Preventive monetary and macroprudential policy response to anticipated shocks to financial stability*

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Abstract

In this paper, we develop a simple framework to study optimal macroprudential and monetary policy interactions as a response to a financial shock. Our model combines together nominal rigidities and capital accumulation – features that were usually studied separately in the previous literature. In our model, we show that the agents do not internalize how their assets purchases affect assets prices. Thus, when a crisis occurs, there are fire sales – less demand for capital further reduces their price and agents are worse off. Policy intervention (both monetary and macroprudential) can improve allocations by restricting borrowing ex-ante (during the accumulation of risks and imbalances) and stimulating the economy ex-post (during a crisis). As a result, we find a complementarity relation between ex-ante monetary policy and preventive macroprudential policy. We also compare this result with a flexible prices model and frictionless model, and conduct several sensitivity analyses exercises.

Keywords: Macroprudential policy, monetary policy, pecuniary externalities, nominal rigidities, financial frictions, capital accumulation.

JEL Codes: E44, E58, G28, D62

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1 INTRODUCTION

Monetary policy is widely recognized as a central bank's main tool to manage inflation. Indeed, before the financial crises (e.g. Financial crisis of 2007–2008) interest rate was the primary instrument to achieve central banks' objectives.

However, after the Financial Crisis 2008 the role of macroprudential policies has increased dramatically because Financial crises have shown that financial stability is still an important cause for concern for Central banks. The initial economic shock arising from the US housing market raised the prospect of a higher probability of default, but most importantly led to a vicious spiral where default concern led to a fall in asset prices. This fall in asset prices then reinforced concerns about banks and other financial intermediaries' solvency, and this further reduced liquidity in a range of asset markets, with a variety of self-amplifying spirals then bringing the whole financial system to its knees.

Lack of liquidity dries up key financial markets, thus preventing institutions from restructuring their portfolios, adapting their strategies, and steering away from potential dangers caused by exogenous economic shocks. In turn, defaults start accumulating, and the domino effect leads to further reductions in liquidity and ultimately causes financial institutions, corporations, and other non-financial bodies to fail to meet their contractual obligations.

In this paper we respond to this challenge to analyse the simultaneous usage of both monetary and macroprudential policies. We study a three-period model with nominal rigidities and financial frictions. We refer to the first and the second periods in the model as medium-run and the third period is a metaphor for a long-run period. In the first period agents are free to choose their allocations (consumption and borrowing decisions), then in the second period a crisis can come with a certain probability. In other words, financial/borrowing constraint (which limits the ability to borrow up to a fraction of market values of capital) may bind, and we refer to a crisis period when this constraint binds.

To reduce the severity of the crisis policymaker can use ex-ante macroprudential and monetary policy policies to limit borrowing, and ex-post monetary policy to stimulate an economy during a recession. In this framework we analyze the optimal coordination between preventive monetary and macroprudential policies seeking to reduce the impact of anticipated financial shocks.

To do it, we assume the following environment with a continuum of agents, namely, households and firms with linear technology. Households are endowed with some amount of a capital good, also they invest part of the final good to create new capital in the first period and after consume all the final. Households fund firms with linear technology and receive profits from them. Besides, households own their inferior concave technology. These concave or linear technologies are used for the production of raw good using capital as inputs. Linear technology firms are more effective users of capital but they can be financially constrained which is the source of pecuniary externality. This externality influences the equilibrium prices because the firms do not internalize the effects of their private decisions on aggregate prices. Thus, in "normal" times firms borrow too much which leads to inefficient fire-sales during recessions. This is the source of the first inefficiency in our model which motivates using preventive macroprudential policy. Households with concave technology are inefficient users of capital, thus, in an economy without any frictions households are not willing to hold any capital while firms with linear technology are willing to hold all capital.

So, it is an important result in our paper, which is initially are not driven by our assumptions. In our model we assume that the agents are small enough to not have any impact on aggregate prices. Therefore, when the agents make their decisions, they do not take into account that their decisions can lead to pecuniary externalities. So, in our model we do not assume that the agents do not internalize externalities but it is obtained from the micro-foundations of individual decisions of agents.

Finally, the raw good, which is produced from the capital by any of these two technologies, combines with the labor and is used in the production of a final good. In our baseline version we assume that the prices of the final good are fixed in the first and the second period and fully flexible in the third. The price rigidity leads to an aggregate demand externality. In usual circumstances agents do not internalise how their private decisions affect aggregate demand which determines the price level in an economy which in the case of rigid prices influences consumption choice.

To put it differently, overborrowing and rigid prices increase the severity of a crisis leading to inefficient allocations. It creates a rationale for using both monetary policy and macroprudential policy to target both aims – price and financial stability.

That is, the policymaker's tool are a macroprudential tax (Pigouvian taxes) (which influence the cost of debt of firms with linear technology in the pre-crisis period, thus it reduces the size of fire-sales), and ex-ante state non-contingent monetary policy nominal rate in the pre-crisis period and ex-post state-contingent nominal rates in post-crisis periods. These rates work as usual monetary policy tools and deal with nominal rigidities in the model.

Finally, we show that Social Planner who internalize the externalities can improve allocation and reducing the severity of the crisis. To solve the Social Planner problem we use numerical methods because our model does not have a closed-form solution. Moreover, we also exploit the "simplicity" of our model which allows us to solve our model without any log-linearization techniques, thus we abstract from any quadratic form policy objective. Therefore, Social Planner can simply maximize the weighed utility of agents. This "simplicity" comes from a three-period framework, which gives us a non-linear system to solve. The result of solving this problem is the four optimal parameters (one macroprudential tax and three monetary policy rates) which deliver the maximum utility of agents.

The resulting framework has a rich structure, allowing us to find an answer to our main research question, namely, to characterize optimal coordination between monetary and macroprudential policies in economies with pecuniary and aggregate demand externalities and financial frictions. In other words, in this paper we what to show how monetary and macroprudential policies interact. And we ask, are these policies substitutes or complements?

Under the presence of multiple externalities and the evolving nature of markets, in particular, for short-term funding, the forthcoming regulatory architecture should recognise that there are markets that are "too important to fail" and not only banks that are "too big to fail". Hence, regulation, and policy more generally, should also be focused on "systemic markets" as well as "systemic institutions".

The ability to adjust monetary policy appropriately to economic conditions requires the measurement of inflation and output growth. Likewise, maintenance of financial stability requires equivalent measures of financial fragility. Such measures could be used as a yardstick to assess the success of the regulatory policy.

Recent crises have actively promoted thinking not only about price stability but also about financial stability (Cerutti, Claessens, and Laeven 2017). For instance, the average number of macroprudential instruments[1] almost doubled and these instruments have been tightened more after GFC. Therefore, most central banks actually have been targeting two aims: price and financial stability using two sets of instruments: monetary and macroprudential measures.

Thus, many countries face a trade-off between price and financial stability, namely, a question of either these two policies are substitutes or complements. Hence, to target both aims it is important to understand the optimal interaction between monetary and macroprudential policies. The purpose of this paper is to provide a useful framework for policymakers regarding of joint use of monetary and macroprudential policies.

From the perspectives of economic literature, the focus of many previous papers was on looking at either optimal monetary policy or analysing macroprudential policy. Optimal monetary policy design is well studied in economic literature (Woodford 2003; Clarida, Gali, and Gertler 1999). Moreover, recent adventures on optimal monetary policy are focused on the analysis of heterogeneous agents models (Kaplan, Moll, and

[1] Caps on loan-to-value and debt-to-income ratios, limits on credit growth and other balance sheet restrictions, (countercyclical) capital and reserve requirements and surcharges, and Pigouvian levies.

Violante 2018) in which the agents are heterogeneous in the amounts of liquid and illiquid assets.

Also, the question of whether macroprudential policy can be a substitute or complement to monetary policy arises when monetary policy is limited or restricted in certain sense. These inefficiencies of monetary policy can take different forms. For instance, when a monetary policy is bounded below (such as in Zero Lower Bound, ZLB, Eggertsson and Krugman 2012) expansionary fiscal policy becomes effective. Fixed exchange rate and "fear of floating" (Calvo and Reinhart 2002; Dávila and Korinek 2017) are other examples when monetary policy becomes inapplicable. Finally, when monetary tightening is used for financial stability purposes ("leaning against the wind" (Borio 2014; Woodford 2012) in fact leads to a weaker economy with higher unemployment and lower inflation but with a lower probability of a crisis. While if a crisis occurs, the severity increases.

Many papers now are devoted to justify the macroprudential policy and estimate its effects on the economy. From the theoretical side the closest papers to our paper study optimal time-consistent monetary policy in an economy with pecuniary externality (Bianchi and Mendoza 2018). They show that usual taxes on debt are ineffective, and they construct time-consistent policy rule which delivers the maximum utility. Stavrakeva 2020 in a similar model with pecuniary externality studies the optimal capital requirements and finds that countries with larger fiscal capacity should implement lower bank capital requirements. The interaction between capital and liquidity regulations was studied in Kara and Ozsoy 2016, where the authors show that more strict capital requirements reduce fire-sales but the banks are responding to this tightening by decreasing liquidity. Macroprudential policy on liquidity requirements can improve these allocations. Moreover, capital regulation is also studied in economies which is subject to default (Clerc et al. 2015). From an empirical perspective, the authors (Auer and Ongena 2019) study the introduction of the countercyclical capital buffer (CCyB) in Switzerland. They find that such macroprudential policy leads to higher growth in commercial lending, higher interest rates and higher fees charged to the firms. The importance of loan-to-value (LTV) limits is studies by Alam et al. 2019, the authors use a huge panel of 134 countries from January 1990 to December 2016 and find nonlinear effects on household credit of LTV changes, meaning that the effect of LTV tightening is diminishing with the size of LTV.

Besides, there is a growing strand in the literature studying the combinations of monetary and macroprudential policies. The most influential paper for us written by Basu et al. 2020. We are also familiar with Adrian et al. 2020 but their paper focuses on quantitative results while we go in line with Basu et al. 2020 and choose quality or descriptive approach and develop a model with a rich structure that gives us a qualitative answer, not a quantitative one. However, we differ from Basu et al. 2020 because they fix the amount of capital in their model (land in their terminology), while we allow agents to endogenously accumulate capital. It gives us room to study the transmission of macroprudential policy using two channels. The first one is the same as in Basu et al. 2020, the preventive macroprudential policy restricts the amount of borrowing in the pre-crisis period. Thus, with positive macroprudential taxes firms with linear technology own only a part of available capital in the economy and the rest is owned by households (who have inferior technology). The second channel is a new one. With endogenous capital accumulation, positive macroprudential taxes also limit the amount of this accumulation. In other words, macroprudential taxes constrain the creation of new capital. Therefore, macroprudential policy works in two directions to prevent excessive fire-sales.

Also, there are studies where authors examine both monetary and macroprudential regulation together. For instance, Cozzi et al. 2020 study the interactions between monetary and macroprudential policies. Van der Ghote 2021 study monetary and macro-prudential policy intervention in a general equilibrium economy with recurrent boom-bust cycles. The author shows that there are welfare gains from policy coordination, in particular, it is optimal to intervene with macroprudential measures during the boom phase which also has effects during busts. Moreover, contractionary monetary policy during booms also helps to make bust less severe. Furthermore, Darracq Pariès, Kok Sørensen, and Rottner 2020 find the strategic complementarities between monetary and macroprudential policy.

