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In[1]:= Needs["Notation`"];
In[2]:= Needs["MaTeX`"];
In[3]:= (*$RecursionLimit=100*)

In[4]:= (*p1 = {p10,p11,p12,p13};
          p2 = {p20,p21,p22,p23};
          q = {q0,q1,q2,q3};
          l = {l0, l1,l2, l3};
          b = {b0, b1,b2,b3};
          Q = {Q0,Q1,Q2,Q3};*)

In[5]:= (*eta = DiagonalMatrix[{1,-1,-1,-1}];*)

In[6]:= (*ClearAll[p1,p2,q,l,b, Q, eta];
          vecRules = {p1 → {p10,p11,p12,p13},
                      p2 → {p20,p21,p22,p23},
                      q → {q0,q1,q2,q3},
                      l → {l0, l1,l2, l3},
                      b → {b0, b1,b2,b3},
                      (*Q → {Q0,Q1,Q2,Q3},*)
                      Qp → {Qp0, Qp1, Qp2, Qp3},
                      eta → DiagonalMatrix[{1,-1,-1,-1}]};*)

In[7]:= (*RealQ[num_] := TrueQ[Refine[Element[num, Reals]]];
          IsReal[x_] := FreeQ[RealQ[x],x∈Reals];*)

In[8]:= (*scalars = {x1, x2, m1, m2, S};
          IsScalar[x_]:=MemberQ[scalars, x]||NumberQ[x]*)
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In[1]:= ClearAll[scalars];
scalars = {x1, x2, m1, m2, S, ħ, ρ, γ, ε};

$Assumptions = And @@ (Element[#, Reals] & /@ scalars);

IsScalar[expr_] := Module[{vars, dotExp},
  vars = Variables[expr];
  dotExp = Cases[vars, holdDot[_], _];
  vars = Complement[vars, dotExp];
  vars = Join[vars, HoldForm /@ dotExp];

  (vars === {} && NumericQ[expr]) || (vars != {} && SubsetQ[scalars, vars])
];

(*IsScalar[expr_]:=Module[{vars},
  vars=Variables[expr];

  (vars==={}&&NumericQ[expr])||(vars!={}&&SubsetQ[scalars,vars])
];*)

In[2]:= scalars
Out[2]= {x1, x2, m1, m2, S, ħ, ρ, γ, ε}

In[3]:= AddToScalars[expr_] :=
  (If[! MemberQ[scalars, HoldForm[expr]], scalars = Append[scalars, HoldForm[expr]]];
   expr);

AddDotProdToScalars[vectors_] := Module[{vecPairs},
  (*vecPairs = Subsets[vectors,{2}];*)
  vecPairs = DeleteDuplicatesBy[Tuples[vectors, 2], Sort];
  dotProdPairs = holdDot @@ # & /@ vecPairs;
  Do[AddToScalars[dotProd], {dotProd, dotProdPairs}];
];

```

```

In[=]:= holdDotDef := (
  ClearAll[holdDot];
  (*holdDot[a_,b_]:=Module[{aa=a,bb=b},If[OrderedQ[{aa,bb}],a.eta.b,b.eta.a]];*)
  (*holdDot/:holdDot[a_,b_]:=If[OrderedQ[{a,b}],holdDot[a,b],holdDot[b,a]];*)
  (*holdDot/:holdDot[a_,b_]:=0/(PossibleZeroQ[a]||PossibleZeroQ[b]);*)
  holdDot/:holdDot[a_,b_]:=holdDot@@Sort[{a,b}]/;!OrderedQ[{a,b}];*)
  holdDot[a_, b_] := 0 /; (PossibleZeroQ[a] || PossibleZeroQ[b]);
  holdDot[a_, b_] := holdDot @@ Sort[{a, b}] /; ! OrderedQ[{a, b}];
  holdDot[a_ + b_, x_] := holdDot[a, x] + holdDot[b, x];
  holdDot[x_, a_ + b_] := holdDot[a, x] + holdDot[b, x];
  holdDot[x_, c_?IsScalar y_] := c holdDot[x, y];
  holdDot[c_?IsScalar x_, y_] := c holdDot[x, y];
  holdDot[x_, Times[holdDot[a_, b_], y_]] := holdDot[a, b] x holdDot[x, y];
  holdDot[Times[holdDot[a_, b_] x_], y_] := holdDot[a, b] x holdDot[x, y];
  (*holdDot[c_ x_,y_]:=c holdDot[x,y]/;FreeQ[c,_List];*)
  holdDot[x_,c_ y_]:=c holdDot[x,y]/;FreeQ[c,_List];*)

Notation[ParsedBoxWrapper[RowBox[{"a_", "⊕", "b_"}]]  $\leftrightarrow$ 
  ParsedBoxWrapper[RowBox[{"holdDot", "[", "a_", ",", "b_", "]"}]]];
)

In[=]:= holdDotDer := (
  (*ClearAll[holdDot];*)

  SetOptions[D, NonConstants → {holdDot, Wedge}];
  (*D[expr_holdDot,vars_]:=Module[{x,y,z},*)
  holdDot /: D[holdDot[a_, b_], vars_] := Module[{x = a, y = b, z},
    (*{x,y}=List@@expr;*)
    z = {vars}[1];
    Which[
      IsScalar[z] && x === q && y === q,
      holdDot[D[Q, {vars}[1]], y] + holdDot[x, D[Q, {vars}[1]]],
      IsScalar[z] && x === q,
      holdDot[D[Q, {vars}[1]], y],
      IsScalar[z] && y === q,
      holdDot[x, D[Q, {vars}[1]]],
      ! IsScalar[{vars}[1]],
      x x D[y, {vars}[1]] + y x D[x, {vars}[1]],
      True, 0]];
  )
)

```

In[\circ]:= **holdDotDef**

Notation: Future versions of the Notation package will no longer support \leftrightarrow , instead they will use \iff . Please make this change to all your Notations.

In[\circ]:= **vectors** = { $p_1, p_2, q, l, l_1, l_2, Q_p$ }

Out[\circ]= { $p_1, p_2, q, l, l_1, l_2, Q_p$ }

In[\circ]:= **AddDotProdToScalars[vectors]**

In[\circ]:= **scalars**

Out[\circ]= { $x_1, x_2, m_1, m_2, S, \hbar, \rho, \gamma, \epsilon, p_1 \odot p_1, p_1 \odot p_2, p_1 \odot q, l \odot p_1, l_1 \odot p_1, l_2 \odot p_1, p_1 \odot Q_p, p_2 \odot p_2, p_2 \odot q, l \odot p_2, l_1 \odot p_2, l_2 \odot p_2, p_2 \odot Q_p, q \odot q, l \odot q, l_1 \odot q, l_2 \odot q, q \odot Q_p, l \odot l, l \odot l_1, l \odot l_2, l \odot Q_p, l_1 \odot l_1, l_1 \odot l_2, l_1 \odot Q_p, l_2 \odot l_2, l_2 \odot Q_p, Q_p \odot Q_p$ }

In[\circ]:= **ClearAll[Wedge];**

Wedge[x_-, y_-] := $0 /; (x == y \text{ || PossibleZeroQ}[x] \text{ || PossibleZeroQ}[y]);$
Wedge[$a_- + b_-, x_-$] := **Wedge**[a, x] + **Wedge**[b, x];
Wedge[$x_-, a_- + b_-$] := **Wedge**[x, a] + **Wedge**[x, b];
Wedge[$x_-, c_- \text{? IsScalar } y_-$] := $c \text{Wedge}[x, y];$
Wedge[$c_- \text{? IsScalar } x_-, y_-$] := $c \text{Wedge}[x, y];$
Wedge[x_-, y_-] := $- \text{Wedge}[y, x] /; \text{OrderedQ}[\{y, x\}];$
Wedge[$x_-, \text{Times}[\text{holdDot}[a_-, b_-], y_-]$] := **holdDot**[a, b] $\times \text{Wedge}[x, y];$
Wedge[$\text{Times}[\text{holdDot}[a_-, b_-] x_-], y_-$] := **holdDot**[a, b] $\times \text{Wedge}[x, y];$

In[\circ]:= **wedgeDer** := (**SetOptions**[D, NonConstants \rightarrow {**holdDot**, **Wedge**}]);

(***D**[expr_Wedge, vars_]^:=Module[{x,y,z},*)

Wedge /: **D**[**Wedge**[a_-, b_- , vars_]] := **Module**[{ $x = a, y = b, z$ },

(*{ $x, y}$ =List@@expr;*)

$z = \{vars\}[[1]]$;

Which[

IsScalar[z] $\&&$ $x == q \&& y == q$,
Wedge[**D**[Q, {vars}[[1]]], y] + **Wedge**[x, **D**[Q, {vars}[[1]]]],
IsScalar[z] $\&&$ $x == q$,
Wedge[**D**[Q, {vars}[[1]]], y],
IsScalar[z] $\&&$ $y == q$,
Wedge[x, **D**[Q, {vars}[[1]]]],
! IsScalar[{vars}[[1]]],
Wedge[x, **D**[y, {vars}[[1]]]] + **Wedge**[**D**[x, {vars}[[1]]], y],
True, 0]];

)

```

In[=]:= aWedgeDb[expr_, a_, b_] := Wedge[a, D[expr, b]];
aWedgeDb[expr_holdDot, a_, b_] := Module[{x, y}, {x, y} = List @@ expr;
                                         Wedge[a, y D[x, b]] + Wedge[a, x D[y, b]]];
                                         
```

*In[=]:= (*ClearAll[holdDot]*)
(*SetAttributes[holdDot, HoldAll]*)*

*In[=]:= (*holdDot[a_,b_]:=*

```

Module[{aa=a,bb=b},If[OrderedQ[{aa,bb}],HoldForm[a.eta.b],HoldForm[b.eta.a]]];*)
(*holdDot[a_,b_]:=Module[{aa=a,bb=b},If[OrderedQ[{aa,bb}],a.eta.b,b.eta.a]];
holdDot[a+b_, x_]:=holdDot[a,x]+holdDot[b,x];
holdDot[x_, a+b_]:=holdDot[a,x]+holdDot[b,x];
holdDot[x_, c_?NumberQ y_]:= c holdDot[x,y];
holdDot[c_?NumberQ x_, y_]:= c holdDot[x,y];
holdDot[c_ x_,y_]:=c holdDot[x,y]/;FreeQ[c,_List];
holdDot[x_,c_ y_]:=c holdDot[x,y]/;FreeQ[c,_List];*)
```

*In[=]:= (*UpValues[holdDot]={};*

```

holdDot,Dt[holdDot[x_,y_], z,__]:=Which[
x==q&&y==q,
holdDot[D[Q, z],y]+holdDot[x,D[Q, z]],
x==q,
holdDot[D[Q,z],y],
y==q,
holdDot[x,D[Q, z]],
True,
θ ]*)
```

*(*holdDot/:D[holdDot[x_,y_],z_,NonConstants→_]:=D[holdDot[x,y],z]*)*

*In[=]:= (*wrapDot[expr_]:=expr //.{(p1.eta.p1/.vecRules)→m₁²,(-p1.eta.p1/.vecRules)→-m₁²,*

```

(p2.eta.p2/.vecRules)→m22,(-p2.eta.p2/.vecRules)→-m22,
(p1.eta.p2/.vecRules)→γ,(-p1.eta.p2/.vecRules)→-γ,
(p1.eta.q/.vecRules)→p1.eta.q,(-p1.eta.q/.vecRules)→-p1.eta.q,
(p2.eta.q/.vecRules)→p2.eta.q,(-p2.eta.q/.vecRules)→-p2.eta.q,
(p1.eta.l/.vecRules)→p1.eta.l,(-p1.eta.l/.vecRules)→-p1.eta.l,
(p2.eta.l/.vecRules)→p2.eta.l,(-p2.eta.l/.vecRules)→-p2.eta.l,
(l.eta.q/.vecRules)→l.eta.q,(-l.eta.q/.vecRules)→-l.eta.q,
(l.eta.l/.vecRules)→l.eta.l,(-l.eta.l/.vecRules)→-l.eta.l,
(q.eta.q/.vecRules)→q.eta.q,(-q.eta.q/.vecRules)→-q.eta.q})*)
```

```
In[ ]:= (*holdDot[a_, b_]:=a.eta.b//  
FullSimplify//.{p1.eta.p2→HoldForm[HoldPattern[y],-p1.eta.p2→-HoldForm[y],  
p1.eta.q→HoldForm[p1.eta.q],-p1.eta.q→-HoldForm[p1.eta.q],  
p2.eta.q→HoldForm[p2.eta.q],-p2.eta.q→-HoldForm[p2.eta.q],  
p1.eta.l→HoldForm[p1.eta.l],-p1.eta.l→-HoldForm[p1.eta.l],  
p2.eta.l→HoldForm[p2.eta.l],-p2.eta.l→-HoldForm[p2.eta.l],  
l.eta.q→HoldForm[l.eta.q],-l.eta.q→-HoldForm[l.eta.q]}]*)  
  
In[ ]:= (*wrapDot[expr_, ] :=expr //.{p1.eta.p2→HoldForm[y],-p1.eta.p2→-HoldForm[y],  
p1.eta.q→HoldForm[p1.eta.q],-p1.eta.q→-HoldForm[p1.eta.q],  
p2.eta.q→HoldForm[p2.eta.q],-p2.eta.q→-HoldForm[p2.eta.q],  
p1.eta.l→HoldForm[p1.eta.l],-p1.eta.l→-HoldForm[p1.eta.l],  
p2.eta.l→HoldForm[p2.eta.l],-p2.eta.l→-HoldForm[p2.eta.l],  
l.eta.q→HoldForm[l.eta.q],-l.eta.q→-HoldForm[l.eta.q],  
l.eta.l→HoldForm[l.eta.l],-l.eta.l→-HoldForm[l.eta.l],  
q.eta.q→HoldForm[q.eta.q],-q.eta.q→-HoldForm[q.eta.q]}]*)  
  
In[ ]:= (*holdDot[expr_] :=  
Simplify[expr ,{p1.eta.p2→HoldForm[y],p1.eta.q→HoldForm[p1.eta.q],  
p2.eta.q→HoldForm[p2.eta.q],p1.eta.l→HoldForm[p1.eta.l],  
p2.eta.l→HoldForm[p2.eta.l],l.eta.q→HoldForm[l.eta.q]}]*)
```

In[]:= **holdDotDef**

Notation: Future versions of the Notation package will no longer support \leftrightarrow , instead they will use \iff . Please make this change to all your Notations.

```
In[ ]:= α1 = (γ x2 - m2^2 x1) / s;  
α2 = (γ x1 - m1^2 x2) / s;  
Q = α1 p1 + α2 p2 + Qp;  
q == Q;
```

In[]:= **holdDotDer**

In[]:= **wedgeDer**

In[]:= D[holdDot[p1, q]/holdDot[p2, q], x1]

Out[]=

$$-\frac{p1 \odot q \left(-\frac{m2^2 p1 \odot p2}{s} + \frac{\gamma p2 \odot p2}{s}\right)}{(p2 \odot q)^2} + \frac{-\frac{m2^2 p1 \odot p1}{s} + \frac{\gamma p1 \odot p2}{s}}{p2 \odot q}$$

In[]:= **onShellP[expr_] :=**

$\text{expr} // . \{p1 \odot q \rightarrow 0, p2 \odot q \rightarrow 0, p1 \odot p2 \rightarrow \gamma, p1 \odot p1 \rightarrow m1^2, p2 \odot p2 \rightarrow m2^2\}$

(*onShellP[expr_]:=expr//.{HoldForm[p1.eta.q]→0, HoldForm[p2.eta.q]→0}*)

```

In[1]:= (*simplifiedForm[expr_]:=expr/.{(-m1^2 m2^2+s^2)/s->1,(m1^2 m2^2-s^2)/s->-1}*)

{(coef1_ \[Epsilon]+coef2_ lOp1)^n->(coef2)^n*(\[Epsilon]*(coef1/coef2)/Abs[coef1/coef2]+lOp1)^n,
 (coef1_ \[Epsilon]+coef2_ lOp2)^n->(coef2)^n*(\[Epsilon]*(coef1/coef2)/Abs[coef1/coef2]+lOp2)^n}*)

In[2]:= 
$$\left( -\frac{m1^2 m2^2}{w s} + \frac{\gamma^2}{w s} \right) /. (T1 : -m1^2 m2^2 / c_.) + (T2 : \gamma^2 / c_.) \Rightarrow s / c$$


Out[2]= 
$$-\frac{m1^2 m2^2}{s w} + \frac{\gamma^2}{s w}$$


In[3]:= simplifiedForm[expr_] :=

expr /. {
$$\left( -\frac{m1^2 m2^2}{s} + \frac{\gamma^2}{s} \right) \rightarrow 1, \left( \frac{m1^2 m2^2}{s} - \frac{\gamma^2}{s} \right) \rightarrow -1, -m1^2 m2^2 + \gamma^2 \rightarrow s, m1^2 m2^2 - \gamma^2 \rightarrow -s} //.

{(coef1_. \[Epsilon]+coef2_. l1dot_.+rest_.)^n \[Rule] (coef2)^n * 
(\[Epsilon]*(coef1/coef2)/Abs[coef1/coef2]+l1dot+Distribute[rest/coef2])^n /;
l1dot == lOp1 || l1dot == lOp2 || l1dot == l1Op1 || l1dot == l1Op2 ||
l1dot == l2Op1 || l1dot == l2Op2}

In[4]:= cancelFactors[expr_] :=

expr // . {dot1 : d1_ ^ (n_.) * (c1_. \[Epsilon] + dot2 : d2_ + d3_) ^ (m_.) /; d1 == d2 + d3 \[Rule]
If[Abs[n] < Abs[m], (c1 \[Epsilon] + d1) ^ (m+n), d1 ^ (m+n)]}

In[5]:= factorRho[exp_] :=

exp /. {(expr_Plus)^n \[Rule] Factor[expr]^n /; ContainsAll[Variables[expr], {\[Epsilon], lOp1}] ||
ContainsAll[Variables[expr], {\[Epsilon], lOp2}]}

In[6]:= simplifyTerm[expr_] := expr // . {((num_ a_) / (Power[den_, -n_]) \[Rule]
num / (den ^ (-n-1)) /; ContainsAll[Variables[den], {\[Epsilon], a}] /; a == lOp1 || a == lOp2)}

In[7]:= simplifyTerm[expr_] := expr // .
{((num_Power[a_, m_:1]) / (Power[den_, -n_:1]) \[Rule] num Power[a, m-1] / (den ^ (-n-1)) /;
ContainsAll[Variables[den], {\[Epsilon], a}] /; a == lOp1 || a == lOp2)}

In[8]:= simplifyTerm[expr_] := (lst = {};
Do[AppendTo[lst, expr[[i]] // . {((num_Power[a_, m_:1]) / (Power[den_, -n_:1]) \[Rule]
num Power[a, m-1] / (den ^ (-n-1)) /; ContainsAll[Variables[den], {\[Epsilon], a}] /;
a == lOp1 || a == lOp2}], {i, 1, Length[expr]}];
Total[lst]);

In[9]:= cancelFactors[expr_] :=

expr // . {dot1 : d1_ ^ (n_.) * (c1_. \[Epsilon] + dot2 : d2_ + d3_) ^ (m_.) /; d1 == d2 + d3 \[Rule]
If[Abs[n] < Abs[m], (c1 \[Epsilon] + d1) ^ (m+n), d1 ^ (m+n)]}$$

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In[8]:= wedgeToDot[expr_] :=
  expr /. {HoldPattern[Wedge[l1, c_]] :> l ⊗ c, HoldPattern[Wedge[c_, l1]] :> -l ⊗ c}

In[9]:= extractLog[expr_, expnt_] := Coefficient[factorRho[
  simplifyTerm[Expand[simplifiedForm[onShellP[expr]]]] /. {l → ρ l, ε → ρ ε}], ρ, expnt]

In[10]:= delx := (D[#, {x1, 1}] - D[#, {x2, 1}]) &;

In[11]:= L[1] = 1 / ((p1 ⊗ l + i ε) (p2 ⊗ l - i ε));
L[2] = 1 / ((p1 ⊗ l + i ε) (p2 ⊗ (q - l) - i ε));
L[3] = 1 / ((p1 ⊗ (q - l) + i ε) (p2 ⊗ (q - l) - i ε));
L[4] = 1 / ((p1 ⊗ (q - l) + i ε) (p2 ⊗ l - i ε));

In[12]:= Cf[1] = (1/2) 
$$\left( 2 \gamma + \hbar (p2 - p1) \odot l - \frac{\hbar * \gamma l \odot l}{(p1 \odot l + i \epsilon)} + \frac{\hbar * \gamma l \odot l}{(p2 \odot l - i \epsilon)} \right);$$

Cf[2] = (1/2) 
$$\left( 2 \gamma + \hbar (p2 + p1) \odot l - 2 \hbar * p1 \odot q - \frac{\hbar * \gamma l \odot l}{(p1 \odot l + i \epsilon)} + \frac{\hbar * \gamma (q - l) \odot (q - l)}{(p2 \odot (q - l) - i \epsilon)} \right);$$

Cf[3] = (1/2) 
$$\left( 2 \gamma + \hbar (p2 - p1) \odot (2 q - l) - \frac{\hbar * \gamma (q - l) \odot (q - l)}{(p1 \odot (q - l) + i \epsilon)} + \frac{\hbar * \gamma (q - l) \odot (q - l)}{(p2 \odot (q - l) - i \epsilon)} \right);$$

Cf[4] = (1/2) 
$$\left( 2 \gamma - \hbar (p2 + p1) \odot l + 2 \hbar * p2 \odot q - \frac{\hbar * \gamma (q - l) \odot (q - l)}{(p1 \odot (q - l) + i \epsilon)} + \frac{\hbar * \gamma l \odot l}{(p2 \odot l - i \epsilon)} \right);$$


In[13]:= Aθ[1] = i 
$$\frac{(2 p1 + \hbar (q + \rho * l)) \odot (2 p2 - \hbar (q + \rho * l))}{(q - \rho * l) \odot (q - \rho * l)};$$

Aθ[2] = i 
$$\frac{(2 p1 + \hbar (q + \rho * l)) \odot (2 p2 - \hbar (q - \rho * l))}{(q - \rho * l) \odot (q - \rho * l)};$$

Aθ[3] = i 
$$\frac{(2 p1 + \hbar (q - \rho * l)) \odot (2 p2 - \hbar (q - \rho * l))}{(q - \rho * l) \odot (q - \rho * l)};$$

Aθ[4] = i 
$$\frac{(2 p1 + \hbar (q - \rho * l)) \odot (2 p2 - \hbar (q + \rho * l))}{(q - \rho * l) \odot (q - \rho * l)};$$


In[14]:= Do[Do[Cfhbar[i][n] = Coefficient[Normal[Series[Cf[i], {h, 0, n}]], h, n];, {i, 1, 4}], {n, 0, 2}];

In[15]:= Do[Do[Aθhbar[i][n] = Coefficient[Normal[Series[Aθ[i], {h, 0, n}]], h, n];, {i, 1, 4}], {n, 0, 1}];

In[16]:= Do[Do[Do[Aθhbarl[i][n, m] = Coefficient[Normal[Series[Aθhbar[i][n], {ρ, 0, m}]], ρ, m], {i, 1, 4}];, {m, 0, 3}], {n, 0, 1}];

```

```

In[=]:= D[p1 ⊙ q, x1]

Out[=]= - m2^2 p1 ⊙ p1 / s + γ p1 ⊙ p2 / s

In[=]:= a1 = aWedgeDb[A0[1] /. {ρ → 0}, l1, p2] /.
{HoldPattern[Wedge[l1, c_]] → l ⊙ c, HoldPattern[Wedge[c_, l1]] → -l ⊙ c};

a2 = aWedgeDb[aWedgeDb[A0[1] /. {ρ → 0}, l1, p1] /.
{HoldPattern[Wedge[l1, c_]] → l ⊙ c, HoldPattern[Wedge[c_, l1]] → -l ⊙ c}, p1, p1];

a3 = aWedgeDb[aWedgeDb[A0[1] /. {ρ → 0}, l1, p2] /.
{HoldPattern[Wedge[l1, c_]] → l ⊙ c, HoldPattern[Wedge[c_, l1]] → -l ⊙ c}, p1, p1];

b1 = D[Wedge[p1, q] * (aWedgeDb[A0[1] /. {ρ → 0}, l1, p1] /.
{HoldPattern[Wedge[l1, c_]] → l ⊙ c, HoldPattern[Wedge[c_, l1]] → -l ⊙ c}), x1];

b2 = D[Wedge[p1, q] * (aWedgeDb[A0[1] /. {ρ → 0}, l1, p2] /.
{HoldPattern[Wedge[l1, c_]] → l ⊙ c, HoldPattern[Wedge[c_, l1]] → -l ⊙ c}), x1];

a1 = simplifiedForm@onShellP@a1;
a2 = simplifiedForm@onShellP@a2;
a3 = simplifiedForm@onShellP@a3;
b1 = simplifiedForm@onShellP@b1;
b2 = simplifiedForm@onShellP@b2;

In[=]:= sum = 0;
Do[sum += L[i]*Cf[i], {i, {1, 2, 3, 4}}];

