Preventive monetary and macroprudential policy response to anticipated shocks to financial stability*

Konstantin Styrin† Alexander Tishin‡

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Abstract

In this paper, we develop a simple framework to study the optimal macroprudential and monetary policy interactions in response to financial shocks. Our model combines nominal rigidities and capital accumulation, features that have usually been studied separately in previous literature. In our model, we show that agents do not internalise how their asset purchases affect asset prices. Policy interventions (both monetary and macroprudential) can improve allocations by restricting borrowing ex-ante (during the accumulation of risks and imbalances) and stimulating the economy ex-post (during crises). As a result, we find a complementary relationship between ex-ante monetary policy and preventive macroprudential policy.

Keywords: Macroprudential policy, monetary policy, pecuniary externalities, nominal rigidities, financial frictions, capital accumulation.

JEL Codes: E44, E58, G28, D62

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†Bank of Russia, Adviser, Research and Forecasting department, and New Economic School, Senior Lecturer, StyrinKA@mail.cbr.ru

‡Bank of Russia, Economist, Research and Forecasting department, TishinAV@mail.cbr.ru; corresponding author

1 INTRODUCTION

Monetary policy is widely recognised as central banks' main tool to manage inflation. Indeed, before the recent financial crises (e.g., the financial crisis of 2007–2008), interest rates were the primary instrument central banks used to achieve their objectives.

However, since the 2008 financial crisis, the role of macroprudential policies has increased dramatically, because financial crises have shown that financial stability is still an important cause of concern for central banks. The initial economic shock arising from the US housing market raised the prospect of higher probabilities of default, but most importantly led to a vicious spiral where fears of default led to a fall in asset prices. This fall in asset prices reinforced concerns about the solvency of banks and other financial intermediaries, and this further reduced liquidity in a range of asset markets, with a variety of self-amplifying spirals then bringing the whole financial system to its knees.

A lack of liquidity dries up key financial markets and prevents institutions from restructuring their portfolios, adapting their strategies, and steering away from potential dangers caused by exogenous economic shocks. In turn, defaults start accumulating, and the domino effect leads to further reductions in liquidity and ultimately causes financial institutions, corporations, and other non-financial bodies to fail to meet their contractual obligations.

In this paper, we respond to the challenge of analysing the simultaneous use of monetary and macroprudential policies. We study a three-period model with nominal rigidities and financial frictions. We consider the first and second periods in the model to be the medium-run, with the third period representing the long-run. In the first period, agents are free to choose their allocations (consumption and borrowing decisions). In the second period, there is a certain probability of a crisis. That is, a financial or borrowing constraint (which limits agents' ability to borrow to a certain fraction of the market value of capital) may arise, and we consider the period when this constraint holds to be a crisis period.

To reduce the severity of the crisis, the policymaker may use ex-ante macroprudential and monetary policy policies to limit borrowing and ex-post monetary policy to stimulate the economy during the recession. In this framework, we analyse the optimal coordination between preventive monetary and macroprudential policies seeking to reduce the impact of anticipated financial shocks.

To do so, we assume an environment with a continuum of agents, namely, households and firms with linear technology. Households are endowed with some amount of a capital good. In the first period, they invest part of their final goods in the creation of new capital and they consume all the rest. Households fund firms with linear technology and receive profits from them. Additionally, households have their own, inferior, concave technology. The concave technologies of households and the linear technologies of firms are used to produce raw goods using capital as inputs. Firms, with their linear technology, are more effective users of capital, but they may be constrained financially, which is a source of pecuniary externality. That externality influences equilibrium prices because firms do not internalise the effects of their private decisions on aggregate prices. Thus, in 'normal' times, firms borrow too much, which leads to inefficient fire sales during recessions. This is the source of the first inefficiency in our model, which motivates the use of preventive macroprudential policy. Households, with their concave technology, are inefficient users of capital, and thus, in a frictionless economy, households are not willing to hold any capital, while firms with linear technology are willing to hold all capital.

This is an important result for our paper which is not initially driven by our assumptions. In our model, we assume that agents are small enough not to have any individual impact on aggregate prices. When agents make their decisions, they do not take into account that their decisions may lead to pecuniary externalities. Thus, our model does not assume that agents do not internalise externalities; this aspect is deduced, rather, from the micro-foundations of the individual decisions of agents.

Finally, raw goods, which are produced from the capital by either of the two technologies, combine with labour and are used in the production of final goods. In our baseline version, we assume that the prices of final goods are fixed in the first and second periods and fully flexible in the third. Price rigidity leads to an aggregate demand externality. In usual circumstances, agents do not internalise how their private decisions affect the aggregate demand determining the price level in an economy, which, in the case of rigid prices, influences consumption choices.

To put it differently, overborrowing and rigid prices increase the severity of crises leading to inefficient allocation. This creates the rationale for using both monetary policy and macroprudential policy to target both aims of price stability and financial stability.

Thus, the policymaker's tools are macroprudential taxes (Pigouvian taxes) (which influence the cost of debt for firms with linear technology in the pre-crisis period, thus reducing the size of fire sales) and ex-ante state-non-contingent monetary policy nominal rates in the pre-crisis period and ex-post state-contingent nominal rates in the post-crisis period. These rates function as usual monetary policy tools and deal with the nominal rigidities in the model.

Finally, we show that a social planner who internalises externalities can improve allocation and reduce the severity of the crisis. We use numerical methods to solve the social planner problem because our model does not have a closed-form solution. Moreover, we also exploit the 'simplicity' of our model, which allows us to solve our model without log-linearisation techniques, and thus we eliminate any quadratic forms of policy objective. Therefore, the social planner can simply maximise the weighed utility of agents. This 'simplicity' is a result of the three-period framework, which gives us a non-linear system to solve. The result of solving this problem is the four optimal parameters (one macroprudential tax and three monetary policy rates) which deliver maximum utility to agents.

The resulting framework has a rich structure, allowing us to find an answer to our main research question, namely, to characterise the optimal coordination between monetary and macroprudential policies in economies with pecuniary and aggregate demand externalities and financial frictions. In other words, in this paper, we set out to show how monetary and macroprudential policies interact, and we ask whether these policies substitute or complement one another.

In the presence of multiple externalities and the evolving nature of markets, in particular, the market for short-term funding, any proposed regulatory architecture should recognise that there are markets that are 'too important to fail', and not only banks that are 'too big to fail'. Hence, regulation, and policy more generally, should be focused on 'systemic markets' in addition to 'systemic institutions'.

The ability to adjust monetary policy appropriately to economic conditions requires the measurement of inflation and output growth. Likewise, the maintenance of financial stability requires equivalent measures of financial fragility. Such measures could be used as a yardstick to assess the success of the regulatory policy.

Recent crises have actively promoted thinking not only about price stability but also about financial stability (Cerutti et al., 2017). For instance, the average number of macroprudential instruments[1] used has almost doubled, and these instruments have been tightened since the global financial crisis. Most central banks have thus actually been targeting two aims, price stability and financial stability, using two sets of instruments: monetary and macroprudential measures.

Many countries, therefore, face a trade-off between price stability and financial stability, the question of whether macroprudential and monetary policies substitute or complement one another. Hence, to target both aims it is important to understand the optimal

[1] Caps on loan-to-value and debt-to-income ratios, limits on credit growth and other balance sheet restrictions, (countercyclical) capital and reserve requirements and surcharges, and Pigouvian levies.

interaction between monetary and macroprudential policies. The purpose of this paper is to provide a useful framework for policymakers regarding the joint use of monetary and macroprudential policies.

From the perspective of economic literature, the focus of many previous papers has been on either optimising monetary policy or analysing macroprudential policy. Optimal monetary policy design is well studied in economic literature (Clarida et al., 1999; Woodford & Walsh, 2005). Additionally, recent discussions on optimal monetary policy have been focused on the analysis of heterogeneous agent models (Kaplan et al., 2018), in which agents are heterogeneous in their amounts of liquid and illiquid assets.

The question of whether macroprudential policy is a substitute for or a complement to monetary policy arises when monetary policy is limited or restricted in some sense. Such inefficiencies of monetary policy can take different forms. For instance, when a monetary policy is bounded below (such as in Zero Lower Bound, ZLB; Eggertsson and Krugman (2012)), expansionary fiscal policy becomes effective. Fixed exchange rates or 'fear of floating' (Calvo & Reinhart, 2002; Dávila & Korinek, 2017) are other examples of when monetary policy becomes inapplicable. When monetary tightening is used for financial stability purposes ('leaning against the wind' (Borio, 2014; Woodford, 2012), this, in fact, leads to a weaker economy with higher unemployment and lower inflation, but with a lower probability of a crisis. If a crisis occurs, however, it will be of greater severity.

Many recent papers have been devoted to justifying macroprudential policy and estimating its effects on the economy. From the theoretical perspective, the papers nearest to ours study optimal time-consistent monetary policy in an economy with pecuniary externalities (Bianchi & Mendoza, 2018). They show that usual taxes on debt are ineffective, and they construct a time-consistent policy rule which delivers maximum utility. Stavrakeva (2020), in a similar model with pecuniary externalities, studies optimal capital requirements and finds that countries with larger fiscal capacity should implement lower bank capital requirements. The interaction between capital and liquidity regulations has been studied by Kara and Ozsoy (2020), and those authors show that stricter capital requirements reduce fire sales, but that banks respond to this tightening by decreasing liquidity. Macroprudential policy on liquidity requirements can improve these allocations. Capital regulations have also been studied in economies subject to default (Clerc et al., 2015). From an empirical perspective (Auer & Ongena, 2019) have studied the introduction of the countercyclical capital buffer (CCyB) in Switzerland. They find that such macroprudential policy leads to higher growth in commercial lending, higher interest rates, and higher fees charged to firms. The importance of loan-to-value (LTV) limits has been studied by Alam et al. (2019). The authors use a large sample of 134 countries from January 1990 to December 2016 and find that changes in LTV limits have a non-linear effect on household credit, namely that the effect of a tightening of LTV limits diminishes with the size of LTV limits.

There is a growing trend in the literature studying combinations of monetary and macroprudential policies. The most influential to us paper is written by Basu et al. (2020). We are also familiar with the work of Adrian et al. (2020), but their paper focuses on quantitative results, while we follow Basu et al. (2020) and take a descriptive approach, developing a model with a rich structure that gives us a qualitative, rather than quantitative, solution. However, we differ from Basu et al. (2020) in the fact that, while they fix the amount of capital in their model ('land' in their terminology), we allow agents to endogenously accumulate capital. This gives us the opportunity to study the transmission of macroprudential policy via two channels. The first one is the same as that used by Basu et al. (2020), a preventive macroprudential policy which restricts the amount of borrowing in the pre-crisis period. In this instance, because of positive macroprudential taxes, firms with linear technology own only a part of the capital available in the economy, while the rest is owned by households (which have inferior technology). Our second channel is new. With endogenous capital accumulation, positive macroprudential taxes also limit the amount of accumulation. In other words, macroprudential taxes constrain the creation of new capital. Macroprudential policy thus works in two directions to prevent excessive fire sales.

There are also studies in which the authors examine monetary and macroprudential regulation together. For instance, Cozzi et al. (2020) has studied the interactions between monetary and macroprudential policies. Van der Ghote (2021) has studied monetary and macroprudential policy intervention in a general equilibrium economy with recurrent boom-bust cycles. The author shows that there are welfare gains from policy coordination. In particular, it is optimal to intervene with macroprudential measures during the boom phase, which has effects during the bust period. Additionally, contractionary monetary policy during booms also helps to make busts less severe. Further, Darracq Pariès et al. (2020) find the strategic complementarities between monetary and macroprudential policy.

In line with these papers, we also study the interaction of monetary and macroprudential policies. We build our model based on the ideas of pecuniary externality (Dávila & Korinek, 2017) and aggregate demand externality (Farhi & Werning, 2016). We differ from these papers in two ways: we study capital accumulation in a framework with sticky prices, whereas the previous papers study either sticky prices without capital accumulation or capital accumulation without sticky prices. We take an approach similar to that of Dávila and Korinek (2017), in which we consider similar economies with pecuniary externalities and financial frictions. Our work does still differ from that of Dávila and Korinek (2017), as, although they allow endogenous capital accumulation, there are no nominal rigidities, while we consider an economy with nominal rigidities.

