Preventive monetary and macroprudential policy response to anticipated shocks to financial stability*

Konstantin Styrin † Alexander Tishin ‡

May 28, 2021

Abstract

In this paper, we develop a simple framework to study optimal macroprudential and monetary policy interactions as a response to a financial shock. Our model combines together nominal rigidities and capital accumulation - features that usually studied separately in the previous literature. In our model, we show that the agents do not internalize how their assets purchases affect assets prices. Thus, when a crisis occurs, there are fire sales – less demand for capital further reduces their price and agents are worse off. Policy intervention (both monetary and macroprudential) can improve allocations by restricting borrowing ex-ante (during the accumulation of risks and imbalances) and stimulating the economy ex-post (during the implementation of risks). As a result, we find a complementarity relation between ex-ante monetary policy and preventive macroprudential policy. We also compare this result with a flexible prices model and conduct several sensitivity analysis exercises.

Keywords: Macroprudential policy, monetary policy, pecuniary externalities, nominal rigidities, financial frictions, capital accumulation.

JEL Codes: E44, E58, G28, D62

---

*Preliminary and incomplete. Please do not circulate. The authors are grateful to Andrey Sinyakov and the broader Bank of Russia colleagues for their comments, in particular to Udara Peiris for insightful discussion, participants of the XXII April International Academic Conferences at the Higher School of Economics in Moscow. All remaining errors are the authors' responsibility. The views expressed in this paper are solely those of the authors and do not necessarily reflect the official position of the Bank of Russia. The Bank of Russia assumes no responsibility for the contents of the paper.

†Bank of Russia, StyrinKA@mail.cbr.ru

‡Bank of Russia, TishinAV@mail.cbr.ru; corresponding author

1 INTRODUCTION

Monetary policy is widely recognized as a central bank's main tool to manage inflation. Indeed, before the financial crises (e.g. Financial crisis of 2007–2008) interest rate was the primary instrument to achieve central banks' objectives.

However, after the Financial Crisis 2008 and the recent COVID-2019 crisis the role of macroprudential policies has increased dramatically because Financial crises have shown that financial stability is still an important cause for concern for Central banks. The initial economic shock arising from the US housing market raised the prospect of a higher probability of default, but most importantly led to a vicious spiral where default concern led to a fall in asset prices. This fall in asset prices then reinforced concerns about banks and other financial intermediaries' solvency, and this further reduced liquidity in a range of asset markets, with a variety of self-amplifying spirals then bringing the whole financial system to its knees.

Lack of liquidity dries up key financial markets, thus preventing institutions from restructuring their portfolios, adapting their strategies, and steering away from potential dangers caused by exogenous economic shocks. In turn, defaults start accumulating, and the domino effect leads to further reductions in liquidity and ultimately causes financial institutions, corporations, and other non-financial bodies to fail to meet their contractual obligations.

Under the presence of multiple externalities and the evolving nature of markets, in particular, for short-term funding, the forthcoming regulatory architecture should recognise that there are markets that are "too important to fail" and not only banks that are "too big to fail". Hence, regulation, and policy more generally, should also be focused on "systemic markets" as well as "systemic institutions".

The ability to adjust monetary policy appropriately to economic conditions requires the measurement of inflation and output growth. Likewise, maintenance of financial stability requires equivalent measures of financial fragility. Such measures could be used as a yardstick to assess the success of the regulatory policy.

As a result, these crises have actively promoted thinking not only about price stability but also about financial stability (Cerutti, Claessens, and Laeven 2017). For instance, the average number of macroprudential instruments[1] almost doubled and these instruments have been tightened more after GFC. Therefore, most central banks actually have been targeting two aims: price and financial stability using two sets of instruments: monetary and macroprudential measures.

Thus, many countries face a trade-off between price and financial stability, namely, a question of either these two policies are substitutes or complements. Hence, to target both aims it is important to understand the optimal interaction between monetary and macroprudential policies. The purpose of this paper is to provide a useful framework for policymakers regarding of joint use of monetary and macroprudential policies.

In this paper we study a three-period model with nominal rigidities and financial frictions. We refer to the first and the second periods in the model as short-run and the third period is a metaphor for a long-run period. In the first period agents are free to choose their allocations (consumption and borrowing decisions), then in the second period a crisis can come with a certain probability. In other words, financial/borrowing constraint (which limits the ability to borrow up to a fraction of new capital bought), we refer to a crisis period when this constraint binds.

To reduce the severity of the crisis policymaker can use ex-ante macroprudential policy to limit borrowing and ex-post monetary policy to stimulate an economy during a recession. In this framework we analyze the optimal coordination between preventive monetary and macroprudential policies seeking to reduce the impact of anticipated financial shocks.

To do it, we assume the following environment with a continuum of agents, namely, households and firms with linear technology. Households invest part of the final good to create new capital in the first period. In subsequent periods, households consume all the final good. Households fund these firms and receive profits from them. Besides, households own their inferior concave technology. These concave or linear technology

[1] Caps on loan-to-value and debt-to-income ratios, limits on credit growth and other balance sheet restrictions, (countercyclical) capital and reserve requirements and surcharges, and Pigouvian levies.

is used for the production of raw good using capital as inputs. Linear technology firms are more effective users of capital but they can be financially constrained which is the source of pecuniary externality. This externality influences the equilibrium prices because the firms do not internalize the effects of their private decisions on aggregate prices. Thus, in "normal" times firms borrow too much which leads to inefficient fire-sales during recessions. This is the source of the first inefficiency in our model which motivates using preventive macroprudential policy. Households with concave technology are inefficient users of capital, thus, in an economy without any frictions households are not willing to hold any capital while firms with linear technology are willing to hold all capital.

Finally, the raw good, which is produced from the capital by any of these two technologies, combines with the labor and is used in the production of a final good. In our baseline version we assume that the prices of the final good are fixed in the first and the second period and fully flexible in the third. The price rigidity leads to an aggregate demand externality. In usual circumstances agents do not internalise how their private decisions affect aggregate demand which determines the price level in an economy which in the case of rigid prices influences consumption choice.

To put it differently, overborrowing and rigid prices increase the severity of the crisis leading to inefficient allocations. It creates a rationale for using both monetary policy and macroprudential policy to target both aims – price and financial stability.

That is, the policymaker's tool are a macroprudential tax (Pigouvian taxes) (which influence the cost of debt of firms with linear technology in the pre-crisis period, thus it reduces the size of fire-sales), and ex-ante state non-contingent monetary policy nominal rate in the pre-crisis period and ex-post state-contingent nominal rates in post-crisis periods. These rates work as usual monetary policy tools and deal with nominal rigidities in the model.

Finally, we show that Social Planner who internalize the externalities can improve allocation and reducing the severity of the crisis. To solve the Social Planner problem we use numerical methods because our model does not have a closed-form solution. Moreover, we also exploit the fact that we solve our model without any log-linearization techniques, thus it allows us to abstract from any quadratic form policy objective. Therefore, Social Planner can simply maximize the weighed utility of agents. As a result of solving this problem is the four optimal parameters (one macroprudential tax and three monetary policy rates) which deliver the maximum utility of agents.

The resulting framework has a rich structure, allowing us to find an answer to our main research question, namely, to characterize optimal coordination between monetary and macroprudential policies with inefficient pecuniary and aggregate demand externalities in economies with financial frictions. In other words, in this paper we what to show how monetary and macroprudential policies interact. And we ask, will these policies behave like substitutes or compliments?

From the perspectives of economic literature, the focus of many previous papers was on looking at either optimal monetary policy or analysing macroprudential policy. Optimal monetary policy design is well studied in economic literature (Woodford 2003; Clarida, Gali, and Gertler 1999). Moreover, recent adventures on optimal monetary policy are focused on the analysis of heterogeneous agents models (Kaplan, Moll, and Violante 2018) in which the agents are heterogeneous in the amounts of liquid and illiquid assets.

Also, the question of whether macroprudential policy can be a substitute or complement to monetary policy arises when monetary policy is limited or restricted in certain sense. These inefficiencies of monetary policy can take different forms. For instance, when a monetary policy is bounded below (such as in Zero Lower Bound, ZLB, Eggertsson and Krugman 2012) expansionary fiscal policy becomes effective. Fixed exchange rate and "fear of floating" (Calvo and Reinhart 2002; Dávila and Korinek 2017) are other examples when monetary policy becomes inapplicable. Finally, when monetary tightening is used for financial stability purposes ("leaning against the wind" (Borio 2014; Woodford 2012)) in fact leads to a weaker economy with higher unemployment and lower inflation but with a lower probability of a crisis. While if a crisis occurs, the severity increases.

Many papers now are devoted to justify the macroprudential policy and estimate its effects on the economy. From the theoretical side the closest papers to our paper study optimal time-consistent monetary policy in an economy with pecuniary externality (Bianchi and Mendoza 2018). They show that usual taxes on debt are ineffective while they construct time-consistent policy rule which delivers the maximum utility. Stavrakeva 2020 in a similar model with pecuniary externality studies the optimal capital requirements and finds that countries with larger fiscal capacity should implement lower bank capital requirements. The interaction between capital and liquidity regulations was studied in Kara and Ozsoy 2016, where the authors show that more strict capital requirements reduce fire-sales but the banks are responding to this tightening by decreasing liquidity. Macroprudential policy on liquidity requirement can improve these allocations. Moreover, capital regulation is also studied in economies which is subject to default (Clerc et al. 2015). From an empirical perspective, the authors (Auer and Ongena 2019) study the introduction of the countercyclical capital buffer (CCyB) in Switzerland. They find that such macroprudential policy leads to higher growth in commercial lending, higher interest rates and higher fees charged to the firms. The importance of loan-to-value (LTV) limits is studies by Alam et al. 2019, the authors use a huge panel of 134 countries from January 1990 to December 2016 and find nonlinear effects on household credit of LTV changes, meaning that the effect of LTV tightening is diminishing with the size of LTV.

