m^2 l \otimes p2 l \otimes p2 + l \otimes l - 2\gamma l \otimes p1 l \otimes p2)$ ]]];

In[\*]:= **K4**[ $\frac{2\gamma \hbar l \otimes q l \wedge p1}{(-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)}$ ]

Out[\*]=

$$\begin{aligned} & \frac{2\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)^2} - \frac{m^2 \gamma \hbar l \otimes p2 p1 \wedge q}{S(-i\epsilon + l \otimes p1)^3} - \\ & \frac{\gamma \hbar l \otimes l p1 \wedge q}{S(-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)} - \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} \end{aligned}$$

```
In[*]:= b = extractLog[simplifiedForm[onShellP[wsp]], -2];
```

```
In[*]:= extractLog[simplifiedForm[onShellP[delx[sum]]], -2]
```

```
Out[*]=
```

$$\begin{aligned}
& -\frac{\gamma \hbar l \odot l}{2(-i\epsilon + l \odot p1)^2(-i\epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot l}{(-i\epsilon + l \odot p1)^3(-i\epsilon + l \odot p2)} + \\
& \frac{3\hbar}{2(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} - \frac{m^2 \gamma \hbar}{S(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} - \frac{\gamma^2 \hbar}{S(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} + \\
& \frac{\gamma \hbar l \odot l}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^3} - \frac{\gamma \hbar l \odot l}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^3} - \frac{\gamma \hbar l \odot l}{(-i\epsilon + l \odot p1)^2(i\epsilon + l \odot p2)^2} - \\
& \frac{\gamma \hbar l \odot l}{2(i\epsilon + l \odot p1)^2(i\epsilon + l \odot p2)^2} + \frac{\gamma \hbar l \odot l}{(-i\epsilon + l \odot p1)^3(i\epsilon + l \odot p2)} - \frac{\hbar}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} + \\
& \frac{m^2 \gamma \hbar}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} + \frac{m^2 \gamma \hbar}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} + \frac{2\gamma^2 \hbar}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} + \\
& \frac{3\hbar}{2(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} - \frac{m^2 \gamma \hbar}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)} - \frac{\gamma^2 \hbar}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)}
\end{aligned}$$

```
In[*]:= sum = 0;
```

```
Do[sum += L[i]*Cf[i]/. {γ → p1 ⊙ p2}, {i, {1, 2}}];
```

```
Expand[
```

```

  simplifiedForm@Coefficient[factorRho[simplifyTerm@Expand[onShellP[aWedgeDb[
    simplifiedForm@sum, l1, q]] /. {Wedge[l1, c_] → (γ p2 - m^2 p1) ⊙ c / S,
    Wedge[c_, l1] → -(γ p2 - m^2 p1) ⊙ c / S}]] /. {l → ρ l, ε → ρ ε}]], ρ, -3]]

```

```
Out[*]=
```

```
$Aborted
```

```
In[*]:= aWedgeDb[Expand[
```

```

  L[i]*Cf[i]*(A0hbarl[i][0, 0]+A0hbarl[i][0, 1]+A0hbarl[i][0, 2]+ħ A0hbarl[i][1, 0]+
  ħ A0hbarl[i][1, 1]+ħ A0hbarl[i][1, 2])/. {i → 1}]
  , p1, p1]

```

```
In[*]:= ClearAll[sum10, sum11, sum20, sum21, sum]
```

```

In[*]:= sum10 = 0;
Do[term10 =
  L[i] * Cf[i] * (A0hbarL[i][0, 0] + A0hbarL[i][0, 1] + A0hbarL[i][0, 2] +  $\hbar$  A0hbarL[i][1, 0] +
     $\hbar$  A0hbarL[i][1, 1] +  $\hbar$  A0hbarL[i][1, 2]) /. { $\gamma \rightarrow p1 \odot p2$ };

  sum10 += simplifyTerm[Expand[
    Coefficient[factorRho[simplifiedForm[onShellP[aWedgeDb[term10, p1, p1]]] /.
      { $l \rightarrow \rho l$ ,  $\epsilon \rightarrow \rho \epsilon$ }},  $\rho$ , -2]], {i, 1, 4}];

In[*]:= sum11 = 0;
Do[term11 = q  $\odot$  q * L[i] * Cf[i] * (A0hbarL[i][0, 0] + A0hbarL[i][0, 1] + A0hbarL[i][0, 2] +
   $\hbar$  A0hbarL[i][1, 0] +  $\hbar$  A0hbarL[i][1, 1] +  $\hbar$  A0hbarL[i][1, 2]) /. { $\gamma \rightarrow p1 \odot p2$ };

  sum11 += simplifyTerm@Expand[Coefficient[
    factorRho[simplifiedForm@onShellP[D[aWedgeDb[term11, p1, p1], x1] -
      D[aWedgeDb[term11, p1, p1], x2]] /. { $l \rightarrow \rho l$ ,  $\epsilon \rightarrow \rho \epsilon$ }},  $\rho$ , -2]], {i, 1, 4}];

In[*]:= sum20 = 0;
Do[
  term20 = Wedge[p1, q] * L[i] * Cf[i] * (A0hbarL[i][0, 0] + A0hbarL[i][0, 1] + A0hbarL[i][0, 2] +
     $\hbar$  A0hbarL[i][1, 0] +  $\hbar$  A0hbarL[i][1, 1] +  $\hbar$  A0hbarL[i][1, 2]) /. { $\gamma \rightarrow p1 \odot p2$ };

  sum20 += simplifyTerm[
    Expand[Coefficient[factorRho[simplifiedForm@onShellP[D[term20, x1]] /.
      { $l \rightarrow \rho l$ ,  $\epsilon \rightarrow \rho \epsilon$ }},  $\rho$ , -2]], {i, 1, 4}];

In[*]:= sum21 = 0;
Do[term21 =
  q  $\odot$  q * Wedge[p1, q] * L[i] * Cf[i] * (A0hbarL[i][0, 0] + A0hbarL[i][0, 1] + A0hbarL[i][0, 2] +
     $\hbar$  A0hbarL[i][1, 0] +  $\hbar$  A0hbarL[i][1, 1] +  $\hbar$  A0hbarL[i][1, 2]) /. { $\gamma \rightarrow p1 \odot p2$ };

  sum21 += simplifyTerm@Expand[Coefficient[
    factorRho[simplifiedForm@onShellP[D[D[term21, x1], x1] - D[D[term21, x2], x1]] /.
      { $l \rightarrow \rho l$ ,  $\epsilon \rightarrow \rho \epsilon$ }},  $\rho$ , -2]], {i, 1, 4}];

In[*]:= simplifyTerm@Expand[
  ((sum10 - sum20) /  $\hbar$  - (sum11 - sum21) / 2 /. {( $l \odot q$ )2  $\rightarrow$   $l \odot l * q \odot q$ } /.
    {HoldPattern[Wedge[l, c_]]  $\rightarrow$   $\frac{1}{S}$  (Wedge[p1, c] * holdDot[l,  $\gamma * p2 - m^2 p1$ ] +
      Wedge[p2, c] * holdDot[l,  $\gamma p1 - m^2 p2$ ])}] /.
    { $l \odot q \rightarrow 0$ } /. {Power[ $\hbar$ , -1]  $\rightarrow$  0, Power[ $\hbar$ , 1]  $\rightarrow$  0}

```

In[\*]:= Coefficient[simplifyTerm[Expand[sum10/h]], h, 0]

Out[\*]=

$$-\frac{2i\gamma^2 l \odot l p1 \wedge q}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)^2 q \odot q} + \frac{6i\gamma p1 \wedge q}{(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2) q \odot q} -$$

$$\frac{2i\gamma p1 \wedge q}{(i\epsilon + l \odot p1)(-i\epsilon + l \odot p2) q \odot q} - \frac{2i\gamma p1 \wedge q}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q} + \frac{6i\gamma p1 \wedge q}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q}$$

In[\*]:= Coefficient[simplifyTerm@Expand[(sum11 - sum21)], h, 0]

Out[\*]=

$$-\frac{16i\gamma^2 l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2) q \odot q} - \frac{8i\gamma^2 l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2 q \odot q} +$$

$$\frac{8i\gamma^2 l \odot q l \wedge p1}{(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2 q \odot q} + \frac{16i\gamma^2 l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2) q \odot q} -$$

$$\frac{16i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2) q \odot q} + \frac{8i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2) q \odot q} -$$

$$\frac{16i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} + \frac{8i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} +$$

$$\frac{16i\gamma l \odot q p1 \wedge p2}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} - \frac{8i\gamma^3 l \odot q p1 \wedge p2}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} +$$

$$\frac{16i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2) q \odot q} - \frac{8i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2) q \odot q} -$$

$$\frac{8im^2\gamma^2 p1 \wedge q}{S(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2) q \odot q} + \frac{8im^2\gamma^2 p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q} +$$

$$\frac{8i\gamma^3 p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q} - \frac{8i\gamma^3 p1 \wedge q}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q}$$

In[\*]:= b = simplifyTerm@Expand[(sum10 - sum20)/h /. {(l \odot q)^2 \to l \odot l \* q \odot q} /.

$$\left\{ \text{HoldPattern}[\text{Wedge}[l, c\_]] \Rightarrow \frac{1}{S} (\text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m^2 p1] + \right.$$

$$\left. \text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m^2 p2]) \right\} /. \{l \odot q \rightarrow 0\}$$

```

In[*]:= a1 = i γ Wedge[p1, l] × δ[p2 ⊙ l] (A0[l] /. {ρ → 0})  $\left( \frac{1}{(p1 ⊙ l - i \epsilon)^2} - \frac{1}{(p1 ⊙ l + i \epsilon)^2} \right);$ 

a2 = ħ γ l ⊙ q Wedge[p1, l] (A0[l] /. {ρ → 0})
 $\left( \frac{-2 i \delta[p2 ⊙ l]}{(p1 ⊙ l - i \epsilon)^3} - \frac{1}{(p1 ⊙ l - i \epsilon)^2 (p2 ⊙ l + i \epsilon)^2} + \frac{1}{(p1 ⊙ l + i \epsilon)^2 (p2 ⊙ l + i \epsilon)^2} \right);$ 

a3 = -ħ γ q ⊙ q Wedge[p1, l]
 $\left( i \frac{\delta[p2 ⊙ l]}{(p1 ⊙ l - i \epsilon)^3} + \frac{1}{2 (p1 ⊙ l - i \epsilon)^2 (p2 ⊙ l + i \epsilon)^2} - \frac{1}{2 (p1 ⊙ l + i \epsilon)^2 (p2 ⊙ l + i \epsilon)^2} \right)$ 

onShellP[aWedgeDb[A0[l] /. {ρ → 0}, l1, q] /.
  {HoldPattern[Wedge[l1, c_]] :> l ⊙ c, HoldPattern[Wedge[c_, l1]] :> -l ⊙ c});

a4 = -  $\left( \hbar \gamma l \odot q \text{Wedge}[p1, q] \left( \frac{1}{(p1 \odot l - i \epsilon)^2 (p2 \odot l + i \epsilon)^2} + \frac{2 i \delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^3} \right) - \right.$ 
 $\left. i \gamma \text{Wedge}[p1, q] \frac{\delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^2} \right)$  onShellP[aWedgeDb[A0[l] /. {ρ → 0}, l1, q] /.
  {HoldPattern[Wedge[l1, c_]] :> l ⊙ c, HoldPattern[Wedge[c_, l1]] :> -l ⊙ c});

a5 =  $\left( \frac{-i \gamma \delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^2} + \hbar \gamma l \odot q \left( \frac{1}{(p1 \odot l - i \epsilon)^2 (p2 \odot l + i \epsilon)^2} + \frac{2 i \delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^3} \right) \right)$ 
Wedge[p1, q] × onShellP[aWedgeDb[A0[l] /. {ρ → 0}, l1, q] /.
  {HoldPattern[Wedge[l1, c_]] :> l ⊙ c, HoldPattern[Wedge[c_, l1]] :> -l ⊙ c});

a6 = i ħ γ  $\frac{q \odot q}{2} \left( \frac{\delta[p1 \odot l]}{(p2 \odot l + i \epsilon)^2} + \frac{\delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^2} \right)$ 
onShellP[aWedgeDb[(aWedgeDb[A0[l] /. {ρ → 0}, l1, q] /.
  {HoldPattern[Wedge[l1, c_]] :> l ⊙ c, HoldPattern[Wedge[c_, l1]] :> -l ⊙ c}),
  p1, p1] - D[Wedge[p1, q] (aWedgeDb[A0[l] /. {ρ → 0}, l1, q] /.
  {HoldPattern[Wedge[l1, c_]] :> l ⊙ c, HoldPattern[Wedge[c_, l1]] :> -l ⊙ c}), x1]];

In[*]:= Expand[Coefficient[simplifyTerm[Expand[onShellP[a1 + a2 + a3 + a4 + a5 + a6] / ħ]] /.
  {(l ⊙ p1)^n_ . δ[l ⊙ p1] → 0, (l ⊙ p2)^m_ . δ[l ⊙ p2] → 0}, ħ, 0] /.
  {δ[l ⊙ p2] →  $\frac{i}{(p2 \odot l + i \epsilon)} - \frac{i}{(p2 \odot l - i \epsilon)}$ , δ[l ⊙ p1] →  $\frac{i}{(p1 \odot l + i \epsilon)} - \frac{i}{(p1 \odot l - i \epsilon)}$ }]

In[*]:= a =
  simplifyTerm[Expand[onShellP[a1 + a2 + a3 + a4 + a5] / ħ /. {HoldPattern[Wedge[l, c_]] :>  $\frac{1}{s}$ 
    (Wedge[p1, c] * holdDot[l, γ * p2 - m2^2 p1] + Wedge[p2, c] * holdDot[l, γ p1 - m1^2 p2])}]]]

```

In[ ]:= Expand[a /. {(l ⊗ p1)^n\_ . δ[l ⊗ p1] → 0, (l ⊗ p2)^m\_ . δ[l ⊗ p2] → 0} /. {l ⊗ q → 0} /.

$$\left\{ \delta[l \otimes p2] \rightarrow \frac{i}{(p2 \otimes l + i \epsilon)} - \frac{i}{(p2 \otimes l - i \epsilon)}, \delta[l \otimes p1] \rightarrow \frac{i}{(p1 \otimes l + i \epsilon)} - \frac{i}{(p1 \otimes l - i \epsilon)} \right\}]$$

Out[ ]:=

$$\begin{aligned} & \frac{i m^2 \gamma \hbar p1 \wedge p2}{S(-i \epsilon + l \otimes p1)^2} - \frac{i m^2 \gamma \hbar p1 \wedge p2}{S(i \epsilon + l \otimes p1)^2} + \frac{i \gamma^2 \hbar p1 \wedge p2}{S(-i \epsilon + l \otimes p1)(-i \epsilon + l \otimes p2)} + \frac{i \gamma^2 \hbar p1 \wedge p2}{S(i \epsilon + l \otimes p1)(-i \epsilon + l \otimes p2)} - \\ & \frac{i m^2 \gamma \hbar p1 \wedge p2}{S(-i \epsilon + l \otimes p1)(i \epsilon + l \otimes p2)} - \frac{2 i \gamma^2 \hbar p1 \wedge p2}{S(-i \epsilon + l \otimes p1)(i \epsilon + l \otimes p2)} + \frac{i m^2 \gamma \hbar p1 \wedge p2}{S(i \epsilon + l \otimes p1)(i \epsilon + l \otimes p2)} + \\ & \frac{4 i \gamma^3 p1 \wedge p2}{S \hbar(-i \epsilon + l \otimes p1)(-i \epsilon + l \otimes p2) q \otimes q} - \frac{4 i \gamma^3 p1 \wedge p2}{S \hbar(i \epsilon + l \otimes p1)(-i \epsilon + l \otimes p2) q \otimes q} - \\ & \frac{4 i \gamma^3 p1 \wedge p2}{S \hbar(-i \epsilon + l \otimes p1)(i \epsilon + l \otimes p2) q \otimes q} + \frac{4 i \gamma^3 p1 \wedge p2}{S \hbar(i \epsilon + l \otimes p1)(i \epsilon + l \otimes p2) q \otimes q} \end{aligned}$$

$$\begin{aligned}
\text{In}[*]:= & \text{a} = \text{simplifyTerm}\left[\text{Expand}\left[\left(\text{simplifyTerm}@\text{Expand}\left[-\gamma * q \odot q * \text{Wedge}[p1, l]\right.\right.\right.\right. \\
& \left.\left.\left.\left(\frac{i \delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^3} + \frac{1}{2 (p1 \odot l - i \epsilon)^2 (p2 \odot l + i \epsilon)^2} - \frac{1}{2 (p1 \odot l + i \epsilon)^2 (p2 \odot l + i \epsilon)^2}\right) * \right.\right.\right. \\
& \left.\left.\left.\text{onShellP}[a\text{WedgeDb}[A0[1] /. \{\rho \rightarrow 0\}, l1, q] /. \{\text{HoldPattern}[\right.\right. \\
& \left.\left.\left.\text{Wedge}[l1, c\_]] \Rightarrow l \odot c / S, \text{HoldPattern}[\text{Wedge}[c_, l1]] \Rightarrow -l \odot c / S\}\right]\right] + \right. \\
& \left.\text{simplifyTerm}@\text{Expand}\left[i \gamma \frac{q \odot q}{2} \left(\frac{\delta[p1 \odot l]}{(p2 \odot l + i \epsilon)^2} + \frac{\delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^2}\right) * \text{onShellP}\right.\right. \\
& \left.\left.\text{aWedgeDb}[a\text{WedgeDb}[A0[1] /. \{\rho \rightarrow 0\}, l1, q] /. \{\text{HoldPattern}[\text{Wedge}[l1, c\_]] \Rightarrow \right.\right. \\
& \left.\left.\left.l \odot c / S, \text{HoldPattern}[\text{Wedge}[c_, l1]] \Rightarrow -l \odot c / S\}, p1, p1] - \right.\right. \\
& \left.\left.\text{D}[\text{Wedge}[p1, q] * (a\text{WedgeDb}[A0[1] /. \{\rho \rightarrow 0\}, l1, q] /. \{\text{HoldPattern}[\text{Wedge}[l1, \right.\right. \\
& \left.\left.\left.c\_]] \Rightarrow l \odot c / S, \text{HoldPattern}[\text{Wedge}[c_, l1]] \Rightarrow -l \odot c / S\}), x1]\right]\right] /.
\end{aligned}$$

$$\begin{aligned}
& \left\{ \text{HoldPattern}[\text{Wedge}[l, c\_]] \Rightarrow \frac{1}{S} \left( \text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m^2 p1] + \right.\right. \\
& \left.\left. \text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m^2 p2] \right) \right\} /.
\end{aligned}$$

$$\begin{aligned}
& \{l \odot q \rightarrow 0\} /. \{(l \odot p1) \wedge n\_ . \delta[l \odot p1] \rightarrow 0, (l \odot p2) \wedge m\_ . \\
& \delta[ \\
& l \odot p2] \rightarrow 0\}
\end{aligned}$$

Out[\*]=

$$\begin{aligned}
& \frac{i m^2 \gamma \hbar p1 \wedge p2}{S^2 (-i \epsilon + l \odot p1)^2} - \frac{i m^2 \gamma \hbar p1 \wedge p2}{S^2 (i \epsilon + l \odot p1)^2} - \frac{i m^2 \gamma \hbar p1 \wedge p2}{S^2 (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{i \gamma^2 \hbar p1 \wedge p2}{S^2 (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \\
& \frac{i m^2 \gamma \hbar p1 \wedge p2}{S^2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{i \gamma^2 \hbar p1 \wedge p2}{S^2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{m^2 \gamma \hbar p1 \wedge p2 \delta[l \odot p1]}{S^2 (i \epsilon + l \odot p2)} + \\
& \frac{\gamma^2 \hbar p1 \wedge p2 \delta[l \odot p1]}{S^2 (i \epsilon + l \odot p2)} - \frac{4 \gamma^3 p1 \wedge q \delta[l \odot p1]}{S^2 (i \epsilon + l \odot p2) q \odot q} - \frac{2 \gamma^2 \hbar p1 \wedge p2 \delta[l \odot p2]}{S^2 (-i \epsilon + l \odot p1)} + \frac{4 m^2 \gamma^2 p1 \wedge q \delta[l \odot p2]}{S^2 (-i \epsilon + l \odot p1) q \odot q}
\end{aligned}$$

$$\begin{aligned}
\text{In}[*]:= & \text{Expand}\left[a /. \left\{ \delta[l \odot p2] \rightarrow \frac{i}{(p2 \odot l + i \epsilon)} - \frac{i}{(p2 \odot l - i \epsilon)}, \delta[l \odot p1] \rightarrow \frac{i}{(p1 \odot l + i \epsilon)} - \frac{i}{(p1 \odot l - i \epsilon)} \right\} \right] /. \\
& \{\hbar \rightarrow 0\}
\end{aligned}$$