In[=]:= ClearAll[a1, a2, a3, a4, a5, a6]

```

```
In[=]:= a1 = Coefficient[extractLog[
  sum*(aWedgeDb[A0[1] /. {ρ → 0}, p1, p1] - D[Wedge[p1, q]* (A0[1] /. {ρ → 0}), x1]), -2], ℏ, 1]
Out[=]=
```

$$\frac{4i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2) q \odot q} - \frac{4i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2) q \odot q} +$$

$$\frac{4i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} - \frac{4i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} -$$

$$\frac{4i\gamma l \odot q p1 \wedge p2}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} + \frac{4i\gamma^3 l \odot q p1 \wedge p2}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} -$$

$$\frac{4i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2) q \odot q} + \frac{4i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2) q \odot q}$$


```
In[=]:= a2 = Coefficient[
  extractLog[(aWedgeDb[sum /. {γ → p1 ⊙ p2}, p1, p1] - Wedge[p1, q]*D[sum, x1]) *
  (A0[1] /. {ρ → 0}), -2], ℏ, 1]
Out[=]=
```

$$\frac{8i\gamma^2 l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2) q \odot q} + \frac{4i\gamma^2 l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2 q \odot q} -$$

$$\frac{4i\gamma^2 l \odot q l \wedge p1}{(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2 q \odot q} - \frac{8i\gamma^2 l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2) q \odot q} +$$

$$\frac{4i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2) q \odot q} + \frac{4i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} -$$

$$\frac{4i\gamma l \odot q p1 \wedge p2}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} - \frac{4i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2) q \odot q} +$$

$$\frac{4im2^2 \gamma^2 p1 \wedge q}{S(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2) q \odot q} - \frac{4im2^2 \gamma^2 p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q} -$$

$$\frac{4i\gamma^3 p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q} + \frac{4i\gamma^3 p1 \wedge q}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q}$$

```

In[]:= a3 = Coefficient[
  extractLog[-(aWedgeDb[sum /. {γ → p1 ⊕ p2}, p1, p1] - Wedge[p1, q] * D[sum, x1]) *
  (wedgeToDot[aWedgeDb[A0[1] /. ρ → 0, l1, q]], -2], ħ, 1]

Out[]=

$$-\frac{8i\gamma^2l\odot q\l\wedge p1}{(-i\epsilon+l\odot p1)^3(-i\epsilon+l\odot p2)q\odot q} - \frac{4i\gamma^2l\odot q\l\wedge p1}{(-i\epsilon+l\odot p1)^2(i\epsilon+l\odot p2)^2q\odot q} +$$


$$\frac{4i\gamma^2l\odot q\l\wedge p1}{(i\epsilon+l\odot p1)^2(i\epsilon+l\odot p2)^2q\odot q} + \frac{8i\gamma^2l\odot q\l\wedge p1}{(-i\epsilon+l\odot p1)^3(i\epsilon+l\odot p2)q\odot q} -$$


$$\frac{4i\gamma l\odot q\, p1\wedge p2}{(-i\epsilon+l\odot p1)^2(-i\epsilon+l\odot p2)q\odot q} - \frac{4i\gamma l\odot q\, p1\wedge p2}{(-i\epsilon+l\odot p1)(i\epsilon+l\odot p2)^2q\odot q} +$$


$$\frac{4i\gamma l\odot q\, p1\wedge p2}{(i\epsilon+l\odot p1)(i\epsilon+l\odot p2)^2q\odot q} + \frac{4i\gamma l\odot q\, p1\wedge p2}{(-i\epsilon+l\odot p1)^2(i\epsilon+l\odot p2)q\odot q}$$


In[]:= a4 = Coefficient[
  extractLog[-sum * (aWedgeDb[wedgeToDot[aWedgeDb[A0[1] /. ρ → 0, l1, p1]], p1, p1] -
  D[Wedge[p1, q] * wedgeToDot[aWedgeDb[A0[1] /. ρ → 0, l1, p1]], x1]), -2], ħ, 1]

Out[]=

$$-\frac{2i\gamma^2p1\wedge p2}{S(-i\epsilon+l\odot p1)(i\epsilon+l\odot p2)} + \frac{2i\gamma^2p1\wedge p2}{S(i\epsilon+l\odot p1)(i\epsilon+l\odot p2)}$$


In[]:= a2 + a3 + a4

Out[]=

$$-\frac{2i\gamma^2p1\wedge p2}{S(-i\epsilon+l\odot p1)(i\epsilon+l\odot p2)} + \frac{2i\gamma^2p1\wedge p2}{S(i\epsilon+l\odot p1)(i\epsilon+l\odot p2)} +$$


$$\frac{4im2^2\gamma^2p1\wedge q}{S(-i\epsilon+l\odot p1)(-i\epsilon+l\odot p2)q\odot q} - \frac{4im2^2\gamma^2p1\wedge q}{S(-i\epsilon+l\odot p1)(i\epsilon+l\odot p2)q\odot q} -$$


$$\frac{4i\gamma^3p1\wedge q}{S(-i\epsilon+l\odot p1)(i\epsilon+l\odot p2)q\odot q} + \frac{4i\gamma^3p1\wedge q}{S(i\epsilon+l\odot p1)(i\epsilon+l\odot p2)q\odot q}$$


In[]:= sum = 0;
Do[sum += L[i]*Cf[i], {i, {1, 2, 3, 4}}];

In[]:= b1 = extractLog[D[sum * q ⊙ q * aWedgeDb[A0[1] /. ρ → 0, p1, p1], x1] -
D[sum * q ⊙ q * aWedgeDb[A0[1] /. ρ → 0, p1, p1], x2], -2];

In[]:= b2 = extractLog[D[(A0[1] /. ρ → 0) * q ⊙ q * aWedgeDb[sum, p1, p1], x1] -
D[(A0[1] /. ρ → 0) * q ⊙ q * aWedgeDb[sum, p1, p1], x2], -2];

In[]:= b3 = -extractLog[
  D[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]] * q ⊙ q * aWedgeDb[sum, p1, p1], x1] -
  D[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]] * q ⊙ q * aWedgeDb[sum, p1, p1], x2], -2];

```

```

In[1]:= b4 = -extractLog[
  D[sum * q ⊙ q * aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], p1, p1], x1] -
  D[sum * q ⊙ q * aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], p1, p1],
  x2], -2];

In[2]:= b5 =  $\frac{1}{2}$  extractLog[
  D[wedgeToDot[aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], l1, q]] *
  q ⊙ q * aWedgeDb[sum, p1, p1], x1] -
  D[wedgeToDot[aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], l1, q]] *
  q ⊙ q * aWedgeDb[sum, p1, p1], x2], -2];

In[3]:= c1 = extractLog[D[D[sum * Wedge[p1, q] * (A0[1] /. ρ → 0), x1] -
  D[sum * Wedge[p1, q] * (A0[1] /. ρ → 0), x2], x1], -2];

In[4]:= c2 = -extractLog[
  D[D[sum * Wedge[p1, q] * q ⊙ q * wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], x1] - D[sum *
  Wedge[p1, q] * q ⊙ q * wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], x2], x1], -2];

In[5]:= c3 =
 $\frac{1}{2}$  extractLog[D[D[sum * Wedge[p1, q] * q ⊙ q * wedgeToDot[aWedgeDb[wedgeToDot[aWedgeDb[
  (A0[1] /. ρ → 0), l1, q]], l1, q]], x1] - D[sum * Wedge[p1, q] * q ⊙ q * wedgeToDot[
  aWedgeDb[wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]], l1, q]], x2], x1], -2];

In[6]:= Coefficient[Expand[b1 + b2 + b3 + b4 + b5], ħ, 0]

In[7]:= Coefficient[Expand[c1 + c2 + c3], ħ, 0] - Coefficient[Expand[b1 + b2 + b3 + b4 + b5], ħ, 0]

In[8]:= wedgeToDot[aWedgeDb[(A0[1] /. ρ → 0), l1, q]]

Out[8]= 
$$\frac{i(-2\hbar l \odot p_1 + 2\hbar l \odot p_2 - 2\hbar^2 l \odot q)}{q \odot q} - \frac{2i l \odot q (4p_1 \odot p_2 - 2\hbar p_1 \odot q + 2\hbar p_2 \odot q - \hbar^2 q \odot q)}{(q \odot q)^2}$$


```

In[8]:= **Coefficient[Expand[b3], \hbar , 0]**

Out[8]=

$$\begin{aligned}
 & -\frac{16 i \gamma^2 l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2) q \odot q} - \frac{8 i \gamma^2 l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 q \odot q} + \\
 & \frac{8 i \gamma^2 l \odot q l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 q \odot q} + \frac{16 i \gamma^2 l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2) q \odot q} - \\
 & \frac{8 i m1^2 \gamma^2 l \odot q p1 \wedge p2}{S (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) q \odot q} - \frac{8 i \gamma^3 l \odot q p1 \wedge p2}{S (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) q \odot q} + \\
 & \frac{8 i m1^2 \gamma^2 l \odot q p1 \wedge p2}{S (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) q \odot q} + \frac{8 i \gamma^3 l \odot q p1 \wedge p2}{S (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) q \odot q} + \\
 & \frac{8 i m2^2 \gamma^2 p1 \wedge q}{S (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) q \odot q} + \frac{8 i \gamma^3 p1 \wedge q}{S (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) q \odot q} - \\
 & \frac{8 i m2^2 \gamma^2 p1 \wedge q}{S (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q} - \frac{8 i \gamma^3 p1 \wedge q}{S (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q}
 \end{aligned}$$

In[9]:= **extractLog[simplifyTerm[simplifiedForm[onShellP[Expand[sum]]]], -2]**

Out[9]=

$$\begin{aligned}
 & -\frac{\gamma}{(-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{\gamma}{(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \\
 & \frac{\gamma}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{\gamma}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{\gamma \hbar l \odot q}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \\
 & \frac{\gamma \hbar l \odot q}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot q}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot q}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)}
 \end{aligned}$$

```
In[1]:= extractLog[Total[Select[
  List @@ simplifyTerm[simplifiedForm[Expand[onShellP[aWedgeDb[sum, p1, p1]]]]],
  ContainsAll[List @@ Numerator[##], {p1 \wedge q} &]]] - 
  simplifyTerm[Expand[simplifiedForm[onShellP[p1 \wedge q D[sum, x1]]]]], -2]

Out[1]=

$$\frac{\gamma \hbar l \odot l p_1 \wedge q}{2 (-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2)^2} + \frac{\gamma \hbar l \odot l p_1 \wedge q}{(-i \epsilon + l \odot p_1)^3 (-i \epsilon + l \odot p_2)} -$$


$$\frac{\hbar p_1 \wedge q}{2 (-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2)} + \frac{m^2 \gamma \hbar p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2)} + \frac{\gamma \hbar l \odot l p_1 \wedge q}{2 (-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)^2} -$$


$$\frac{\gamma \hbar l \odot l p_1 \wedge q}{(-i \epsilon + l \odot p_1)^3 (i \epsilon + l \odot p_2)} + \frac{\hbar p_1 \wedge q}{2 (-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)} - \frac{m^2 \gamma \hbar p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)} -$$


$$\frac{\gamma^2 \hbar p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)} - \frac{\hbar p_1 \wedge q}{(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)} + \frac{\gamma^2 \hbar p_1 \wedge q}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)}$$


In[2]:= ClearAll[sum];
sum = 0;
Do[sum += L[i]*Cf[i] /. {γ → p1 ⊕ p2}, {i, {1, 2, 3, 4}}];
```

In[8]:= **extractLog[delx[(aWedgeDb[sum /. {γ → p1 ⊕ p2}, p1, p1] - Wedge[p1, q] * D[sum, x1])], -3]**
Out[8]=

$$\begin{aligned}
& - \frac{2\gamma l \wedge p1}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} - \frac{\gamma l \wedge p1}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} + \\
& \frac{\gamma l \wedge p1}{(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} + \frac{2\gamma l \wedge p1}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2)} + \frac{6\gamma\hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^4 (-i\epsilon + l \odot p2)} - \\
& \frac{2\gamma\hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^3} + \frac{2\gamma\hbar l \odot q l \wedge p1}{(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^3} + \frac{4\gamma\hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2)^2} - \\
& \frac{6\gamma\hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^4 (i\epsilon + l \odot p2)} - \frac{p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} - \frac{p1 \wedge p2}{(-i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2} + \\
& \frac{p1 \wedge p2}{(i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2} + \frac{p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} + \frac{2\hbar l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} - \\
& \frac{2\hbar l \odot q p1 \wedge p2}{(i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^3} + \frac{2\hbar l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} - \\
& \frac{2\hbar l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2)} + \frac{m2^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \\
& \frac{m2^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2} + \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2} + \frac{m2^2 \gamma \hbar p1 \wedge q}{S(i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2} - \\
& \frac{\gamma^2 \hbar p1 \wedge q}{S(i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2} - \frac{m2^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)}
\end{aligned}$$

In[9]:= **extractLog[simplifiedForm[onShellP[delx[sum]]], -5]**

Out[9]=

0

In[10]:= **simplifyTerm[Expand[extractLog[simplifiedForm[onShellP[aWedgeDb[sum, p1, p1] - Wedge[p1, q] * D[sum, x1]]], -2]]]**

Out[10]=

$$\begin{aligned}
& - \frac{\gamma l \wedge p1}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \frac{\gamma l \wedge p1}{(i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \\
& \frac{2\gamma\hbar l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} - \frac{p1 \wedge p2}{(-i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)} + \frac{p1 \wedge p2}{(i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)} + \\
& \frac{\hbar l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \frac{m2^2 \gamma \hbar p1 \wedge q}{S(-i\epsilon + l \odot p1) (-i\epsilon + l \odot p2)}
\end{aligned}$$

```

In[=]:= b1 = simplifyTerm[Expand[extractLog[
  simplifiedForm[onShellP[aWedgeDb[sum12, p1, p1] - Wedge[p1, q] * D[sum12, x1]], -3]]]
Out[=]=

$$\frac{\gamma \hbar q \odot q l \wedge p1}{2(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2} + \frac{\hbar q \odot q p1 \wedge p2}{2(i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2}$$


In[=]:= b2 = simplifyTerm[Expand[extractLog[
  simplifiedForm[onShellP[aWedgeDb[sum14, p1, p1] - Wedge[p1, q] * D[sum14, x1]], -3]]]
Out[=]=

$$-\frac{\gamma \hbar q \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2)} - \frac{\hbar q \odot q p1 \wedge p2}{2(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)}$$


In[=]:= ClearAll[b3];
b3 = extractLog[simplifiedForm[onShellP[sum]], -3]
Out[=]=

$$-\frac{\gamma \hbar q \odot q}{2(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} - \frac{\gamma \hbar q \odot q}{2(-i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2} +$$


$$\frac{\gamma \hbar q \odot q}{2(i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2} + \frac{\gamma \hbar q \odot q}{2(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)}$$


In[=]:= Coefficient[simplifyTerm[
  Expand[onShellP[(aWedgeDb[wedgeToDot[aWedgeDb[A0[1] /. {\rho \rightarrow 0}, l1, q]], p1, p1] - D[p1 \wedge q * wedgeToDot[aWedgeDb[A0[1] /. {\rho \rightarrow 0}, l1, q]], x1]) * b3]], \hbar, 1]
Out[=]=

$$\frac{4i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2) q \odot q} - \frac{4i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2) q \odot q} +$$


$$\frac{4i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2 q \odot q} - \frac{4i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2 q \odot q} -$$


$$\frac{4i\gamma l \odot q p1 \wedge p2}{(i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2 q \odot q} + \frac{4i\gamma^3 l \odot q p1 \wedge p2}{S(i\epsilon + l \odot p1) (i\epsilon + l \odot p2)^2 q \odot q} -$$


$$\frac{4i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2) q \odot q} + \frac{4i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2) q \odot q} +$$


$$\frac{4im2^2\gamma^2 p1 \wedge q}{S(-i\epsilon + l \odot p1) (-i\epsilon + l \odot p2) q \odot q} - \frac{4im2^2\gamma^2 p1 \wedge q}{S(-i\epsilon + l \odot p1) (i\epsilon + l \odot p2) q \odot q} -$$


$$\frac{4i\gamma^3 p1 \wedge q}{S(-i\epsilon + l \odot p1) (i\epsilon + l \odot p2) q \odot q} + \frac{4i\gamma^3 p1 \wedge q}{S(i\epsilon + l \odot p1) (i\epsilon + l \odot p2) q \odot q}$$


```

In[1]:= b3 = Total[Select[List @@ b3, ContainsAll[List @@ Numerator[##], {l \wedge p1}] &]]

Out[1]=

$$\begin{aligned} & -\frac{\gamma l \wedge p1}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \frac{\gamma l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \\ & \frac{\gamma l \wedge p1}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} - \frac{\gamma l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} + \frac{2 \gamma \hbar l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} + \\ & \frac{\gamma \hbar l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot q l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \frac{2 \gamma \hbar l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} \end{aligned}$$

In[2]:= Do[If[ContainsAll[List @@ Numerator[b3[[i]]], {l \odot q, l \wedge p1}], b3[[i]] = K4[b3[[i]]]], {i, 1, Length[b3]}]

In[3]:= simplifyTerm[Expand[b3 /. {HoldPattern[Wedge[l, c_]] :> $\frac{1}{S} (\text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m2^2 p1] + \text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m1^2 p2])$ }]]

Out[3]=

$$\begin{aligned} & \frac{\gamma^2 p1 \wedge p2}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{\gamma^2 p1 \wedge p2}{S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{\gamma^2 p1 \wedge p2}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \\ & \frac{\gamma^2 p1 \wedge p2}{S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} - \frac{2 m1^2 \gamma \hbar l \odot q p1 \wedge p2}{S(-i \epsilon + l \odot p1)^3} + \frac{\gamma^2 \hbar l \odot q p1 \wedge p2}{S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)^2} + \\ & \frac{2 \gamma^2 \hbar l \odot q p1 \wedge p2}{S(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} - \frac{m1^2 \gamma \hbar l \odot q p1 \wedge p2}{S(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} + \frac{2 \gamma^2 \hbar p1 \wedge q}{S(-i \epsilon + l \odot p1)^2} + \frac{m1^2 \gamma \hbar p1 \wedge q}{2 S(i \epsilon + l \odot p1)^2} - \\ & \frac{m1^2 \gamma \hbar l \odot p2 p1 \wedge q}{S(-i \epsilon + l \odot p1)^3} - \frac{\gamma \hbar l \odot l p1 \wedge q}{S(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \frac{m2^2 \gamma \hbar p1 \wedge q}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \\ & \frac{m2^2 \gamma \hbar p1 \wedge q}{2 S(i \epsilon + l \odot p2)^2} + \frac{\gamma \hbar l \odot l p1 \wedge q}{2 S(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \frac{\gamma^2 \hbar p1 \wedge q}{S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} \end{aligned}$$

In[4]:= extractLog[simplifyTerm[Expand[simplifiedForm[onShellP[sum]]]], -2]

Out[4]=

$$\begin{aligned} & -\frac{\gamma}{(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \frac{\gamma}{(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \\ & \frac{\gamma}{(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} - \frac{\gamma}{(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \frac{\gamma \hbar l \odot q}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \\ & \frac{\gamma \hbar l \odot q}{(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot q}{(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot q}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} \end{aligned}$$

```
In[1]:= simplifiedForm[onShellP[sum12]]
Out[1]=

$$\frac{2 \gamma - \frac{\gamma \hbar l \ominus l}{i \epsilon + l \ominus p1} + \frac{\gamma \hbar l \ominus l}{-i \epsilon + l \ominus p2} + \hbar (-l \odot p1 + l \odot p2) - 2 \gamma - \frac{\gamma \hbar l \ominus l}{i \epsilon + l \ominus p1} + \hbar (l \odot p1 + l \odot p2) - \frac{\gamma \hbar (l \ominus l - 2 l \odot q + q \odot q)}{i \epsilon + l \ominus p2}}{2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2) - 2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)}$$


In[2]:= a =
extractLog[simplifiedForm[onShellP[aWedgeDb[sum, p1, p1] - Wedge[p1, q] * D[sum, x1]]],
-2] /. {HoldPattern[Wedge[l, c_]] :>

$$\frac{1}{S} (\text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m2^2 p1] + \text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m1^2 p2])\}$$
];

In[3]:= simplifyTerm[
Expand[Total[Select[List @@ a, ContainsAll[List @@ Numerator[##], {p1 \wedge p2, l \odot q}] *
(*&&ContainsNone[List @@ Numerator[##], {l \odot q}]*)(*&&
ContainsAll[List @@ Denominator[##], {S^2}]*)] /. {HoldPattern[Wedge[l, c_]] :>

$$\frac{1}{S} (\text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m2^2 p1] + \text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m1^2 p2])}]]

Out[3]=

$$\begin{aligned} & \frac{\hbar l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \frac{2 \gamma^2 \hbar l \odot q p1 \wedge p2}{S (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \\ & \frac{\hbar l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \frac{\gamma^2 \hbar l \odot q p1 \wedge p2}{S (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \\ & \frac{\hbar l \odot q p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \frac{\gamma^2 \hbar l \odot q p1 \wedge p2}{S (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \frac{\hbar l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} + \\ & \frac{m1^2 \gamma \hbar l \odot q p1 \wedge p2}{S (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} + \frac{2 \gamma^2 \hbar l \odot q p1 \wedge p2}{S (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} - \frac{m1^2 \gamma \hbar l \odot q p1 \wedge p2}{S (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} \end{aligned}$$


In[4]:= K4[expr_] := simplifyTerm[Expand[ $\frac{1}{2 S} (\text{expr} /. \{l \odot q \text{l} \wedge \text{p1} \rightarrow -\text{p1} \wedge \text{q}\})$ 

$$(m2^2 l \odot p1 l \odot p1 + m1^2 l \odot p2 l \odot p2 + l \odot l - 2 \gamma l \odot p1 l \odot p2)\]];$$$$

```

In[5]:= K4[$\frac{2 \gamma \hbar l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)}$]

Out[5]=
$$\begin{aligned} & \frac{2 \gamma^2 \hbar p1 \wedge q}{S (-i \epsilon + l \odot p1)^2} - \frac{m1^2 \gamma \hbar l \odot p2 p1 \wedge q}{S (-i \epsilon + l \odot p1)^3} - \\ & \frac{\gamma \hbar l \odot l p1 \wedge q}{S (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \frac{m2^2 \gamma \hbar p1 \wedge q}{S (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} \end{aligned}$$

```

In[]:= b = extractLog[simplifiedForm[onShellP[wsp]], -2];
In[]:= extractLog[simplifiedForm[onShellP[delx[sum]]], -2]
Out[]=

$$\begin{aligned}
& - \frac{\gamma \hbar l \ominus l}{2 (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \ominus l}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} + \\
& \frac{3 \hbar}{2 (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{m2^2 \gamma \hbar}{S (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{\gamma^2 \hbar}{S (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \\
& \frac{\gamma \hbar l \ominus l}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^3} - \frac{\gamma \hbar l \ominus l}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^3} - \frac{\gamma \hbar l \ominus l}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \\
& \frac{\gamma \hbar l \ominus l}{2 (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{\gamma \hbar l \ominus l}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} - \frac{\hbar}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \\
& \frac{m1^2 \gamma \hbar}{S (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{m2^2 \gamma \hbar}{S (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{2 \gamma^2 \hbar}{S (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \\
& \frac{3 \hbar}{2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{m1^2 \gamma \hbar}{S (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{\gamma^2 \hbar}{S (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)}
\end{aligned}$$


```

```

In[]:= sum = 0;
Do[sum += L[i]*Cf[i] /. {γ → p1 ⊕ p2}, {i, {1, 2}}];

Expand[
simplifiedForm@Coefficient[factorRho@simplifyTerm@Expand[onShellP[aWedgeDb[
simplifiedForm@sum, l1, q] /. {Wedge[l1, c_] → (γ p2 - m2^2 p1) ⊙ c / s,
Wedge[c_, l1] → -(γ p2 - m2^2 p1) ⊙ c / s}], l → ρ l, ε → ρ ε]], ρ, -3]]

```

```

Out[]= $Aborted

```

```

In[]:= aWedgeDb[Expand[
L[i]*Cf[i]*(A0hbarl[i][0, 0] + A0hbarl[i][0, 1] + A0hbarl[i][0, 2] + h A0hbarl[i][1, 0] +
h A0hbarl[i][1, 1] + h A0hbarl[i][1, 2]) /. {i → 1}],
, p1, p1]

```

```

In[]:= ClearAll[sum10, sum11, sum20, sum21, sum]

```

```

In[8]:= sum10 = 0;
Do[term10 =
  L[i]*Cf[i]*(A0hbarl[i][0, 0] + A0hbarl[i][0, 1] + A0hbarl[i][0, 2] + \[hbar] A0hbarl[i][1, 0] +
  \[hbar] A0hbarl[i][1, 1] + \[hbar] A0hbarl[i][1, 2]) /. {\[gamma] \[rightarrow] p1 \[Circle] p2};

sum10 += simplifyTerm[Expand[
  Coefficient[factorRho[simplifiedForm[onShellP[aWedgeDb[term10, p1, p1]]] /.
  {l \[rightarrow] \rho l, \epsilon \[rightarrow] \rho \epsilon}], \rho, -2]], {i, 1, 4}];

In[9]:= sum11 = 0;
Do[term11 = q \[Circle] q * L[i]*Cf[i]*(A0hbarl[i][0, 0] + A0hbarl[i][0, 1] + A0hbarl[i][0, 2] +
  \[hbar] A0hbarl[i][1, 0] + \[hbar] A0hbarl[i][1, 1] + \[hbar] A0hbarl[i][1, 2]) /. {\[gamma] \[rightarrow] p1 \[Circle] p2};

sum11 += simplifyTerm@Expand[Coefficient[
  factorRho[simplifiedForm@onShellP[D[aWedgeDb[term11, p1, p1], x1] - D[aWedgeDb[term11, p1, p1], x2]] /. {l \[rightarrow] \rho l, \epsilon \[rightarrow] \rho \epsilon}], \rho, -2]], {i, 1, 4}];

