UNIVERSITY OF PERADENIYA

FACULTY OF SCIENCE

DEPARTMENT OF STATISTICS & COMPUTER SCIENCE

ST 405 – MULTIVARIATE METHODS II

MINI PROJECT – 02

CANONICAL CORRELATION ANALYSIS

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1. Introduction

The automobile industry is constantly evolving, with manufacturers striving to produce vehicles that are powerful and efficient. One crucial aspect in understanding the performance of automobiles is analyzing the relationship between their specifications and characteristics. This study aim to investigate the relationship between car specifications (such as miles per gallon, displacement, horsepower and Rear axle ratio) and characteristics (such as weight, Type of Transmission, Number of Cylinders, Gears and Corburetors) using Canonical Correlation Analysis (CCA).

Understanding these relationships can provide valuable insights to optimize vehicle designs for improved performance and fuel efficiency. Additionally, this study contributes to the field of multivariate statistics by applying CCA to real-world automotive data.

2. Methodology

2.1. Dataset

The dataset using in this project, known as the **`mtcars`** dataset, Which is inbuilt in R provides a comprehensive overview of various cars attributes. The data was extracted from the 1974 *Motor Trend* US magazine, and comprises fuel consumption and 10 aspects of automobile design and performance for 32 automobiles. This dataset consists of 11 columns and for the analysis in this project, I am specifically interested in a subset of columns that include:

- **mpg**: This column indicates the miles per gallon, representing each car's fuel efficiency.
- **cyl**: It denotes the number of cylinders in each car's engine.
- **disp**: This column represents the engine displacement, reflecting the engine's size and capacity.
- **hp**: It signifies the horsepower, which measures the engine's power output.

- **drat**: This column indicates the rear axle ratio, affecting the car's acceleration and top speed.
- **wt**: It represents the weight of the car.
- **am**: This binary column indicates the type of transmission, with 0 denoting an automatic transmission and 1 representing a manual transmission.
- **gear**: It denotes the number of forward gears.
- **carb**: This column indicates the number of carburetors in the engine.

These columns collectively provide insights into crucial aspects of each car's specifications and characteristics, which allows analyze their relationship using Canonical Correlation Analysis (CCA). This analysis aims to uncover patterns and connections between these attributes, which can inform decisions in automotive engineering and design.

2.2. Data Pre-processing

Data Cleaning: Removing any incomplete or inconsistent records from the dataset.

Then, separate the selected 9 variables into two groups. The first group includes Car Specifications and the second group has Characteristic variables

Car Specifications

- **mpg** - miles per gallon
- **disp** - displacement
- **hp** - horsepower
- **drat** - rear axle ratio

Car Characteristics

- **cyl** - number of cylinders in engine
- **wt** - weight of the car.
- **Am** – Transmission (0 – automatic / 1 – manual)
- **Gear** - number of forward gears
- **Carb** - number of carburetors in the engine

Then I standardize the variables in dataset before applying Canonical Correlation Analysis (CCA), here what does is transform each variable to have a mean of 0 and a standard deviation of 1, ensuring that all variables are on the same scale

2.3. Canonical Correlation Analysis

The Spearman correlation coefficient is useful for examining the relationship between individual X and Y variables but falls short in assessing correlations within groups of X and Y variables. This is due to the numerous correlations that exist between the two groups, making it difficult to understand the overall relationship. Canonical Correlation Analysis (CCA), on the other hand, provides a multivariate approach to this problem. By using a method similar to principal component analysis, CCA reduces dimensionality and extracts principal components from both groups. It then maximizes the correlation between these components while ensuring they are uncorrelated within their respective groups. This approach allows CCA to offer a comprehensive view of the linear correlation between two sets of variables (Que, Luo, Wang, Zhou, & Yuan, 2022).

2.4. Wilks' Lambda Test

Wilks' Lambda test used in multivariate analysis to assess the significance of the relationship between two sets of variables. It measures the proportion of variance in one set of variables that cannot be accounted for by the other set.

Wilks' Lambda can help determine the overall significance of the relationship between the specifications and characteristics of cars.

Null hypothesis: there is no significant relationship between car specifications and characteristics.

Alternative hypothesis: there is a significant relationship between car specifications and characteristics.