Out[\*]=

$$\begin{aligned}
& - \frac{4 i m^2 \gamma^2 p1 \wedge q}{S^2 (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) q \odot q} + \frac{4 i m^2 \gamma^2 p1 \wedge q}{S^2 (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q} + \\
& \frac{4 i \gamma^3 p1 \wedge q}{S^2 (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q} - \frac{4 i \gamma^3 p1 \wedge q}{S^2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q}
\end{aligned}$$



```
In[*]:= Dg =
  Simplify[Expand[Solve[A*(m1^2 m2^2 + γ^2) + B*m1^2 γ + C*γ*m2^2 + D*γ == I12 && 2 A γ m1^2 +
    B m1^4 + C γ^2 + D m1^2 == I11 && 2 A m2^2 γ + B γ^2 + C m2^4 + D m2^2 == I22 &&
    2 A γ + B m1^2 + C m2^2 + 4 D == I00, {A, B, C, D}, Reals][[1]][[4]][[2]] /.
    {2 m1^4 m2^4 - 4 m1^2 m2^2 γ^2 + 2 γ^4 → 2 S^2}], γ^2 - m1^2 m2^2 == S]
```

```
Out[*]:=
  I22 m1^2 + I11 m2^2 + I00 S - 2 I12 γ
  -----
  2 S
```

```
In[*]:= I1 = Expand[Dg /. {I00 →  $\frac{l \otimes l}{(p1 \otimes l + i \epsilon)^2 (p2 \otimes l + i \epsilon)^2}$ ,
  I12 →  $\frac{1}{(p1 \otimes l + i \epsilon)(p2 \otimes l + i \epsilon)}$ , I11 →  $\frac{1}{(p2 \otimes l + i \epsilon)^2}$ , I22 →  $\frac{1}{(p1 \otimes l + i \epsilon)^2}$ }}];
```

```
In[*]:= I2 = Expand[Dg /. {I00 →  $-\frac{l \otimes l}{(p1 \otimes l - i \epsilon)^2 (p2 \otimes l + i \epsilon)^2}$ ,
  I12 →  $-\frac{1}{(p1 \otimes l - i \epsilon)(p2 \otimes l + i \epsilon)}$ , I11 →  $-\frac{1}{(p2 \otimes l + i \epsilon)^2}$ , I22 →  $-\frac{1}{(p1 \otimes l - i \epsilon)^2}$ }}];
```

```
In[*]:= I3 = Expand[Dg /. {I00 →  $\frac{2 l \otimes l}{(p1 \otimes l - i \epsilon)^3 (p2 \otimes l + i \epsilon)}$ ,
  I12 →  $\frac{2}{(p1 \otimes l - i \epsilon)^2}$ , I11 →  $\frac{2}{(p1 \otimes l - i \epsilon)(p2 \otimes l + i \epsilon)}$ , I22 →  $\frac{2 p2 \otimes l}{(p1 \otimes l - i \epsilon)^3}$ }}];
```

```
In[*]:= I4 = Expand[Dg /. {I00 →  $-\frac{2 l \otimes l}{(p1 \otimes l - i \epsilon)^3 (p2 \otimes l - i \epsilon)}$ ,
  I12 →  $-\frac{2}{(p1 \otimes l - i \epsilon)^2}$ , I11 →  $-\frac{2}{(p1 \otimes l - i \epsilon)(p2 \otimes l - i \epsilon)}$ , I22 →  $-\frac{2 p2 \otimes l}{(p1 \otimes l - i \epsilon)^3}$ }}];
```

In[\*]:= **I1 + I2 + I3 + I4**

Out[\*]=

$$\begin{aligned}
 & -\frac{m1^2}{2S(-i\epsilon+l\odot p1)^2} + \frac{m1^2}{2S(i\epsilon+l\odot p1)^2} - \frac{l\odot l}{(-i\epsilon+l\odot p1)^3(-i\epsilon+l\odot p2)} - \\
 & \frac{m2^2}{S(-i\epsilon+l\odot p1)(-i\epsilon+l\odot p2)} - \frac{l\odot l}{2(-i\epsilon+l\odot p1)^2(i\epsilon+l\odot p2)^2} + \\
 & \frac{l\odot l}{2(i\epsilon+l\odot p1)^2(i\epsilon+l\odot p2)^2} + \frac{l\odot l}{(-i\epsilon+l\odot p1)^3(i\epsilon+l\odot p2)} + \\
 & \frac{m2^2}{S(-i\epsilon+l\odot p1)(i\epsilon+l\odot p2)} + \frac{\gamma}{S(-i\epsilon+l\odot p1)(i\epsilon+l\odot p2)} - \frac{\gamma}{S(i\epsilon+l\odot p1)(i\epsilon+l\odot p2)}
 \end{aligned}$$

```
In[*]:= ClearAll[a1, a2, a3, b1, b2, b3, c, d];
```

```
a1 = Expand[
  aWedgeDb[aWedgeDb[A0hbar[1][0] /.  $\rho \rightarrow 0$ , l, p1] /. HoldPattern[Wedge[l, c_]]  $\Rightarrow$  l  $\otimes$  c,
  l1, q] /. {HoldPattern[Wedge[l1, c_]]  $\Rightarrow$  ( $\gamma$  p2 - m2 p1)  $\otimes$  c / S,
  HoldPattern[Wedge[c_, l1]]  $\Rightarrow$  -( $\gamma$  p2 - m2 p1)  $\otimes$  c / S};

a2 = Expand[
  aWedgeDb[aWedgeDb[A0hbar[1][0] /.  $\rho \rightarrow 0$ , l, p2] /. HoldPattern[Wedge[l, c_]]  $\Rightarrow$  l  $\otimes$  c,
  l1, q] /. {HoldPattern[Wedge[l1, c_]]  $\Rightarrow$  ( $\gamma$  p2 - m2 p1)  $\otimes$  c / S,
  HoldPattern[Wedge[c_, l1]]  $\Rightarrow$  -( $\gamma$  p2 - m2 p1)  $\otimes$  c / S};

a3 = Expand[
  aWedgeDb[aWedgeDb[A0hbar[1][0] /.  $\rho \rightarrow 0$ , l, q] /. HoldPattern[Wedge[l, c_]]  $\Rightarrow$  l  $\otimes$  c,
  l1, q] /. {HoldPattern[Wedge[l1, c_]]  $\Rightarrow$  ( $\gamma$  p2 - m2 p1)  $\otimes$  c / S,
  HoldPattern[Wedge[c_, l1]]  $\Rightarrow$  -( $\gamma$  p2 - m2 p1)  $\otimes$  c / S};

b1 = aWedgeDb[L[1] * Cf[1] + L[2] * Cf[2], l1, q] /.
  {HoldPattern[Wedge[l1, c_]]  $\Rightarrow$  ( $\gamma$  p2 - m2 p1)  $\otimes$  c / S,
  HoldPattern[Wedge[c_, l1]]  $\Rightarrow$  -( $\gamma$  p2 - m2 p1)  $\otimes$  c / S};

b2 = aWedgeDb[L[1] * Cf[1] + L[4] * Cf[4], l1, q] /.
  {HoldPattern[Wedge[l1, c_]]  $\Rightarrow$  ( $\gamma$  p2 - m2 p1)  $\otimes$  c / S,
  HoldPattern[Wedge[c_, l1]]  $\Rightarrow$  -( $\gamma$  p2 - m2 p1)  $\otimes$  c / S};

b3 = aWedgeDb[L[1] * Cf[1] + L[2] * Cf[2] + L[3] * Cf[3] + L[4] * Cf[4], l1, q] /.
  {HoldPattern[Wedge[l1, c_]]  $\Rightarrow$  ( $\gamma$  p2 - m2 p1)  $\otimes$  c / S,
  HoldPattern[Wedge[c_, l1]]  $\Rightarrow$  -( $\gamma$  p2 - m2 p1)  $\otimes$  c / S};

c = 0;
Do[c += L[i] * Cf[i], {i, 1, 4}];
c = simplifyTerm[simplifiedForm[c]];
c = aWedgeDb[c, l1, q] /. {HoldPattern[Wedge[l1, c_]]  $\Rightarrow$  ( $\gamma$  p2 - m2 p1)  $\otimes$  c / S,
  HoldPattern[Wedge[c_, l1]]  $\Rightarrow$  -( $\gamma$  p2 - m2 p1)  $\otimes$  c / S};

d = aWedgeDb[aWedgeDb[A0hbar[1][0] /.  $\rho \rightarrow 0$ , l, q] /. HoldPattern[Wedge[l, c_]]  $\Rightarrow$  l  $\otimes$  c,
  l1, q] /. {HoldPattern[Wedge[l1, c_]]  $\Rightarrow$  ( $\gamma$  p2 - m2 p1)  $\otimes$  c / S,
  HoldPattern[Wedge[c_, l1]]  $\Rightarrow$  -( $\gamma$  p2 - m2 p1)  $\otimes$  c / S};
```

```

In[*]:= a1 = onShellP@a1;
a2 = onShellP@a2;
a3 = onShellP@a3;
c = simplifyTerm[Expand[simplifiedForm[onShellP[c]]]];
d = onShellP@d;
b1 = simplifyTerm@Expand[simplifiedForm@onShellP@b1];
b2 = simplifyTerm@Expand[simplifiedForm@onShellP@b2];
b3 = simplifyTerm@Expand[simplifiedForm[onShellP[b3]]];

In[*]:= simplifyTerm@Expand[onShellP[c] /. {-m1^2 m2^2 + γ^2 → S, m1^2 m2^2 - γ^2 → -S}]

In[*]:= Coefficient[
  factorRho[simplifyTerm@Expand[onShellP[c] /. {-m1^2 m2^2 + γ^2 → S, m1^2 m2^2 - γ^2 → -S}] /.
    {l → ρ l, ε → ρ ε}], ρ, -2]

In[*]:= sum = Coefficient[
  factorRho[simplifyTerm[Expand[-ħ * a3 * b3 - c * d]] /. {l → ρ l, ε → ρ ε}], ρ, -2]

In[*]:= (*ClearAll[holdDot]
  SetOptions[D, NonConstants→{holdDot}];*)

In[*]:= (*Product rule when holdDot is inside product*)
(*holdDot/:D[expr_Times,z_] /; MemberQ[expr,_holdDot,{0,Infinity}]:=
  Module[{factors=List@@expr},
    Total[Table[D[factors[[i]],z]*Times@@Delete[factors,i],{i,Length[factors]}]]];*)

In[*]:= T = 0;

Do[T += L[i] * D[A0hbar[i][0], {x1, 2}], {i, 1, 4}]

cancelFactors@simplifiedForm@Expand@onShellP@T /. {ρ → 1} /.
  {l ⊗ l - 2 l ⊗ q + q ⊗ q → qm l2}

```

Out[\*]=

$$\begin{aligned}
& \frac{8 i m_1^2 m_2^4 \gamma}{q m l^2 S^2 (-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2)} - \frac{8 i m_2^2 \gamma^3}{q m l^2 S^2 (-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2)} - \\
& \frac{8 i m_1^2 m_2^4 \gamma}{q m l^2 S^2 (i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2)} + \frac{8 i m_2^2 \gamma^3}{q m l^2 S^2 (i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2)} - \\
& \frac{8 i m_1^2 m_2^4 \gamma}{q m l^2 S^2 (-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)} + \frac{8 i m_2^2 \gamma^3}{q m l^2 S^2 (-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)} + \\
& \frac{8 i m_1^2 m_2^4 \gamma}{q m l^2 S^2 (i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)} - \frac{8 i m_2^2 \gamma^3}{q m l^2 S^2 (i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)}
\end{aligned}$$

$In[ ] := \text{onShellP@Expand@D}[A0[1] /. \{\rho \rightarrow 1\} /. \{l \otimes l - 2 l \otimes q + q \otimes q \rightarrow qml2\}, x1]$

$Out[ ] :=$

$$\frac{2 i m_1^2 m_2^2 \hbar}{qml2 S} - \frac{2 i \gamma^2 \hbar}{qml2 S} + \frac{2 i m_2^2 \hbar^2 l \otimes p_1}{qml2 S} - \frac{2 i \gamma \hbar^2 l \otimes p_2}{qml2 S}$$

$In[ ] := \text{Expand}[A0[1]] /. \{\rho \rightarrow 1\} /. \{l \otimes l - 2 l \otimes q + q \otimes q \rightarrow qml2\}$

$Out[ ] :=$

$$-\frac{i \hbar^2 l \otimes l}{qml2} - \frac{2 i \hbar l \otimes p_1}{qml2} + \frac{2 i \hbar l \otimes p_2}{qml2} -$$

$$\frac{2 i \hbar^2 l \otimes q}{qml2} + \frac{4 i p_1 \otimes p_2}{qml2} - \frac{2 i \hbar p_1 \otimes q}{qml2} + \frac{2 i \hbar p_2 \otimes q}{qml2} - \frac{i \hbar^2 q \otimes q}{qml2}$$

$In[ ] := \text{onShellP@Expand@D}[A0[2] /. \{\rho \rightarrow 1\} /. \{l \otimes l - 2 l \otimes q + q \otimes q \rightarrow qml2\}, x1]$

$Out[ ] :=$

$$\frac{2 i m_1^2 m_2^2 \hbar}{qml2 S} - \frac{2 i \gamma^2 \hbar}{qml2 S}$$

$In[ ] := \text{onShellP@Expand@D}[A0[3] /. \{\rho \rightarrow 1\} /. \{l \otimes l - 2 l \otimes q + q \otimes q \rightarrow qml2\}, x1]$

$Out[ ] :=$

$$\frac{2 i m_1^2 m_2^2 \hbar}{qml2 S} - \frac{2 i \gamma^2 \hbar}{qml2 S} - \frac{2 i m_2^2 \hbar^2 l \otimes p_1}{qml2 S} + \frac{2 i \gamma \hbar^2 l \otimes p_2}{qml2 S}$$

$In[ ] := \text{onShellP@Expand@D}[A0[4] /. \{\rho \rightarrow 1\} /. \{l \otimes l - 2 l \otimes q + q \otimes q \rightarrow qml2\}, x1]$

$Out[ ] :=$

$$\frac{2 i m_1^2 m_2^2 \hbar}{qml2 S} - \frac{2 i \gamma^2 \hbar}{qml2 S}$$

$In[ ] := \text{Expand}[A0[1] - A0[3] /. \{\rho \rightarrow 1\} /. \{l \otimes l - 2 l \otimes q + q \otimes q \rightarrow qml2\}]$

$Out[ ] :=$

$$-\frac{4 i \hbar l \otimes p_1}{qml2} + \frac{4 i \hbar l \otimes p_2}{qml2} - \frac{4 i \hbar^2 l \otimes q}{qml2}$$

$In[ ] := \text{Expand}[A0hbarl[1][1, 1] - A0hbarl[3][1, 1]]$

$Out[ ] :=$

$$-\frac{4 i l \otimes p_1}{q \otimes q} + \frac{4 i l \otimes p_2}{q \otimes q}$$

$In[ ] := \text{Expand}[A0[2] + A0[4] /. \{\rho \rightarrow 1\} /. \{l \otimes l - 2 l \otimes q + q \otimes q \rightarrow qml2\}]$

$Out[ ] :=$

$$\frac{2 i \hbar^2 l \otimes l}{qml2} + \frac{8 i p_1 \otimes p_2}{qml2} - \frac{4 i \hbar p_1 \otimes q}{qml2} + \frac{4 i \hbar p_2 \otimes q}{qml2} - \frac{2 i \hbar^2 q \otimes q}{qml2}$$

In[\*]:= **Expand@simplifiedForm@onShellP@D[A0hbar[1][1], x1]**

Out[\*]=

$$\frac{4 i m^2 \rho^2 (l \odot p_1)^2}{s \left( \rho^2 l \odot l - 2 \rho l \odot q + q \odot q \right)^2} - \frac{4 i m^2 \rho^2 l \odot p_1 l \odot p_2}{s \left( \rho^2 l \odot l - 2 \rho l \odot q + q \odot q \right)^2} -$$

$$\frac{4 i \gamma \rho^2 l \odot p_1 l \odot p_2}{s \left( \rho^2 l \odot l - 2 \rho l \odot q + q \odot q \right)^2} + \frac{4 i \gamma \rho^2 (l \odot p_2)^2}{s \left( \rho^2 l \odot l - 2 \rho l \odot q + q \odot q \right)^2} - \frac{2 i}{\rho^2 l \odot l - 2 \rho l \odot q + q \odot q}$$

In[\*]:=

In[\*]:= **a = D[p1 ⊙ q, x1]**

**b = D[p1 ⊙ q, x2]**

Out[\*]=

$$-\frac{m^2 p_1 \odot p_1}{s} + \frac{\gamma p_1 \odot p_2}{s}$$

Out[\*]=

$$\frac{\gamma p_1 \odot p_1}{s} - \frac{m^2 p_1 \odot p_2}{s}$$

In[\*]:= **A0hbarL[1][0, 0]**

Out[\*]=

$$\frac{4 i p_1 \odot p_2}{q \odot q}$$

## One Loop: Calculation 1

```

In[ ]:= MaTeX["\\begin{aligned} R_2 &= -\\partial_{x_1}(p_1\\wedge \\q\\,,_A_{1,1})+\\frac{1}{2}(\\partial_{x_1}^2-\\partial_{x_1}\\partial_{x_2})(q^2\\,,_p_1\\wedge \\q\\,,_A_{1,0}))\\\\=-\\sum_i L_{iC_{i,0}}\\partial_{x_1}(p_1\\wedge \\q\\,,_i\\tilde{A}_{0,1})+C_{i,1}\\partial_{x_1}(p_1\\wedge \\q\\,,_i\\tilde{A}_{0,0})+\\frac{1}{2}\\sum_i L_{iC_{i,0}}(\\partial_{x_i}^2-\\partial_{x_1}\\partial_{x_2})(p_1\\wedge \\q\\,,_q^2\\,,_i\\tilde{A}_{0,0}))\\\\-\\partial_{x_1}(p_1\\wedge \\q\\sum_i(\\,,_i\\tilde{A}_{0,1})\\partial_{x_1}L_{iC_{i,0}}+\\,,_i\\tilde{A}_{0,0})\\partial_{x_1}L_{iC_{i,1}})+\\frac{(q^2)^2}{2}p_1\\wedge \\q\\sum_i\\,,_i\\tilde{A}_{0,0})(\\partial_{x_1}^2-\\partial_{x_1}\\partial_{x_2})L_{iC_{i,0}})\\\\+\\sum_i\\partial_{x_1}(L_{iC_{i,0}})\\partial_{x_1}(p_1\\wedge \\q\\,,_q^2\\,,_i\\tilde{A}_{0,0})-\\frac{1}{2}\\sum_i[\\partial_{x_1}(L_{iC_{i,0}})\\partial_{x_2}(p_1\\wedge \\q\\,,_q^2\\,,_i\\tilde{A}_{0,0})+\\partial_{x_2}(L_{iC_{i,0}})\\partial_{x_1}(p_1\\wedge \\q\\,,_q^2\\,,_i\\tilde{A}_{0,0})]\\\\\\end{aligned}"]
"DisplayStyle" → True

```

Out[•]=

$$\begin{aligned}
R_2 &= -\partial_{x_1}(p_1 \wedge q A_{1,1}) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})(q^2 p_1 \wedge q A_{1,0}) \\
&= -\sum_i L_i[C_{i,0}\partial_{x_1}(p_1 \wedge q_i \tilde{A}_{0,1}) + C_{i,1}\partial_{x_1}(p_1 \wedge q_i \tilde{A}_{0,0})] + \frac{1}{2}\sum_i L_i C_{i,0}(\partial_{x_i}^2 - \partial_{x_1}\partial_{x_2})(p_1 \wedge q q^2{}_i \tilde{A}_{0,0}) \\
&\quad - p_1 \wedge q \sum_i ({}_i \tilde{A}_{0,1}\partial_{x_1} L_i C_{i,0} + {}_i \tilde{A}_{0,0}\partial_{x_1} L_i C_{i,1}) + \frac{q^2}{2} p_1 \wedge q \sum_i {}_i \tilde{A}_{0,0}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2}) L_i C_{i,0} \\
&\quad + \sum_i \partial_{x_1}(L_i C_{i,0})\partial_{x_1}(p_1 \wedge q q^2{}_i \tilde{A}_{0,0}) - \frac{1}{2}\sum_i [\partial_{x_1}(L_i C_{i,0})\partial_{x_2}(p_1 \wedge q q^2{}_i \tilde{A}_{0,0}) + \partial_{x_2}(L_i C_{i,0})\partial_{x_1}(p_1 \wedge q q^2{}_i \tilde{A}_{0,0})]
\end{aligned}$$

```
ln[*]:= ol = 1;  
        fromDiag = 1;  
        toDiag = 4;
```

$$\begin{aligned} X1 \sum_i L_{iC_{i,0}} \partial_{x_1} (p_1 \wedge q) \\ \partial_{x_1} (p_1 \wedge q) \sum_i L_{iC_{i,0}} \\ \partial_{x_1} (p_1 \wedge q) \sum_i L_{iC_{i,0}} \end{aligned}$$