In[10]:= sum20 = 0;
Do[
  term20 = Wedge[p1, q]*L[i]*Cf[i]*(A0hbarl[i][0, 0] + A0hbarl[i][0, 1] + A0hbarl[i][0, 2] +
  \[hbar] A0hbarl[i][1, 0] + \[hbar] A0hbarl[i][1, 1] + \[hbar] A0hbarl[i][1, 2]) /. {\[gamma] \[rightarrow] p1 \[Circle] p2};

sum20 += simplifyTerm[
  Expand[Coefficient[factorRho[simplifiedForm@onShellP[D[term20, x1]] /.
  {l \[rightarrow] \rho l, \epsilon \[rightarrow] \rho \epsilon}], \rho, -2]], {i, 1, 4}];

In[11]:= sum21 = 0;
Do[term21 =
  q \[Circle] q * Wedge[p1, q]*L[i]*Cf[i]*(A0hbarl[i][0, 0] + A0hbarl[i][0, 1] + A0hbarl[i][0, 2] +
  \[hbar] A0hbarl[i][1, 0] + \[hbar] A0hbarl[i][1, 1] + \[hbar] A0hbarl[i][1, 2]) /. {\[gamma] \[rightarrow] p1 \[Circle] p2};

sum21 += simplifyTerm@Expand[Coefficient[
  factorRho[simplifiedForm@onShellP[D[D[term21, x1], x1] - D[D[term21, x2], x1]] /.
  {l \[rightarrow] \rho l, \epsilon \[rightarrow] \rho \epsilon}], \rho, -2]], {i, 1, 4}];

In[12]:= simplifyTerm@Expand[((sum10 - sum20)/\hbar - (sum11 - sum21)/2 /. {(l \[Circle] q)^2 \[rightarrow] l \[Circle] l * q \[Circle] q}) /.
  {HoldPattern[Wedge[l, c_]] \[leftrightarrow] \frac{1}{S} (Wedge[p1, c]*holdDot[l, \gamma p2 - m2^2 p1] +
  Wedge[p2, c]*holdDot[l, \gamma p1 - m1^2 p2])}] /. {l \[Circle] q \[rightarrow] 0} /. {Power[\hbar, -1] \[rightarrow] 0, Power[\hbar, 1] \[rightarrow] 0}

```

```

In[]:= Coefficient[simplifyTerm[Expand[sum10 / h]], h, 0]
Out[]= - $\frac{2i\gamma^2 l \odot p1 \wedge q}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)^2 q \odot q} + \frac{6i\gamma p1 \wedge q}{(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2) q \odot q} -$ 
 $\frac{2i\gamma p1 \wedge q}{(i\epsilon + l \odot p1)(-i\epsilon + l \odot p2) q \odot q} - \frac{2i\gamma p1 \wedge q}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q} + \frac{6i\gamma p1 \wedge q}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q}$ 

In[]:= Coefficient[simplifyTerm@Expand[(sum11 - sum21)], h, 0]
Out[=] - $\frac{16i\gamma^2 l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2) q \odot q} - \frac{8i\gamma^2 l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2 q \odot q} +$ 
 $\frac{8i\gamma^2 l \odot q l \wedge p1}{(i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2 q \odot q} + \frac{16i\gamma^2 l \odot q l \wedge p1}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2) q \odot q} -$ 
 $\frac{16i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2) q \odot q} + \frac{8i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2) q \odot q} -$ 
 $\frac{16i\gamma l \odot q p1 \wedge p2}{(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} + \frac{8i\gamma^3 l \odot q p1 \wedge p2}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} +$ 
 $\frac{16i\gamma l \odot q p1 \wedge p2}{(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} - \frac{8i\gamma^3 l \odot q p1 \wedge p2}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2 q \odot q} +$ 
 $\frac{8i m2^2 \gamma^2 p1 \wedge q}{S(-i\epsilon + l \odot p1)(-i\epsilon + l \odot p2) q \odot q} + \frac{8i m2^2 \gamma^2 p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q} +$ 
 $\frac{8i\gamma^3 p1 \wedge q}{S(-i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q} - \frac{8i\gamma^3 p1 \wedge q}{S(i\epsilon + l \odot p1)(i\epsilon + l \odot p2) q \odot q}$ 

In[]:= b = simplifyTerm@Expand[((sum10 - sum20) / h /. {(l \odot q)^2 \rightarrow l \odot l * q \odot q}) /.
  {HoldPattern[Wedge[l, c_]] \rightarrow  $\frac{1}{S} (Wedge[p1, c] * holdDot[l, \gamma * p2 - m2^2 p1] +$ 
 $Wedge[p2, c] * holdDot[l, \gamma p1 - m1^2 p2])$ } /. {l \odot q \rightarrow 0}]

```

$$\text{In[}]:= \text{a1} = i \gamma \text{Wedge}[p1, l] \times \delta[p2 \odot l] (\text{A0}[1] /. \{\rho \rightarrow 0\}) \left(\frac{1}{(p1 \odot l - i \epsilon)^2} - \frac{1}{(p1 \odot l + i \epsilon)^2} \right);$$

$$\text{a2} = \hbar \gamma l \odot q \text{Wedge}[p1, l] (\text{A0}[1] /. \{\rho \rightarrow 0\})$$

$$\left(\frac{-2 i \delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^3} - \frac{1}{(p1 \odot l - i \epsilon)^2 (p2 \odot l + i \epsilon)^2} + \frac{1}{(p1 \odot l + i \epsilon)^2 (p2 \odot l + i \epsilon)^2} \right);$$

$$\text{a3} = -\hbar \gamma q \odot q \text{Wedge}[p1, l]$$

$$\left(i \frac{\delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^3} + \frac{1}{2 (p1 \odot l - i \epsilon)^2 (p2 \odot l + i \epsilon)^2} - \frac{1}{2 (p1 \odot l + i \epsilon)^2 (p2 \odot l + i \epsilon)^2} \right)$$

$$\text{onShellP[aWedgeDb[A0[1] /. \{\rho \rightarrow 0\}, l1, q] /.}$$

$$\{\text{HoldPattern}[\text{Wedge}[l1, c_]] \rightarrow l \odot c, \text{HoldPattern}[\text{Wedge}[c_, l1]] \rightarrow -l \odot c\};$$

$$\text{a4} = -\left(\hbar \gamma l \odot q \text{Wedge}[p1, q] \left(\frac{1}{(p1 \odot l - i \epsilon)^2 (p2 \odot l + i \epsilon)^2} + \frac{2 i \delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^3} \right) - \right.$$

$$\left. i \gamma \text{Wedge}[p1, q] \frac{\delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^2} \right) \text{onShellP[aWedgeDb[A0[1] /. \{\rho \rightarrow 0\}, l1, q] /.}$$

$$\{\text{HoldPattern}[\text{Wedge}[l1, c_]] \rightarrow l \odot c, \text{HoldPattern}[\text{Wedge}[c_, l1]] \rightarrow -l \odot c\};$$

$$\text{a5} = \left(\frac{-i \gamma \delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^2} + \hbar \gamma l \odot q \left(\frac{1}{(p1 \odot l - i \epsilon)^2 (p2 \odot l + i \epsilon)^2} + \frac{2 i \delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^3} \right) \right)$$

$$\text{Wedge}[p1, q] \times \text{onShellP[aWedgeDb[A0[1] /. \{\rho \rightarrow 0\}, l1, q] /.}$$

$$\{\text{HoldPattern}[\text{Wedge}[l1, c_]] \rightarrow l \odot c, \text{HoldPattern}[\text{Wedge}[c_, l1]] \rightarrow -l \odot c\};$$

$$\text{a6} = i \hbar \gamma \frac{q \odot q}{2} \left(\frac{\delta[p1 \odot l]}{(p2 \odot l + i \epsilon)^2} + \frac{\delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^2} \right)$$

$$\text{onShellP[aWedgeDb[(aWedgeDb[A0[1] /. \{\rho \rightarrow 0\}, l1, q] /.}$$

$$\{\text{HoldPattern}[\text{Wedge}[l1, c_]] \rightarrow l \odot c, \text{HoldPattern}[\text{Wedge}[c_, l1]] \rightarrow -l \odot c\},$$

$$p1, p1] - D[\text{Wedge}[p1, q] (\text{aWedgeDb[A0[1] /. \{\rho \rightarrow 0\}, l1, q] /.}$$

$$\{\text{HoldPattern}[\text{Wedge}[l1, c_]] \rightarrow l \odot c, \text{HoldPattern}[\text{Wedge}[c_, l1]] \rightarrow -l \odot c\}), x1]];$$

$$\text{In[}]:= \text{Expand}[\text{Coefficient}[\text{simplifyTerm}[\text{Expand}[\text{onShellP[a1 + a2 + a3 + a4 + a5 + a6] / \hbar]] /.}$$

$$\{(l \odot p1)^n_. \delta[l \odot p1] \rightarrow 0, (l \odot p2)^m_. \delta[l \odot p2] \rightarrow 0\}, \hbar, 0] /.$$

$$\{\delta[l \odot p2] \rightarrow \frac{i}{(p2 \odot l + i \epsilon)} - \frac{i}{(p2 \odot l - i \epsilon)}, \delta[l \odot p1] \rightarrow \frac{i}{(p1 \odot l + i \epsilon)} - \frac{i}{(p1 \odot l - i \epsilon)}\}]$$

$$\text{In[}]:= \text{a} =$$

$$\text{simplifyTerm}[\text{Expand}[\text{onShellP[a1 + a2 + a3 + a4 + a5] / \hbar /. \{\text{HoldPattern}[\text{Wedge}[l, c_]] \rightarrow \frac{1}{S}$$

$$(\text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m2^2 p1] + \text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m1^2 p2]\})]]$$

In[1]:= $\text{Expand}\left[a / . \left\{(l \odot p1) \wedge n_-. \delta[l \odot p1] \rightarrow 0, (l \odot p2) \wedge m_-. \delta[l \odot p2] \rightarrow 0\right\} / . \{l \odot q \rightarrow 0\} / . \left\{\delta[l \odot p2] \rightarrow \frac{i}{(p2 \odot l + i \epsilon)}, \delta[l \odot p1] \rightarrow \frac{i}{(p1 \odot l + i \epsilon)} - \frac{i}{(p1 \odot l - i \epsilon)}\right\}\right]$

Out[1]=

$$\begin{aligned} & \frac{i m1^2 \gamma \hbar p1 \wedge p2}{S(-i \epsilon + l \odot p1)^2} - \frac{i m1^2 \gamma \hbar p1 \wedge p2}{S(i \epsilon + l \odot p1)^2} + \frac{i \gamma^2 \hbar p1 \wedge p2}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \frac{i \gamma^2 \hbar p1 \wedge p2}{S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \\ & \frac{i m1^2 \gamma \hbar p1 \wedge p2}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} - \frac{2 i \gamma^2 \hbar p1 \wedge p2}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \frac{i m1^2 \gamma \hbar p1 \wedge p2}{S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \\ & \frac{4 i \gamma^3 p1 \wedge p2}{S \hbar (-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} - \frac{4 i \gamma^3 p1 \wedge p2}{S \hbar (i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} - \\ & \frac{4 i \gamma^3 p1 \wedge p2}{S \hbar (-i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q} + \frac{4 i \gamma^3 p1 \wedge p2}{S \hbar (i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q} \end{aligned}$$

```
In[8]:= a = simplifyTerm[Expand[simplifyTerm@Expand[-γ*q ⊙ q * Wedge[p1, l]

$$\left( \frac{i \delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^3} + \frac{1}{2(p1 \odot l - i \epsilon)^2 (p2 \odot l + i \epsilon)^2} - \frac{1}{2(p1 \odot l + i \epsilon)^2 (p2 \odot l + i \epsilon)^2} \right) *$$

onShellP[aWedgeDb[A0[1] /. {ρ → 0}, l1, q] /. {HoldPattern[
Wedge[l1, c_] → l ⊙ c / S, HoldPattern[Wedge[c_, l1]] → -l ⊙ c / S]}] +
simplifyTerm@Expand[i γ  $\frac{q \odot q}{2} \left( \frac{\delta[p1 \odot l]}{(p2 \odot l + i \epsilon)^2} + \frac{\delta[p2 \odot l]}{(p1 \odot l - i \epsilon)^2} \right)$ ]*onShellP[
aWedgeDb[aWedgeDb[A0[1] /. {ρ → 0}, l1, q] /. {HoldPattern[Wedge[l1, c_]] →
l ⊙ c / S, HoldPattern[Wedge[c_, l1]] → -l ⊙ c / S}, p1, p1] -
D[Wedge[p1, q] * (aWedgeDb[A0[1] /. {ρ → 0}, l1, q] /. {HoldPattern[Wedge[l1,
c_]] → l ⊙ c / S, HoldPattern[Wedge[c_, l1]] → -l ⊙ c / S}), x1]]] ] /.

$$\left\{ \text{HoldPattern}[\text{Wedge}[l, c_]] \rightarrow \frac{1}{S} (\text{Wedge}[p1, c] * \text{holdDot}[l, \gamma * p2 - m2^2 p1] +
\text{Wedge}[p2, c] * \text{holdDot}[l, \gamma p1 - m1^2 p2]) \right\} ] /.$$


$$\{l \odot q \rightarrow 0\} / . \{ (l \odot p1)^n . \delta[l \odot p1] \rightarrow 0, (l \odot p2)^m .$$


$$\delta[l \odot p2] \rightarrow 0 \}$$

```

Out[8]=

$$\begin{aligned}
& \frac{i m1^2 \gamma \hbar p1 \wedge p2}{S^2 (-i \epsilon + l \odot p1)^2} - \frac{i m1^2 \gamma \hbar p1 \wedge p2}{S^2 (i \epsilon + l \odot p1)^2} - \frac{i m1^2 \gamma \hbar p1 \wedge p2}{S^2 (-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} - \frac{i \gamma^2 \hbar p1 \wedge p2}{S^2 (-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \\
& \frac{i m1^2 \gamma \hbar p1 \wedge p2}{S^2 (i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \frac{i \gamma^2 \hbar p1 \wedge p2}{S^2 (i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} - \frac{m1^2 \gamma \hbar p1 \wedge p2 \delta[l \odot p1]}{S^2 (i \epsilon + l \odot p2)} + \\
& \frac{\gamma^2 \hbar p1 \wedge p2 \delta[l \odot p1]}{S^2 (i \epsilon + l \odot p2)} - \frac{4 \gamma^3 p1 \wedge q \delta[l \odot p1]}{S^2 (i \epsilon + l \odot p2) q \odot q} - \frac{2 \gamma^2 \hbar p1 \wedge p2 \delta[l \odot p2]}{S^2 (-i \epsilon + l \odot p1)} + \frac{4 m2^2 \gamma^2 p1 \wedge q \delta[l \odot p2]}{S^2 (-i \epsilon + l \odot p1) q \odot q}
\end{aligned}$$

```
In[9]:= Expand[a /. {δ[l ⊙ p2] →  $\frac{i}{(p2 \odot l + i \epsilon)}$ , δ[l ⊙ p1] →  $\frac{i}{(p1 \odot l + i \epsilon)} - \frac{i}{(p1 \odot l - i \epsilon)}$ }] /.
```

{ $\hbar \rightarrow 0$ }

Out[9]=

$$\begin{aligned}
& - \frac{4 i m2^2 \gamma^2 p1 \wedge q}{S^2 (-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} + \frac{4 i m2^2 \gamma^2 p1 \wedge q}{S^2 (-i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q} + \\
& \frac{4 i \gamma^3 p1 \wedge q}{S^2 (-i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q} - \frac{4 i \gamma^3 p1 \wedge q}{S^2 (i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q}
\end{aligned}$$

```

In[8]:= Dg =
Simplify[Expand[Solve[A*(m1^2 m2^2 + y^2) + B*m1^2 y + C*y*m2^2 + D*y == I12 && 2 A y m1^2 +
B m1^4 + C y^2 + D m1^2 == I11 && 2 A m2^2 y + B y^2 + C m2^4 + D m2^2 == I22 &&
2 A y + B m1^2 + C m2^2 + 4 D == I00, {A, B, C, D}, Reals][[1]][[4]][[2]]].
{2 m1^4 m2^4 - 4 m1^2 m2^2 y^2 + 2 y^4 == 2 S^2}], y^2 - m1^2 m2^2 == S]

Out[8]=
I22 m1^2 + I11 m2^2 + I00 S - 2 I12 y
-----
2 S

In[9]:= I1 = Expand[Dg /. {I00 -> l ⊗ l
                           (p1 ⊙ l + i ε)^2 (p2 ⊙ l + i ε)^2,
                           I12 -> 1
                           (p1 ⊙ l + i ε) (p2 ⊙ l + i ε),
                           I11 -> 1
                           (p2 ⊙ l + i ε)^2,
                           I22 -> 1
                           (p1 ⊙ l + i ε)^2}];

In[10]:= I2 = Expand[Dg /. {I00 -> -l ⊗ l
                           (p1 ⊙ l - i ε)^2 (p2 ⊙ l + i ε)^2,
                           I12 -> -1
                           (p1 ⊙ l - i ε) (p2 ⊙ l + i ε),
                           I11 -> -1
                           (p2 ⊙ l + i ε)^2,
                           I22 -> -1
                           (p1 ⊙ l - i ε)^2}];

In[11]:= I3 = Expand[Dg /. {I00 -> 2 l ⊗ l
                           (p1 ⊙ l - i ε)^3 (p2 ⊙ l + i ε),
                           I12 -> 2
                           (p1 ⊙ l - i ε)^2,
                           I11 -> 2
                           (p1 ⊙ l - i ε) (p2 ⊙ l + i ε),
                           I22 -> 2 p2 ⊙ l
                           (p1 ⊙ l - i ε)^3}];

In[12]:= I4 = Expand[Dg /. {I00 -> -2 l ⊗ l
                           (p1 ⊙ l - i ε)^3 (p2 ⊙ l - i ε),
                           I12 -> -2
                           (p1 ⊙ l - i ε)^2,
                           I11 -> -2
                           (p1 ⊙ l - i ε) (p2 ⊙ l - i ε),
                           I22 -> -2 p2 ⊙ l
                           (p1 ⊙ l - i ε)^3}];

```

In[8]:= **I1 + I2 + I3 + I4**

Out[8]=

$$\begin{aligned}
 & -\frac{m1^2}{2 S(-i \epsilon + l \odot p1)^2} + \frac{m1^2}{2 S(i \epsilon + l \odot p1)^2} - \frac{l \odot l}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \\
 & \frac{m2^2}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{l \odot l}{2 (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \\
 & \frac{l \odot l}{2 (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{l \odot l}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \\
 & \frac{m2^2}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \frac{\gamma}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} - \frac{\gamma}{S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)}
 \end{aligned}$$

```

In[]:= ClearAll[a1, a2, a3, b1, b2, b3, c, d];

a1 = Expand[
  aWedgeDb[aWedgeDb[A0hbar[1][0] /. ρ → 0, l, p1] /. HoldPattern[Wedge[l, c_]] :> l ⊙ c,
  l1, q] /. {HoldPattern[Wedge[l1, c_]] :> (γ p2 - m2^2 p1) ⊙ c / s,
  HoldPattern[Wedge[c_, l1]] :> -(γ p2 - m2^2 p1) ⊙ c / s}];

a2 = Expand[
  aWedgeDb[aWedgeDb[A0hbar[1][0] /. ρ → 0, l, p2] /. HoldPattern[Wedge[l, c_]] :> l ⊙ c,
  l1, q] /. {HoldPattern[Wedge[l1, c_]] :> (γ p2 - m2^2 p1) ⊙ c / s,
  HoldPattern[Wedge[c_, l1]] :> -(γ p2 - m2^2 p1) ⊙ c / s}];

a3 = Expand[
  aWedgeDb[aWedgeDb[A0hbar[1][0] /. ρ → 0, l, q] /. HoldPattern[Wedge[l, c_]] :> l ⊙ c,
  l1, q] /. {HoldPattern[Wedge[l1, c_]] :> (γ p2 - m2^2 p1) ⊙ c / s,
  HoldPattern[Wedge[c_, l1]] :> -(γ p2 - m2^2 p1) ⊙ c / s}];

b1 = aWedgeDb[L[1]*Cf[1] + L[2]*Cf[2], l1, q] /.
  {HoldPattern[Wedge[l1, c_]] :> (γ p2 - m2^2 p1) ⊙ c / s,
  HoldPattern[Wedge[c_, l1]] :> -(γ p2 - m2^2 p1) ⊙ c / s};

b2 = aWedgeDb[L[1]*Cf[1] + L[4]*Cf[4], l1, q] /.
  {HoldPattern[Wedge[l1, c_]] :> (γ p2 - m2^2 p1) ⊙ c / s,
  HoldPattern[Wedge[c_, l1]] :> -(γ p2 - m2^2 p1) ⊙ c / s};

b3 = aWedgeDb[L[1]*Cf[1] + L[2]*Cf[2] + L[3]*Cf[3] + L[4]*Cf[4], l1, q] /.
  {HoldPattern[Wedge[l1, c_]] :> (γ p2 - m2^2 p1) ⊙ c / s,
  HoldPattern[Wedge[c_, l1]] :> -(γ p2 - m2^2 p1) ⊙ c / s};

c = 0;
Do[c += L[i]*Cf[i], {i, 1, 4}];
c = simplifyTerm[simplifiedForm[c]];
c = aWedgeDb[c, l1, q] /. {HoldPattern[Wedge[l1, c_]] :> (γ p2 - m2^2 p1) ⊙ c / s,
  HoldPattern[Wedge[c_, l1]] :> -(γ p2 - m2^2 p1) ⊙ c / s};

d = aWedgeDb[aWedgeDb[A0hbar[1][0] /. ρ → 0, l, q] /. HoldPattern[Wedge[l, c_]] :> l ⊙ c,
  l1, q] /. {HoldPattern[Wedge[l1, c_]] :> (γ p2 - m2^2 p1) ⊙ c / s,
  HoldPattern[Wedge[c_, l1]] :> -(γ p2 - m2^2 p1) ⊙ c / s};

```

```

In[1]:= a1 = onShellP@a1;
a2 = onShellP@a2;
a3 = onShellP@a3;
c = simplifyTerm[Expand[simplifiedForm[onShellP[c]]]];
d = onShellP@d;
b1 = simplifyTerm@Expand[simplifiedForm@onShellP[b1]];
b2 = simplifyTerm@Expand[simplifiedForm@onShellP[b2]];
b3 = simplifyTerm@Expand[simplifiedForm@onShellP[b3]];

In[2]:= simplifyTerm@Expand[onShellP[c] /. { -m1^2 m2^2 + \gamma^2 \rightarrow S, m1^2 m2^2 - \gamma^2 \rightarrow -S }]

In[3]:= Coefficient[
  factorRho[simplifyTerm@Expand[onShellP[c] /. { -m1^2 m2^2 + \gamma^2 \rightarrow S, m1^2 m2^2 - \gamma^2 \rightarrow -S }] /.
  {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho, -2]

In[4]:= sum = Coefficient[
  factorRho[simplifyTerm[Expand[-\hbar * a3 * b3 - c * d]] /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho, -2]

In[5]:= (*ClearAll[holdDot]
SetOptions[D, NonConstants \rightarrow {holdDot}];*)

(*Product rule when holdDot is inside product*)
(*holdDot/:D[expr_Times,z_]:=MemberQ[expr,_holdDot,{0,Infinity}]:=Module[{factors=List@@expr},
Total[Table[D[factors[[i]],z]*Times@@Delete[factors,i],{i,Length[factors]}]]];*)

In[6]:= T = 0;

Do[T += L[i]*D[A0hbar[i][0], {x1, 2}], {i, 1, 4}]

cancelFactors@simplifiedForm@Expand@onShellP@T /. {\rho \rightarrow 1} /.
{l \odot l - 2 l \odot q + q \odot q \rightarrow q \cdot l^2}

Out[6]=

$$\frac{8 i m1^2 m2^4 \gamma}{q \cdot l^2 S^2 (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{8 i m2^2 \gamma^3}{q \cdot l^2 S^2 (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} -$$


$$\frac{8 i m1^2 m2^4 \gamma}{q \cdot l^2 S^2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{8 i m2^2 \gamma^3}{q \cdot l^2 S^2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} -$$


$$\frac{8 i m1^2 m2^4 \gamma}{q \cdot l^2 S^2 (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{8 i m2^2 \gamma^3}{q \cdot l^2 S^2 (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} +$$


$$\frac{8 i m1^2 m2^4 \gamma}{q \cdot l^2 S^2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{8 i m2^2 \gamma^3}{q \cdot l^2 S^2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)}$$


```

```

In[1]:= onShellP@Expand@D[A0[1] /. {ρ → 1} /. {l ⊗ l - 2 l ⊙ q + q ⊙ q → qml2}, x1]
Out[1]=

$$\frac{2 i m1^2 m2^2 \hbar}{qml2 S} - \frac{2 i \gamma^2 \hbar}{qml2 S} + \frac{2 i m2^2 \hbar^2 l \odot p1}{qml2 S} - \frac{2 i \gamma \hbar^2 l \odot p2}{qml2 S}$$