Besides, there is a growing strand in the literature studying the combinations of monetary and macroprudential policies. The most influential paper for us written by Basu et al. 2020. We are also familiar with Adrian et al. 2020 but their paper focuses on quantitative results while we go in line with Basu et al. 2020 and choose a quality or descriptive approach and develop a model with a rich structure that gives us a qualitative answer, not a quantitative one. However, we differ from Basu et al. 2020 because they fix the amount of capital in their model (land in their terminology), while we allow agents to endogenously accumulate capital. It gives us room to study the transmission of macroprudential policy using two channels. The first one is the same as in Basu et al. 2020, the preventive macroprudential policy restricts the amount of borrowing in the pre-crisis period. Thus, with positive macroprudential taxes firms with linear technology own only a part of available capital in the economy and the rest is owned by households (who have inferior technology). The second channel is a new one. With endogenous capital accumulation, positive macroprudential taxes also limit the amount of this accumulation. In other words, macroprudential taxes constrain the creation of new capital. Therefore, macroprudential policy works in two directions to prevent excessive fire-sales.

Also, there are studies where authors examine both monetary and macroprudential regulation together. For instance, Cozzi et al. 2020 study the interactions between monetary and macroprudential policies. Van der Ghote 2021 study monetary and macro-prudential policy intervention in a general equilibrium economy with recurrent boom-bust cycles. The author shows that there are welfare gains from policy coordination, in particular, it is optimal to intervene with macroprudential measures during the boom phase which also has effects during busts. Moreover, contractionary monetary policy during booms also helps to make bust less severe.

In line with these papers, we also study the interaction between monetary and macroprudential policies. We build our model based on ideas of pecuniary externality (Dávila and Korinek 2017) and aggregate demand externality (Farhi and Werning 2016). We differ from these papers in two ways: we study capital accumulation in a framework with sticky prices whereas in the previous papers there are either no capital accumulation but there are sticky prices or there is capital accumulation without sticky prices. We are similar to Dávila and Korinek 2017 because we consider similar economies with pecuniary externalities and financial frictions. Hence Dávila and Korinek 2017 allow endogenous capital accumulation, we still differ from their work because we consider our economy with nominal rigidities, while in (Dávila and Korinek 2017) there are no nominal rigidities. The second complementary strand of literature focuses on economies with nominal rigidities (as in (Farhi and Werning 2016; Schmitt-Grohé and Uribe 2016; Korinek and Simsek 2016)). The main driving force of inefficiencies in these models is aggregate demand externality. Similar to them we add nominal rigidities in the form of sticky prices. However, these models usually assume a fixed amount of capital while we allow endogenous capital accumulation.

Many previous cited papers use the infinite-horizon model, thus, to solve such models, authors use different approximation/log-linearization techniques. We contributed to it by using a three-period framework which allows us to solve the model straightforward without any approximation and log-linearization techniques. It means we find the most accurate solution for our model and do not lose any information because of different non-linearities. Actually, we solve a fully non-linear model.

Finally, for the case of policy implications we are not restricted by a three-period framework. For instance, we can extend the number of periods when the financial constraint can bind. We also can add an ex-post macroprudential policy to study if there is room to improve allocations if a crisis has already occurred. So, there are a lot of ways to extend this framework at a small price, we leave it to further research.

The rest of this paper proceeds as follows. In Section 2 we describe the baseline model. In Section 3 we present our main results. In Section 4 we provide sensitivity analysis (show how the results differ when we change parameters values). In section 5 we discuss policy implications. In Section 6 we state our remarks and limitations of the current model. Finally, in Section 7 we conclude.

2 MODEL

In this section, we construct a closed economy model in a New Keynesian way in the spirit of Dávila and Korinek 2017; Farhi and Werning 2016. The model consists of households, final goods firms and capital sector firms. Effectively, we have a simple three-period framework but additionally, assume that after the last period all variables stay at their steady states. Thus, we write optimisation problems of agents as three-period problems. In this section, we describe a baseline version of our model. In Appendix B one can find FOCs for a fixed prices model and in Appendix C for a flexible prices.

CONSUMERS Households maximize a utility function over consumption and linear disutility of labor.

$$\max_{c_t,h_t,k_t^{concave},b_t,K,inv_0} \mathbb{E}_0 \sum_{t=0}^{2} \beta^t \left[\log c_{t,s} - h_{t,s}\right], \tag{1}$$

Their maximization is subject to a budget constraint:

$$c_0 + b_1 + inv_0(1 + \frac{\phi}{2}\frac{inv_0}{k_{-1}}) + q_0(k_0^{concave} - inv_0 - k_{-1}) + n_0^{linear}$$
$$= w_0 h_0 + p_{x,0}k_{-1} + \Pi_0^{final}, \tag{2}$$
$$c_{1,s} + b_{2,s} + q_{1,s}(k_{1,s}^{concave} - k_0^{concave})$$
$$= w_{1,s}h_{1,s} + (1+i_1)b_1 + p_{x,1,s}A_{1,s}\log(1+k_0^{concave}) + \Pi_{1,s}^{final}, \tag{3}$$
$$c_{2,s} + b_3 + q_{2,s}(k_2^{concave} - k_{1,s}^{concave})$$
$$= w_{2,s}h_{2,s} + (1+i_{2,s})b_{2,s} + p_{x,2,s}A_{2,s}\log(1+k_{1,s}^{concave}) + n_2^{linear} + \Pi_{2,s}^{final}, \tag{4}$$

where c_t is the consumption, h_t is labor supply of households, inv_0 is the investments, made in $t = 0$, k_{-1} is the initial endowment of capital, q_t is the price of capital in t, $k_t^{concave}$ is the amount of capital that is in the hands of households (i.e. the amount of capital that is used for production with concave technology), n_0^{linear} is the initial financing from households to firms with linear technology, w_t is the wage, $p_{x,t}$ is the relative price of raw inputs (intermediate good that is used in the production of the final good), b_t is the loans to firms with linear technology, Π_t^{final} is the profits of final good producer, n_2^{linear} is a net worth of firms with linear technology given back to households (like dividends/profits). Moreover, in optimisation problem of households there are also $i_{t,s}$ is the nominal interest rate, which is governed by the policymaker.

Note, that subindex s indicates state-contingent variables and takes values $s = L$ if the state is low (bad, when financial constraint binds) and $s = H$ if the state is high (good, when financial constraint does not bind). The variables without subindex s denote that these variables are non-contingent, thus their values are independent of a state of the nature. In other words, we assume that markets are incomplete before a crisis.

LINEAR FIRMS Linear sector firms are perfectly competitive and initially financed by households with an amount of n_0^{linear}. Linear firms are unconstrained at $t = 0$ and $t = 2$ but might by financially constrained with positive probability at $t = 1$. Each period linear firms can raise the amount of capital at the market price up to the amount of their net worth (n_t^{linear}) and required amount of borrowing (d_t) according to equation (6). In turn, net worth is the amount of the produced good (at the market price) plus the market price of owned capital net of debt repayments with interest ($(1 + i_t)d_t$).

$$\max_{\{n_t^{linear}, d_t, k_t^{linear}\}_{t=1}^{2}} \beta^2 \frac{\lambda_2}{\lambda_0} n_2^{linear} \tag{5}$$

subject to the following constraints:

$$q_{t,s} k_{t,s}^{linear} = n_{t,s}^{linear} + d_{t+1,s}^{linear} \forall t \geq 0 \tag{6}$$

$$d_{2,L}^{linear} \leq \kappa q_{1,L} k_{1,L}^{linear} \tag{7}$$

$$n_{1,s}^{linear} = p_{x,1,s} A_{1,s} k_0^{linear} + q_{1,s} k_0^{linear} - (1 + i_1) d_1^{linear} (1 + \theta_1) + t_1 \tag{8}$$

$$n_{2,s}^{linear} = p_{x,2,s} A_{2,s} k_{1,s}^{linear} + q_{2,s} k_{1,s}^{linear} - (1 + i_{2,s}) d_{2,s}^{linear} \tag{9}$$

$$t_1 = -\theta_1 (1 + i_1) d_1^{linear}, \tag{10}$$

where (6) represents balance constraint for each period, (7) is financial constraint which binds in $t = 1$ with a positive probability, the next equations for n_t^{linear} represent the evolution of net worth of linear firms.

