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Realization of dS Vacua in String Theory

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Abstract: The possibility of metastable de Sitter vacua in string theory has been a long-standing mystery. This paper presents a very brief review of progress and puzzles in the stringy constructions of such dS vacua. We start with an introduction to the KKLT mechanism and criticisms to the KKLT scenario. We then review debate surrounding the swampland dS conjecture and its ramifications. One particular possibility that we explore is the realization of quintessence models in string theory. Leaving glimpses of open problems throughout the review, we conclude with some closing remarks on the importance and consequences of proving/disproving the existence of dS vacua in string theory.

Contents

1 Introduction

Nobel winning experimental data from [1] and [2] observed as early as 1998 heavily suggest that the late-time cosmology of our universe requires a very tiny positive cosmological constant Λ (and that therefore our universe might be asymptotically de Sitter). If we believe that string theory consistently describes our universe, it is natural to then expect that de Sitter (dS) vacua are realizable in string theory. It has therefore been of great interest to the string phenomenology community to study de Sitter vacua in supergravity and string theory (in order to build late-time models of our universe).

We pick up the story of dS vacua from this experimental awakening. One of the earliest work on dS vacua after this series of experimental observations was the no-go theorem seen in [3]. The authors used the equations of motion in warped compactifications to invalidate dS compactifications of supergravity theories that have: a gravity action with no higher curvature corrections, a non-positive scalar potential, an effective Newton's gravitational constant that is finite, and massless scalar fields with positive kinetic terms. In some sense, the no-go theorem of [3] blocks all naive dS compactifications one might expect. Soon after the no-go theorem came out, [4] provided a construction of dS vacua in supercritical string theory that violate the hypotheses of the theorem.

dS vacua, if they exist in critical string theory, are believed to be metastable at best [5]. So we focus our story on meta-stable dS vacua. A monumental step in the direction of dS vacua was the outline given by S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi (KKLT) and is dubbed the KKLT mechanism [6]. Sweeping a lot of details under the rug, their famous recipe involves 2 steps:

- Freeze all moduli in a warped IIB compactification with non-trivial NS and RR 3-form fluxes to yield a supersymmetric (SUSY) AdS vacuum with exponentially small cosmological constant ($|\Lambda| \ll 1$).

- Introduce a small number of anti-D3 ($\overline{\mathrm{D3}}$) branes to break supersymmetry to achieve a controlled uplift of the AdS minimum state to a dS ground state.[1]

If the recipe works as planned, the resulting dS vacuum will have a lifetime larger than the cosmological timescale. [6] arrives at a highly non-trivial and stunning result: moduli stabilization, scale separation, and a de Sitter vacuum in the same construction. An alternative to the KKLT mechanism, known as the Large Volume Scenario (LVS) construction was first proposed in [7] and [8]. This construction involves looking at the scalar potential associated with IIB Calabi-Yau compactifications in the large volume limit. In [8], the authors argued that there generally exists a limit in which a non-SUSY AdS vacuum emerges, and that this vacuum can be uplifted (*a la* KKLT, by adding a few $\overline{\mathrm{D3}}$-branes) to a tachyon-free dS vacuum, fixing all moduli. Other known popular approaches follow a similar two-step scheme: find a SUSY AdS vacuum, uplift to a dS vacuum using a controlled and small SUSY breaking effect. These constructions however have drawn significant attention due to various possibilities of instability. Despite the large flurry of works on moduli-stabilization and dS uplifts following the KKLT and LVS constructions, there is not one single explicit construction or computation of a moduli-stabilized dS vacuum in string theory, nor a universally accepted outline for such a construction. This difficulty that dS vacua proposes, among other things, has led several physicists to wonder if metastable dS vacua populate the string theory landscape or not.

In this paper, we conduct a very brief review of the KKLT mechanism to construct meta-stable dS vacua and swampland dS conjectures. In section 2, we present the KKLT scenario for the construction of dS vacua. In 3 we survey the criticism of the KKLT picture, particularly from the perspectives of $\overline{\mathrm{D3}}$-brane backreaction and holography (with recent arguments proposed in [9]). In 4 we present swampland dS conjectures proposed in [10]

[1] There is also a less widely known third step: *cross your fingers*

and [11]. In 5 we present quintessence models as the popular alternative to meta-stable dS vacua, and a list of challenges for stringy realizations of quintessence models. Having hinted at various open problems throughout the paper, we finally conclude in 6 by reemphasizing the importance of resolving the uncertainty surrounding dS vacua in string theory. The author acknowledges his inexperience in the area and the page limitations of this paper, and therefore refers the readers to [12] [13] [14] [15] for reviews with a similar structure.

2 An Overview of KKLT

The KKLT mechanism for the construction of meta-stable dS vacua first appeared in [6] (following pioneering work in [16] [17] [18] on orientifolds and flux compactifications) and has since been analyzed extensively in the literature (by both 'believers' and 'non-believers'). In this review, we adapt the presentation of the KKLT mechanism as seen in [6], [9], and [12].