In line with these papers, we also study the interaction between monetary and macroprudential policies. We build our model based on ideas of pecuniary externality Dávila and Korinek 2017 and aggregate demand externality (Farhi and Werning 2016). We differ from these papers in two ways: we study capital accumulation in a framework with sticky prices whereas in the previous papers there are either no capital accumulation but there are sticky prices or there is capital accumulation without sticky prices. We are similar to Dávila and Korinek 2017 because we consider similar economies with pecuniary externalities and financial frictions. Hence Dávila and Korinek 2017 allow endogenous capital accumulation, we still differ from their work because we consider our economy with nominal rigidities, while in Dávila and Korinek 2017 there are no nominal rigidities. The second complementary strand of literature focuses on economies with nominal rigidities (as in Farhi and Werning 2016; Schmitt-Grohé and Uribe 2016; Korinek and Simsek 2016). The main driving force of inefficiencies in these models is aggregate demand externality. Similar to them we add nominal rigidities in the form of sticky prices. However, these models usually assume a fixed amount of capital while we allow endogenous capital accumulation.

Need to mention that in papers with financial/collateral constraints there is a possibility of multiple equilibria, for instance, Schmitt-Grohé and Uribe 2021 determine such possibility. Although, in our paper we focus on over-borrowing equilibria, Schmitt-Grohé and Uribe 2021 show that there also exist equilibria with under-borrowing. While it is an important issue to discuss, we leave it to further research.

Many previous cited papers use the infinite-horizon model, thus, to solve such models, authors use different approximation/log-linearization techniques. We contributed to it by using a three-period framework which allows us to solve the model straightforward without any approximation and log-linearization techniques. It means we find the most accurate solution for our model and do not lose any information because of different non-linearities.

Finally, for the case of policy implications this three-period framework is easily extendable to any other number of periods. For instance, we can extend the number of periods when the financial constraint can bind. We also can add an ex-post macroprudential policy to study if there is room to improve allocations if a crisis has already occurred. So, there are a lot of ways to extend this framework at a small price, we leave it to further research.

The rest of this paper proceeds as follows. In Section 2 we describe the baseline model. In Section 3 we present our main results. In Section 4 we provide sensitivity analysis (show how the results differ when we change parameters values). In section 5 we discuss policy implications. In Section 6 we state our remarks and limitations of the current model. Finally, in Section 7 we conclude.

2 MODEL

In this section, we construct a closed economy model in a New Keynesian way in the spirit of Dávila and Korinek 2017; Farhi and Werning 2016. The model consists of households, final goods firms and capital sector firms. Effectively, we have a simple three-period framework but additionally, assume that after the last period all variables stay at their steady states. Thus, we write optimization problems of agents as three-period problems. In this section, we describe a baseline version of our model. In Appendix B one can find FOCs for a fixed prices model and in Appendix C for a flexible prices.

CONSUMERS Households maximize a utility function over consumption and linear disutility of labor.

$$\max_{\{c_t,h_t,b_t\}_{t=0}^2,\{k_t^{concave}\}_{t=1}^2,inv_0} \mathbb{E}_0 \sum_{t=0}^{2} \beta^t \left[\log c_{t,s} - h_{t,s}\right], \tag{1}$$

Their maximization is subject to a budget constraint:

$$c_0 + b_1 + inv_0(1 + \frac{\phi}{2}\frac{inv_0}{k_{-1}}) + q_0(k_0^{concave} - inv_0 - k_{-1}) + n_0^{linear}$$
$$= w_0 h_0 + p_{x,0} k_{-1} + \Pi_0^{final}, \tag{2}$$
$$c_{1,s} + b_{2,s} + q_{1,s}(k_{1,s}^{concave} - k_0^{concave})$$
$$= w_{1,s} h_{1,s} + (1 + i_1) b_1 + p_{x,1,s} A_{1,s} \log(1 + k_0^{concave}) + \Pi_{1,s}^{final}, \tag{3}$$
$$c_{2,s} + b_3 + q_{2,s}(k_2^{concave} - k_{1,s}^{concave})$$
$$= w_{2,s} h_{2,s} + (1 + i_{2,s}) b_{2,s} + p_{x,2,s} A_{2,s} \log(1 + k_{1,s}^{concave}) + n_2^{linear} + \Pi_{2,s}^{final}, \tag{4}$$

where c_t is the consumption, h_t is labor supply of households, inv_0 is the investments, made in $t = 0$, k_{-1} is the initial endowment of capital, q_t is the price of capital in t, $k_t^{concave}$ is the amount of capital that is in the hands of households (i.e. the amount of capital that is used for production with concave technology), n_0^{linear} is the initial financing from households to firms with linear technology, w_t is the wage, $p_{x,t}$ is the relative price of raw inputs (intermediate good that is used in the production of the final good), b_t is the one-period loans to firms with linear technology, Π_t^{final} is the profits of a final good producer, n_2^{linear} is a net worth of firms with linear technology given back to households (like dividends/profits). Moreover, in the optimization problem of households there are also $i_{t,s}$ is the nominal interest rate, which is governed by the policymaker.

Note, that subindex s indicates state-contingent variables and takes values $s = L$ if the state is low (bad, when financial constraint binds) and $s = H$ if the state is high (good, when financial constraint does not bind). The variables without subindex s denote that these variables are non-contingent, thus their values are independent of a state of nature. In other words, we assume that markets are incomplete.

LINEAR FIRMS Linear sector firms are perfectly competitive and initially financed by households with an amount of n_0^{linear}. Linear firms are unconstrained at $t = 0$ and $t = 2$ but might by financially constrained with positive probability at $t = 1$.

A typical firm maximizes its discounted profits:

$$\max_{\{n_t^{linear}, d_t, k_t^{linear}\}_{t=1}^2} \beta^2 \frac{\lambda_2}{\lambda_0} n_2^{linear} \tag{5}$$

subject to the following constraints:

$$q_{t,s} k_{t,s}^{linear} = n_{t,s}^{linear} + d_{t+1,s}^{linear} \; \forall t \geq 0 \tag{6}$$

$$d_{2,L}^{linear} \leq \kappa q_{1,L} k_{1,L}^{linear} \tag{7}$$

$$n_{1,s}^{linear} = p_{x,1,s} A_{1,s} k_0^{linear} + q_{1,s} k_0^{linear} - (1+i_1) d_1^{linear} (1+\theta_1) + t_1 \tag{8}$$

$$n_{2,s}^{linear} = p_{x,2,s} A_{2,s} k_{1,s}^{linear} + q_{2,s} k_{1,s}^{linear} - (1+i_{2,s}) d_{2,s}^{linear} \tag{9}$$

$$t_1 = -\theta_1 (1+i_1) d_1^{linear}, \tag{10}$$

where (6) represents balance constraint for each period, (7) is financial constraint which binds in $t = 1$ with a positive probability, the next equations (8), (9) for n_t^{linear} represent the evolution of net worth of linear firms and show that net worth consists of revenues from the production, plus the market price of owned capital, net of debt repayments with interest, $(1 + i_t)d_t$, and reverse tax transfers (equation (10).

In each period linear firms can raise the amount of capital at the market price up to the amount of their net worth (n_t^{linear}) and required amount of borrowing (d_t) according to equation (6).

FINAL GOODS Here we will follow the New Keynesian tradition and assume a monopolistic competition between firms. Firms use the raw inputs (x_t) and labor (h_t) to produce a final good (y_t). The inputs are produced using either linear or concave technology and are traded at a relative price $p_{x,t}$, labor is supplied by households for a wage w_t. Then:

$$y_t(j) = A_t^H h_t(j)^\alpha k_t(j)^{1-\alpha} \tag{11}$$

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{12}$$

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{-\epsilon} y_t \tag{13}$$

For now in $t = 0;1$ we assume fully fixed prices, we take this idea from Lorenzoni 2008. It means that these prices are inherited and determined in some previous periods $t = -1$. However, starting from $t = 2$ we assume fully flexible prices. Therefore, for flexible prices we can write:

$$1 = \frac{\epsilon}{\epsilon - 1} \left(\frac{w_t}{\alpha} \right)^\alpha \left(\frac{p_{t,s}}{1-\alpha} \right)^{1-\alpha} \tag{14}$$

We conclude the description of the model with market clearing conditions.

MARKET CLEARING

$$c_0 + inv_0 (1 + \frac{\phi}{2} \frac{inv_0}{k_{-1}}) = k_{-1}^\alpha h_0^{1-\alpha} \tag{15}$$

$$c_{t,s} = x_{t,s}^\alpha h_{t,s}^{1-\alpha} \forall t \geq 1, \tag{16}$$

$$x_{t,s} = \log(1 + k_{t,s}^{concave}) + k_{t,s}^{linear}, \tag{17}$$

$$k_{t,s}^{linear} + k_{t,s}^{concave} = K, \tag{18}$$

$$K = k_{-1} + inv_0, \tag{19}$$

$$b_{t,s} = d_{t,s}, \tag{20}$$

where equation (15) shows resource constraint for the $t = 0$, which means that agents spend all output in the economy on their consumption and producing new capital inv_0. Equation (16) shows resource constraint for the rest periods, when agents spend all output in the economy on their consumption. Equation (17) indicates the total amount of raw inputs x_t, which equals to the sum of capital-intensive output using two technologies: concave and linear. Equation (18) determines that total amount of capital is divided between firms with different technologies. Equation (19) specifies that total amount of capital in the economy, which sum of initial endowment k_{-1} and new produced capital inv_0. Finally, equation (20) establishes that consumers fully finance linear sector firms.

EQUILIBRIUM The equilibrium in the economy is the following. A real allocation $\{\{c_{t,s}, x_{t,s}, h_{t,s}, y_{t,s}\}_{t=0}^{2}, \{inv_0\}, \{k_{t,s}^{linear}, k_{t,s}^{concave}\}_{t=0}^{1}, \{d_{t,s}, b_{t,s}\}_{t=1}^{2}\}$ an asset allocation with a set of prices $\{\{w_{t,s}, q_{t,s}, p_{x,t,s}\}_{t=0}^{2}\}$, when agents solve their optimization problem (1) and (5) subject to their constraints, and all markets are clear, i.e. equations (15), (16), (17), (18), (19), (20) hold for all dates and states.

SOCIAL PLANNER To find optimal values we introduce a Social Planner which internalizes externalities. The optimization problem of Social Planner of the following. Social Planner takes the utility of all agents in the economy with corresponding Pareto weights and resource and financial constraints in the economy. In our case, the Social Planner problem becomes similar to Ramsey problem of finding optimal values of "taxes". While some papers assume that a Social Planner chooses only initial allocations (in $t = 0$), and this may be enough to correct externalities. In our paper we assume that a Social Planner maximizes a whole lifetime utility of agents. From our perspective, it does not affect the analysis.