3. Results and Discussion

3.1. Canonical Correlation

The canonical correlation coefficients for the study are 0.9747788, 0.8550909, 0.4771323 and 0.3174629. The squared canonical correlations are 0.9501937, 0.7311804, 0.2276552 and 0.1007827 for canonical variates. Respectively, these coefficients indicate that approximately 95%, 73.12%, 22.8% and 10% of the variance in the respective sets of variables is shared between the canonical variates of the two sets.

3.2. Wilk's Lambda Test

Wilks' Lambda, using F-approximation (Rao's F):

	stat	approx	df1	df2	p.value
1 to 4:	0.009298678	11.962258	20	77.23224	2.220446e-16
2 to 4:	0.186696743	4.708472	12	63.78953	1.920850e-05
3 to 4:	0.694505794	1.666225	6	50.00000	1.489075e-01
4 to 4:	0.899217319	1.457017	2	26.00000	2.513279e-01

Figure 1: Wilk's Lambda using F-approximation

The Wilks' Lambda test results indicate that the first and second sets of canonical correlations are statistically significant, suggesting a meaningful relationship between the two sets of variables for these canonical variates. In contrast, the third and fourth sets of canonical correlations are not statistically significant, indicating that these canonical variates do not contribute significantly to the relationship between the variable sets.

3.3. Canonical Loadings

3.3.1. Car Specifications Canonical Loadings

	[,1]	[,2]	[,3]	[,4]
mpg	0.2769325	0.1932924	-1.7989608	-1.046130
disp	-0.7169487	1.0298140	-1.6049850	1.080137
hp	0.1003727	-1.4480923	-0.3885621	-1.082230
drat	0.1508174	-0.3139453	-0.2825614	1.491155

Figure 2: Car Specifications Canonical Loadings

The standardized canonical loadings for the car specifications show the relationships between the original variables (mpg, disp, hp, drat) and the canonical variate pairs.

In the first canonical variate, mpg has a positive loading (0.2769325), disp has a negative loading (-0.7169487), hp has a small positive loading (0.1003727), and drat has a small positive loading (0.1508174). This suggests that the first canonical variate is primarily characterized by a strong negative contribution from disp and smaller positive contributions from mpg, hp, and drat.

In the second canonical variate, mpg has a moderate positive loading (0.1932924), disp has a large positive loading (1.0298140), hp has a large negative loading (-1.4480923), and drat has a small negative loading (-0.3139453). This indicates that the second canonical variate is mainly influenced by high positive contributions from disp, moderate positive contributions from mpg, and small negative contributions from hp and drat.

For the third canonical variate, mpg has a large negative loading (-1.7989608), disp has a large negative loading (-1.6049850), hp has a small negative loading (-0.3885621), and drat has a small negative loading (-0.2825614). This suggests that the third canonical variate is predominantly characterized by high negative contributions from mpg and disp, and smaller negative contributions from hp and drat.

For the fourth canonical variate, mpg has a moderate negative loading (-1.046130), disp has a large positive loading (1.080137), hp has a large negative loading (-1.082230), and drat has a large positive loading (1.491155). This indicates that the fourth canonical variate is primarily influenced by large positive contributions from disp and drat, and large negative contributions from mpg and hp.

3.3.2. Car Characteristics Canonical Loadings

	[,1]	[,2]	[,3]	[,4]
cyl	-0.5937477838	-0.27785984	-1.2972413	-1.18600680
wt	-0.5892256398	0.48681193	-0.6233610	1.86688856
am	-0.0000253653	-0.03901582	-1.0278586	0.68503654
gear	-0.0491271803	-0.18112746	-0.8321884	0.22944247
carb	0.1944597889	-0.89517086	1.4291001	-0.09880977

Figure 3: Car Characteristics Canonical Loadings

The standardized canonical coefficients for the car characteristics depict the relationships between the original variables (cyl, wt, am, gear, carb) and the canonical variate pairs.

In the first canonical variate, cyl has a negative loading (-0.5937477838), wt has a negative loading (-0.5892256398), am has a small negative loading (-0.0000253653), gear has a small negative loading (-0.0491271803), and carb has a moderate positive loading (0.1944597889). This indicates that the first canonical variate is predominantly characterized by negative contributions from cyl and wt, with a moderate positive contribution from carb.