2.1 An AdS vacuum with small cosmological constant

We start with F-theory compactified on an elliptically fibered Calabi-Yau fourfold X_4. Under the orientifold limit, we take X_4 to be $(X_3 \times \mathbb{T}^2)/\mathbb{Z}_2$, where X_3 is a Calabi-Yau threefold. This orientifold in general contains orientifold O3 and O7 planes[2] in the fibration. The effective D3-brane charges induced by the O3 plane must obey the D3-brane tadpole cancellation condition (2.4) (this tadpole cancellation is achieved by inserting spacetime-filling D3-branes or an appropriate combination of the IIB 3-form fluxes H_3 and F_3, which correspond to the NS flux and the RR flux in Type IIB string theory respectively). We know that F-Theory compactified on $X_4 \times S^1$ is dual to M-theory on X_4 [19]. It will be useful to us to use the M-Theory language to derive flux consistency conditions (such as flux quantization and the tadpole). The type IIB fluxes map to the M-Theory four form flux G_4 by $G_4 = F_3 \wedge a + H_3 \wedge b$ where a and b are 1-forms on $\mathbb{T}^2$ dual to the A and the B cycles of the torus:

$$\int_A \Omega_1 = 1 \qquad \int_B \Omega_1 = \tau \tag{2.1}$$

where Ω_1 is the holomorphic $(1,0)$ form on $\mathbb{T}^2$, and τ is the complex structure parameter of $\mathbb{T}^2$. As explained in [20], the G_4 flux (which we assume to be properly normalized) must obey the shifted flux quantization condition:

$$[G_4] + \frac{c_2(X_4)}{2} \in H^4(X_4, \mathbb{Z}). \tag{2.2}$$

where $c_2(X_4)$ is the second Chern class of X_4 (equivalently one may look at $-p_1(X_4)$, the first Pontryagin class of X_4). We also obtain an M2-brane tadpole condition in the language of M-Theory [18] [17]:

$$\frac{\chi(X_4)}{24} = N_{\mathrm{M2}} + \frac{1}{2}\int_{X_4} G_4 \wedge G_4 \tag{2.3}$$

[2]The $\mathbb{Z}_2$ action in the orientifold involves an involution that usually has fixed loci, which are conventionally interpreted as O3 or O7 planes in the theory.

where $\chi(X_4)$ is the Euler characteristic of X_4 and $N_{\rm M2}$ is the number of space-time filling M2-branes. As we uplift from M-Theory to F-theory, the M2-branes map to space-time filling D3-branes and (2.3) translates to a D3-brane tadpole[3]:

$$\frac{\chi(X_4)}{24} = N_{\rm D3} + \frac{1}{2}\int_{X_3} H_3 \wedge F_3. \tag{2.4}$$

Here $\chi(X_4)$ is the Euler characteristic of the Calabi-Yau fourfold X_4 and $N_{\rm D3}$ the effective number of D3-branes (i.e. number of D3-branes minus the number of $\overline{\rm D3}$-branes). Note that although we haven't added any $\overline{\rm D3}$ branes yet, the general tadpole condition will come in handy when we uplift to dS. The reader may wish to think of (2.4) in the following manner: the left-hand side counts the number of $O3/O7$ planes in the theory, the first term on the right-hand side counts the effective number of space-time filling D3-branes, and the last term quantifies the contribution from the NS and RR fluxes.

The presence of non-trivial fluxes induces a superpotential on the Calabi-Yau fourfold, which upto leading order is given by the Gukov-Vafa-Witten superpotential [21]:

$$W = \int_{X_4} G_4 \wedge \Omega_4 \tag{2.5}$$

Here Ω_4 is the holomorphic $(4,0)$-form on X_4, given by $\Omega_3 \wedge \Omega_1$ (where Ω_3 is the holomorphic (3,0) form on X_3). Using (2.1), we can rewrite[4] (2.5) as:

$$W = \int_{X_3} G_3 \wedge \Omega_3, \qquad G_3 = F_3 - \tau H_3 \tag{2.6}$$

where τ is the IIB axio-dilaton. At weak-string coupling, we obtain the tree-level Kähler potential [16]:

$$K = -3\log(-i(\rho - \overline{\rho})) - \log(-i(\tau - \overline{\tau})) - \log\left(i\int_{X_3} \Omega_3 \wedge \overline{\Omega}_3\right) \tag{2.7}$$

Here ρ is the Kähler volume modulus (in the simplifying case we will now assume, we assert that this is our only Kähler modulus). In $\mathcal{N}=1$ supergravity (SUGRA), the superpotential generates a scalar potential which is given by the familiar formula:

$$V = e^K \left(g^{a\bar{b}} D_a W \overline{D_b W} - 3|W|^2\right) \tag{2.8}$$

where D_a is the Kähler covariant derivative $D_a = \partial_a + \partial_a K$, and a, b run over all moduli fields including the complex moduli, the single Kähler modulus, and the IIB axio-dilaton. Since the superpotential (2.6) does not depend on the volume modulus ρ (which describes the volume of the internal geometry), we may exclude its contribution from the first term in (2.8). Furthermore, the no-scale property of the Kähler potential dictates