Out[•]=

\$Aborted

$$In[\bullet] := \mathbf{X1} = 0;$$

```
Do[X1 += -D[Wedge[p1, q]*A0hbarL[i][1, ol], x1] x L[i] x Cfhbar[i][0],
  {i, fromDiag, toDiag}];
```

```
X1 = simplifiedForm@onShellP[X1];
```

```
In[*]:= cancelFactors@Coefficient[Expand@factorRho[X1 /. {l → ρ l, ε → ρ ε}], ρ, -2]
```

```
Out[*]:=
```

0

```
In[*]:= MaTeX["\\begin{aligned}X2\\cong\\sum_i L_iC_{i,1}\\partial_{x_1}(p_1\\wedge q\\,,_i\\tilde{A}_{0,0})\\equiv\\partial_{x_1}(p_1\\wedge q)\\sum_i L_iC_{i,1}\\,,_i\\tilde{A}_{0,0}+p_1\\wedge q\\sum_i L_iC_{i,1}\\,,\\partial_{x_1}\\,,_i\\tilde{A}_{0,0}\\end{aligned}"]
```

```
Out[*]:=
```

$$X2 \cong \sum_i L_i C_{i,1} \partial_{x_1} (p_1 \wedge q_i \tilde{A}_{0,0})$$

$$= \partial_{x_1} (p_1 \wedge q) \sum_i L_i C_{i,1} \tilde{A}_{0,0} + p_1 \wedge q \sum_i L_i C_{i,1} \partial_{x_1} \tilde{A}_{0,0}$$

```
In[*]:= X2 = 0;
```

```
Do[X2 += -D[Wedge[p1, q]*A0hbarl[i][0, ol], x1]*L[i]*Cfhbar[i][1], {i, fromDiag, toDiag}];
```

```
X2 = simplifiedForm@onShellP[X2];
```

```
In[*]:= cancelFactors@Coefficient[Expand@factorRho[X2 /. {l → ρ l, ε → ρ ε}], ρ, -2]
```

```
Out[*]:=
```

$$\frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} -$$

$$\frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} - \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} -$$

$$\frac{4 i m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) q \odot q} + \frac{4 i m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} +$$

$$\frac{4 i \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} - \frac{4 i \gamma^3 p_1 \wedge q}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q}$$



$$\begin{aligned} & \partial_{x_1} \partial_{x_2} (p_1 \wedge q^2 \wedge \tilde{A}_{\{0,0\}}) = -\frac{1}{2} \partial_{x_1}^2 (p_1 \wedge q) \sum_{i \in I} \partial_{x_i} \tilde{A}_{\{0,0\}} - \frac{1}{2} \partial_{x_1} (p_1 \wedge q) \sum_{i \in I} \partial_{x_i} \tilde{A}_{\{0,0\}} - \frac{1}{2} \partial_{x_1}^2 (p_1 \wedge q) \sum_{i \in I} \partial_{x_i} \tilde{A}_{\{0,0\}} - \frac{1}{2} \partial_{x_1}^2 (p_1 \wedge q) \sum_{i \in I} \partial_{x_i} \tilde{A}_{\{0,0\}} \\ & \partial_{x_1} \partial_{x_2} (p_1 \wedge q^2 \wedge \tilde{A}_{\{0,0\}}) = -\frac{1}{2} \partial_{x_1}^2 (p_1 \wedge q) \sum_{i \in I} \partial_{x_i} \tilde{A}_{\{0,0\}} - \frac{1}{2} \partial_{x_1}^2 (p_1 \wedge q) \sum_{i \in I} \partial_{x_i} \tilde{A}_{\{0,0\}} - \frac{1}{2} \partial_{x_1}^2 (p_1 \wedge q) \sum_{i \in I} \partial_{x_i} \tilde{A}_{\{0,0\}} - \frac{1}{2} \partial_{x_1}^2 (p_1 \wedge q) \sum_{i \in I} \partial_{x_i} \tilde{A}_{\{0,0\}} \end{aligned}$$

Out[•]=

$$\begin{aligned} X3 &\cong -\frac{1}{2}\sum_i L_i C_{i,0}(\partial_{x_1}^2 - \\ &= -\frac{1}{2}\partial_{x_1}^2(p_1 \wedge q) \sum_i L_i C_{i,0} q^2 {}_i\tilde{A}_{0,0} - \partial_{x_1}(p_1 \wedge q) \sum_i L_i C_{i,0} \partial_{x_1}(q^2 {}_i\tilde{A}_{0,0}) - \frac{1}{2}(p_1 \wedge q) \\ &-\frac{1}{2}\partial_{x_1}\partial_{x_2}(p_1 \wedge q) \sum_i L_i C_{i,0} {}_i\tilde{A}_{0,0} - \frac{1}{2}\partial_{x_1}(p_1 \wedge q) \sum_i L_i C_{i,0} \partial_{x_2}(q^2 {}_i\tilde{A}_{0,0}) - \frac{1}{2}\partial_{x_2}(p_1 \wedge q) \sum_i L_i C_{i,0} \partial_{x_1}(q^2 {}_i\tilde{A}_{0,0}) - \frac{1}{2}(p_1 \wedge q) \sum_i \end{aligned}$$

$$\text{In}[\bullet] := \text{Do}[X306[n] = 0, \{n, 0, 6\}];$$

$$\begin{aligned} & \text{Do}\left[\text{Do}\left[\text{X306}[n] += \frac{-\text{Binomial}[2, n]}{2} \text{D}[q \otimes q \times \text{ToExpression}["A0" \leftrightarrow \text{ToString}[i] \leftrightarrow \text{"hbar"}]][0], \right. \\ & \quad \{x1, n\} \times \text{ToExpression}["L" \leftrightarrow \text{ToString}[i]] \times \\ & \quad \left. \text{ToExpression}["C" \leftrightarrow \text{ToString}[i] \leftrightarrow \text{"hbar"}]][0], \{i, 1, 2\}, \{n, 0, 2\}\right]; \\ & \text{Do}\left[\text{Do}\left[\text{X306}[2n+m+3] += \right. \\ & \quad \frac{-1}{2} \text{D}[q \otimes q \times \text{ToExpression}["A0" \leftrightarrow \text{ToString}[i] \leftrightarrow \text{"hbar"}]][0], \{x1, n\}, \{x2, m\} \times \\ & \quad \text{ToExpression}["L" \leftrightarrow \text{ToString}[i]] \times \\ & \quad \left. \text{ToExpression}["C" \leftrightarrow \text{ToString}[i] \leftrightarrow \text{"hbar"}]][0], \{i, 1, 2\}, \{n, 0, 1\}, \{m, 0, 1\}\right] \end{aligned}$$

$$In[\bullet] := (*X3[0] \text{ and } X3[3] \text{ are zero}*)$$

```
X306[0] = 0;
```

```
X306[3] = 0;
```

$$In[\bullet] := \text{Do}[X306[n] = \text{onShellLP}[X306[n]], \{n, 0, 6\}]$$

$$X3 = -\frac{\gamma \text{Wedge}[p1, p2]}{S} X306[1] - \text{Wedge}[p1, q] \times X306[2] -$$

$$\frac{\gamma \text{Wedge}[p1, p2]}{S} X306[4] + \frac{m1^2 \text{Wedge}[p1, p2]}{S} X306[5] - \text{Wedge}[p1, q] \times X306[6];$$

In[\*]:= X3 = 0;

$$\text{Do}[X3 += \frac{1}{2} L[i] \times \text{Cfhbar}[i][0] \times (D[D[\text{Wedge}[p1, q] \times q \odot q \times A0\text{hbar}[i][0, ol], x1], x1] -$$

$$D[D[\text{Wedge}[p1, q] \times q \odot q \times A0\text{hbar}[i][0, ol], x2], x1]), \{i, \text{fromDiag}, \text{toDiag}\}];$$

X3 = simplifiedForm@onShellP@X3;

In[\*]:= cancelFactors@Coefficient[Expand@factorRho[X3 /. {l → ρ l, ε → ρ ε}], ρ, -2]

Out[\*]=

0

In[\*]:= MaTeX[

$$"X4 \cong p_1 \wedge q \sum_i \tilde{A}_{0,1} \partial_{x_1} L_i C_{i,0}"]$$

Out[\*]=

$$X4 \cong p_1 \wedge q \sum_i \tilde{A}_{0,1} \partial_{x_1} L_i C_{i,0}$$

In[\*]:= X4 = 0;

$$\text{Do}[X4 += -\text{Wedge}[p1, q] \times A0\text{hbar}[i][1, ol] \times D[L[i] \times \text{Cfhbar}[i][0], x1],$$

$$\{i, \text{fromDiag}, \text{toDiag}\}];$$

X4 = simplifiedForm@onShellP[X4];

In[\*]:= -simplifiedForm[onShellP[Wedge[p1, q] × A0hbar[4][1, 1] × D[L[4] × Cfhbar[4][0], x1]]]

Out[\*]=

$$-\frac{2i\gamma(l \odot p1 + l \odot p2)p1 \wedge q}{(-i\epsilon + l \odot p1)^2(-i\epsilon + l \odot p2)q \odot q}$$

In[\*]:= D[L[4] × Cfhbar[4][0], x1]

Out[\*]=

$$-\frac{\gamma \left( -\frac{m2^2 p1 \odot p1}{S} + \frac{\gamma p1 \odot p2}{S} \right)}{(-i\epsilon + l \odot p2)(i\epsilon - l \odot p1 + p1 \odot q)^2}$$

In[\*]:= Cfhbar[4][0]

Out[\*]=

γ

```
In[*]:= cancelFactors@Coefficient[Expand@factorRho[X4 /. {l → ρ l, ε → ρ ε}], ρ, -2]
```

```
Out[*]=
```

$$-\frac{2i\gamma p_1 \wedge q}{(-i\epsilon + l \odot p_1)(-i\epsilon + l \odot p_2)q \odot q} - \frac{2i\gamma p_1 \wedge q}{(-i\epsilon + l \odot p_1)(i\epsilon + l \odot p_2)q \odot q}$$

```
MaTeX[
```

```
"X5\\cong\\,p_1\\wedge q\\sum_i\\,_{i}\\tilde{A}_{0,0}\\partial_{x_1}L_iC_{i,1}"]
```

```
Out[*]=
```

$$X5 \cong p_1 \wedge q \sum_i \tilde{A}_{0,0} \partial_{x_1} L_i C_{i,1}$$

```
In[*]:= s = 0;
```

```
Do[s += aWedgeDb[L[i]*Cf[i], p1, p1], {i, 1, 4}];
```

```
In[*]:= cancelFactors@Coefficient[
```

```
Expand@factorRho[simplifiedForm[onShellP[s]] /. {l → ρ l, ε → ρ ε}], ρ, -3]
```

$$\begin{aligned} & -\frac{\gamma \hbar q \odot q l \wedge p_1}{2(-i\epsilon + l \odot p_1)^2(i\epsilon + l \odot p_2)^2} + \frac{\gamma \hbar q \odot q l \wedge p_1}{2(i\epsilon + l \odot p_1)^2(i\epsilon + l \odot p_2)^2} - \\ & \frac{\gamma p_1 \wedge q}{(-i\epsilon + l \odot p_1)^2(-i\epsilon + l \odot p_2)} + \frac{\gamma p_1 \wedge q}{(-i\epsilon + l \odot p_1)^2(i\epsilon + l \odot p_2)} \\ & \frac{\gamma l \wedge p_1}{(i\epsilon + l \odot p_1)^2(-i\epsilon + l \odot p_2)} + \frac{\gamma l \wedge p_1}{(-i\epsilon + l \odot p_1)^2(i\epsilon + l \odot p_2)} + \\ & \frac{\hbar p_1 \wedge q}{2(-i\epsilon + l \odot p_1)(-i\epsilon + l \odot p_2)} - \frac{\hbar p_1 \wedge q}{2(-i\epsilon + l \odot p_1)(i\epsilon + l \odot p_2)} + \frac{\hbar p_1 \wedge q}{(i\epsilon + l \odot p_1)(i\epsilon + l \odot p_2)} \end{aligned}$$

```
In[*]:= X5 = 0;
```

```
Do[X5 += -Wedge[p1, q] × A0hbarl[i][0, ol] × D[L[i] × Cf[hbar[i][1], x1],  
{i, fromDiag, toDiag}];
```

```
X5 = simplifiedForm@onShellP[X5];
```

```
In[*]:= cancelFactors@Coefficient[Expand@factorRho[X5 /. {l -> rho l, epsilon -> rho epsilon}], rho, -2]
```

```
Out[*]=
```

$$\begin{aligned} & \frac{2 i \gamma^2 l \otimes l p_1 \wedge q}{(-i \epsilon + l \otimes p_1)^2 (-i \epsilon + l \otimes p_2)^2 q \otimes q} + \\ & \frac{4 i \gamma^2 l \otimes l p_1 \wedge q}{(-i \epsilon + l \otimes p_1)^3 (-i \epsilon + l \otimes p_2) q \otimes q} - \frac{2 i \gamma p_1 \wedge q}{(-i \epsilon + l \otimes p_1) (-i \epsilon + l \otimes p_2) q \otimes q} + \\ & \frac{4 i m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \otimes p_1) (-i \epsilon + l \otimes p_2) q \otimes q} + \frac{2 i \gamma^2 l \otimes l p_1 \wedge q}{(-i \epsilon + l \otimes p_1)^2 (i \epsilon + l \otimes p_2)^2 q \otimes q} - \\ & \frac{4 i \gamma^2 l \otimes l p_1 \wedge q}{(-i \epsilon + l \otimes p_1)^3 (i \epsilon + l \otimes p_2) q \otimes q} + \frac{2 i \gamma p_1 \wedge q}{(-i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2) q \otimes q} - \\ & \frac{4 i m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2) q \otimes q} - \frac{4 i \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2) q \otimes q} - \\ & \frac{4 i \gamma p_1 \wedge q}{(i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2) q \otimes q} + \frac{4 i \gamma^3 p_1 \wedge q}{S(i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2) q \otimes q} \end{aligned}$$

```
In[*]:= MaTeX["X6\\cong-\\frac{1}{2}p_1\\wedge q\\sum_i q^2\\_i\\tilde{A}_{0,0}(\\partial_{x_1}^2-\\partial_{x_1}\\partial_{x_2})L_iC_{i,0}\\_L_{iC}_{i,0}"]
```

```
Out[*]=
```

$$X6 \cong -\frac{1}{2}p_1 \wedge q \sum_i q^2 {}_i\tilde{A}_{0,0}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})L_iC_{i,0}$$

```
In[*]:= X6 = 0;
```

```
Do[X6 += \frac{Wedge[p1, q] \times q \otimes q}{2} A0hbar l[i][0, o l] \times (D[D[L[i] \times Cfhbar[i][0], x1], x1] - D[D[L[i] \times Cfhbar[i][0], x2], x1]), {i, fromDiag, toDiag}];
```

```
X6 = simplifiedForm@onShellP[X6];
```

```
In[*]:= cancelFactors@Coefficient[Expand@factorRho[X6 /. {l -> rho l, epsilon -> rho epsilon}], rho, -2]
```

```
Out[*]=
```

0

```
In[*]:= MaTeX[
  "\\begin{aligned}X7\\cong-\\sum_i\\partial_{x_1}(L_iC_{i,0})\\partial_{x_1}(p_1\\wedge q\\wedge q^2\\wedge \\_i\\tilde{A}_{0,0})\\=-\\partial_{x_1}(p_1\\wedge q)\\sum_iq^2\\_i\\tilde{A}_{0,0}\\partial_{x_1}(L_iC_{i,0})-p_1\\wedge q\\sum_i\\partial_{x_1}(L_iC_{i,0})\\partial_{x_1}(q^2\\_i\\tilde{A}_{0,0})\\end{aligned}"
```

Out[\*]=

$$X7 \cong - \sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_1}(p_1 \wedge q q^2 {}_i \tilde{A}_{0,0})$$

$$= -\partial_{x_1}(p_1 \wedge q) \sum_i q^2 {}_i \tilde{A}_{0,0} \partial_{x_1}(L_i C_{i,0}) - p_1 \wedge q \sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_1}(q^2 {}_i \tilde{A}_{0,0})$$

```
In[*]:= X7 = 0;
```

```
Do[X7 += D[Wedge[p1, q] * q * A0hbar[i][0, 0], x1] * D[L[i] * Cf[i][0], x1],
  {i, fromDiag, toDiag}];
```

```
X7 = simplifiedForm@onShellP[X7];
```

```
In[*]:= X7 = 0;
```

```
Do[X7 += Wedge[p1, q] * D[q * A0[i] /. {ρ → 1}, x1] * D[L[i] * Cf[i], x1], {i, 1, 4}];
```

```
X7 = simplifiedForm@onShellP[X7];
```

```
In[*]:= a = Total@Select[List@@Expand[X7 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2}, Exponent[#, h] ≤ 1 &];
```

```
In[*]:= b = {};
Do[AppendTo[b, simplifiedForm[a[[i]]], {i, 1, Length[a]}];
ClearAll[a];
a = Total[b];
ClearAll[b];
```

```
In[*]:= b = {};
Do[AppendTo[b, simplifyTerm[a[[i]]], {i, 1, Length[a]}];
ClearAll[a];
a = Total[b];
ClearAll[b];
```

```
Expand[Series[Coefficient[factorRho[a /. {l → ρ l, ε → ρ ε}], ρ, -3] /. {qml2 →
  1 / (Normal[Series[1 / (ρ² l ⊗ l + q ⊗ q - 2 ρ l ⊗ q), {ρ, 0, 1}]] /. {ρ → 1})}, {ρ, 0, 1}]]
Expand[Series[Coefficient[factorRho[a /. {l → ρ l, ε → ρ ε}], ρ, -3] /. {qml2 →
  1 / (Normal[Series[1 / (ρ² l ⊗ l + q ⊗ q - 2 ρ l ⊗ q), {ρ, 0, 1}]] /. {ρ → 1})}, {ρ, 0, 1}]]
```

Out[\*]=

0

```

In[*]:= simplifyTerm[
  Expand[simplifiedForm@Normal[Series[X7 /. l -> l - 2 l ⊗ q + q ⊗ q -> q m l 2, {ħ, 0, 1}]]]]
In[*]:= cancelFactors@Coefficient[Expand@factorRho[X7 /. {l -> ρ l, ε -> ρ ε}], ρ, -2]
Out[*]:=
0
In[*]:= MaTeX[
  "\\begin{aligned} X8 \\cong \\frac{1}{2} \\sum_i \\partial_{x_1} (L_i C_{i,0}) \\partial_{x_2} (p_1 \\wedge q^2 \\sum_i q^2 \\partial_{x_1} (L_i C_{i,0}) \\partial_{x_2} (p_1 \\wedge q) \\sum_i q^2 \\partial_{x_1} (L_i C_{i,0}) + \\frac{1}{2} p_1 \\wedge q \\sum_i \\partial_{x_1} (L_i C_{i,0}) \\partial_{x_2} (q^2 \\sum_i q^2 \\partial_{x_1} (L_i C_{i,0}) \\partial_{x_2} (p_1 \\wedge q) \\sum_i q^2 \\partial_{x_1} (L_i C_{i,0}) \\end{aligned}" ]

```

Out[\*]=

$$\begin{aligned}
 X8 &\cong \frac{1}{2} \sum_i \partial_{x_1} (L_i C_{i,0}) \partial_{x_2} (p_1 \wedge q q^2 {}_i \tilde{A}_{0,0}) \\
 &= \frac{1}{2} \partial_{x_2} (p_1 \wedge q) \sum_i q^2 {}_i \tilde{A}_{0,0} \partial_{x_1} (L_i C_{i,0}) + \frac{1}{2} p_1 \wedge q \sum_i \partial_{x_1} (L_i C_{i,0}) \partial_{x_2} (q^2 {}_i \tilde{A}_{0,0})
 \end{aligned}$$

```

In[*]:= X8 = 0;

Do[X8 += -1/2 D[Wedge[p1, q] * q ⊗ q * A0hbarl[i][0, ol], x2] * D[L[i] * Cfhbar[i][0], x1],
  {i, fromDiag, toDiag}];

```

```
X8 = simplifiedForm@onShellP[X8];
```

```

In[*]:= cancelFactors@Coefficient[Expand@factorRho[X8 /. {l -> ρ l, ε -> ρ ε}], ρ, -2]
Out[*]:=
0

```

```

In[*]:= X8 = 0;

Do[X8 += -1/2 D[Wedge[p1, q] * q ⊗ q * (A0hbarl[i][0, ol] + ħ * A0hbarl[i][1, ol]), x2] *
  D[L[i] * (Cfhbar[i][0] + ħ * Cfhbar[i][1]), x1], {i, fromDiag, toDiag}];