The second complementary strand of literature focuses on economies with nominal rigidities (as in Farhi and Werning (2016), Korinek and Simsek (2016), and Schmitt-Grohé and Uribe (2016)). The main driver of inefficiencies in these models is aggregate demand externality. Similarly to them, we add nominal rigidities in the form of sticky prices. However, these models usually assume a fixed amount of capital, while we allow endogenous capital accumulation.

It is necessary to mention that in papers with financial or collateral constraints there is the possibility of multiple equilibria, as has been determined, for example, by Schmitt-Grohé and Uribe (2021) Although in our paper we focus on over-borrowing equilibria, Schmitt-Grohé and Uribe (2021) show that there also exist equilibria with under-borrowing. Although this is an important discussion issue, we leave it to further research.

Many previously cited papers have used infinite-horizon models, and their authors have thus used different approximation or log-linearisation techniques to solve their models. Our contribution is the use of a three-period framework, which allows us to solve our model straightforwardly with no approximation or log-linearisation techniques. This means we may find the most accurate solution for our model with no loss of information owing to different non-linearities.

Finally, this three-period framework is easily extendable to any number of periods for the study of policy implications. For instance, we can increase the number of periods during which the financial constraint holds. We also can add an ex-post macroprudential policy to determine whether it is possible to improve allocations after a crisis has already occurred. There are thus many ways to extend this framework at a small price, although we leave this to further research.

The rest of this paper proceeds as follows. In Section 2 we describe the baseline model. In Section 3 we present our main results. In Section 4 we provide sensitivity analysis. In section 5 we discuss policy implications. In Section 6 we state our remarks on and the limitations of the current model. Finally, in Section 7 we conclude.

2 MODEL

In this section, we construct a closed economy model in a New Keynesian way in the spirit of Dávila and Korinek (2017) and Farhi and Werning (2016). The model consists of households, final goods firms, and capital sector firms. Effectively, we have a simple three-period framework, but we additionally assume that after the last period all variables stay at their steady states. Thus, we write optimisation problems for our agents as three-period problems. In this section, we describe the baseline version of our model. FOCs are given in Appendix B and in Appendix C for fixed-price and flexible-price models, respectively.

CONSUMERS Households maximise a utility function of consumption and linear disutility of labour.

$$\max_{\{c_t,h_t,b_t\}_{t=0}^2,\{k_t^{concave}\}_{t=1}^2,inv_0} \mathbb{E}_0 \sum_{t=0}^{2} \beta^t \left[\log c_{t,s} - h_{t,s}\right], \tag{1}$$

Their maximisation is subject to a budget constraint:

$$c_0 + b_1 + inv_0(1+\frac{\phi}{2}\frac{inv_0}{k_{-1}}) + q_0(k_0^{concave} - inv_0 - k_{-1}) + n_0^{linear}$$
$$= w_0 h_0 + p_{x,0}k_{-1} + \Pi_0^{final}, \tag{2}$$
$$c_{1,s} + b_{2,s} + q_{1,s}(k_{1,s}^{concave} - k_0^{concave})$$
$$= w_{1,s}h_{1,s} + (1+i_1)b_1 + p_{x,1,s}A_{1,s}\log(1+k_0^{concave}) + \Pi_{1,s}^{final}, \tag{3}$$
$$c_{2,s} + b_3 + q_{2,s}(k_2^{concave} - k_{1,s}^{concave})$$
$$= w_{2,s}h_{2,s} + (1+i_{2,s})b_{2,s} + p_{x,2,s}A_{2,s}\log(1+k_{1,s}^{concave}) + n_2^{linear} + \Pi_{2,s}^{final}, \tag{4}$$

where c_t is the consumption, h_t is the labour supply of households, inv_0 is the investments, made at time $t = 0$, k_{-1} is the initial endowment of capital, q_t is the price of capital at time t, $k_t^{concave}$ is the amount of capital that is in the hands of households (i.e. the amount of capital that is used for production with concave technology), n_0^{linear} is the initial financing from households to firms with linear technology, w_t is the wages, $p_{x,t}$ is the relative price of raw inputs (intermediate goods that are used in the production of the final goods), b_t is the one-period loans to firms with linear technology, Π_t^{final} is the profits of producers of final goods, n_2^{linear} is the net worth of firms with linear technology returned back to households (like dividends or profits). The household optimisation problem also includes the nominal interest rate, $i_{t,s}$, which is governed by the policymaker.

Note, that subindex s indicates state-contingent variables and takes values of $s = L$ if the state is low (bad, when the financial constraint holds) or $s = H$ if the state is high (good, when the financial constraint does not hold). Variables without subindex s are non-contingent, and thus their values are independent of the state. In other words, we assume that markets are incomplete.

LINEAR FIRMS Firms with linear technology are perfectly competitive and initially financed by households in the amount of n_0^{linear}. Linear firms are unconstrained at times $t = 0$ and $t = 2$, but have a positive probability of financial constraint at time $t = 1$.

A typical firm maximises its discounted profits:

$$\max_{\{n_t^{linear}, d_t, k_t^{linear}\}_{t=1}^{2}} \beta^2 \frac{\lambda_2}{\lambda_0} n_2^{linear} \tag{5}$$

subject to the following constraints:

$$q_{t,s} k_{t,s}^{linear} = n_{t,s}^{linear} + d_{t+1,s}^{linear} \;\forall t \geq 0 \tag{6}$$

$$d_{2,L}^{linear} \leq \kappa q_{1,L} k_{1,L}^{linear} \tag{7}$$

$$n_{1,s}^{linear} = p_{x,1,s} A_{1,s} k_0^{linear} + q_{1,s} k_0^{linear} - (1 + i_1) d_1^{linear} (1 + \theta_1) + t_1 \tag{8}$$

$$n_{2,s}^{linear} = p_{x,2,s} A_{2,s} k_{1,s}^{linear} + q_{2,s} k_{1,s}^{linear} - (1 + i_{2,s}) d_{2,s}^{linear} \tag{9}$$

$$t_1 = -\theta_1 (1 + i_1) d_1^{linear}, \tag{10}$$

where (6) represents the balance constraint for each period, (7) is the financial constraint which holds at time $t = 1$ with a positive probability. Equations (8) and (9) for n_t^{linear} represent the evolution of the net worth of linear firms and show that net worth consists of revenues from the production, plus the market price of owned capital, net of debt repayments with interest, $(1 + i_t) d_t$, and reverse tax transfers (equation (10)), finally, $lambda_t$ is the Lagrange multiplier.

In each period, linear firms can raise an amount of capital, at the market price, up to the value of their net worth (n_t^{linear}) and the required amount of borrowing (d_t) according to equation (6).

FINAL GOODS Here we will follow the New Keynesian tradition and assume a monopolistic competition between firms. Firms use the raw inputs (x_t) and labour (h_t) to produce a final goods (y_t). Inputs are produced using either linear or concave technology and are traded at a relative price $p_{x,t}$. Labour is supplied by households for a wage w_t. Thus:

$$y_t(j) = A_t^H h_t(j)^\alpha k_t(j)^{1-\alpha} \tag{11}$$

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon - 1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon - 1}} \tag{12}$$

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{-\epsilon} y_t \tag{13}$$

For times $t = 0$ and $t = 1$, we assume fully fixed prices, taking the idea from Lorenzoni (2008). This means that these prices are determined in and inherited from some previous period $t = -1$. However, starting from time $t = 2$, we assume fully flexible prices. Therefore, for flexible prices we know:

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_t}{\alpha}\right)^{\alpha}\left(\frac{p_{t,s}}{1-\alpha}\right)^{1-\alpha} \tag{14}$$

We conclude the description of the model with the market clearing conditions.

MARKET CLEARING

$$c_0 + inv_0(1 + \frac{\phi}{2}\frac{inv_0}{k_{-1}}) = k_{-1}^{\alpha} h_0^{1-\alpha} \tag{15}$$

$$c_{t,s} = x_{t,s}^{\alpha} h_{t,s}^{1-\alpha} \forall t \geq 1, \tag{16}$$

$$x_{t,s} = \log(1 + k_{t,s}^{concave}) + k_{t,s}^{linear}, \tag{17}$$

$$k_{t,s}^{linear} + k_{t,s}^{concave} = K, \tag{18}$$

$$K = k_{-1} + inv_0, \tag{19}$$

$$b_{t,s} = d_{t,s}, \tag{20}$$

where equation (15) shows the resource constraint for time $t = 0$, when agents spend all economic output on consumption and on the production of new capital inv_0. Equation (16) shows the resource constraint for the remaining periods, when agents spend all economic output on consumption. Equation (17) indicates the total amount of raw inputs x_t, which equals to the sum of capital-intensive output using two technologies: concave and linear. Equation (18) determines that the total amount of capital is divided between firms with different technologies. Equation (19) specifies the total amount of capital in the economy, which is the sum of initial endowment k_{-1} and newly produced capital

inv_0. Finally, equation (20) establishes that consumers fully finance firms with linear technology.

EQUILIBRIUM The equilibrium in the economy is the following. A real allocation $\{\{c_{t,s}, x_{t,s}, h_{t,s}, y_{t,s}\}_{t=0}^{2}, \{inv_0\}, \{k_{t,s}^{linear}, k_{t,s}^{concave}\}_{t=0}^{1}, \{d_{t,s}, b_{t,s}\}_{t=1}^{2}\}$ with a set of prices $\{\{w_{t,s}, q_{t,s}, p_{x,t,s}\}_{t=0}^{2}\}$, when agents solve their optimisation problems (1) and (5) subject to their constraints, and all markets are clear, i.e. equations (15), (16), (17), (18), (19), (20) hold for all times and states.

SOCIAL PLANNER To find the optimal values, we introduce a social planner which internalises externalities. The optimisation problem of the social planner is the following. The social planner takes into account the utility of all agents in the economy with corresponding Pareto weights and the resource and financial constraints in the economy. In our case, the social planner problem becomes similar to the Ramsey problem of finding the optimal values of 'taxes'. While some papers assume that a social planner chooses only initial allocations (at time $t = 0$), and this may be enough to correct externalities, in our paper, we assume that the social planner maximises agents' whole lifetime utility. From our perspective, this does not affect the analysis.

We note that the problem cannot be solved in closed form, and thus we use numerical methods to solve it. The full algorithm is described in Appendix D. The main idea is that we use a non-linear solver to find the solution for a fixed set of policy parameters: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$. This non-linear solver gives us the optimal allocations of the endogenous variables. Then, we apply a minimise function to this solver to find the set of policy parameters that finds the minimum of the negative value of utility. This 'minimiser' uses the simplex search method of Lagarias et al. (1998), as implemented in Matlab. The minimise function searches along the set of policy parameters θ_1, i_1, $i_{2,L}$ and $i_{2,H}$ to determine the minimum. Thus, this set is the solution that gives the maximum value of utility (minimum of negative utility), i.e. the values of the policy parameters.

3 RESULTS

First, we need to establish the version with which we will compare our results. A natural choice is to take a similar model without price rigidities (such model is presented in Appendix C) (we refer to this model as the 'flexible-price' version). The main difference with the model in the previous section is that, with fully flexible prices, we do not need monetary policy, and the real rates are determined endogenously in the model. Additionally, in our baseline fixed-price model, the policymaker manages nominal rates and, in fact, sets the real rates (because of the fully fixed prices). Thus, we take the equilibrium real interest rates from the flexible-price model as an indicator for a 'neutral' monetary policy. To show the numerical results, we use the parameters indicated in Table 1 in Appendix A.

Although some of these parameters are commonly used in the literature, it is necessary to discuss some of our choices. We apply a discount factor, β, equal to 0.8. While this is not a commonly used value, it is used by Basu et al. (2020). Our reasoning in using such a low discount factor follows from the high degree of stylisation in our model. Our three-period framework represents the medium-run (the first two periods) and the long-run (the last period), and therefore these periods should be discounted more than in classical models (where a step of one year or one quarter is usually assumed between the periods). On the other hand, our model actually requires relatively high interest rates because, again, the difference between our periods is much larger than a year. Thus, we match the steady-state interest rate of $1/\beta - 1 = 0.25$. The share of capital used in production, α, is equal to $1/3$, which is a value commonly used in the literature representing a long-lasting phenomenon. We calibrate the price markup to 6 to fit an average price markup of 20%.