In the second canonical variate, cyl has a small negative loading (-0.27785984), wt has a moderate positive loading (0.48681193), am has a small negative loading (-0.03901582), gear has a small negative loading (-0.18112746), and carb has a large negative loading (-0.89517086). This suggests that the second canonical variate is mainly influenced by a large negative contribution from carb and moderate positive contributions from wt.

For the third canonical variate, cyl has a large negative loading (-1.2972413), wt has a small negative loading (-0.6233610), am has a large negative loading (-1.0278586), gear has a moderate negative loading (-0.8321884), and carb has a large positive loading (1.4291001). This indicates that the third canonical variate is predominantly characterized by large negative contributions from cyl, am, and gear, and a large positive contribution from carb.

For the fourth canonical variate, cyl has a large negative loading (-1.18600680), wt has a large positive loading (1.86688856), am has a moderate positive loading (0.68503654), gear has a small positive loading (0.22944247), and carb has a small negative loading (-0.09880977). This suggests that the fourth canonical variate is primarily influenced by large positive contributions from wt and am, and large negative contributions from cyl

4. Conclusion and recommendation

The canonical correlation analysis between General Blood Tests and Liver Function Tests revealed significant relationships, indicating shared variance and potential dependencies between the two sets of variables. The standardized canonical coefficients and canonical loadings provided valuable insights into the specific contributions of the original tests to each canonical variate pair. To gain deeper insights, further investigation into the key variables driving the associations is recommended, as this can offer valuable clinical information regarding the interplay between general health indicators and liver functionality. Additionally, exploring the clinical significance of these relationships and validating the findings through independent datasets or cross-validation techniques will enhance the robustness and applicability of the analysis in medical research and decision-making

5. References

Que, S., Luo, H., Wang , L., Zhou , W., & Yuan, S. (2022, February 9). Canonical Correlation Study on the Relationship between Shipping Development and Water Environment of the Yangtze River. pp. 3-4

('Using the Canonical Correlation Analysis Method to Study Students' Levels in Face-to-Face and Online Education in Jordan', 2023)

Afifi, A, Clark, V and May, S. 2004. Computer-Aided Multivariate Analysis. 4th ed. Boca Raton, Fl: Chapman & Hall/CRC.

González, I., Déjean, S., Martin, P. G., & Baccini, A. (2008). CCA: AnRPackage to Extend Canonical Correlation Analysis. Journal of Statistical Software, 23(12).