```

```
X8 = simplifiedForm@onShellP[X8];
```

```
In[*]:= X8 = 0;
```

```
Do[X8 +=  $\frac{-1}{2}$  D[Wedge[p1, q] × q ⊗ q × A0[i], x2] × D[L[i] × Cf[i], x1], {i, 1, 4}];
```

```
X8 = simplifiedForm@onShellP[X8];
```

```
In[*]:= a = Total@Select[
    List@@Expand[X8 /. {ρ → 1} /. l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2], Exponent[#, h] ≤ 1 &];
```

```
In[*]:= b = {};
Do[AppendTo[b, simplifiedForm[a[[i]]]], {i, 1, Length[a]};
ClearAll[a];
a = Total[b];
ClearAll[b];
```

```
In[*]:= b = {};
Do[AppendTo[b, simplifyTerm[a[[i]]]], {i, 1, Length[a]};
ClearAll[a];
a = Total[b];
ClearAll[b];
```

```
In[*]:= Coefficient[factorRho[a /. {l → ρ l, ε → ρ ε}], ρ, -2]
```

```
Out[*]=
```

$$\begin{aligned}
 & -\frac{i m^2 \gamma^2 \hbar l \otimes l q \otimes q p_1 \wedge p_2}{q m l^2 S(-i \epsilon + l \otimes p_1)^2 (-i \epsilon + l \otimes p_2)^2} + \\
 & \frac{2 i m^2 \gamma \hbar q \otimes q p_1 \wedge p_2}{q m l^2 S(-i \epsilon + l \otimes p_1)(-i \epsilon + l \otimes p_2)} - \frac{2 i m^2 m^2 \gamma^2 \hbar q \otimes q p_1 \wedge p_2}{q m l^2 S^2(-i \epsilon + l \otimes p_1)(-i \epsilon + l \otimes p_2)} + \\
 & \frac{2 i m^2 m^2 \gamma^2 \hbar q \otimes q p_1 \wedge p_2}{q m l^2 S^2(-i \epsilon + l \otimes p_1)(i \epsilon + l \otimes p_2)} + \frac{2 i m^2 \gamma^3 \hbar q \otimes q p_1 \wedge p_2}{q m l^2 S^2(-i \epsilon + l \otimes p_1)(i \epsilon + l \otimes p_2)} + \\
 & \frac{2 i m^2 \gamma \hbar q \otimes q p_1 \wedge p_2}{q m l^2 S(i \epsilon + l \otimes p_1)(i \epsilon + l \otimes p_2)} - \frac{2 i m^2 \gamma^3 \hbar q \otimes q p_1 \wedge p_2}{q m l^2 S^2(i \epsilon + l \otimes p_1)(i \epsilon + l \otimes p_2)} + \\
 & \frac{4 i \gamma^3 q \otimes q p_1 \wedge q}{q m l^2 S(-i \epsilon + l \otimes p_1)(-i \epsilon + l \otimes p_2)} - \frac{4 i \gamma^3 q \otimes q p_1 \wedge q}{q m l^2 S(-i \epsilon + l \otimes p_1)(i \epsilon + l \otimes p_2)}
 \end{aligned}$$

```
In[*]:= MaTeX[
  "\\begin{aligned}X9\\cong\\frac{1}{2}\\sum_i\\partial_{x_2}(L_{iC_{i,0}})\\partial_{x_1}(p_1\\wedge
  q\\,,q^2\\,,_i\\tilde{A}_{0,0})\\\\=\\frac{1}{2}\\partial_{x_1}(p_1\\wedge
  q)\\sum_iq^2\\,,_i\\tilde{A}_{0,0})\\partial_{x_2}(L_{iC_{i,0}})+\\frac{1}{2}p_1\\wedge
  q\\sum_i\\partial_{x_2}(L_{iC_{i,0}})\\partial_{x_1}(q^2\\,,_i\\tilde{A}_{0,0})\\end{aligned}"
```

Out[\*]=

$$X9 \cong \frac{1}{2} \sum_i \partial_{x_2}(L_i C_{i,0}) \partial_{x_1}(p_1 \wedge q q^2{}_i \tilde{A}_{0,0})$$

$$= \frac{1}{2} \partial_{x_1}(p_1 \wedge q) \sum_i q^2{}_i \tilde{A}_{0,0} \partial_{x_2}(L_i C_{i,0}) + \frac{1}{2} p_1 \wedge q \sum_i \partial_{x_2}(L_i C_{i,0}) \partial_{x_1}(q^2{}_i \tilde{A}_{0,0})$$

```
In[*]:= X9 = 0;
```

```
Do[X9 += -1/2 D[Wedge[p1, q] x q @ q x A0hbar[l][0, ol], x1] x D[L[i] x Cf hbar[i][0, x2],
  {i, fromDiag, toDiag}];
```

```
X9 = simplifiedForm@onShellP[X9];
```

```
In[*]:= cancelFactors@Coefficient[Expand@factorRho[X9 /. {l -> rho l, epsilon -> rho epsilon}], rho, -2]
```

Out[\*]=

0

```
In[*]:= XSuml0 = 0;
```

```
Do[XSuml0 += ToExpression["X" <-> ToString[n]], {n, 1, 9}];
```

```
In[*]:= YSuml0 = Y1 + Y2 + Y3;
```



In[\*]:= **cancelFactors@**

**Coefficient[Expand@factorRho[(XSuml0 + YSuml0) /. {l → ρ l, ε → ρ ε}], ρ, -2]**

Out[\*]=

$$\begin{aligned}
& \frac{8 i \gamma^2 l \odot q l \wedge p_1}{(-i \epsilon + l \odot p_1)^3 (-i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^2 l \odot q l \wedge p_1}{(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)^2 q \odot q} - \\
& \frac{4 i \gamma^2 l \odot q l \wedge p_1}{(i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)^2 q \odot q} - \frac{8 i \gamma^2 l \odot q l \wedge p_1}{(-i \epsilon + l \odot p_1)^3 (i \epsilon + l \odot p_2) q \odot q} + \\
& \frac{4 i \gamma l \odot q p_1 \wedge p_2}{(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} - \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} + \\
& \frac{4 i \gamma l \odot q p_1 \wedge p_2}{(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} - \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} - \\
& \frac{4 i \gamma l \odot q p_1 \wedge p_2}{(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} + \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} - \\
& \frac{4 i \gamma l \odot q p_1 \wedge p_2}{(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} + \\
& \frac{4 i m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) q \odot q} - \frac{4 i m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} - \\
& \frac{4 i \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^3 p_1 \wedge q}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q}
\end{aligned}$$

In[\*]:= **MaTeX["\\begin{aligned} Y=**

\\sum\_{ip\_1} \\wedge \\partial\_{p\_1} A\_{1,1} + \\frac{1}{2} \\sum\_i (\\partial\_{x\_1} - \\partial\_{x\_2}) (q^2 p\_1 \\wedge \\partial\_{p\_1} A\_{1,0}) \\end{aligned} = \\sum\_{ip\_1} \\wedge \\partial\_{p\_1} (L\_i C\_{i,0} A\_{0,1} + L\_i C\_{i,1} A\_{0,0}) - \\frac{1}{2} \\sum\_i (\\partial\_{x\_1} - \\partial\_{x\_2}) (q^2 p\_1 \\wedge \\partial\_{p\_1} (L\_i C\_{i,0} A\_{0,1} + L\_i C\_{i,1} A\_{0,0})) \\end{aligned}"]

Out[\*]=

$$\begin{aligned}
Y &= \sum_i p_1 \wedge \partial_{p_1} A_{1,1} + \frac{1}{2} \sum_i (\partial_{x_1} - \partial_{x_2}) (q^2 p_1 \wedge \partial_{p_1} A_{1,0}) \\
&= \sum_i p_1 \wedge \partial_{p_1} (L_i C_{i,0} A_{0,1} + L_i C_{i,1} A_{0,0}) - \frac{1}{2} \sum_i (\partial_{x_1} - \partial_{x_2}) (q^2 p_1 \wedge \partial_{p_1} (L_i C_{i,0} A_{0,1} + L_i C_{i,1} A_{0,0}))
\end{aligned}$$

In[\*]:= **ol = 1;**

**fromDiag = 1;**

**toDiag = 4;**

In[\*]:= Y1 = 0;

Do[Y1 += aWedgeDb[L[i] x Cfhbar[i][0], p1, p1] x A0hbarl[i][1, ol], {i, fromDiag, toDiag}];

Y1 = simplifiedForm@onShellP@Y1;

In[\*]:= Y2 = 0;

Do[Y2 += aWedgeDb[L[i] x Cfhbar[i][1] x A0hbarl[i][0, ol], p1, p1], {i, fromDiag, toDiag}];

Y2 = simplifiedForm@onShellP@Y2;

In[\*]:= Y3 = 0;

Do[Y3 +=  $\frac{-1}{2}$   
(D[q ⊗ q \* aWedgeDb[L[i] x Chbar[i][0] x A0hbarl[i][0, ol], p1, p1], x1] - D[q ⊗ q \* aWedgeDb[L[i] x Cfhbar[i][0] x A0hbarl[i][0, ol], p1, p1], x2]), {i, fromDiag, toDiag}];

Y3 = simplifiedForm@onShellP@Y3;

In[\*]:= cancelFactors@Coefficient[Expand@factorRho[Y1 /. {l → ρ l, ε → ρ ε}], ρ, -2]

Out[\*]=

$$\frac{2 i \gamma p_1 \wedge q}{(-i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2) q \odot q} + \frac{2 i \gamma p_1 \wedge q}{(-i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2) q \odot q} \\ - \frac{2 i \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2) q \odot q} + \frac{6 i \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2) q \odot q}$$

In[\*]:= X4 = 0;

Do[X4 += -Wedge[p1, q] x A0hbarl[i][1, ol] x D[L[i] x Cfhbar[i][0], x1],  
{i, fromDiag, toDiag}];

X4 = simplifiedForm@onShellP[X4];

In[\*]:= MaTeX["\\begin{aligned} R\_2 = -\\partial\_{x\_1}(p\_1\\wedge \\q\\,,A\_{1,1})+\\frac{1}{2}(\\partial\_{x\_1}^2-\\partial\_{x\_1}\\partial\_{x\_2})(q^2p\_1\\wedge q\\,,p\_1\\wedge q\\,,A\_{1,0}))\\\\=-\\partial\_{x\_1}(p\_1\\wedge q\\,,L\_i(C\_{i,1}A\_{0,0}+C\_{i,0}A\_{0,1}))+\\frac{1}{2}(\\partial\_{x\_1}^2-\\partial\_{x\_1}\\partial\_{x\_2})(q^2p\_1\\wedge q\\,,L\_iC\_{i,0}A\_{0,0}))\\end{aligned}"]

Out[\*]=

$$R_2 = -\partial_{x_1}(p_1 \wedge q A_{1,1}) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1} \partial_{x_2})(q^2 p_1 \wedge q A_{1,0}) \\ = -\partial_{x_1}(p_1 \wedge q L_i(C_{i,1}A_{0,0} + C_{i,0}A_{0,1})) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1} \partial_{x_2})(q^2 p_1 \wedge q L_i C_{i,0}A_{0,0})$$

```

(*ol=0;
fromDiag=1;
toDiag=2;

ClearAll[X];
Do[X[i]=0;,{i,1,9}];

Do[X[1]+=D[Wedge[p1,q]*A0hbarL[i][1,ol],x1]*L[i]*Cfhbar[i][0];

X[2]+=D[Wedge[p1,q]*A0hbarL[i][0,ol],x1]*L[i]*Cfhbar[i][1];

X[3]+=-1/2 L[i]*Cfhbar[i][0]*(D[D[Wedge[p1,q]*qQq*A0hbarL[i][0,ol],x1],x1]-
D[D[Wedge[p1,q]*qQq*A0hbarL[i][0,ol],x2],x1]);

X[4]+=Wedge[p1,q]*A0hbarL[i][1,ol]*D[L[i]*Cfhbar[i][0],x1];

X[5]+=Wedge[p1,q]*A0hbarL[i][0,ol]*D[L[i]*Cfhbar[i][1],x1];

X[6]+=-Wedge[p1,q]*qQq/2 A0hbarL[i][0,ol]*
(D[D[L[i]*Cfhbar[i][0],x1],x1]-D[D[L[i]*Cfhbar[i][0],x2],x1]);

X[7]+=-D[Wedge[p1,q]*qQq*A0hbarL[i][0,ol],x1]*D[L[i]*Cfhbar[i][0],x1];

X[8]+=-1/2 D[Wedge[p1,q]*qQq*A0hbarL[i][0,ol],x2]*D[L[i]*Cfhbar[i][0],x1];

X[9]+=-1/2 D[Wedge[p1,q]*qQq*A0hbarL[i][0,ol],x1]*D[L[i]*Cfhbar[i][0],x2];
,{i,fromDiag,toDiag}];

Do[X[i]=simplifiedForm@onShellP[X[i]];,{i,1,9}];

XSum=0;
Do[XSum+=X[i],{i,1,9}];*)

```

```

ol = 2;
fromDiag = 1;
toDiag = 4;

ClearAll[X];
Do[X[i] = 0;, {i, 1, 3}];

Do[X[1] += -D[Wedge[p1, q] × L[i] × Cfhbar[i][0] × A0hbarl[i][1, ol], x1];
  X[2] += -D[Wedge[p1, q] × L[i] × Cfhbar[i][1] × A0hbarl[i][0, ol], x1];
  X[3] +=
     $\frac{1}{2}$  D[(D[q ⊙ q × Wedge[p1, q] × L[i] × Cfhbar[i][0] × A0hbarl[i][0, ol], x1] - D[q ⊙ q × Wedge[p1,
      q] × L[i] × Cfhbar[i][0] × A0hbarl[i][0, ol], x2]), x1], {i, fromDiag, toDiag}];

Do[X[i] = simplifiedForm@onShellP@X[i];, {i, 1, 3}];

XSum = 0;
Do[XSum += X[i], {i, 1, 3}];

In[*]:= ol = 0;
fromDiag = 1;
toDiag = 4;

ClearAll[Z];
Do[Z[i] = 0;, {i, 1, 3}];

Do[Z[1] += aWedgeDb[L[i] × Cfhbar[i][0] × A0hbarl[i][1, ol], p1, p1];
  Z[2] += aWedgeDb[L[i] × Cfhbar[i][1] × A0hbarl[i][0, ol], p1, p1];
  Z[3] +=  $\frac{-1}{2}$  (D[q ⊙ q × aWedgeDb[L[i] × Cfhbar[i][0] × A0hbarl[i][0, ol], p1, p1], x1] -
    D[q ⊙ q × aWedgeDb[L[i] × Cfhbar[i][0] × A0hbarl[i][0, ol], p1, p1],
      x2]), {i, fromDiag, toDiag}];

Do[Z[i] = simplifiedForm@onShellP@Z[i];, {i, 1, 3}];

ZSum = 0;
Do[ZSum += Z[i], {i, 1, 3}];

In[*]:= cancelFactors@Coefficient[Expand@factorRho[(XSum + ZSum) /. {l → ρ l, ε → ρ ε}], ρ, -2]

In[*]:= cancelFactors@Coefficient[Expand@factorRho[(ZSum) /. {l → ρ l, ε → ρ ε}], ρ, -2]

```

$$\begin{aligned}
& \frac{4 \hbar \gamma p_1 \wedge q}{(-\hbar \epsilon + l \odot p_1)(-\hbar \epsilon + l \odot p_2) q \odot q} - \frac{2 \hbar \gamma p_1 \wedge q}{(\hbar \epsilon + l \odot p_1)(-\hbar \epsilon + l \odot p_2) q \odot q} - \\
& \frac{4 \hbar \gamma p_1 \wedge q}{(-\hbar \epsilon + l \odot p_1)(\hbar \epsilon + l \odot p_2) q \odot q} + \frac{6 \hbar \gamma p_1 \wedge q}{(\hbar \epsilon + l \odot p_1)(\hbar \epsilon + l \odot p_2) q \odot q} \\
& (*ZSum: l^0, l*) \\
& - \frac{2 \hbar \gamma p_1 \wedge q}{(\hbar \epsilon + l \odot p_1)(-\hbar \epsilon + l \odot p_2) q \odot q} + \frac{6 \hbar \gamma p_1 \wedge q}{(\hbar \epsilon + l \odot p_1)(\hbar \epsilon + l \odot p_2) q \odot q} \\
& \frac{2 \hbar \gamma p_1 \wedge q}{(-\hbar \epsilon + l \odot p_1)(-\hbar \epsilon + l \odot p_2) q \odot q} + \frac{2 \hbar \gamma p_1 \wedge q}{(-\hbar \epsilon + l \odot p_1)(\hbar \epsilon + l \odot p_2) q \odot q} \\
& (*XSum: l^0, l*) \\
& \frac{2 \hbar \gamma p_1 \wedge q}{(\hbar \epsilon + l \odot p_1)(-\hbar \epsilon + l \odot p_2) q \odot q} - \frac{6 \hbar \gamma p_1 \wedge q}{(\hbar \epsilon + l \odot p_1)(\hbar \epsilon + l \odot p_2) q \odot q} \\
& - \frac{2 \hbar \gamma p_1 \wedge q}{(-\hbar \epsilon + l \odot p_1)(-\hbar \epsilon + l \odot p_2) q \odot q} - \frac{2 \hbar \gamma p_1 \wedge q}{(-\hbar \epsilon + l \odot p_1)(\hbar \epsilon + l \odot p_2) q \odot q}
\end{aligned}$$

## First Pair

$$\begin{aligned}
In[*]:= Bx1 &= \frac{(2 p_1 + \hbar l) \odot (2 p_2 - \hbar l) (2 p_1 + \hbar q + \hbar l) \odot (2 p_2 - \hbar q - \hbar l)}{(q - l) \odot (q - l) (2 p_1 \odot l + \hbar l \odot l + \hbar \epsilon) (2 p_2 \odot l - \hbar l \odot l - \hbar \epsilon)}; \\
Cx1 &= ((2 p_1 + \hbar q + \hbar l) \odot (2 p_2 - \hbar q + \hbar l) (2 p_1 + \hbar l) \odot (2 p_2 - 2 \hbar q + \hbar l)) / \\
& ((q - l) \odot (q - l) (2 p_1 \odot l + \hbar l \odot l + \hbar \epsilon) (2 p_2 \odot (q - l) - \hbar (q - l) \odot (q - l) - \hbar \epsilon)); \\
In[*]:= B1C1 &= Bx1 + Cx1 /. \{l \odot l - 2 l \odot q + q \odot q \rightarrow qm l^2, -l \odot l + 2 l \odot q - q \odot q \rightarrow -qm l^2\}; \\
In[*]:= B1C1 \\
Out[*]:= & ((\hbar^2 l \odot l + 2 \hbar l \odot p_1 + 2 \hbar l \odot p_2 - 2 \hbar^2 l \odot q + 4 p_1 \odot p_2 - 4 \hbar p_1 \odot q) \\
& (\hbar^2 l \odot l + 2 \hbar l \odot p_1 + 2 \hbar l \odot p_2 + 4 p_1 \odot p_2 - 2 \hbar p_1 \odot q + 2 \hbar p_2 \odot q - \hbar^2 q \odot q)) / \\
& (qm l^2 (\hbar \epsilon + \hbar l \odot l + 2 l \odot p_1) (-\hbar \epsilon - qm l^2 \hbar + 2 (-l \odot p_2 + p_2 \odot q))) + \\
& ((-\hbar^2 l \odot l - 2 \hbar l \odot p_1 + 2 \hbar l \odot p_2 + 4 p_1 \odot p_2) \\
& (-\hbar^2 l \odot l - 2 \hbar l \odot p_1 + 2 \hbar l \odot p_2 - 2 \hbar^2 l \odot q + 4 p_1 \odot p_2 - 2 \hbar p_1 \odot q + 2 \hbar p_2 \odot q - \hbar^2 q \odot q)) / \\
& (qm l^2 (\hbar \epsilon + \hbar l \odot l + 2 l \odot p_1) (-\hbar \epsilon - \hbar l \odot l + 2 l \odot p_2)) \\
In[*]:= Do[Bx1hbar[n] &= Coefficient[Normal[Series[Bx1, {\hbar, 0, 1}]], \hbar, n]; \\
& Cx1hbar[n] = Coefficient[Normal[Series[Cx1, {\hbar, 0, 1}]], \hbar, n], {n, 0, 1}];
\end{aligned}$$

```
In[*]:= simplifiedForm@Bx1hbar[0]
```

```
Out[*]=
```

$$\frac{4 (p_1 \odot p_2)^2}{(i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) (l \odot l - 2 l \odot q + q \odot q)}$$

```
In[*]:= simplifiedForm@Cx1hbar[0]
```

```
Out[*]=
```

$$-\frac{4 (p_1 \odot p_2)^2}{(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2 - p_2 \odot q) (l \odot l - 2 l \odot q + q \odot q)}$$

