Inverses

I am going to review a few fundamental properties of inverses of functions. First, a way to think about them. Next, a way to compute them. A geometric interpretation of them as a reflection about the line $y = x$ and derive this.

And wrap it off with the computing derivative of inverse function.

Audience

This is targeted at, or at least assumes, basic precalculus understanding and the optional ending section differential

Motivation, Definition, Computation

Say I have a function, f , of x. f takes in x and produces $f(x)$. So I have this pair $(x, f(x))$. Todo draw picture of x and an arrow over a box, f, that points to $f(x)$. Now the inverse reverses the direction of the arrow and completes a loop. Given $f(x)$ as the input, it outputs x. I have heard the terms 'image' and 'preimage':$f(x)$ is the image, and x is the preimage. And functions take in preimages and return their images. Inverses then take in images and return the preimages whose image is in the input image.

There is some relation between x and $f(x)$ that the function f defines. Now the inverse is not some magical, and especially some new relation I pulled out of thin air. Rather, it is but the exact same relation at play except with this idea of swapping.

Computation: Swapping Simply switch x and y to compute the inverse function.

Derivative of inverses

TODO and implicit differentiation same idea y = x^n. x = y^n. Again seeing swapping at play. The input x, has corresponding y such that y^n = x. And in this relation, the derivative nature of the variables are swapped. Small x delta induces large y delta in original fn. But in the inverse, small y associated with large x change as x = y^n shows. Already notions of their derivaties being recriprocals of each other are suggestive to me.