[3] Refer to A for more details on this step
[4] Refer to A for more details on this step

that $g^{a\bar{b}}\partial_a K \partial_b K = 3$. Finally, since the Kähler potential does not depend on the complex moduli, we can rewrite (2.8) as:

$$V = e^K g^{i\bar{j}} D_i W \overline{D_j W} \tag{2.9}$$

where i, j run only over the complex moduli and the axio-dilaton. To find minima for this potential, we need to solve the F-term equation $D_i W = 0$ for all i above (which would imply $V = 0$). This procedure would fix all the complex moduli (and the axio-dilaton), but it does not fix the Kähler volume modulus. Therefore we need to employ another procedure to fix this modulus. The condition $D_I W = 0$ for all I implies that G_4 is self-dual (i.e. $*G_4 = G_4$) by (2.6), which translates to G_3 being imaginary self-dual (i.e. $*G_3 = iG_3$). Given the Hodge decomposition of $H^3(X_3, \mathbb{Z})$ as $H^{3,0} \oplus H^{2,1} \oplus H^{1,2} \oplus H^{0,3}$, the fact that SUSY vacua have $W = 0$ constrains G_3 to be type $(2,1)$, whereas for non-SUSY vacua, we may have a non-vanishing $(0,3)$ component.[5]. *And now KKLT begins.*

Consider a set of integral flux choices H_3, F_3, aand use (2.9) to fix all complex moduli, including the axio-dilaton (so that $D_I W = 0$). Let the flux setup be so that at the point in moduli space where $D_I W = 0$, the G_3 flux has a non-vanishing (0,3) component (which would imply that $W|_{D_I W=0} \propto G_{0,3} \neq 0$, let $W_0 := W|_{D_I W}$ be the constant determined by $G_{0,3}$). Furthermore, this would imply that the F-term equation for the Kähler modulus is not satisfied, $D_\rho W = (\partial_\rho K) W \propto W \neq 0$. Finally, we assume that the set of points in the complex moduli space where G_3 is imaginary self-dual is made of isolated points where the complex structure moduli and the axio-dilaton are rendered massive (more particularly, as shown in work prior to KKLT, these masses and the string coupling g_s may be fixed at the scale $m \sim \alpha'/R^3$ for reasonably small flux quanta). So we may now worry only about an effective field theory for ρ. The single Kähler direction remains a flat direction and must now be stabilized somehow to obtain a SUSY AdS vacuum.

This is done by incorporating non-perturbative corrections to the superpotential (2.6) given by:

$$W = W_0 + W_{\text{n.p.}} = W_0 + \sum_{\mathbf{k}} \mathcal{A}_{\mathbf{k}}(z^i, \tau) e^{ik^\alpha T_\alpha} \tag{2.10}$$

where $\mathbf{k}$ runs through effective divisors of X_3 (and the $\mathcal{A}_{\mathbf{k}}$ prefactors are often called the Pfaffian prefactors), and T_α are the Kähler moduli (although in our case, we have only one). This non-perturbative expansion was first motivated in [22] as arising from Euclidean D3-brane instantons wrapped on divisors of X_3 of arithmetic genus 1. The authors of [6] also motivated this correction using gaugino condensates (or fractional instantons), a source for much debate as we will see later in the paper. Since the complex structure moduli and the axio-dilaton have been fixed at the mass scale described above, we may integrate out them out and rewrite these corrections as:

[5] The Hodge theory details involved in this portion makes it terse, but the reader is encouraged to think of this as just motivation for what is about to come next. The fact that SUSY vacua force G_3 to be type (2,1) is actually true in a broader context.

$$W = W_0 + \sum_{\mathbf{k}} \mathcal{A}_{\mathbf{k}} e^{ik^\alpha T_\alpha} \tag{2.11}$$

We further simplify things by considering a single correction to the super potential (so one effective divisor of X_3), and since we are only looking at a single Kähler modulus ρ:

$$W = W_0 + \mathcal{A} e^{ia\rho} \tag{2.12}$$

where we have relabelled some terms just for notational convenience. We pause to make a remark here. We might choose to consider corrections for the Kähler potential K too, but as we will see shortly, we will fix the Kähler modulus at large values compared to string scale (so that we may indeed neglect such corrections). Another remark is in order, we consider only a single correction to the superpotential (in the language of KKLT this would correspond to a single gaugino condensate), the racetrack models explore corrections due to more than one gaugino condensate.