We need to note that the problem cannot be solved in the closed-form, thus we use numerical methods to solve it. The full algorithm is described in appendix D. The main idea is that we use non-linear solve to find the solution for a fix set of policy parameters θ_1, i_1, $i_{2,L}$ and $i_{2,H}$. This non-linear solver gives us the optimal allocations of endogenous variables. Then, we apply minimize function to this solver to find the set of policy parameters that delivers a minimum of the negative value of utility. This "minimizer" uses the simplex search method of Lagarias et al. 1998, implemented in

Matlab. The minimize function searches along a set of policy parameters θ_1, i_1, $i_{2,L}$ and $i_{2,H}$ to determine the minimum. Thus, this set is our solution that gives the maximum value of utility (minimum of negative utility), i.e. the values of policy parameters.

3 RESULTS

In this section, we present our main results. First of all, we need to establish a version with which we will compare our results. A natural choice is to take a similar model without any price rigidities (you can find this model in Appendix C) (we refer to this model as flexible prices version). The main difference with the model in the previous section is that with fully flexible prices we do not need any monetary policy and the real rates determine endogenously in the model. Moreover, in our baseline model with fixed prices, the policymaker manages the nominal rates but, in fact, it sets the real rates (because of fully fixed prices). Hence, we take the equilibrium real interest rates from the model with flexible prices as an indicator of "neutral" monetary policy. To show numerical results we use parameters which are indicated in the table 1 in appendix A.

Although part of these parameters is commonly used in the literature, there is a need to discuss the choice of some of them. We take a discount factor, β, equals 0.8. While it is not a "popular" value, it is used in Basu et al. 2020. The motivation to use such a low discount factor follows from the high degree of stylization of the model. Our three-period framework represents medium (the first two periods) and long-run (the last period), therefore, these periods should be discounted more than in classical models (where a step of one year/quarter is usually assumed between the periods). On another hand, we actually need to have relatively high interest rates because, again, the difference between our periods is much larger than a year, thus, we match the steady-state interest rate to $\frac{1}{\beta} - 1 = 0.25$. Share of capital in production, α equals to $\frac{1}{3}$, which common value in the literature, representing this long-lasting phenomenon. We calibrate the price markup to 6 to fit the average price markup of 20%.

We fit an initial endowment of capital goods of households, k_{-1}, to 0.85, which reflects the average value of a ratio of net worth to total assets of households and nonprofit organizations according to the US data from 1995 to 2020. We think that it is the closest economic indicator for our model because the endowment of capital goods of households stands for their initial wealth. We take this idea from the quantitative model of Bernanke, Gertler, and Gilchrist 1999. Also, this calibration, to some extent, is similar to Basu et al. 2020, yet we differ from their paper. In Basu et al. 2020, the authors assume a fixed amount of capital (or land according to their terminology), in our paper there is endogenous capital accumulation and, therefore, we use the calibrated value as an initial value for capital endowment.

For initial financing of firms with linear technology, n_0^{linear}, we take a calibrated value of 0.53 from the US data, representing a ratio of net worth to total assets of the nonfinancial corporate business on a sample from 1995 to 2020 (the idea is taken from Bernanke, Gertler, and Gilchrist 1999). We take this ratio because it is the closest economic indicator for our model, since n_0^{linear} represent the source of net worth for firms. In this case, there is no direct and observed economic indicator to calibrate this value because of the high degree of stylization of our model. Furthermore, there is no direct mapping from our model to Basu et al. 2020 (the closest to our model). Having the natural restriction that the initial financing of firms with linear technology can not be larger than the initial endowment of households (our calibration satisfies this restriction), we also can compare our value with an initial debt level from Basu et al. 2020 of 0.6. So, our calibration is quite close to it.

Moreover, we also take values for probability of a bad/good shock, ρ_L, from Basu et al. 2020 and set it to 0.5. This choice may look controversial, however, the reader should not consider this probability as a probability of a crisis in multi-period models (which are usually calibrated to 3%-5%). In our model, it reflects the probability of two states in the medium term similar to Basu et al. 2020.

A parameter that governs the toughness of the debt limit in equation (7), κ, calibrated to 0.025, also according to Basu et al. 2020. This low value indicates that a crisis will be really tough requiring firms with linear technology to borrow only 2.5% from their current market value of a capital stock. Finally, capital adjustment we set to 1, similar to Uribe and Schmitt-Grohé 2017.

Before showing our result we clarify our terminology. In the current and the next sections we consider several versions of the model from section 2. As a baseline, we refer to a model with nominal rigidities and financial frictions. Besides the baseline, we step by step abandon our assumptions and consider the next versions: a model with flexible prices and financial frictions, a frictionless model with nominal rigidities, and a frictionless model with flexible prices. These four versions cover all cases with inefficiencies: both financial frictions and rigid prices. Therefore, to show how inefficiencies actually disturb optimal allocations we need to compare the results between these four versions.

Formally, a baseline version is fully described in section 2. In a model with flexible prices and financial frictions, we relax the assumption about fully fixed prices in periods $t = 0; 1$. It means that equation (14) holds for all periods $t \geq 0$. Moreover, with fully flexible prices there is no need for monetary policy, thus, nominal interest rates (which are exogenous for a baseline) become endogenous and real rates under flexible prices: $r_1, r_{2,L}$ and $r_{2,H}$. This does not affect macroprudential policy θ_1 which is left as the only exogenous policy tool. These two versions are main in our analysis, frictionless models are supportive. In a frictionless model with nominal rigidities, we relax the assumption about binding financial constraints in a "low" state. So, we just remove equation (7) from our model, meaning that in both "low" and "high" states firms with linear production technology never meet this constraint. While we keep the assumption about fully rigid prices in periods $t = 0; 1$. In a frictionless model with flexible prices, we simultaneously remove both assumptions about rigid prices and binding financial constraints. This is a fully frictionless economy, which delivers the first-best allocation in our model.

RESULT 1. To present our main result we compare optimal policies in the baseline economy with the other three versions, discussed above. Our finding is presented in Result 1 and in figure 1. In this figure we show 4 sub-figures for each of the policy instruments, also a grey dashed line shows a steady-state value ($\frac{1}{\beta} - 1$) for corresponding sub-figures. In each sub-figure we plot four bars for each of the models. The first bar (dark red) is our baseline model with nominal rigidities and financial frictions. The second bar (dark blue) is a model with flexible prices and financial frictions, the third bar (light red) is a frictionless model with nominal rigidities. Finally, the fourth bar (light blue) is a frictionless model with flexible prices. To be simple: dark and light red bars differ in frictions/frictionless models for fixed prices. Dark and light blue bars differ in frictions/frictionless models for flexible prices.

Also, we need to mention that we go along with other articles and study constrained efficient allocation because only this model gives us a rich structure of inefficiencies that require macroprudential and monetary intervention. Moreover, our model is built around the emergence of financial friction which is the source of pecuniary externality in this framework. Therefore, from the perspective of our analysis, there is no need to consider models without financial frictions. Thus, it is natural to compare allocations with fixed and flexible prices with the presence of financial frictions. However, for clarity of our analysis and to understand the outcome of the model in an ideal (i.e., frictionless) environment we also show the results for fixed and flexible prices without the presence of financial frictions in figure 1. Further in the paper, we examine how the different frictions in the model affect the outcome and discuss models only with financial frictions and allocations which are only constrained efficient.

Result 1 (Complementarity in static) *In response to anticipated financial shocks, it is optimal to tighten both monetary and macroprudential policy. This Result is illustrated in figure 1.*

We begin with a discussion of the frictionless economy in figure 1 it is the last two light red and blue bars for fixed and flexible price models. We see that optimal macroprudential policy, θ_1, is larger for the first-best model (i.e. model without any

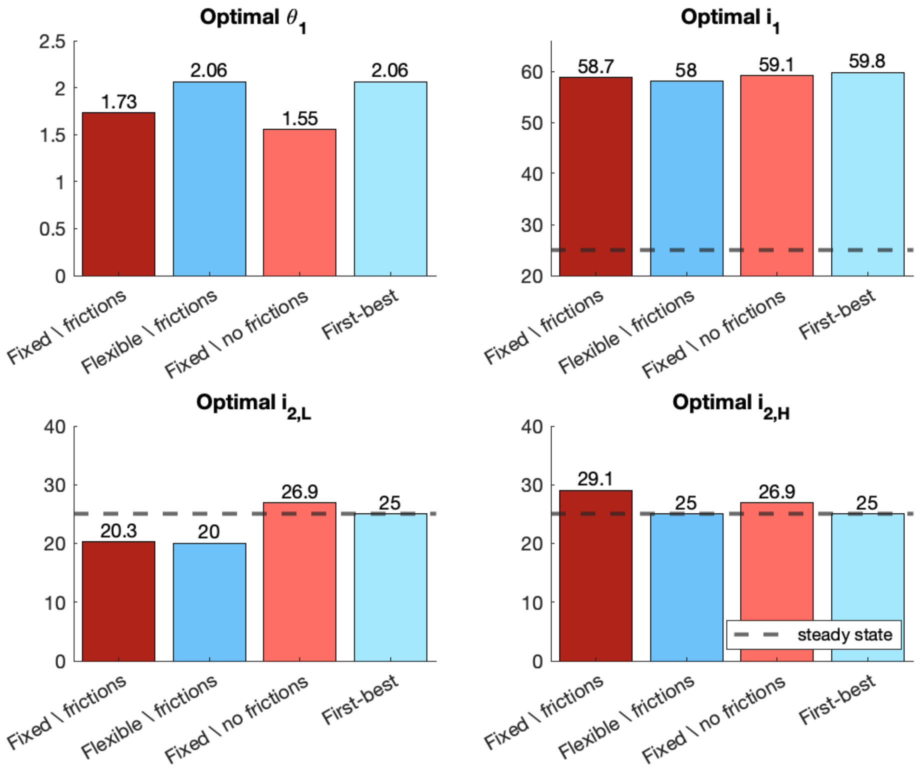

Figure 1: Complementarity in static:
Optimal policy
fixed prices vs flexible prices

frictions and rigidities). The optimal ex-ante monetary policy, i_1, is almost the same. However, we see some differences for ex-post rates. For first-best allocations optimal ex-post interest rate, $i_{2,L} = i_{2,H}$, sets at its steady-state level: $\frac{1}{\beta} - 1$. For a frictionless economy with fixed prices optimal ex-post interest rate, $i_{2,L} = i_{2,H}$, becomes slightly higher than steady-state value. Hence, both models are frictionless, "low" and "high" states are almost the same but nominal rigidities are important. This is an expected result because in the frictionless without nominal rigidities there are no reasons to deviate from steady-state values and it is not the case, when we consider the frictionless economy with nominal rigidities. It indicates the inability of firms to reset their prices to optimal ones. Thus, requiring monetary policy intervention.

Now, we turn to more interesting models with financial friction. It is the first two dark red and blue bars in Figure 1 for fixed and flexible price models. We see that in both versions optimal θ_1 is positive and in a flexible prices version, it is significantly higher. Moreover, optimal θ_1 coincides in the model with financial friction and flexible prices and in the first-best model. A reader may ask, why θ_1 is positive even in the first-best model? The reason is that the economy in period $t = 0$ start from some initial values (for example, calibrated values of k_{-1} and n_0^{linear}). These values may not always be at their steady states. Therefore, the economy sees the need for macroprudential (and monetary) intervention to change allocation in $t = 0$. Moreover, the fact that optimal θ_1's coincide in model with financial friction and flexible prices and in the first-best model may indicate that not only the emergence of financial frictions motivates to use macroprudential tools but also the sub-optimal initial allocations may be enough to intervene.