We fix the initial endowment of capital goods of households, k_{-1}, at 0.85, which reflects the average value of the ratio of net worth to total assets of households and non-profit organisations according to US data from 1995 to 2020. We think that this is the closest economic indicator for our model because households' endowment of capital goods represents their initial wealth. We take this idea from the quantitative model of Bernanke et al. (1999). This calibration is also to some extent similar to that used by Basu et al. (2020), although ours differs from theirs. Basu et al. (2020) assume a fixed amount of capital (or 'land' according to their terminology), while our paper includes endogenous capital accumulation and, therefore, we use the calibrated value as the initial value for capital endowment.

For the initial financing of firms with linear technology, n_0^{linear}, we use a calibrated value of 0.53 from US data, representing the ratio of net worth to total assets of non-financial corporate businesses in a sample running from 1995 to 2020 (this idea is taken from Bernanke et al. (1999)). We take this ratio because it is the economic indicator closest to the value in our model, since n_0^{linear} represents the source of firms' net worth. There is no direct and observed economic indicator to calibrate the value of this variable because of the high degree of stylisation of our model. There is also no direct mapping from our model to the model of Basu et al. (2020) which is closest to our model. Having the natural restriction that the initial financing of firms with linear technology cannot be larger than the initial endowment of households (our calibration satisfies this restriction), we can also compare our value with the initial debt level of 0.6 used by Basu et al. (2020). Our calibration is thus quite close to theirs.

We also take values for the probability of a good or bad shock, ρ_L, from Basu et al. (2020) and set it at 0.5. This choice may be debated, however, this probability should not be interpreted as the probability of a crisis used in multi-period models (which are usually calibrated at 3%-5%). In our model, it reflects the probability of two states in the medium term, as similarly used by Basu et al. (2020).

The parameter that governs the toughness of the debt limit in equation (7), κ, is calibrated to 0.025, also according to Basu et al. (2020). This low value indicates that any crisis will be quite tough, constraining firms with linear technology to borrowing only up to 2.5% of the current market value of their capital stocks. Finally, we set capital adjustments at 1, similarly to Uribe and Schmitt-Grohé (2017).

Before providing our results, we must clarify our terminology. In this and the next section, we consider several versions of the model from Section 2. As a baseline, we use a model with nominal rigidities and financial frictions. In addition to the baseline, we abandon our assumptions one by one and consider alternate versions: a model with flexible prices and financial frictions, a frictionless model with nominal rigidities, and a frictionless model with flexible prices. These four versions cover all cases of the inefficiencies of financial frictions and rigid prices. To show how inefficiencies actually affect optimal allocations, we compare the results of the four versions.

The baseline version is fully described in Section 2. For the model with flexible prices and financial frictions, we relax the assumption of the fully fixed price at times $t = 0$ and $t = 1$. This means that equation (14) holds for all periods when $t \geq 0$. Moreover, with fully flexible prices there is no need for monetary policy, and thus the nominal interest rates (which are exogenous for the baseline) become endogenous and real rates under flexible prices: $r_1, r_{2,L}$ and $r_{2,H}$. This does not affect macroprudential policy θ_1, which is left as the only exogenous policy tool. These two versions are the most important in our analysis, while the frictionless models are supportive. In the frictionless model with nominal rigidities, we relax the assumption of binding financial constraints in the 'low' state. So, we simply remove equation (7) from our model, meaning that in both the 'low' and 'high' states, firms with linear production technology never meet this constraint. We retain the assumption of fully rigid prices in the periods for times $t = 0$ and $t = 1$. In the frictionless model with flexible prices, we simultaneously remove both the assumption of rigid prices and of binding financial constraints. This is a fully frictionless economy, which delivers the best allocation in our model.

RESULT 1. To present our main result, we compare the optimal policies of the baseline economy with those of the other three versions described above. Our findings are presented in Result 1 and in Figure 1. The figure includes 4 sub-figures for each policy instrument, along with a grey dashed line showing the steady-state value $(1/\beta - 1)$ for the corresponding sub-figures. In each sub-figure. we plot four bars, one for each model. The first bar (dark red) is our baseline model with nominal rigidities and financial frictions. The second bar (dark blue) is the model with flexible prices and financial frictions. The third bar (light red) is the frictionless model with nominal rigidities. Finally, the fourth bar (light blue) is the frictionless model with flexible prices.

It is also necessary to mention that we take the approach used in other articles and study constrained efficient allocation because only this model gives us a rich structure of inefficiencies that require macroprudential and monetary intervention. Additionally, our model is built around the emergence of financial friction, which is the source of pecuniary externality in this framework. From the perspective of our analysis, there is no need to consider models without financial frictions. Thus, it is natural to compare allocations with fixed and flexible prices in the presence of financial frictions. However, for clarity of analysis and to understand the outcome of the model in an ideal (i.e., frictionless) environment, we also show the results for fixed and flexible prices without the presence of financial frictions in Figure 1. Further in the paper, we examine how the different frictions in the model affect the outcome and discuss only models with financial frictions and only allocations which are constrained efficient.

Result 1 (Complementarity in static) *In response to anticipated financial shocks, it is optimal to tighten both monetary and macroprudential policy. This Result is illustrated in Figure 1.*

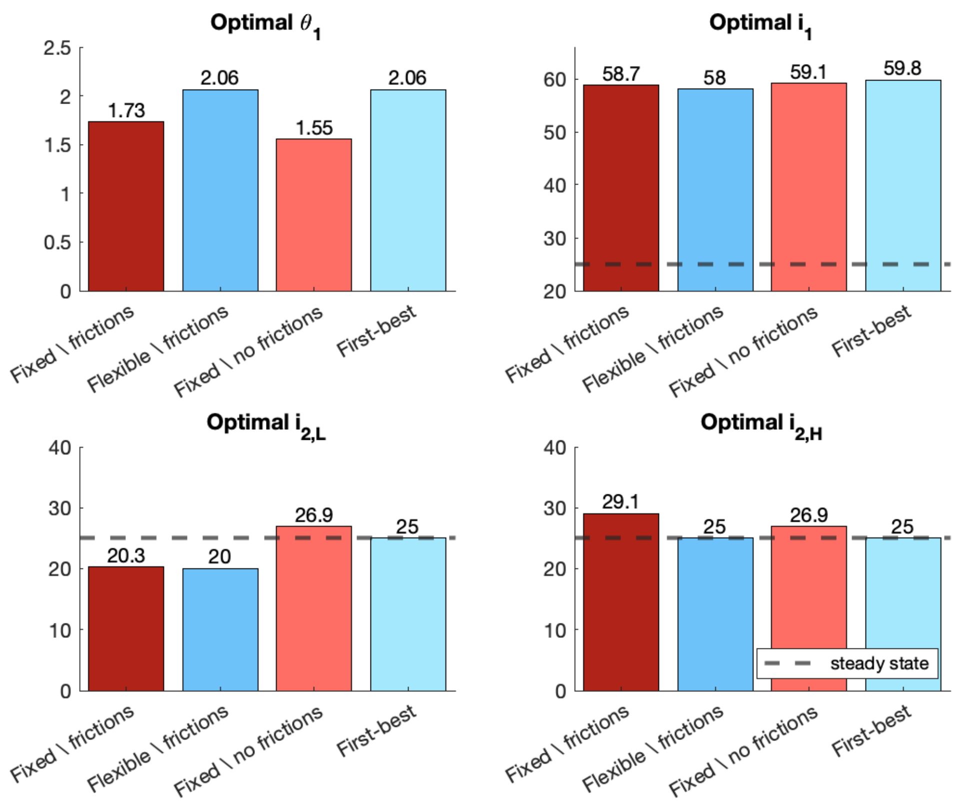

Figure 1: Complementarity in static:
Optimal policy
fixed prices vs flexible prices

We begin with a discussion of the frictionless economy in Figure 1 represented by the light red and blue bars for fixed and flexible price models. We see that the optimal macroprudential policy, θ_1, is larger for the first-best model (i.e. the model without any frictions or rigidities). The situation for the optimal ex-ante monetary policy, i_1, is similar. However, we see some differences with the ex-post rates. For the first-best allocations, the optimal ex-post interest rate, $i_{2,L} = i_{2,H}$, is its steady-state level: $1/\beta - 1$. For a frictionless economy with fixed prices, the optimal ex-post interest rate, $i_{2,L} = i_{2,H}$, is slightly higher than its steady-state value. Hence, both models are frictionless and their 'low' and 'high' states are almost the same, but nominal rigidities are important. This is an expected result, because, in a frictionless model without nominal rigidities, there are no reasons to deviate from the steady-state values, but this is not the case when we consider a frictionless economy with nominal rigidities, which implies the inability of firms to reset their prices to the optimal level, thus requiring monetary policy intervention.

Now, we turn to the more interesting models with financial friction. These are represented by the dark red and blue bars in Figure 1 for the fixed and flexible price models. We see that the optimal θ_1 is positive in both versions and it is significantly higher with flexible prices. The optimal θ_1 in the model with financial friction and flexible prices is equal to that in the first-best model. Why is θ_1 positive even in the first-best model? The reason is that the economy at period $t = 0$ begins from certain initial values (for example, the calibrated values of k_{-1} and n_0^{linear}). These values may not always be at their steady-state values, and the economy, therefore, has a need for macroprudential (and monetary) intervention to change the allocations existing at time $t = 0$. The fact that the optimal θ_1 values in the model with financial friction and flexible prices and in the first- best model are equal may indicate that not only the emergence of financial friction motivates the use of macroprudential tools, but also those sub-optimal initial allocations may be enough to spur intervention.

The optimal i_1 is almost the same in both versions (with financial frictions), but it is slightly higher in the version with fixed prices. The optimal $i_{2,L}$ values are almost the same. The optimal i2,H for flexible prices takes its steady-state value $(1/\beta - 1)$, but it is significantly higher with fixed prices.

The economic insight is the following: in response to anticipated binding collateral constraints, it is optimal to tighten both ex-ante macroprudential and monetary policies and continue tightening if the crisis does not occur, while, if the crisis occurs, it is optimal to soften ex-post monetary policy. Tightening policy before the crisis prevents agents from borrowing excessively, and thus, if a crisis occurs, fire sales are less extreme. This tightening happens for both fixed- and flexible-price economies with financial frictions. If a crisis does occur, then loosening monetary policy stimulates agents to consume and spend more. This also holds for both fixed- and flexible-price economies with financial frictions. If a crisis does not occur, the policy remains tight only if prices are rigid and firms cannot set their prices optimally (in response to the no-crisis state). This means that under a fixed-prices paradigm, in trying to soften a crisis ex-ante (by tightening macroprudential and monetary policy), the policymaker may end up with tight monetary policy even if there is no crisis, while if prices are fully flexible the economy will fully recover from previous actions (even if no crisis occurs). This result highlights the importance of assessing the degree of price rigidity in the economy to better understand its behaviour during a crisis and the speed of its post-crisis recovery.

We call this result 'complementarity in static' because, as we see, both macroprudential policy and monetary policy are tougher before a crisis. Both policies (θ_1 and i_1) operate in a similar manner and in the same direction.

Note that the values look unrealistic. This is due to the calibration chosen and the simplifying assumptions made. Because the model is highly stylised, it is impossible to choose parameters that match realistic values of interest rates. This is why we consider our model to be qualitative, and not quantitative.

RESULT 2. We wish to show how each instrument behaves as a function of all the other policy instruments. Visualising this would require the plotting of a 4-d figure, however, which is impossible. Instead, we set fixed values for two of the 4 instruments and plot the others as functions of one another.

Since our paper is mainly focused on the preventive policy we plot θ_1 as a function of i_1 while keeping $i_{2,L}$ and $i_{2,H}$ at their optimal values (at their optimal values for every pair of θ_1 and i_1). Likewise, we plot i_1 as a function of θ_1. The interaction of these two lines indicates our optimal solution. We can see this in Figure 2 and in Result 2. The other functions are available upon request. The idea behind the figures in the appendix is the same: we show the behaviour of one policy instrument as a function of another policy instrument while fixing the remaining instruments at their optimal values.