In[2]:= Expand[A0[1]] /. {ρ → 1} /. {l ⊗ l - 2 l ⊙ q + q ⊙ q → qml2}
Out[2]=

$$-\frac{i \hbar^2 l \odot l}{qml2} - \frac{2 i \hbar l \odot p1}{qml2} + \frac{2 i \hbar l \odot p2}{qml2} -$$


$$\frac{2 i \hbar^2 l \odot q}{qml2} + \frac{4 i p1 \odot p2}{qml2} - \frac{2 i \hbar p1 \odot q}{qml2} + \frac{2 i \hbar p2 \odot q}{qml2} - \frac{i \hbar^2 q \odot q}{qml2}$$


In[3]:= onShellP@Expand@D[A0[2] /. {ρ → 1} /. {l ⊗ l - 2 l ⊙ q + q ⊙ q → qml2}, x1]
Out[3]=

$$\frac{2 i m1^2 m2^2 \hbar}{qml2 S} - \frac{2 i \gamma^2 \hbar}{qml2 S}$$


In[4]:= onShellP@Expand@D[A0[3] /. {ρ → 1} /. {l ⊗ l - 2 l ⊙ q + q ⊙ q → qml2}, x1]
Out[4]=

$$\frac{2 i m1^2 m2^2 \hbar}{qml2 S} - \frac{2 i \gamma^2 \hbar}{qml2 S} - \frac{2 i m2^2 \hbar^2 l \odot p1}{qml2 S} + \frac{2 i \gamma \hbar^2 l \odot p2}{qml2 S}$$


In[5]:= onShellP@Expand@D[A0[4] /. {ρ → 1} /. {l ⊗ l - 2 l ⊙ q + q ⊙ q → qml2}, x1]
Out[5]=

$$\frac{2 i m1^2 m2^2 \hbar}{qml2 S} - \frac{2 i \gamma^2 \hbar}{qml2 S}$$


In[6]:= Expand[A0[1] - A0[3] /. {ρ → 1} /. {l ⊗ l - 2 l ⊙ q + q ⊙ q → qml2}]
Out[6]=

$$-\frac{4 i \hbar l \odot p1}{qml2} + \frac{4 i \hbar l \odot p2}{qml2} - \frac{4 i \hbar^2 l \odot q}{qml2}$$


In[7]:= Expand[A0hbarl[1][1, 1] - A0hbarl[3][1, 1]]
Out[7]=

$$-\frac{4 i l \odot p1}{q \odot q} + \frac{4 i l \odot p2}{q \odot q}$$


In[8]:= Expand[A0[2] + A0[4] /. {ρ → 1} /. {l ⊗ l - 2 l ⊙ q + q ⊙ q → qml2}]
Out[8]=

$$\frac{2 i \hbar^2 l \odot l}{qml2} + \frac{8 i p1 \odot p2}{qml2} - \frac{4 i \hbar p1 \odot q}{qml2} + \frac{4 i \hbar p2 \odot q}{qml2} - \frac{2 i \hbar^2 q \odot q}{qml2}$$


```

```
In[8]:= Expand@simplifiedForm@onShellP@D[A0hbar[1][1], x1]
Out[8]=

$$\frac{4 i m2^2 \rho^2 (l \odot p1)^2}{s (\rho^2 l \odot l - 2 \rho l \odot q + q \odot q)^2} - \frac{4 i m2^2 \rho^2 l \odot p1 l \odot p2}{s (\rho^2 l \odot l - 2 \rho l \odot q + q \odot q)^2} -$$


$$\frac{4 i \gamma \rho^2 l \odot p1 l \odot p2}{s (\rho^2 l \odot l - 2 \rho l \odot q + q \odot q)^2} + \frac{4 i \gamma \rho^2 (l \odot p2)^2}{s (\rho^2 l \odot l - 2 \rho l \odot q + q \odot q)^2} - \frac{2 i}{\rho^2 l \odot l - 2 \rho l \odot q + q \odot q}$$


In[9]:= 
a = D[p1 \Theta q, x1]
b = D[p1 \Theta q, x2]

Out[9]=

$$-\frac{m2^2 p1 \odot p1}{s} + \frac{\gamma p1 \odot p2}{s}$$


Out[10]=

$$\frac{\gamma p1 \odot p1}{s} - \frac{m1^2 p1 \odot p2}{s}$$


In[11]:= A0hbarl[1][0, 0]
Out[11]=

$$\frac{4 i p1 \odot p2}{q \odot q}$$

```

One Loop: Calculation 1

```
In[1]:= MaTeX["\\begin{aligned} R_2 = -\\partial_{x_1}(p_1\\wedge q\\wedge A_{1,1})+\\frac{1}{2}(\\partial_{x_1}^2-\\partial_{x_1}\\partial_{x_2})(q^2\\wedge p_1\\wedge q\\wedge A_{1,0})\\\\=\\sum_i L_i[C_{i,0}\\partial_{x_1}(p_1\\wedge q\\wedge _i\\tilde{A}_{0,1})+C_{i,1}\\partial_{x_1}(p_1\\wedge q\\wedge _i\\tilde{A}_{0,0})]+\\frac{1}{2}\\sum_i L_i C_{i,0}(\\partial_{x_i}^2-\\partial_{x_1}\\partial_{x_2})(p_1\\wedge q\\wedge ,q^2\\wedge _i\\tilde{A}_{0,0})\\\\-\\sum_i(_i\\tilde{A}_{0,1}\\partial_{x_1}L_i C_{i,0}+,_i\\tilde{A}_{0,0}\\partial_{x_1}L_i C_{i,1})+\\frac{q^2}{2}p_1\\wedge \\sum_i\\sum_j L_i C_{i,0}(_j\\tilde{A}_{0,0})(\\partial_{x_1}^2-\\partial_{x_1}\\partial_{x_2})\\partial_{x_1}(p_1\\wedge L_i C_{i,0})\\\\+\\frac{1}{2}\\sum_i L_i C_{i,0}(_i\\tilde{A}_{0,0}-\\frac{1}{2}\\sum_j L_j C_{j,0})\\partial_{x_1}(p_1\\wedge \\partial_{x_2}(p_1\\wedge q\\wedge ,q^2\\wedge _i\\tilde{A}_{0,0})+\\partial_{x_2}(L_i C_{i,0})\\partial_{x_1}(p_1\\wedge wedge q\\wedge ,q^2\\wedge _i\\tilde{A}_{0,0})]\\end{aligned}", "DisplayStyle" \rightarrow True]
```

$$\begin{aligned}
R_2 &= -\partial_{x_1}(p_1 \wedge q A_{1,1}) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})(q^2 p_1 \wedge q A_{1,1})) \\
&= -\sum_i L_i [C_{i,0}\partial_{x_1}(p_1 \wedge q_i \tilde{A}_{0,1}) + C_{i,1}\partial_{x_1}(p_1 \wedge q_i \tilde{A}_{0,0})] + \frac{1}{2}\sum_i L_i C_{i,0}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})(p_1 \wedge q q^2 i \tilde{A}_{0,0}) \\
&\quad - p_1 \wedge q \sum_i (i \tilde{A}_{0,1}\partial_{x_1}L_i C_{i,0} + i \tilde{A}_{0,0}\partial_{x_1}L_i C_{i,1}) + \frac{q^2}{2}p_1 \wedge q \sum_i i \tilde{A}_{0,0}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})L_i C_{i,0} \\
&+ \sum_i \partial_{x_1}(L_i C_{i,0})\partial_{x_1}(p_1 \wedge q q^2 i \tilde{A}_{0,0}) - \frac{1}{2}\sum_i [\partial_{x_1}(L_i C_{i,0})\partial_{x_2}(p_1 \wedge q q^2 i \tilde{A}_{0,0}) + \partial_{x_2}(L_i C_{i,0})\partial_{x_1}(p_1 \wedge q q^2 i \tilde{A}_{0,0})]
\end{aligned}$$

```
In[•]:= ol = 1;  
        fromDiag = 1;  
        toDiag = 4;
```

```
In[=]:= MaTeX["\\begin{aligned} X1 \\cong \\sum_i L_{iC_{i,0}} \\partial_{x_1}(p_1 \\wedge q) \\_, i \\tilde{A}_{0,1}) \\_ \\_ \\_ = \\partial_{x_1}(p_1 \\wedge q) \\wedge q \\sum_i L_{iC_{i,0}} \\_, i \\tilde{A}_{0,1} + (p_1 \\wedge q) \\sum_i L_{iC_{i,0}} \\_, \\partial_{x_1} \\_, i \\tilde{A}_{0,1} \\end{aligned}"]
```

Out[•]=

In[•]:= X1 = 0;

```
Do[X1 += -D[Wedge[p1, q] * A0hbarl[i][1, ol], x1] * L[i] * Cfbar[i][0],  
{i, fromDiag, toDiag}];
```

```
X1 = simplifiedForm@onShellP[X1];
```

```
In[1]:= cancelFactors@Coefficient[Expand@factorRho[x1 /. {l → ρ l, ε → ρ ε}], ρ, -2]
Out[1]= 0

In[2]:= MaTeX["\begin{aligned} & \text{q} \wedge \sum_i L_{iC[i,1]} \partial_{x_1}(p_1 \wedge q) \\ & = \sum_i L_{iC[i,1]} \partial_{x_1}(p_1 \wedge q) + p_1 \wedge \sum_i L_{iC[i,1]} \partial_{x_1}(q) \end{aligned}"]
Out[2]=

$$X2 \cong \sum_i L_i C_{i,1} \partial_{x_1}(p_1 \wedge q) \tilde{A}_{0,0}$$


$$= \partial_{x_1}(p_1 \wedge q) \sum_i L_i C_{i,1} \tilde{A}_{0,0} + p_1 \wedge q \sum_i L_i C_{i,1} \partial_{x_1} q \tilde{A}_{0,0}$$


In[3]:= X2 = 0;

Do[X2 += -D[Wedge[p1, q] * A0hbarl[i][0, ol], x1] * L[i] * Cfbar[i][1],
{i, fromDiag, toDiag}];

X2 = simplifiedForm@onShellP[X2];

In[4]:= cancelFactors@Coefficient[Expand@factorRho[X2 /. {l → ρ l, ε → ρ ε}], ρ, -2]
Out[4]=

$$\frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} -$$


$$- \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} - \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} -$$


$$+ \frac{4 i m2^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) q \odot q} + \frac{4 i m2^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} +$$


$$- \frac{4 i \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} - \frac{4 i \gamma^3 p_1 \wedge q}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q}$$

```

```
In[1]:= MaTeX["\\begin{aligned} & X3 \\cong -\\frac{1}{2}\\sum_i L_i C_{i,0}(\\partial_{x_1})^2 - \\partial_{x_1}\\partial_{x_2}(p_1\\wedge q)\\wedge, q^2\\wedge, _i\\tilde{A}_{0,0})\\\\& = -\\frac{1}{2}\\partial_{x_1}^2(p_1\\wedge q)\\wedge\\sum_i L_i C_{i,0}\\wedge, q^2\\wedge, _i\\tilde{A}_{0,0}-\\partial_{x_1}(p_1\\wedge q)\\wedge\\sum_i L_i C_{i,0}\\wedge, \\partial_{x_1}(q^2\\wedge, _i\\tilde{A}_{0,0})-\\frac{1}{2}(p_1\\wedge q)\\wedge\\sum_i L_i C_{i,0}\\wedge, \\partial_{x_1}^2(q^2\\wedge, _i\\tilde{A}_{0,0})\\\\& -\\frac{1}{2}\\partial_{x_1}\\partial_{x_2}(p_1\\wedge q)\\wedge, q^2\\wedge, _i\\tilde{A}_{0,0}-\\frac{1}{2}\\partial_{x_1}(p_1\\wedge q)\\wedge\\sum_i L_i C_{i,0}\\wedge, \\partial_{x_2}(q^2\\wedge, _i\\tilde{A}_{0,0})-\\frac{1}{2}(p_1\\wedge q)\\wedge\\sum_i L_i C_{i,0}\\wedge, \\partial_{x_1}(q^2\\wedge, _i\\tilde{A}_{0,0})-\\frac{1}{2}(p_1\\wedge q)\\wedge\\sum_i L_i C_{i,0}\\wedge, \\partial_{x_2}(q^2\\wedge, _i\\tilde{A}_{0,0})\\\\& \\end{aligned}"]
```

$$\begin{aligned} Out[=] &= X3 \cong -\frac{1}{2} \sum_i L_i C_{i,0} (\partial_{x_1}^2 - \\ &= -\frac{1}{2} \partial_{x_1}^2 (p_1 \wedge q) \sum_i L_i C_{i,0} q^2 {}_i \tilde{A}_{0,0} - \partial_{x_1} (p_1 \wedge q) \sum_i L_i C_{i,0} \partial_{x_1} (q^2 {}_i \tilde{A}_{0,0}) - \frac{1}{2} (p_1 \wedge q) \\ &\quad - \frac{1}{2} \partial_{x_1} \partial_{x_2} (p_1 \wedge q) \sum_i L_i C_{i,0} {}_i \tilde{A}_{0,0} - \frac{1}{2} \partial_{x_1} (p_1 \wedge q) \sum_i L_i C_{i,0} \partial_{x_2} (q^2 {}_i \tilde{A}_{0,0}) - \frac{1}{2} \partial_{x_2} (p_1 \wedge q) \sum_i L_i C_{i,0} \partial_{x_1} (q^2 {}_i \tilde{A}_{0,0}) - \frac{1}{2} (p_1 \wedge q) \sum_i \end{aligned}$$

```
In[•]:= Do[X306[n] = 0, {n, 0, 6}];
```

```

Do[Do[X306[n] +=  $\frac{-\text{Binomial}[2, n]}{2} D[q \odot q \times \text{ToExpression}["A0" \& ToString[i] \& "hbar"][[0],$ 
      {x1, n}] \times \text{ToExpression}["L" \& ToString[i]] \times
      \text{ToExpression}["C" \& ToString[i] \& "hbar"][[0], {i, 1, 2}], {n, 0, 2}];

Do[Do[X306[2 n + m + 3] +=  $\frac{-1}{2} D[q \odot q \times \text{ToExpression}["A0" \& ToString[i] \& "hbar"][[0], {x1, n}, {x2, m}] \times$ 
      \text{ToExpression}["L" \& ToString[i]] \times
      \text{ToExpression}["C" \& ToString[i] \& "hbar"][[0], {i, 1, 2}], {n, 0, 1}, {m, 0, 1}]

```

In[•]:= (*X3[0] and X3[3] are zero*)

X306[0] = 0;
X306[3] = 0;

```
In[•]:= Do[X306[n] = onShellP[X306[n]], {n, 0, 6}]
```

```

X3 = -  $\frac{\gamma \text{Wedge}[p1, p2]}{s} X306[1] - \text{Wedge}[p1, q] \times X306[2] -$ 
       $\frac{\gamma \text{Wedge}[p1, p2]}{s} X306[4] + \frac{m1^2 \text{Wedge}[p1, p2]}{s} X306[5] - \text{Wedge}[p1, q] \times X306[6];$ 

In[=]:= X3 = 0;

Do[X3 +=  $\frac{1}{2} L[i] \times Cfhbar[i][0] * (D[D[Wedge[p1, q]] * q \ominus q * A0hbarl[i][0, ol], x1], x1) -$ 
     D[D[Wedge[p1, q]] * q \ominus q * A0hbarl[i][0, ol], x2], {i, fromDiag, toDiag}];

X3 = simplifiedForm@onShellP@X3;

In[=]:= cancelFactors@Coefficient[Expand@factorRho[X3 /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho, -2]

Out[=]= 0

In[=]:= MaTeX[
  "X4 \cong p_1 \wedge q \sum_i i \tilde{A}_{0,1} \partial_{x_1} L_i C_{i,0}"]

Out[=]= X4 \cong p_1 \wedge q \sum_i i \tilde{A}_{0,1} \partial_{x_1} L_i C_{i,0}

In[=]:= X4 = 0;

Do[X4 += -Wedge[p1, q] * A0hbarl[i][1, ol] \times D[L[i] \times Cfhbar[i][0], x1],
 {i, fromDiag, toDiag}];

X4 = simplifiedForm@onShellP[X4];

In[=]:= -simplifiedForm[onShellP[Wedge[p1, q] * A0hbarl[4][1, 1] \times D[L[4] \times Cfhbar[4][0], x1]]]

Out[=]= 
$$-\frac{2 i \gamma (l \odot p1 + l \odot p2) p1 \wedge q}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) q \ominus q}$$


In[=]:= D[L[4] \times Cfhbar[4][0], x1]

Out[=]= 
$$-\frac{\gamma \left(-\frac{m2^2 p1 \odot p1}{s} + \frac{\gamma p1 \odot p2}{s}\right)}{(-i \epsilon + l \odot p2) (i \epsilon - l \odot p1 + p1 \odot q)^2}$$


In[=]:= Cfhbar[4][0]

Out[=]= \gamma

```

```

In[1]:= cancelFactors@Coefficient[Expand@factorRho[x4 /. {l → ρ l, ε → ρ ε}], ρ, -2]

Out[1]=

$$-\frac{2 i \gamma p1 \wedge q}{(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} - \frac{2 i \gamma p1 \wedge q}{(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q}$$


MaTeX[
 "X5\\cong\\,p_1\\wedge q\\sum_i\\,\\tilde{A}_{0,0}\\partial_{x_1}L_iC_{i,1}"
]

Out[2]=

$$X5 \cong p_1 \wedge q \sum_i \tilde{A}_{0,0} \partial_{x_1} L_i C_{i,1}$$


In[3]:= s = 0;
Do[s += aWedgeDb[L[i]*Cf[i], p1, p1], {i, 1, 4}];

In[4]:= cancelFactors@Coefficient[
 Expand@factorRho[simplifiedForm[onShellP[s]] /. {l → ρ l, ε → ρ ε}], ρ, -3]


$$-\frac{\gamma \hbar q \odot q l \wedge p1}{2 (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{\gamma \hbar q \odot q l \wedge p1}{2 (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} -$$


$$\frac{\gamma p1 \wedge q}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \frac{\gamma p1 \wedge q}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)}$$


$$\frac{\gamma l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \frac{\gamma l \wedge p1}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} +$$


$$\frac{\hbar p1 \wedge q}{2 (-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{\hbar p1 \wedge q}{2 (-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \frac{\hbar p1 \wedge q}{(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)}$$


In[5]:= X5 = 0;

Do[X5 += -Wedge[p1, q] × A0hbarl[i][0, ol] × D[L[i] × Cfbar[i][1], x1],
 {i, fromDiag, toDiag}];

X5 = simplifiedForm@onShellP[X5];

```

In[8]:= `cancelFactors@Coefficient[Expand@factorRho[X5 /. {l → ρ l, ε → ρ ε}], ρ, -2]`

Out[8]=

$$\begin{aligned} & \frac{2 i \gamma^2 l \odot l p_1 \wedge q}{(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2)^2 q \odot q} + \\ & \frac{4 i \gamma^2 l \odot l p_1 \wedge q}{(-i \epsilon + l \odot p_1)^3 (-i \epsilon + l \odot p_2) q \odot q} - \frac{2 i \gamma p_1 \wedge q}{(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) q \odot q} + \\ & \frac{4 i m_2^2 \gamma^2 p_1 \wedge q}{S (-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) q \odot q} + \frac{2 i \gamma^2 l \odot l p_1 \wedge q}{(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)^2 q \odot q} - \\ & \frac{4 i \gamma^2 l \odot l p_1 \wedge q}{(-i \epsilon + l \odot p_1)^3 (i \epsilon + l \odot p_2) q \odot q} + \frac{2 i \gamma p_1 \wedge q}{(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} - \\ & \frac{4 i m_2^2 \gamma^2 p_1 \wedge q}{S (-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} - \frac{4 i \gamma^3 p_1 \wedge q}{S (-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} - \\ & \frac{4 i \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^3 p_1 \wedge q}{S (i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} \end{aligned}$$

In[9]:= `MaTeX["X6\\cong-\\frac{1}{2}p_1\\wedge q\\sum_i q^2 {}_i\tilde{A}_{0,0}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})L_i C_{i,0}"]`

Out[9]=

$$X6 \cong -\frac{1}{2} p_1 \wedge q \sum_i q^2 {}_i\tilde{A}_{0,0} (\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2}) L_i C_{i,0}$$

In[10]:= `X6 = 0;`

`Do[X6 += $\frac{\text{Wedge}[p_1, q] \times q \odot q}{2}$ A0hbarl[i][0, ol] \times (D[D[L[i] \times Cfbar[i][0], x1], x1] - D[D[L[i] \times Cfbar[i][0], x2], x1]), {i, fromDiag, toDiag}];`

`X6 = simplifiedForm@onShellP[X6];`

In[11]:= `cancelFactors@Coefficient[Expand@factorRho[X6 /. {l → ρ l, ε → ρ ε}], ρ, -2]`

Out[11]=

$$0$$

```
In[1]:= MaTeX[
  "\begin{aligned} X7 & \cong -\sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_1}(p_1 \wedge q) q^2 {}_i \tilde{A}_{0,0} \\
  & = -\partial_{x_1}(p_1 \wedge q) \sum_i q^2 {}_i \tilde{A}_{0,0} \partial_{x_1}(L_i C_{i,0}) - p_1 \wedge q \sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_1}(q^2 {}_i \tilde{A}_{0,0}) \\
  & = 0;
\end{aligned}"
]

Out[1]=
X7 \cong -\sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_1}(p_1 \wedge q) q^2 {}_i \tilde{A}_{0,0}
= -\partial_{x_1}(p_1 \wedge q) \sum_i q^2 {}_i \tilde{A}_{0,0} \partial_{x_1}(L_i C_{i,0}) - p_1 \wedge q \sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_1}(q^2 {}_i \tilde{A}_{0,0})
= 0;

In[2]:= X7 = 0;

Do[X7 += D[Wedge[p1, q] \times q \otimes q \times A0hbarl[i][0, ol], x1] \times D[L[i] \times Cfbar[i][0], x1],
{i, fromDiag, toDiag}];

X7 = simplifiedForm@onShellP[X7];

In[3]:= X7 = 0;

Do[X7 += Wedge[p1, q] \times D[q \otimes q \times A0[i] /. {\rho \rightarrow 1}, x1] \times D[L[i] \times Cf[i], x1], {i, 1, 4}];

X7 = simplifiedForm@onShellP[X7];

In[4]:= a = Total@Select[List @@ Expand[X7 /. l \otimes l - 2 l \otimes q + q \otimes q \rightarrow qml2], Exponent[#, \hbar] \leq 1 &];

In[5]:= b = {};
Do[AppendTo[b, simplifiedForm[a[[i]]]], {i, 1, Length[a]}];
ClearAll[a];
a = Total[b];
ClearAll[b];

In[6]:= b = {};
Do[AppendTo[b, simplifyTerm[a[[i]]]], {i, 1, Length[a]}];
ClearAll[a];
a = Total[b];
ClearAll[b];

Expand[Series[Coefficient[factorRho[a /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho, -3] /. {qml2 \rightarrow
1 / (Normal[Series[1 / (\rho^2 l \otimes l + q \otimes q - 2 \rho l \otimes q), {\rho, 0, 1}]] /. {\rho \rightarrow 1})}, {\rho, 0, 1}]]]

Expand[Series[Coefficient[factorRho[a /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho, -3] /. {qml2 \rightarrow
1 / (Normal[Series[1 / (\rho^2 l \otimes l + q \otimes q - 2 \rho l \otimes q), {\rho, 0, 1}]] /. {\rho \rightarrow 1})}, {\rho, 0, 1}]]]

Out[6]=
0
```

```
In[=]:= simplifyTerm[  
  Expand[simplifiedForm@Normal[Series[X7 /. l ⊕ l - 2 l ⊗ q + q ⊗ q → q m l2, {h, 0, 1}]]]]  
  
In[=]:= cancelFactors@Coefficient[Expand@factorRho[X7 /. {l → ρ l, ε → ρ ε}], ρ, -2]  
  
Out[=]= 0  
  
In[=]:= MaTeX[  
  " \begin{aligned} X8 & \cong \frac{1}{2} \sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_2}(p_1 \wedge q q^2 {}_i \tilde{A}_{0,0}) \\ & = \frac{1}{2} \partial_{x_2}(p_1 \wedge q) \sum_i q^2 {}_i \tilde{A}_{0,0} \partial_{x_1}(L_i C_{i,0}) + \frac{1}{2} p_1 \wedge q \sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_2}(q^2 {}_i \tilde{A}_{0,0}) \end{aligned}" ]  
  
Out[=]=  

$$\begin{aligned} X8 &\cong \frac{1}{2} \sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_2}(p_1 \wedge q q^2 {}_i \tilde{A}_{0,0}) \\ &= \frac{1}{2} \partial_{x_2}(p_1 \wedge q) \sum_i q^2 {}_i \tilde{A}_{0,0} \partial_{x_1}(L_i C_{i,0}) + \frac{1}{2} p_1 \wedge q \sum_i \partial_{x_1}(L_i C_{i,0}) \partial_{x_2}(q^2 {}_i \tilde{A}_{0,0}) \end{aligned}$$
  
  
In[=]:= X8 = 0;  
  
Do[X8 +=  $\frac{-1}{2} D[Wedge[p1, q] \times q \odot q \times A0hbarl[i][0, ol], x2] \times D[L[i] \times Cfbar[i][0], x1]$ ,  
 {i, fromDiag, toDiag}];  
  
X8 = simplifiedForm@onShellP[X8];  
  