6. Appendices

```
library(CCA)

library(CCP)
library(candisc)

library(ggplot2)
library(dplyr)

data("mtcars")
df <- mtcars %>%
  select(mpg,cyl,disp,hp,drat,wt,am,gear,carb)

head(df)

##                    mpg cyl disp  hp drat    wt am gear carb
## Mazda RX4         21.0   6  160 110 3.90 2.620  1    4    4
## Mazda RX4 Wag     21.0   6  160 110 3.90 2.875  1    4    4
## Datsun 710        22.8   4  108  93 3.85 2.320  1    4    1
## Hornet 4 Drive    21.4   6  258 110 3.08 3.215  0    3    1
## Hornet Sportabout 18.7   8  360 175 3.15 3.440  0    3    2
## Valiant           18.1   6  225 105 2.76 3.460  0    3    1

# Check for null values in each column
missing_values <- df %>%
 summarise_all(~ sum(is.na(.)))
# Print the number of missing values in each column
print(missing_values)

##   mpg cyl disp hp drat wt am gear carb
## 1   0   0    0  0    0  0  0    0    0

car_spec <- cbind(df[,c(1,3:5)])
car_charac <- cbind(df[,c(2,6:9)])

car_spec

##                      mpg  disp  hp drat
## Mazda RX4           21.0 160.0 110 3.90
## Mazda RX4 Wag       21.0 160.0 110 3.90
## Datsun 710          22.8 108.0  93 3.85
## Hornet 4 Drive      21.4 258.0 110 3.08
## Hornet Sportabout   18.7 360.0 175 3.15
## Valiant             18.1 225.0 105 2.76
## Duster 360          14.3 360.0 245 3.21
## Merc 240D           24.4 146.7  62 3.69
## Merc 230            22.8 140.8  95 3.92
## Merc 280            19.2 167.6 123 3.92
## Merc 280C           17.8 167.6 123 3.92
## Merc 450SE          16.4 275.8 180 3.07
## Merc 450SL          17.3 275.8 180 3.07
## Merc 450SLC         15.2 275.8 180 3.07
## Cadillac Fleetwood  10.4 472.0 205 2.93
## Lincoln Continental 10.4 460.0 215 3.00
```

```
## Chrysler Imperial   14.7 440.0 230 3.23
## Fiat 128            32.4  78.7  66 4.08
## Honda Civic         30.4  75.7  52 4.93
## Toyota Corolla      33.9  71.1  65 4.22
## Toyota Corona       21.5 120.1  97 3.70
## Dodge Challenger    15.5 318.0 150 2.76
## AMC Javelin         15.2 304.0 150 3.15
## Camaro Z28          13.3 350.0 245 3.73
## Pontiac Firebird    19.2 400.0 175 3.08
## Fiat X1-9           27.3  79.0  66 4.08
## Porsche 914-2       26.0 120.3  91 4.43
## Lotus Europa        30.4  95.1 113 3.77
## Ford Pantera L      15.8 351.0 264 4.22
## Ferrari Dino        19.7 145.0 175 3.62
## Maserati Bora       15.0 301.0 335 3.54
## Volvo 142E          21.4 121.0 109 4.11
```

```
car_charac
```

```
##                     cyl    wt am gear carb
## Mazda RX4             6 2.620  1    4    4
## Mazda RX4 Wag         6 2.875  1    4    4
## Datsun 710            4 2.320  1    4    1
## Hornet 4 Drive        6 3.215  0    3    1
## Hornet Sportabout     8 3.440  0    3    2
## Valiant               6 3.460  0    3    1
## Duster 360            8 3.570  0    3    4
## Merc 240D             4 3.190  0    4    2
## Merc 230              4 3.150  0    4    2
## Merc 280              6 3.440  0    4    4
## Merc 280C             6 3.440  0    4    4
## Merc 450SE            8 4.070  0    3    3
## Merc 450SL            8 3.730  0    3    3
## Merc 450SLC           8 3.780  0    3    3
## Cadillac Fleetwood    8 5.250  0    3    4
## Lincoln Continental   8 5.424  0    3    4
## Chrysler Imperial     8 5.345  0    3    4
## Fiat 128              4 2.200  1    4    1
## Honda Civic           4 1.615  1    4    2
## Toyota Corolla        4 1.835  1    4    1
## Toyota Corona         4 2.465  0    3    1
## Dodge Challenger      8 3.520  0    3    2
## AMC Javelin           8 3.435  0    3    2
## Camaro Z28            8 3.840  0    3    4
## Pontiac Firebird      8 3.845  0    3    2
## Fiat X1-9             4 1.935  1    4    1
## Porsche 914-2         4 2.140  1    5    2
## Lotus Europa          4 1.513  1    5    2
## Ford Pantera L        8 3.170  1    5    4
## Ferrari Dino          6 2.770  1    5    6
## Maserati Bora         8 3.570  1    5    8
## Volvo 142E            4 2.780  1    4    2
```