FINAL GOODS Here we will follow the New Keynesian tradition and assume a monopolistic competition between firms. Firms use the raw inputs (x_t) and labor (h_t) to produce a final good (y_t). The inputs are produced using either linear or concave technology for a relative price $p_{x,t}$, labor is supplied by households for a wage w_t. Then:

$$y_t(j) = A_t^H h_t(j)^\alpha k_t(j)^{1-\alpha} \tag{11}$$

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{12}$$

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{-\epsilon} y_t \tag{13}$$

For now in $t = 0; 1$ we assume fully fixed prices, we take this idea from Lorenzoni 2008. It means that these prices are inherited and determined in some previous periods $t = -1$. However, starting from $t = 2$ we assume fully flexible prices. Therefore, for flexible prices we can write:

$$1 = \frac{\epsilon}{\epsilon - 1} \left(\frac{w_t}{\alpha} \right)^\alpha \left(\frac{p_{t,s}}{1-\alpha} \right)^{1-\alpha} \tag{14}$$

We conclude the description of the model with market clearing conditions.

MARKET CLEARING

$$c_0 + inv_0(1 + \frac{\phi}{2}\frac{inv_0}{k_{-1}}) = k_{-1}^{\alpha} h_0^{1-\alpha} \tag{15}$$

$$c_{t,s} = x_{t,s}^{\alpha} h_{t,s}^{1-\alpha} \forall t \geq 1, \tag{16}$$

$$x_{t,s} = \log(1 + k_{t,s}^{concave}) + k_{t,s}^{linear}, \tag{17}$$

$$k_{t,s}^{linear} + k_{t,s}^{concave} = K, \tag{18}$$

$$K = k_{-1} + inv_0, \tag{19}$$

$$b_{t,s} = d_{t,s}, \tag{20}$$

where equation (15) shows resource constraint for the $t = 0$, which means that agents spend all output in the economy on their consumption and producing new capital inv_0. Equation (16) shows resource constraint for the rest periods, when agents spend all output in the economy on their consumption. Equation (17) indicates the total amount of raw inputs x_t, which equals to the sum of capital-intensive output using two technologies: concave and linear. Equation (18) determines that total amount of capital is divided between firms with different technologies. Equation (19) specifies that total amount of capital in the economy, which sum of initial endowment k_{-1} and new produced capital inv_0. Finally, equation (20) establishes that consumers fully finance linear sector firms.

EQUILIBRIUM The equilibrium is the following. A real allocation $\{c_{t,s}, x_{t,s}, h_{t,s}, y_{t,s}\}_{t=0}^{2}$, $\{inv_0\}$, $\{k_{t,s}^{linear}, k_{t,s}^{concave}\}_{t=0}^{1}$, $\{d_{t,s}, b_{t,s}\}$ an asset allocation with a set of prices $\{w_{t,s}, q_{t,s}, p_{x,t,s}\}_{t=0}^{2}$, when agents solve their optimization problem (1) and (5) and all markets are clear, i.e. equations (15), (16), (17), (18), (19), (20) hold for all dates and states.

SOCIAL PLANNER To find optimal values we introduce a Social Planner which internalizes externalities. The optimization problem of Social Planner of the following. Social Planner takes the utility of all agents in the economy with corresponding Pareto weights and resource and financial constraints in the economy. In our case, the Social

Planner problem becomes similar to Ramsey problem of finding optimal values of “taxes”.

We need to note that the problem cannot be solved in the closed-form, thus we use numerical methods to solve it. The full algorithm is described in appendix D. The main idea is that we use non-linear solve to find the solution for a fix set of policy parameters θ_1, i_1, $i_{2,L}$ and $i_{2,H}$. This non-linear solver gives us the optimal allocations. Then, we apply minimize function to this solver to find the set of policy parameters that delivers a minimum of the negative value of utility. This “minimizer” uses the simplex search method of Lagarias et al. 1998, implemented in Matlab. The minimize function searches along a set of policy parameters θ_1, i_1, $i_{2,L}$ and $i_{2,H}$ to determine the minimum. Thus, this set is our solution that gives the maximum value of utility (minimum of negative utility), i.e. the values of policy parameters.

3 RESULTS

In this section, we present our main results. First of all, we need to establish a version with which we will compare our results. A natural choice is to take a similar model without any price rigidities (you can find this model in Appendix C) (we refer to this model as flexible prices version). The main difference with the model in the previous section is that with fully flexible prices we do not need any monetary policy and the real rates determine endogenously in the model. Therefore, in our baseline model with fixed prices, the policymaker manages the nominal rates but, in fact, it sets the real rates. Hence, we take the equilibrium real interest rates from the model with flexible prices as an indicator of neutral monetary policy. To show numerical results we use parameters which are indicated in the table in appendix A.

Result 1 (Complementarity in static) *In response to anticipated financial shocks, it is optimal to tighten both monetary and macroprudential policy. This Result is illustrated in figure 1.*

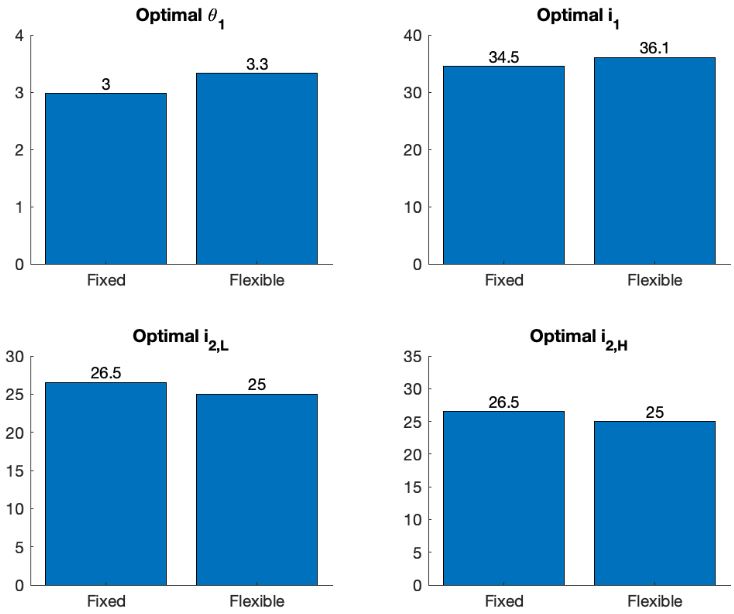

Figure 1: Complementarity in static:
Optimal policy
fixed prices vs flexible prices

To present our main result we compare optimal policies in the economy with fixed and flexible prices. Our finding is presented in Result 1 and in figure 1. Since in the model with fixed prices, there is no inflation, thus, nominal interest rates equal to real, we compare allocations of real interest rates from a model with flexible prices (in which these real interest rates are endogenous) to our optimal policies in the model with fixed prices (in which these rates are exogenous). In other words, in the model with flexible prices, no need to have the monetary policy, thus the only policy parameter is θ_1, while in the model with fixed prices we have our usual 4 policy parameters.

We see that in both versions optimal θ_1 is positive and in a flexible prices version, it is significantly higher. Optimal i_1 is almost the same in both versions but for fixed prices, it is slightly higher. Optimal $i_{2,L}$ are almost the same. While optimal $i_{2,H}$ for flexible prices takes the steady-state value $(\frac{1}{\beta} - 1)$ but for fixed prices it is significantly higher.

Note that the values look not realistic, this is because of current calibration and simplifying assumptions. Due to the fact, that the model is highly stylized, it is

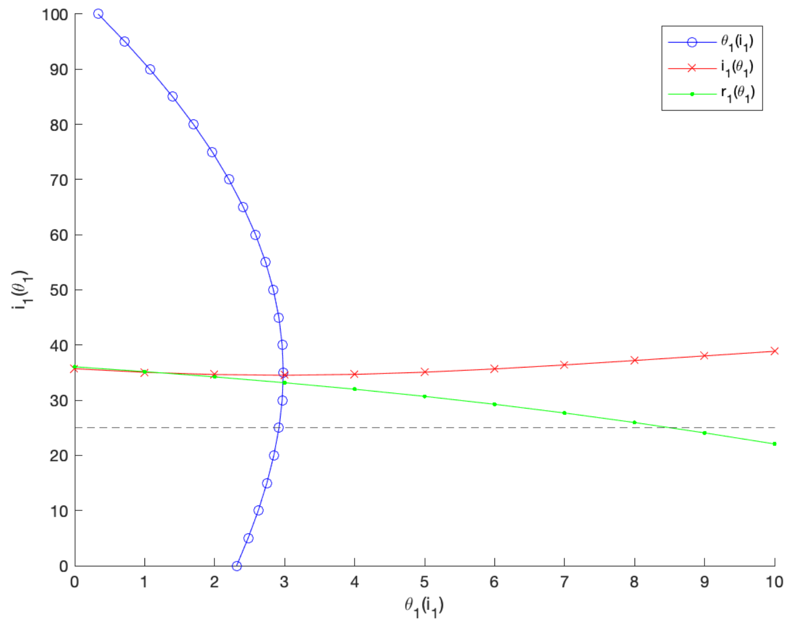

Figure 2: Semi-complementarity in dynamic

impossible to choose such parameters to match the realistic values of interest rates. Therefore, we consider our model as qualitative, not quantitative.

Result 2 (Semi-complementarity in dynamic) *In response to higher ex-ante monetary policy, optimal macroprudential policy shows an inverse U-shape curve, while in response to higher macroprudential policy, optimal monetary policy remains almost stable. This Result is illustrated in figure 2.*

The next important result that we want to emphasise is the following. We want to see how each of the instruments behave as a function of all other policy instruments. However, to see it, we need to plot a 4-d figure, which is impossible. Thus, we fix two out of 4 instruments and plot the rest like functions from each other.