In[=]:= cancelFactors@Coefficient[Expand@factorRho[X8 /. {l → ρ l, ε → ρ ε}], ρ, -2]  
  
Out[=]= 0  
  
In[=]:= X8 = 0;  
  
Do[X8 +=  $\frac{-1}{2} D[Wedge[p1, q] \times q \odot q \times (A0hbarl[i][0, ol] + \hbar * A0hbarl[i][1, ol]), x2] \times$   
  $D[L[i] \times (Cfbar[i][0] + \hbar * Cfbar[i][1]), x1]$ , {i, fromDiag, toDiag}];  
  
X8 = simplifiedForm@onShellP[X8];
```

```

In[8]:= X8 = 0;

Do[X8 += -1/2 D[Wedge[p1, q] \times q \Theta q \times A0[i], x2] \times D[L[i] \times Cf[i], x1], {i, 1, 4}];

X8 = simplifiedForm@onShellP[X8];

In[9]:= a = Total@Select[
  List@@Expand[X8 /. {\rho \rightarrow 1} /. l \odot l - 2 l \odot q + q \odot q \rightarrow qml2], Exponent[#, \hbar] \leq 1 &];

In[10]:= b = {};
Do[AppendTo[b, simplifiedForm[a[[i]]]], {i, 1, Length[a]}];
ClearAll[a];
a = Total[b];
ClearAll[b];

In[11]:= b = {};
Do[AppendTo[b, simplifyTerm[a[[i]]]], {i, 1, Length[a]}];
ClearAll[a];
a = Total[b];
ClearAll[b];

In[12]:= Coefficient[factorRho[a /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho, -2]

Out[12]=

$$\begin{aligned}
& -\frac{i m1^2 \gamma^2 \hbar l \odot l q \odot q p1 \wedge p2}{qml2 S(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} + \\
& \frac{2 i m1^2 \gamma \hbar q \odot q p1 \wedge p2}{qml2 S(-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{2 i m1^2 m2^2 \gamma^2 \hbar q \odot q p1 \wedge p2}{qml2 S^2(-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \\
& \frac{2 i m1^2 m2^2 \gamma^2 \hbar q \odot q p1 \wedge p2}{qml2 S^2(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{2 i m1^2 \gamma^3 \hbar q \odot q p1 \wedge p2}{qml2 S^2(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \\
& \frac{2 i m1^2 \gamma \hbar q \odot q p1 \wedge p2}{qml2 S(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{2 i m1^2 \gamma^3 \hbar q \odot q p1 \wedge p2}{qml2 S^2(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \\
& \frac{4 i \gamma^3 q \odot q p1 \wedge q}{qml2^2 S(-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{4 i \gamma^3 q \odot q p1 \wedge q}{qml2^2 S(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)}
\end{aligned}$$


```

```
In[1]:= MaTeX[  

  " \begin{aligned} X9 &\cong \frac{1}{2} \sum_i \partial_{x_2} (L_i C_{i,0}) \partial_{x_1} (p_1 \wedge q) q^2 {}_i \tilde{A}_{0,0} \\ &= \frac{1}{2} \partial_{x_1} (p_1 \wedge q) \sum_i q^2 {}_i \tilde{A}_{0,0} \partial_{x_2} (L_i C_{i,0}) + \frac{1}{2} p_1 \wedge q \sum_i \partial_{x_2} (L_i C_{i,0}) \partial_{x_1} (q^2 {}_i \tilde{A}_{0,0}) \end{aligned}" ]  

Out[1]=  


$$X9 \cong \frac{1}{2} \sum_i \partial_{x_2} (L_i C_{i,0}) \partial_{x_1} (p_1 \wedge q) q^2 {}_i \tilde{A}_{0,0}$$
  


$$= \frac{1}{2} \partial_{x_1} (p_1 \wedge q) \sum_i q^2 {}_i \tilde{A}_{0,0} \partial_{x_2} (L_i C_{i,0}) + \frac{1}{2} p_1 \wedge q \sum_i \partial_{x_2} (L_i C_{i,0}) \partial_{x_1} (q^2 {}_i \tilde{A}_{0,0})$$
  

In[2]:= X9 = 0;  

  

Do[ $X9 += \frac{-1}{2} D[Wedge[p1, q] \times q \Theta q \times A0hbarl[i][0, o1], x1] \times D[L[i] \times Cfbar[i][0], x2],$   

  {i, fromDiag, toDiag}];  

  

X9 = simplifiedForm@onShellP[X9];  

In[3]:= cancelFactors@Coefficient[Expand@factorRho[X9 /. {l -> \rho l, \epsilon -> \rho \epsilon}], \rho, -2]  

Out[3]=  

0  

In[4]:= XSuml0 = 0;  

  

Do[ $XSuml0 += ToExpression["X" \& ToString[n]], \{n, 1, 9\}]  

In[5]:= YSuml0 = Y1 + Y2 + Y3;$ 
```

```
In[8]:= cancelFactors@  
Coefficient[Expand@factorRho[(XSuml0 + YSuml0) /. {l → ρ l, ε → ρ ε}], ρ, -2]  
Out[8]=
```

$$\begin{aligned} & \frac{8 i \gamma^2 l \odot q l \wedge p_1}{(-i \epsilon + l \odot p_1)^3 (-i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^2 l \odot q l \wedge p_1}{(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)^2 q \odot q} - \\ & \frac{4 i \gamma^2 l \odot q l \wedge p_1}{(i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2)^2 q \odot q} - \frac{8 i \gamma^2 l \odot q l \wedge p_1}{(-i \epsilon + l \odot p_1)^3 (i \epsilon + l \odot p_2) q \odot q} + \\ & \frac{4 i \gamma l \odot q p_1 \wedge p_2}{(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} - \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2) q \odot q} + \\ & \frac{4 i \gamma l \odot q p_1 \wedge p_2}{(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} - \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} - \\ & \frac{4 i \gamma l \odot q p_1 \wedge p_2}{(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} + \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2)^2 q \odot q} - \\ & \frac{4 i \gamma l \odot q p_1 \wedge p_2}{(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^3 l \odot q p_1 \wedge p_2}{S(-i \epsilon + l \odot p_1)^2 (i \epsilon + l \odot p_2) q \odot q} + \\ & \frac{4 i m2^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (-i \epsilon + l \odot p_2) q \odot q} - \frac{4 i m2^2 \gamma^2 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} - \\ & \frac{4 i \gamma^3 p_1 \wedge q}{S(-i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} + \frac{4 i \gamma^3 p_1 \wedge q}{S(i \epsilon + l \odot p_1) (i \epsilon + l \odot p_2) q \odot q} \end{aligned}$$

```
In[9]:= MaTeX["\\begin{aligned} Y=\\sum_ip_1\\wedge\\partial_{p_1}A_{1,1}+\\frac{1}{2}\\sum_i(\\partial_{x_1}-\\partial_{x_2})(q^2p_1\\wedge\\sum_ip_1\\wedge\\partial_{p_1}(L_iC_{i,0}A_{0,1}+L_iC_{i,1}A_{0,0})-\\frac{1}{2}\\sum_i(L_iC_{i,0}A_{0,1}+L_iC_{i,1}A_{0,0}))\\end{aligned}"]
```

```
Out[9]=
```

$$\begin{aligned} Y &= \sum_i p_1 \wedge \partial_{p_1} A_{1,1} + \frac{1}{2} \sum_i (\partial_{x_1} - \partial_{x_2})(q^2 p_1 \wedge \sum_i p_1 \wedge \partial_{p_1} (L_i C_{i,0} A_{0,1} + L_i C_{i,1} A_{0,0})) \\ &= \sum_i p_1 \wedge \partial_{p_1} (L_i C_{i,0} A_{0,1} + L_i C_{i,1} A_{0,0}) - \frac{1}{2} \sum_i (\partial_{x_1} - \partial_{x_2})(q^2 p_1 \wedge \partial_{p_1} (L_i C_{i,0} A_{0,1} + L_i C_{i,1} A_{0,0})) \end{aligned}$$

```
In[10]:= ol = 1;  
fromDiag = 1;  
toDiag = 4;
```

```

In[=]:= Y1 = 0;

Do[Y1 += aWedgeDb[L[i] × Cfbar[i][0], p1, p1] × A0hbarl[i][1, ol], {i, fromDiag, toDiag}];

Y1 = simplifiedForm@onShellP@Y1;

In[=]:= Y2 = 0;

Do[Y2 += aWedgeDb[L[i] × Cfbar[i][1] × A0hbarl[i][0, ol], p1, p1], {i, fromDiag, toDiag}];

Y2 = simplifiedForm@onShellP@Y2;

In[=]:= Y3 = 0;

Do[Y3 += -1/2
  (D[q ⊗ q * aWedgeDb[L[i] × Chbar[i][0] × A0hbarl[i][0, ol], p1, p1], x1] - D[q ⊗ q * aWedgeDb[
    L[i] × Cfbar[i][0] × A0hbarl[i][0, ol], p1, p1], x2]), {i, fromDiag, toDiag}];

Y3 = simplifiedForm@onShellP@Y3;

In[=]:= cancelFactors@Coefficient[Expand@factorRho[Y1 /. {l → ρ l, ε → ρ ε}], ρ, -2]

Out[=]=

$$\frac{2 i \gamma p_1 \wedge q}{(-i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2) q \odot q} + \frac{2 i \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2) q \odot q}$$


$$- \frac{2 i \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2) q \odot q} + \frac{6 i \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2) q \odot q}$$


In[=]:= X4 = 0;

Do[X4 += -Wedge[p1, q] * A0hbarl[i][1, ol] × D[L[i] × Cfbar[i][0], x1],
{i, fromDiag, toDiag}];

X4 = simplifiedForm@onShellP[X4];

In[=]:= MaTeX["\begin{aligned} R_2 &= -\partial_{x_1}(p_1 \wedge q A_{1,1}) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1} \partial_{x_2})(q^2 p_1 \wedge q \not\wedge \\
&\quad q \wedge A_{1,1}) + \frac{1}{2}(\partial_{x_1} \partial_{x_2} - \partial_{x_2} \partial_{x_1})(q^2 p_1 \wedge q \not\wedge \\
&\quad p_1 \wedge q \wedge A_{1,0}) \\ &\quad + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1} \partial_{x_2})(q^2 p_1 \wedge q \not\wedge \\
&\quad q \wedge L_i(C_{i,1}A_{0,0} + C_{i,0}A_{0,1})) + \frac{1}{2}(\partial_{x_1} \partial_{x_2} - \partial_{x_2} \partial_{x_1})(q^2 p_1 \wedge q \not\wedge \\
&\quad q \wedge L_i(C_{i,0}A_{0,0} + C_{i,1}A_{0,1})) \end{aligned}"]

Out[=]=

$$R_2 = -\partial_{x_1}(p_1 \wedge q A_{1,1}) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1} \partial_{x_2})(q^2 p_1 \wedge q \not\wedge q \wedge A_{1,1})$$


$$+ \frac{1}{2}(\partial_{x_1} \partial_{x_2} - \partial_{x_2} \partial_{x_1})(q^2 p_1 \wedge q \not\wedge p_1 \wedge q \wedge A_{1,0})$$


$$+ \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1} \partial_{x_2})(q^2 p_1 \wedge q \not\wedge q \wedge L_i(C_{i,0}A_{0,0} + C_{i,1}A_{0,1}))$$


$$+ \frac{1}{2}(\partial_{x_1} \partial_{x_2} - \partial_{x_2} \partial_{x_1})(q^2 p_1 \wedge q \not\wedge q \wedge L_i(C_{i,1}A_{0,0} + C_{i,0}A_{0,1}))$$


```

```

(*ol=0;
fromDiag=1;
toDiag=2;

ClearAll[X];
Do[X[i]=0;, {i,1,9}];

Do[X[1]+=D[Wedge[p1,q]*A0hbarl[i][1,ol],x1]*L[i]*Cfhbar[i][0];
X[2]+=D[Wedge[p1,q]*A0hbarl[i][0,ol],x1]*L[i]*Cfhbar[i][1];
X[3]+=- $\frac{1}{2}$ L[i]*Cfhbar[i][0]*(D[D[Wedge[p1,q]*qOq*A0hbarl[i][0,ol],x1],x1]-
D[D[Wedge[p1,q]*qOq*A0hbarl[i][0,ol],x2],x1]);
X[4]+=Wedge[p1,q]*A0hbarl[i][1,ol]*D[L[i]*Cfhbar[i][0],x1];
X[5]+=Wedge[p1,q]*A0hbarl[i][0,ol]*D[L[i]*Cfhbar[i][1],x1];
X[6]+=- $\frac{-Wedge[p1,q]*qOq}{2}$ A0hbarl[i][0,ol]*
(D[D[L[i]*Cfhbar[i][0],x1],x1]-D[D[L[i]*Cfhbar[i][0],x2],x1]);
X[7]+=-D[Wedge[p1,q]*qOq*A0hbarl[i][0,ol],x1]*D[L[i]*Cfhbar[i][0],x1];
X[8]+= $\frac{1}{2}$ D[Wedge[p1,q]*qOq*A0hbarl[i][0,ol],x2]*D[L[i]*Cfhbar[i][0],x1];
X[9]+= $\frac{1}{2}$ D[Wedge[p1,q]*qOq*A0hbarl[i][0,ol],x1]*D[L[i]*Cfhbar[i][0],x2];
,{i,fromDiag,toDiag}];

Do[X[i]=simplifiedForm@onShellP[X[i]];,{i,1,9}];

XSum=0;
Do[XSum+=X[i],{i,1,9}];*)

```

```

ol = 2;
fromDiag = 1;
toDiag = 4;

ClearAll[X];
Do[X[i] = 0;, {i, 1, 3}];

Do[X[1] += -D[Wedge[p1, q] × L[i] × Cfbar[i][0] × A0hbarl[i][1, ol], x1];
X[2] += -D[Wedge[p1, q] × L[i] × Cfbar[i][1] × A0hbarl[i][0, ol], x1];
X[3] +=
  1
  - D[(D[q ⊗ q * Wedge[p1, q] × L[i] × Cfbar[i][0] × A0hbarl[i][0, ol], x1] - D[q ⊗ q * Wedge[p1,
  2
    q] × L[i] × Cfbar[i][0] × A0hbarl[i][0, ol], x2]), x1], {i, fromDiag, toDiag}];

Do[X[i] = simplifiedForm@onShellP@X[i];, {i, 1, 3}];

XSum = 0;
Do[XSum += X[i], {i, 1, 3}];

In[=]:= ol = 0;
fromDiag = 1;
toDiag = 4;

ClearAll[Z];
Do[Z[i] = 0;, {i, 1, 3}];

Do[Z[1] += aWedgeDb[L[i] × Cfbar[i][0] × A0hbarl[i][1, ol], p1, p1];
Z[2] += aWedgeDb[L[i] × Cfbar[i][1] × A0hbarl[i][0, ol], p1, p1];
Z[3] +=
  -1
  (D[q ⊗ q * aWedgeDb[L[i] × Cfbar[i][0] × A0hbarl[i][0, ol], p1, p1], x1] -
  2
    D[q ⊗ q * aWedgeDb[L[i] × Cfbar[i][0] × A0hbarl[i][0, ol], p1, p1],
    x2]), {i, fromDiag, toDiag}];

Do[Z[i] = simplifiedForm@onShellP@Z[i];, {i, 1, 3}];

ZSum = 0;
Do[ZSum += Z[i], {i, 1, 3}];

In[=]:= cancelFactors@Coefficient[Expand@factorRho[(XSum + ZSum) /. {l → ρ l, ε → ρ ε}], ρ, -2]

In[=]:= cancelFactors@Coefficient[Expand@factorRho[(ZSum) /. {l → ρ l, ε → ρ ε}], ρ, -2]

```

$$\begin{aligned}
& \frac{4 \bar{i} \gamma p1 \wedge q}{(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} - \frac{2 \bar{i} \gamma p1 \wedge q}{(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} - \\
& \frac{4 \bar{i} \gamma p1 \wedge q}{(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q} + \frac{6 \bar{i} \gamma p1 \wedge q}{(i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q} \\
(*ZSum: l^0, l*) \\
& - \frac{2 \bar{i} \gamma p1 \wedge q}{(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} + \frac{6 \bar{i} \gamma p1 \wedge q}{(i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q} \\
& \frac{2 \bar{i} \gamma p1 \wedge q}{(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} + \frac{2 \bar{i} \gamma p1 \wedge q}{(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q} \\
(*XSum: l^0, l*) \\
& \frac{2 \bar{i} \gamma p1 \wedge q}{(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} - \frac{6 \bar{i} \gamma p1 \wedge q}{(i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q} \\
& - \frac{2 \bar{i} \gamma p1 \wedge q}{(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2) q \odot q} - \frac{2 \bar{i} \gamma p1 \wedge q}{(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2) q \odot q}
\end{aligned}$$

First Pair

$$\begin{aligned}
In[1]:= & \text{Bx1} = \frac{(2 p1 + \hbar l) \odot (2 p2 - \hbar l) (2 p1 + \hbar q + \hbar l) \odot (2 p2 - \hbar q - \hbar l)}{(q - l) \odot (q - l) (2 p1 \odot l + \hbar l \odot l + i \epsilon) (2 p2 \odot l - \hbar l \odot l - i \epsilon)}; \\
Cx1 = & \left((2 p1 + \hbar q + \hbar l) \odot (2 p2 - \hbar q + \hbar l) (2 p1 + \hbar l) \odot (2 p2 - 2 \hbar q + \hbar l) \right) / \\
& \left((q - l) \odot (q - l) (2 p1 \odot l + \hbar l \odot l + i \epsilon) (2 p2 \odot (q - l) - \hbar (q - l) \odot (q - l) - i \epsilon) \right); \\
In[2]:= & \text{B1C1} = \text{Bx1} + \text{Cx1} /. \{l \odot l - 2 l \odot q + q \odot q \rightarrow qml2, -l \odot l + 2 l \odot q - q \odot q \rightarrow -qml2\}; \\
In[3]:= & \text{B1C1} \\
Out[3]= & \left(\begin{aligned} & (\hbar^2 l \odot l + 2 \hbar l \odot p1 + 2 \hbar l \odot p2 - 2 \hbar^2 l \odot q + 4 p1 \odot p2 - 4 \hbar p1 \odot q) \\ & (\hbar^2 l \odot l + 2 \hbar l \odot p1 + 2 \hbar l \odot p2 + 4 p1 \odot p2 - 2 \hbar p1 \odot q + 2 \hbar p2 \odot q - \hbar^2 q \odot q) \end{aligned} \right) / \\
& \left(qml2 (i \epsilon + \hbar l \odot l + 2 l \odot p1) (-i \epsilon - qml2 \hbar + 2 (-l \odot p2 + p2 \odot q)) \right) + \\
& \left(\begin{aligned} & (-\hbar^2 l \odot l - 2 \hbar l \odot p1 + 2 \hbar l \odot p2 + 4 p1 \odot p2) \\ & (-\hbar^2 l \odot l - 2 \hbar l \odot p1 + 2 \hbar l \odot p2 - 2 \hbar^2 l \odot q + 4 p1 \odot p2 - 2 \hbar p1 \odot q + 2 \hbar p2 \odot q - \hbar^2 q \odot q) \end{aligned} \right) / \\
& \left(qml2 (i \epsilon + \hbar l \odot l + 2 l \odot p1) (-i \epsilon - \hbar l \odot l + 2 l \odot p2) \right) \\
In[4]:= & \text{Do}[\text{Bx1hbar}[n] = \text{Coefficient}[\text{Normal}[\text{Series}[\text{Bx1}, \{\hbar, 0, 1\}]], \hbar, n]; \\
& \text{Cx1hbar}[n] = \text{Coefficient}[\text{Normal}[\text{Series}[\text{Cx1}, \{\hbar, 0, 1\}]], \hbar, n], \{n, 0, 1\}];
\end{aligned}$$

```
In[8]:= simplifiedForm@Bx1hbar[0]
Out[8]=

$$\frac{4 (p_1 \odot p_2)^2}{(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2)(l \odot l - 2 l \odot q + q \odot q)}$$


In[9]:= simplifiedForm@Cx1hbar[0]
Out[9]=

$$-\frac{4 (p_1 \odot p_2)^2}{(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2 - p_2 \odot q)(l \odot l - 2 l \odot q + q \odot q)}$$

```

R_2

```
In[10]:= MaTeX["\begin{aligned}R_2 = -\partial_{x_1}(p_1 \wedge q(B_0 + C_0)) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})(q^2 p_1 \wedge q(B_{-1} + C_{-1}))
+ q \left( -\partial_{x_1}(B_0 + C_0) + \frac{1}{2}\partial_{x_1}^2(q^2(B_{-1} + C_{-1})) - \frac{1}{2}\partial_{x_1}\partial_{x_2}(q^2(B_{-1} + C_{-1})) \right)
+ \partial_{x_1}(p_1 \wedge q) \left( -(B_0 + C_0) + \partial_{x_1}(q^2(B_{-1} + C_{-1})) - \frac{1}{2}\partial_{x_2}(q^2(B_{-1} + C_{-1})) \right)
- \frac{1}{2}\partial_{x_2}(p_1 \wedge q)\partial_{x_1}(q^2(B_{-1} + C_{-1})) \end{aligned}"]
Out[10]=

$$R_2 = -\partial_{x_1}(p_1 \wedge q(B_0 + C_0)) + \frac{1}{2}(\partial_{x_1}^2 - \partial_{x_1}\partial_{x_2})(q^2 p_1 \wedge q(B_{-1} + C_{-1}))$$


$$= p_1 \wedge q \left( -\partial_{x_1}(B_0 + C_0) + \frac{1}{2}\partial_{x_1}^2(q^2(B_{-1} + C_{-1})) - \frac{1}{2}\partial_{x_1}\partial_{x_2}(q^2(B_{-1} + C_{-1})) \right)$$


$$+ \partial_{x_1}(p_1 \wedge q) \left( -(B_0 + C_0) + \partial_{x_1}(q^2(B_{-1} + C_{-1})) - \frac{1}{2}\partial_{x_2}(q^2(B_{-1} + C_{-1})) \right)$$


$$- \frac{1}{2}\partial_{x_2}(p_1 \wedge q)\partial_{x_1}(q^2(B_{-1} + C_{-1}))$$

```

```

In[1]:= R21D1 = -simplifiedForm@onShellP[
  D[Wedge[p1, q]*Expand[(Bx1hbar[1] + Cx1hbar[1]) /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2,
    -l ⊙ l + 2 l ⊙ q - q ⊙ q → -qml2}] /. qml2 → l ⊕ l - 2 l ⊙ q + q ⊙ q], x1]];

```

$$\text{R22D1} = (\text{D}[\text{D}[q \odot q \times \text{Wedge}[p1, q] * (\text{Bx1hbar}[0] + \text{Cx1hbar}[0]), x1], x1] - \text{D}[\text{D}[q \odot q \times \text{Wedge}[p1, q] * (\text{Bx1hbar}[0] + \text{Cx1hbar}[0]), x2], x1]) //.$$

$$\left\{ \begin{aligned} & \left(-\frac{m2^2 p1 \odot p1}{s} + \frac{\gamma p1 \odot p2}{s} \right) \rightarrow 1, \left(\frac{m2^2 p1 \odot p1}{s} - \frac{\gamma p1 \odot p2}{s} \right) \rightarrow -1, \\ & \left(\frac{\gamma p1 \odot p2}{s} - \frac{m1^2 p2 \odot p2}{s} \right) \rightarrow 1, \left(-\frac{\gamma p1 \odot p2}{s} + \frac{m1^2 p2 \odot p2}{s} \right) \rightarrow -1, \\ & \left(\frac{\gamma p1 \odot p1}{s} - \frac{m1^2 p1 \odot p2}{s} \right) \rightarrow 0, \left(-\frac{\gamma p1 \odot p1}{s} + \frac{m1^2 p1 \odot p2}{s} \right) \rightarrow 0, \\ & \left(-\frac{m2^2 p1 \odot p2}{s} + \frac{\gamma p2 \odot p2}{s} \right) \rightarrow 0, \left(\frac{m2^2 p1 \odot p2}{s} - \frac{\gamma p2 \odot p2}{s} \right) \rightarrow 0 \end{aligned} \right\};$$

$$\text{R22D1} = \frac{1}{2} \text{simplifiedForm}@onShellP[\text{R22D1}];$$

$$(*\text{ClearAll}[\text{workspace1}, \text{workspace2}];*)$$

$$\text{R22D1} = \frac{1}{2} (\text{D}[\text{D}[q \odot q \times \text{Wedge}[p1, q] * (\text{Bx1hbar}[0] + \text{Cx1hbar}[0]), x1], x1] - \text{D}[\text{D}[q \odot q \times \text{Wedge}[p1, q] * (\text{Bx1hbar}[0] + \text{Cx1hbar}[0]), x2], x1]);$$

$$\text{R2D1} = \text{simplifiedForm}@onShellP[\text{R22D1}];$$

$$\text{R2D1} = \text{R21D1} + \text{R22D1};$$

$$\text{R2D1} = \text{Collect}[\text{factorRho}@\left(\text{simplifiedForm}[\text{R2D1}] /. \{l \odot l - 2 l \odot q + q \odot q \rightarrow qml2, -l \odot l + 2 l \odot q - q \odot q \rightarrow -qml2\} /. \{l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon\}\right), \rho];$$

$$\text{R2D1Div} = \text{Total}@\text{Select}[\text{List}@@\text{R2D1}, \text{Exponent}[\#, \rho] < -1 &];$$

$$\text{R2D1Div}$$

$$\text{R2D1Div} = \text{R2D1Div} /. \left\{ \left(\text{num_a}_- \right) / \left(\text{Power}[\text{den}_-, -n_-] \right) \rightarrow \text{num} / (\text{den}^{(-n - 1)}) /; \text{ContainsAll}[\text{Variables}[\text{den}], \{\epsilon, a\}] /; a == l \odot p1 || a == l \odot p2 \right\}$$

$$\text{Expand}[\text{simplifiedForm}[\text{onShellP}[\text{D}[\text{D}[\text{Bx1hbar}[0] + \text{Cx1hbar}[0], x1], x1] - \text{D}[\text{D}[\text{Bx1hbar}[0] + \text{Cx1hbar}[0], x1], x2]]]]$$