Result 2 (Semi-complementarity in dynamic) *In response to higher ex-ante monetary policy, optimal macroprudential policy shows an inverse U-shaped curve, while in response to higher macroprudential policy, optimal monetary policy remains almost stable. This Result is illustrated in Figure 2.*

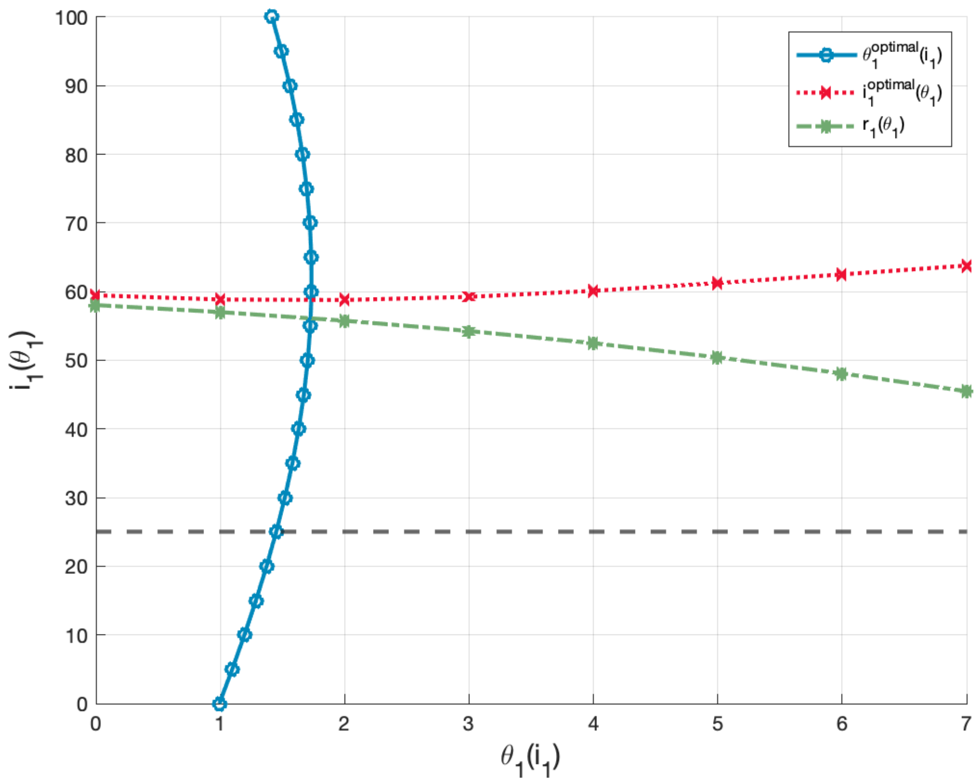

Figure 2: Semi-complementarity in dynamic

In Figure 2, the solid blue line (with circles) indicates θ_1 as a function of i_1. That is, for every value of i_1, our algorithm finds the optimal θ_1[2]. The inverse, the red dashed (with crosses) indicates i_1 as a function of θ_1, i.e. for every possible value of θ_1 we find the optimum value of i_1. The green dash-dotted line (with asterisks) shows the optimal value of r_1 in the model with flexible prices (remember that in these two versions of the model with flexible and fixed prices i_1=r_1). Finally, the grey dashed line indicates the steady state value of these interest rates ($1/\beta - 1$).

[2]We also find optimal $i_{2,L}$ and $i_{2,H}$ which are not shown. They are available upon request.

We see that the red line (with crosses) is almost insensitive to changes in θ_1, while the blue line (with circles) shows a U-shaped curve, i.e. θ_1 increases when i_1 is small, but declines with higher values of i_1. In other words, for small i_1 values, we tighten macroprudential policy but ease it for high i_1 values. We call this result 'semi-complementarity in dynamic' because the red line (with crosses) remains almost constant (compared with the green line (with asterisks) that shows policy easing). So in the model with fixed prices, monetary policy remains tight, but does not tighten for every possible value of macroprudential policy θ_1. These policies thus remain tight, but the tightening of one policy does not necessarily result in the tightening of the other policy. In a dynamic state, then, these policies are only partly complementary, because we see the non-monotonic direction of optimal macroprudential policy as a function of ex-ante monetary policy. This result highlights the fact that policymakers should use macroprudential policy with caution: if the interest rates in the economy are already high, it may be not optimal to continue tightening macroprudential policy.

RESULT 3. We contrast our results with those of Basu et al. (2020) and look at both ex-ante (i.e. preventive) and ex-post policies. The economic insight is the following: every policymaker desires to prevent economic instability using a variety of available tools which potentially reduce the probability of a crisis or the severity of a possible crisis (in our model, these are θ_1 and i_1). However, during a crisis, the policymaker has a desire to stimulate the economy to help get through the crisis. In our model, these ex-post instruments are interest rates ($i_{2,L}$ and $i_{2,H}$). Our findings in the use of these ex-ante and ex-post rates are shown in Result 3 and Figure 3.

Result 3 (Ex-ante vs ex-post policies) *In response to anticipated financial shocks, it is also optimal to tighten monetary policy before the crisis and soften expected interest rates during the crisis. While there is almost no difference between the fixed and flexible models in the tightening* *of monetary policy before a crisis, the easing during the crisis is larger for the flexible models, as illustrated in Figure 3.*

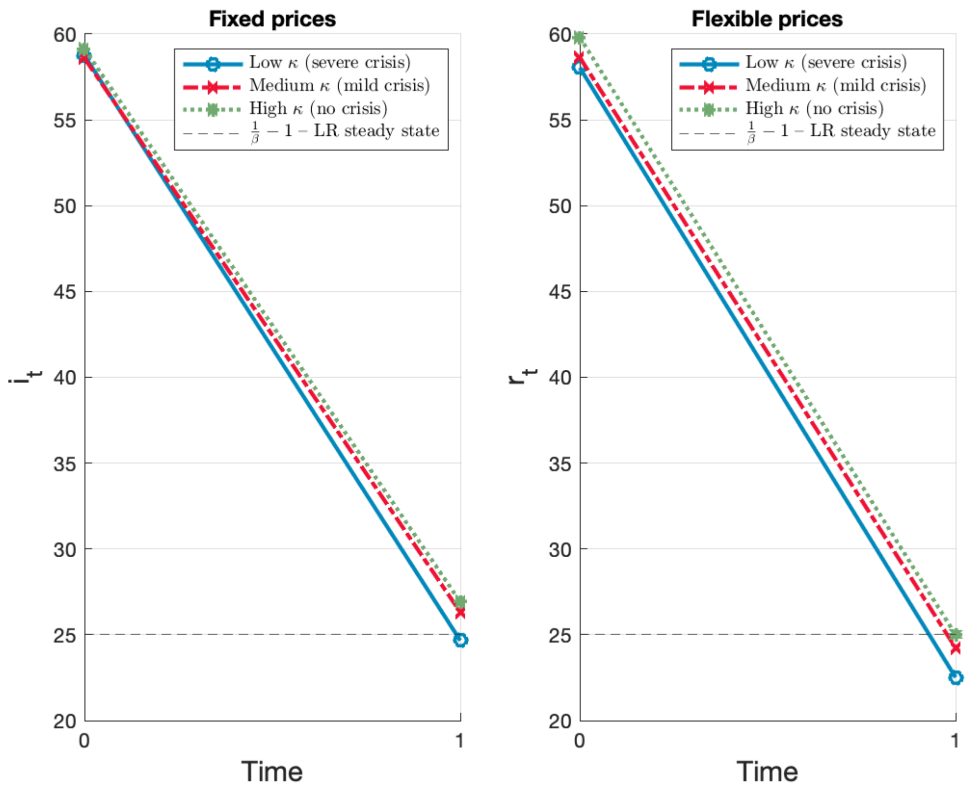

Figure 3: Ex-ante vs ex-post policies

In Figure 3 we compare our results for fixed prices (left) and flexible prices (right). We plot three lines on each chart. The solid blue lines (with circles) show the optimal policy for a low κ (= 0.025), which indicates a severe crisis. The dashed red lines (with crosses) show the optimal policy for a medium κ (= 0.25), which indicates a mild crisis. The green dash-dotted lines (with asterisks) show the optimal policy if no financial constraint holds (no crisis occurs). The idea behind showing these lines for different κ values is that in the model, κ indicates the severity of a crisis. The smaller the kappa, the more severe the crisis. So, we plot extreme cases, for a rather serious crisis and for a much less severe crisis. Finally, the dashed grey line indicates the steady-state value of these interest rates ($1/\beta - 1$).

From Figure 3, we again see that ex-ante (during the accumulation of risks and imbalances), monetary policy tightens, while it eases ex-post (during a crisis). We see a difference between the models and their κ values. In the fixed-price model, the monetary policy tightening in the pre-crisis ($t = 0$) period is almost the same for all κ values, while in the flexible-price model, monetary policy tightening is stronger for high κ values.

For the fixed-price model in a crisis period ($t = 1$), we see that easing is smaller for high κ values[3] and larger low κ values (even falling below the steady-state level). It is important to note that the green line remains above its steady-state level, which indicates the importance of nominal rigidities (otherwise, it would reach its steady-state value).

For the flexible-price model, we see that easing is even greater for low κ values and that interest rates fall significantly below their steady-state level. For high κ values, the ex-post interest rate exactly reaches its steady-state level (if there is no crisis, there is no need for ex-post policies). The red line indicating a medium crisis always falls between the extreme cases.

This highlights the fact that our model does not, ex-ante, pay much attention to the 'size' of a crisis (since the i_1 values are very similar in both versions), but we see much larger ex-post policy easing if κ is low.

To conclude this section, we find that in response to anticipated financial shocks in the form of debt limits, it is optimal to preventively tighten both monetary and macroprudential policy, while in a crisis period it is optimal to support the economy by easing interest rates.

[3] Here, as an ex-post interest rate we plot the weighted interest rate: $i_2 = \rho_l i_{2,L} + \rho_H i_{2,H}$.

4 SENSITIVITY ANALYSIS

In this section, we present and discuss a series of supplemental results. Specifically, we provide sensitivity analysis. We offer this discussion because our model is more qualitative than quantitative, and therefore we cannot plausibly calibrate all of our parameters, which may be important in the determination of optimal policy. Thus, in this section, we vary the initial capital endowment k_{-1}, the initial amount of financing firms with linear technology n_0^{linear}, the debt or financing constraint limit κ, the capital adjustment costs ϕ, and the probability of a crisis ρ_L. In the following figures, we adhere to the following notation: the solid blue lines (with circles) are the optimal policies in the fixed-price model, the dashed red lines (with crosses) are the optimal policies in the flexible-price model, and the dashed grey lines indicate the steady-state value of interest rates $(1/\beta - 1)$.

SUPPLEMENTAL RESULT 1 We begin this section with the role of initial households' endowment, k_{-1} on the optimal values of the policy parameter. We vary k_{-1} from 0.5 to 3, which seem to us to be reasonable values for this parameter. Our findings are described in Supplemental Result 1 and illustrated in Figure 4. In this analysis, we highlight the importance of initial household endowment in the determination of the optimal policy.