Since our paper is mainly focused on the preventive policy we plot θ_1 as a function of i_1 while keeping $i_{2,L}$ and $i_{2,H}$ at their optimal values (at their optimal values for every pair if θ_1 and i_1), and the opposite, we plot i_1 as a function of θ_1. The interaction of these two lines indicates our optimal solution. We can see it in figure 2 and in Result 2.

In figure 2 the blue line (with circles sign) indicates θ_1 as a function of i_1, i.e. for every value if i_1 our algorithm finds optimal θ_1[2]. And the opposite, the red line (with cross sign) indicates i_1 as a function of θ_1, i.e. for every possible values of θ_1 we find an optimum value of i_1. The green line (with point sign) shows the optimal value of r_1 in the model with flexible prices (remember that in these two versions of model with flexible and fixed prices i_1=r_1). Finally, the dashed line points steady state value of these interest rates ($\frac{1}{\beta} - 1$).

We see that the red line (with cross sign) is almost insensitive to changes in θ_1 while the blue line (with circles sign) shows a U-shape curve, i.e. θ_1 grows when i_1 is small but with higher values of i_1 it declines. In other words, for small i_1 we tighten macroprudential policy but for high i_1 we ease. We call this result semi-complementarity in dynamic because the red line (with cross sign) remains almost constant (comparing with the green line (with point sign) which shows policy easing), so in the model, with fixed prices, monetary policy remains tight, but not tightening for every possible value of macroprudential policy θ_1. So, these policies remain tight but with the tightening of one policy, the other policy does not necessarily also tighten. Thus, in dynamics these policies only partly complement.

Result 3 (Ex-ante vs ex-post policies) *In response to anticipated financial shocks, it is also optimal to tighten monetary policy in the absence of crisis and to loosen monetary policy in a crisis. This Result is illustrated in figure 3.*

We also contrast our results with Basu et al. 2020 and look at both ex-ante (i.e. preventive) and ex-post policies. The economic intuition is the following. While every policymaker desire to prevent any economic instability using a variety of available tools which potentially reduce a probability of a crisis and severity of a crisis, however, there is still a need to have instruments during a crisis (actually, for high κ a financial constraint does not bind). In our model, it is interest rates. Our findings in using these ex-ante and ex-post rates are highlighted in Result 3 and in figure 3.

[2]We also find optimal $i_{2,L}$ and $i_{2,H}$ which are not shown.

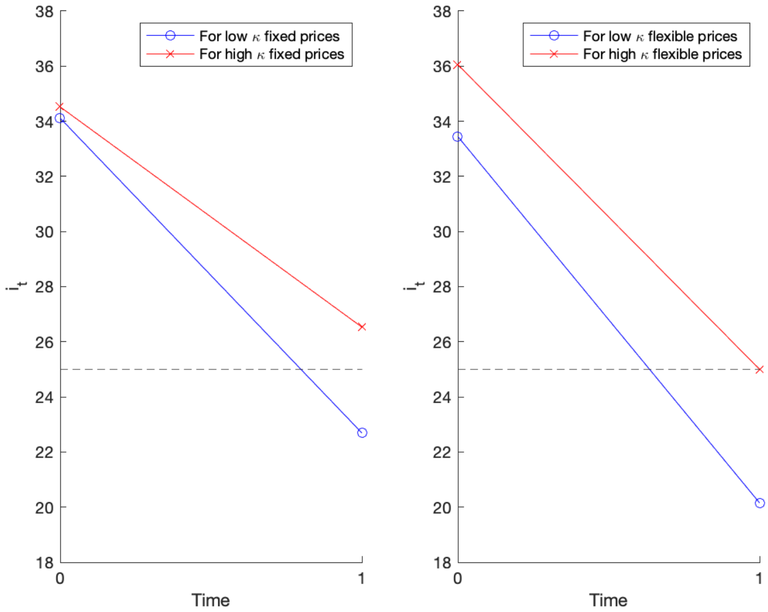

Figure 3: Ex-ante vs ex-post policies

In figure 3 we compare our results for fixed prices (left figure) and flexible prices (right figure). Moreover, we also plot two lines on each of the figures. Blue lines (with circle sign) show optimal policy for a low value of κ ($= 0.025$) and red lines (with cross sign) show optimal policy for a high value of κ ($= 0.9$). The idea of showing two lines for different κ's is the following. In the model, κ indicates the severity of a crisis. The smaller the kappa, the more severe the crisis. So, we plot extreme cases, when there is a serious crisis, and the opposite when a crisis is not so heavy. Finally, the dashed line points steady-state value of these interest rates ($\frac{1}{\beta} - 1$).

From figure 3 we again see that ex-ante (during the accumulation of risks and imbalances) monetary policy tightens while ex-post (during the risk implementation period) monetary policy eases. Moreover, we see a striking difference both between two κ's and two models. In the fixed prices model monetary policy tightening in the pre-crisis ($t = 0$) period is almost the same for both κ's, meanwhile in the model with flexible prices monetary policy tightening for low κ is stronger. However, in the crisis period ($t = 1$) we see almost the same results: for high κ easing is smaller, for low κ easing is larger. This highlights the fact that our model does not ex-ante pay much attention to the "size" of a crisis (because for low κ policy tightening is smaller), however ex-post we see much larger easing if κ is low.

To conclude this section, we find that in response to anticipated financial shocks in the form of debt limits, it is optimal to preventive tighten both monetary and macroprudential policies while in a crisis period it is optimal to support the economy and ease interest rates.

4 SENSITIVITY ANALYSIS

In this section, we present and discuss a series of supplemental results. Specifically, we provide sensitivity analysis. This discussion is based on the fact that our model is more qualitative than quantitative. Therefore, we cannot plausible calibrate all of our parameters which can be important when we determine optimal policy. Thus, in this section we vary initial capital endowment k_{-1}, an initial amount of financing firms with linear technology n_0^{linear}, debt/financing constraint limit κ, capital adjustment costs ϕ, a probability of a crisis ρ_L. In the next figures, we adhere to the following notation: blue lines (with circle sign) are optimal policies in fixed price model, red lines (with cross sign) are optimal policies in a flexible model, dashed lines indicated a steady-state value of interest rates ($\frac{1}{\beta} - 1$).

Supplemental Result 1 (Change in k_{-1}) *With a larger endowment of k_{-1}, a policymaker needs to significantly tighten macroprudential policy, θ_1, ease ex-ante monetary policy, i_1, and also ex-post monetary policy in a crisis period, $i_{2,L}$, tighten ex-post monetary policy if the crisis does not occur, $i_{2,H}$. This Supplemental Result is illustrated in figure 4.*

The main economic take-away is the following. The initial households capital endowment is important when a policymaker use macroprudential policy and monetary policy to soften the severity of a crisis. We see that if households are richer initially (have more k_{-1}), then a policymaker have to set quite a high ex-ante macroprudential

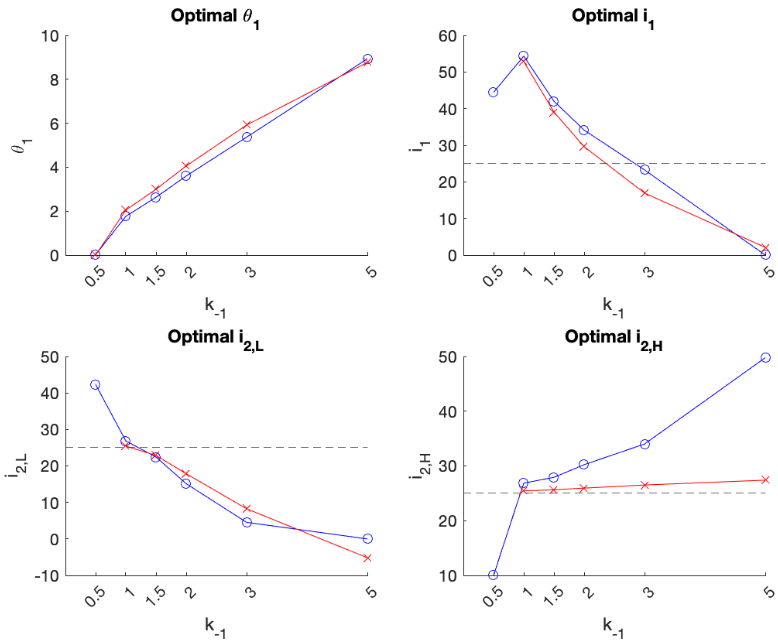

Figure 4: Change in k_{-1}

policy, θ_1 and ex-post monetary policy, $i_{2,H}$. While it have to set ex-ante monetary policy i_1 and ex-post monetary policy, $i_{2,L}$ quite low.

Moreover, we see that for θ_1, i_1 and $i_{2,L}$ optimal policies in the fixed price model are very close to macroprudential tax and interest rates in the flexible prices model. However, for $i_{2,H}$ we see that for fixed prices a policymaker has to significantly increase an interest rate. Furthermore, we see (especially for i_1) that for some high values of k_{-1} an optimal policy falls below a steady-state.