$R_2$

```
In[*]:= MaTeX["\\begin{aligned}R_2 = -\\partial_{x_1}(p_1\\wedge \\q\\,,(B_0+C_0))+\\frac{1}{2}(\\partial_{x_1}^2-\\partial_{x_1}\\partial_{x_2})(q^2\\p_1\\wedge \\q\\,,(B_{-1}+C_{-1}))\\\\=p_1\\wedge \\q\\,,\\left(-\\partial_{x_1}(B_0+C_0)+\\frac{1}{2}\\partial_{x_1}^2(q^2\\,,(B_{-1}+C_{-1}))-\\frac{1}{2}\\partial_{x_1}\\partial_{x_2}(q^2\\,,(B_{-1}+C_{-1}))\\right)\\\\+\\partial_{x_1}(p_1\\wedge \\q)\\left(-(B_0+C_0)+\\partial_{x_1}(q^2\\,,(B_{-1}+C_{-1}))-\\frac{1}{2}\\partial_{x_2}(q^2\\,,(B_{-1}+C_{-1}))\\right)\\\\-\\frac{1}{2}\\partial_{x_2}(p_1\\wedge \\q)\\partial_{x_1}(q^2\\,,(B_{-1}+C_{-1}))\\end{aligned}"]
```

```
Out[*]=
```

$$\begin{aligned} R_2 &= -\partial_{x_1}(p_1 \wedge q (B_0 + C_0)) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})(q^2 p_1 \wedge q (B_{-1} + C_{-1})) \\ &= p_1 \wedge q \left( -\partial_{x_1}(B_0 + C_0) + \frac{1}{2}\partial_{x_1}^2(q^2 (B_{-1} + C_{-1})) - \frac{1}{2}\partial_{x_1}\partial_{x_2}(q^2 (B_{-1} + C_{-1})) \right) \\ &\quad + \partial_{x_1}(p_1 \wedge q) \left( -(B_0 + C_0) + \partial_{x_1}(q^2 (B_{-1} + C_{-1})) - \frac{1}{2}\partial_{x_2}(q^2 (B_{-1} + C_{-1})) \right) \\ &\quad - \frac{1}{2}\partial_{x_2}(p_1 \wedge q) \partial_{x_1}(q^2 (B_{-1} + C_{-1})) \end{aligned}$$

```

In[*]:= R21D1 = -simplifiedForm[onShellP[
  D[Wedge[p1, q] * (Expand[(Bx1hbar[1] + Cx1hbar[1]) /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2,
    -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}]] /. qml2 → l ⊗ l - 2 l ⊗ q + q ⊗ q), x1]]];

R22D1 = (D[D[q ⊗ q × Wedge[p1, q] * (Bx1hbar[0] + Cx1hbar[0]), x1], x1] -
  D[D[q ⊗ q × Wedge[p1, q] * (Bx1hbar[0] + Cx1hbar[0]), x2], x1]) /.
  {
     $\left(-\frac{m^2 p_1 \otimes p_1}{s} + \frac{\gamma p_1 \otimes p_2}{s}\right) \rightarrow 1, \left(\frac{m^2 p_1 \otimes p_1}{s} - \frac{\gamma p_1 \otimes p_2}{s}\right) \rightarrow -1,$ 
     $\left(\frac{\gamma p_1 \otimes p_2}{s} - \frac{m^2 p_2 \otimes p_2}{s}\right) \rightarrow 1, \left(-\frac{\gamma p_1 \otimes p_2}{s} + \frac{m^2 p_2 \otimes p_2}{s}\right) \rightarrow -1,$ 
     $\left(\frac{\gamma p_1 \otimes p_1}{s} - \frac{m^2 p_1 \otimes p_2}{s}\right) \rightarrow 0, \left(-\frac{\gamma p_1 \otimes p_1}{s} + \frac{m^2 p_1 \otimes p_2}{s}\right) \rightarrow 0,$ 
     $\left(-\frac{m^2 p_1 \otimes p_2}{s} + \frac{\gamma p_2 \otimes p_2}{s}\right) \rightarrow 0, \left(\frac{m^2 p_1 \otimes p_2}{s} - \frac{\gamma p_2 \otimes p_2}{s}\right) \rightarrow 0$ 
  };

R22D1 =  $\frac{1}{2}$  simplifiedForm@onShellP[R22D1];
(*ClearAll[workspace1, workspace2];*)

In[*]:= R22D1 =  $\frac{1}{2}$  (D[D[q ⊗ q × Wedge[p1, q] * (Bx1hbar[0] + Cx1hbar[0]), x1], x1] -
  D[D[q ⊗ q × Wedge[p1, q] * (Bx1hbar[0] + Cx1hbar[0]), x2], x1]);

In[*]:= R22D1 = simplifiedForm[onShellP[R22D1]];

In[*]:= R2D1 = R21D1 + R22D1;

In[*]:= R2D1 = Collect[factorRho@(simplifiedForm[R2D1] /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2,
  -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2} /. {l → ρ l, ε → ρ ε}), ρ];

In[*]:= R2D1Div = Total@Select[List @@ R2D1, Exponent[#, ρ] < -1 &];

In[*]:= R2D1Div

In[*]:= R2D1Div = R2D1Div /. {(num_ a_) / (Power[den_, -n_]) =>
  num / (den ^ (-n - 1)) /; ContainsAll[Variables[den], {ε, a}] /; a == l ⊗ p1 || a == l ⊗ p2}

In[*]:= Expand[simplifiedForm[
  onShellP[D[Bx1hbar[0] + Cx1hbar[0], x1], x1] - D[D[Bx1hbar[0] + Cx1hbar[0], x1], x2]]]]

```

```
In[*]:= Coefficient[
  Collect[factorRho[simplifyTerm[Expand[R22D1]] /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2,
    - l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2} /. {l → ρ l, ε → ρ ε}], ρ], ρ, -2]
```

```
Out[*]=
```

$$\begin{aligned}
& -\frac{4 m^2 \gamma^2 p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)} - \frac{4 \gamma^3 p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)} + \\
& \frac{4 m^2 \gamma^2 p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)} + \frac{4 \gamma^3 p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)} + \\
& \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)} + \frac{4 \gamma^3 q \odot q p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)} - \\
& \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_2)^2} - \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)} \\
& C \rightarrow \frac{4 m^2 \gamma^2 p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)} + \frac{4 \gamma^3 p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)} - \\
& \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_2)^2} - \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)} \\
& B \rightarrow -\frac{4 m^2 \gamma^2 p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)} - \frac{4 \gamma^3 p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)} + \\
& \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)} + \frac{4 \gamma^3 q \odot q p_1 \wedge q}{q m l^2 S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)}
\end{aligned}$$

$R_1$

```
In[*]:= MaTeX["\\begin{aligned}R_1= (p_1\\wedge \\partial_{p_1})(B_0+C_0)-\\frac{1}{2}(\\partial_{x_1}-\\partial_{x_2})(q^2(p_1\\wedge \\partial_{p_1})(B_{-1}+C_{-1}))\\end{aligned}"]
```

```
Out[*]=
```

$$R_1 = (p_1 \wedge \partial_{p_1})(B_0 + C_0) - \frac{1}{2}(\partial_{x_1} - \partial_{x_2})(q^2(p_1 \wedge \partial_{p_1})(B_{-1} + C_{-1}))$$



```

In[*]:= R11D1 = simplifiedForm[
  onShellP[aWedgeDb[Expand[(Bx1hbar[1]+Cx1hbar[1]) /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2,
    -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}]] /. qml2 → l ⊗ l - 2 l ⊗ q + q ⊗ q, p1, p1]]];
R12D1 =
  -1/2 simplifiedForm[onShellP[D[q ⊗ q × aWedgeDb[Bx1hbar[0]+Cx1hbar[0], p1, p1], x1] -
    D[q ⊗ q × aWedgeDb[Bx1hbar[0]+Cx1hbar[0], p1, p1], x2]]];
In[*]:= simplifyTerm@
  Coefficient[Collect[factorRho@(Expand[R22D1] /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2,
    -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2} /. {l → ρ l, ε → ρ ε}), ρ], ρ, -2]
Out[*]:=
  0
In[*]:= R1D1 = R11D1 + R12D1;

```

In[\*]:= R12D1

Out[\*]=

$$\begin{aligned}
& \frac{1}{2} \left( -q \odot q \left( \frac{8 \gamma^2 \left( -\frac{m^2 l_{\odot p1}}{s} + \frac{\gamma l_{\odot p2}}{s} \right) l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \right. \right. \\
& \quad \frac{8 \gamma^2 \left( -\frac{m^2 l_{\odot p1}}{s} + \frac{\gamma l_{\odot p2}}{s} \right) l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} + \\
& \quad \frac{16 \gamma \left( -\frac{m^2 l_{\odot p1}}{s} + \frac{\gamma l_{\odot p2}}{s} \right) p1 \wedge p2}{(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \quad \left. \frac{16 \gamma \left( -\frac{m^2 l_{\odot p1}}{s} + \frac{\gamma l_{\odot p2}}{s} \right) p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} \right) + \\
& q \odot q \left( \frac{8 \gamma^2 \left( \frac{\gamma l_{\odot p1}}{s} - \frac{m^2 l_{\odot p2}}{s} \right) l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \right. \\
& \quad \frac{8 \gamma^2 \left( \frac{\gamma l_{\odot p1}}{s} - \frac{m^2 l_{\odot p2}}{s} \right) l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \quad \frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 (l \odot l - 2 l \odot q + q \odot q)} + \\
& \quad \frac{16 \gamma \left( \frac{\gamma l_{\odot p1}}{s} - \frac{m^2 l_{\odot p2}}{s} \right) p1 \wedge p2}{(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \quad \frac{16 \gamma \left( \frac{\gamma l_{\odot p1}}{s} - \frac{m^2 l_{\odot p2}}{s} \right) p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \quad \left. \left. \frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 (l \odot l - 2 l \odot q + q \odot q)} \right) \right)
\end{aligned}$$

Singular terms

In[\*]:= S1 = simplifiedForm@onShellP[aWedgeDb[Bx1hbar[0]+Cx1hbar[0], p1, p1]]

Out[\*]=

$$\begin{aligned} & \frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \\ & \frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} + \\ & \frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} \end{aligned}$$

In[\*]:= simplifiedForm[expr\_] := expr /. { $\left(-\frac{m1^2 m2^2}{s} + \frac{\gamma^2}{s}\right) \rightarrow 1$ ,  $\left(\frac{m1^2 m2^2}{s} - \frac{\gamma^2}{s}\right) \rightarrow -1$ } /. { $\epsilon \rightarrow 2 \epsilon$ } /.

$$\begin{aligned} & \left\{ (coef1_ \epsilon + coef2_ l \odot p1)^{(n_)} \rightarrow (coef2)^n * (\epsilon * coef1 / coef2 + l \odot p1)^n, \right. \\ & \left. (coef1_ \epsilon + coef2_ l \odot p2)^{(n_)} \rightarrow (coef2)^n * (\epsilon * coef1 / coef2 + l \odot p2)^n \right\} \end{aligned}$$

In[\*]:= S2 = -simplifiedForm@onShellP[Expand[D[Wedge[p1, q] \* (Bx1hbar[0]+Cx1hbar[0]), x1]]] /.

$$\begin{aligned} & \left\{ (num\_a_) / (Power[den_, -n_]) \rightarrow \right. \\ & \left. num / (den^{-(n-1)}) /; \text{ContainsAll}[\text{Variables}[den], \{\epsilon, a\}] /; a == l \odot p1 \parallel a == l \odot p2 \right\} \end{aligned}$$

Out[\*]=

$$\begin{aligned} & - \frac{4 \gamma^3 p1 \wedge p2}{s (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} + \\ & \frac{4 \gamma^3 p1 \wedge p2}{s (i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} + \\ & \frac{8 m2^2 \gamma^2 p1 \wedge q}{s (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \frac{8 m2^2 \gamma^2 p1 \wedge q}{s (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} \end{aligned}$$

In[\*]:= **S1 + S2**

Out[\*]=

$$\begin{aligned}
 & \frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \\
 & \frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} + \\
 & \frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \\
 & \frac{4 \gamma^3 p1 \wedge p2}{S(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \\
 & \frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} + \\
 & \frac{4 \gamma^3 p1 \wedge p2}{S(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} + \\
 & \frac{8 m^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \frac{8 m^2 \gamma^2 p1 \wedge q}{S(i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2}
 \end{aligned}$$

In[\*]:= **R1D1 = Collect[factorRho@{SimplifiedForm[R1D1] /. {l \odot l - 2 l \odot q + q \odot q \rightarrow qml2, -l \odot l + 2 l \odot q - q \odot q \rightarrow -qml2} /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}}, \rho];**

In[\*]:= **R1D1Div = Total@Select[List@@R1D1, Exponent[#, \rho] < -1 &];**

In[\*]:= **R1D1Div = R1D1Div /. {(num\_ a\_) / (Power[den\_, -n\_]) \rightarrow num / (den ^ (-n - 1)) /; ContainsAll[Variables[den], {\epsilon, a}] /; a == l \odot p1 || a == l \odot p2}**

Out[\*]=

$$\begin{aligned}
 & \frac{\frac{2 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \frac{2 \gamma^2 q \odot q l \wedge p1}{qml2 (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{4 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \frac{4 \gamma q \odot q p1 \wedge p2}{qml2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2}}{\rho^3} + \\
 & - \frac{\frac{2 \gamma p1 \wedge q}{qml2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{6 \gamma p1 \wedge q}{qml2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)}}{\rho^2}
 \end{aligned}$$

```
In[*]:= Collect[factorRho[simplifyTerm[R11D1]] /.
  {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2} /. {l → ρ l, ε → ρ ε}], ρ]
```

```
Out[*]=
```

$$\frac{\frac{2 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{4 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2}}{\rho^3} +$$

$$\frac{1}{\rho} \left( \frac{2 \gamma^2 l \odot l \wedge p1}{qml2 (i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} - \frac{4 \gamma^2 l \odot l \wedge p1}{qml2 (i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} + \right.$$

$$\frac{4 \gamma^2 l \odot l \wedge p1}{qml2 (i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \frac{4 \gamma l \odot l p1 \wedge p2}{qml2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)^2} - \frac{4 p1 \wedge p2}{qml2 (-i \epsilon + l \odot p2)} -$$

$$\left. \frac{4 \gamma l \odot l p1 \wedge p2}{qml2 (i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \frac{4 p1 \wedge p2}{qml2 (i \epsilon + l \odot p2)} + \frac{4 \gamma l \odot l p1 \wedge p2}{qml2 (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} \right) +$$

$$- \frac{2 \gamma p1 \wedge q}{qml2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{6 \gamma p1 \wedge q}{qml2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)}$$

$$\rho^2$$

## Second Pair

### R\_2 for the last two diagrams

```
In[*]:= Bx2 = ((2 p1 + h q - h l) ⊗ (2 p2 - h q + h l) (2 p1 + 2 h q - h l) ⊗ (2 p2 - 2 h q + h l)) /
  ((q - l) ⊗ (q - l) (2 p1 ⊗ (q - l) + h (q - l) ⊗ (q - l) + i ε) (2 p2 ⊗ (q - l) - h (q - l) ⊗ (q - l) - i ε));
Cx2 = (2 p1 + 2 h q - h l) ⊗ (2 p2 - h l) (2 p1 + h q - h l) ⊗ (2 p2 - h q - h l)
  (q - l) ⊗ (q - l) (2 p1 ⊗ (q - l) + h (q - l) ⊗ (q - l) + i ε) (2 p2 ⊗ l - h l ⊗ l - i ε);
```

```
B2C2 = Bx2 + Cx2;
```

```
In[*]:= Do[Bx2hbar[n] = Coefficient[Normal[Series[Bx2, {h, 0, 1}]], h, n];
  Cx2hbar[n] = Coefficient[Normal[Series[Cx2, {h, 0, 1}]], h, n], {n, 0, 1};
```

```
In[*]:= Distribute@simplifiedForm@B2xhbar[1]
```

```
Out[*]=
```

$$\frac{16 (p1 \odot p2)^2}{(2 \epsilon + 2 i l \odot p1 - 2 i p1 \odot q)^2 (-2 i \epsilon - 2 l \odot p2 + 2 p2 \odot q)} +$$

$$\frac{\frac{8 p1 \odot p2 (l \odot p1 - l \odot p2 - p1 \odot q + p2 \odot q) + 4 p1 \odot p2 (2 l \odot p1 - 2 l \odot p2 - 4 p1 \odot q + 4 p2 \odot q)}{-2 i \epsilon - 2 l \odot p2 + 2 p2 \odot q} + \frac{16 (p1 \odot p2)^2 (-l \odot l + 2 l \odot q - q \odot q)}{(2 \epsilon - 2 i l \odot p2 + 2 i p2 \odot q)^2}}{(2 i \epsilon - 2 l \odot p1 + 2 p1 \odot q) (l \odot l - 2 l \odot q + q \odot q)}$$

```
In[*]:= Distribute@simplifiedForm@C2xhbar[1]
```

```
Out[*]=
```

$$\frac{8 (p_1 \odot p_2)^2}{(-i \epsilon + l \odot p_2) (2 \epsilon + 2 i l \odot p_1 - 2 i p_1 \odot q)^2} + \frac{4 p_1 \odot p_2 \left( \frac{l \odot p_1 \odot p_2}{(-i \epsilon + l \odot p_2)^2} - \frac{l \odot p_1 + l \odot p_2 - 2 p_2 \odot q}{-i \epsilon + l \odot p_2} \right) - \frac{4 p_1 \odot p_2 (l \odot p_1 + l \odot p_2 + p_1 \odot q - p_2 \odot q)}{-i \epsilon + l \odot p_2}}{(2 i \epsilon - 2 l \odot p_1 + 2 p_1 \odot q) (l \odot l - 2 l \odot q + q \odot q)}$$

```
In[*]:= MaTeX["\\begin{aligned}R_2 = -\\partial_{x_1}(p_1\\wedge q\\,,(B_0+C_0))+\\frac{1}{2}(\\partial_{x_1}^2-\\partial_{x_1}\\partial_{x_2})(q^2p_1\\wedge q\\,,(B_{-1}+C_{-1}))\\\\=p_1\\wedge q\\,,\\left(-\\partial_{x_1}(B_0+C_0)+\\frac{1}{2}\\partial_{x_1}^2(q^2(B_{-1}+C_{-1}))-\\partial_{x_1}\\partial_{x_2}(q^2(B_{-1}+C_{-1}))\\right)\\\\+\\partial_{x_1}(p_1\\wedge q)\\left(-(B_0+C_0)+\\partial_{x_1}(q^2(B_{-1}+C_{-1}))-\\partial_{x_2}(q^2(B_{-1}+C_{-1}))\\right)\\\\-\\frac{1}{2}\\partial_{x_2}(p_1\\wedge q)\\partial_{x_1}(q^2(B_{-1}+C_{-1}))\\end{aligned}"]
```

```
Out[*]=
```

$$\begin{aligned} R_2 &= -\partial_{x_1}(p_1 \wedge q (B_0 + C_0)) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1} \partial_{x_2})(q^2 p_1 \wedge q (B_{-1} + C_{-1})) \\ &= p_1 \wedge q \left( -\partial_{x_1}(B_0 + C_0) + \frac{1}{2} \partial_{x_1}^2 (q^2 (B_{-1} + C_{-1})) - \frac{1}{2} \partial_{x_1} \partial_{x_2} (q^2 (B_{-1} + C_{-1})) \right) \\ &\quad + \partial_{x_1}(p_1 \wedge q) \left( -(B_0 + C_0) + \partial_{x_1}(q^2 (B_{-1} + C_{-1})) - \frac{1}{2} \partial_{x_2}(q^2 (B_{-1} + C_{-1})) \right) \\ &\quad - \frac{1}{2} \partial_{x_2}(p_1 \wedge q) \partial_{x_1}(q^2 (B_{-1} + C_{-1})) \end{aligned}$$

```
R21D2 = -simplifiedForm[onShellP[
  D[Wedge[p1, q]*Expand[(Bx2hbar[1]+Cx2hbar[1])/. {l \odot l - 2 l \odot q + q \odot q \to qml2,
    -l \odot l + 2 l \odot q - q \odot q \to -qml2}]/. qml2 \to l \odot l - 2 l \odot q + q \odot q), x1]]];
```

```
R22D2 = D[q \odot q \times Wedge[p1, q]* (Bx2hbar[0]+Cx2hbar[0]), x1];
```

```
workspace = D[R22D2, x2];
```

```
R22D2 = D[R22D2, x1] - workspace;
```

```
R22D2 = \frac{1}{2} simplifiedForm[onShellP[R22D2]];
```

```
ClearAll[workspace];
```

```
In[*]:= R2D2 = R21D2 + R22D2;
```

```
In[*]:= R2D2 = Collect[
  factorRho@ (R2D2 /. {l \odot l - 2 l \odot q + q \odot q \to qml2, -l \odot l + 2 l \odot q - q \odot q \to -qml2}) /.
  {l \to \rho l, \epsilon \to \rho \epsilon}, \rho];
```

```
In[*]:= R2D2Div = Total@Select[List@@R2D2, Exponent[#, ρ] < -1 &];
```

```
In[*]:= R2D2Div = R2D2Div /. { (num_ a_) / (Power[den_, -n_]) =>
  num / (den ^ (-n - 1)) /; ContainsAll[Variables[den], {ε, a}] /; a == l ⊗ p1 || a == l ⊗ p2};
```

```
In[*]:= Coefficient[R1D2Div + R2D2Div, ρ, -4]
```

```
Out[*]:=
```

$$\frac{8 \gamma^2 q \otimes q p1 \wedge q}{q m l 2 (-i \epsilon + l \otimes p1)^3 (-i \epsilon + l \otimes p2)} +$$

$$\frac{4 \gamma^2 q \otimes q p1 \wedge q}{q m l 2 (-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)^2} - \frac{8 \gamma^2 q \otimes q p1 \wedge q}{q m l 2 (-i \epsilon + l \otimes p1)^3 (i \epsilon + l \otimes p2)}$$

```
In[*]:= Collect[FactorRho[SimplifyTerm[Expand[R1D2 /.
```

```
{l ⊗ l - 2 l ⊗ q + q ⊗ q → q m l 2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -q m l 2}]] /. {l → ρ l, ε → ρ ε}], ρ]
```

```
Out[*]:=
```

$$-\frac{2 \gamma^2 l \otimes l p1}{q m l 2 (-i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)^2} - \frac{4 \gamma l \otimes l p1 p2}{q m l 2 (-i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)^2} + \frac{4 p1 p2}{q m l 2 (-i \epsilon + l \otimes p2)} + \frac{4 p1 p2}{q m l 2 (i \epsilon + l \otimes p2)}$$

$$\frac{1}{\rho} +$$

$$\frac{1}{\rho^3} \left( -\frac{4 \gamma^2 l \wedge p1}{(-i \epsilon + l \otimes p1)^3 (-i \epsilon + l \otimes p2)} - \frac{2 \gamma^2 l \wedge p1}{(-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)^2} + \frac{4 \gamma^2 l \wedge p1}{(-i \epsilon + l \otimes p1)^3 (i \epsilon + l \otimes p2)} - \right.$$

$$\left. \frac{4 \gamma p1 \wedge p2}{(-i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)} - \frac{4 \gamma p1 \wedge p2}{(-i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2} + \frac{4 \gamma p1 \wedge p2}{(-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)} \right) +$$

$$-\frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \otimes p1)^3 (-i \epsilon + l \otimes p2)} - \frac{2 \gamma^2 p1 \wedge q}{(-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)^2} + \frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \otimes p1)^3 (i \epsilon + l \otimes p2)}$$

$$\frac{1}{\rho^4} +$$

$$-\frac{2 \gamma^2 l \otimes l p1 \wedge q}{q m l 2 (-i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)^2} + \frac{6 \gamma p1 \wedge q}{q m l 2 (-i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)} - \frac{2 \gamma p1 \wedge q}{q m l 2 (-i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)}$$

$$\frac{1}{\rho^2}$$

```
In[*]:= R22D2 = 1/2 (D[D[q ⊗ q × Wedge[p1, q] * (Bx2hbar[0] + Cx2hbar[0]), x1], x1] -
```

```
D[D[q ⊗ q × Wedge[p1, q] * (Bx2hbar[0] + Cx2hbar[0]), x2], x1]);
```

```
In[*]:= R22D2 = simplifiedForm[onShellP[R22D2]];]
```

```
In[*]:= R22 = R22D1 + R22D2;
```