Next, optimal i_1 is almost the same in both versions (with financial frictions) but for fixed prices, it is slightly higher. Optimal $i_{2,L}$ are almost the same. Optimal $i_{2,H}$ for flexible prices takes the steady-state value ($\frac{1}{\beta} - 1$) but for fixed prices it is significantly higher.

The economic intuition is the following. In response to anticipated binding collateral constraints, it is optimal to tighten both ex-ante macroprudential and monetary policies, continue tightening if a crisis does not occur, while if a crisis occurs it is optimal to soften ex-post monetary policy. So, tightening policy before the crisis limits agents from excessive borrowing, therefore, if a crisis occurs, fire sales are less. This tightening happens for both fixed and flexible prices in economies with financial frictions. Moreover, if a crisis occurs, then lowing monetary policy stimulates agents to consume and spend more. It also holds for both fixed and flexible prices in economies with financial frictions. While if a crisis does not occur, the policy remains tight only if prices are rigid which indicates the inability of firms to set prices optimally (responding to a no crisis state). It means that under fixed prices paradigm trying to soften a crisis ex-ante (by tightening macroprudential and monetary policy), a policymaker may end up with tight monetary policy even there is no crisis, while if prices are fully flexible the economy is fully restored from previous actions (even if a crisis noes not occur). This result also highlights the importance of assessing the degree of price rigidity in the economy to better understand the behaviour of the economy during a crisis and the speed of recovery after the crisis.

We call this result as complementarity in static because as we see both macroprudential policy and monetary policy are tougher before a crisis. So, both policies (θ_1 and i_1) work in a similar manner and in the same direction.

Note that the values look not realistic, this is because of current calibration and simplifying assumptions. Due to the fact, that the model is highly stylized, it is impossible to choose such parameters to match the realistic values of interest rates. Therefore, we consider our model as qualitative, not quantitative.

RESULT 2. The next important result that we want to emphasise is the following. We want to see how each of the instruments behave as a function of all other policy instruments. However, to see it, we need to plot a 4-d figure, which is impossible. Thus, we fix two out of 4 instruments and plot the rest like functions from each other.

Since our paper is mainly focused on the preventive policy we plot θ_1 as a function of i_1 while keeping $i_{2,L}$ and $i_{2,H}$ at their optimal values (at their optimal values for every pair of θ_1 and i_1), and the opposite, we plot i_1 as a function of θ_1. The interaction of these two lines indicates our optimal solution. We can see it in figure 2 and in Result 2. The other functions you can find in figures 9 and 10 in appendix E. The idea for figures in the appendix is the same: we show the behaviour of one particular policy instrument as a function of another policy instrument (keeping the rest instruments to their optimal values).

Result 2 (Semi-complementarity in dynamic) *In response to higher ex-ante monetary policy, optimal macroprudential policy shows an inverse U-shape curve, while in response to higher macroprudential policy, optimal monetary policy remains almost stable. This Result is illustrated in figure 2.*

In figure 2 the blue solid line (with circles sign) indicates θ_1 as a function of i_1, i.e. for every value if i_1 our algorithm finds optimal θ_1[2]. And the opposite, a red dashed

[2] We also find optimal $i_{2,L}$ and $i_{2,H}$ which are not shown. See appendix E for details

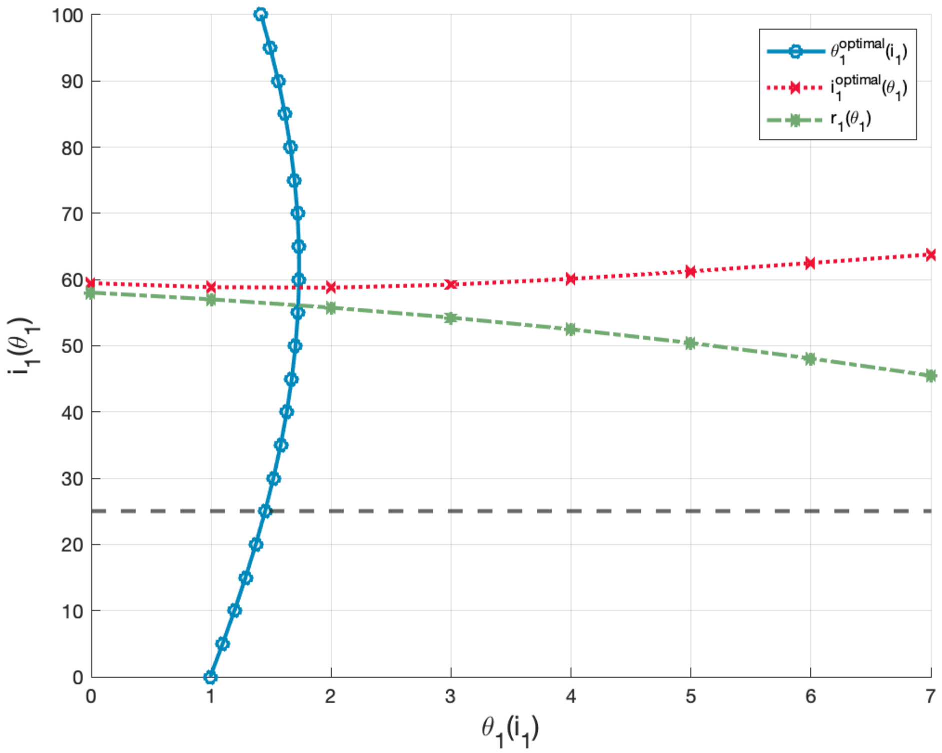

Figure 2: Semi-complementarity in dynamic

line (with cross sign) indicates i_1 as a function of θ_1, i.e. for every possible value of θ_1 we find an optimum value of i_1. A green dash-doted line (with asterisk sign) shows the optimal value of r_1 in the model with flexible prices (remember that in these two versions of model with flexible and fixed prices i_1=r_1). Finally, a grey dashed line points steady state value of these interest rates ($\frac{1}{\beta} - 1$).

We see that the red line (with cross sign) is almost insensitive to changes in θ_1 while the blue line (with circles sign) shows a U-shape curve, i.e. θ_1 grows when i_1 is small but with higher values of i_1 it declines. In other words, for small i_1 we tighten macroprudential policy but for high i_1 we ease. We call this result semi-complementarity in dynamic because the red line (with cross sign) remains almost constant (comparing with the green line (with asterisk sign) which shows policy easing), so in the model, with fixed prices, monetary policy remains tight, but not tightening for every possible value of macroprudential policy θ_1. So, these policies remain tight but with the tightening of one policy, the other policy does not necessarily also tighten. Thus, in dynamics these policies only partly complement, because we see the non-monotonic direction of the optimal macroprudential policy as a function of ex-ante monetary policy. This result highlights the fact that a policymaker should use macroprudential policy with caution because if the interest rates in the economy are already high, then it may be not optimal to continue tightening macroprudential policy.

RESULT 3. We also contrast our results with Basu et al. 2020 and look at both ex-ante (i.e. preventive) and ex-post policies. The economic intuition is the following. Every policymaker desire to prevent any economic instability using a variety of available tools which potentially reduce a probability of a crisis and the severity of a crisis (in our model it is θ_1 and i_1). However, during a crisis a policymaker has a desire to stimulate the economy to help to get through a crisis. In our model, these ex-post instruments are interest rates ($i_{2,L}$ and $i_{2,H}$). Our findings in using these ex-ante and ex-post rates are highlighted in Result 3 and in figure 3.

Result 3 (Ex-ante vs ex-post policies) *In response to anticipated financial shocks, it is also optimal to tighten monetary policy before the crisis and soften the expected interest rates during the crisis. While there is almost no difference in tighten of monetary policy before the crisis between fixed and flexible models, yet for flexible prices easing during a crisis is larger. This Result is illustrated in figure 3.*

In figure 3 we compare our results for fixed prices (left figure) and flexible prices (right figure). Moreover, we also plot three lines on each of the figures. Blue solid lines (with circle sign) show optimal policy for a low value of κ ($= 0.025$), which indicate a serve crisis, red dashed lines (with cross sign) show optimal policy for a medium value of κ ($= 0.25$), which indicates a mild crisis, and green dash-dotted lines (with asterisk sign) show optimal policy if a financial constraint does nor bind (crisis does not occur). The idea of showing these lines for different κ's is the following. In the model, κ indicates the severity of a crisis. The smaller the kappa, the more severe the crisis. So, we plot extreme cases, when there is a serious crisis, and the opposite when

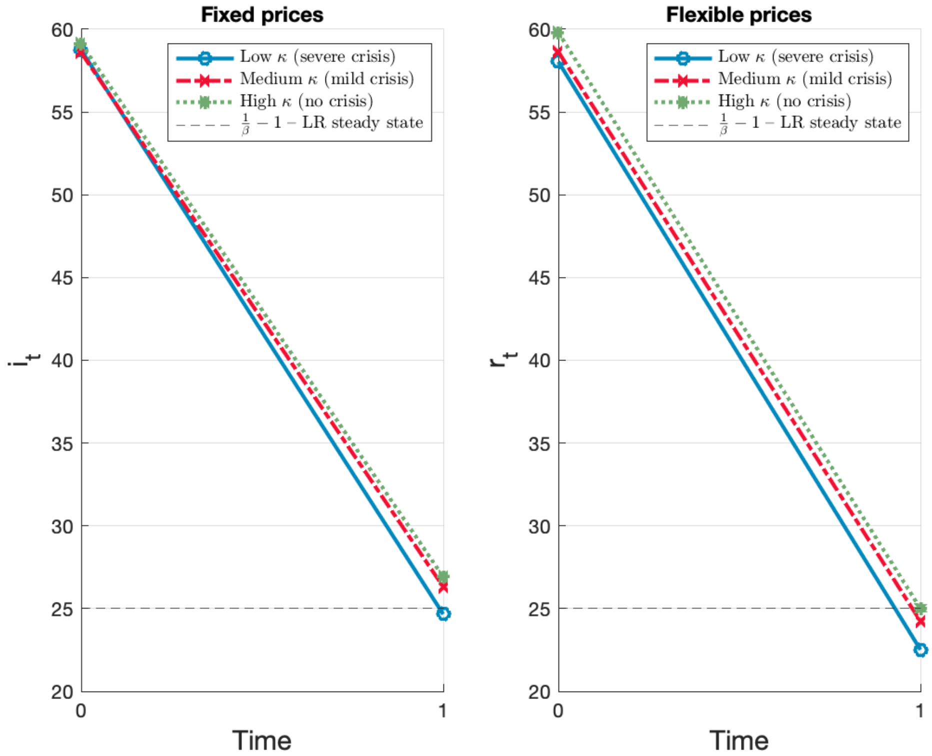

Figure 3: Ex-ante vs ex-post policies

a crisis is not so heavy. Finally, a dashed grey line points indicate the steady-state value of these interest rates ($\frac{1}{\beta} - 1$).

From figure 3 we again see that ex-ante (during the accumulation of risks and imbalances) monetary policy tightens while ex-post (during a crisis) monetary policy eases. Moreover, we see a difference both between these κ's and models. In the fixed prices model monetary policy tightening in the pre-crisis ($t = 0$) period is almost the same for all κ's, meanwhile in the model with flexible prices monetary policy tightening for high κ is stronger.