Supplemental Result 1 (Change in k_{-1}) *With a larger endowment of k_{-1}, the policymaker must significantly tighten macroprudential policy, θ_1, ease ex-ante monetary policy, i_1, and also ease ex-post monetary policy in the crisis period, $i_{2,L}$, or tighten ex-post monetary policy if the crisis does not occur, $i_{2,H}$. This Supplemental Result is illustrated in Figure 4.*

The main economic take-away is the following. The initial households capital endowment is important when a policymaker uses macroprudential policy and monetary policy to soften the severity of a crisis. We see that if households are richer initially (have

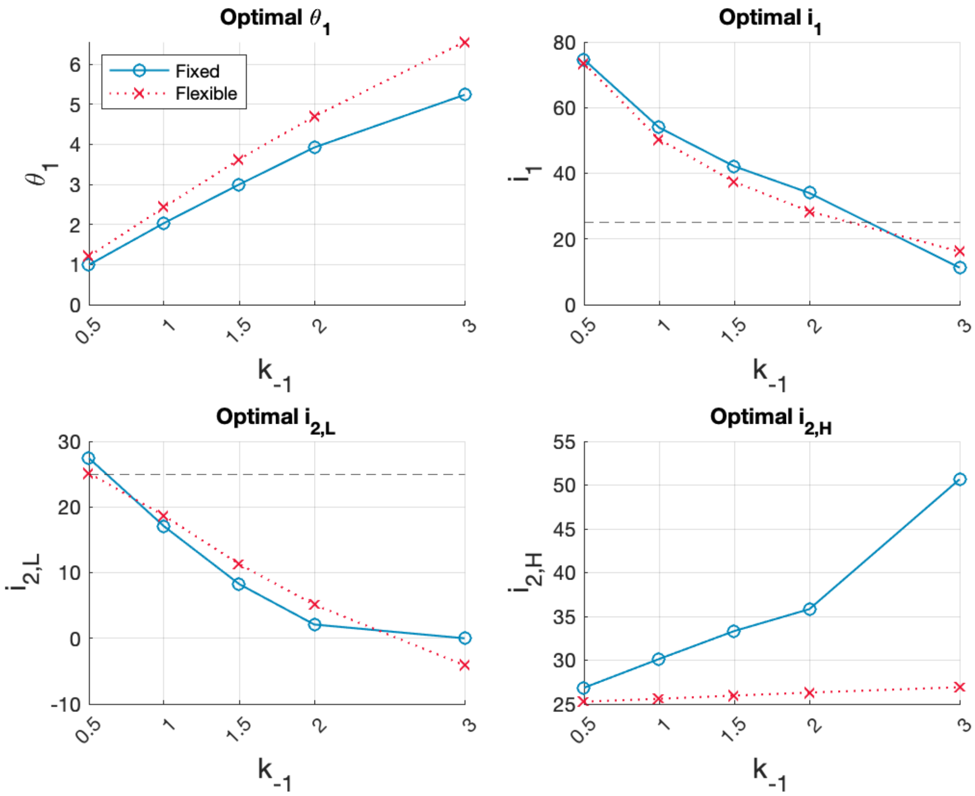

Figure 4: Supplemental Result 1 – change in k_{-1}

greater k_{-1} value), then the policymaker must set ex-ante macroprudential policy, θ_1, and ex-post monetary policy, $i_{2,H}$, quite high, while it must set ex-ante monetary policy, i_1, and ex-post monetary policy, $i_{2,L}$ quite low. The policymaker softens ex-ante monetary policy to reduce the effect of high macroprudential taxes on aggregate demand. Also, it seems that such financial crises are more harmful to richer households, because the policymaker significantly lowers $i_{2,L}$ in a crisis state and raises $i_{2,H}$ in a non-crisis state. The opposite is true for poorer households (when the initial household capital endowment is low): the policymaker sets macroprudential policy quite low, and there is no need to compensate for macroprudential taxes with low ex-ante interest rates (which remain quite high). The policymaker softens its ex-post policy noticeably less ($i_{2,L}$ is rather higher for poorer households than for richer households) if a crisis occurs and sets the $i_{2,H}$ rate very close to its steady-state level.

Additionally, we see that optimal policies for θ_1, i_1 and $i_{2,L}$ in the fixed-price model are very close to the macroprudential tax and interest rates in the flexible-price model. For $i_{2,H}$ in the fixed-price model, however, we see that the policymaker must significantly increase the interest rate if households are rich initially. Furthermore, we see (especially for i_1 and $i_{2,L}$) that for some values of k_{-1}, the optimal policy falls below the steady-state level.

SUPPLEMENTAL RESULT 2 Next, we analyse the role of the initial amount of financing of firms with linear technology, n_0^{linear}. To see the effect of n_0^{linear} on the optimal values of the policy parameters, we vary it from 0.1 to k_{-1}. Taking into account that n_0^{linear} must be less than the initial household capital endowment, this figure is a natural upper bound. Our findings are presented in Supplemental Result 2 and illustrated in Figure 5.

Supplemental Result 2 (Change in n_0^{linear}) *With a larger amount of financing for firms with linear technology, n_0^{linear}, the optimal macroprudential policy shows an inverse U-shaped curve, i.e., macroprudential policy tightens when n_0^{linear} is low and it loosens macroprudential policy (lower, but positive, θ_1) when n_0^{linear} is higher. The policymaker also must tighten ex-ante monetary policy, i_1, and also ex-post monetary policy in a crisis period, $i_{2,L}$, but loosen ex-post monetary policy if there is no crisis, $i_{2,H}$. This Supplemental Result is illustrated in Figure 5.*

This Supplemental Result shows two important features. First, we see that depending on n_0^{linear}, the optimal θ_1 is non-linear in the fixed-price model. It rises when n_0^{linear} is low and it decreases when n_0^{linear} is higher. Second, we also see that there is a limit for interest rates and a lower bound for θ_1. This means that it is optimal for a larger n_0^{linear} to tighten both rates up to a certain limit. The opposite is true for the easing of $i_{2,H}$, it is optimal to reduce the rate down to some limit. This highlights the result that, if the economy consists of 'rich' firms, monetary policy may be a constraint both ex-ante (during the accumulation of risks and imbalances) and ex-post (during a crisis). This limit is closely related to the upper limit of n_0^{linear}. By construction, firms with linear technology cannot

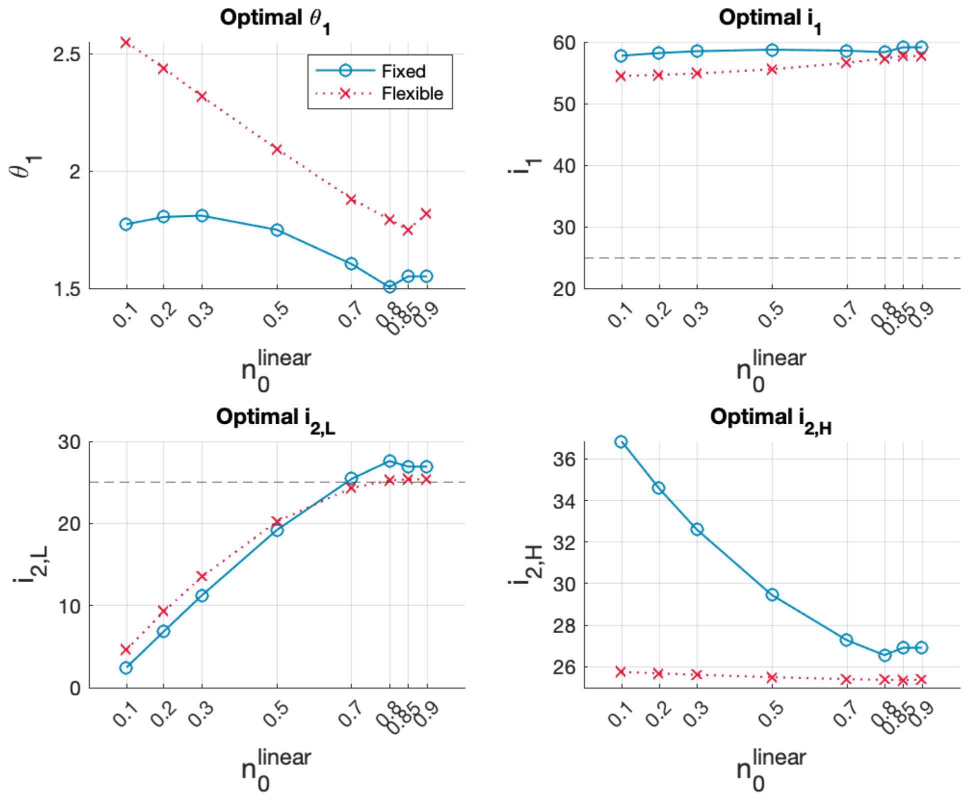

Figure 5: Supplemental Result 2 – change in n_0^{linear}

receive financing greater than the initial household endowment. Figure 5 supports this claim: because we see some strange behaviour around $k_{-1} = 0.85$, and we consider this the limit for n_0^{linear}.

Figure 5 highlights important differences between the fixed- and flexible-price models. If firms are initially poor (under-financed), then fixed prices may require slightly different policies (especially, for θ_1 and $i_{2,H}$).

SUPPLEMENTAL RESULT 3 We analyse the role of debt limit κ. To see the effect of κ on the optimal values of the policy parameters, we vary it from 0.025 to 0.975. By constriction, κ is the limit of the value of new debt, given as a ratio of the market value of a firm's capital. Our findings are described in in Supplemental Result 3 and illustrated in Figure 6.

Supplemental Result 3 (Change in κ) *With a higher κ value (less strict debt/financial constraint), the policymaker must implement softer macroprudential regulation (lower, but positive, θ_1), tighten ex-ante monetary policy, i_1, and also ex-post monetary policy in a crisis period, $i_{2,L}$, but loosen ex-post monetary policy if the crisis does not occur, $i_{2,H}$. Note that there is an upper limit for κ, beyond which there is no variation in optimal policies. This Supplemental Result is illustrated in Figure 6.*

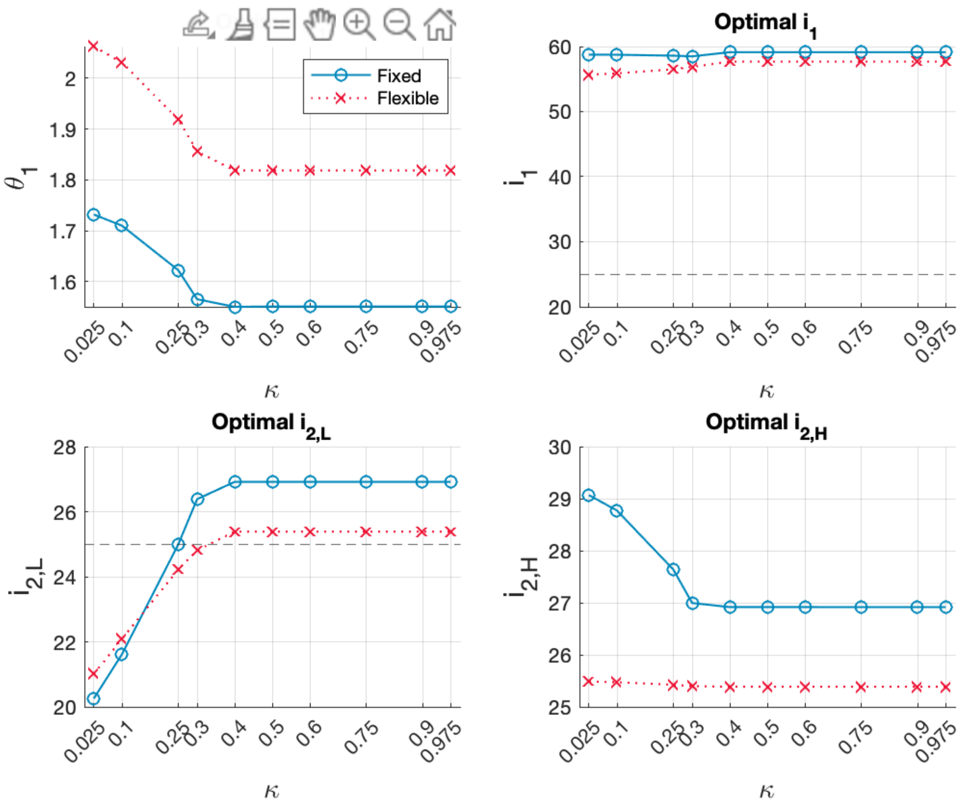

Figure 6: Supplemental Result 3 – change in κ

When we consider the debt limit κ, firstly, we see that there is an upper limit for κ, above which there is no reaction from any policy. The insight is this: If the debt limit is not strict, then the financial constraint does not hold because firms with linear technology are willing to take on less debt and so do not meet the financial constraint.

We can determine the approximate threshold for κ in the current calibration. From Figure 6, we can see that macroprudential tax and all interest rates become flat around a value of 0.4, which is the limit for κ. This figure shows that in our economy, firms with linear technology never borrow more than 40% of the market value of their capital stock.

Moreover, we see that the optimal policies for the fixed- and flexible-price models move in more or less the same directions with changes in κ. However, we note that the optimal θ_1 is softer for the fixed-price model than for the flexible-price model. The optimal values for $i_{2,L}$ and $i_{2,H}$ are stricter, while the optimal i_1 is almost the same.