We are finally in a position to freeze the Kähler modulus and reap a SUSY AdS vacuum. Recall that the Kähler potential was given by $K = -3\log\left(-i(\rho - \overline{\rho})\right)$. We ensure that our tadpole condition has been satisfied without additional D3-branes so far (i.e. flux only). For a SUSY vacuum we need $D_\rho W = 0$. We consider the simplifying case $\rho = i\sigma$. Finally, we consider $W_0, \mathcal{A}, a \in \mathbb{R}$, and $W_0 < 0$. Under these considerations $D_\rho W = 0$ implies:

$$\begin{aligned} 0 &= \partial_\rho (W_0 + \mathcal{A} e^{ia\rho}) + \partial_\rho K (W_0 + \mathcal{A} e^{ia\rho}) \\ &= ia\mathcal{A} e^{ia\rho} + \frac{3i}{-i(\rho - \overline{\rho})} (W_0 + \mathcal{A} e^{ia\rho}) \\ &= i\left(a\mathcal{A} e^{-a\sigma} + \frac{3}{2\sigma}(W_0 + \mathcal{A} e^{-a\sigma})\right) \end{aligned} \tag{2.13}$$

So if σ_0 is the critical value of σ, we get:

$$W_0 = -\mathcal{A} e^{-a\sigma_0} \left(\frac{2a\sigma_0}{3} + 1\right) \tag{2.14}$$

Going back to our formula for the scalar potential, we get:

$$V = -3e^K |W(\sigma_0)|^2 = -\frac{3}{8\sigma_0^3}(W_0 + \mathcal{A} e^{-a\sigma_0})^2 = -\frac{a^2 \mathcal{A}^2 e^{-2a\sigma_0}}{6\sigma_0} < 0 \tag{2.15}$$

We have just arrived at a moduli-stabilized supersymmetric AdS vacuum. Furthermore, we expect that by tuning our fluxes, we can ensure that $W_0 \ll 1$ (and $V \ll 1$ at the critical point) and therefore $a\sigma_0 > 1$; by taking $a < 1$ after that, we can ensure $\sigma_0 \gg 1$. This ensures that our calculation was consistent (because we have used the SUGRA approximation for the scalar potential, and the tree-level Kähler potential, the large volume modulus ensures control).

2.2 Breaking supersymmetry to yield a dS vacuum

In the previous subsection, we satisfied the tadpole (2.4) by only turning on flux. Now suppose we turn on too much flux in our theory. Just enough flux so that we need a single $\overline{D3}$-brane to satisfy our tadpole condition (we take advantage and insert this brane). The tadpole is now satisfied, but there are extra energy contributions from the extra flux and the tension of the $\overline{D3}$-brane. As in [23], this extra energy is given by:

$$\delta V \sim \frac{2A}{(\Im(\rho))^3} \tag{2.16}$$

Where A depends quartically on the warp factor at the location of the $\overline{D3}$-brane. Again, as in [23], these $\overline{D3}$-branes are driven to the tip of the Klebanov-Strassler (KS) throat where the warp factor is minimal. In fact, by turning on the right amount of flux, it is possible to construct exponentially small warp factors at the tip of the KS throat, and therefore the extra energy added per $\overline{D3}$-brane is exponentially suppressed. The 2 prefactor in (2.16) comes from the fact that the extra flux energy is equal to the $\overline{D3}$ tension. Now we introduce a small number of $\overline{D3}$-branes, so that the additional energy is $\frac{8D}{(\Im(\rho))^3}$ (here, the 8 was added *a posteriori*). Where D depends on the number of $\overline{D3}$-branes and the warp factor at the tip of the KS throat (i.e. discretely tuneable). Adding this to our previous result for the scalar potential we get:

$$V = \frac{a\mathcal{A}e^{-a\sigma}}{2\sigma^2}\left(W_0 + \mathcal{A}e^{-a\sigma} + \frac{1}{3}\sigma a\mathcal{A}e^{-a\sigma}\right) + \frac{D}{\sigma^3} \tag{2.17}$$

This scalar potential uplifts the previous negative AdS minimum to a positive dS minimum (for a suitable D). Moreover, we may fine-tune D to a small enough values to set this minimum very close to 0.

3 Criticism of the KKLT Scenario

Since the publication of the KKLT Mechanism, several questions about whether the procedure yields stable dS vacua (among other things) have been raised. The review of KKLT criticism presented here is adapted from sections of [12].

3.1 $\overline{\text{D3}}$-brane Backreaction in the Internal Geometry

Much of the debate around $\overline{\text{D3}}$-brane backreaction in the internal geometry stems from the monumental discoveries of [24] about singularities near the $\overline{D3}$-branes at the tip of the KS throat (which were later argued to be sources for perturbative instability and brane-flux annihilation). Before we proceed addressing these issues, we first go back to the source of the problem.