In a crisis period ($t = 1$) for a fixed prices model we see that for high κ easing is smaller[3], for a low κ easing is larger (even falls below its steady-state). It is important to note that a green line remains above its steady-state, which indicates the importance of nominal rigidities (otherwise it would have reached a steady-state).

While for flexible prices version we see that for low κ easing is even larger and the interest rates fall significantly below its steady-state. For a high κ ex-post interest rate

[3]Here, as an ex-post interest rate we plot the weighted interest rate: $i_2 = \rho_l i_{2,L} + \rho_H i_{2,H}$.

reaches exactly steady-state (no crisis – no need for ex-post policies). A red line that indicates a medium crisis always locates between extreme cases.

This highlights the fact that our model does not ex-ante pay much attention to the "size" of a crisis (because i_1's are very similar for both versions), however ex-post we see a much larger policy easing if κ is low.

To conclude this section, we find that in response to anticipated financial shocks in the form of debt limits, it is optimal to preventive tighten both monetary and macroprudential policies while in a crisis period it is optimal to support the economy and ease interest rates.

4 SENSITIVITY ANALYSIS

In this section, we present and discuss a series of supplemental results. Specifically, we provide sensitivity analysis. This discussion is based on the fact that our model is more qualitative than quantitative. Therefore, we cannot plausible calibrate all of our parameters which can be important when we determine optimal policy. Thus, in this section we vary initial capital endowment k_{-1}, an initial amount of financing firms with linear technology n_0^{linear}, debt/financing constraint limit κ, capital adjustment costs ϕ, a probability of a crisis ρ_L. In the next figures, we adhere to the following notation: blue solid lines (with circle sign) are optimal policies in fixed price model, red dashed lines (with cross sign) are optimal policies in a flexible model, grey dashed lines indicated a steady-state value of interest rates ($\frac{1}{\beta} - 1$).

SUPPLEMENTAL RESULT 1 We begin this section with the role of initial households' endowment, k_{-1}, to see the effect of k_{-1} on optimal values of policy parameters we vary it from 0.5 to 3. It seems more or less reasonable values for this parameter. Our findings are highlighted in Supplemental Result 1 and illustrated in figure 4.

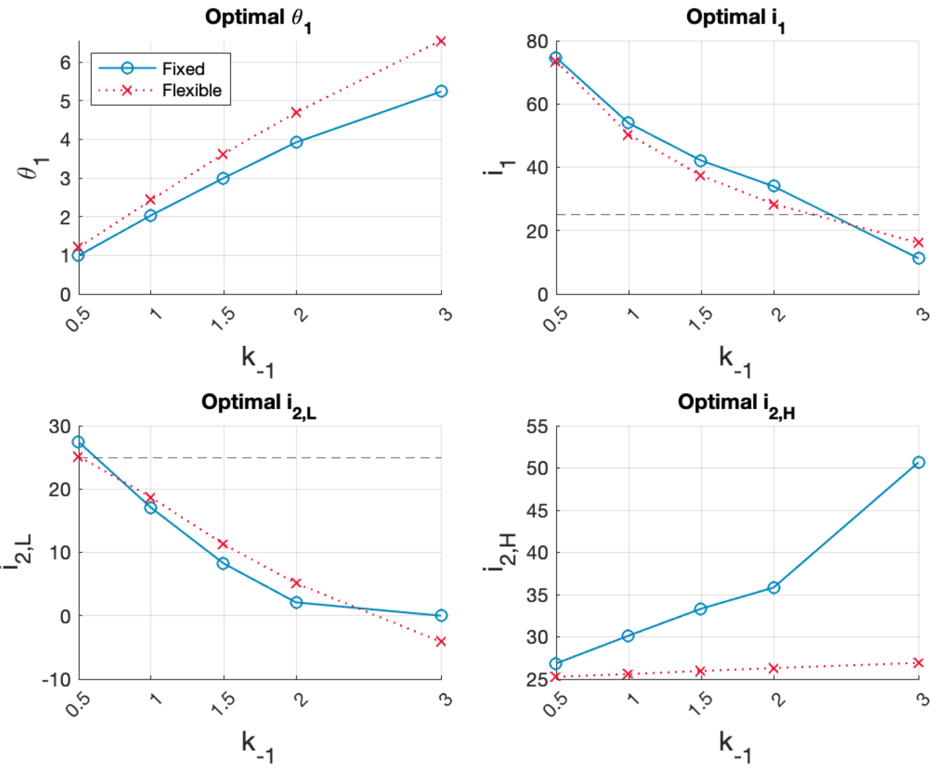

Figure 4: Supplemental Result 1 – change in k_{-1}

With this analysis we highlight the importance of initial households' endowment in determining the optimal policy.

Supplemental Result 1 (Change in k_{-1}) *With a larger endowment of k_{-1}, a policymaker needs to significantly tighten macroprudential policy, θ_1, ease ex-ante monetary policy, i_1, and also ex-post monetary policy in a crisis period, $i_{2,L}$, tighten ex-post monetary policy if the crisis does not occur, $i_{2,H}$. This Supplemental Result is illustrated in figure 4.*

The main economic take-away is the following. The initial households capital endowment is important when a policymaker uses macroprudential policy and monetary policy to soften the severity of a crisis. We see that if households are richer initially (have more k_{-1}), then a policymaker have to set quite a high ex-ante macroprudential policy, θ_1 and ex-post monetary policy, $i_{2,H}$. While it have to set ex-ante monetary policy i_1 and ex-post monetary policy, $i_{2,L}$ quite low. Policymaker softens ex-ante monetary policy to reduce the effect of high macroprudential taxes on aggregate demand. Also, it looks that such a financial crisis is more harmful for rich households because a policymaker significantly lowers $i_{2,L}$ in a crisis state and raise $i_{2,H}$ in no crisis state. And the opposite is true for poor households (when initial households capital endowment is low): policymaker sets macroprudential policy quite low, there is no need to compensate macroprudential taxes with low ex-ante interest rates (which stays quite high). Hence, a policymaker softens its ex-post policy noticeably less ($i_{2,L}$ is quite high for poor than for rich households) if a crisis occurs and sets a rate, $i_{2,H}$, very close to a steady-state.

Moreover, we see that for θ_1, i_1 and $i_{2,L}$ optimal policies in the fixed price model are very close to macroprudential tax and interest rates in the flexible prices model. However, for $i_{2,H}$ we see that for fixed prices a policymaker has to significantly increase an interest rate if households are rich initially. Furthermore, we see (especially for i_1 and $i_{2,L}$) that for some values of k_{-1} an optimal policy falls below a steady-state.

SUPPLEMENTAL RESULT 2 Next, we analyse the role of initial amount of financing firms with linear technology, n_0^{linear}, to see the effect of n_0^{linear} on optimal values of policy parameters we vary it from 0.1 to k_{-1}. Taking into account that n_0^{linear} should be less than the initial households capital endowment, it is our natural upper bound. Our findings are highlighted in Supplemental Result 2 and illustrated in figure 5.

Supplemental Result 2 (Change in n_0^{linear}) *With a larger amount of financing firms with linear technology, n_0^{linear}, the optimal macroprudential policy shows the inverse U-shape curve, i.e. it tightens when n_0^{linear} is low and it loosens macroprudential policy (lower but positive θ_1) when n_0^{linear} becomes higher. Also, a policymaker needs to tighten ex-ante monetary policy, i_1, and also ex-post monetary policy in a crisis period, $i_{2,L}$, and loosing ex-post monetary policy if the crisis does not occur, $i_{2,H}$. This Supplemental Result is illustrated in figure 5.*

This Supplemental Result shows two important features. First of all, we see that depending on n_0^{linear}, optimal θ_1 is non-linear for the fixed prices model. It rises when n_0^{linear} is low and it decreases when n_0^{linear} becomes higher. And two, we also see that there is a limit for interest rates and a low bound for θ_1. That means that for both these rates, we see that for larger n_0^{linear} it is optimal to tighten rates up to a certain limit.

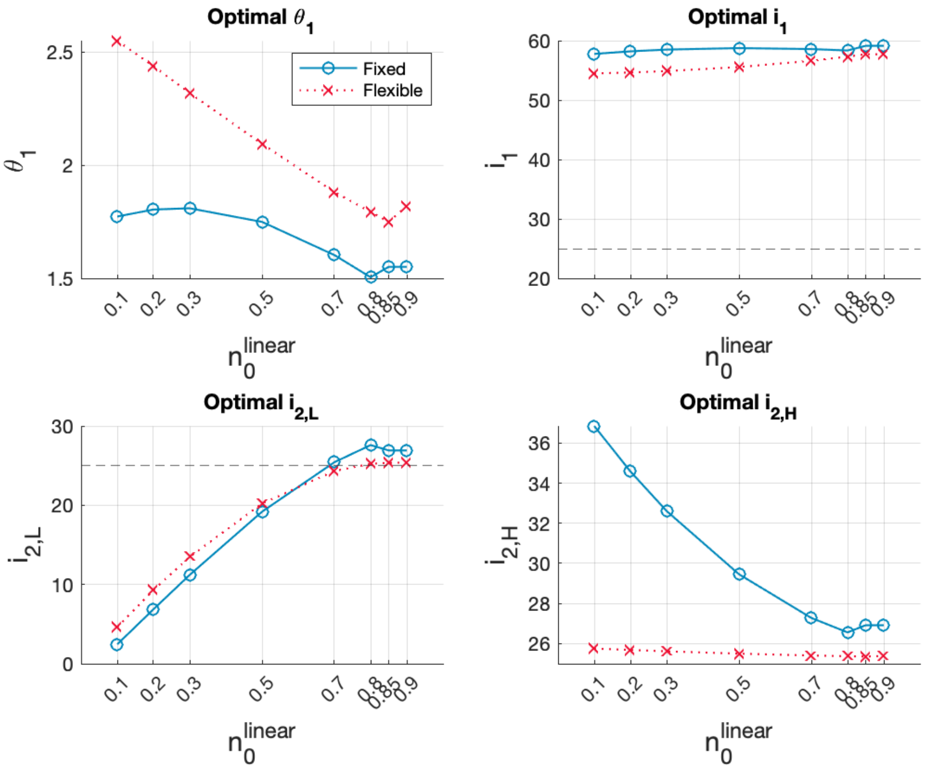

Figure 5: Supplemental Result 2 – change in n_0^{linear}

The opposite is true for easing of $i_{2,H}$, it is optimal to low the rate down to some limit. This highlights the result that if the economy consists of "rich" firms then monetary policy could be a constraint in both ex-ante (during the accumulation of risks and imbalances) and ex-post (during a crisis). Moreover, this limit is closely related to the upper limit of n_0^{linear}. By construction, firms with linear technology cannot receive financing more than the initial endowment of households. Figure 5 supports this claim because we see some strange behaviour around $k_{-1} = 0.85$, we consider this phenomenon as a limit for n_0^{linear}.

Figure 5 highlights important differences for fixed and flexible prices. If firms initially are poor (under-financed) then for fixed prices it may require slightly different policies (especially, for $\theta_1, i_{2,H}$).

