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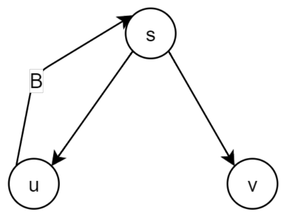
4. [4 points: 2 points each part.]

(a) Give a counterexample to the following: if there's a path from u to v and $u.d < v.d$, then v is a descendant of u.

Answer:

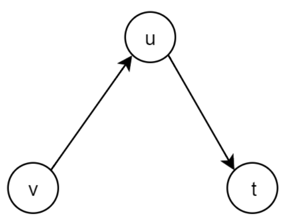


If you consider the above graph, with *dfs* starting at s, then -
we'll discover u first ($s \rightarrow u$).
Then we'll find $u \rightarrow s$. But as s has already been discovered, we label the edge as *backedge* B.
Then we come back to s, and go to v next. ($s \rightarrow v$).
No more nodes to be explored. This completes the traversal of the graph.

Now, in this traversal starting with node s, we can see clearly that v is discovered later than u. Hence $u.d < v.d$.
But v is not a descendant of u. Both share a common parent s; but no direct relation.

(b) How can a vertex u appear as the only vertex in a DFS tree even though it has both incoming and outgoing edges? Give an example.

Answer:



Consider a case like in the figure above. Let us assume *dfs* traversal of the above graph, starting in alphabetical order. Hence we start at t.
We find that there are no outgoing edges from t, we are finished with t with discovery/finish time being 1/2.
We move on to u. We see that u has an outgoing edge to t; but as t has already been marked as visited, no further traversal from u. We mark u.d/u.f as 3/4.
Next we move on to v. As all other nodes pointed by outgoing edges from v are visited, we are done with v and mark it as 5/6. As all nodes have been visited, DFS traversal is complete.

As you can see from above, u has an outgoing edge (and an incoming edge as well); but in DFS, it still ends up as a tree with itself being the only vertex.
This happened because outgoing edge from u pointed to an already visited vertex, and incoming edges to u were traversed after u was already visited.