```
In[8]:= Coefficient[
  Collect[factorRho[simplifyTerm[Expand[R22D1]] /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2,
    -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2} /. {l → ρ l, ε → ρ ε}], ρ], ρ, -2]

Out[8]=

$$\begin{aligned}
 & -\frac{4 m2^2 \gamma^2 p1 \wedge q}{qml2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{4 \gamma^3 p1 \wedge q}{qml2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \\
 & \frac{4 m2^2 \gamma^2 p1 \wedge q}{qml2 S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \frac{4 \gamma^3 p1 \wedge q}{qml2 S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \\
 & \frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{qml2^2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \frac{4 \gamma^3 q \odot q p1 \wedge q}{qml2^2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \\
 & \frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{qml2^2 S(i \epsilon + l \odot p2)^2} - \frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{qml2^2 S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} \\
 C \rightarrow & \frac{4 m2^2 \gamma^2 p1 \wedge q}{qml2 S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} + \frac{4 \gamma^3 p1 \wedge q}{qml2 S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} - \\
 & \frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{qml2^2 S(i \epsilon + l \odot p2)^2} - \frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{qml2^2 S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} \\
 B \rightarrow & -\frac{4 m2^2 \gamma^2 p1 \wedge q}{qml2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{4 \gamma^3 p1 \wedge q}{qml2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \\
 & \frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{qml2^2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \frac{4 \gamma^3 q \odot q p1 \wedge q}{qml2^2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)}
 \end{aligned}$$


```

 R_1

```
In[9]:= MaTeX["\\begin{aligned}R_1= (p_1 \\wedge
\\partial_{p_1})(B_0+C_0)-\\frac{1}{2}(\\partial_{x_1}-\\partial_{x_2})(q^2(p_1 \\wedge \\partial_{p_1})(B_{-1}+C_{-1}))\\end{aligned}"]

Out[9]=

$$R_1 = (p_1 \wedge \partial_{p_1})(B_0 + C_0) - \frac{1}{2}(\partial_{x_1} - \partial_{x_2})(q^2(p_1 \wedge \partial_{p_1})(B_{-1} + C_{-1}))$$


```

```

In[°]:= R11D1 = simplifiedForm[
  onShellP[aWedgeDb[Expand[(Bx1hbar[1] + Cx1hbar[1]) /. {l ⊙ l - 2 l ⊙ q + q ⊙ q → qml2,
    -l ⊙ l + 2 l ⊙ q - q ⊙ q → -qml2}] /. qml2 → l ⊙ l - 2 l ⊙ q + q ⊙ q, p1, p1]]];
R12D1 =

$$\frac{-1}{2} \text{simplifiedForm}\left[\text{onShellP}\left[D\left[q \odot q \times \text{aWedgeDb}\left[Bx1hbar[0] + Cx1hbar[0], p1, p1\right], x1\right] - D\left[q \odot q \times \text{aWedgeDb}\left[Bx1hbar[0] + Cx1hbar[0], p1, p1\right], x2\right]\right]\right];$$

In[°]:= simplifyTerm@
  Coefficient[Collect[factorRho@(Expand[R22D1] /. {l ⊙ l - 2 l ⊙ q + q ⊙ q → qml2,
    -l ⊙ l + 2 l ⊙ q - q ⊙ q → -qml2}) /. {l → ρ l, ε → ρ ε}), ρ], ρ, -2]
Out[°]=
0
In[°]:= R1D1 = R11D1 + R12D1;

```

R12D1

Out[]=

$$\begin{aligned}
& \frac{1}{2} \left(-q \odot q \left(\frac{8 \gamma^2 \left(-\frac{m2^2 l \odot p1}{s} + \frac{\gamma l \odot p2}{s} \right) l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \right. \right. \\
& \quad \frac{8 \gamma^2 \left(-\frac{m2^2 l \odot p1}{s} + \frac{\gamma l \odot p2}{s} \right) l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} + \\
& \quad \frac{16 \gamma \left(-\frac{m2^2 l \odot p1}{s} + \frac{\gamma l \odot p2}{s} \right) p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \quad \left. \left. \frac{16 \gamma \left(-\frac{m2^2 l \odot p1}{s} + \frac{\gamma l \odot p2}{s} \right) p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} \right) + \right. \\
& q \odot q \left(\frac{8 \gamma^2 \left(\frac{\gamma l \odot p1}{s} - \frac{m1^2 l \odot p2}{s} \right) l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \right. \\
& \quad \frac{8 \gamma^2 \left(\frac{\gamma l \odot p1}{s} - \frac{m1^2 l \odot p2}{s} \right) l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \quad \frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 (l \odot l - 2 l \odot q + q \odot q)} + \\
& \quad \frac{16 \gamma \left(\frac{\gamma l \odot p1}{s} - \frac{m1^2 l \odot p2}{s} \right) p1 \wedge p2}{(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \quad \frac{16 \gamma \left(\frac{\gamma l \odot p1}{s} - \frac{m1^2 l \odot p2}{s} \right) p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \\
& \quad \left. \left. \frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 (l \odot l - 2 l \odot q + q \odot q)} \right) \right)
\end{aligned}$$

Singular terms

```

In[1]:= S1 = simplifiedForm@onShellP[aWedgeDb[Bx1hbar[0] + Cx1hbar[0], p1, p1]]
Out[1]=


$$\frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} -$$


$$\frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} +$$


$$\frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)}$$


In[2]:= simplifiedForm[expr_] := expr //.
   $\left\{ \left( -\frac{m1^2 m2^2}{s} + \frac{\gamma^2}{s} \right) \rightarrow 1, \left( \frac{m1^2 m2^2}{s} - \frac{\gamma^2}{s} \right) \rightarrow -1 \right\} /. \{ \epsilon \rightarrow 2 \epsilon \} /.$ 
   $\left\{ (coef1_{\epsilon} + coef2_{l \odot p1})^{(n_1)} \rightarrow (coef2)^n * (\epsilon * coef1 / coef2 + l \odot p1)^n,$ 
   $(coef1_{\epsilon} + coef2_{l \odot p2})^{(n_2)} \rightarrow (coef2)^n * (\epsilon * coef1 / coef2 + l \odot p2)^n \right\}$ 

In[3]:= S2 = -simplifiedForm@onShellP[Expand[D[Wedge[p1, q] * (Bx1hbar[0] + Cx1hbar[0]), x1]] /. 
   $\left( \frac{\text{num\_a}_-}{\text{Power}[\text{den}_-, -n_-]} \right) \rightarrow$ 
  num / (den^(n-1)) /; ContainsAll[Variables[den], {\epsilon, a}] /; a == l \odot p1 || a == l \odot p2]
Out[3]=


$$-\frac{4 \gamma^3 p1 \wedge p2}{s (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} +$$


$$\frac{4 \gamma^3 p1 \wedge p2}{s (i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} +$$


$$\frac{8 m2^2 \gamma^2 p1 \wedge q}{s (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \frac{8 m2^2 \gamma^2 p1 \wedge q}{s (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2}$$


```

In[8]:= **S1 + S2**

Out[8]=

$$\begin{aligned}
 & \frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \\
 & \frac{4 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} + \\
 & \frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \\
 & \frac{4 \gamma^3 p1 \wedge p2}{S(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} - \\
 & \frac{8 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} + \\
 & \frac{4 \gamma^3 p1 \wedge p2}{S(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} + \\
 & \frac{8 m2^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2} - \frac{8 m2^2 \gamma^2 p1 \wedge q}{S(i \epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)^2}
 \end{aligned}$$

In[9]:= **R1D1 = Collect[factorRho@(simplifiedForm[R1D1] /. {l \odot l - 2 l \odot q + q \odot q \rightarrow qml2, -l \odot l + 2 l \odot q - q \odot q \rightarrow -qml2}) /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho];**

In[10]:= **R1D1Div = Total@Select[List@@R1D1, Exponent[\#, \rho] < -1 &];**

In[11]:= **R1D1Div = R1D1Div /. {(num_a_) / (Power[den_, -n_]) \(\rightarrow\) num / (den ^ (-n - 1)) /; ContainsAll[Variables[den], {\epsilon, a}] /; a == l \odot p1 || a == l \odot p2}**

Out[11]=

$$\begin{aligned}
 & \frac{2 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \frac{2 \gamma^2 q \odot q l \wedge p1}{qml2 (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{4 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \frac{4 \gamma q \odot q p1 \wedge p2}{qml2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \\
 & \rho^3 \\
 & - \frac{2 \gamma p1 \wedge q}{qml2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{6 \gamma p1 \wedge q}{qml2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} \\
 & \rho^2
 \end{aligned}$$

```
In[=]:= Collect[factorRho[simplifyTerm[R11D1] /.
  {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2} /. {l → ρ l, ε → ρ ε}], ρ]

Out[=]=

$$\frac{\frac{2 \gamma^2 l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{4 \gamma p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2}}{\rho^3} +$$


$$\frac{1}{\rho} \left( \frac{2 \gamma^2 l \odot l \wedge p1}{qml2 (i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} - \frac{4 \gamma^2 l \odot l \wedge p1}{qml2 (i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} + \right.$$


$$\frac{4 \gamma^2 l \odot l \wedge p1}{qml2 (i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \frac{4 \gamma l \odot l p1 \wedge p2}{qml2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)^2} - \frac{4 p1 \wedge p2}{qml2 (-i \epsilon + l \odot p2)} -$$


$$\frac{4 \gamma l \odot l p1 \wedge p2}{qml2 (i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \frac{4 p1 \wedge p2}{qml2 (i \epsilon + l \odot p2)} + \frac{4 \gamma l \odot l p1 \wedge p2}{qml2 (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} \right) +$$


$$-\frac{2 \gamma p1 \wedge q}{qml2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{6 \gamma p1 \wedge q}{qml2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)}$$


```

Second Pair

R_2 for the last two diagrams

```
In[=]:= Bx2 = ((2 p1 + ħ q - ħ l) ⊙ (2 p2 - ħ q + ħ l) (2 p1 + 2 ħ q - ħ l) ⊙ (2 p2 - 2 ħ q + ħ l)) /
  ((q - l) ⊙ (q - l) (2 p1 ⊙ (q - l) + ħ (q - l) ⊙ (q - l) + ī ε) (2 p2 ⊙ (q - l) - ħ (q - l) ⊙ (q - l) - ī ε));
Cx2 = -(2 p1 + 2 ħ q - ħ l) ⊙ (2 p2 - ħ l) (2 p1 + ħ q - ħ l) ⊙ (2 p2 - ħ q - ħ l) /
  ((q - l) ⊙ (q - l) (2 p1 ⊙ (q - l) + ħ (q - l) ⊙ (q - l) + ī ε) (2 p2 ⊙ l - ħ l ⊙ l - ī ε));
```

B2C2 = Bx2 + Cx2;

```
In[=]:= Do[Bx2hbar[n] = Coefficient[Normal[Series[Bx2, {ħ, 0, 1}]], ħ, n];
Cx2hbar[n] = Coefficient[Normal[Series[Cx2, {ħ, 0, 1}]], ħ, n], {n, 0, 1}];
```

In[=]:= Distribute@simplifiedForm@B2xhbar[1]

Out[=]=

$$\frac{16 (p1 \odot p2)^2}{(2 \epsilon + 2 i l \odot p1 - 2 i p1 \odot q)^2 (-2 i \epsilon - 2 l \odot p2 + 2 p2 \odot q)} +$$

$$\frac{8 p1 \odot p2 (l \odot p1 - l \odot p2 - p1 \odot q) + 4 p1 \odot p2 (2 l \odot p1 - 2 l \odot p2 - 4 p1 \odot q + 4 p2 \odot q)}{-2 i \epsilon - 2 l \odot p2 + 2 p2 \odot q} + \frac{16 (p1 \odot p2)^2 (-l \odot l + 2 l \odot q - q \odot q)}{(2 \epsilon - 2 i l \odot p2 + 2 i p2 \odot q)^2}$$

$$\frac{(2 i \epsilon - 2 l \odot p1 + 2 p1 \odot q) (l \odot l - 2 l \odot q + q \odot q)}{(2 i \epsilon - 2 l \odot p1 + 2 p1 \odot q) (l \odot l - 2 l \odot q + q \odot q)}$$

In[\circ]:= **Distribute@simplifiedForm@C2xhbar[1]**

Out[\circ]=

$$\frac{\frac{8 (p_1 \odot p_2)^2}{(-i \epsilon + l \odot p_2) (2 \epsilon + 2 i l \odot p_1 - 2 i p_1 \odot q)^2} + \frac{4 p_1 \odot p_2 \left(\frac{l \odot l p_1 \odot p_2}{(-i \epsilon + l \odot p_2)^2} - \frac{l \odot p_1 + l \odot p_2 - 2 p_2 \odot q}{-i \epsilon + l \odot p_2} \right) - \frac{4 p_1 \odot p_2 (l \odot p_1 + l \odot p_2 + p_1 \odot q - p_2 \odot q)}{-i \epsilon + l \odot p_2}}{(2 i \epsilon - 2 l \odot p_1 + 2 p_1 \odot q) (l \odot l - 2 l \odot q + q \odot q)}$$

In[\circ]:= **MaTeX["\begin{aligned} R_2 = -\partial_{x_1}(p_1 \wedge q (B_0 + C_0)) + \frac{1}{2} (\partial_{x_1}^2 - \partial_{x_1} \partial_{x_2}) (q^2 p_1 \wedge q (B_{-1} + C_{-1})) \\ = p_1 \wedge q \left(-\partial_{x_1}(B_0 + C_0) + \frac{1}{2} \partial_{x_1}^2 (q^2 (B_{-1} + C_{-1})) - \frac{1}{2} \partial_{x_1} \partial_{x_2} (q^2 (B_{-1} + C_{-1})) \right) \\ + \partial_{x_1}(p_1 \wedge q) \left(-(B_0 + C_0) + \partial_{x_1}(q^2 (B_{-1} + C_{-1})) - \frac{1}{2} \partial_{x_2}(q^2 (B_{-1} + C_{-1})) \right) \\ - \frac{1}{2} \partial_{x_2}(p_1 \wedge q) \partial_{x_1}(q^2 (B_{-1} + C_{-1})) \end{aligned}"]**

Out[\circ]=

$$\begin{aligned} R_2 &= -\partial_{x_1}(p_1 \wedge q (B_0 + C_0)) + \frac{1}{2} (\partial_{x_1}^2 - \partial_{x_1} \partial_{x_2}) (q^2 p_1 \wedge q (B_{-1} + C_{-1})) \\ &= p_1 \wedge q \left(-\partial_{x_1}(B_0 + C_0) + \frac{1}{2} \partial_{x_1}^2 (q^2 (B_{-1} + C_{-1})) - \frac{1}{2} \partial_{x_1} \partial_{x_2} (q^2 (B_{-1} + C_{-1})) \right) \\ &\quad + \partial_{x_1}(p_1 \wedge q) \left(-(B_0 + C_0) + \partial_{x_1}(q^2 (B_{-1} + C_{-1})) - \frac{1}{2} \partial_{x_2}(q^2 (B_{-1} + C_{-1})) \right) \\ &\quad - \frac{1}{2} \partial_{x_2}(p_1 \wedge q) \partial_{x_1}(q^2 (B_{-1} + C_{-1})) \end{aligned}$$

R21D2 = -simplifiedForm[onShellP[
 D[Wedge[p1, q]*Expand[(Bx2hbar[1] + Cx2hbar[1]) /. {l \odot l - 2 l \odot q + q \odot q \rightarrow qml2, -l \odot l + 2 l \odot q - q \odot q \rightarrow -qml2}] /. qml2 \rightarrow l \odot l - 2 l \odot q + q \odot q], x1]]];

R22D2 = D[q \odot q \times Wedge[p1, q]* (Bx2hbar[0] + Cx2hbar[0]), x1];

workspace = D[R22D2, x2];

R22D2 = D[R22D2, x1] - workspace;

R22D2 = $\frac{1}{2}$ simplifiedForm[onShellP[R22D2]]; ClearAll[workspace];

In[\circ]:= R2D2 = R21D2 + R22D2;

In[\circ]:= R2D2 = Collect[
 factorRho@(R2D2 /. {l \odot l - 2 l \odot q + q \odot q \rightarrow qml2, -l \odot l + 2 l \odot q - q \odot q \rightarrow -qml2} /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}), \rho];

```

In[1]:= R2D2Div = Total@Select[List@@R2D2, Exponent[#, ρ] < -1 &];

In[2]:= R2D2Div = R2D2Div /. {num_ a_)/(Power[den_, -n_]) :>
  num/(den^(n - 1)); ContainsAll[Variables[den], {ε, a}] /; a == l ⊕ p1 || a == l ⊕ p2};

In[3]:= Coefficient[R1D2Div + R2D2Div, ρ, -4]

Out[3]=

$$\frac{8 \gamma^2 q \odot q p1 \wedge q}{qm12 (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} + \frac{4 \gamma^2 q \odot q p1 \wedge q}{qm12 (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \frac{8 \gamma^2 q \odot q p1 \wedge q}{qm12 (-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)}$$


In[4]:= Collect[factorRho[simplifyTerm[Expand[R11D2 /.
  {l ⊕ l - 2 l ⊕ q + q ⊕ q → qm12, -l ⊕ l + 2 l ⊕ q - q ⊕ q → -qm12}]] /.
  {l → ρ l, ε → ρ ε}], ρ]

Out[4]=

$$\begin{aligned} & -\frac{2 \gamma^2 l \odot l p1}{qm12 (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} - \frac{4 \gamma l \odot l p1 \wedge p2}{qm12 (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)^2} + \frac{4 p1 \wedge p2}{qm12 (-i \epsilon + l \odot p2)} + \frac{4 p1 \wedge p2}{qm12 (i \epsilon + l \odot p2)} + \\ & \rho^3 \left( -\frac{4 \gamma^2 l \wedge p1}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \frac{2 \gamma^2 l \wedge p1}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{4 \gamma^2 l \wedge p1}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} - \right. \\ & \left. \frac{4 \gamma p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \frac{4 \gamma p1 \wedge p2}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \frac{4 \gamma p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} \right) + \\ & -\frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \frac{2 \gamma^2 p1 \wedge q}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \\ & \rho^4 \left( -\frac{2 \gamma^2 l \odot l p1 \wedge q}{qm12 (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} + \frac{6 \gamma p1 \wedge q}{qm12 (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{2 \gamma p1 \wedge q}{qm12 (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} \right. \\ & \left. \rho^2 \right) \\ In[5]:= R22D2 = \frac{1}{2} (D[D[q ⊕ q × Wedge[p1, q]] * (Bx2hbar[0] + Cx2hbar[0]), x1], x1] - \\ & D[D[q ⊕ q × Wedge[p1, q]] * (Bx2hbar[0] + Cx2hbar[0]), x2], x1]; \\ In[6]:= R22D2 = simplifiedForm[onShellP[R22D2]]; \\ In[7]:= R22 = R22D1 + R22D2; \end{aligned}$$


```

R1 for the last two diagrams

```
In[1]:= MaTeX["\\begin{aligned}R_1= & (p_1 \\wedge \\\partial_{p_1})(B_0+C_0)-\\frac{1}{2}(\\partial_{x_1}-\\partial_{x_2})(q^2(p_1 \\wedge \\\partial_{p_1})(B_{-1}+C_{-1}))\\end{aligned}"]
```

Out[1]=

$$R_1 = (p_1 \wedge \partial_{p_1})(B_0 + C_0) - \frac{1}{2}(\partial_{x_1} - \partial_{x_2})(q^2(p_1 \wedge \partial_{p_1})(B_{-1} + C_{-1}))$$

```
In[2]:= B2xhbar[1]
```

Out[2]=

$$\frac{\frac{8 p_1 \partial p_2 (\partial p_1 - \partial p_2 - p_1 \partial q + p_2 \partial q) + 4 p_1 \partial p_2 (2 \partial p_1 - 2 \partial p_2 - 4 p_1 \partial q + 4 p_2 \partial q)}{-i \epsilon - 2 \partial p_2 + 2 p_2 \partial q} + \frac{16 (p_1 \partial p_2)^2 (-\partial l + 2 \partial q - q \partial q)}{(-2 i \partial p_2 + 2 i p_2 \partial q)^2}}{i \epsilon - 2 \partial p_1 + 2 p_1 \partial q} + \frac{\frac{16 (p_1 \partial p_2)^2 (l \partial l - 2 \partial q + q \partial q)}{(\epsilon + 2 i \partial p_1 - 2 i p_1 \partial q)^2 (-i \epsilon - 2 \partial p_2 + 2 p_2 \partial q)}}{l \odot l - 2 l \odot q + q \odot q}$$

```
(*R11D2 = aWedgeDb[B2xhbar[1]+C2xhbar[1], p1,p1];
```

```
R12D2 = -\frac{1}{2} D[q \odot q \times aWedgeDb[B2xhbar[0]+C2xhbar[0], p1,p1], x1] +
\frac{1}{2} D[q \odot q \times aWedgeDb[B2xhbar[0]+C2xhbar[0], p1,p1], x2];*)
```

```
In[3]:= R11D2 = aWedgeDb[Expand[(Bx2hbar[1] + Cx2hbar[1]) /. {l \odot l - 2 l \odot q + q \odot q \rightarrow qml2,
-l \odot l + 2 l \odot q - q \odot q \rightarrow -qml2}] /. {qml2 \rightarrow l \odot l - 2 l \odot q + q \odot q}, p1, p1];
```

```
R12D2 = -\frac{1}{2} D[q \odot q \times aWedgeDb[Bx2hbar[0] + Cx2hbar[0], p1, p1], x1] +
\frac{1}{2} D[q \odot q \times aWedgeDb[Bx2hbar[0] + Cx2hbar[0], p1, p1], x2];
```

```
R11D2 = onShellP[R11D2];
```

```
R12D2 = onShellP[R12D2];
```

```
R11D2 = simplifiedForm@R11D2;
```

```
R12D2 = simplifiedForm@R12D2;
```

```
R1D2 = R11D2 + R12D2;
```

```
In[4]:= Coefficient[Collect[factorRho@
(simplifyTerm[Expand[simplifiedForm[R11D2]]]) /. {l \odot l - 2 l \odot q + q \odot q \rightarrow qml2,
-l \odot l + 2 l \odot q - q \odot q \rightarrow -qml2}] /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho, -4]
```

Out[4]=

$$-\frac{4 \gamma^2 p_1 \wedge q}{(-2 i \epsilon + l \odot p_1)^3 (-2 i \epsilon + l \odot p_2)} - \frac{2 \gamma^2 p_1 \wedge q}{(-2 i \epsilon + l \odot p_1)^2 (2 i \epsilon + l \odot p_2)^2} + \frac{4 \gamma^2 p_1 \wedge q}{(-2 i \epsilon + l \odot p_1)^3 (2 i \epsilon + l \odot p_2)}$$

```
In[5]:= R1D2 = simplifiedForm@R1D2;
```

```

In[1]:= R1D2 = Collect[
  factorRho@{R1D2 /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → q m l2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -q m l2} /.
    {l → ρ l, ε → ρ ε}}, ρ];

In[2]:= R1D2Div = Total@Select[List @@ R1D2, Exponent[#, ρ] < -1 &];

R1D2Div = R1D2Div /. {(num_a_) / (Power[den_, -n_]) :>
  num / (den^(n - 1)) /; ContainsAll[Variables[den], {ε, a}] /; a == l ⊙ p1 || a == l ⊙ p2};

In[3]:= - 4 m2^2 γ^2 l ⊙ p2 q ⊙ q p1 ∧ q
          q m l2^2 S(-i ε + l ⊙ p1)^2 (-i ε + l ⊙ p2) /. {(num_a_) / (Power[den_, -n_]) :>
  num / (den^(n - 1)) /; ContainsAll[Variables[den], {ε, a}] /; a == l ⊙ p1 || a == l ⊙ p2}

Out[3]= - 4 m2^2 γ^2 q ⊙ q p1 ∧ q
          q m l2^2 S(-i ε + l ⊙ p1)^2

In[4]:= Coefficient[R1D2Div + R2D2Div, ρ, -2]

Out[4]= 4 γ^3 p1 ∧ q
          q m l2 S(-i ε + l ⊙ p1)(-i ε + l ⊙ p2) - 4 γ^3 p1 ∧ q
          q m l2 S(-i ε + l ⊙ p1)(i ε + l ⊙ p2) -
  4 γ^3 q ⊙ q p1 ∧ q
          q m l2^2 S(-i ε + l ⊙ p1)(-i ε + l ⊙ p2) + 4 m2^2 γ^2 q ⊙ q p1 ∧ q
          q m l2^2 S(i ε + l ⊙ p2)^2