SUPPLEMENTAL RESULT 3 Also, we analyse the role of the debt limit κ, to see the effect of κ on optimal values of policy parameters we vary it from 0.025 to 0.975. So, by constriction, κ is a fraction that limits the amount of new debt to a part of a market

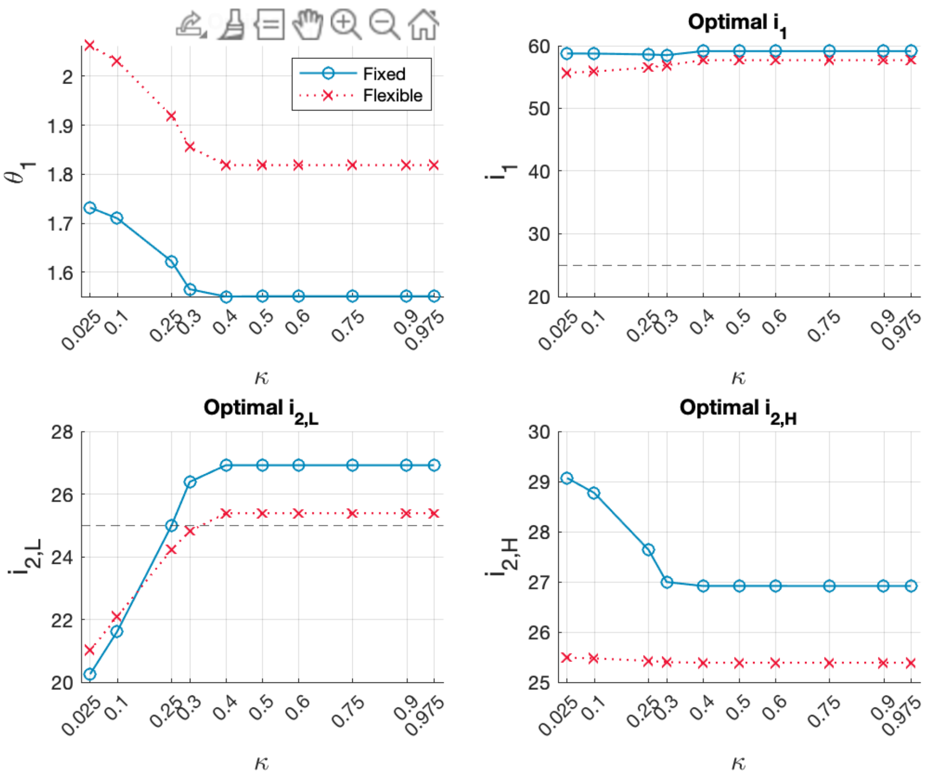

Figure 6: Supplemental Result 3 – change in κ

value of capital. Our findings are highlighted in Supplemental Result 3 and illustrated in figure 6.

Supplemental Result 3 (Change in κ) *With higher κ (less strict debt/financial constraint), a policymaker needs to implement softer macroprudential regulation (lower but positive θ_1), to tighten ex-ante monetary policy, i_1, and also ex-post monetary policy in a crisis period, $i_{2,L}$, and loosing ex-post monetary policy if the crisis does not occur, $i_{2,H}$. Note that there is an upper limit in κ after which all optimal policies are constant. This Supplemental Result is illustrated in figure 6.*

When we consider the debt limit κ, firstly, we see that there is an upper limit in κ after which all policies do not react. The intuition is the following. If the debt limit is not tough, then the financial constraint does not bind because firms with linear technology are willing to take less debt and so do not meet this financial constraint.

Also, we may determine the approximate threshold for κ in current calibration. From figure 6 we see that macroprudential tax and all interest rates become flat around

0.4, which is the limit for κ. This picture shows that in our economy firms with linear technology never borrow more than 40% of their market value of capital stock.

Moreover, we see that optimal policies for fixed and flexible prices more or less show the same directions when κ changes. However, we note that optimal θ_1 is softer for fixed prices than for flexible prices model. Optimal $2_{2,L}$ and $i_{2,H}$ are stricter and optimal i_1 is almost the same.

SUPPLEMENTAL RESULT 4 Also, we analyse the role of investment costs ϕ, to see the effect of ϕ on optimal values of policy parameters we vary it from 0.5 to 5. With higher ϕ, it becomes more difficult to create new capital in the economy. Our findings are highlighted in Supplemental Result 4 and illustrated in figure 7.

Supplemental Result 4 (Change in ϕ) *With larger investments costs, ϕ, a policymaker needs to implement softer macroprudential regulation (lower but positive θ_1). It is optimal to soften ex-ante monetary policy, i_1, and ex-post monetary policy, both $i_{2,L}$ and $i_{2,H}$. This Supplemental Result is illustrated in figure 7.*

If we change the capital adjustment costs parameter, we see the expected result. With larger ϕ it is more difficult to create a new unit of capital, thus, agents accumulate less capital and it requires a smaller amount of intervention from a policymaker. However, we note the difference in optimal interest rates between fixed and flexible models. The difference for $i_{2,L}$ and $i_{2,H}$ is much larger for fixed prices model.

SUPPLEMENTAL RESULT 5 Also, we analyse the role of the probability of a crisis, ρ_L, to see the effect of ρ_L on optimal values of policy parameters we vary it from 0.1 to 0.9. The larger value indicates the larger probability of a crisis. Our findings are highlighted in Supplemental Result 5 and illustrated in figure 8.

Supplemental Result 5 (Change in ρ_L) *With the growing probability of a crisis (ρ_L), a policymaker needs to implement a tighter macroprudential policy (higher θ_1). A policymaker needs to soften ex-ante monetary policy, i_1. It is optimal to tighten ex-post monetary policy, both $i_{2,L}$ and $i_{2,H}$. This Supplemental Result is illustrated in figure 8.*

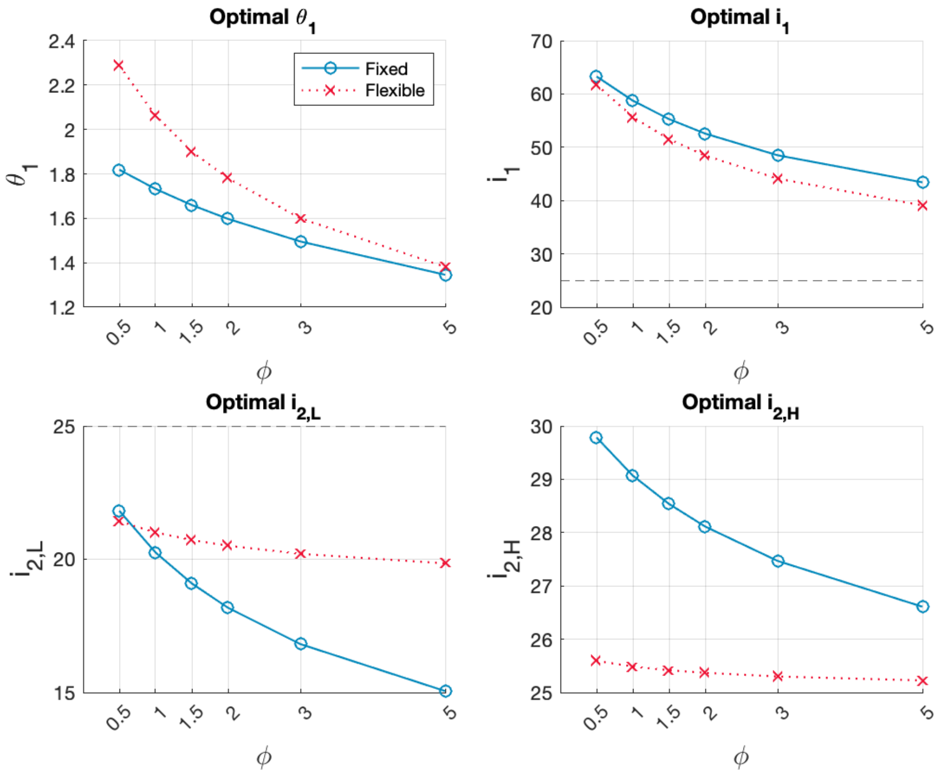

Figure 7: Supplemental Result 4 – change in ϕ

The most striking result is the following. When the probability of a crisis, ρ_L, rises, a policymaker tightens macroprudential policy but slightly loosens ex-ante monetary policy. Moreover, it is optimal to tighten ex-post monetary policy. We also note that for fixed prices i_1 falls a little in contrast to the flexible prices model. And the same, $i_{2,H}$ rises a lot for fixed prices model while this interest rate remains the same for flexible prices.

This result is striking because with a higher probability of a crisis, we expect to tighten ex-ante monetary policy to prevent excessive capital accumulation (in addition to tightening macroprudential policy). Loosening ex-ante monetary policy with a growing probability of a crisis, firstly, looks counter-intuitive because it seems that if a crisis is imminent, a policymaker needs to make every effort to reduce the severity of the crisis. However, we can provide the following economic intuition. If a crisis is inevitable, a policymaker restricts borrowing by firms with linear technology using θ_1. This more tight restriction suppresses the demand for capital. So, there is a contraction in aggregate demand. Therefore, a policymaker needs to provide expansionary monetary policy, thus, optimal ex-ante interest rate lowers reflecting this expansion but monetary policy overall remains tight because it is still above fixed prices optimal values. So, similar to Supplemental Result 1 ex-ante monetary policy and ex-ante macroprudential policy work in the opposite directions: tightening macroprudential policy requires softening monetary policy to stimulate aggregate demand.

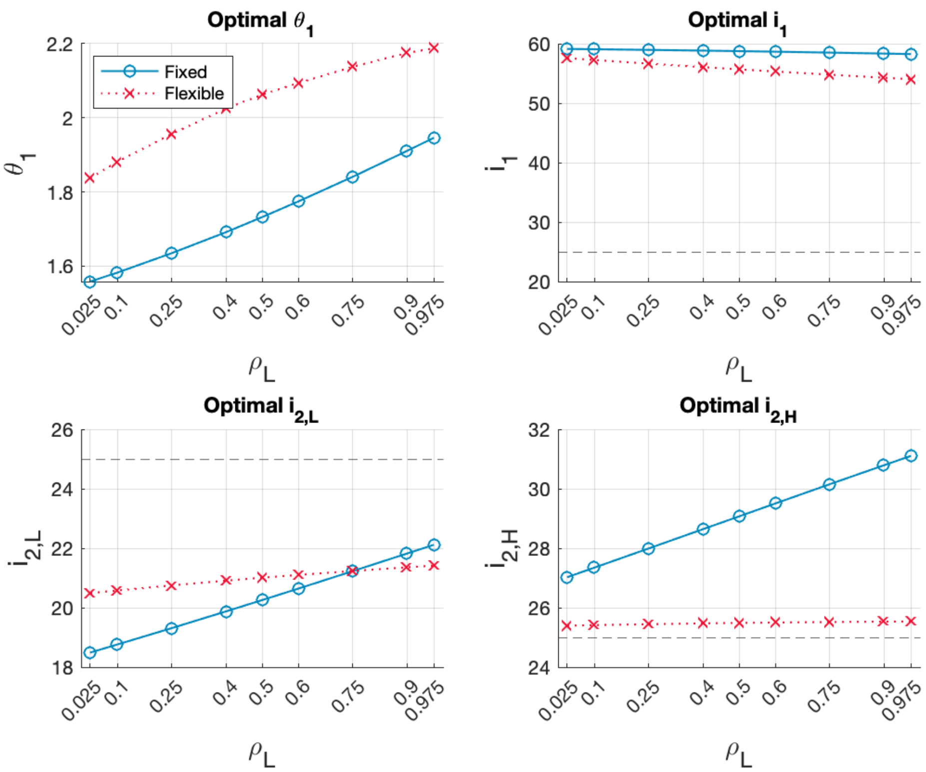

Figure 8: Supplemental Result 5 – change in ρ_L

To conclude this section, it is important to determine the point to which we compare monetary tightening/easing. If we compare fixed prices to flexible prices then the Results above are true. Furthermore, these results remain the same for several sensitivity tests (when we change calibration values) and which are indicated in a number of Supplemental Results. For most values parameters values, a policymaker still has to tighten macroprudential policy (but less than for flexible prices), set ex-ante monetary policy higher than for flexible prices. Ease even more ex-post monetary policy than for flexible prices if a crisis occurs and tighten more if a crisis does not occur.