Supplemental Result 2 (Change in n_0^{linear}) *With a larger amount of financing firms with linear technology, n_0^{linear}, the optimal macroprudential policy shows the inverse U-shape curve, i.e. it tightens when n_0^{linear} is low and it loosens macroprudential policy (lower but positive θ_1) when n_0^{linear} becomes higher. Also, a policymaker needs to tighten ex-ante monetary policy, i_1, and also ex-post monetary policy in a crisis period, $i_{2,L}$, and loosing ex-post monetary policy if the crisis does not occur, $i_{2,H}$. This Supplemental Result is illustrated in figure 5.*

This Supplemental Result shows two important features. First of all, we see that depending on n_0^{linear}, optimal θ_1 is non-linear. It rises when n_0^{linear} is low and it decreases when n_0^{linear} becomes higher. And two, we also see that there is an upper

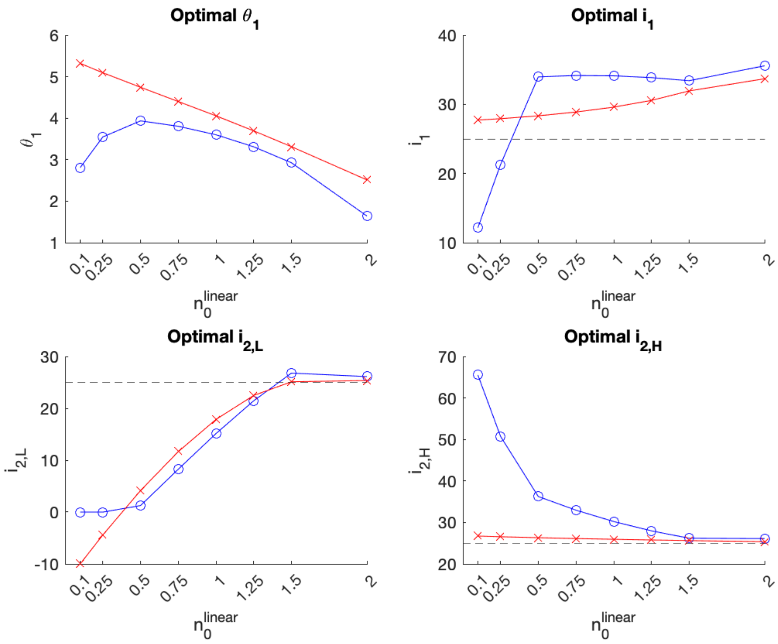

Figure 5: Change in n_0^{linear}

limit for i_1 and $i_{2,L}$. For both these rates, we see that for larger n_0^{linear} it is optimal to tighten rates up to a certain limit. The opposite is true for easing of $i_{2,H}$, it is optimal to low the rate down to some limit (actually, to steady-state). This highlights the result that if the economy consists of "rich" firms then monetary policy could be a constraint in either ex-ante (during the accumulation of risks and imbalances) and ex-post (during the implementation of risks).

Figure 5 highlights important differences for fixed and flexible prices. If firms initially are poor (underfinanced) then for fixed prices it may require slightly different policies (especially, for $\theta_1, i_1, i_{2,H}$).

Supplemental Result 3 (Change in κ) *With higher κ (less strict debt/financial constraint), a policymaker needs to implement softer macroprudential regulation (lower but positive θ_1), to tighten ex-ante monetary policy, i_1, and also ex-post monetary policy in a crisis period, $i_{2,L}$, and loosing ex-post monetary policy if the crisis does not occur, $i_{2,H}$. Note that there is an upper limit in κ after which all optimal policies are constant. This Supplemental Result is illustrated in figure 6.*

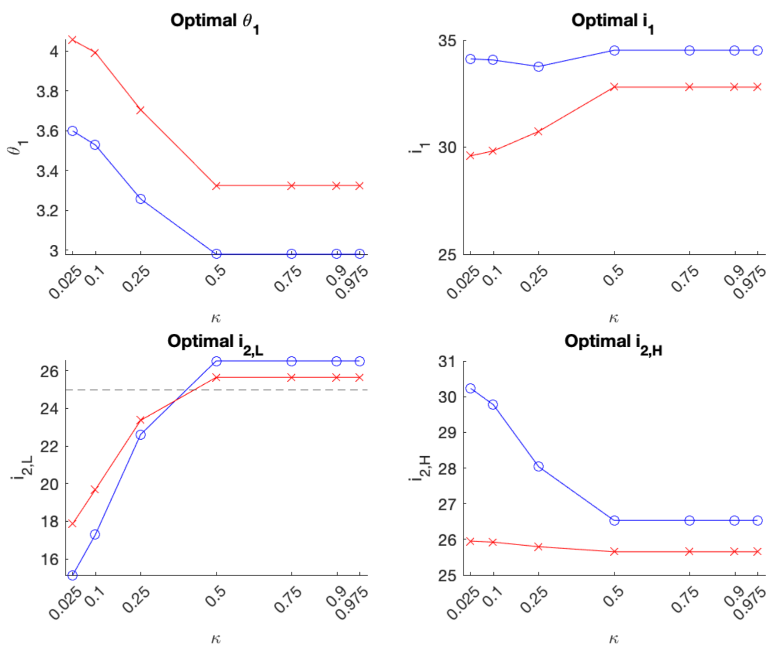

Figure 6: Change in κ

When we consider the debt limit κ, firstly, we see that there is an upper limit in κ after which all policies do not react. The intuition is the following. If the debt limit is not tough, then the financial constraint does not bind because firms with linear technology are willing to take less debt and so do not meet this financial constraint.

Moreover, we see that optimal policies for fixed and flexible prices more or less show the same directions when κ changes. However, we note that for fixed prices, optimal θ_1 is softer than for flexible prices model. Optimal i_1 and $i_{2,H}$ are stricter and optimal $i_{2,L}$ is almost the same.

Supplemental Result 4 (Change in ϕ) *With larger investments costs, ϕ, a policymaker needs to implement softer macroprudential regulation (lower but positive θ_1). Optimal ex-ante monetary policy, i_1, shows inverse U-shape, i.e. it increases when ϕ is low and decreases when ϕ is high. It is optimal to soften ex-post monetary policy, both $i_{2,L}$ and $i_{2,H}$. This Supplemental Result is illustrated in figure 7.*

If we change the capital adjustment costs parameter, we see the most expected result. With larger ϕ it is more difficult to create a new unit of capital, thus, agents accumulate less capital and it requires a smaller amount of intervention from a

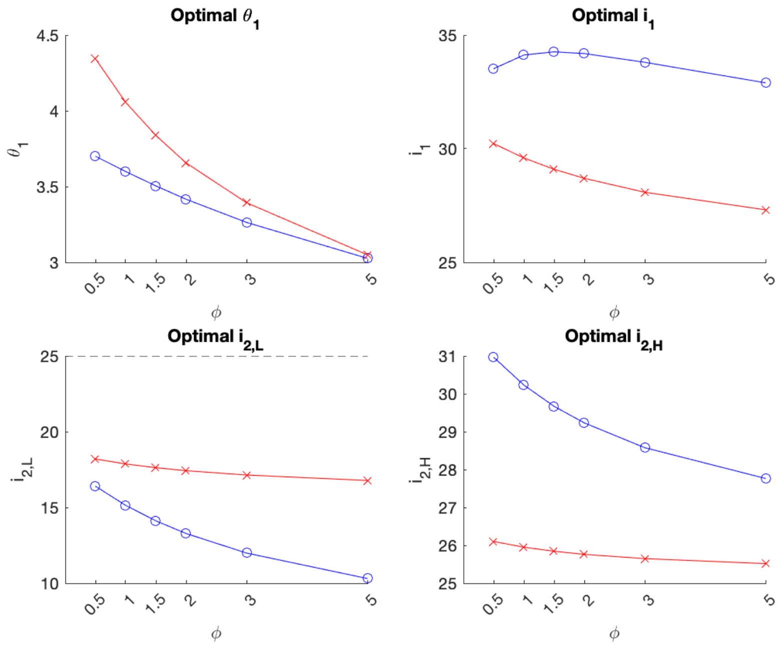

Figure 7: Change in ϕ

policymaker. However, we note the difference for optimal interest rates between fixed and flexible models. The tightening of i_1 and $i_{2,H}$ is much stronger for fixed prices model, while for θ_1 and $i_{2,L}$ the result is the opposite.

Supplemental Result 5 (Change in ρ_L) *With the growing probability of a crisis (ρ_L), a policymaker needs to implement a tighter macroprudential policy (higher θ_1). A policymaker needs to soften ex-ante monetary policy, i_1. It is optimal to tighten ex-post monetary policy, both $i_{2,L}$ and $i_{2,H}$. This Supplemental Result is illustrated in figure 8.*

The most striking result is the following. When the probability of a crisis ρ_L rises, a policymaker tightens macroprudential policy but slightly loosen ex-ante monetary policy. Moreover, it is optimal to tighten ex-post monetary policy. We also note that for fixed prices i_1 falls a little in contrast to the flexible prices model. And the same, $i_{2,H}$ rises a lot for fixed prices model while this interest rate remains the same for flexible prices.

This result is striking because with a higher probability of a crisis, we expect to tighten ex-ante monetary policy to prevent excessive capital accumulation (in addition to tightening macroprudential policy). Loosening ex-ante monetary policy with a growing probability of a crisis looks counter-intuitive because it seems that if a crisis is imminent, a policymaker needs to make every effort to reduce the severity of the crisis.