SUPPLEMENTAL RESULT 4 We analyse the role of investment costs ϕ. To see the effect of ϕ on the optimal values of the policy parameters, we vary it from 0.5 to 5. With higher ϕ, it becomes more difficult to create new capital in the economy. Our findings are demonstrated in Supplemental Result 4 and illustrated in Figure 7.

Supplemental Result 4 (Change in ϕ) *With larger investments costs, ϕ, the policymaker must implement softer macroprudential regulation (lower, but positive, θ_1). It is optimal to soften ex-ante monetary policy, i_1, and both ex-post monetary policies $i_{2,L}$ and $i_{2,H}$. This Supplemental Result is illustrated in Figure 7.*

We see the expected result as we vary the capital costs parameter. With larger ϕ it is more difficult to create a new unit of capital, agents accumulate less capital, and less intervention is required from the policymaker. However, we note the difference in the optimal interest rates between the fixed- and flexible price models. The difference between $i_{2,L}$ and $i_{2,H}$ is much larger in the fixed-price model.

SUPPLEMENTAL RESULT 5 We analyse the role of the probability of a crisis, ρ_L. To see the effect of ρ_L on the optimal values of the policy parameters, we vary it from 0.1 to 0.9. A larger value indicates a higher probability of a crisis. Our findings are given in in Supplemental Result 5 and illustrated in Figure 8.

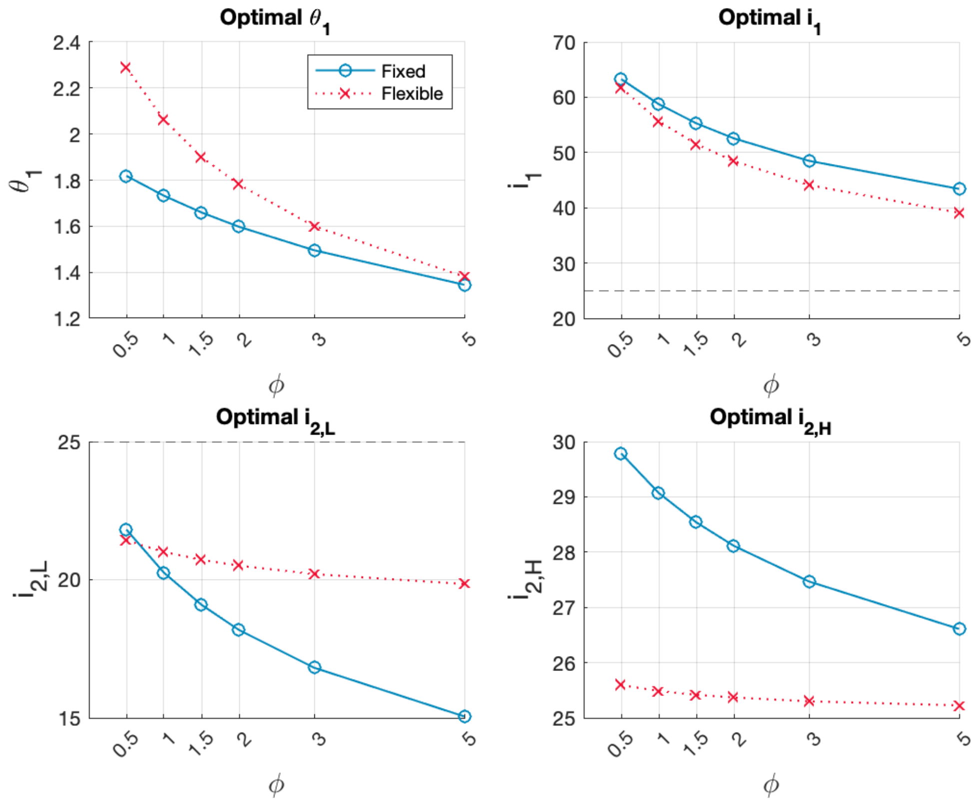

Figure 7: Supplemental Result 4 – change in ϕ

Supplemental Result 5 (Change in ρ_L) *With an increase in the probability of a crisis (higher ρ_L), the policymaker must implement a tighter macroprudential policy (higher θ_1). and soften ex-ante monetary policy, i_1. It is optimal to tighten both ex-post monetary policies $i_{2,L}$ and $i_{2,H}$. This Supplemental Result is illustrated in Figure 8.*

The most striking result is that when the probability of a crisis, ρ_L, rises, the policymaker tightens macroprudential policy but slightly loosens ex-ante monetary policy. It is optimal to tighten ex-post monetary policy. We also note that in the fixed-price model, i_1 falls less in comparison with the flexible-price model, while $i_{2,H}$ rises a lot in the fixed-price model while it remains relatively the same in the face of flexible prices.

This result is striking because, with a higher probability of a crisis, we expect a tightening in ex-ante monetary policy to prevent excessive capital accumulation (in addition to a tightening of macroprudential policy). A loosening of ex-ante monetary policy with the growing probability of a crisis looks counter-intuitive, because it would seem that if a crisis is imminent, the policymaker must make every effort to reduce the severity of the crisis. However, we can provide the following economic insight: If a crisis is inevitable, the policymaker restricts borrowing by firms with linear technology using macroprudential policy θ_1. This tighter restriction suppresses the demand for capital, so there is a contraction in aggregate demand. The policymaker must therefore implement expansionary monetary policy. The optimal ex-ante interest rate decreases reflecting this expansion, but monetary policy remains tight overall, because the interest rate is still higher than the optimal value for fixed prices. So, similar to Supplemental Result 1, ex-ante monetary policy and ex-ante macroprudential policy work in opposite directions: a tightening of macroprudential policy requires a softening of monetary policy to stimulate aggregate demand.

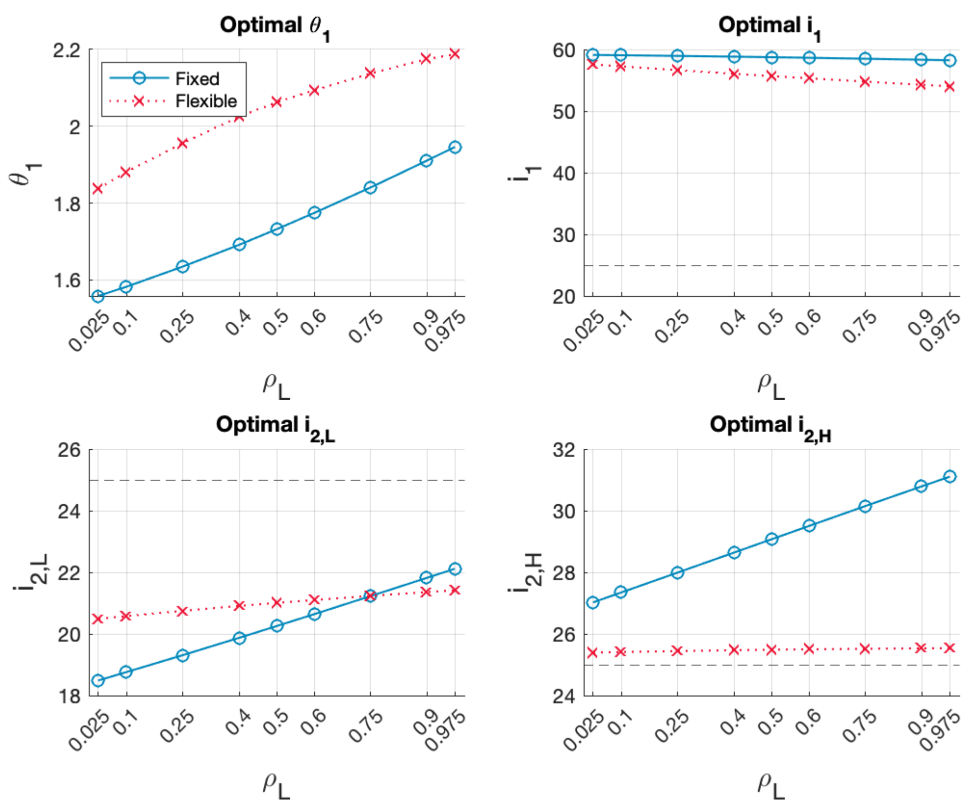

Figure 8: Supplemental Result 5 – change in ρ_L

To conclude this section, it is important to determine the point at which we compare monetary tightening and easing. If we compare fixed prices with flexible prices, then the above results hold true. These results remain the same for several sensitivity tests (when we change the calibration values), as is indicated in several of our Supplemental Results. For most parameter values, the policymaker still must tighten macroprudential policy (but less than for flexible prices) and set ex-ante monetary policy higher than for flexible prices. Ex-post monetary policy must be eased even more than for flexible prices if a crisis occurs and tightened more if a crisis does not occur.

5 POLICY IMPLICATIONS

In terms of the usefulness of our analysis for the implementation of policy, we see several implications. In practice, it is important for policymakers to understand or estimate the initial household endowment k_{-1}, and the amount of financing of firms with linear technology, n_0^{linear}. Both of these variables indicate households' or firms' resources, but the optimal policy responses for each are diametrically opposed. Thus, it is essential to be accurate in determining or estimating these endowments.

It is also important for policymakers to correctly estimate the severity of a potential crisis. If the debt limit is not very tight, then it is possible that the financial constraint may not be met, which leads to a slightly different optimal policy response from the policymaker. It is also important to understand the actual difficulty of creating new capital, because high difficulty of capital creation requires much less policy intervention. The probability of a crisis undoubtedly matters in the determination of the optimal policy intervention.

Additionally, in our exercises with fixed- and flexible-price models, we have sought to demonstrate extreme cases in order to compare the behaviours of macroprudential and monetary policy tools. In these experiments, we highlight that price rigidity especially matters ex-ante to prevent crises. While with fully fixed prices the 'size' of a crisis does not matter (optimal ex-ante monetary policy is almost insensitive to the toughness of the financial constraint), the difference in the optimal policies in the flexible-price model is large. Therefore, it is important for policymakers to understand not only the severity of a potential crisis but also the degree of price rigidity.

6 FURTHER REMARKS

In this section, we identify several limitations of our model. First, it is a model of a closed economy. This limits the space available for analysis. Nevertheless, there are still plenty of interesting results discussed in the previous sections, although there is room for improvement. The model still lacks a rich banking sector and the possible externalities that emerge in this sector. Many interesting features are also difficult to incorporate, such as interest rates that affect the liquidity or efficiency of trades in addition to real interest rates.

We also do not take into account any effects from the external sector. For that consideration, we would need to extend our model to a small open economy with a commodity-exporting sector. We plan to consider a small open economy model in a separate paper.

We exploit fully fixed prices here (but not Calvo or Rotemberg pricing), which allows us to compare the exogenous policy instruments $(i_1, i_{2,L}, i_{2,H})$ of our fixed-price model with the endogenous variables $(r_1, r_{2,L}, r_{2,H})$ of our flexible-price model. In other words, when a policymaker manipulates $i_1, i_{2,L}, i_{2,H}$, it is effectively setting $r_1, r_{2,L}, r_{2,H}$. Intuitively, both policies have the same effect of increasing the real interest rate because of fixed-price equilibria. With some sluggishness in nominal prices, this would be different. There are likely some interesting trade-offs.

There are also a couple of 'modelling' limitations, such as the quasi-linear utility function ($\log(c_t) - h_t$) and the form of the capital adjustment function ($inv_0(1 + inv_0/k_{-1})$). Some of our results may have been caused by these particular forms. However, as a robustness check, we plan to consider the more general utility function ($c_{t,s}^{1-\sigma}/(1-\sigma) - \omega h_{t,s}^{1+\eta}/(1+\eta)$).

Furthermore, our framework allows us to compare time-consistent and time-inconsistent monetary policies. Is there a time-consistent function for macroprudential or monetary policy? We also plan to conduct this analysis in the future.

Finally, our model is a descriptive one, and we can advise only on the direction of policy and not on the exact rates. Thus, it is important to understand what the choice of one policy instrument implies about the choice of the other, given the large trade-offs? We arrive at some understanding of this in Result 2, but there is still room for improvement in these findings.