In[5]:= RD2LogDiv =
  8 π i γ^3 p1 ∧ q δ[l ⊙ p2]
          q m l2 S(-i ε + l ⊙ p1) - 4 γ^3 q ⊙ q p1 ∧ q
          q m l2^2 S(-i ε + l ⊙ p1)(-i ε + l ⊙ p2) + 4 m2^2 γ^2 q ⊙ q p1 ∧ q
          q m l2^2 S(i ε + l ⊙ p2)^2;

In[6]:= S2

Out[6]= - 4 γ^3 p1 ∧ p2
          S(i ε + l ⊙ p1)(-i ε + l ⊙ p2)(l ⊙ l - 2 l ⊙ q + q ⊙ q) +
  4 γ^3 p1 ∧ p2
          S(i ε + l ⊙ p1)(i ε + l ⊙ p2)(l ⊙ l - 2 l ⊙ q + q ⊙ q) +
  8 m2^2 γ^2 p1 ∧ q
          S(-i ε + l ⊙ p2)(l ⊙ l - 2 l ⊙ q + q ⊙ q)^2 - 8 m2^2 γ^2 p1 ∧ q
          S(i ε + l ⊙ p2)(l ⊙ l - 2 l ⊙ q + q ⊙ q)^2

```

```

In[8]:= S3 = simplifiedForm@onShellP[aWedgeDb[B2xhbar[0] + C2xhbar[0], p1, p1]]
Out[8]=


$$-\frac{8 \gamma p1 \wedge p2}{(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)} +$$


$$\frac{8 \gamma p1 \wedge p2}{(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)} -$$


$$\frac{2 i \gamma^2 (-2 i l \wedge p1 - 2 i p1 \wedge q)}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)} +$$


$$\frac{2 \gamma^2 (2 l \wedge p1 + 2 p1 \wedge q)}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)}$$


In[9]:= S4 = -Expand[simplifiedForm@onShellP[D[Wedge[p1, q] * (B2xhbar[0] + C2xhbar[0]), x1]] /.
  {(num_a_) / (Power[den_, -n_]) :>
    num / (den^(n - 1)) /; ContainsAll[Variables[den], {epsilon, a}] /; a == l \odot p1 || a == l \odot p2}]

In[10]:= Expand[S1 + S2 + S3 + S4]
In[11]:= simplifyTerm[Total@Select[List @@ Expand[R12D1 + R12D2], ContainsAll[List @@ Numerator[##], {p1 \wedge q}] &]]
Out[11]=


$$-\frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} -$$


$$\frac{4 \gamma^3 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} +$$


$$\frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} +$$


$$\frac{4 \gamma^3 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} +$$


$$\frac{4 \gamma^2 q \odot q p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)} +$$


$$\frac{2 \gamma^2 q \odot q p1 \wedge q}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 (l \odot l - 2 l \odot q + q \odot q)} -$$


$$\frac{4 \gamma^2 q \odot q p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)}$$


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```
In[8]:= simplifyTerm[Total@
Select[List @@ Expand[R22D1 + R22D2], ContainsAll[List @@ Numerator[##], {p1 \wedge q}] &]

Out[8]=

$$\frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} +$$


$$\frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} +$$


$$\frac{4 \gamma^3 q \odot q p1 \wedge q}{S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} -$$


$$\frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} -$$


$$\frac{4 \gamma^3 q \odot q p1 \wedge q}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} -$$


$$\frac{4 m2^2 \gamma^2 q \odot q p1 \wedge q}{S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} +$$


$$\frac{4 m2^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} +$$


$$\frac{4 \gamma^3 p1 \wedge q}{S(-i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} -$$


$$\frac{4 m2^2 \gamma^2 p1 \wedge q}{S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} -$$


$$\frac{4 \gamma^3 p1 \wedge q}{S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} -$$


$$\frac{4 m2^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} -$$


$$\frac{4 \gamma^3 p1 \wedge q}{S(-i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} +$$


$$\frac{4 m2^2 \gamma^2 p1 \wedge q}{S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} +$$


$$\frac{4 \gamma^3 p1 \wedge q}{S(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)(l \odot l - 2 l \odot q + q \odot q)^2} -$$

```

$$\frac{4 \gamma^2 q \odot q p1 \wedge q}{(-i\epsilon + l \odot p1)^3 (-i\epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)} -$$

$$\frac{2 \gamma^2 q \odot q p1 \wedge q}{(-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)^2 (l \odot l - 2 l \odot q + q \odot q)} +$$

$$\frac{4 \gamma^2 q \odot q p1 \wedge q}{(-i\epsilon + l \odot p1)^3 (i\epsilon + l \odot p2) (l \odot l - 2 l \odot q + q \odot q)}$$

R_2 for first two diagrams

$$In[=]:= Bx = \frac{(2 p1 + \hbar l) \odot (2 p2 - \hbar l) (2 p1 + \hbar q + \hbar l) \odot (2 p2 - \hbar q - \hbar l)}{(q - l) \odot (q - l) (2 p1 \odot l + \hbar l \odot l + i\epsilon) (2 p2 \odot l - \hbar l \odot l - i\epsilon)};$$

$$Cx = \frac{(2 p1 + \hbar q + \hbar l) \odot (2 p2 - \hbar q + \hbar l) (2 p1 + \hbar l) \odot (2 p2 - 2 \hbar q + \hbar l)}{(q - l) \odot (q - l) (2 p1 \odot l + \hbar l \odot l + i\epsilon) (2 p2 \odot (q - l) - \hbar (q - l) \odot (q - l) - i\epsilon)};$$

$$In[=]:= BC = Bx + Cx;$$

$$In[=]:= Do[Bxhbar[n] = Coefficient[Normal[Series[Bx, {hbar, 0, 1}]], hbar, n];$$

$$Cxhbar[n] = Coefficient[Normal[Series[Cx, {hbar, 0, 1}]], hbar, n], {n, 0, 1}];$$

$$In[=]:= R21 = -D[Bxhbar[1] + Cxhbar[1], x1];$$

$$R22 = \frac{1}{2} D[q \odot q \times (Bxhbar[0] + Cxhbar[0]), \{x1, 2\}];$$

$$R23 = \frac{-1}{2} D[q \odot q \times (Bxhbar[0] + Cxhbar[0]), \{x1, 1\}, \{x2, 1\}];$$

$$In[=]:= R21 = onShellP[R21];$$

$$R22 = onShellP[R22];$$

$$R23 = onShellP[R23];$$

$$In[=]:= Rpwmq = R21 + R22 + R23;$$

$$In[=]:= factorRho[exp_] :=$$

$$exp /. \{(expr_Plus)^n_ \rightarrow Factor[expr]^n /; ContainsAll[Variables[expr], \{\epsilon, l \odot p1\}] \&$$

$$ContainsAll[Variables[expr], \{\epsilon, l \odot p2\}]\}$$

$$In[=]:= Coefficient[factorRho@$$

$$Expand[simplifiedForm[Rpwmq] /. \{l \odot l - 2 l \odot q + q \odot q \rightarrow qml2\} /. \{l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon\}], \rho, -2]$$

$$\text{In[}]:= \text{R2D1Div} =$$

$$\frac{\text{Wedge}[p1, q]}{q m l^2} \left(\frac{2 \gamma}{(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{6 \gamma}{(i \epsilon + l \odot p1)(i \epsilon + l \odot p2)} - \frac{8 \pi i \gamma^3 \delta[l \odot p2]}{S(i \epsilon + l \odot p1)} \right) +$$

$$\frac{\text{Wedge}[p1, q] q \odot q}{S q m l^2} \left(\frac{4 \gamma^3}{(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{4 m^2 \gamma^2}{(i \epsilon + l \odot p2)^2} + \frac{8 \pi i m^2 \gamma^2 \delta[l \odot p2]}{(i \epsilon + l \odot p1)} \right);$$

R_1 for the first two diagrams

```
In[ ]:= R11 = aWedgeDb[Bxhbar[1]+Cxhbar[1], p1, p1];
R12 = -1/2 D[q \odot q \times aWedgeDb[Bxhbar[0]+Cxhbar[0], p1, p1], x1] +
1/2 D[q \odot q \times aWedgeDb[Bxhbar[0]+Cxhbar[0], p1, p1], x2];

In[ ]:= R11 = onShellP[R11];
R12 = onShellP[R12];

In[ ]:= R1D1Log = Coefficient[factorRho@Expand[
simplifiedForm[R11+R12]/. {l \odot l - 2 l \odot q + q \odot q \rightarrow q m l^2} /. {l \rightarrow \rho l, \epsilon \rightarrow \rho \epsilon}], \rho, -2]
```

$$\text{Out[}]= -\frac{2 \gamma p1 \wedge q}{q m l^2 (i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \frac{6 \gamma p1 \wedge q}{q m l^2 (i \epsilon + l \odot p1)(i \epsilon + l \odot p2)}$$

$$\text{In[}]:= \text{RD1LogDiv} = \text{Expand}[R1D1Log + R2D1Div]$$

$$\text{Out[}]=$$

$$\frac{4 \gamma^3 q \odot q p1 \wedge q}{q m l^2 S(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{4 m^2 \gamma^2 q \odot q p1 \wedge q}{q m l^2 S(i \epsilon + l \odot p2)^2} -$$

$$\frac{8 i \pi \gamma^3 p1 \wedge q \delta[l \odot p2]}{q m l^2 S(i \epsilon + l \odot p1)} + \frac{8 i m^2 \pi \gamma^2 q \odot q p1 \wedge q \delta[l \odot p2]}{q m l^2 S(i \epsilon + l \odot p1)}$$

$$\text{In[}]:= \text{RD1LogDiv} + \text{RD2LogDiv}$$

$$\text{Out[}]=$$

$$-\frac{8 \pi i \gamma^3 q \odot q p1 \wedge q \delta[l \odot p1]}{q m l^2 S(-i \epsilon + l \odot p1)} + \frac{8 i \pi \gamma^3 p1 \wedge q \delta[l \odot p2]}{q m l^2 S(-i \epsilon + l \odot p1)} -$$

$$\frac{8 i \pi \gamma^3 p1 \wedge q \delta[l \odot p2]}{q m l^2 S(i \epsilon + l \odot p1)} + \frac{8 i m^2 \pi \gamma^2 q \odot q p1 \wedge q \delta[l \odot p2]}{q m l^2 S(i \epsilon + l \odot p1)}$$

```
In[1]:= Coefficient[Collect[
  factorRho@({R1D2 /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2} /.
    {l → ρ l, ε → ρ ε}}, ρ], ρ, -4]

R1D2 → - $\frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \frac{2 \gamma^2 p1 \wedge q}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} +$ 
 $\frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \frac{4 \gamma^2 q \odot q p1 \wedge q}{qml2 (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} +$ 
 $\frac{2 \gamma^2 q \odot q p1 \wedge q}{qml2 (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} - \frac{4 \gamma^2 q \odot q p1 \wedge q}{qml2 (-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)}$ 

R2D2 →  $\frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} + \frac{2 \gamma^2 p1 \wedge q}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} -$ 
 $\frac{4 \gamma^2 p1 \wedge q}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} - \frac{4 \gamma^2 q \odot q p1 \wedge q}{qml2 (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} -$ 
 $\frac{2 \gamma^2 q \odot q p1 \wedge q}{qml2 (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} + \frac{4 \gamma^2 q \odot q p1 \wedge q}{qml2 (-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)}$ 

In[2]:= Normal[Series[1/(l ⊕ l - 2 l ⊙ q + q ⊙ q) /. {l → ρ l}, {ρ, 0, 2}]]

Out[2]=  $\frac{2 \rho l \odot q}{(q \odot q)^2} + \frac{1}{q \odot q} + \frac{\rho^2 (4 (l \odot q)^2 - l \odot l q \odot q)}{(q \odot q)^3}$ 

wsp = q ⊙ q * aWedgeDb[(Bx1hbar[0] + Bx2hbar[0] + Cx1hbar[0] + Cx2hbar[0]), p1, p1];

t1 = Expand[simplifiedForm[onShellP[D[wsp, x1] - D[wsp, x2]]]];
ClearAll[wsp];
wsp = List @@ t1;
ClearAll[t1];
t1 = {};
Do[AppendTo[t1, simplifyTerm[wsp[[i]]]], {i, 1, Length[wsp]}];
t1 = Total[t1];
ClearAll[wsp];
```

```

In[1]:= wsp = q ⊙ q * Wedge[p1, q] * (Bx1hbar[0] + Bx2hbar[0] + Cx1hbar[0] + Cx2hbar[0]);

t2 = Expand[simplifiedForm[onShellP[D[D[wsp, x1] - D[wsp, x2], x1]]]];
ClearAll[wsp];
wsp = List @@ t2;
ClearAll[t2];
t2 = {};
Do[AppendTo[t2, simplifyTerm[wsp[[i]]]], {i, 1, Length[wsp]}];
t2 = Total[t2];
ClearAll[wsp];

In[2]:= t1 = factorRho[t1 /. {l → ρ l, ε → ρ ε}];
t2 = factorRho[t2 /. {l → ρ l, ε → ρ ε}];

In[3]:= t1 = t1 /. {(ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q) →
  1 / Normal[Series[1 / (ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q), {ρ, 0, 2}]]];
t2 = t2 /. {(ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q) → 1 / Normal[Series[1 / (ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q), {ρ, 0, 2}]]};

In[4]:= Coefficient[Expand[t1], ρ, -2] - Coefficient[Expand[t2], ρ, -2]
Out[4]=

$$\begin{aligned}
& -\frac{16 \gamma^2 l \odot q \, l \wedge p1}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2) q \odot q} - \frac{8 \gamma^2 l \odot q \, l \wedge p1}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 q \odot q} + \\
& \frac{8 \gamma^2 l \odot q \, l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2 q \odot q} + \frac{16 \gamma^2 l \odot q \, l \wedge p1}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2) q \odot q} - \\
& \frac{16 \gamma l \odot q \, p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) q \odot q} + \frac{8 \gamma^3 l \odot q \, p1 \wedge p2}{S(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2) q \odot q} - \\
& \frac{16 \gamma l \odot q \, p1 \wedge p2}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} + \frac{8 \gamma^3 l \odot q \, p1 \wedge p2}{S(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} + \\
& \frac{16 \gamma l \odot q \, p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} - \frac{8 \gamma^3 l \odot q \, p1 \wedge p2}{S(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2 q \odot q} + \\
& \frac{16 \gamma l \odot q \, p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) q \odot q} - \frac{8 \gamma^3 l \odot q \, p1 \wedge p2}{S(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2) q \odot q} - \\
& \frac{8 m2^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2) q \odot q} + \frac{8 m2^2 \gamma^2 p1 \wedge q}{S(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q} + \\
& \frac{8 \gamma^3 p1 \wedge q}{S(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q} - \frac{8 \gamma^3 p1 \wedge q}{S(i \epsilon + l \odot p1) (i \epsilon + l \odot p2) q \odot q}
\end{aligned}$$


```

```

In[1]:= s1 = simplifyTerm@Expand[simplifiedForm[
  onShellP[aWedgeDb[(Bx1hbar[1] + Bx2hbar[1] + Cx1hbar[1] + Cx2hbar[1]), p1, p1]]];
s2 = simplifyTerm@Expand[simplifiedForm[
  onShellP[D[Wedge[p1, q] * (Bx1hbar[1] + Bx2hbar[1] + Cx1hbar[1] + Cx2hbar[1]), x1]]];

In[2]:= s1 = factorRho[s1 /. {l → ρ l, ε → ρ ε}];
s2 = factorRho[s2 /. {l → ρ l, ε → ρ ε}];

In[3]:= s1 = s1 /. {(ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q) →
  1 / Normal[Series[1 / (ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q), {ρ, 0, 3}]]];
s2 = s2 /. {(ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q) → 1 / Normal[Series[1 / (ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q), {ρ, 0, 3}]]};

In[4]:= Coefficient[Expand[s1], ρ, -2]
Out[4]=

$$-\frac{2 \gamma^2 l \odot l p_1 \wedge q}{(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2)^2 q \odot q} + \frac{6 \gamma p_1 \wedge q}{(-i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2) q \odot q} -$$


$$\frac{2 \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2) q \odot q} - \frac{2 \gamma p_1 \wedge q}{(-i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2) q \odot q} + \frac{6 \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2) q \odot q}$$


In[5]:= Coefficient[Expand[s2], ρ, -2]
Out[5]=

$$-\frac{2 \gamma^2 l \odot l p_1 \wedge q}{(-i \epsilon + l \odot p_1)^2 (-i \epsilon + l \odot p_2)^2 q \odot q} + \frac{6 \gamma p_1 \wedge q}{(-i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2) q \odot q} -$$


$$\frac{2 \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1)(-i \epsilon + l \odot p_2) q \odot q} - \frac{2 \gamma p_1 \wedge q}{(-i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2) q \odot q} + \frac{6 \gamma p_1 \wedge q}{(i \epsilon + l \odot p_1)(i \epsilon + l \odot p_2) q \odot q}$$


In[6]:= sum = 0;
Do[sum += L[i] × Cf[i] (A0hbarl[i][0, 0] + A0hbarl[i][0, 1] + A0hbarl[i][0, 2] +
  ħ A0hbarl[i][1, 0] + ħ A0hbarl[i][1, 1] + ħ A0hbarl[i][1, 2]), {i, 1, 4}]

In[7]:= sum = 0;
Do[sum += L[i] × Cf[i] × A0[i], {i, 1, 4}]

In[8]:= Expand[simplifyTerm[Expand[simplifiedForm[sum]]] /. {(ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q) →
  1 / Normal[Series[1 / (ρ² l ⊙ l - 2 ρ l ⊙ q + q ⊙ q), {ρ, 0, 2}]]}] /. {ρ → 1}]

```

```

In[=]:= simplifyTerm[  

  Expand[Expand[simplifiedForm[Bx1hbar[1] + Bx2hbar[0] + Cx1hbar[0] + Cx2hbar[1]] /.  

    {l → ρ l, ε → ρ ε} /. {Power[(ρ² l ⊕ l - 2 ρ l ⊕ q + q ⊕ q), -1] →  

     1 / Normal[Series[1 / (ρ² l ⊕ l - 2 ρ l ⊕ q + q ⊕ q), {ρ, 0, 2}]], Power[(ρ² l ⊕ l - 2 ρ l ⊕ q + q ⊕ q), -2] →  

     Normal[Series[1 / (ρ² l ⊕ l - 2 ρ l ⊕ q + q ⊕ q)², {ρ, 0, 2}]]}], {ρ → 1}]] -  

  simplifyTerm[simplifiedForm[Expand[sum]]]]  

  

In[=]:= simplifyTerm[simplifiedForm[Expand[sum]]]  

  

In[=]:= a = simplifyTerm[  

  simplifiedForm[factorRho[Expand[Expand[Bx1hbar[1]] /. {l → ρ l, ε → ρ ε} /.  

    {Power[(ρ² l ⊕ l - 2 ρ l ⊕ q + q ⊕ q), -1] → Normal[Series[1 /  

      (ρ² l ⊕ l - 2 ρ l ⊕ q + q ⊕ q), {ρ, 0, 2}], Power[(ρ² l ⊕ l - 2 ρ l ⊕ q + q ⊕ q), -2] →  

      Normal[Series[1 / (ρ² l ⊕ l - 2 ρ l ⊕ q + q ⊕ q)², {ρ, 0, 2}]]}], {ρ → 1}];  

  

In[=]:= b = simplifyTerm[simplifiedForm[  

  Expand[(L[i] × Cf[i]) (A0hbarl[i][0, 0] + A0hbarl[i][0, 1] + A0hbarl[i][0, 2] +  

    ħ A0hbarl[i][1, 0] + ħ A0hbarl[i][1, 1] + ħ A0hbarl[i][1, 2]) /. {i → 1})]]];  

  

In[=]:= Length[a]  

Length[Coefficient[b, ħ, 1]]  

Length[factorRho@(Expand[i a] - Coefficient[b, ħ, 1]) /. p1 ⊕ p2 → γ]  

Out[=]= 24  

  

Out[=]= 28  

Out[=]= 4  

  

In[=]:= (Expand[i a] - Coefficient[b, ħ, 0]) /. p1 ⊕ p2 → γ  

Out[=]= 0  

  

In[=]:= factorRho[Expand[i a] /. p1 ⊕ p2 → γ /. {l → ρ l, ε → ρ ε}]  

  

In[=]:= Complement[List @@ factorRho[Coefficient[b, ħ, 1] /. p1 ⊕ p2 → γ /. {l → ρ l, ε → ρ ε}],  

  List @@ factorRho[Expand[i a] /. p1 ⊕ p2 → γ /. {l → ρ l, ε → ρ ε}]]  

Out[=]=  $\left\{ \frac{8 i \gamma \rho (l \odot q)^2}{(i \epsilon + l \odot p1) (q \odot q)^3}, -\frac{8 i \gamma \rho (l \odot q)^2}{(-i \epsilon + l \odot p2) (q \odot q)^3}, -\frac{2 i \gamma \rho l \odot l}{(i \epsilon + l \odot p1) (q \odot q)^2}, \frac{2 i \gamma \rho l \odot l}{(-i \epsilon + l \odot p2) (q \odot q)^2} \right\}$ 

```

```
In[=]:= Complement[List @@ factorRho[Expand[i a] /. p1 ⊕ p2 → γ /. {l → ρ l, ε → ρ ε}],  
List @@ factorRho[Coefficient[b, ℏ, 1] /. p1 ⊕ p2 → γ /. {l → ρ l, ε → ρ ε}]]  
  
Out[=]=  
{ $\frac{16 i \gamma \rho (l \odot q)^2}{(i \epsilon + l \odot p1) (q \odot q)^3}, -\frac{16 i \gamma \rho (l \odot q)^2}{(-i \epsilon + l \odot p2) (q \odot q)^3}, -\frac{4 i \gamma \rho l \odot l}{(i \epsilon + l \odot p1) (q \odot q)^2}, \frac{4 i \gamma \rho l \odot l}{(-i \epsilon + l \odot p2) (q \odot q)^2}\}$   
  
In[=]:= Coefficient[Expand[simplifyTerm[Expand[L[1] × Cf[1]]] (A0 hbar[1][0] + ℏ A0 hbar[1][1])], ℏ, 1]  
  
In[=]:= Expand[L[1] × Cf[1]]  
  
Out[=]=  

$$\frac{\gamma \hbar l \odot l}{2(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)^2} - \frac{\gamma \hbar l \odot l}{2(i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} +$$
  

$$\frac{\gamma}{(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} - \frac{\hbar l \odot p1}{2(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)} + \frac{\hbar l \odot p2}{2(i \epsilon + l \odot p1)(-i \epsilon + l \odot p2)}$$

```

One Loop: Calculation 3

```
In[=]:= MaTeX["  
"\\begin{aligned}Q_1=&J_1|_{O(l^{-2}),O(\hbar^0)},\\;\\;Q_2=&J_1|_{O(l^{-2}),0(\hbar)},\\\\\\;Q_3=&J_2|_{O(l^{-2}),0(\hbar^0)}\\\\Q_4=&J_3|_{O(l^{-2}),0(\hbar^0)},\\;\\;Q_5=&J_3|_{O(l^{-2}),0(\hbar)}\\\\Q_6=&J_4|_{O(l^{-2}),0(\hbar^0)}\\\\Q_7=&J_1|_{O(l^{-3}),O(\hbar^0)},\\;\\;Q_8=&J_1|_{O(l^{-3}),O(\hbar)},\\;\\;Q_9=&J_2|_{O(l^{-3}),0(\hbar^0)}\\\\Q_{10}=&J_3|_{O(l^{-3}),O(\hbar^0)},\\;\\;Q_{11}=&J_3|_{O(l^{-3}),O(\hbar)}\\\\Q_{12}=&J_4|_{O(l^{-3}),O(\hbar^0)}\\\\Q_{13}=&J_1|_{O(l^{-4}),O(\hbar^0)},\\;\\;Q_{14}=&J_1|_{O(l^{-4}),O(\hbar)},\\;\\;Q_{15}=&J_2|_{O(l^{-4}),O(\hbar^0)}\\\\Q_{16}=&J_3|_{O(l^{-4}),O(\hbar^0)},\\;\\;Q_{17}=&J_3|_{O(l^{-4}),O(\hbar)}\\\\Q_{18}=&J_4|_{O(l^{-4}),O(\hbar^0)}\\end{aligned}"]  
  
Out[=]=  
Q1 = J1|O(l-2),O(h0), Q2 = J1|O(l-2),O(h),  
Q3 = J2|O(l-2),O(h0)  
Q4 = J3|O(l-2),O(h0), Q5 = J3|O(l-2),O(h)  
Q6 = J4|O(l-2),O(h0)  
Q7 = J1|O(l-3),O(h0), Q8 = J1|O(l-3),O(h),  
Q9 = J2|O(l-3),O(h0)  
Q10 = J3|O(l-3),O(h0), Q11 = J3|O(l-3),O(h)  
Q12 = J4|O(l-3),O(h0)  
Q13 = J1|O(l-4),O(h0), Q14 = J1|O(l-4),O(h),  
Q15 = J2|O(l-4),O(h0)  
Q16 = J3|O(l-4),O(h0), Q17 = J3|O(l-4),O(h)  
Q18 = J4|O(l-4),O(h0)
```