5 POLICY IMPLICATIONS

In terms of the usefulness of our analysis for policy implementation, we see several policy implications. In practice, for a policymaker it is important to understand/estimate an initial household endowment, k_{-1}, and the amount of financing of firms with linear technology, n_0^{linear}. Both of these variables indicate households' or firms resources but the optimal policy response is diametrically opposite. Thus, one has to be accurate in determining/estimating these endowments.

Also, for a policymaker, it is important to correctly estimate the severity of a crisis. If the debt limit is not so tight, then it may be possible that the financial consultant does not bind and, thus, requires a slightly different optimal policy from a policymaker. It is also important to understand how difficult to create new capital in practice because if it is difficult then it requires much less policy interventions. Finally, the probability of a crisis, undoubtedly, matters, when we determine the optimal policy interventions.

Moreover, in exercises with models with fixed and flexible prices, we want to show extreme cases to compare behaviours of macroprudential and monetary policy tools. In these experiments we highlight that price rigidity matters especially ex-ante to prevent a crisis. While with fully fixed prices the "size" of a crisis does not matter (optimal ex-ante monetary policy almost insensitive to the toughness of the financial constraint), yet the difference in optimal policies in the model with flexible prices is large. Therefore, for a policymaker it is important not only to understand the severity of a crisis but also the degree of price rigidity.

6 FURTHER REMARKS

In this section, we point out several limitations that there are in our model. First of all, this is a model of a closed economy. It limits our available space for analysis. Nevertheless, there is still plenty of interesting results that are discussed in the previous sections. However, there is room for improvement. We still lack of rich banking sector and possible externalities that emerge in this sector. Many interesting features are also difficult to incorporate: interest rates affecting the liquidity/efficiency of trades as well as real interest rates.

We also do not take into account any effects from the external sector, to consider it, we need to extend our model to a small open economy with a commodity-exporting sector. We plan to consider a small open economy model in a separate paper.

Also, we exploit fully fixed prices (but not a-la Calvo or Rottenberg pricing), which allows us to compare exogenous policy instruments $(i_1, i_{2,L}, i_{2,H})$ in the model with fixed prices with endogenous variables $(r_1, r_{2,L}, r_{2,H})$ in the model with flexible prices. In other words, when policymaker manipulates $i_1, i_{2,L}, i_{2,H}$ it effectively sets $r_1, r_{2,L}, r_{2,H}$. Intuitively both policies do the same thing in increasing real interest rate: because of fixed price equilibria. With some sluggishness in nominal prices, this would be different. There should be some interesting trade-offs.

Moreover, there is a couple of "modelling" limitations, such as quasi-linear utility function ($\log(c_t) - h_t$) and a form of capital adjustment function ($inv_0(1 + \frac{inv_0}{k_{-1}})$). Some of our results may be caused by these particular forms. However, as a robustness check we plan to consider more general utility function ($\frac{c_{t,s}^{1-\sigma}}{1-\sigma} - \omega \frac{h_{t,s}^{1+\eta}}{1+\eta}$).

Furthermore, our framework allows us to compare time-consistent and time-inconsistent monetary policy. Is there a time consistent function for macroprudential/monetary policies? We also plan to implement this analysis in the future.

Finally, our model is a descriptive one, we only can advise a direction of policy but not the exact rates. Thus, it is important to understand what does the choice of one policy instrument, imply about the choice of the other given richer trade-offs? We partly do it in Result 2. However, there is still room for improvement in these findings.

7 CONCLUSION

In this paper we develop a closed economy model that features multiple externalities, especially, we characterize optimal monetary and macroprudential policies in the economy with fire-sales and financially constrained agents. Effectively, our model has three periods and two generalized types of agents: lenders (households/owners of raw-inputs/final goods producers) and borrowers (firms with linear capital production technology). Flows of real assets in our model are the following. In the first period households are endowment with an amount of capital and in the same period households decide the amount of new capital creation (accumulation). The creation of new capital occurs only in the first period. The capital, initially owned by households, are sold to firms. These firms use capital to produce a raw input good which is, combined with labor, used in the production of a final good. This final good is fully consumed by households. Financial flows are the following. Firms with linear technology are initially financed by households, then they debt each period and reinvest their net wealth (profits). After the last period, all net wealth is paid back to households.

At date 1 (the second period), there is a shock with a certain probability. Markets are incomplete. Net wealth determines capital goods prices. At date 1, borrowers have borrowing constraints - can only borrow up to a fraction of new capital bought. In a low state (bad realization of uncertainty) capital price falls and constraint binds. A binding constraint generates an additional cost to debt and benefit to capital (collateral premium). Firms with linear technology are forced to take less debt and sell part of the capital (show less demand for capital). Less demand for capital further reduces their price which leads to pecuniary externality/fire sales. This intuition works backwards: the pecuniary externality causes too much lending at date 0, and not enough at date 1. Thus, we ex-ante/preventive examine the macroprudential and monetary policy to restrict agents to accumulate too much capital.

There are two policies available: tax on borrowing or increase the borrowing interest rate. Both policies raise the real interest rate, (and prices of capital) suppressing lending and investment but because of pecuniary externality, may lift capital prices so the collateral constraint does not bind leaving room for Pareto improvement. Intuitively both policies do the same thing in increasing real interest rate: because of fixed price equilibria. The difference is that changing the cost of debt affects both lenders and borrowers directly while the tax affects the borrower only at a first-order level.

In our baseline calibration, we find that it is optimal to tighten both ex-ante macroprudential and monetary policy, ease monetary policy if a crisis occurs and again tighten if a does not crisis occur. Furthermore, macroprudential tightening for fixed prices is less than for flexible prices. These results do not only show the optimal direction of policy instruments in response to financial shock but also highlights the importance of price rigidities. Thus, we compare only two extreme cases, which shows a big difference in determining the optimal policy. Moreover, we note that if we compare the optimal rates to the steady-state values, then some of these results depends on parameters values.

Hence, the model is too stylized to give plausible quantitative (in terms of exact interest rates) recommendations but it still gives interesting qualitative results that may motivate a larger quantitative model for practical policy applications. And despite making some simplifying assumptions, our model has a very rich for its structure which allows us to draw important results for economic literature and policymaking.

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A PARAMETERS

Table 1: Parameter Values for baseline model

Parameter	Description	Value	Source
β	Discount factor	0.8	Basu et al. 2020
α	Share of capital	$\frac{1}{3}$	Commonly used in the literature
ϵ	Price Markup	6	Commonly used in the literature
n_0^{linear}	Initial financing	0.53	US Data[4]
k_{-1}	Initial endowment	0.85	US Data[5]
ϕ	Capital adjustment	1	Uribe and Schmitt-Grohé 2017
ρ_L	Probability of bad shock	0.5	Basu et al. 2020
κ	Debt limit	0.025	Basu et al. 2020

[4] A ratio of Nonfinancial Corporate Business; Net Worth to Nonfinancial Corporate Business; Total Assets. Source: FRED, Federal Reserve Bank of St. Louis

[5] A ratio of Households and Nonprofit Organizations; Net Worth to Households and Nonprofit Organizations; Total Assets. Source: FRED, Federal Reserve Bank of St. Louis

B FOCS FOR MODEL WITH FIXED PRICES

FOR $t = 0$.

$$c_0^{-1} = \beta(1+r_1)(\rho_L c_{1,L}^{-1} + \rho_H c_{1,H}^{-1}), \tag{21}$$

$$c_0 = w_0, \tag{22}$$

$$\frac{\alpha}{1-\alpha}\frac{k_{-1}}{h_0} = \frac{w_0}{p_{x,0}}, \tag{23}$$

$$c_0 + inv_0(\frac{\phi}{2}\frac{inv_0}{k_{-1}}) = k_{-1}^{\alpha} h_0^{1-\alpha}, \tag{24}$$

$$1 + \phi\frac{inv_0}{k_{-1}} = q_0, \tag{25}$$

$$c_0^{-1} q_0 = \beta[\rho_L c_{1,L}^{-1}(\frac{p_{x,1,L}A_{1,L}}{1+k_0^{concave}} + q_{1,L}) + \rho_H c_{1,H}^{-1}(\frac{p_{x,1,H}A_{1,H}}{1+k_0^{concave}} + q_{1,H})], \tag{26}$$

$$q_0 c_0^{-1} = \beta\frac{\rho_L c_{1,L}^{-1}[p_{x,1,L}A_{1,L} + q_{1,L}] + \rho_H c_{1,H}^{-1}[p_{x,1,H}A_{1,H} + q_{1,H}]}{(1+\theta_1)}, \tag{27}$$

$$inv_0 + k_{-1} = K, \tag{28}$$

$$k_0^{linear} + k_0^{concave} = K, \tag{29}$$

$$\tag{30}$$

9 equations for 9 unknowns: $c_0, h_0, w_0, q_0, k_0^{concave}, k_0^{linear}, K, inv_0, p_{x,0}$.

FOR $t = 1, s = L$.

$$c_{1,L}^{-1} = \beta(1 + i_{2,L})c_{2,L}^{-1}, \tag{31}$$

$$c_{1,L} = w_{1,L}, \tag{32}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,L}}{h_{1,L}} = \frac{w_{1,L}}{p_{x,1,L}}, \tag{33}$$

$$c_{1,L} = x_{1,L}^{\alpha} h_{1,L}^{1-\alpha}, \tag{34}$$

$$c_{1,L}^{-1} q_{1,L} = \beta c_{2,L}^{-1}\left(\frac{p_{x,2,L}A_{2,L}}{1 + k_{1,L}^{concave}} + q_{2,L}\right), \tag{35}$$

$$x_{1,L} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{36}$$

$$q_{1,L}k_{1,L}^{linear} = \frac{n_{1,L}^{linear}}{1-\kappa}, \tag{37}$$

$$n_{1,L}^{linear} = p_{x,1,s}A_{1,s}k_0^{linear} + q_{1,s}k_0^{linear} - (1 + i_1)(q_0 k_0^{linear} - n_0^{linear}), \tag{38}$$

$$k_{1,L}^{linear} + k_{1,L}^{concave} = K, \tag{39}$$

9 equations for 9 unknowns: $c_{1,L}, h_{1,L}, w_{1,L}, x_{1,L}, p_{x,1,L}, q_{1,L}, n_{1,L}^{linear}, k_{1,L}^{linear}, k_{1,L}^{concave}$.