Our mechanism for the dS uplift was the introduction of a small number of $\overline{\text{D3}}$-branes. The uplift energy and perturbative stability associated with these dynamical objects were argued for by calculations in [23]. These calculations use the 'probe argument,' which holds the KS geometry fixed and neglects effects of the $\overline{\text{D3}}$-branes on the internal geometry (i.e. backreactions). The question that arises, is whether we can go beyond the probe approximation (with results that match the approximation at appropriate limits). This analysis would evidently be the study of the backreaction of the $\overline{\text{D3}}$-branes on the local KS throat geometry. But this is a tall order! As [12] points out, to perform this analysis, we don't even know explicit compact Calabi-Yau metrics, let alone metrics after SUSY breaking. One approach to solving this problem, is to just solve the supergravity equations with $\overline{\text{D3}}$-branes and see what happens. Another tall order! To attack this problem, we simplify the supergravity equations and introduce some approximations. The following were the simplifying assumptions made: one first works in non-compact spaces with explicit metrics (the canonical choice is then the famous KS solution, which pops up in local approximations of compact Calabi-Yau metrics, as in [16]), one makes the supergravity equations ODEs by smearing the $\overline{D3}$-branes on a subspace of the throat, one finally linearizes these ODEs. This system of equations was solved in [24], and the results showed an IR singularity (not sourced by D-branes or NS5-branes and not a consequence of brane-flux annihilation etc.) near the $\overline{\text{D3}}$-branes at the tip of the KS throat (coming from H_3 and F_3 field strengths). While [24] maintains neutrality about the physicality of this singularity, [12] classifies this as an unphysical singularity. An independent argument of [25] arrives at the same conclusion of a singularity sourced by H_3 and F_3. [26] studies these singularities, and argue that they drive the system toward brane-flux annihilation.

These singularities were finally shown to be resolved if one considered spherical NS5 or D5 branes instead of $\overline{\text{D3}}$-branes [27]. Presumably this solves the issue. *Almost*. There are several other reported sources for instabilities that will not be reviewed in this paper (e.g. 'clumping' of flux sources by virtue of their charges [28],).

3.2 $\overline{\text{D3}}$-brane Backreaction on the moduli

We now focus on the backreaction of $\overline{D3}$-branes with the moduli. We wonder whether the SUSY breaking mechanism affects the volume stabilization mechanism proposed by KKLT. Particularly, we focus on whether the $\overline{\text{D3}}$-branes interact with the gaugino condensates. If such interactions were non-negligible, then our formula for the post-uplift scalar potential (2.17) wouldn't be the simple sum of the 'extra energy' and the 'AdS' potential. Indeed it was shown in a 10D computation that in the 1 Kähler modulus KKLT picture (stabilized by a gaugino condensate) with $\overline{D3}$-branes we can never get dS vacua because of the backreaction of $\overline{D3}$ with the volume modulus [29]. Instead, what happens is that the uplift energy 'flattens out' and we get a meta-stable SUSY breaking AdS vacuum.

There appears to be a way out: *racetrack*. Racetrack models for stabilization (with two gaugino condensates stabilizing the volume modulus) appear to be able to produce SUSY AdS vacuum (*a la* KKLT), with dS uplifts that have negligible backreaction with the volume modulus. Unfortunately, it was shown that racetrack models are in high tension with strong versions of the Weak Gravity Conjecture (see [30]), which are widely believed to be true [31].

3.3 Holography

This brief subsection of the paper is a summary of the recent work [9], which aims to show the impossibility of the first step of the KKLT mechanism using conventional holography principles. The first step of KKLT produces a moduli-stabilized scale-separated SUSY AdS_4 vacuum with $|\Lambda| \ll 1$. [9] initially focusses on a KKLT procedure to achieve an AdS_3 vacuum, and generalize their results to an AdS_4 vacuum. They first justify why one would expect a microscopic 1+1 dual theory to this AdS_3 vacuum. They then argue that the SUSY AdS_3 vacuum obtained by compactifying M-Theory on X_4 in the presence of a G_4 flux (*a la* KKLT) is holographically dual to a CFT realized on the domain wall obtained by wrapping M5 branes on a special Lagrangian (SLag) 4-cycle in X_4 (Poincare dual to the G_4 flux). They place bounds on the central charge of this CFT by studying these SLag cycles (and using the AdS/CFT dictionary, the AdS length l_{AdS} corresponds to the central charge). These bounds show that $l_{\mathrm{AdS}}^2 < \chi(X_4)$ and using the fact that $\chi(X_4) \lesssim l_s^2$ (since $\chi(X_4)$ counts the number of light degrees of freedom) they establish $l_{\mathrm{AdS}} \lesssim l_s$, where l_{AdS} is the length scale of a KKLT-like AdS vacuum, and l_s is the species length scale. Under their assumptions, this shows that the effective field theory associated to the AdS_3 or AdS_4 vacuum breaks down.

4 Swampland dS conjectures

Given the notorious difficulty of finding explicit dS vacua constructions in string theory, the authors of [10] argued the following remarkable conjecture.

The de Sitter conjecture: *The scalar potential of a consistent theory of quantum gravity must obey the following constraint*

$$|\nabla V| \geq \frac{c}{M_p} V \tag{4.1}$$

where $c > 0$ is a universal $\mathcal{O}(1)$ constant.