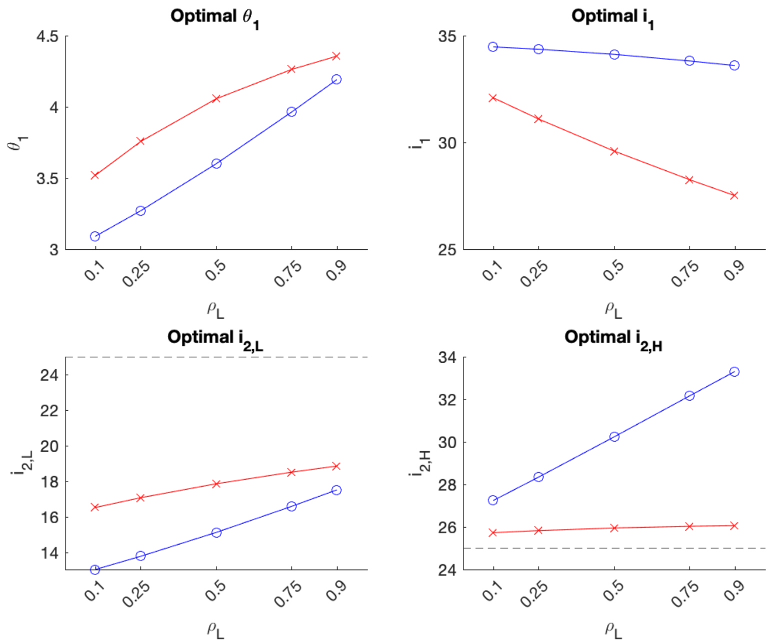

Figure 8: Change in ρ_L

Moreover, it is important to determine the point to which we compare monetary tightening/easing. If we compare fixed prices to flexible prices then the Results above are true. Furthermore, these results remain the same for a number of sensitivity tests (when we change calibration values) and which indicated in a number of Supplemental Results. For most values parameters values, a policymaker still has to tighten macroprudential policy (but less than for flexible prices), set ex-ante monetary policy higher than for flexible prices. Ease even more ex-post monetary policy than for flexible prices if a crisis occurs and tighten more if a crisis does not occur.

However, if we compare this result to the steady-state value of interest rate ($\frac{1}{\beta} - 1$), we find that our results are highly dependent on calibration values. The main results remain the same, it is still optimal to tighten ex-ante macroprudential and to tighten (set above steady-state) monetary policy, to ease (set below steady-state) ex-post monetary policy in a crisis state and tighten (set above steady-state) ex-post monetary policy in no crisis state. Nevertheless, these results do not hold for several parameters values. For instance, when initial households' capital endowment is very large, then it is optimal to set the ex-ante policy rate below steady-state. We may apply the same story when the amount of initial financing of firms with linear technology is small, it again needs to set ex-ante interest rate below steady state.

5 POLICY IMPLICATIONS

To conclude this section, we see several policy implications. In practice, for a policymaker it is important to understand/estimate an initial household endowment, k_{-1}, and the amount of financing of firms with linear technology, n_0^{linear}. Both of these variables indicate households' or firms resources but the optimal policy response is diametrically opposite. Thus, one has to be accurate in determining these endowments.

Also, for a policymaker, it is important to correctly estimate the severity of a crisis. If the debt limit is not so tight, then it may be possible that the financial consultant does not bind and, thus, requires a slightly different optimal policy from a policymaker. It is also important to understand how difficult to create new capital in practice because if it is difficult then it requires much less policy interventions. Finally, the probability of a crisis, undoubtedly, matters, when we determine the optimal policy interventions.

6 FURTHER REMARKS

In this section, we point out several limitations that there are in our model. First of all, this is a model of a closed economy. It limits our available space for analysis. Nevertheless, there is still plenty of interesting results that are discussed in the previous sections. However, there is room for improvement. We still lack of rich banking sector and possible externalities that emerge in this sector. There are many interesting features that are difficult to incorporate: interest rates affecting the liquidity/efficiency of trades as well as real interest rates.

We also do not take into account any effects from the external sector, to consider it, we need to extend our model to a small open economy with a commodity-exporting sector. We plan to consider a small open economy model in a separate paper.

Also, we exploit fully fixed prices (but not a-la Calvo or Rottenberg pricing), which allows us to compare exogenous policy instruments ($i_1, i_{2,L}, i_{2,H}$) in the model with fixed prices with endogenous variables ($r_1, r_{2,L}, r_{2,H}$) in the model with flexible prices. In other words, when policymaker manipulates $i_1, i_{2,L}, i_{2,H}$ it effectively sets $r_1, r_{2,L}, r_{2,H}$. Intuitively both policies do the same thing in increasing real interest rate: because of fixed price equilibria. With some sluggishness in nominal prices, this would be different. There should be some interesting trade-offs.

Moreover, there is a couple of "modelling" limitations, such as quasi-linear utility function ($\log(c_t) - h_t$) and a form of capital adjustment function ($inv_0(1 + \frac{inv_0}{k_{-1}})$). Some of our results may be caused by these particular forms. However, as a robustness check we plan to consider more general utility function ($\frac{c_{t,s}^{1-\sigma}}{1-\sigma} - \frac{h_{t,s}^{1+\eta}}{1+\eta}$).

Furthermore, our framework allows us to compare time-consistent and time-inconsistent monetary policy. Is there a time consistent function for macroprudential/monetary policies? We also plan to implement this analysis in the future.

Finally, our model is a descriptive one, we only can advise a direction of policy but not the exact rates. Thus, it is important to understand what does the choice of one policy instrument, imply about the choice of the other given richer trade-offs? We partly do it in Result 2. However, there is still room for improvement in these findings.

7 CONCLUSION

In this paper we develop a closed economy model that features multiple externalities, especially, we characterize optimal monetary and macroprudential policies in the economy with fire-sales and financially constrained agents. Effectively, our model has three periods and two generalized types of agents: lenders (households/owners of raw-inputs/final goods producers) and borrowers (firms with linear capital production technology). Flows of real assets in our model are the following. In the first period households are endowment with an amount of capital and in the same period households decide the amount of new capital creation (accumulation). The creation of new capital occurs only in the first period. The capital, initially owned by households, are sold to firms. These firms use capital to produce a raw input good which is, combined with labor, used in the production of a final good. This final good is fully consumed by households. Financial flows are the following. Firms with linear technology are initially financed by households, then they debt each period and reinvest their net wealth (profits). After the last period, all net wealth is paid back to households.

At date 1 (the second period), there is a shock with a certain probability. Markets are incomplete. Net wealth determines capital goods prices. At date 1, borrowers have borrowing constraint - can only borrow up to a fraction of new capital bought. In a low state (bad realization of uncertainty) capital price falls and constraint binds. A binding constraint generates an additional cost to debt and benefit to capital (collateral premium). Firms with linear technology are forced to take less debt and sell part of the capital (show less demand for capital). Less demand for capital further reduces their price which leads to pecuniary externality/fire sales. This intuition works backwards: the pecuniary externality causes too much lending at date 0, and not enough at date 1. Thus, we ex-ante/preventive examine the macroprudential and monetary policy to restrict agents to accumulate too much capital.

There are two policies available: tax on borrowing or increase the borrowing interest rate. Both policies raise the real interest rate, (and prices of capital) suppressing lending and investment but because of pecuniary externality, may lift capital prices so the collateral constraint does not bind leaving room for Pareto improvement. Intuitively both policies do the same thing in increasing real interest rate: because of fixed price equilibria. The difference is that changing the cost of debt affects both lenders and borrowers directly while the tax affects the borrower only at a first-order level.

In our baseline calibration, we find that it is optimal to tighten both ex-ante macroprudential and monetary policy, ease monetary policy if a crisis occurs and again tighten if a does not crisis occur. Furthermore, macroprudential tightening for fixed prices is less than for flexible prices. These results do not only show the optimal direction of policy instruments in response to financial shock but also highlights the importance of price rigidities. Thus, we compare only two extreme cases, which shows a big difference in determining the optimal policy. Moreover, we note that if we compare the optimal rates to the steady-state values, then some of these results depends on parameters values.

Despite making some simplifying assumptions, our model has a very rich for its structure which allows us to draw important results for economic literature and policymaking.

REFERENCES

Adrian, Tobias et al. (2020). "A Quantitative Model for the Integrated Policy Framework".

Alam, Zohair et al. (2019). *Digging deeper–Evidence on the effects of macroprudential policies from a new database*. International Monetary Fund.

Auer, Raphael and Steven Ongena (2019). "The countercyclical capital buffer and the composition of bank lending".

Basu, Suman Sambha et al. (2020). "A Conceptual Model for the Integrated Policy Framework".

Bianchi, Javier and Enrique G Mendoza (2018). "Optimal time-consistent macroprudential policy". *Journal of Political Economy* 126.2, pp. 588–634.

Borio, Claudio (2014). "The financial cycle and macroeconomics: What have we learnt?" *Journal of Banking & Finance* 45, pp. 182–198.

Calvo, Guillermo A and Carmen M Reinhart (2002). "Fear of floating". *The Quarterly journal of economics* 117.2, pp. 379–408.

Cerutti, Eugenio, Stijn Claessens, and Luc Laeven (2017). "The use and effectiveness of macroprudential policies: New evidence". *Journal of Financial Stability* 28, pp. 203–224.