7 CONCLUSION

In this paper, we develop a closed economy model that features multiple externalities. In particular, we characterise the optimal monetary and macroprudential policies in an economy with fire sales and financially constrained agents. Our model includes three periods and two generalised types of agents: lenders (households/owners of raw inputs/producers of final goods) and borrowers (firms with linear capital production technology). The flows of real assets in our model are the following. In the first period, households are endowed with an amount of capital and in the same period households decide on the amount of new capital creation (accumulation). The creation of new capital occurs only in the first period. Capital, initially owned by households, is sold to firms. These firms use the capital to produce raw input goods which are, together with labour, used in the production of final goods. These final goods are fully consumed by households. The financial flows are the following. Firms with linear technology are initially financed by households, they take on and pay debt each period and reinvest their net wealth (profits). After the last period, all net wealth is paid back to households.

At time $t = 1$ (the second period), there is a certain probability of a shock. Markets are incomplete and net wealth determines the price of capital goods. At time $t = 1$, borrowers face borrowing constraints – they can borrow only a certain fraction of the new capital they have bought. In the low state (bad realisation of uncertainty), the capital price falls and the constraint binds. A binding constraint generates an additional cost to debt and an additional benefit to capital (collateral premium). Firms with linear technology are forced to take less debt and sell part of their capital (show less demand for capital). Lower demand for capital further reduces its price, which leads to pecuniary externalities or fire sales. This intuition works backwards: the pecuniary externality causes too much lending at time $t = 0$, and not enough at time $t = 1$. Thus, we ex-ante/preventively examine macroprudential and monetary policy to restrict agents from accumulating too much capital.

There are two policies available: a tax on borrowing or an increase in the borrowing interest rate. Both policies raise the real interest rate (and the price of capital) and suppress lending and investment, but, because of pecuniary externalities, may raise capital prices such that the collateral constraint does not bind, leaving room for Pareto improvement. Intuitively, both policies have the same effect of increasing the real interest rate because of fixed-price equilibria. The difference is that changing the cost of debt affects both lenders and borrowers directly, while a tax affects only the borrower at the first-order level.

In our baseline calibration, we find that it is optimal to tighten both ex-ante macroprudential and monetary policy, and then ease monetary policy if a crisis occurs or tighten it again if a crisis does not occur. Less macroprudential tightening is necessary for fixed prices than for flexible prices. These results not only demonstrate the optimal direction of policy instruments in response to financial shocks, but also highlight the importance of price rigidities. Thus, we compare only two extreme cases, which shows a big difference in the determination of the optimal policy. We note that if we compare the optimal rates to their steady-state values, some of these results depend on the values of our parameters.

Our model is therefore too stylised to yield plausible quantitative recommendations (in terms of exact interest rates), but it still gives interesting qualitative results that may motivate a larger quantitative model for practical policy applications. Despite making some simplifying assumptions, our model has a very rich structure, which allows us to draw some important conclusions for economic literature and policymaking.

REFERENCES

Adrian, T., Erceg, C., Lindé, J., Zabczyk, P., & Zhou, J. (2020). A quantitative model for the integrated policy framework.

Alam, Z., Alter, M. A., Eiseman, J., Gelos, M. R., Kang, M. H., Narita, M. M., Nier, E., & Wang, N. (2019). Digging deeper–evidence on the effects of macroprudential policies from a new database.

Auer, R., & Ongena, S. (2019). The countercyclical capital buffer and the composition of bank lending.

Basu, S. S., Boz, E., Gopinath, G., Roch, F., & Unsal, F. (2020). A conceptual model for the integrated policy framework.

Bernanke, B. S., Gertler, M., & Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of macroeconomics*, *1*, 1341–1393.

Bianchi, J., & Mendoza, E. G. (2018). Optimal time-consistent macroprudential policy. *Journal of Political Economy*, *126*(2), 588–634.

Borio, C. (2014). The financial cycle and macroeconomics: What have we learnt? *Journal of Banking & Finance*, *45*, 182–198.

Calvo, G. A., & Reinhart, C. M. (2002). Fear of floating. *The Quarterly journal of economics*, *117*(2), 379–408.

Cerutti, E., Claessens, S., & Laeven, L. (2017). The use and effectiveness of macroprudential policies: New evidence. *Journal of Financial Stability*, *28*, 203–224.

Clarida, R., Gali, J., & Gertler, M. (1999). The science of monetary policy: A new keynesian perspective. *Journal of economic literature*, *37*(4), 1661–1707.

Clerc, L., Derviz, A., Mendicino, C., Moyen, S., Nikolov, K., Stracca, L., Suarez, J., Vardoulakis, A. P., et al. (2015). Capital regulation in a macroeconomic model with three layers of default. *International Journal of Central Banking*, *11*(3), 9–63.

Cozzi, G., Darracq Paries, M., Karadi, P., Körner, J., Kok, C., Mazelis, F., Nikolov, K., Rancoita, E., Van der Ghote, A., & Weber, J. (2020). Macroprudential policy measures: Macroeconomic impact and interaction with monetary policy.

Darracq Pariès, M., Kok Sørensen, C., & Rottner, M. (2020). Reversal interest rate and macroprudential policy.

Dávila, E., & Korinek, A. (2017). Pecuniary Externalities in Economies with Financial Frictions. *The Review of Economic Studies*, *85*(1), 352–395.

Eggertsson, G. B., & Krugman, P. (2012). Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach. *The Quarterly Journal of Economics*, *127*(3), 1469–1513.

Farhi, E., & Werning, I. (2016). A theory of macroprudential policies in the presence of nominal rigidities. *Econometrica*, *84*(5), 1645–1704.

Kaplan, G., Moll, B., & Violante, G. L. (2018). Monetary policy according to hank. *American Economic Review*, *108*(3), 697–743.

Kara, G. I., & Ozsoy, S. M. (2020). Bank regulation under fire sale externalities. *The Review of Financial Studies*, *33*(6), 2554–2584.

Korinek, A., & Simsek, A. (2016). Liquidity trap and excessive leverage. *American Economic Review*, *106*(3), 699–738.

Lagarias, J. C., Reeds, J. A., Wright, M. H., & Wright, P. E. (1998). Convergence properties of the nelder–mead simplex method in low dimensions. *SIAM Journal on optimization*, *9*(1), 112–147.

Lorenzoni, G. (2008). Inefficient credit booms. *The Review of Economic Studies*, *75*(3), 809–833.

Schmitt-Grohé, S., & Uribe, M. (2016). Downward nominal wage rigidity, currency pegs, and involuntary unemployment. *Journal of Political Economy*, *124*(5), 1466–1514.

Schmitt-Grohé, S., & Uribe, M. (2021). Multiple equilibria in open economies with collateral constraints. *The Review of Economic Studies*, *88*(2), 969–1001.

Stavrakeva, V. (2020). Optimal bank regulation and fiscal capacity. *The Review of Economic Studies*, *87*(2), 1034–1089.

Uribe, M., & Schmitt-Grohé, S. (2017). *Open economy macroeconomics*. Princeton University Press.

Van der Ghote, A. (2021). Interactions and coordination between monetary and macroprudential policies. *American Economic Journal: Macroeconomics*, *13*(1), 1–34.

Woodford, M. (2012). Inflation targeting and financial stability.

Woodford, M., & Walsh, C. E. (2005). Interest and prices: Foundations of a theory of monetary policy. *Macroeconomic Dynamics*, *9*(3), 462–468.

A PARAMETERS

Table 1: Parameter values for baseline model

Parameter	Description	Value	Source
β	Discount factor	0.8	Basu et al. (2020)
α	Share of capital	1/3	Commonly used in the literature
ϵ	Price markup	6	Commonly used in the literature
n_0^{linear}	Initial financing	0.53	US Data[a]
k_{-1}	Initial endowment	0.85	US Data[b]
ϕ	Capital adjustment	1	Uribe and Schmitt-Grohé (2017)
ρ_L	Probability of bad shock	0.5	Basu et al. (2020)
κ	Debt limit	0.025	Basu et al. (2020)

[a] A ratio of Nonfinancial Corporate Business; Net Worth to Nonfinancial Corporate Business; Total Assets. Source: FRED, Federal Reserve Bank of St. Louis

[b] A ratio of Households and Nonprofit Organizations; Net Worth to Households and Nonprofit Organizations; Total Assets. Source: FRED, Federal Reserve Bank of St. Louis

B FOCS FOR FIXED-PRICE MODEL

FOR $t = 0$.

$$c_0^{-1} = \beta(1 + r_1)(\rho_L c_{1,L}^{-1} + \rho_H c_{1,H}^{-1}), \tag{21}$$

$$c_0 = w_0, \tag{22}$$

$$\frac{\alpha}{1-\alpha}\frac{k_{-1}}{h_0} = \frac{w_0}{p_{x,0}}, \tag{23}$$

$$c_0 + inv_0(\frac{\phi}{2}\frac{inv_0}{k_{-1}}) = k_{-1}^{\alpha} h_0^{1-\alpha}, \tag{24}$$

$$1 + \phi\frac{inv_0}{k_{-1}} = q_0, \tag{25}$$

$$c_0^{-1} q_0 = \beta[\rho_L c_{1,L}^{-1}(\frac{p_{x,1,L}A_{1,L}}{1 + k_0^{concave}} + q_{1,L}) + \rho_H c_{1,H}^{-1}(\frac{p_{x,1,H}A_{1,H}}{1 + k_0^{concave}} + q_{1,H})], \tag{26}$$

$$q_0 c_0^{-1} = \beta\frac{\rho_L c_{1,L}^{-1}[p_{x,1,L}A_{1,L} + q_{1,L}] + \rho_H c_{1,H}^{-1}[p_{x,1,H}A_{1,H} + q_{1,H}]}{(1 + \theta_1)}, \tag{27}$$

$$inv_0 + k_{-1} = K, \tag{28}$$

$$k_0^{linear} + k_0^{concave} = K, \tag{29}$$

$$\tag{30}$$

9 equations with 9 unknowns: $c_0, h_0, w_0, q_0, k_0^{concave}, k_0^{linear}, K, inv_0, p_{x,0}$.

FOR $t = 1, s = L$.

$$c_{1,L}^{-1} = \beta(1 + i_{2,L})c_{2,L}^{-1}, \tag{31}$$

$$c_{1,L} = w_{1,L}, \tag{32}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,L}}{h_{1,L}} = \frac{w_{1,L}}{p_{x,1,L}}, \tag{33}$$

$$c_{1,L} = x_{1,L}^{\alpha} h_{1,L}^{1-\alpha}, \tag{34}$$

$$c_{1,L}^{-1} q_{1,L} = \beta c_{2,L}^{-1}(\frac{p_{x,2,L}A_{2,L}}{1 + k_{1,L}^{concave}} + q_{2,L}), \tag{35}$$

$$x_{1,L} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{36}$$

$$q_{1,L} k_{1,L}^{linear} = \frac{n_{1,L}^{linear}}{1-\kappa}, \tag{37}$$

$$n_{1,L}^{linear} = p_{x,1,s} A_{1,s} k_0^{linear} + q_{1,s} k_0^{linear} - (1+i_1)(q_0 k_0^{linear} - n_0^{linear}), \tag{38}$$

$$k_{1,L}^{linear} + k_{1,L}^{concave} = K, \tag{39}$$

9 equations with 9 unknowns: $c_{1,L}, h_{1,L}, w_{1,L}, x_{1,L}, p_{x,1,L}, q_{1,L}, n_{1,L}^{linear}, k_{1,L}^{linear}, k_{1,L}^{concave}$.

FOR $t = 1, s = H$.

$$c_{1,H}^{-1} = \beta(1 + i_{2,H}) c_{2,H}^{-1}, \tag{40}$$

$$c_{1,H} = w_{1,H}, \tag{41}$$

$$\frac{\alpha}{1-\alpha} \frac{x_{1,H}}{h_{1,H}} = \frac{w_{1,H}}{p_{x,1,H}}, \tag{42}$$

$$c_{1,H} = x_{1,H}^{\alpha} h_{1,H}^{1-\alpha}, \tag{43}$$

$$c_{1,H}^{-1} q_{1,H} = \beta c_{2,H}^{-1} \left(\frac{p_{x,2,H} A_{2,H}}{1 + k_{1,H}^{concave}} + q_{2,H} \right), \tag{44}$$

$$x_{1,H} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{45}$$

$$q_{1,H} = \frac{p_{x,2,H} A_{2,H} + q_{2,H}}{(1 + i_{2,H})}, \tag{46}$$

$$k_{1,H}^{linear} + k_{1,H}^{concave} = K \tag{47}$$

8 equations with 8 unknowns: $c_{1,H}, x_{1,H}, w_{1,H}, h_{1,H}, q_{1,H}, p_{x,1,L}, k_{1,H}^{linear}, k_{1,H}^{concave}$.

