GDM HW#5 PGM

P ①

$$\begin{tikzcd} & w \arrow[dl] \arrow[d] & \\ q & x \arrow[dr] & & z \arrow[dl] \\ & & y \arrow[d] & \\ & & p & \end{tikzcd}$$

(a) $y \perp\!\!\!\perp w \mid x$

Only one path $w \rightarrow x \rightarrow y$

h2t model $x \in$ Observed
Blocked by II.

$\therefore$ TRUE.

(b) $x \perp\!\!\!\perp q \mid w$

Only one path $q \leftarrow w \rightarrow x$

t2t model $w \in$ Observed.
Blocked by II.

$\therefore$ TRUE.

1 (c) $z \perp\!\!\!\perp x, w, q \mid p$.

q $\quad q \leftarrow w \rightarrow x \rightarrow y \leftarrow z$ $\quad$ Blocked by I_w.

w $\quad w \rightarrow x \rightarrow y \leftarrow z$ $\quad I_y$

x $\quad x \rightarrow y \leftarrow z$ $\quad I_y$

For all three cases above,
$w, y \notin$ Observed. $\therefore$ Blocked by I.

$\therefore$ TRUE.

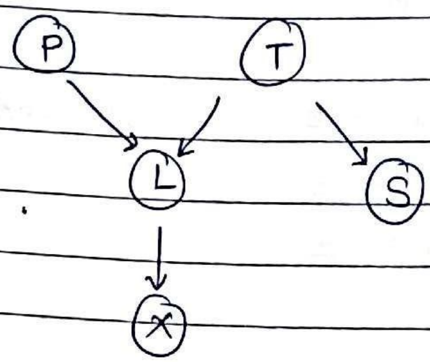
(d) $y \perp\!\!\!\perp w$.

As w is a parent of y,
y is not conditionally indep. of w.
(marginally)

$\therefore$ FALSE.

(2)

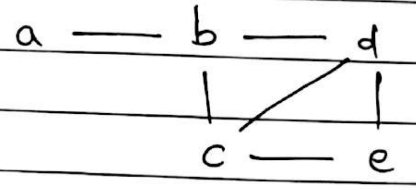
(i)



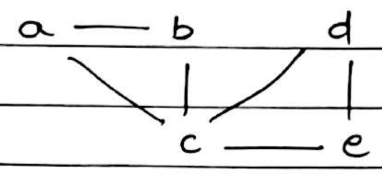
(2) $P(P, L, T, S, X)$

$$= P(P) \cdot P(T) \cdot P(L \mid P, T) \cdot P(S \mid T) \cdot P(X \mid L)$$

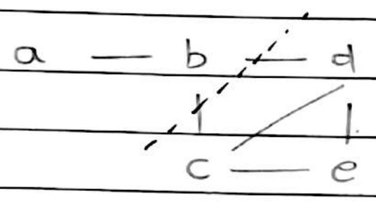
(3) 3. A



3. B



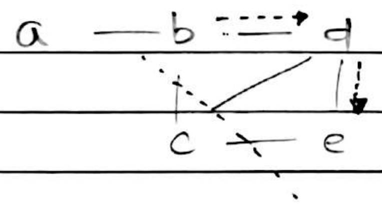
(1) $d, e \perp\!\!\!\perp a \mid b$.



Every path to 'a' from 'd' and 'e' goes through 'b'.

∴ TRUE.

$b \perp\!\!\!\perp e \mid c$



As you can see, even if we cut 'c' out, we can reach 'e' from 'b' through 'd'.

∴ FALSE.

classmate

Date

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3 (2) 3.A

$p(a, b, c, d, e)$

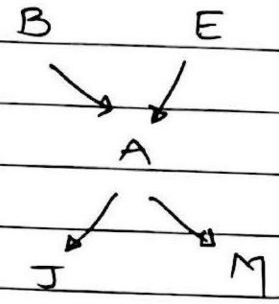
$= \prod \psi_1(a, b) \times \psi_2(b, c, d) \times \psi_3(c, d, e)$

3.B

$p(a, b, c, d, e)$

$= \prod \psi_1(a, b, c) \times \psi_2(c, d, e)$.

(4)



(1) $P(B, \sim E, A, J, \sim M)$

$= P(B) \times P(\sim E) \times P(A \mid B, \sim E) \times P(J \mid A) \times P(\sim M / A)$

$= P(B) \times [1 - \underset{0.002}{P(E)}] \times P(A \mid B, \sim E) \times P(J \mid A) \times [1 - \underset{0.7}{P(M / A)}]$

$= 0.001 \times 0.998 \times 0.94 \times 0.9 \times 0.3$

(Substituting values.)

$= 0.000253$ //

(2) $\cancel{P(A) = [P(A \mid B) \times P(B)] + [P(A \mid M) \times P(M)]}$

$\cancel{=}$

(2) $P(A) = \dfrac{P(A|B,E) \times P(B,E)}{P(B,E|A)}$.

[∴ Using Bayes' Thm.]

Now, $P(A|B,E) = 0.95$

$P(B,E) = P(B) \cdot P(E)$. (Independent)

$= 0.001 \times 0.002$

$= 0.000002$

$P(B,E|A) = 1$

As 'A' has happened,
one of its parents must've happened.
∴ The parents' joint prob given 'A' is '1'.

$\therefore P(A) = \dfrac{0.95 \times 0.000002}{1}$

$= 0.0000019$,,

[OR $P(B,E|A) = P(A|B,E) + P(A|B,\sim E) + P(A|\sim B,E) + P(A|\sim B,\sim E)$]

(5)

$$\begin{tikzcd} y_1 \arrow[r, dash] \arrow[d, dash] & y_2 \arrow[r] \arrow[d, dash] & y_3 \arrow[r, dash] \arrow[d, dash] & \cdots & y_n \arrow[d, dash] \\ x_1 & x_2 & x_3 & & x_n \end{tikzcd}$$

$P(x_1, x_2 \ldots x_n, y_1 y_2 \ldots y_n)$

$$= \prod_{i=1}^{N} \psi_1(y_i, y_{i-1}) * \psi_2(x_i, y_i)$$

where $\psi(y_1, y_0) = \psi(y_1)$. //

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(6) Summing up over values of 'c'

a	b	p(a, b)
0	0	0.336
0	1	0.264
1	0	0.256
1	1	0.144

(1) If $a \perp\!\!\!\perp b$,
then $p(a,b) = p(a) * p(b)$.

From above table, let us test if this eq^n holds.
For $a=0$ and $b=0$,

$p(a=0, b=0) = 0.336$.

$p(a=0) = p(a=0 \mid b=0) + p(a=0 \mid b=1)$
$= 0.336 + 0.264$
$= 0.600$

$p(b=0) = p(b=0 \mid a=0) + p(b=0 \mid a=1)$
$= 0.336 + 0.256$
$= 0.692$.

Now, $p(a=0) * p(b=0) = 0.6 \times 0.692$
$= 0.4152$
$\neq 0.336$
$\neq p(a=0, b=0)$

$\therefore$ FALSE

(2) If $a \perp\!\!\!\perp b \mid c$,

then $p(a,b \mid c) = p(a \mid c) \times p(b \mid c)$

Lets test for $c = 0$.

$$p(a,b \mid c=0) = 0.192 + 0.048 + 0.192 + 0.048$$
$$= 0.48$$

$p(a \mid c=0) =$