We note that the constant c above is allowed to depend on d the macroscopic dimension of spacetime, and $|\nabla V|$ is the norm defined by the metric on the field space. It is also evident that as $M_p \to \infty$, the condition $|\nabla V| \geq 0$ is trivially satisfied. This is what we would expect of any swampland criterion (since as $M_p \to \infty$, quantum gravity effects remain unseen and any theory should be consistent). At risk of stating the obvious, the dS conjecture proposes (among other things) that all dS vacua are in the swampland (since $V > 0$ and $|\nabla V| = 0$ for dS vacua).

4.1 Motivation

The natural motivation for the conjecture above was the famous difficulty of constructing dS vacua in string theory. As the authors of [10] remark, if one plays with the idea of the impossibility of dS vacua, they might naively be tempted to propose the conjecture: $|\nabla V| > A$ for some universal constant $A > 0$. But the existence of SUSY flux vacua with flat directions would violate this conjecture, since $|\nabla V| = 0$ (for recent work on such constructions, see e.g. [32]). A better approach might be to suggest the bound $|\nabla V| \geq A(\phi)$

where $A(\phi) \leq 0$ for SUSY vacua. The simplest such bound that one could think of would be $A(\phi) = \frac{c}{M_p}V(\phi)$ (since $V(\phi) \leq 0$ for SUSY vacua). So far, this might seem like an awful lot of guesswork, but we now motivate this conjecture with some non-trivial examples. We explicitly work out a simple but revealing example (the first example worked out in [10]), while mentioning results from analyzing several other highly non-trivial examples.

We look at a SUSY vacuum with zero cosmological constant ($V(\phi) = 0$) and a single flat direction ($\nabla V(\phi) = 0$). In this case, the ratio $|\nabla V(\phi)|/V(\phi)$ is indeterminate, so we must add deformations to our theory before we test the inequality. To this end, we must consider a massive continuous deformation to the vacuum of the form: $V = \frac{1}{2}m^2\phi^2$. Then for any $\phi \neq 0$, we get: $\frac{|\nabla V|}{V} = \frac{m^2|\phi|}{\frac{1}{2}m^2\phi^2} = \frac{2}{|\phi|}$. This initially appears problematic, since for large ϕ the conjectured bound appears to be violated. However, swampland criteria (such as those in [33] [34]) ensure that for large ϕ, a tower of light states appear and that therefore our naive massive deformation is now invalid. It follows that our potential is only valid when ϕ remains within the Planck scale (i.e. $|\phi| \lesssim 1$ in Planck units, which implies $\frac{2}{|\phi|} \gtrsim 1$ in Planck units). So indeed, $|\nabla V|/V > \mathcal{O}(1)$ in Planck units, as expected by the conjectured bound.

Another easy check to conduct is the case of a SUSY vacuum that crosses $V = 0$. That is, a vacuum such that $V = 0$ at some point but $|\nabla V|$ is a non-zero constant when $V = 0$. Then the ratio $|\nabla V|/V$ diverges about the zero point, so the conjectured bound still holds.

After discussing these preliminary tests, [10] conducts several highly non-trivial checks. The example we briefly present here is that of warped M-theory compactifications of 11 dimensional supergravity to d dimensions. Using scaling properties of the potential contributions from the fluxes and the Ricci curvature of the internal geometry, the authors of [10] showed that:

$$\frac{|\nabla V|}{V} \geq \frac{6}{\sqrt{(d-2)(11-d)}} \tag{4.2}$$

This bound is a significant extension of the landmark no-go theorem of [3]. It shows not only that one cannot obtain a dS vacuum (as the no-go theorem tells us), but that one cannot even come close to a dS vacuum. The reader may wish to evaluate the above bound for $d = 4$, we get $\frac{6}{\sqrt{14}} \approx 1.6$.

4.2 Challenges

Apart from the general worry about existence of dS vacua, a number of other possible counter-examples for the dS conjecture have been suggested over the years (see for e.g. [35] [36] [37] [38]). We first point out that the conjectured bound in (4.1) not only outlaws all dS vacua, but any positive hilltop potential (i.e. a potential with a positive maximum, the Higgs field and the axion in QCD *par exemple*). In [35], the authors explore the fate of the Higgs field in a post-dS-conjecture world. While it is immediately evident that the Higgs potential is invalidated by the dS conjecture, the question remains if we can consistently couple the Higgs field to a quintessence field (the apparently de facto alternative to dS vacua briefly reviewed in 5). While we review the Higgs potential in the context of quintessence in 5, we skip to the point now and spoil to the reader that even after coupling the Higgs field to quintessence, the resulting potential sits in high tension with (4.1) (and the only ways out, crudely put, require a large amount of fine-tuning and seem unnatural). Another counter-example of much interest to particle physics is that of the QCD axion (a predicted solution to the so-called CP problem) shown in [37] to be inconsistent with the dS conjecture (even after coupling to quintessence). The final possible counter-example that we point out is that provided by [36]. It appears that the dS conjecture not only invalidates all dS vacua, but in a large number of racetrack and KKLT models, it also invalidates Minkowski or AdS vacua. Particularly, [36] shows that the 1-modulus KKLT AdS vacuum outlined initially in 2 is inconsistent with (4.1).