Clarida, Richard, Jordi Gali, and Mark Gertler (1999). "The science of monetary policy: a new Keynesian perspective". *Journal of economic literature* 37.4, pp. 1661–1707.

Clerc, Laurent et al. (2015). "Capital Regulation in a Macroeconomic Model with Three Layers of Default". *International Journal of Central Banking* 11.3, pp. 9–63.

Cozzi, Guido et al. (2020). "Macroprudential Policy Measures: Macroeconomic Impact and Interaction With Monetary Policy".

Dávila, Eduardo and Anton Korinek (Feb. 2017). "Pecuniary Externalities in Economies with Financial Frictions". *The Review of Economic Studies* 85.1, pp. 352–395. ISSN: 0034-6527.

Eggertsson, Gauti B and Paul Krugman (2012). "Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach". *The Quarterly Journal of Economics* 127.3, pp. 1469–1513.

Farhi, Emmanuel and Iván Werning (2016). "A theory of macroprudential policies in the presence of nominal rigidities". *Econometrica* 84.5, pp. 1645–1704.

Kaplan, Greg, Benjamin Moll, and Giovanni L Violante (2018). "Monetary policy according to HANK". *American Economic Review* 108.3, pp. 697–743.

Kara, Gazi and S Mehmet Ozsoy (2016). "Bank regulation under fire sale externalities".

Korinek, Anton and Alp Simsek (2016). "Liquidity trap and excessive leverage". *American Economic Review* 106.3, pp. 699–738.

Lagarias, Jeffrey C et al. (1998). "Convergence properties of the Nelder–Mead simplex method in low dimensions". *SIAM Journal on optimization* 9.1, pp. 112–147.

Lorenzoni, Guido (2008). "Inefficient credit booms". *The Review of Economic Studies* 75.3, pp. 809–833.

Schmitt-Grohé, Stephanie and Martin Uribe (2016). "Downward nominal wage rigidity, currency pegs, and involuntary unemployment". *Journal of Political Economy* 124.5, pp. 1466–1514.

Stavrakeva, Vania (2020). "Optimal bank regulation and fiscal capacity". *The Review of Economic Studies* 87.2, pp. 1034–1089.

Uribe, Martin and Stephanie Schmitt-Grohé (2017). *Open economy macroeconomics*. Princeton University Press.

Van der Ghote, Alejandro (2021). "Interactions and coordination between monetary and macroprudential policies". *American Economic Journal: Macroeconomics* 13.1, pp. 1–34.

Woodford, Michael (2003). "Interest and prices".

— (2012). *Inflation targeting and financial stability*. Tech. rep. National Bureau of Economic Research.

A PARAMETERS

Table 1: Parameter Values for baseline model

Parameter	Description	Value	Source
β	Discount factor	0.8	Basu et al. 2020
α	Share of labor	$\frac{1}{3}$	Commonly used in the literature
ϵ	Price Markup	6	Commonly used in the literature[3]
n_0^{linear}	Initial financing	1	Our choice
k_{-1}	Initial endowment	2	Our choice
ϕ	Capital adjustment	1	Uribe and Schmitt-Grohé 2017
ρ_L	Probability of bad shock	0.5	Basu et al. 2020
κ	Debt limit	0.025	Basu et al. 2020

[3] To fit average price markup of 20%

B FOCS FOR MODEL WITH FIXED PRICES

FOR $t = 0$.

$$c_0^{-1} = \beta(1+r_1)(\rho_L c_{1,L}^{-1} + \rho_H c_{1,H}^{-1}), \tag{21}$$

$$c_0 = w_0, \tag{22}$$

$$\frac{\alpha}{1-\alpha}\frac{k_{-1}}{h_0} = \frac{w_0}{p_{x,0}}, \tag{23}$$

$$c_0 + inv_0(\frac{\phi}{2}\frac{inv_0}{k_{-1}}) = k_{-1}^{\alpha} h_0^{1-\alpha}, \tag{24}$$

$$1 + \phi\frac{inv_0}{k_{-1}} = q_0, \tag{25}$$

$$c_0^{-1} q_0 = \beta[\rho_L c_{1,L}^{-1}(\frac{p_{x,1,L}A_{1,L}}{1+k_0^{concave}} + q_{1,L}) + \rho_H c_{1,H}^{-1}(\frac{p_{x,1,H}A_{1,H}}{1+k_0^{concave}} + q_{1,H})], \tag{26}$$

$$q_0 c_0^{-1} = \beta\frac{\rho_L c_{1,L}^{-1}[p_{x,1,L}A_{1,L} + q_{1,L}] + \rho_H c_{1,H}^{-1}[p_{x,1,H}A_{1,H} + q_{1,H}]}{(1+\theta_1)}, \tag{27}$$

$$inv_0 + k_{-1} = K, \tag{28}$$

$$k_0^{linear} + k_0^{concave} = K, \tag{29}$$

$$\tag{30}$$

9 equations for 9 unknowns: $c_0, h_0, w_0, q_0, k_0^{concave}, k_0^{linear}, K, inv_0, p_{x,0}$.

FOR $t = 1, s = L$.

$$c_{1,L}^{-1} = \beta(1 + i_{2,L})c_{2,L}^{-1}, \tag{31}$$

$$c_{1,L} = w_{1,L}, \tag{32}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,L}}{h_{1,L}} = \frac{w_{1,L}}{p_{x,1,L}}, \tag{33}$$

$$c_{1,L} = x_{1,L}^{\alpha} h_{1,L}^{1-\alpha}, \tag{34}$$

$$c_{1,L}^{-1} q_{1,L} = \beta c_{2,L}^{-1}\left(\frac{p_{x,2,L}A_{2,L}}{1 + k_{1,L}^{concave}} + q_{2,L}\right), \tag{35}$$

$$x_{1,L} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{36}$$

$$q_{1,L} k_{1,L}^{linear} = \frac{n_{1,L}^{linear}}{1-\kappa}, \tag{37}$$

$$n_{1,L}^{linear} = p_{x,1,s}A_{1,s}k_0^{linear} + q_{1,s}k_0^{linear} - (1 + i_1)(q_0 k_0^{linear} - n_0^{linear}), \tag{38}$$

$$k_{1,L}^{linear} + k_{1,L}^{concave} = K, \tag{39}$$

9 equations for 9 unknowns: $c_{1,L}, h_{1,L}, w_{1,L}, x_{1,L}, p_{x,1,L}, q_{1,L}, n_{1,L}^{linear}, k_{1,L}^{linear}, k_{1,L}^{concave}$.

FOR $t = 1, s = H$.

$$c_{1,H}^{-1} = \beta(1 + i_{2,H})c_{2,H}^{-1}, \tag{40}$$

$$c_{1,H} = w_{1,H}, \tag{41}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,H}}{h_{1,H}} = \frac{w_{1,H}}{p_{x,1,H}}, \tag{42}$$

$$c_{1,H} = x_{1,H}^{\alpha} h_{1,H}^{1-\alpha}, \tag{43}$$

$$c_{1,H}^{-1} q_{1,H} = \beta c_{2,H}^{-1}\left(\frac{p_{x,2,H}A_{2,H}}{1 + k_{1,H}^{concave}} + q_{2,H}\right), \tag{44}$$

$$x_{1,H} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{45}$$

$$q_{1,H} = \frac{p_{x,2,H}A_{2,H} + q_{2,H}}{(1 + i_{2,H})}, \tag{46}$$

$$k_{1,H}^{linear} + k_{1,H}^{concave} = K \tag{47}$$

8 equations for 8 unknowns: $c_{1,H}, x_{1,H}, w_{1,H}, h_{1,H}, q_{1,H}, p_{x,1,L}, k_{1,H}^{linear}, k_{1,H}^{concave}$.

FOR $t = 2, s$. For $s = H$ allocations are the same as on $t \geq 3$.

$$c_{2,s}^{-1} = \beta(1 + r_{3,s})c^{-1}, \tag{48}$$

$$c_{2,s} = w_{2,s}, \tag{49}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{2,s}}{h_{2,s}} = \frac{w_{2,s}}{p_{x,2,s}}, \tag{50}$$

$$c_{2,s} = x_{2,s}^{\alpha} h_{2,s}^{1-\alpha}, \tag{51}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_{2,s}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,2,s}}{1-\alpha}\right)^{1-\alpha}, \tag{52}$$

$$x_{2,s} = \log(1 + k_{1,s}^{concave}) + k_{1,s}^{linear}, \tag{53}$$

$$q_{2,s} = \frac{p_x A + q}{(1 + r_{3,s})}, \tag{54}$$

$$\tag{55}$$

7 equations for 7 unknowns: $c_{2,s}, w_{2,s}, h_{2,s}, x_{2,s}, q_{2,s}, p_{x,2,s}, r_{3,s}$.

FOR $t \geq 3$. All variables are at steady states. Thus, we remove the times indexes. And new prices and fully flexible, thus, interest rate becomes endogenous r.

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{p_x}{1-\alpha}\right)^{1-\alpha}, \tag{56}$$

$$q = \frac{\beta}{1-\beta}p_x, \tag{57}$$

$$r = \frac{1}{\beta} - 1, \tag{58}$$

$$c = x^{\alpha}h^{1-\alpha}, \tag{59}$$

$$x = K, \tag{60}$$

$$c = w, \tag{61}$$

$$\frac{\alpha}{1-\alpha}\frac{x}{h} = \frac{w}{p_x}, \tag{62}$$

7 equations for 7 unknowns: c, w, h, x, p_x, r, q.