## R1 for the last two diagrams

```
In[*]:= MaTeX["\\begin{aligned}R_1= (p_1\\wedge
```

$$\\partial_{p_1})(B_0+C_0)-\\frac{1}{2}(\\partial_{x_1}-\\partial_{x_2})(q^2(p_1\\wedge\\partial_{p_1})(B_{-1}+C_{-1}))\\end{aligned}"]$$

Out[\*]=

$$R_1 = (p_1 \wedge \partial_{p_1})(B_0 + C_0) - \frac{1}{2}(\partial_{x_1} - \partial_{x_2})(q^2(p_1 \wedge \partial_{p_1})(B_{-1} + C_{-1}))$$

```
In[*]:= B2xhbar[1]
```

Out[\*]=

$$\frac{\frac{8 p_1 p_2 (\log p_1 - \log p_2 - p_1 \log p_2 \log q + 4 p_1 p_2 (2 \log p_1 - 2 \log p_2 - 4 p_1 \log p_2 \log q + 4 p_2 \log q))}{-i \epsilon - 2 \log p_2 + 2 p_2 \log q} + \frac{16 (p_1 p_2)^2 (-\log p_1 + 2 \log q - q \log q)}{(\epsilon - 2 i \log p_2 + 2 i p_2 \log q)^2}}{i \epsilon - 2 \log p_1 + 2 p_1 \log q} + \frac{16 (p_1 p_2)^2 (\log p_1 - 2 \log q + q \log q)}{(\epsilon + 2 i \log p_1 - 2 i p_1 \log q)^2 (-i \epsilon - 2 \log p_2 + 2 p_2 \log q)}$$

$$\log p_1 - 2 \log q + q \log q$$

```
(*R11D2 = aWedgeDb[B2xhbar[1]+C2xhbar[1], p1,p1];
```

```
R12D2 = -1/2 D[q \odot q \times aWedgeDb[B2xhbar[0]+C2xhbar[0], p1,p1], x1] +
```

$$\frac{1}{2} D[q \odot q \times aWedgeDb[B2xhbar[0]+C2xhbar[0], p1,p1], x2]; *)$$

```
In[*]:= R11D2 = aWedgeDb[Expand[(Bx2hbar[1]+Cx2hbar[1]) /. {log p1 - 2 log q + q log q -> qml2,
```

$$-\log p_1 + 2 \log q - q \log q \rightarrow -q m l^2}]] /. \{q m l^2 \rightarrow \log p_1 - 2 \log q + q \log q\}, p1, p1];$$

```
R12D2 = -1/2 D[q \odot q \times aWedgeDb[Bx2hbar[0]+Cx2hbar[0], p1, p1], x1] +
```

$$\frac{1}{2} D[q \odot q \times aWedgeDb[Bx2hbar[0]+Cx2hbar[0], p1, p1], x2];$$

```
R11D2 = onShellP[R11D2];
```

```
R12D2 = onShellP[R12D2];
```

```
R11D2 = simplifiedForm@R11D2;
```

```
R12D2 = simplifiedForm@R12D2;
```

```
R1D2 = R11D2 + R12D2;
```

```
In[*]:= Coefficient[Collect[FactorRho@
```

$$(\text{simplifyTerm}[\text{Expand}[\text{simplifiedForm}[R11D2]]] /. \{\log p_1 - 2 \log q + q \log q \rightarrow q m l^2, \\ -\log p_1 + 2 \log q - q \log q \rightarrow -q m l^2\} /. \{l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon\}), \rho], \rho, -4]$$

Out[\*]=

$$-\frac{4 \gamma^2 p_1 \wedge q}{(-2 i \epsilon + \log p_1)^3 (-2 i \epsilon + \log p_2)} - \frac{2 \gamma^2 p_1 \wedge q}{(-2 i \epsilon + \log p_1)^2 (2 i \epsilon + \log p_2)^2} + \frac{4 \gamma^2 p_1 \wedge q}{(-2 i \epsilon + \log p_1)^3 (2 i \epsilon + \log p_2)}$$

```
In[*]:= R1D2 = simplifiedForm@R1D2;
```



```
In[*]:= R1D2 = Collect[
  factorRho@{R1D2 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2} /.
    {l → ρ l, ε → ρ ε}}, ρ];
```

```
In[*]:= R1D2Div = Total@Select[List@@R1D2, Exponent[#, ρ] < -1 &];
```

```
R1D2Div = R1D2Div /. {(num_ a_) / (Power[den_, -n_] =>
  num / (den ^ (-n - 1)) /; ContainsAll[Variables[den], {ε, a}] /; a == l ⊗ p1 || a == l ⊗ p2};
```

```
In[*]:= - (4 m2^2 γ^2 l ⊗ p2 q ⊗ q p1 ∧ q) / (qml2^2 S(-i ε + l ⊗ p1)^2 (-i ε + l ⊗ p2)) /. {(num_ a_) / (Power[den_, -n_] =>
  num / (den ^ (-n - 1)) /; ContainsAll[Variables[den], {ε, a}] /; a == l ⊗ p1 || a == l ⊗ p2}
```

```
Out[*]:=
```

$$-\frac{4 m^2 \gamma^2 q \otimes q p_1 \wedge q}{q m l^2 S(-i \epsilon + l \otimes p_1)^2}$$

```
In[*]:= Coefficient[R1D2Div + R2D2Div, ρ, -2]
```

```
Out[*]:=
```

$$\frac{4 \gamma^3 p_1 \wedge q}{q m l^2 S(-i \epsilon + l \otimes p_1)(-i \epsilon + l \otimes p_2)} - \frac{4 \gamma^3 p_1 \wedge q}{q m l^2 S(-i \epsilon + l \otimes p_1)(i \epsilon + l \otimes p_2)} -$$

$$\frac{4 \gamma^3 q \otimes q p_1 \wedge q}{q m l^2 S(-i \epsilon + l \otimes p_1)(-i \epsilon + l \otimes p_2)} + \frac{4 m^2 \gamma^2 q \otimes q p_1 \wedge q}{q m l^2 S(i \epsilon + l \otimes p_2)^2}$$

```
In[*]:= RD2LogDiv =
```

$$\frac{8 \pi i \gamma^3 p_1 \wedge q \delta[l \otimes p_2]}{q m l^2 S(-i \epsilon + l \otimes p_1)} - \frac{4 \gamma^3 q \otimes q p_1 \wedge q}{q m l^2 S(-i \epsilon + l \otimes p_1)(-i \epsilon + l \otimes p_2)} + \frac{4 m^2 \gamma^2 q \otimes q p_1 \wedge q}{q m l^2 S(i \epsilon + l \otimes p_2)^2};$$

```
In[*]:= S2
```

```
Out[*]:=
```

$$-\frac{4 \gamma^3 p_1 \wedge p_2}{S(i \epsilon + l \otimes p_1)(-i \epsilon + l \otimes p_2)(l \otimes l - 2 l \otimes q + q \otimes q)} +$$

$$\frac{4 \gamma^3 p_1 \wedge p_2}{S(i \epsilon + l \otimes p_1)(i \epsilon + l \otimes p_2)(l \otimes l - 2 l \otimes q + q \otimes q)} +$$

$$\frac{8 m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \otimes p_2)(l \otimes l - 2 l \otimes q + q \otimes q)^2} - \frac{8 m^2 \gamma^2 p_1 \wedge q}{S(i \epsilon + l \otimes p_2)(l \otimes l - 2 l \otimes q + q \otimes q)^2}$$

```
In[*]:= S3 = simplifiedForm@onShellP[aWedgeDb[B2xhbar[0]+C2xhbar[0], p1, p1]]
```

```
Out[*]:=
```

$$\begin{aligned}
& -\frac{8 \gamma p1 \wedge p2}{(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)} + \\
& \frac{8 \gamma p1 \wedge p2}{(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)} - \\
& \frac{2 i \gamma^2 (-2 i l \wedge p1 - 2 i p1 \wedge q)}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)} + \\
& \frac{2 \gamma^2 (2 l \wedge p1 + 2 p1 \wedge q)}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)}
\end{aligned}$$

```
In[*]:= S4 = -Expand[simplifiedForm@onShellP[D[Wedge[p1, q] * (B2xhbar[0] + C2xhbar[0]), x1]]] /.
  { {num_ a_} / (Power[den_, -n_]) ->
    num / (den ^ (-n - 1)) }; ContainsAll[Variables[den], {epsilon, a}] /; a == l \odot p1 || a == l \odot p2 }
```

```
In[*]:= Expand[S1 + S2 + S3 + S4]
```

```
In[*]:= simplifyTerm[Total@
  Select[List @@ Expand[R12D1 + R12D2], ContainsAll[List @@ Numerator[##], {p1 \wedge q}] &]]
```

```
Out[*]:=
```

$$\begin{aligned}
& -\frac{4 m^2 \gamma^2 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \frac{4 \gamma^3 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} + \\
& \frac{4 m^2 \gamma^2 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} + \\
& \frac{4 \gamma^3 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} + \\
& \frac{4 \gamma^2 q \odot q p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)} + \\
& \frac{2 \gamma^2 q \odot q p1 \wedge q}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 (l \odot l - 2 l \odot q + q \odot q)} - \\
& \frac{4 \gamma^2 q \odot q p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)}
\end{aligned}$$

In[\*]:= **simplifyTerm**[Total@

**Select**[List@@Expand[R22D1+R22D2], ContainsAll[List@@Numerator[**#**], {p1∧q}] &]]

Out[\*]=

$$\begin{aligned}
& \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{S(-i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)^2} + \\
& \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)^2} + \\
& \frac{4 \gamma^3 q \odot q p_1 \wedge q}{S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{S(-i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \frac{4 \gamma^3 q \odot q p_1 \wedge q}{S(-i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \frac{4 m^2 \gamma^2 q \odot q p_1 \wedge q}{S(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)^2} + \\
& \frac{4 m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)} + \\
& \frac{4 \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)} - \\
& \frac{4 m^2 \gamma^2 p_1 \wedge q}{S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)} - \\
& \frac{4 \gamma^3 p_1 \wedge q}{S(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)} - \\
& \frac{4 m^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)} - \\
& \frac{4 \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)} + \\
& \frac{4 m^2 \gamma^2 p_1 \wedge q}{S(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)} + \\
& \frac{4 \gamma^3 p_1 \wedge q}{S(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)} -
\end{aligned}$$

$$\begin{aligned}
& \frac{4 \gamma^2 q \odot q p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \\
& \frac{2 \gamma^2 q \odot q p1 \wedge q}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 (l \odot l - 2 l \odot q + q \odot q)} + \\
& \frac{4 \gamma^2 q \odot q p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)}
\end{aligned}$$

## R\_2 for first two diagrams

$$\begin{aligned}
In[*]:= Bx &= \frac{(2 p1 + \hbar l) \odot (2 p2 - \hbar l) (2 p1 + \hbar q + \hbar l) \odot (2 p2 - \hbar q - \hbar l)}{(q - l) \odot (q - l) (2 p1 \odot l + \hbar l \odot l + i \epsilon) (2 p2 \odot l - \hbar l \odot l - i \epsilon)}; \\
Cx &= \frac{(2 p1 + \hbar q + \hbar l) \odot (2 p2 - \hbar q + \hbar l) (2 p1 + \hbar l) \odot (2 p2 - 2 \hbar q + \hbar l)}{(q - l) \odot (q - l) (2 p1 \odot l + \hbar l \odot l + i \epsilon) (2 p2 \odot (q - l) - \hbar (q - l) \odot (q - l) - i \epsilon)};
\end{aligned}$$

$$In[*]:= BC = Bx + Cx;$$

$$\begin{aligned}
In[*]:= Do[Bxhbar[n] &= Coefficient[Normal[Series[Bx, {\hbar, 0, 1}]]], \hbar, n]; \\
Cxhbar[n] &= Coefficient[Normal[Series[Cx, {\hbar, 0, 1}]]], \hbar, n], {n, 0, 1}];
\end{aligned}$$

$$In[*]:= R21 = -D[Bxhbar[1] + Cxhbar[1], x1];$$

$$R22 = \frac{1}{2} D[q \odot q \times (Bxhbar[0] + Cxhbar[0]), \{x1, 2\}];$$

$$R23 = \frac{-1}{2} D[q \odot q \times (Bxhbar[0] + Cxhbar[0]), \{x1, 1\}, \{x2, 1\}];$$

$$In[*]:= R21 = onShellP[R21];$$

$$R22 = onShellP[R22];$$

$$R23 = onShellP[R23];$$

$$In[*]:= Rpwq = R21 + R22 + R23;$$

$$In[*]:= factorRho[exp_] :=$$

$$\begin{aligned}
& \text{exp} /. \{(expr\_Plus)^(n\_)\rightarrow \text{Factor}[expr]^\wedge n /; \text{ContainsAll}[\text{Variables}[expr], \{\epsilon, l \odot p1\}]\} \\
& \text{ContainsAll}[\text{Variables}[expr], \{\epsilon, l \odot p2\}]\}
\end{aligned}$$

$$In[*]:= Coefficient[factorRho@$$

$$\text{Expand}[\text{simplifiedForm}[Rpwq] /. \{l \odot l - 2 l \odot q + q \odot q \rightarrow qml2\} /. \{l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon\}], \rho, -2]$$

In[\*]:= R2D1Div =

$$\frac{\text{Wedge}[p1, q]}{qm l^2} \left( \frac{2 \gamma}{(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{6 \gamma}{(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} - \frac{8 \pi i \gamma^3 \delta[l \odot p2]}{S(i \epsilon + l \odot p1)} \right) +$$

$$\frac{\text{Wedge}[p1, q] q \odot q}{S q m l^2} \left( \frac{4 \gamma^3}{(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{4 m^2 \gamma^2}{(i \epsilon + l \odot p2)^2} + \frac{8 \pi i m^2 \gamma^2 \delta[l \odot p2]}{(i \epsilon + l \odot p1)} \right);$$

## R\_1 for the first two diagrams

In[\*]:= R11 = aWedgeDb[Bxhbar[1]+Cxhbar[1], p1, p1];

$$R12 = \frac{-1}{2} D[q \odot q \times aWedgeDb[Bxhbar[0]+Cxhbar[0], p1, p1], x1] +$$

$$\frac{1}{2} D[q \odot q \times aWedgeDb[Bxhbar[0]+Cxhbar[0], p1, p1], x2];$$

In[\*]:= R11 = onShellP[R11];

R12 = onShellP[R12];

In[\*]:= R1D1Log = Coefficient[factorRho@Expand[  
simplifiedForm[R11+R12]/. {l \odot l - 2 l \odot q + q \odot q \rightarrow q m l^2}/. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho, -2]

Out[\*]=

$$-\frac{2 \gamma p1 \wedge q}{qm l^2 (i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \frac{6 \gamma p1 \wedge q}{qm l^2 (i \epsilon + l \odot p1)(i \epsilon + l \odot p2)}$$

In[\*]:= RD1LogDiv = Expand[R1D1Log+R2D1Div]

Out[\*]=

$$\frac{4 \gamma^3 q \odot q p1 \wedge q}{qm l^2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{4 m^2 \gamma^2 q \odot q p1 \wedge q}{qm l^2 S(i \epsilon + l \odot p2)^2} -$$

$$\frac{8 i \pi \gamma^3 p1 \wedge q \delta[l \odot p2]}{qm l^2 S(i \epsilon + l \odot p1)} + \frac{8 i m^2 \pi \gamma^2 q \odot q p1 \wedge q \delta[l \odot p2]}{qm l^2 S(i \epsilon + l \odot p1)}$$