4.3 The Refined dS conjecture

Given the great amount of concern voiced against the dS conjecture, and the realization that it sits in tension with particle physics models such as the Higgs field, the QCD axion, etc., the authors of [11] formulated the following refined version of the conjecture which evades possible counter-examples.

The refined de Sitter conjecture: *The scalar potential of a consistent theory of quantum gravity obeys one of the following constraints*

$$\begin{gathered} |\nabla V| \geq \frac{c}{M_p} V \\ \textit{or} \\ \min \nabla_i \nabla_j V \leq -\frac{c'}{M_p^2} V \end{gathered} \tag{4.3}$$

where $\min \nabla_i \nabla_j V$ *is the minimal eigenvalue of the Hessian of* V *in an orthonormal frame, and* $c, c' > 0$ *are universal* $\mathcal{O}(1)$ *constants*

We note some trivial cases. We see that the above conjecture holds trivially if V is non-positive (since the first inequality is always true), and as before, if $M_p \to \infty$. Somewhat less trivially, this conjecture avoids trouble with the Higgs field, the QCD axion, etc. Finally, the refined dS conjecture still outlaws all dS vacua since $\nabla\nabla V > 0$ for a dS vacuum and $V > 0$.

[11] derives this conjecture under two assumptions: the Large Distance Conjecture (see [34]) and the fact that we are in a sufficiently weak-coupling regime where light-states dominate the Hilbert space of the low energy theory. This last assumption is motivated again by the large distance conjecture. The authors of [11] expect that when the tower of exponentially light states (by virtue of the large distance conjecture) pops up, the number of states in the Hilbert space increases monotonically (whether this is actually the case, at the moment, is unclear to the author). Under these assumptions, one can calculate the Gibbons-Hawking entropy of the dS vacuum (as initially proposed in [39]) by computing the logarithm of the number of states in the causal domain (the authors extend this entropy to general classes of accelerating universes). By comparing this entropy to the natural entropy of the part of the accelerating universe, set by the scalar potential, they arrive at the refined dS conjecture (4.3).

5 Quintessence Models

If the de Sitter conjectures hold, and we really do not have any dS vacua in string theory, the naive question one should ask is: "what becomes of string theory now?"[6] One possibility is that string theory is simply wrong or it does not describe our universe, failing to model experimental truth. Another possibility is that we have misinterpreted experiments for quite a while now, and maybe there is no late-time accelerated expansion of our universe. There is the third possibility that we are able to reconcile experimental evidence with string theory in some way.

From a cosmological point of view, the simplest and most popular alternative to the cosmological constant picture is the quintessence scenario. For a comprehensive review of quintessence and quintessence in string theory, we point the reader to [40] and [13]. The brief review of quintessence presented here is adapted from sections of [13] and [41].

Let us suppose the cosmological constant picture is thrown out of the window (i.e. that dark energy is no longer modeled by a small constant Λ). Assuming the validity of the refined dS conjecture we see that our universe is necessarily in some form of a 'quintessence epoch.' We embrace the possibility that the late-time acceleration and dark energy of our universe are explained by the dynamics of a light 'quintessence' scalar-field ϕ slow-rolling to minimum potential ($V = 0$). Perhaps the simplest scalar potential associated to the quintessence field is given by $V(\phi) = V_0 e^{-\frac{\lambda}{M_P}\phi}$. In any quintessence model, we need $V(\phi_{today}) \sim \Lambda^4$, and $m \sim 10^{-32} eV$.

If quintessence is supposed to work, one would ask: "can we find consistent embeddings of quintessence in string theory?" This problem is not only notoriously hard, it is arguably harder than finding dS vacua in the landscape. We briefly survey some challenges here. *Finetuning*: As seen above, the quintessence field requires two levels of finetuning, one for the potential V and the other for its mass m (compared to dS models which only need one fine-tuning for V). Furthermore, in stringy supergravity, after SUSY breaking, the quintessence fields receive corrections on orders above $(\text{TeV})^4$ [13]. *Fifth-force constraints*: If the quintessence field is a scalar, it must obey phenomenological/experimental fifth-force bounds. *Time-varying constants and phenomenological issues*: For the Higgs field to work with the dS conjecture, it must couple directly to the quintessence field ϕ. Under these circumstances, [38] shows that the Higgs field gets a ϕ-dependent VEV. This ϕ-dependence induces a long-range force, which is again constrained by fifth-force constraints. Furthermore, as the quintessence field moves, the Higgs VEV becomes time-dependent, this behavior is severely constrained by the time-dependence of the proton-to-electron mass ratio. To evade all these constraints, one needs an obscene level of fine-tuning, which brings up phenomenological issues. Even if one does this fine-tuning at tree-level, it may be the case that radiative corrections and instability could throw it off. More specifically in string theory the fifth-force constraints and time-dependent constraints severely restrict the interaction of the quintessence field with the visible sector. It has not been exceedingly

[6] An alternative question: "*dS vacua are dead. dS vacua remain dead. And we have killed them. How shall we comfort ourselves, the murderers of all murderers? What was holiest and mightiest of all that the world has yet owned has bled to death under our knives: who will wipe this blood off us?*"

clear whether we can naturally evade the fifth-force constraints and time-dependence of fundamental constants in string theory.

