Homework: **Probabilistic Graphical Models (PGM)**

1. Check the Assignment Schedule for the DUE date.
2. Submit via Moodle.

Problem 1

Given the following directed graph:

w -> q
w -> x
x -> y
z -> y
y -> p

Which of the statement(s) are true? Justify your answers.

a. y is conditionally independent of w given x.
b. x is conditionally independent of q given w.
c. z is conditionally independent of x, w, and q given p.
d. y is conditionally independent of w.

Problem 2

1. Draw a DAG for the following toy problem:
    - Pneumonia (P) and tuberculosis (T) may cause a patient to have lung infiltrates (L).
    - An x-ray test (X) can be taken to indicate whether the patient has lung infiltrates (L).
    - A sputum smear test (S) can be taken to check for tuberculosis (T).
2. Write down the factorization of the joint probability function for this DAG.

Problem 3

Graph 3.A	Graph 3.B
a – b	a – b
b – c	a – c
b – d	b – c
c – d	c – d
c – e	c – e
d – e	d – e

1. Justify if these statements are true for Figure 3.A:
    - d and e are conditionally independent of a given b.
    - b is conditionally independent of e given c.

2. Write down the joint probability distribution p(a,b,c,d,e) for both 3.A and 3.B.

Problem 4

Given the following PGM with provided marginal and conditional probabilities:

Graph:
B->A
E->A
A->J
A->M

Probabilities:
P(B) = 0.001
P(E) = 0.002
P(A|B,E) = 0.95
P(A|B,~E) = 0.94
P(A|~B,E) = 0.29
P(J|A) = 0.9
P(J|~A) = 0.05
P(M|A) = 0.7
P(M|~A) = 0.01

1. Calculate the joint probability P(B,~E,A,J,~M). Assume each random variable is True or False. That is, P(~E) = 1 – P(E) and P(~M|A) = 1 – P(M|A)
2. What is the marginal probability P(A)?

Problem 5

1. Write down the joint probability function for the following undirected graph:

    y1 – y2, y2 – y3,…, y(n-1) – yn
    y1 – x1, y2 – x2,…, yn – xn

Problem 6

Suppose the probability table for a joint probability function is given below:

a	b	c	p(a,b,c)
0	0	0	0.192
0	0	1	0.144
0	1	0	0.048
0	1	1	0.216
1	0	0	0.192
1	0	1	0.064
1	1	0	0.048
1	1	1	0.096

Which of the following statements are true? Justify your answers.

1. a is conditionally independent of b.
2. a is conditionally independent of b given c for both c = 0 and c = 1.