In[\*]:= RD1LogDiv + RD2LogDiv

$$-\frac{8 \pi i \gamma^3 q \odot q p1 \wedge q \delta[l \odot p1]}{qm l^2 S(-i \epsilon + l \odot p1)} + \frac{8 i \pi \gamma^3 p1 \wedge q \delta[l \odot p2]}{qm l^2 S(-i \epsilon + l \odot p1)} -$$

$$\frac{8 i \pi \gamma^3 p1 \wedge q \delta[l \odot p2]}{qm l^2 S(i \epsilon + l \odot p1)} + \frac{8 i m^2 \pi \gamma^2 q \odot q p1 \wedge q \delta[l \odot p2]}{qm l^2 S(i \epsilon + l \odot p1)}$$

```
In[*]:= Coefficient[Collect[
  factorRho@{R1D2 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qm12, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qm12} /.
    {l → ρ l, ε → ρ ε}}, ρ], ρ, -4]
```

$$\begin{aligned}
 R1D2 \rightarrow & -\frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \otimes p1)^3 (-i \epsilon + l \otimes p2)} - \frac{2 \gamma^2 p1 \wedge q}{(-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)^2} + \\
 & \frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \otimes p1)^3 (i \epsilon + l \otimes p2)} + \frac{4 \gamma^2 q \otimes q p1 \wedge q}{qm12 (-i \epsilon + l \otimes p1)^3 (-i \epsilon + l \otimes p2)} + \\
 & \frac{2 \gamma^2 q \otimes q p1 \wedge q}{qm12 (-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)^2} - \frac{4 \gamma^2 q \otimes q p1 \wedge q}{qm12 (-i \epsilon + l \otimes p1)^3 (i \epsilon + l \otimes p2)} \\
 R2D2 \rightarrow & \frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \otimes p1)^3 (-i \epsilon + l \otimes p2)} + \frac{2 \gamma^2 p1 \wedge q}{(-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)^2} - \\
 & \frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \otimes p1)^3 (i \epsilon + l \otimes p2)} - \frac{4 \gamma^2 q \otimes q p1 \wedge q}{qm12 (-i \epsilon + l \otimes p1)^3 (-i \epsilon + l \otimes p2)} - \\
 & \frac{2 \gamma^2 q \otimes q p1 \wedge q}{qm12 (-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)^2} + \frac{4 \gamma^2 q \otimes q p1 \wedge q}{qm12 (-i \epsilon + l \otimes p1)^3 (i \epsilon + l \otimes p2)}
 \end{aligned}$$

```
In[*]:= Normal[Series[1 / (l ⊗ l - 2 l ⊗ q + q ⊗ q) /. {l → ρ l}, {ρ, 0, 2}]]
```

```
Out[*]=
```

$$\frac{2 \rho l \otimes q}{(q \otimes q)^2} + \frac{1}{q \otimes q} + \frac{\rho^2 (4 (l \otimes q)^2 - l \otimes l q \otimes q)}{(q \otimes q)^3}$$

```
wsp = q ⊗ q * aWedgeDb[(Bx1hbar[0] + Bx2hbar[0] + Cx1hbar[0] + Cx2hbar[0]), p1, p1];
```

```
t1 = Expand[simplifiedForm[onShellP[D[wsp, x1] - D[wsp, x2]]];
```

```
ClearAll[wsp];
```

```
wsp = List@@t1;
```

```
ClearAll[t1];
```

```
t1 = {};
```

```
Do[AppendTo[t1, simplifyTerm[wsp[[i]]], {i, 1, Length[wsp]}];
```

```
t1 = Total[t1];
```

```
ClearAll[wsp];
```

```
In[*]:= wsp = q ⊗ q * Wedge[p1, q] * (Bx1hbar[0] + Bx2hbar[0] + Cx1hbar[0] + Cx2hbar[0]);
```

```
t2 = Expand[simplifiedForm[onShellP[D[D[wsp, x1] - D[wsp, x2], x1]]];
ClearAll[wsp];
wsp = List @@ t2;
ClearAll[t2];
t2 = {};
Do[AppendTo[t2, simplifyTerm[wsp[[i]]], {i, 1, Length[wsp]}];
t2 = Total[t2];
ClearAll[wsp];
```

```
In[*]:= t1 = factorRho[t1 /. {l → ρ l, ε → ρ ε}];
t2 = factorRho[t2 /. {l → ρ l, ε → ρ ε}];
```

```
In[*]:= t1 = t1 /. {(ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q) →
  1 / Normal[Series[1 / (ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), {ρ, 0, 2}]]];
t2 = t2 /.
  {(ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q) → 1 / Normal[Series[1 / (ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), {ρ, 0, 2}]]];
```

```
In[*]:= Coefficient[Expand[t1], ρ, -2] - Coefficient[Expand[t2], ρ, -2]
```

```
Out[*]:=
```

$$\begin{aligned}
& -\frac{16 \gamma^2 l \otimes q l \wedge p1}{(-i \epsilon + l \otimes p1)^3 (-i \epsilon + l \otimes p2) q \otimes q} - \frac{8 \gamma^2 l \otimes q l \wedge p1}{(-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)^2 q \otimes q} + \\
& \frac{8 \gamma^2 l \otimes q l \wedge p1}{(i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)^2 q \otimes q} + \frac{16 \gamma^2 l \otimes q l \wedge p1}{(-i \epsilon + l \otimes p1)^3 (i \epsilon + l \otimes p2) q \otimes q} - \\
& \frac{16 \gamma l \otimes q p1 \wedge p2}{(-i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2) q \otimes q} + \frac{8 \gamma^3 l \otimes q p1 \wedge p2}{S(-i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2) q \otimes q} - \\
& \frac{16 \gamma l \otimes q p1 \wedge p2}{(-i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2 q \otimes q} + \frac{8 \gamma^3 l \otimes q p1 \wedge p2}{S(-i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2 q \otimes q} + \\
& \frac{16 \gamma l \otimes q p1 \wedge p2}{(i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2 q \otimes q} - \frac{8 \gamma^3 l \otimes q p1 \wedge p2}{S(i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2 q \otimes q} + \\
& \frac{16 \gamma l \otimes q p1 \wedge p2}{(-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2) q \otimes q} - \frac{8 \gamma^3 l \otimes q p1 \wedge p2}{S(-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2) q \otimes q} - \\
& \frac{8 m^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2) q \otimes q} + \frac{8 m^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2) q \otimes q} + \\
& \frac{8 \gamma^3 p1 \wedge q}{S(-i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2) q \otimes q} - \frac{8 \gamma^3 p1 \wedge q}{S(i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2) q \otimes q}
\end{aligned}$$

```

In[*]:= s1 = simplifyTerm@Expand[simplifiedForm[
  onShellP[aWedgeDb[(Bx1hbar[1] + Bx2hbar[1] + Cx1hbar[1] + Cx2hbar[1]), p1, p1]]];
s2 = simplifyTerm@Expand[simplifiedForm[
  onShellP[D[Wedge[p1, q] * (Bx1hbar[1] + Bx2hbar[1] + Cx1hbar[1] + Cx2hbar[1]), x1]]];

In[*]:= s1 = factorRho[s1 /. {l → ρ l, ε → ρ ε}];
s2 = factorRho[s2 /. {l → ρ l, ε → ρ ε}];

In[*]:= s1 = s1 /. {(ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q) →
  1 / Normal[Series[1 / (ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), {ρ, 0, 3}]]];
s2 = s2 /.
  {(ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q) → 1 / Normal[Series[1 / (ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), {ρ, 0, 3}]]];

In[*]:= Coefficient[Expand[s1], ρ, -2]
Out[*]=

$$\begin{aligned}
& -\frac{2 \gamma^2 l \otimes l p_1 \wedge q}{(-i \epsilon + l \otimes p_1)^2 (-i \epsilon + l \otimes p_2)^2 q \otimes q} + \frac{6 \gamma p_1 \wedge q}{(-i \epsilon + l \otimes p_1) (-i \epsilon + l \otimes p_2) q \otimes q} - \\
& \frac{2 \gamma p_1 \wedge q}{(i \epsilon + l \otimes p_1) (-i \epsilon + l \otimes p_2) q \otimes q} - \frac{2 \gamma p_1 \wedge q}{(-i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2) q \otimes q} + \frac{6 \gamma p_1 \wedge q}{(i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2) q \otimes q}
\end{aligned}$$


In[*]:= Coefficient[Expand[s2], ρ, -2]
Out[*]=

$$\begin{aligned}
& -\frac{2 \gamma^2 l \otimes l p_1 \wedge q}{(-i \epsilon + l \otimes p_1)^2 (-i \epsilon + l \otimes p_2)^2 q \otimes q} + \frac{6 \gamma p_1 \wedge q}{(-i \epsilon + l \otimes p_1) (-i \epsilon + l \otimes p_2) q \otimes q} - \\
& \frac{2 \gamma p_1 \wedge q}{(i \epsilon + l \otimes p_1) (-i \epsilon + l \otimes p_2) q \otimes q} - \frac{2 \gamma p_1 \wedge q}{(-i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2) q \otimes q} + \frac{6 \gamma p_1 \wedge q}{(i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2) q \otimes q}
\end{aligned}$$


In[*]:= sum = 0;
Do[sum += L[i] × Cf[i] (A0hbarl[i][0, 0] + A0hbarl[i][0, 1] + A0hbarl[i][0, 2] +
  ħ A0hbarl[i][1, 0] + ħ A0hbarl[i][1, 1] + ħ A0hbarl[i][1, 2]), {i, 1, 4}]

In[*]:= sum = 0;
Do[sum += L[i] × Cf[i] × A0[i], {i, 1, 4}]

In[*]:= Expand[simplifyTerm[Expand[simplifiedForm[sum]]] /. {(ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q) →
  1 / Normal[Series[1 / (ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), {ρ, 0, 2}]]} /. {ρ → 1}]

```



```

In[*]:= simplifyTerm[
  Expand[Expand[simplifiedForm[Bx1hbar[1] + Bx2hbar[0] + Cx1hbar[0] + Cx2hbar[1]]] /.
    {l → ρ l, ε → ρ ε} /. {Power[(ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), -1] →
      1 / Normal[Series[1 / (ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), {ρ, 0, 2}]]}] /. {ρ → 1}] -
  simplifyTerm[simplifiedForm[Expand[sum]]]

In[*]:= simplifyTerm[simplifiedForm[Expand[sum]]]

In[*]:= a = simplifyTerm[
  simplifiedForm[factorRho[Expand[Expand[Bx1hbar[1]]] /. {l → ρ l, ε → ρ ε} /.
    {Power[(ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), -1] → Normal[Series[1 /
      (ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), {ρ, 0, 2}]], Power[(ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q), -2] →
      Normal[Series[1 / (ρ² l ⊗ l - 2 ρ l ⊗ q + q ⊗ q)², {ρ, 0, 2}]]]}]]] /. {ρ → 1}];

In[*]:= b = simplifyTerm[simplifiedForm[
  Expand[(L[i] × Cf[i] (A0hbarl[i][0, 0] + A0hbarl[i][0, 1] + A0hbarl[i][0, 2] +
    ħ A0hbarl[i][1, 0] + ħ A0hbarl[i][1, 1] + ħ A0hbarl[i][1, 2]) /. {i → 1})]]];

In[*]:= Length[a]
Length[Coefficient[b, ħ, 1]]
Length[factorRho@Expand[ĥ a] - Coefficient[b, ħ, 1]] /. p1 ⊗ p2 → γ

Out[*]:=
24

Out[*]:=
28

Out[*]:=
4

In[*]:= (Expand[ĥ a] - Coefficient[b, ħ, 0]) /. p1 ⊗ p2 → γ
Out[*]:=
0

In[*]:= factorRho[Expand[ĥ a] /. p1 ⊗ p2 → γ /. {l → ρ l, ε → ρ ε}]

In[*]:= Complement[List @@ factorRho[Coefficient[b, ħ, 1] /. p1 ⊗ p2 → γ /. {l → ρ l, ε → ρ ε}],
  List @@ factorRho[Expand[ĥ a] /. p1 ⊗ p2 → γ /. {l → ρ l, ε → ρ ε}]]

Out[*]:=

$$\left\{ \frac{8 i \gamma \rho (l \otimes q)^2}{(i \epsilon + l \otimes p1)(q \otimes q)^3}, -\frac{8 i \gamma \rho (l \otimes q)^2}{(-i \epsilon + l \otimes p2)(q \otimes q)^3}, -\frac{2 i \gamma \rho l \otimes l}{(i \epsilon + l \otimes p1)(q \otimes q)^2}, \frac{2 i \gamma \rho l \otimes l}{(-i \epsilon + l \otimes p2)(q \otimes q)^2} \right\}$$


```

```
In[*]:= Complement[List @@ factorRho[Expand[i a] /. p1 ⊗ p2 → γ /. {l → ρ l, ε → ρ ε}],
  List @@ factorRho[Coefficient[b, ħ, 1] /. p1 ⊗ p2 → γ /. {l → ρ l, ε → ρ ε}]]
```

```
Out[*]=
```

$$\left\{ \frac{16 i \gamma \rho (l \otimes q)^2}{(i \epsilon + l \otimes p1)(q \otimes q)^3}, -\frac{16 i \gamma \rho (l \otimes q)^2}{(-i \epsilon + l \otimes p2)(q \otimes q)^3}, -\frac{4 i \gamma \rho l \otimes l}{(i \epsilon + l \otimes p1)(q \otimes q)^2}, \frac{4 i \gamma \rho l \otimes l}{(-i \epsilon + l \otimes p2)(q \otimes q)^2} \right\}$$

```
In[*]:= Coefficient[Expand[simplifyTerm[Expand[L[1] × Cf[1]]] (A0ħbar[1][0] + ħ A0ħbar[1][1])], ħ, 1]
```

```
In[*]:= Expand[L[1] × Cf[1]]
```

```
Out[*]=
```

$$\begin{aligned} & \frac{\gamma \hbar l \otimes l}{2 (i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)^2} - \frac{\gamma \hbar l \otimes l}{2 (i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)} + \\ & \frac{\gamma}{(i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)} - \frac{\hbar l \otimes p1}{2 (i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)} + \frac{\hbar l \otimes p2}{2 (i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)} \end{aligned}$$

## One Loop: Calculation 3

```
In[*]:= MaTeX[
```

```
"\\begin{aligned}Q_1&=J_1|_{O(l^{-2}),O(\hbar^0)}, \\Q_2&=J_1|_{O(l^{-2}),O(\hbar)}, \\Q_3&=J_2|_{O(l^{-2}),O(\hbar^0)}, \\Q_4&=J_2|_{O(l^{-2}),O(\hbar)}, \\Q_5&=J_3|_{O(l^{-2}),O(\hbar^0)}, \\Q_6&=J_3|_{O(l^{-2}),O(\hbar)}, \\Q_7&=J_1|_{O(l^{-3}),O(\hbar^0)}, \\Q_8&=J_1|_{O(l^{-3}),O(\hbar)}, \\Q_9&=J_2|_{O(l^{-3}),O(\hbar^0)}, \\Q_{10}&=J_2|_{O(l^{-3}),O(\hbar)}, \\Q_{11}&=J_3|_{O(l^{-3}),O(\hbar^0)}, \\Q_{12}&=J_3|_{O(l^{-3}),O(\hbar)}, \\Q_{13}&=J_1|_{O(l^{-4}),O(\hbar^0)}, \\Q_{14}&=J_1|_{O(l^{-4}),O(\hbar)}, \\Q_{15}&=J_2|_{O(l^{-4}),O(\hbar^0)}, \\Q_{16}&=J_2|_{O(l^{-4}),O(\hbar)}, \\Q_{17}&=J_3|_{O(l^{-4}),O(\hbar^0)}, \\Q_{18}&=J_3|_{O(l^{-4}),O(\hbar)}\\end{aligned}"
```

```
Out[*]=
```

$$\begin{aligned} Q_1 &= J_1|_{O(l^{-2}),O(\hbar^0)}, & Q_2 &= J_1|_{O(l^{-2}),O(\hbar)}, \\ Q_3 &= J_2|_{O(l^{-2}),O(\hbar^0)}, \\ Q_4 &= J_3|_{O(l^{-2}),O(\hbar^0)}, & Q_5 &= J_3|_{O(l^{-2}),O(\hbar)}, \\ Q_6 &= J_4|_{O(l^{-2}),O(\hbar^0)}, \\ Q_7 &= J_1|_{O(l^{-3}),O(\hbar^0)}, & Q_8 &= J_1|_{O(l^{-3}),O(\hbar)}, \\ Q_9 &= J_2|_{O(l^{-3}),O(\hbar^0)}, \\ Q_{10} &= J_3|_{O(l^{-3}),O(\hbar^0)}, & Q_{11} &= J_3|_{O(l^{-3}),O(\hbar)}, \\ Q_{12} &= J_4|_{O(l^{-3}),O(\hbar^0)}, \\ Q_{13} &= J_1|_{O(l^{-4}),O(\hbar^0)}, & Q_{14} &= J_1|_{O(l^{-4}),O(\hbar)}, \\ Q_{15} &= J_2|_{O(l^{-4}),O(\hbar^0)}, \\ Q_{16} &= J_3|_{O(l^{-4}),O(\hbar^0)}, & Q_{17} &= J_3|_{O(l^{-4}),O(\hbar)}, \\ Q_{18} &= J_4|_{O(l^{-4}),O(\hbar^0)} \end{aligned}$$

```
In[*]:= MaTeX["J1=\sum_i L_i C_i, \; ; J2=\partial_{\Delta x} \sum_i L_i C_i, \; ; J3=p_1 \wedge \partial_{p_1} \sum_i L_i C_i - p_1 \wedge q \partial_{x_1} \sum_i L_i C_i, \; ; J4=\partial_{\Delta x} (p_1 \wedge \partial_{p_1} \sum_i L_i C_i - p_1 \wedge q \partial_{x_1} \sum_i L_i C_i)"]
```

Out[\*]:=

$$J1 = \sum_i L_i C_i, \quad J2 = \partial_{\Delta x} \sum_i L_i C_i, \quad J3 = p_1 \wedge \partial_{p_1} \sum_i L_i C_i - p_1 \wedge q \partial_{x_1} \sum_i L_i C_i, \quad J4 = \partial_{\Delta x} \left( p_1 \wedge \partial_{p_1} \sum_i L_i C_i - p_1 \wedge q \partial_{x_1} \sum_i L_i C_i \right)$$

```
In[*]:= ClearAll[sum];
sum = 0;
Do[sum += L[i]*Cf[i] /. {y -> p1 \otimes p2}, {i, 1, 4}]
```

```
In[*]:= ClearAll[J1, J2, J3, J4];
J1 = sum;
J2 = delx[J1];
J3 = aWedgeDb[sum, p1, p1] - p1 \wedge q * D[sum, x1];
J4 = delx[J3];
```

```
J1 = simplifiedForm[onShellP[J1]];
J2 = simplifiedForm[onShellP[J2]];
J3 = simplifiedForm[onShellP[J3]];
J4 = simplifiedForm[onShellP[J4]];
```

```
In[*]:= ClearAll[J1lm2, J2lm2, J3lm2, J4lm2];
J1lm2 = extractLog[J1, -2];
J2lm2 = extractLog[J2, -2];
J3lm2 = extractLog[J3, -2];
J4lm2 = extractLog[J4, -2];
```

```
In[*]:= ClearAll[J1lm3, J2lm3, J3lm3, J4lm3];
J1lm3 = extractLog[J1, -3];
J2lm3 = extractLog[J2, -3];
J3lm3 = extractLog[J3, -3];
J4lm3 = extractLog[J4, -3];
```

```
In[*]:= ClearAll[J1lm4, J2lm4, J3lm4, J4lm4];
J1lm4 = extractLog[J1, -4];
J2lm4 = extractLog[J2, -4];
J3lm4 = extractLog[J3, -4];
J4lm4 = extractLog[J4, -4];
```

$$\begin{aligned} R_{(1)} &= \frac{1}{\hbar} \Big( p_1 \wedge \\ & \partial_{p_1} p_1 - \partial_{x_1} (p_1 \wedge \\ & q, A_0) \Big) J_1 - \frac{1}{2} \partial_{\Delta x} \Big( q^2, p_1 \wedge \\ & \partial_{p_1} p_1 - \partial_{x_1} (q^2, p_1 \wedge \\ & q, A_0) \Big) J_1 - \frac{1}{2} \Big( q^2, p_1 \wedge \\ & \partial_{p_1} p_1 - \partial_{x_1} (q^2, p_1 \wedge \\ & q, A_0) \Big) J_2 + \frac{1}{\hbar} A_0 J_3 - \frac{1}{2} \partial_{\Delta x} (q^2, A_0) J_3 - \frac{1}{2} q^2, A_0, J_4 - \frac{1}{\hbar} l^{(1)} \Big( J_3 \partial_{q, A_0} + J_1 (p_1 \partial_{p_1}, \partial_{q A_0} - \partial_{x_1} (p_1 \wedge \\ & q, \partial_{q, A_0}) \Big) \Big) + \frac{1}{2} l^{(1)} \Big( J_1, \partial_{\Delta x} (q^2, p_1 \wedge \partial_{p_1}, \partial_{q A_0} - \partial_{x_1} (q^2, p_1 \wedge \\ & q, \partial_{q, A_0}) + J_2 (q^2, p_1 \wedge \partial_{p_1}, \partial_{q A_0} - \partial_{x_1} (q^2, p_1 \wedge \\ & q, \partial_{q, A_0}) \Big) + J_3 \partial_{\Delta x} (q^2, \partial_{q, A_0} + J_4 q^2, \partial_{q, A_0} \Big) + \frac{1}{2} \hbar l^{(2)} \Big( J_3 \partial_{q^2, A_0} + J_1 (p_1 \partial_{p_1}, \partial_{q^2 A_0} - \partial_{x_1} (p_1 \wedge \\ & q, \partial_{q^2, A_0}) \Big) \Big) - \frac{1}{4} l^{(2)} \Big( J_1, \partial_{\Delta x} (q^2, p_1 \wedge \partial_{p_1}, \partial_{q^2 A_0} - \partial_{x_1} (q^2, p_1 \wedge \\ & q, \partial_{q^2, A_0}) + J_2 (q^2, p_1 \wedge \partial_{p_1}, \partial_{q^2 A_0} - \partial_{x_1} (q^2, p_1 \wedge \\ & q, \partial_{q^2, A_0}) \Big) + J_3 \partial_{\Delta x} (q^2, \partial_{q^2, A_0} + J_4 q^2, \partial_{q^2, A_0} \Big) \end{aligned}$$

Out[•]=

$$\begin{aligned}
R_{(1)} = & \frac{1}{\hbar} \left( p_1 \wedge \partial_{p_1} A_0 - \partial_{x_1} (p_1 \wedge q A_0) \right) J_1 - \frac{1}{2} \partial_{\Delta x} \left( q^2 p_1 \wedge \partial_{p_1} A_0 - \partial_{x_1} (q^2 p_1 \wedge q A_0) \right) J_1 \\
& - \frac{1}{2} \left( q^2 p_1 \wedge \partial_{p_1} A_0 - \partial_{x_1} (q^2 p_1 \wedge q A_0) \right) J_2 \\
& + \frac{1}{\hbar} A_0 J_3 - \frac{1}{2} \partial_{\Delta x} (q^2 A_0) J_3 \\
& - \frac{1}{2} q^2 A_0 J_4 \\
& - \frac{1}{\hbar} l^{(1)} \left[ J_3 \partial_q A_0 + J_1 (p_1 \partial_{p_1} \partial_q A_0 - \partial_{x_1} (p_1 \wedge q \partial_q A_0)) \right] \\
& + \frac{1}{2} l^{(1)} \left[ J_1 \partial_{\Delta x} (q^2 p_1 \wedge \partial_{p_1} \partial_q A_0 - \partial_{x_1} (q^2 p_1 \wedge q \partial_q A_0)) + J_2 (q^2 p_1 \wedge \partial_{p_1} \partial_q A_0 - \partial_{x_1} (q^2 p_1 \wedge q \partial_q A_0)) \right. \\
& \left. + J_3 \partial_{\Delta x} (q^2 \partial_q A_0) + J_4 q^2 \partial_q A_0 \right] \\
& + \frac{1}{2\hbar} l^{(2)} \left[ J_3 \partial_q^2 A_0 + J_1 (p_1 \partial_{p_1} \partial_q^2 A_0 - \partial_{x_1} (p_1 \wedge q \partial_q^2 A_0)) \right] \\
& - \frac{1}{4} l^{(2)} \left[ J_1 \partial_{\Delta x} (q^2 p_1 \wedge \partial_{p_1} \partial_q^2 A_0 - \partial_{x_1} (q^2 p_1 \wedge q \partial_q^2 A_0)) + J_2 (q^2 p_1 \wedge \partial_{p_1} \partial_q^2 A_0 - \partial_{x_1} (q^2 p_1 \wedge q \partial_q^2 A_0)) \right. \\
& \left. + J_3 \partial_{\Delta x} (q^2 \partial_q^2 A_0) + J_4 q^2 \partial_q^2 A_0 \right]
\end{aligned}$$

