Introduction

In this note(s), I will work with a matrix A with m rows and n columns. A can be viewed as a function that maps a vector, $\vec{v}$, in $\mathbb{R}^n$ to $A\vec{v}$ in $\mathbb{R}^m$. I have two, closely related, goals for this note concerning this matrix A. The first is to prove the Rank Nullity Theorem, $N = \mathrm{Dim}(\mathrm{Range}(A)) + \mathrm{Dim}(\mathrm{Nullspace}(A))$. And the second is to discus decomposing the domain of A (viewed as a function), $\mathbb{R}^n$, as the direct sum of the nullspace of A and any complementary subspace to it in $\mathbb{R}^n$. Note that for the second goal, one such complementary subspace to the nullspace is the rowspace, its orthogonal complement in $\mathbb{R}^n$.

Axler proof

Key idea: Redundancy and NullSpace $A\vec{u} = A\vec{v} \Rightarrow A\vec{u} - A\vec{v} = \vec{0} \Rightarrow A(\vec{u} - \vec{v}) = 0$ Where last implication follows from linearity. So if $\vec{u} \neq \vec{v}$, I have found a non-trivial vector in the nullspace of A, $\vec{u} - \vec{v}$.