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1. [4 points] Come up with a counterexample to the following conjecture.

   > Let $G = (V, E)$ be a connected, undirected graph with integer-valued weight function w defined on E. Let $A \subseteq E$ be such that $A \subseteq T$ for some minimum spanning tree T of G. Suppose $(S, V - S)$ is a cut of G that respects A and let uv be a safe edge for A crossing $(S, V - S)$. Then uv is a light edge for $(S, V - S)$.

   See slide 5 of the MST slides (or the textbook) for definitions of these terms.

Answer:

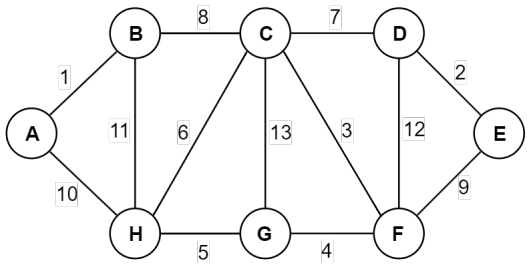
We know that -
An edge e is a **safe** edge if,

$(V, A \in e) \subseteq$ some MST of Graph G

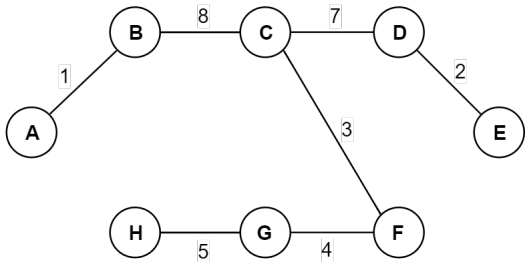
and
An edge e is a **light** edge if,

it crosses the cut $(S, V - S)$, is of minimum weight of all edges crossing the cut and respects the cut.

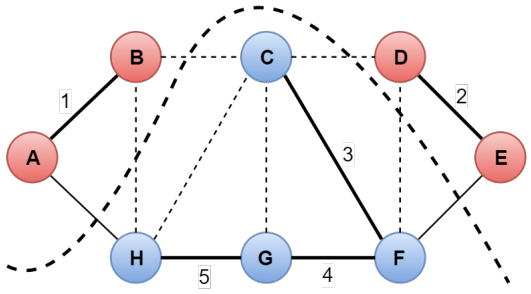
Let $G(V, E)$ be a connected, undirected graph with integer valued weight function w defined on E. The graph is given as follows -



Since all edges are unique, we have a single MST T defined as follows -



Let $A = AB, DE, CF, FG, GH$ and the cut $(S, V - S)$ defined as follows -



Consider the edge $BC(8)$ for A, crossing the cut $(S, V - S)$.
Since $(V, A \in BC \subseteq T$,
BC is a safe edge; but is not a light edge, as there is another edge $CD(7)$ which crosses the cut $(S, V - S)$ and is of minimum weight.

Hence, the conjecture is $false$.