FOR $t = 2, s$. For $s = H$ allocations are the same as at time $t \geq 3$.

$$c_{2,s}^{-1} = \beta(1 + r_{3,s}) c^{-1}, \tag{48}$$

$$c_{2,s} = w_{2,s}, \tag{49}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{2,s}}{h_{2,s}} = \frac{w_{2,s}}{p_{x,2,s}}, \tag{50}$$

$$c_{2,s} = x_{2,s}^{\alpha} h_{2,s}^{1-\alpha}, \tag{51}$$

$$1 = \frac{\epsilon}{\epsilon-1}\left(\frac{w_{2,s}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,2,s}}{1-\alpha}\right)^{1-\alpha}, \tag{52}$$

$$x_{2,s} = \log(1+k_{1,s}^{concave}) + k_{1,s}^{linear}, \tag{53}$$

$$q_{2,s} = \frac{p_x A + q}{(1+r_{3,s})}, \tag{54}$$

$$\tag{55}$$

7 equations with 7 unknowns: $c_{2,s}, w_{2,s}, h_{2,s}, x_{2,s}, q_{2,s}, p_{x,2,s}, r_{3,s}$.

FOR $t \geq 3$. All variables are at their steady-state values. Thus, we remove the time indices. New prices are fully flexible, thus, the interest rate becomes the endogenous r.

$$1 = \frac{\epsilon}{\epsilon-1}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{p_x}{1-\alpha}\right)^{1-\alpha}, \tag{56}$$

$$q = \frac{\beta}{1-\beta}p_x, \tag{57}$$

$$r = \frac{1}{\beta} - 1, \tag{58}$$

$$c = x^{\alpha}h^{1-\alpha}, \tag{59}$$

$$x = K, \tag{60}$$

$$c = w, \tag{61}$$

$$\frac{\alpha}{1-\alpha}\frac{x}{h} = \frac{w}{p_x}, \tag{62}$$

7 equations with 7 unknowns: c, w, h, x, p_x, r, q.

C FOCS FOR FLEXIBLE-PRICE MODEL

FOR $t = 0$.

$$c_0^{-1} = \beta(1 + r_1)(\rho_L c_{1,L}^{-1} + \rho_H c_{1,H}^{-1}), \tag{63}$$

$$c_0 = w_0, \tag{64}$$

$$\frac{\alpha}{1-\alpha}\frac{k_{-1}}{h_0} = \frac{w_0}{p_{x,0}}, \tag{65}$$

$$c_0 + inv_0(\frac{\phi}{2}\frac{inv_0}{k_{-1}}) = k_{-1}^{\alpha} h_0^{1-\alpha}, \tag{66}$$

$$1 + \phi\frac{inv_0}{k_{-1}} = q_0, \tag{67}$$

$$c_0^{-1} q_0 = \beta[\rho_L c_{1,L}^{-1}(\frac{p_{x,1,L}A_{1,L}}{1 + k_0^{concave}} + q_{1,L}) + \rho_H c_{1,H}^{-1}(\frac{p_{x,1,H}A_{1,H}}{1 + k_0^{concave}} + q_{1,H})], \tag{68}$$

$$q_0 c_0^{-1} = \beta\frac{\rho_L c_{1,L}^{-1}[p_{x,1,L}A_{1,L} + q_{1,L}] + \rho_H c_{1,H}^{-1}[p_{x,1,H}A_{1,H} + q_{1,H}]}{(1 + \theta_1)}, \tag{69}$$

$$inv_0 + k_{-1} = K, \tag{70}$$

$$k_0^{linear} + k_0^{concave} = K, \tag{71}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_0}{\alpha}\right)^{\alpha}\left(\frac{p_{x,0}}{1-\alpha}\right)^{1-\alpha}, \tag{72}$$

10 equations with 10 unknowns: $c_0, h_0, w_0, q_0, k_0^{concave}, k_0^{linear}, K, inv_0, p_{x,0}, r_1$.

FOR $t = 1, s = L$.

$$c_{1,L}^{-1} = \beta(1 + r_{2,L})c_{2,L}^{-1}, \tag{73}$$

$$c_{1,L} = w_{1,L}, \tag{74}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,L}}{h_{1,L}} = \frac{w_{1,L}}{p_{x,1,L}}, \tag{75}$$

$$c_{1,L} = x_{1,L}^{\alpha} h_{1,L}^{1-\alpha}, \tag{76}$$

$$c_{1,L}^{-1} q_{1,L} = \beta c_{2,L}^{-1} \left(\frac{p_{x,2,L} A_{2,L}}{1 + k_{1,L}^{concave}} + q_{2,L} \right), \tag{77}$$

$$x_{1,L} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{78}$$

$$q_{1,L} k_{1,L}^{linear} = \frac{n_{1,L}^{linear}}{1 - \kappa}, \tag{79}$$

$$n_{1,L}^{linear} = p_{x,1,s} A_{1,s} k_0^{linear} + q_{1,s} k_0^{linear} - (1 + r_1)(q_0 k_0^{linear} - n_0^{linear}), \tag{80}$$

$$k_{1,L}^{linear} + k_{1,L}^{concave} = K, \tag{81}$$

$$1 = \frac{\epsilon}{\epsilon - 1} \left(\frac{w_{1,L}}{\alpha} \right)^{\alpha} \left(\frac{p_{x,1,L}}{1 - \alpha} \right)^{1-\alpha}, \tag{82}$$

10 equations with 10 unknowns: $c_{1,L}, h_{1,L}, w_{1,L}, x_{1,L}, p_{x,1,L}, q_{1,L}, n_{1,L}^{linear}, k_{1,L}^{linear}, k_{1,L}^{concave}, r_{2,L}$.

FOR $t = 1, s = H$.

$$c_{1,H}^{-1} = \beta (1 + i_{2,H}) c_{2,H}^{-1}, \tag{83}$$

$$c_{1,H} = w_{1,H}, \tag{84}$$

$$\frac{\alpha}{1 - \alpha} \frac{x_{1,H}}{h_{1,H}} = \frac{w_{1,H}}{p_{x,1,H}}, \tag{85}$$

$$c_{1,H} = x_{1,H}^{\alpha} h_{1,H}^{1-\alpha}, \tag{86}$$

$$c_{1,H}^{-1} q_{1,H} = \beta c_{2,H}^{-1} \left(\frac{p_{x,2,H} A_{2,H}}{1 + k_{1,H}^{concave}} + q_{2,H} \right), \tag{87}$$

$$x_{1,H} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{88}$$

$$q_{1,H} = \frac{p_{x,2,H} A_{2,H} + q_{2,H}}{(1 + i_{2,H})}, \tag{89}$$

$$k_{1,H}^{linear} + k_{1,H}^{concave} = K, \tag{90}$$

$$1 = \frac{\epsilon}{\epsilon - 1} \left(\frac{w_{1,H}}{\alpha} \right)^{\alpha} \left(\frac{p_{x,1,H}}{1 - \alpha} \right)^{1-\alpha}, \tag{91}$$

9 equations with 9 unknowns: $c_{1,H}, x_{1,H}, w_{1,H}, h_{1,H}, q_{1,H}, p_{x,1,L}, k_{1,H}^{linear}, k_{1,H}^{concave}, r_{2,H}$.

FOR $t = 2, s$. For $s = H$ allocations are the same as at time $t \geq 3$.

$$c_{2,s}^{-1} = \beta(1 + r_{3,s})c^{-1}, \tag{92}$$

$$c_{2,s} = w_{2,s}, \tag{93}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{2,s}}{h_{2,s}} = \frac{w_{2,s}}{p_{x,2,s}}, \tag{94}$$

$$c_{2,s} = x_{2,s}^{\alpha} h_{2,s}^{1-\alpha}, \tag{95}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_{2,s}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,2,s}}{1-\alpha}\right)^{1-\alpha}, \tag{96}$$

$$x_{2,s} = \log(1 + k_{1,s}^{concave}) + k_{1,s}^{linear}, \tag{97}$$

$$q_{2,s} = \frac{p_x A + q}{(1 + r_{3,s})}, \tag{98}$$

7 equations with 7 unknowns: $c_{2,s}, w_{2,s}, h_{2,s}, x_{2,s}, q_{2,s}, p_{x,2,s}, r_{3,s}$.

FOR $t \geq 3$. All variables are at their steady-state values. Thus, we remove the time indices.

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{p_x}{1-\alpha}\right)^{1-\alpha}, \tag{99}$$

$$q = \frac{\beta}{1-\beta} p_x, \tag{100}$$

$$r = \frac{1}{\beta} - 1, \tag{101}$$

$$c = x^{\alpha} h^{1-\alpha}, \tag{102}$$

$$x = K, \tag{103}$$

$$c = w, \tag{104}$$

$$\frac{\alpha}{1-\alpha}\frac{x}{h} = \frac{w}{p_x}, \tag{105}$$

7 equations with 7 unknowns: c, w, h, x, p_x, r, q.

D ALGORITHM TO SOLVE MODEL

We solve the model using Matlab software (version 2020a). To solve the model, we first built a system of non-linear equations from (22) to (62) from Appendix B. And a separate system of non-linear equations from (64) to (105) from Appendix C. To both of these systems, we apply the non-linear solver to find solutions for all endogenous variables with given exogenous variables in the following way:

1. Non-linear solver function:
   (a) As input, it takes a vector of the values of policy parameters θ_1, i_1, $i_{2,L}$ and $i_{2,H}$, and a vector of the values of parameters β, α, ϵ, n_0^{linear}, k_{-1}, ϕ, κ and ρ_L.
   (b) (b) It solves the steady-state system of equations and uses the steady-state values as initial values for the non-linear solver.
   (c) Firstly, we assume that financial constraint (7) does not bind and solve the unconstrained version of the model. In other words, we assume that when uncertainty resolves, there is no difference between the low and high states because the financial constraint does not hold.
   (d) Then we check two conditions: whether there is a solution at all, and if there is, then we check if d_1^{linear} is positive. If $d_1^{linear} > 0$ then firms with linear technology are borrowers (which we assume in our model). For some parameters values, these firms may become lenders with $d_1^{linear} < 0$. We do not consider these cases.
   (e) If both previous conditions are verified, we compare $d_{2,L}^{linear}$ and $\kappa q_{1,L} k_{1,L}^{linear}$ (we check if the financial constrain binds).
   (f) If $d_{2,L}^{linear}$ is less than $\kappa q_{1,L} k_{1,L}^{linear}$, then we find a solution, since in this case the amount of borrowing made by firms with linear technology is less than the

possible debt limit, and there is no need to solve the model with a binding financial constraint.

(g) If $d_{2,L}^{linear}$ is greater than $\kappa q_{1,L} k_{1,L}^{linear}$, this means that the amount of borrowing conducted by firms with linear technology is greater than the possible debt limit. This indicates that the financial constraint should bind, so we resolve the model with a binding financial constraint.

(h) (h) As output, this function gives the agents' utility value (1)a vector of the endogenous variables, and an indicator which takes a value of 0 for a given vector of parameters if the financial constraint does not bind, and takes a value of 1 if the financial constraint binds.

The next step is to find the optimal exogenous policy variables, namely $\theta_1, i_1, i_{2,L}, i_{2,H}$. To do so, we minimise the non-linear solver function by varying the policy variables to find the combination which minimise the utility function:

2. Minimize function

(a) As an input, it takes the initial values of policy variables: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$, and a vector of the values of parameters β, α, ϵ, n_0^{linear}, k_{-1}, ϕ, κ and ρ_L.

(b) It takes a negative utility value (from the non-linear solver) and applies a minimise function by varying a vector of policy variables θ_1, i_1, $i_{2,L}$ and $i_{2,H}$H and taking values of parameters β, α, ϵ, n_0^{linear}, k_{-1}, ϕ, κ and ρ_L as given.

(c) As output, it produces the vector of policy variables θ_1, i_1, $i_{2,L}$ and $i_{2,H}$ which gives the minimum value of negative utility. Minimising negative utility is the same as maximizing utility.