In[\*]:= J4lm3

Out[\*]=

$$\begin{aligned}
& -\frac{2\gamma l \wedge p1}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} - \frac{\gamma l \wedge p1}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} + \\
& \frac{\gamma l \wedge p1}{(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} + \frac{2\gamma l \wedge p1}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2)} + \frac{6\gamma \hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^4 (-i\epsilon + l \odot p2)} - \\
& \frac{2\gamma \hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^3} + \frac{2\gamma \hbar l \odot q l \wedge p1}{(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^3} + \frac{4\gamma \hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2)^2} - \\
& \frac{6\gamma \hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^4 (i\epsilon + l \odot p2)} - \frac{p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} - \frac{p1 \wedge p2}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} + \\
& \frac{p1 \wedge p2}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} + \frac{p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} + \frac{2\hbar l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} - \\
& \frac{2\hbar l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^3} + \frac{2\hbar l \odot q p1 \wedge p2}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^3} + \frac{2\hbar l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} - \\
& \frac{2\hbar l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2)} + \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \\
& \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} + \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} + \frac{m^2 \gamma \hbar p1 \wedge q}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} - \\
& \frac{\gamma^2 \hbar p1 \wedge q}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} - \frac{m^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)}
\end{aligned}$$

In[\*]:= ClearAll[B0C0, B1C1];

B0C0 = Bx1hbar[0] + Bx2hbar[0] + Cx1hbar[0] + Cx2hbar[0];

B1C1 = Bx1hbar[1] + Bx2hbar[1] + Cx1hbar[1] + Cx2hbar[1];

In[\*]:= simplifiedForm[B0C0 /. {l ⊙ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊙ l + 2 l ⊙ q - q ⊙ q → -qml2}]

Out[\*]=

$$\begin{aligned}
& \frac{4(p1 \odot p2)^2}{qml2 (i\epsilon + l \odot p1)(-i\epsilon + l \odot p2)} - \frac{4(p1 \odot p2)^2}{qml2 (-i\epsilon + l \odot p2)(-i\epsilon + l \odot p1 - p1 \odot q)} - \\
& \frac{4(p1 \odot p2)^2}{qml2 (i\epsilon + l \odot p1)(i\epsilon + l \odot p2 - p2 \odot q)} + \frac{4(p1 \odot p2)^2}{qml2 (-i\epsilon + l \odot p1 - p1 \odot q)(i\epsilon + l \odot p2 - p2 \odot q)}
\end{aligned}$$

`In[*]:= simplifiedForm[B1C1 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}]`

`Out[*]=`

$$\begin{aligned}
 & -\frac{2 qml2 (p1 \otimes p2)^2}{(-i \epsilon + l \otimes p2) (-i \epsilon + l \otimes p1 - p1 \otimes q)^2} - \frac{4 p1 \otimes p2 \left( \frac{l \otimes p1 \otimes p2}{(-i \epsilon + l \otimes p2)^2} - \frac{l \otimes p1 + l \otimes p2 - 2 p2 \otimes q}{-i \epsilon + l \otimes p2} \right) - \frac{4 p1 \otimes p2 (l \otimes p1 + l \otimes p2 - p1 \otimes q - p2 \otimes q)}{-i \epsilon + l \otimes p2}}{2 (-i \epsilon + l \otimes p1 - p1 \otimes q)} + \\
 & \frac{1}{qml2} \left( 4 p1 \otimes p2 \left( \frac{-2 l \otimes p1 + 2 l \otimes p2}{4 (i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)} + \right. \right. \\
 & \quad \left. \left. 4 \left( \frac{l \otimes l}{8 (i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)^2} - \frac{l \otimes l}{8 (i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)} \right) p1 \otimes p2 \right) - \right. \\
 & \quad \left. \frac{2 p1 \otimes p2 (l \otimes p1 - l \otimes p2 + p1 \otimes q - p2 \otimes q)}{(i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)} \right) + \\
 & \frac{2 qml2 (p1 \otimes p2)^2}{(i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2 - p2 \otimes q)^2} - \frac{4 p1 \otimes p2 \left( -\frac{l \otimes p1 \otimes p2}{(i \epsilon + l \otimes p1)^2} + \frac{l \otimes p1 + l \otimes p2 - 2 p1 \otimes q}{i \epsilon + l \otimes p1} \right) + \frac{4 p1 \otimes p2 (l \otimes p1 + l \otimes p2 - p1 \otimes q + p2 \otimes q)}{i \epsilon + l \otimes p1}}{2 (i \epsilon + l \otimes p2 - p2 \otimes q)} + \\
 & \frac{2 qml2 (p1 \otimes p2)^2}{(-i \epsilon + l \otimes p1 - p1 \otimes q)^2 (i \epsilon + l \otimes p2 - p2 \otimes q)} - \frac{\frac{4 qml2 (p1 \otimes p2)^2}{(i \epsilon + l \otimes p2 - p2 \otimes q)^2} - \frac{8 p1 \otimes p2 (l \otimes p1 - l \otimes p2 - p1 \otimes q + p2 \otimes q) + 4 p1 \otimes p2 (2 l \otimes p1 - 2 l \otimes p2 - 4 p1 \otimes q + 4 p2 \otimes q)}{2 (i \epsilon + l \otimes p2 - p2 \otimes q)}}{2 (-i \epsilon + l \otimes p1 - p1 \otimes q)} \\
 & \qquad \qquad \qquad qml2
 \end{aligned}$$

`In[*]:= Coefficient[Expand[  
simplifiedForm[B1C1] /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}],`

$$\frac{1}{(-i \epsilon + l \otimes p1 - p1 \otimes q) (i \epsilon + l \otimes p2 - p2 \otimes q)}, 1]$$

`Out[*]=`

$$\frac{4 l \otimes p1 p1 \otimes p2}{qml2} - \frac{4 l \otimes p2 p1 \otimes p2}{qml2} - \frac{6 p1 \otimes p2 p1 \otimes q}{qml2} + \frac{6 p1 \otimes p2 p2 \otimes q}{qml2}$$

`In[*]:= ClearAll[R1B1C1];`

`R1B1C1 =`

`simplifyTerm[Expand[simplifiedForm[onShellP[aWedgeDb[B1C1, p1, p1]/l ⊗ l] /.  
{l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}]/4]];`

`ClearAll[R1B1C1p1q, R1B1C1p1p2, R1B1C1p1l];`

`R1B1C1p1q = Coefficient[R1B1C1, p1 ∧ q, 1];`

`R1B1C1p1p2 = Coefficient[R1B1C1, p1 ∧ p2, 1];`

`R1B1C1p1l = Coefficient[R1B1C1, l ∧ p1, 1];`

```
In[*]:= ClearAll[R1B0C0];
R1B0C0 = simplifyTerm[
  Expand[simplifiedForm[onShellP[delx[-q ⊗ q * aWedgeDb[B0C0, p1, p1]]/(8 * l ⊗ l)]]];
```

```
ClearAll[R1B0C0p1q, R1B0C0p1p2, R1B0C0p1l];
R1B0C0p1q = Coefficient[R1B0C0, p1 ∧ q, 1];
R1B0C0p1p2 = Coefficient[R1B0C0, p1 ∧ p2, 1];
R1B0C0p1l = Coefficient[R1B0C0, l ∧ p1, 1];
```

```
In[*]:= ClearAll[R2B1C1];
R2B1C1 = simplifyTerm[Expand[simplifiedForm[
  onShellP[D[-p1 ∧ q * (Expand[B1C1 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q -
    q ⊗ q → -qml2}]] /. {qml2 → l ⊗ l - 2 l ⊗ q + q ⊗ q}], x1]]/(4 * l ⊗ l)]]];
```

```
ClearAll[R2B1C1p1q, R2B1C1p1p2, R2B1C1p1l];
R2B1C1p1q = Coefficient[R2B1C1, p1 ∧ q, 1];
R2B1C1p1p2 = Coefficient[R2B1C1, p1 ∧ p2, 1];
R2B1C1p1l = Coefficient[R2B1C1, l ∧ p1, 1];
```

```
In[*]:= ClearAll[R2B0C0];
R2B0C0 = simplifyTerm[
  Expand[simplifiedForm[onShellP[delx[D[q ⊗ q * p1 ∧ q * B0C0, x1]]/(8 * l ⊗ l)]]];
```

```
ClearAll[R2B0C0p1q, R2B0C0p1p2, R2B0C0p1l];
R2B0C0p1q = Coefficient[R2B0C0, p1 ∧ q, 1];
R2B0C0p1p2 = Coefficient[R2B0C0, p1 ∧ p2, 1];
R2B0C0p1l = Coefficient[R2B0C0, l ∧ p1, 1];
```

```
In[*]:= R1B0C0p1p2 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}
```

Out[\*]=

$$\begin{aligned}
& \frac{\gamma q \otimes q}{q m l^2 l \otimes l (-i \epsilon + l \otimes p_1)^2 (-i \epsilon + l \otimes p_2)} + \frac{m^2 \gamma^2 q \otimes q}{2 q m l^2 S l \otimes l (-i \epsilon + l \otimes p_1)^2 (-i \epsilon + l \otimes p_2)} + \\
& \frac{\gamma^3 q \otimes q}{2 q m l^2 S l \otimes l (-i \epsilon + l \otimes p_1)^2 (-i \epsilon + l \otimes p_2)} + \frac{\gamma q \otimes q}{q m l^2 l \otimes l (-i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2)^2} - \\
& \frac{\gamma q \otimes q}{q m l^2 l \otimes l (i \epsilon + l \otimes p_1) (i \epsilon + l \otimes p_2)^2} - \frac{\gamma q \otimes q}{q m l^2 l \otimes l (-i \epsilon + l \otimes p_1)^2 (i \epsilon + l \otimes p_2)} - \\
& \frac{m^2 \gamma^2 q \otimes q}{2 q m l^2 S l \otimes l (-i \epsilon + l \otimes p_1)^2 (i \epsilon + l \otimes p_2)} - \frac{\gamma^3 q \otimes q}{2 q m l^2 S l \otimes l (-i \epsilon + l \otimes p_1)^2 (i \epsilon + l \otimes p_2)}
\end{aligned}$$

In[\*]:= R2B0C0p1p2 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[\*]=

$$\begin{aligned}
 & -\frac{m1^2 \gamma^2 q \otimes q}{2 qml2 S l \otimes l (-i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)} - \frac{\gamma^3 q \otimes q}{qml2 S l \otimes l (-i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)} - \\
 & \frac{\gamma^3 q \otimes q}{2 qml2 S l \otimes l (-i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2} + \frac{\gamma^3 q \otimes q}{2 qml2 S l \otimes l (i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2} + \\
 & \frac{m1^2 \gamma^2 q \otimes q}{2 qml2 S l \otimes l (-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)} + \frac{\gamma^3 q \otimes q}{qml2 S l \otimes l (-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)}
 \end{aligned}$$

In[\*]:= R1B1C1p1p2 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[\*]=

$$\begin{aligned}
 & -\frac{\gamma}{qml2 (-i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)^2} + \frac{\gamma}{qml2 (i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)^2} - \\
 & \frac{\gamma}{l \otimes l (-i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)} - \frac{\gamma}{qml2 (i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)} - \\
 & \frac{\gamma}{l \otimes l (-i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2} + \frac{\gamma}{l \otimes l (i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2} + \\
 & \frac{\gamma}{l \otimes l (-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)} + \frac{\gamma}{qml2 (i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)}
 \end{aligned}$$

In[\*]:= R2B1C1p1p2 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[\*]=

$$\begin{aligned}
 & \frac{\gamma^3}{2 qml2 S (-i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)^2} - \frac{\gamma^3}{2 qml2 S (i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)^2} + \\
 & \frac{\gamma^3}{2 S l \otimes l (-i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)} + \frac{\gamma^3}{2 qml2 S (i \epsilon + l \otimes p1)^2 (-i \epsilon + l \otimes p2)} + \\
 & \frac{\gamma^3}{2 S l \otimes l (-i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2} - \frac{\gamma^3}{2 S l \otimes l (i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2)^2} - \\
 & \frac{\gamma^3}{2 S l \otimes l (-i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)} - \frac{\gamma^3}{2 qml2 S (i \epsilon + l \otimes p1)^2 (i \epsilon + l \otimes p2)}
 \end{aligned}$$



In[\*]:= R1B0C0p1q /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[\*]=

$$\begin{aligned} & \frac{\gamma^2 q \otimes q}{qml2 \, l \otimes l (-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)} - \\ & \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} - \frac{\gamma^3 q \otimes q}{qml2^2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} + \\ & \frac{\gamma^2 q \otimes q}{2 \, qml2 \, l \otimes l (-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2} - \frac{\gamma^2 q \otimes q}{qml2 \, l \otimes l (-i\epsilon + l \otimes p1)^3 (i\epsilon + l \otimes p2)} + \\ & \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} + \frac{\gamma^3 q \otimes q}{qml2^2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} \end{aligned}$$

In[\*]:= R2B0C0p1q /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[\*]=

$$\begin{aligned} & \frac{m2^2 \gamma^2}{qml2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} + \\ & \frac{\gamma^3}{qml2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} - \frac{m2^2 \gamma^2}{qml2 \, s \, l \otimes l (i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} - \\ & \frac{\gamma^3}{qml2 \, s \, l \otimes l (i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} - \frac{m2^2 \gamma^2}{qml2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} - \\ & \frac{\gamma^3}{qml2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} + \frac{m2^2 \gamma^2}{qml2 \, s \, l \otimes l (i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} + \\ & \frac{\gamma^3}{qml2 \, s \, l \otimes l (i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} - \frac{\gamma^2 q \otimes q}{qml2 \, l \otimes l (-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)} + \\ & \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} + \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, s \, l \otimes l (i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} + \\ & \frac{\gamma^3 q \otimes q}{qml2^2 \, s \, l \otimes l (i\epsilon + l \otimes p1)(-i\epsilon + l \otimes p2)} - \frac{\gamma^2 q \otimes q}{2 \, qml2 \, l \otimes l (-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2} + \\ & \frac{\gamma^2 q \otimes q}{qml2 \, l \otimes l (-i\epsilon + l \otimes p1)^3 (i\epsilon + l \otimes p2)} - \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} - \\ & \frac{\gamma^3 q \otimes q}{qml2^2 \, s \, l \otimes l (-i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} - \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, s \, l \otimes l (i\epsilon + l \otimes p1)(i\epsilon + l \otimes p2)} \end{aligned}$$

In[\*]:= R1B1C1p1q /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[\*]=

$$\begin{aligned}
& -\frac{\gamma^2}{2 \text{qml2} (-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)^2} - \frac{\gamma^2}{l \odot l (-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} + \\
& \frac{3 \gamma}{2 \text{qml2} l \odot l (-i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)} - \frac{\gamma}{2 \text{qml2} l \odot l (i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)} - \\
& \frac{\gamma^2}{2 l \odot l (-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} + \frac{\gamma^2}{l \odot l (-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2)} - \\
& \frac{\gamma}{2 \text{qml2} l \odot l (-i\epsilon + l \odot p1) (i\epsilon + l \odot p2)} + \frac{3 \gamma}{2 \text{qml2} l \odot l (i\epsilon + l \odot p1) (i\epsilon + l \odot p2)}
\end{aligned}$$

In[\*]:= R2B1C1p1q /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[\*]=

$$\begin{aligned}
& \frac{\gamma^2}{2 \text{qml2} (-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)^2} + \frac{\gamma^2}{l \odot l (-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} + \\
& \frac{\gamma^3}{\text{qml2}^2 S(-i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)} - \frac{3 \gamma}{2 \text{qml2} l \odot l (-i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)} - \\
& \frac{m2^2 \gamma^2}{\text{qml2}^2 S(i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)} - \frac{\gamma^3}{\text{qml2}^2 S(i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)} + \\
& \frac{\gamma}{2 \text{qml2} l \odot l (i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)} + \frac{\gamma^2}{2 l \odot l (-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} - \\
& \frac{\gamma^2}{l \odot l (-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2)} + \frac{\gamma}{2 \text{qml2} l \odot l (-i\epsilon + l \odot p1) (i\epsilon + l \odot p2)} + \\
& \frac{m2^2 \gamma^2}{\text{qml2}^2 S(i\epsilon + l \odot p1) (i\epsilon + l \odot p2)} - \frac{3 \gamma}{2 \text{qml2} l \odot l (i\epsilon + l \odot p1) (i\epsilon + l \odot p2)}
\end{aligned}$$

In[ ]:= R1B0C0p11 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[ ]:=

$$\begin{aligned} & \frac{\gamma^2 q \otimes q}{qml2 \, l \otimes l (-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)} - \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, l \otimes l (-i\epsilon + l \otimes p1) (-i\epsilon + l \otimes p2)} - \\ & \frac{\gamma^3 q \otimes q}{qml2^2 \, l \otimes l (-i\epsilon + l \otimes p1) (-i\epsilon + l \otimes p2)} + \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, l \otimes l (i\epsilon + l \otimes p1) (-i\epsilon + l \otimes p2)} + \\ & \frac{\gamma^3 q \otimes q}{qml2^2 \, l \otimes l (i\epsilon + l \otimes p1) (-i\epsilon + l \otimes p2)} + \frac{\gamma^2 q \otimes q}{2 \, qml2 \, l \otimes l (-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2} - \\ & \frac{\gamma^2 q \otimes q}{2 \, qml2 \, l \otimes l (i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2} - \frac{\gamma^2 q \otimes q}{qml2 \, l \otimes l (-i\epsilon + l \otimes p1)^3 (i\epsilon + l \otimes p2)} + \\ & \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, l \otimes l (-i\epsilon + l \otimes p1) (i\epsilon + l \otimes p2)} + \frac{\gamma^3 q \otimes q}{qml2^2 \, l \otimes l (-i\epsilon + l \otimes p1) (i\epsilon + l \otimes p2)} - \\ & \frac{m2^2 \gamma^2 q \otimes q}{qml2^2 \, l \otimes l (i\epsilon + l \otimes p1) (i\epsilon + l \otimes p2)} - \frac{\gamma^3 q \otimes q}{qml2^2 \, l \otimes l (i\epsilon + l \otimes p1) (i\epsilon + l \otimes p2)} \end{aligned}$$

In[ ]:= R2B0C0p11 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[ ]:=

0

In[ ]:= R1B1C1p11 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[ ]:=

$$\begin{aligned} & -\frac{\gamma^2}{2 \, qml2 (-i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)^2} + \frac{\gamma^2}{2 \, qml2 (i\epsilon + l \otimes p1)^2 (-i\epsilon + l \otimes p2)^2} - \\ & \frac{\gamma^2}{l \otimes l (-i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)} - \frac{\gamma^2}{qml2 (i\epsilon + l \otimes p1)^3 (-i\epsilon + l \otimes p2)} - \\ & \frac{\gamma^2}{2 \, l \otimes l (-i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2} + \frac{\gamma^2}{2 \, l \otimes l (i\epsilon + l \otimes p1)^2 (i\epsilon + l \otimes p2)^2} + \\ & \frac{\gamma^2}{l \otimes l (-i\epsilon + l \otimes p1)^3 (i\epsilon + l \otimes p2)} + \frac{\gamma^2}{qml2 (i\epsilon + l \otimes p1)^3 (i\epsilon + l \otimes p2)} \end{aligned}$$

In[ ]:= R2B1C1p11 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}

Out[ ]:=

0

In[\*]:= `simplifiedForm[B0C0 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}]`

Out[\*]=

$$\frac{4 (p1 \otimes p2)^2}{qml2 (i \epsilon + l \otimes p1) (-i \epsilon + l \otimes p2)} - \frac{4 (p1 \otimes p2)^2}{qml2 (-i \epsilon + l \otimes p2) (-i \epsilon + l \otimes p1 - p1 \otimes q)} -$$

$$\frac{4 (p1 \otimes p2)^2}{qml2 (i \epsilon + l \otimes p1) (i \epsilon + l \otimes p2 - p2 \otimes q)} + \frac{4 (p1 \otimes p2)^2}{qml2 (-i \epsilon + l \otimes p1 - p1 \otimes q) (i \epsilon + l \otimes p2 - p2 \otimes q)}$$