Nonetheless, excellent work has been conducted in the stringy quintessence realm. Some quintesence models include the string axion quintessence and stringy embeddings of SUSY Large Extra Dimension (SLED) proposals [41] [42].

6 Conclusion

We have conducted a brief review of the KKLT mechanism and the highly non-trivial challenges that one must overcome to actually construct a dS vacuum in string theory using KKLT. We then discussed more global criticisms to the dS vacua program, namely the swampland conjectures. These conjectures were motivated by non-trivial examples, or other 'trustworthy' conjectures. We finally discussed an alternative to the cosmological constant picture of our universe's expansion, the quintessence scenario. We presented a brief list of challenges that embedding quintessence in string theory imposes.

These theoretical hurdles and a lack of computational control in the study of dS vacua points to how premature our understanding of string compactifications is. As [12] points out, this by no means is a bad sign. In fact, it is a source of excitement for several physicists in the field, to resolve such a stunning puzzle at the heart of string theory. The popular controversy surrounding the existence of dS vacua allows for an influx of various brilliant perspectives that constantly shape our understanding of string theory. A much needed paradigm shift is perhaps in progress.

For a long time, we have been persistently forced the ways of our universe into string theory. Perhaps it is time to listen to string theory now, and see what it has to say about our universe with regards to de Sitter vacua[7].

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A KKLT D3-brane tadpole condition and Gukov-Vafa-Witten Superpotential

A.1 D3-brane tadpole condition

Here we present a quick derivation of the D3-brane tadpole condition seen in (2.4), from (2.3). In the language of the IIB 3-forms we see that $G_4 \wedge G_4 = (F_3 \wedge a + H_3 \wedge b) \wedge (F_3 \wedge a + H_3 \wedge b) = 2(F_3 \wedge H_3) \wedge (b \wedge a)$. Since a, b are the 1-forms dual to the A and B cycles of $\mathbb{T}^2$, we recognize that $b \wedge a$ is just the normalized volume form for the torus. Therefore:

$$\int_{X_4} G_4 \wedge G_4 = 2 \int_{(X_3 \times \mathbb{T}^2)/\mathbb{Z}_2} (F_3 \wedge H_3) \wedge (b \wedge a) = \int_{X_3} F_3 \wedge H_3 \tag{A.1}$$

[7] And what string theory is saying at the moment, at least to the author, is beyond unclear

Where we've used the fact that $\int_{\mathbb{T}^2} b \wedge a = 1$, and the $\mathbb{Z}_2$ orientifold allows us to pull back the $(X_3 \times \mathbb{T}^2)/\mathbb{Z}_2$ to an integral over X_3 at the cost of a 1/2 (where we've integrated over $\mathbb{T}^2$ already so we neglect it). Since the spacetime-filling M2-branes map to spacetime-filling D3 branes in the F-theory uplift, (2.3) translates to:

$$\frac{\chi(X_4)}{24} = N_{\mathrm{D3}} + \frac{1}{2}\int_{X_3} F_3 \wedge H_3 \tag{A.2}$$

A.2 Gukov-Vafa-Witten Superpotential

We present a quick derivation of the Gukov-Vafa-Witten superpotential[8] seen in the form of (2.6) from (2.5). We first compute $G_4 \wedge \Omega_4$ where $G_4 = F_3 \wedge a + H_3 \wedge b$ and $\Omega_4 = \Omega_3 \wedge \Omega_1$, with notation as defined before (2.5).

$$G_4 \wedge \Omega_4 = F_3 \wedge \Omega_3 \wedge \Omega_1 \wedge a + H_3 \wedge \Omega_3 \wedge \Omega_1 \wedge b. \tag{A.3}$$

Recall that the a and b 1-forms are defined to be dual to the A and B 1-cycles on $\mathbb{T}^2$ such that: $\int_{\mathbb{T}^2} \Omega_1 \wedge a = \int_A \Omega_1 = 1$ and $\int_{\mathbb{T}^2} \Omega_1 \wedge b = \int_B \Omega_1 = \tau$. Noting that the orientifold pullback will flip the sign of τ the complex structure parameter of $\mathbb{T}^2$ but not 1, we write:

$$\int_{X_4} G_4 \wedge \Omega_4 = \int_{(X_3 \times \mathbb{T}^2)/\mathbb{Z}_2} G_4 \wedge \Omega_4 = \int_{X_3} (F_3 - \tau H_3) \wedge \Omega_3. \tag{A.4}$$

Where we have utilized the orientifold pullback and split the integral between X_3 and the torus. Recognizing $F_3 - \tau H_3$ as G_3, we obtain:

$$W_0 = \int_{X_3} G_3 \wedge \Omega_3. \tag{A.5}$$

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