```
car_spec <- car_spec %>%
 mutate(across(everything(), scale))
car_charac <- car_charac %>%
 mutate(across(everything(), scale))

matcor(car_spec,car_charac)

## $Xcor
##             mpg       disp         hp       drat
## mpg   1.0000000 -0.8475514 -0.7761684  0.6811719
## disp -0.8475514  1.0000000  0.7909486 -0.7102139
## hp   -0.7761684  0.7909486  1.0000000 -0.4487591
## drat  0.6811719 -0.7102139 -0.4487591  1.0000000
##
## $Ycor
##             cyl         wt          am       gear       carb
## cyl   1.0000000  0.7824958 -0.52260705 -0.4926866 0.52698829
## wt    0.7824958  1.0000000 -0.69249526 -0.5832870 0.42760594
## am   -0.5226070 -0.6924953  1.00000000  0.7940588 0.05753435
## gear -0.4926866 -0.5832870  0.79405876  1.0000000 0.27407284
## carb  0.5269883  0.4276059  0.05753435  0.2740728 1.00000000
##
## $XYcor
##             mpg       disp         hp       drat        cyl
wt
## mpg   1.0000000 -0.8475514 -0.7761684  0.6811719 -0.8521620 -0.86765
94
## disp -0.8475514  1.0000000  0.7909486 -0.7102139  0.9020329  0.88797
99
## hp   -0.7761684  0.7909486  1.0000000 -0.4487591  0.8324475  0.65874
79
## drat  0.6811719 -0.7102139 -0.4487591  1.0000000 -0.6999381 -0.71244
06
## cyl  -0.8521620  0.9020329  0.8324475 -0.6999381  1.0000000  0.78249
58
## wt   -0.8676594  0.8879799  0.6587479 -0.7124406  0.7824958  1.00000
00
## am    0.5998324 -0.5912270 -0.2432043  0.7127111 -0.5226070 -0.69249
53
## gear  0.4802848 -0.5555692 -0.1257043  0.6996101 -0.4926866 -0.58328
70
## carb -0.5509251  0.3949769  0.7498125 -0.0907898  0.5269883  0.42760
59
##               am       gear        carb
## mpg   0.59983243  0.4802848 -0.55092507
## disp -0.59122704 -0.5555692  0.39497686
## hp   -0.24320426 -0.1257043  0.74981247
## drat  0.71271113  0.6996101 -0.09078980
## cyl  -0.52260705 -0.4926866  0.52698829
## wt   -0.69249526 -0.5832870  0.42760594
## am    1.00000000  0.7940588  0.05753435
## gear  0.79405876  1.0000000  0.27407284
## carb  0.05753435  0.2740728  1.00000000
```

```r
# Perform Canonical Correlation Analysis
cc_result <- cc(car_spec, car_charac)

# Display the canonical correlations
print(cc_result$cor)

## [1] 0.9747788 0.8550909 0.4771323 0.3174629

can_variates_spec <- cc_result$xcoef
can_variates_spec

##            [,1]       [,2]       [,3]      [,4]
## mpg   0.2769325  0.1932924 -1.7989608 -1.046130
## disp -0.7169487  1.0298140 -1.6049850  1.080137
## hp    0.1003727 -1.4480923 -0.3885621 -1.082230
## drat  0.1508174 -0.3139453 -0.2825614  1.491155

can_variates_charac <- cc_result$ycoef
can_variates_charac

##               [,1]        [,2]       [,3]        [,4]
## cyl  -0.5937477838 -0.27785984 -1.2972413 -1.18600680
## wt   -0.5892256398  0.48681193 -0.6233610  1.86688856
## am   -0.0000253653 -0.03901582 -1.0278586  0.68503654
## gear -0.0491271803 -0.18112746 -0.8321884  0.22944247
## carb  0.1944597889 -0.89517086  1.4291001 -0.09880977

rho <- cc_result$cor

n = dim(car_spec)[1]
p = length(car_spec)
q = length(car_charac)

n

## [1] 32

p

## [1] 4

q

## [1] 5

p.asym(rho, n, p, q, tstat = 'Wilks')

## Wilks' Lambda, using F-approximation (Rao's F):
##                stat    approx df1      df2      p.value
## 1 to 4:  0.009298678 11.962258  20 77.23224 2.220446e-16
## 2 to 4:  0.186696743  4.708472  12 63.78953 1.920850e-05
## 3 to 4:  0.694505794  1.666225   6 50.00000 1.489075e-01
## 4 to 4:  0.899217319  1.457017   2 26.00000 2.513279e-01

p.asym(rho, n, p, q, tstat = 'Hotelling')
```

```