```

In[=]:= MaTeX["J1=\sum_i L_i C_i,\n; J2=\partial_{\Delta x} \sum_i L_i C_i,\n; J3=p_1 \wedge \partial_{p_1} \sum_i L_i C_i - p_1 \wedge q \partial_{x_1} \sum_i L_i C_i,\n; J4=\partial_{\Delta x} x \Big( p_1 \wedge \partial_{p_1} \sum_i L_i C_i - p_1 \wedge q \partial_{x_1} \sum_i L_i C_i \Big)"]
Out[=]= J1 = Sum[L[i] C[i], {i, 1, 4}], J2 = D[sum, {x, 1}], J3 = p1 \wedge D[p1, {x, 1}], J4 = D[sum, {x, 1}] (p1 \wedge D[p1, {x, 1}] - p1 \wedge D[q, {x, 1}] sum)

In[=]:= ClearAll[sum];
sum = 0;
Do[sum += L[i]*Cf[i] /. {y → p1 ⊕ p2}, {i, 1, 4}]

In[=]:= ClearAll[J1, J2, J3, J4];
J1 = sum;
J2 = delx[J1];
J3 = aWedgeDb[sum, p1, p1] - p1 \wedge q * D[sum, x1];
J4 = delx[J3];

J1 = simplifiedForm[onShellP[J1]];
J2 = simplifiedForm[onShellP[J2]];
J3 = simplifiedForm[onShellP[J3]];
J4 = simplifiedForm[onShellP[J4]];

In[=]:= ClearAll[J1lm2, J2lm2, J3lm2, J4lm2];
J1lm2 = extractLog[J1, -2];
J2lm2 = extractLog[J2, -2];
J3lm2 = extractLog[J3, -2];
J4lm2 = extractLog[J4, -2];

In[=]:= ClearAll[J1lm3, J2lm3, J3lm3, J4lm3];
J1lm3 = extractLog[J1, -3];
J2lm3 = extractLog[J2, -3];
J3lm3 = extractLog[J3, -3];
J4lm3 = extractLog[J4, -3];

In[=]:= ClearAll[J1lm4, J2lm4, J3lm4, J4lm4];
J1lm4 = extractLog[J1, -4];
J2lm4 = extractLog[J2, -4];
J3lm4 = extractLog[J3, -4];
J4lm4 = extractLog[J4, -4];

```

```
In[1]:= MaTeX["\\begin{aligned}R_{(1)}=&\\frac{1}{\\hbar}\\Big(p_1\\wedge\nonumber\\\\\partial_p_1A_0-\\partial_x_1(p_1\\wedge\nonumber\\\\,A_0)\\Big)J1-\\frac{1}{2}\\partial_x\\Big(q^2\\,,p_1\\wedge\nonumber\\\\\partial_p_1A_0-\\partial_x_1(q^2\\,,p_1\\wedge\nonumber\\\\,A_0)\\Big)J1\\\\-\\frac{1}{2}\\Big(q^2\\,,p_1\\wedge\nonumber\\\\\partial_p_1A_0-\\partial_x_1(q^2\\,,p_1\\wedge\nonumber\\\\,A_0)\\Big)J2\\\\+\\frac{1}{\\hbar}A_0J_3-\\frac{1}{2}\\partial_x\\Big(q^2\\,,A_0\\,,J_4\\\\-\\frac{1}{\\hbar}l^{(1)}\\Big)J_3\\partial_q\\,,A_0+J_1(p_1\\partial_{p_1})\\,,\\partial_qA_0-\\partial_x_1(p_1\\wedge\nonumber\\\\,\\partial_q\\,,A_0))\\Big)\\\\+\\frac{1}{2}l^{(1)}\\Big(J_1\\,,\\partial_x\\Big(q^2\\,,p_1\\wedge\\partial_{p_1})\\,,\\partial_qA_0-\\partial_x_1(q^2\\,,p_1\\wedge\nonumber\\\\,q\\,,\\partial_q\\,,A_0)+J_2(q^2\\,,p_1\\wedge\\partial_{p_1})\\,,\\partial_qA_0-\\partial_x_1(q^2\\,,p_1\\wedge\nonumber\\\\,q\\,,\\partial_q\\,,A_0))\\\\+J_3\\,,\\partial_x\\Big(q^2\\,,\\partial_q\\,,A_0)+J_4\\,,q^2\\,,\\partial_q\\,,A_0\\Big)\\\\+\\frac{1}{2}\\hbar l^{(2)}\\Big(J_3\\partial_q\\,,A_0+J_1(p_1\\partial_{p_1})\\,,\\partial_qA_0-\\partial_x_1(p_1\\wedge\nonumber\\\\,q^2\\,,A_0))\\Big)\\\\-\\frac{1}{2}l^{(2)}\\Big(J_1\\,,\\partial_x\\Big(q^2\\,,A_0)\\Big)\\\\-\\frac{1}{2}\\Big(q^2\\,,\\partial_q\\,,A_0)+J_2(q^2\\,,p_1\\wedge\\partial_{p_1})\\,,\\partial_qA_0-\\partial_x_1(q^2\\,,p_1\\wedge\nonumber\\\\,q\\,,\\partial_q\\,,A_0)+J_3\\,,\\partial_x\\Big(q^2\\,,\\partial_q\\,,A_0)+J_4\\,,q^2\\,,\\partial_q\\,,A_0\\Big)\\Big)\\end{aligned}"]
```

$$\begin{aligned}
R_{(1)} = & \frac{1}{\hbar} \left(p_1 \wedge \partial p_1 A_0 - \partial_{x_1} (p_1 \wedge q A_0) \right) J_1 - \frac{1}{2} \partial_{\Delta x} \left(q^2 p_1 \wedge \partial p_1 A_0 - \partial_{x_1} (q^2 p_1 \wedge q A_0) \right) J_1 \\
& - \frac{1}{2} \left(q^2 p_1 \wedge \partial p_1 A_0 - \partial_{x_1} (q^2 p_1 \wedge q A_0) \right) J_2 \\
& + \frac{1}{\hbar} A_0 J_3 - \frac{1}{2} \partial_{\Delta x} (q^2 A_0) J_3 \\
& - \frac{1}{2} q^2 A_0 J_4 \\
& - \frac{1}{\hbar} l^{(1)} \left[J_3 \partial_q A_0 + J_1 (p_1 \partial_{p_1} \partial_q A_0 - \partial_{x_1} (p_1 \wedge q \partial_q A_0)) \right] \\
+ \frac{1}{2} l^{(1)} \left[& J_1 \partial_{\Delta x} (q^2 p_1 \wedge \partial_{p_1} \partial_q A_0 - \partial_{x_1} (q^2 p_1 \wedge q \partial_q A_0)) + J_2 (q^2 p_1 \wedge \partial_{p_1} \partial_q A_0 - \partial_{x_1} (q^2 p_1 \wedge q \partial_q A_0)) \right. \\
& \left. + J_3 \partial_{\Delta x} (q^2 \partial_q A_0) + J_4 q^2 \partial_q A_0 \right] \\
& + \frac{1}{2\hbar} l^{(2)} \left[J_3 \partial_q^2 A_0 + J_1 (p_1 \partial_{p_1} \partial_q^2 A_0 - \partial_{x_1} (p_1 \wedge q \partial_q^2 A_0)) \right] \\
- \frac{1}{4} l^{(2)} \left[& J_1 \partial_{\Delta x} (q^2 p_1 \wedge \partial_{p_1} \partial_q^2 A_0 - \partial_{x_1} (q^2 p_1 \wedge q \partial_q^2 A_0)) + J_2 (q^2 p_1 \wedge \partial_{p_1} \partial_q^2 A_0 - \partial_{x_1} (q^2 p_1 \wedge q \partial_q^2 A_0)) \right. \\
& \left. + J_3 \partial_{\Delta x} (q^2 \partial_q^2 A_0) + J_4 q^2 \partial_q^2 A_0 \right]
\end{aligned}$$

In[1]:= J4lm3

Out[1]=

$$\begin{aligned}
& - \frac{2 \gamma l \wedge p1}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \frac{\gamma l \wedge p1}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \\
& \frac{\gamma l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{2 \gamma l \wedge p1}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \frac{6 \gamma \hbar l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^4 (-i \epsilon + l \odot p2)} - \\
& \frac{2 \gamma \hbar l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^3} + \frac{2 \gamma \hbar l \odot q l \wedge p1}{(i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^3} + \frac{4 \gamma \hbar l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)^2} - \\
& \frac{6 \gamma \hbar l \odot q l \wedge p1}{(-i \epsilon + l \odot p1)^4 (i \epsilon + l \odot p2)} - \frac{p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \frac{p1 \wedge p2}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \\
& \frac{p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \frac{p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} + \frac{2 \hbar l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \\
& \frac{2 \hbar l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^3} + \frac{2 \hbar l \odot q p1 \wedge p2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^3} + \frac{2 \hbar l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \\
& \frac{2 \hbar l \odot q p1 \wedge p2}{(-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \frac{m2^2 \gamma \hbar p1 \wedge q}{S(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \\
& \frac{m2^2 \gamma \hbar p1 \wedge q}{S(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \frac{\gamma^2 \hbar p1 \wedge q}{S(-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \frac{m2^2 \gamma \hbar p1 \wedge q}{S(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \\
& \frac{\gamma^2 \hbar p1 \wedge q}{S(i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \frac{m2^2 \gamma \hbar p1 \wedge q}{S(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} - \frac{\gamma^2 \hbar p1 \wedge q}{S(-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)}
\end{aligned}$$

In[2]:= ClearAll[B0C0, B1C1];

B0C0 = Bx1hbar[0] + Bx2hbar[0] + Cx1hbar[0] + Cx2hbar[0];

B1C1 = Bx1hbar[1] + Bx2hbar[1] + Cx1hbar[1] + Cx2hbar[1];

In[3]:= simplifiedForm[B0C0 /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}]

Out[3]=

$$\begin{aligned}
& \frac{4 (p1 \odot p2)^2}{qml2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{4 (p1 \odot p2)^2}{qml2 (-i \epsilon + l \odot p2) (-i \epsilon + l \odot p1 - p1 \odot q)} - \\
& \frac{4 (p1 \odot p2)^2}{qml2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2 - p2 \odot q)} + \frac{4 (p1 \odot p2)^2}{qml2 (-i \epsilon + l \odot p1 - p1 \odot q) (i \epsilon + l \odot p2 - p2 \odot q)}
\end{aligned}$$

In[8]:= **simplifiedForm[B1C1 /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}]**

Out[8]=

$$\begin{aligned}
 & -\frac{\frac{2 qml2 (p1 \odot p2)^2}{(-i \epsilon + l \odot p2) (-i \epsilon + l \odot p1 - p1 \odot q)} - \frac{4 p1 \odot p2 \left(\frac{l \odot l \odot p2}{(-i \epsilon + l \odot p2)^2} - \frac{l \odot p1 + l \odot p2 - 2 p2 \odot q}{-i \epsilon + l \odot p2} \right) - \frac{4 p1 \odot p2 (l \odot p1 + l \odot p2 + p1 \odot q - p2 \odot q)}{-i \epsilon + l \odot p2}}{2 (-i \epsilon + l \odot p1 - p1 \odot q)} + \\
 & \frac{qml2}{qml2} \\
 & \frac{1}{qml2} \left(4 p1 \odot p2 \left(\frac{-2 l \odot p1 + 2 l \odot p2}{4 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \right. \right. \\
 & \left. \left. 4 \left(\frac{l \odot l}{8 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)^2} - \frac{l \odot l}{8 (i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} \right) p1 \odot p2 \right) - \\
 & \frac{2 p1 \odot p2 (l \odot p1 - l \odot p2 + p1 \odot q - p2 \odot q)}{(i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} \right) + \\
 & \frac{2 qml2 (p1 \odot p2)^2}{(i \epsilon + l \odot p1) (i \epsilon + l \odot p2 - p2 \odot q)^2} - \frac{4 p1 \odot p2 \left(\frac{l \odot l \odot p2}{(i \epsilon + l \odot p1)^2} + \frac{l \odot p1 + l \odot p2 - 2 p1 \odot q}{i \epsilon + l \odot p1} \right) + \frac{4 p1 \odot p2 (l \odot p1 + l \odot p2 - p1 \odot q + p2 \odot q)}{i \epsilon + l \odot p1}}{2 (i \epsilon + l \odot p2 - p2 \odot q)} + \\
 & \frac{qml2}{qml2} \\
 & \frac{2 qml2 (p1 \odot p2)^2}{(-i \epsilon + l \odot p1 - p1 \odot q)^2 (i \epsilon + l \odot p2 - p2 \odot q)} - \frac{\frac{4 qml2 (p1 \odot p2)^2}{(i \epsilon + l \odot p2 - p2 \odot q)^2} - \frac{8 p1 \odot p2 (l \odot p1 - l \odot p2 - p1 \odot q + p2 \odot q) 4 p1 \odot p2 (2 l \odot p1 - 2 l \odot p2 - 4 p1 \odot q + 4 p2 \odot q)}{2 (i \epsilon + l \odot p2 - p2 \odot q)}}{2 (-i \epsilon + l \odot p1 - p1 \odot q)} + \\
 & \frac{qml2}{qml2}
 \end{aligned}$$

In[9]:= **Coefficient[Expand[**

simplifiedForm[B1C1 /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}],

$$\frac{1}{(-i \epsilon + l \odot p1 - p1 \odot q) (i \epsilon + l \odot p2 - p2 \odot q)}, 1]$$

Out[9]=

$$\frac{4 l \odot p1 p1 \odot p2}{qml2} - \frac{4 l \odot p2 p1 \odot p2}{qml2} - \frac{6 p1 \odot p2 p1 \odot q}{qml2} + \frac{6 p1 \odot p2 p2 \odot q}{qml2}$$

In[10]:= **ClearAll[R1B1C1];**

R1B1C1 =

simplifyTerm[Expand[simplifiedForm[onShellP[aWedgeDb[B1C1, p1, p1] / l ⊕ l] /.

{l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}] / 4]];

ClearAll[R1B1C1p1q, R1B1C1p1p2, R1B1C1p1l];

R1B1C1p1q = Coefficient[R1B1C1, p1 ∧ q, 1];

R1B1C1p1p2 = Coefficient[R1B1C1, p1 ∧ p2, 1];

R1B1C1p1l = Coefficient[R1B1C1, l ∧ p1, 1];

```

In[=]:= ClearAll[R1B0C0];
R1B0C0 = simplifyTerm[
  Expand[simplifiedForm[onShellP[delx[-q ⊗ q * aWedgeDb[B0C0, p1, p1]]/(8 * l ⊗ l)]]];

ClearAll[R1B0C0p1q, R1B0C0p1p2, R1B0C0p1l];
R1B0C0p1q = Coefficient[R1B0C0, p1 ∧ q, 1];
R1B0C0p1p2 = Coefficient[R1B0C0, p1 ∧ p2, 1];
R1B0C0p1l = Coefficient[R1B0C0, l ∧ p1, 1];

In[=]:= ClearAll[R2B1C1];
R2B1C1 = simplifyTerm[Expand[simplifiedForm[
  onShellP[D[-p1 ∧ q * (Expand[B1C1 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}] /. {qml2 → l ⊗ l - 2 l ⊗ q + q ⊗ q}], x1] / (4 * l ⊗ l)]]];

ClearAll[R2B1C1p1q, R2B1C1p1p2, R2B1C1p1l];
R2B1C1p1q = Coefficient[R2B1C1, p1 ∧ q, 1];
R2B1C1p1p2 = Coefficient[R2B1C1, p1 ∧ p2, 1];
R2B1C1p1l = Coefficient[R2B1C1, l ∧ p1, 1];

In[=]:= ClearAll[R2B0C0];
R2B0C0 = simplifyTerm[
  Expand[simplifiedForm[onShellP[delx[D[q ⊗ q * p1 ∧ q * B0C0, x1]]/(8 * l ⊗ l)]]];

ClearAll[R2B0C0p1q, R2B0C0p1p2, R2B0C0p1l];
R2B0C0p1q = Coefficient[R2B0C0, p1 ∧ q, 1];
R2B0C0p1p2 = Coefficient[R2B0C0, p1 ∧ p2, 1];
R2B0C0p1l = Coefficient[R2B0C0, l ∧ p1, 1];

In[=]:= R1B0C0p1p2 /. {l ⊗ l - 2 l ⊗ q + q ⊗ q → qml2, -l ⊗ l + 2 l ⊗ q - q ⊗ q → -qml2}
Out[=]=

$$\frac{\gamma q \odot q}{qml2 l \odot l (-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \frac{m1^2 \gamma^2 q \odot q}{2 qml2 S l \odot l (-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} +$$


$$\frac{\gamma^3 q \odot q}{2 qml2 S l \odot l (-i\epsilon + l \odot p1)^2 (-i\epsilon + l \odot p2)} + \frac{\gamma q \odot q}{qml2 l \odot l (-i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} -$$


$$\frac{\gamma q \odot q}{qml2 l \odot l (i\epsilon + l \odot p1)(i\epsilon + l \odot p2)^2} - \frac{\gamma q \odot q}{qml2 l \odot l (-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} -$$


$$\frac{m1^2 \gamma^2 q \odot q}{2 qml2 S l \odot l (-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)} - \frac{\gamma^3 q \odot q}{2 qml2 S l \odot l (-i\epsilon + l \odot p1)^2 (i\epsilon + l \odot p2)}$$


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In[1]:= R2B0C0p1p2 /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}

Out[1]=

$$\begin{aligned}
 & -\frac{m1^2 \gamma^2 q \odot q}{2 qml2 s l \odot l (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \frac{\gamma^3 q \odot q}{qml2 s l \odot l (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \\
 & \frac{\gamma^3 q \odot q}{2 qml2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \frac{\gamma^3 q \odot q}{2 qml2 s l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \\
 & \frac{m1^2 \gamma^2 q \odot q}{2 qml2 s l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} + \frac{\gamma^3 q \odot q}{qml2 s l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)}
 \end{aligned}$$

In[2]:= R1B1C1p1p2 /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}

Out[2]=

$$\begin{aligned}
 & -\frac{\gamma}{qml2 (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)^2} + \frac{\gamma}{qml2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)^2} - \\
 & \frac{\gamma}{l \odot l (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \frac{\gamma}{qml2 (i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} - \\
 & \frac{\gamma}{l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \frac{\gamma}{l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} + \\
 & \frac{\gamma}{l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} + \frac{\gamma}{qml2 (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)}
 \end{aligned}$$

In[3]:= R2B1C1p1p2 /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}

Out[3]=

$$\begin{aligned}
 & \frac{\gamma^3}{2 qml2 s (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)^2} - \frac{\gamma^3}{2 qml2 s (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)^2} + \\
 & \frac{\gamma^3}{2 s l \odot l (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \frac{\gamma^3}{2 qml2 s (i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)} + \\
 & \frac{\gamma^3}{2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \frac{\gamma^3}{2 s l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)^2} - \\
 & \frac{\gamma^3}{2 s l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)} - \frac{\gamma^3}{2 qml2 s (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)}
 \end{aligned}$$

In[1]:= R1B0C0p1q /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}

Out[1]=

$$\begin{aligned} & \frac{\gamma^2 q \odot q}{qml2 l \odot l (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \\ & \frac{m2^2 \gamma^2 q \odot q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{\gamma^3 q \odot q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \\ & \frac{\gamma^2 q \odot q}{2 qml2 l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \frac{\gamma^2 q \odot q}{qml2 l \odot l (-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \\ & \frac{m2^2 \gamma^2 q \odot q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{\gamma^3 q \odot q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} \end{aligned}$$

In[2]:= R2B0C0p1q /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}

Out[2]=

$$\begin{aligned} & \frac{m2^2 \gamma^2}{qml2 s l \odot l (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \\ & \frac{\gamma^3}{qml2 s l \odot l (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{m2^2 \gamma^2}{qml2 s l \odot l (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \\ & \frac{\gamma^3}{qml2 s l \odot l (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{m2^2 \gamma^2}{qml2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \\ & \frac{\gamma^3}{qml2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{m2^2 \gamma^2}{qml2 s l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \\ & \frac{\gamma^3}{qml2 s l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{\gamma^2 q \odot q}{qml2 l \odot l (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} + \\ & \frac{m2^2 \gamma^2 q \odot q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{m2^2 \gamma^2 q \odot q}{qml2^2 s l \odot l (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \\ & \frac{\gamma^3 q \odot q}{qml2^2 s l \odot l (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{\gamma^2 q \odot q}{2 qml2 l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \\ & \frac{\gamma^2 q \odot q}{qml2 l \odot l (-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} - \frac{m2^2 \gamma^2 q \odot q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \\ & \frac{\gamma^3 q \odot q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{m2^2 \gamma^2 q \odot q}{qml2^2 s l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} \end{aligned}$$

In[8]:= R1B1C1p1q /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}

Out[8]=

$$\begin{aligned}
 & -\frac{\gamma^2}{2 qml2 (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} - \frac{\gamma^2}{l \odot l (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} + \\
 & \frac{3 \gamma}{2 qml2 l \odot l (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{\gamma}{2 qml2 l \odot l (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \\
 & \frac{\gamma^2}{2 l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{\gamma^2}{l \odot l (-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} - \\
 & \frac{\gamma}{2 qml2 l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{3 \gamma}{2 qml2 l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)}
 \end{aligned}$$

In[9]:= R2B1C1p1q /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}

Out[9]=

$$\begin{aligned}
 & \frac{\gamma^2}{2 qml2 (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} + \frac{\gamma^2}{l \odot l (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} + \\
 & \frac{\gamma^3}{qml2^2 S (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{3 \gamma}{2 qml2 l \odot l (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \\
 & \frac{m2^2 \gamma^2}{qml2^2 S (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{\gamma^3}{qml2^2 S (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \\
 & \frac{\gamma}{2 qml2 l \odot l (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{\gamma^2}{2 l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \\
 & \frac{\gamma^2}{l \odot l (-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \frac{\gamma}{2 qml2 l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \\
 & \frac{m2^2 \gamma^2}{qml2^2 S (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{3 \gamma}{2 qml2 l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)}
 \end{aligned}$$

In[1]:= R1B0C0p1l/. {l₀ l - 2 l₀ q + q₀ q → qml2, -l₀ l + 2 l₀ q - q₀ q → -qml2}

Out[1]=

$$\begin{aligned} & \frac{\gamma^2 q_{\odot} q}{qml2 l \odot l (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \frac{m2^2 \gamma^2 q_{\odot} q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \\ & \frac{\gamma^3 q_{\odot} q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{m2^2 \gamma^2 q_{\odot} q}{qml2^2 s l \odot l (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \\ & \frac{\gamma^3 q_{\odot} q}{qml2^2 s l \odot l (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} + \frac{\gamma^2 q_{\odot} q}{2 qml2 l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \\ & \frac{\gamma^2 q_{\odot} q}{2 qml2 l \odot l (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} - \frac{\gamma^2 q_{\odot} q}{qml2 l \odot l (-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \\ & \frac{m2^2 \gamma^2 q_{\odot} q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} + \frac{\gamma^3 q_{\odot} q}{qml2^2 s l \odot l (-i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \\ & \frac{m2^2 \gamma^2 q_{\odot} q}{qml2^2 s l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} - \frac{\gamma^3 q_{\odot} q}{qml2^2 s l \odot l (i \epsilon + l \odot p1) (i \epsilon + l \odot p2)} \end{aligned}$$

In[2]:= R2B0C0p1l/. {l₀ l - 2 l₀ q + q₀ q → qml2, -l₀ l + 2 l₀ q - q₀ q → -qml2}

Out[2]=

$$0$$

In[3]:= R1B1C1p1l/. {l₀ l - 2 l₀ q + q₀ q → qml2, -l₀ l + 2 l₀ q - q₀ q → -qml2}

Out[3]=

$$\begin{aligned} & -\frac{\gamma^2}{2 qml2 (-i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} + \frac{\gamma^2}{2 qml2 (i \epsilon + l \odot p1)^2 (-i \epsilon + l \odot p2)^2} - \\ & \frac{\gamma^2}{l \odot l (-i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \frac{\gamma^2}{qml2 (i \epsilon + l \odot p1)^3 (-i \epsilon + l \odot p2)} - \\ & \frac{\gamma^2}{2 l \odot l (-i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \frac{\gamma^2}{2 l \odot l (i \epsilon + l \odot p1)^2 (i \epsilon + l \odot p2)^2} + \\ & \frac{\gamma^2}{l \odot l (-i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} + \frac{\gamma^2}{qml2 (i \epsilon + l \odot p1)^3 (i \epsilon + l \odot p2)} \end{aligned}$$

In[4]:= R2B1C1p1l/. {l₀ l - 2 l₀ q + q₀ q → qml2, -l₀ l + 2 l₀ q - q₀ q → -qml2}

Out[4]=

$$0$$

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In[8]:= simplifiedForm[B0C0 /. {l ⊕ l - 2 l ⊙ q + q ⊙ q → qml2, -l ⊕ l + 2 l ⊙ q - q ⊙ q → -qml2}]

Out[8]=

$$\frac{4 (p1 \odot p2)^2}{qml2 (i \epsilon + l \odot p1) (-i \epsilon + l \odot p2)} - \frac{4 (p1 \odot p2)^2}{qml2 (-i \epsilon + l \odot p2) (-i \epsilon + l \odot p1 - p1 \odot q)} -$$


$$\frac{4 (p1 \odot p2)^2}{qml2 (i \epsilon + l \odot p1) (i \epsilon + l \odot p2 - p2 \odot q)} + \frac{4 (p1 \odot p2)^2}{qml2 (-i \epsilon + l \odot p1 - p1 \odot q) (i \epsilon + l \odot p2 - p2 \odot q)}$$

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