C FOCS FOR MODEL WITH FLEXIBLE PRICES

FOR $t = 0$.

$$c_0^{-1} = \beta(1+r_1)(\rho_L c_{1,L}^{-1} + \rho_H c_{1,H}^{-1}), \tag{63}$$

$$c_0 = w_0, \tag{64}$$

$$\frac{\alpha}{1-\alpha}\frac{k_{-1}}{h_0} = \frac{w_0}{p_{x,0}}, \tag{65}$$

$$c_0 + inv_0(\frac{\phi}{2}\frac{inv_0}{k_{-1}}) = k_{-1}^{\alpha} h_0^{1-\alpha}, \tag{66}$$

$$1 + \phi\frac{inv_0}{k_{-1}} = q_0, \tag{67}$$

$$c_0^{-1} q_0 = \beta[\rho_L c_{1,L}^{-1}(\frac{p_{x,1,L}A_{1,L}}{1+k_0^{concave}} + q_{1,L}) + \rho_H c_{1,H}^{-1}(\frac{p_{x,1,H}A_{1,H}}{1+k_0^{concave}} + q_{1,H})], \tag{68}$$

$$q_0 c_0^{-1} = \beta\frac{\rho_L c_{1,L}^{-1}[p_{x,1,L}A_{1,L} + q_{1,L}] + \rho_H c_{1,H}^{-1}[p_{x,1,H}A_{1,H} + q_{1,H}]}{(1+\theta_1)}, \tag{69}$$

$$inv_0 + k_{-1} = K, \tag{70}$$

$$k_0^{linear} + k_0^{concave} = K, \tag{71}$$

$$1 = \frac{\epsilon}{\epsilon-1}\left(\frac{w_0}{\alpha}\right)^{\alpha}\left(\frac{p_{x,0}}{1-\alpha}\right)^{1-\alpha}, \tag{72}$$

10 equations for 10 unknowns: $c_0, h_0, w_0, q_0, k_0^{concave}, k_0^{linear}, K, inv_0, p_{x,0}, r_1$.

FOR $t = 1, s = L$.

$$c_{1,L}^{-1} = \beta(1 + r_{2,L})c_{2,L}^{-1}, \tag{73}$$

$$c_{1,L} = w_{1,L}, \tag{74}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,L}}{h_{1,L}} = \frac{w_{1,L}}{p_{x,1,L}}, \tag{75}$$

$$c_{1,L} = x_{1,L}^{\alpha} h_{1,L}^{1-\alpha}, \tag{76}$$

$$c_{1,L}^{-1} q_{1,L} = \beta c_{2,L}^{-1}\left(\frac{p_{x,2,L}A_{2,L}}{1 + k_{1,L}^{concave}} + q_{2,L}\right), \tag{77}$$

$$x_{1,L} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{78}$$

$$q_{1,L}k_{1,L}^{linear} = \frac{n_{1,L}^{linear}}{1-\kappa}, \tag{79}$$

$$n_{1,L}^{linear} = p_{x,1,s}A_{1,s}k_0^{linear} + q_{1,s}k_0^{linear} - (1 + r_1)(q_0 k_0^{linear} - n_0^{linear}), \tag{80}$$

$$k_{1,L}^{linear} + k_{1,L}^{concave} = K, \tag{81}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_{1,L}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,1,L}}{1-\alpha}\right)^{1-\alpha}, \tag{82}$$

10 equations for 10 unknowns: $c_{1,L}, h_{1,L}, w_{1,L}, x_{1,L}, p_{x,1,L}, q_{1,L}, n_{1,L}^{linear}, k_{1,L}^{linear}, k_{1,L}^{concave}, r_{2,L}$.

FOR $t = 1, s = H$.

$$c_{1,H}^{-1} = \beta(1 + i_{2,H})c_{2,H}^{-1}, \tag{83}$$

$$c_{1,H} = w_{1,H}, \tag{84}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,H}}{h_{1,H}} = \frac{w_{1,H}}{p_{x,1,H}}, \tag{85}$$

$$c_{1,H} = x_{1,H}^{\alpha} h_{1,H}^{1-\alpha}, \tag{86}$$

$$c_{1,H}^{-1} q_{1,H} = \beta c_{2,H}^{-1}\left(\frac{p_{x,2,H}A_{2,H}}{1 + k_{1,H}^{concave}} + q_{2,H}\right), \tag{87}$$

$$x_{1,H} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{88}$$

$$q_{1,H} = \frac{p_{x,2,H}A_{2,H} + q_{2,H}}{(1 + i_{2,H})}, \tag{89}$$

$$k_{1,H}^{linear} + k_{1,H}^{concave} = K, \tag{90}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_{1,H}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,1,H}}{1-\alpha}\right)^{1-\alpha}, \tag{91}$$

9 equations for 9 unknowns: $c_{1,H}, x_{1,H}, w_{1,H}, h_{1,H}, q_{1,H}, p_{x,1,L}, k_{1,H}^{linear}, k_{1,H}^{concave}, r_{2,H}$.

FOR $t = 2, s$. For $s = H$ allocations are the same as on $t \geq 3$.

$$c_{2,s}^{-1} = \beta(1 + r_{3,s})c^{-1}, \tag{92}$$

$$c_{2,s} = w_{2,s}, \tag{93}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{2,s}}{h_{2,s}} = \frac{w_{2,s}}{p_{x,2,s}}, \tag{94}$$

$$c_{2,s} = x_{2,s}^{\alpha} h_{2,s}^{1-\alpha}, \tag{95}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_{2,s}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,2,s}}{1-\alpha}\right)^{1-\alpha}, \tag{96}$$

$$x_{2,s} = \log(1 + k_{1,s}^{concave}) + k_{1,s}^{linear}, \tag{97}$$

$$q_{2,s} = \frac{p_x A + q}{(1 + r_{3,s})}, \tag{98}$$

7 equations for 7 unknowns: $c_{2,s}, w_{2,s}, h_{2,s}, x_{2,s}, q_{2,s}, p_{x,2,s}, r_{3,s}$.

FOR $t \geq 3$. All variables are at steady states. Thus, we remove the times indexes.

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{p_x}{1-\alpha}\right)^{1-\alpha}, \tag{99}$$

$$q = \frac{\beta}{1-\beta}p_x, \tag{100}$$

$$r = \frac{1}{\beta} - 1, \tag{101}$$

$$c = x^{\alpha}h^{1-\alpha}, \tag{102}$$

$$x = K, \tag{103}$$

$$c = w, \tag{104}$$

$$\frac{\alpha}{1-\alpha}\frac{x}{h} = \frac{w}{p_x}, \tag{105}$$

7 equations for 7 unknowns: c, w, h, x, p_x, r, q.

D ALGORITHM TO SOLVE MODEL

1. Non-linear solver function:
   (a) As input, it takes a vector of values of policy parameters: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$, and a vector of parameter values: β, α, ϵ, n_0^{linear}, k_{-1}, ϕ, κ and ρ_L.
   (b) It solves steady-state system of equations and use steady state values as initial values in non-linear solver.
   (c) Firstly, we assume that financial constraint (7) does not bind and solve unconstrained verion of the model. In other words, we assume that when uncertainty resolves, there is no difference in low and high state because financial constraint does not hold.
   (d) Then we check two conditions: whether there is a solution at all, and if there is, then we check if d_1^{linear} is positive. In other words, $d_1^{linear} > 0$ means that firms with linear technology are borrowers, which we assume in our

model but for some values of parameters these firms may become lenders $d_1^{linear} < 0$. We do not consider these cases.

(e) After solving model without financial constraint, we compare $d_{2,L}^{linear}$ and $\kappa q_{1,L} k_{1,L}^{linear}$.

(f) If $d_{2,L}^{linear}$ is less than $\kappa q_{1,L} k_{1,L}^{linear}$, then we find a solution since in this case the amount of borrowing made by firms with linear technology is less than a possible debt limit, so there is no need to solve a model with binding financial constraint.

(g) If $d_{2,L}^{linear}$ is higher than $\kappa q_{1,L} k_{1,L}^{linear}$, then it means that the amount of borrowing made by firms with linear technology is higher than a possible debt limit. It indicates that a financial constraint should bind, so we resolve the model with binding financial constraint.

(h) As output, this function gives the value of agents utility (1), a vector of endogenous variables, and an indicator, which take 0 is for a given vectors of parameters if financial constraint does not bind, and takes value of 1 if financial constraint binds.

2. Minimize function

(a) As an input, it takes a vector of values of policy parameters: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$, and a vector of parameter values: β, α, ϵ, n_0^{linear}, k_{-1}, ϕ, κ and ρ_L.

(b) Then, it applies non-linear solver with this given vectors, which returns the optimal utility value.

(c) Then, it take negative value of the utility and applies a minimize function varying a vector of policy parameters: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$ and taking parameter values: β, α, ϵ, n_0^{linear}, k_{-1}, ϕ, κ and ρ_L as given

(d) As output, it gives the vector of policy parameters: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$ which receives the minimum value of negative utility. Minimizing the negative utility is the same as maximizing the utility.