##  Hotelling-Lawley Trace, using F-approximation:
##                 stat    approx df1 df2      p.value
## 1 to 4:  22.2045768 23.869920  20  86 0.000000e+00
## 2 to 4:   3.1268042  6.123325  12  94 7.882161e-08
## 3 to 4:   0.4068368  1.729056   6 102 1.216968e-01
## 4 to 4:   0.1120782  1.541076   2 110 2.187380e-01
```

```
p.asym(rho, n, p, q, tstat = 'Pillai')
```

```
##  Pillai-Bartlett Trace, using F-approximation:
##                stat   approx df1 df2      p.value
## 1 to 4:  2.0098120 5.251274  20 104 7.280525e-09
## 2 to 4:  1.0596183 3.363431  12 112 3.297310e-04
## 3 to 4:  0.3284379 1.789091   6 120 1.069197e-01
## 4 to 4:  0.1007827 1.654202   2 128 1.953065e-01
```

```
p.asym(rho, n, p, q, tstat = 'Roy')
```

```
##  Roy's Largest Root, using F-approximation:
##                stat   approx df1 df2      p.value
## 1 to 1:  0.9501937 99.20442   5  26 4.440892e-16
##
##  F statistic for Roy's Greatest Root is an upper bound.
```

```
Wilks(cancor(car_spec,car_charac))
```

```
##
## Test of H0: The canonical correlations in the
## current row and all that follow are zero
##
##      CanR LR test stat approx F numDF  denDF   Pr(> F)
## 1 0.97478      0.00930  11.9623    20 77.232 2.988e-16 ***
## 2 0.85509      0.18670   4.7085    12 63.790 1.921e-05 ***
## 3 0.47713      0.69451   1.6662     6 50.000    0.1489
## 4 0.31746      0.89922      NaN     2    NaN       NaN
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
squared_cc <- rho^2
squared_cc
```

```
## [1] 0.9501937 0.7311804 0.2276552 0.1007827
```

```
loadings <- comput(car_spec,car_charac,cc_result)
```

```
loadings$corr.X.xscores
```

```
##            [,1]        [,2]       [,3]        [,4]
## mpg   0.9094098  0.23058486 -0.3295369 -0.10587647
## disp -0.9793862 -0.05640947 -0.1869268  0.05175901
## hp   -0.7493238 -0.64270403 -0.1349242 -0.08509323
## drat  0.8035998 -0.26382359 -0.1937095  0.49709256
```

```
loadings$corr.Y.xscores
```

```
##            [,1]        [,2]        [,3]        [,4]
## cyl  -0.9047105 -0.22150882 -0.04042523 -0.07882416
```

```
## wt   -0.9182473  0.01648184  0.08103458  0.09154874
## am    0.6730707 -0.36448094 -0.23704924  0.05985649
## gear  0.6242172 -0.51690551 -0.12117217  0.07673665
## carb -0.3741792 -0.75703159  0.09146570  0.05611736

loadings$corr.X.yscores

##             [,1]        [,2]        [,3]        [,4]
## mpg   0.8864734  0.19717101 -0.15723270 -0.03361185
## disp -0.9546849 -0.04823523 -0.08918883  0.01643156
## hp   -0.7304249 -0.54957036 -0.06437668 -0.02701394
## drat  0.7833321 -0.22559315 -0.09242508  0.15780844

loadings$corr.Y.yscores

##             [,1]        [,2]        [,3]       [,4]
## cyl  -0.9281188 -0.25904711 -0.08472542 -0.2482941
## wt   -0.9420058  0.01927496  0.16983672  0.2883762
## am    0.6904856 -0.42624819 -0.49682078  0.1885464
## gear  0.6403681 -0.60450358 -0.25395926  0.2417185
## carb -0.3838606 -0.88532295  0.19169882  0.1767682

# standardized car specification canonical coefficients
s1 <- diag(sqrt(diag(cov(car_spec))))
ss1 <- s1 %*% can_variates_spec
`rownames<-`(ss1,c("mpg","disp","hp","drat"))

##             [,1]       [,2]       [,3]      [,4]
## mpg   0.2769325  0.1932924 -1.7989608 -1.046130
## disp -0.7169487  1.0298140 -1.6049850  1.080137
## hp    0.1003727 -1.4480923 -0.3885621 -1.082230
## drat  0.1508174 -0.3139453 -0.2825614  1.491155

# standardized car characteristics canonical coefficients
s2 <- diag(sqrt(diag(cov(car_charac))))
ss2 <- s2 %*% can_variates_charac
`rownames<-`(ss2,c("cyl", "wt", "am", "gear", "carb"))

##                [,1]        [,2]       [,3]        [,4]
## cyl  -0.5937477838 -0.27785984 -1.2972413 -1.18600680
## wt   -0.5892256398  0.48681193 -0.6233610  1.86688856
## am   -0.0000253653 -0.03901582 -1.0278586  0.68503654
## gear -0.0491271803 -0.18112746 -0.8321884  0.22944247
## carb  0.1944597889 -0.89517086  1.4291001 -0.09880977
```