FOR $t = 1, s = H$.

$$c_{1,H}^{-1} = \beta(1 + i_{2,H})c_{2,H}^{-1}, \tag{40}$$

$$c_{1,H} = w_{1,H}, \tag{41}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,H}}{h_{1,H}} = \frac{w_{1,H}}{p_{x,1,H}}, \tag{42}$$

$$c_{1,H} = x_{1,H}^{\alpha} h_{1,H}^{1-\alpha}, \tag{43}$$

$$c_{1,H}^{-1} q_{1,H} = \beta c_{2,H}^{-1}\left(\frac{p_{x,2,H}A_{2,H}}{1 + k_{1,H}^{concave}} + q_{2,H}\right), \tag{44}$$

$$x_{1,H} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{45}$$

$$q_{1,H} = \frac{p_{x,2,H}A_{2,H} + q_{2,H}}{(1 + i_{2,H})}, \tag{46}$$

$$k_{1,H}^{linear} + k_{1,H}^{concave} = K \tag{47}$$

8 equations for 8 unknowns: $c_{1,H}, x_{1,H}, w_{1,H}, h_{1,H}, q_{1,H}, p_{x,1,L}, k_{1,H}^{linear}, k_{1,H}^{concave}$.

FOR $t = 2, s$. For $s = H$ allocations are the same as on $t \geq 3$.

$$c_{2,s}^{-1} = \beta(1 + r_{3,s})c^{-1}, \tag{48}$$

$$c_{2,s} = w_{2,s}, \tag{49}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{2,s}}{h_{2,s}} = \frac{w_{2,s}}{p_{x,2,s}}, \tag{50}$$

$$c_{2,s} = x_{2,s}^{\alpha} h_{2,s}^{1-\alpha}, \tag{51}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_{2,s}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,2,s}}{1-\alpha}\right)^{1-\alpha}, \tag{52}$$

$$x_{2,s} = \log(1 + k_{1,s}^{concave}) + k_{1,s}^{linear}, \tag{53}$$

$$q_{2,s} = \frac{p_x A + q}{(1 + r_{3,s})}, \tag{54}$$

$$\tag{55}$$

7 equations for 7 unknowns: $c_{2,s}, w_{2,s}, h_{2,s}, x_{2,s}, q_{2,s}, p_{x,2,s}, r_{3,s}$.

FOR $t \geq 3$. All variables are at steady states. Thus, we remove the times' indexes. And new prices and fully flexible, thus, the interest rate becomes endogenous r.

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{p_x}{1-\alpha}\right)^{1-\alpha}, \tag{56}$$

$$q = \frac{\beta}{1-\beta}p_x, \tag{57}$$

$$r = \frac{1}{\beta} - 1, \tag{58}$$

$$c = x^{\alpha}h^{1-\alpha}, \tag{59}$$

$$x = K, \tag{60}$$

$$c = w, \tag{61}$$

$$\frac{\alpha}{1-\alpha}\frac{x}{h} = \frac{w}{p_x}, \tag{62}$$

7 equations for 7 unknowns: c, w, h, x, p_x, r, q.

C FOCS FOR MODEL WITH FLEXIBLE PRICES

FOR $t = 0$.

$$c_0^{-1} = \beta(1+r_1)(\rho_L c_{1,L}^{-1} + \rho_H c_{1,H}^{-1}), \tag{63}$$

$$c_0 = w_0, \tag{64}$$

$$\frac{\alpha}{1-\alpha}\frac{k_{-1}}{h_0} = \frac{w_0}{p_{x,0}}, \tag{65}$$

$$c_0 + inv_0(\frac{\phi}{2}\frac{inv_0}{k_{-1}}) = k_{-1}^{\alpha} h_0^{1-\alpha}, \tag{66}$$

$$1 + \phi\frac{inv_0}{k_{-1}} = q_0, \tag{67}$$

$$c_0^{-1} q_0 = \beta[\rho_L c_{1,L}^{-1}(\frac{p_{x,1,L}A_{1,L}}{1+k_0^{concave}} + q_{1,L}) + \rho_H c_{1,H}^{-1}(\frac{p_{x,1,H}A_{1,H}}{1+k_0^{concave}} + q_{1,H})], \tag{68}$$

$$q_0 c_0^{-1} = \beta\frac{\rho_L c_{1,L}^{-1}[p_{x,1,L}A_{1,L} + q_{1,L}] + \rho_H c_{1,H}^{-1}[p_{x,1,H}A_{1,H} + q_{1,H}]}{(1+\theta_1)}, \tag{69}$$

$$inv_0 + k_{-1} = K, \tag{70}$$

$$k_0^{linear} + k_0^{concave} = K, \tag{71}$$

$$1 = \frac{\epsilon}{\epsilon-1}\left(\frac{w_0}{\alpha}\right)^{\alpha}\left(\frac{p_{x,0}}{1-\alpha}\right)^{1-\alpha}, \tag{72}$$

10 equations for 10 unknowns: $c_0, h_0, w_0, q_0, k_0^{concave}, k_0^{linear}, K, inv_0, p_{x,0}, r_1$.

FOR $t = 1, s = L$.

$$c_{1,L}^{-1} = \beta(1 + r_{2,L})c_{2,L}^{-1}, \tag{73}$$

$$c_{1,L} = w_{1,L}, \tag{74}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,L}}{h_{1,L}} = \frac{w_{1,L}}{p_{x,1,L}}, \tag{75}$$

$$c_{1,L} = x_{1,L}^{\alpha} h_{1,L}^{1-\alpha}, \tag{76}$$

$$c_{1,L}^{-1} q_{1,L} = \beta c_{2,L}^{-1}\left(\frac{p_{x,2,L} A_{2,L}}{1 + k_{1,L}^{concave}} + q_{2,L}\right), \tag{77}$$

$$x_{1,L} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{78}$$

$$q_{1,L} k_{1,L}^{linear} = \frac{n_{1,L}^{linear}}{1-\kappa}, \tag{79}$$

$$n_{1,L}^{linear} = p_{x,1,s} A_{1,s} k_0^{linear} + q_{1,s} k_0^{linear} - (1 + r_1)(q_0 k_0^{linear} - n_0^{linear}), \tag{80}$$

$$k_{1,L}^{linear} + k_{1,L}^{concave} = K, \tag{81}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_{1,L}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,1,L}}{1-\alpha}\right)^{1-\alpha}, \tag{82}$$

10 equations for 10 unknowns: $c_{1,L}, h_{1,L}, w_{1,L}, x_{1,L}, p_{x,1,L}, q_{1,L}, n_{1,L}^{linear}, k_{1,L}^{linear}, k_{1,L}^{concave}, r_{2,L}$.

FOR $t = 1, s = H$.

$$c_{1,H}^{-1} = \beta(1 + i_{2,H})c_{2,H}^{-1}, \tag{83}$$

$$c_{1,H} = w_{1,H}, \tag{84}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{1,H}}{h_{1,H}} = \frac{w_{1,H}}{p_{x,1,H}}, \tag{85}$$

$$c_{1,H} = x_{1,H}^{\alpha}h_{1,H}^{1-\alpha}, \tag{86}$$

$$c_{1,H}^{-1}q_{1,H} = \beta c_{2,H}^{-1}\left(\frac{p_{x,2,H}A_{2,H}}{1 + k_{1,H}^{concave}} + q_{2,H}\right), \tag{87}$$

$$x_{1,H} = \log(1 + k_0^{concave}) + k_0^{linear}, \tag{88}$$

$$q_{1,H} = \frac{p_{x,2,H}A_{2,H} + q_{2,H}}{(1 + i_{2,H})}, \tag{89}$$

$$k_{1,H}^{linear} + k_{1,H}^{concave} = K, \tag{90}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_{1,H}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,1,H}}{1-\alpha}\right)^{1-\alpha}, \tag{91}$$

9 equations for 9 unknowns: $c_{1,H}, x_{1,H}, w_{1,H}, h_{1,H}, q_{1,H}, p_{x,1,L}, k_{1,H}^{linear}, k_{1,H}^{concave}, r_{2,H}$.

FOR $t = 2, s$. For $s = H$ allocations are the same as on $t \geq 3$.

$$c_{2,s}^{-1} = \beta(1 + r_{3,s})c^{-1}, \tag{92}$$

$$c_{2,s} = w_{2,s}, \tag{93}$$

$$\frac{\alpha}{1-\alpha}\frac{x_{2,s}}{h_{2,s}} = \frac{w_{2,s}}{p_{x,2,s}}, \tag{94}$$

$$c_{2,s} = x_{2,s}^{\alpha}h_{2,s}^{1-\alpha}, \tag{95}$$

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w_{2,s}}{\alpha}\right)^{\alpha}\left(\frac{p_{x,2,s}}{1-\alpha}\right)^{1-\alpha}, \tag{96}$$

$$x_{2,s} = \log(1 + k_{1,s}^{concave}) + k_{1,s}^{linear}, \tag{97}$$

$$q_{2,s} = \frac{p_x A + q}{(1 + r_{3,s})}, \tag{98}$$

7 equations for 7 unknowns: $c_{2,s}, w_{2,s}, h_{2,s}, x_{2,s}, q_{2,s}, p_{x,2,s}, r_{3,s}$.

FOR $t \geq 3$. All variables are at steady states. Thus, we remove the times' indexes.

$$1 = \frac{\epsilon}{\epsilon - 1}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{p_x}{1-\alpha}\right)^{1-\alpha}, \tag{99}$$

$$q = \frac{\beta}{1-\beta}p_x, \tag{100}$$

$$r = \frac{1}{\beta} - 1, \tag{101}$$

$$c = x^{\alpha}h^{1-\alpha}, \tag{102}$$

$$x = K, \tag{103}$$

$$c = w, \tag{104}$$

$$\frac{\alpha}{1-\alpha}\frac{x}{h} = \frac{w}{p_x}, \tag{105}$$

7 equations for 7 unknowns: c, w, h, x, p_x, r, q.

D ALGORITHM TO SOLVE MODEL

We solve the model using Matlab software (version 2020a). To solve the model we first built a system of non-linear equations from (22) to (62) from appendix B. And a separate system of non-linear equations from (64) to (105) from appendix C. To both of these systems we apply non-linear solver to find the solution of all endogenous variables with given exogenous in the following way:

1. Non-linear solver function:

    (a) As input, it takes a vector of values of policy parameters: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$, and a vector of parameter values: β, α, ϵ, n_0^{linear}, k_{-1}, ϕ, κ and ρ_L.

    (b) It solves the steady-state system of equations and uses steady-state values as initial values in a non-linear solver.

    (c) Firstly, we assume that financial constraint (7) does not bind and solve the unconstrained version of the model. In other words, we assume that when uncertainty resolves, there is no difference in the low and high state because financial constraint does not hold.

(d) Then we check two conditions: whether there is a solution at all, and if there is, then we check if d_1^{linear} is positive. In other words, $d_1^{linear} > 0$ means that firms with linear technology are borrowers (which we assume in our model) but for some values of parameters these firms may become lenders $d_1^{linear} < 0$. We do not consider these cases.

(e) If both previous conditions are verified, we compare $d_{2,L}^{linear}$ and $\kappa q_{1,L} k_{1,L}^{linear}$ (we check if financial constrain binds).

(f) If $d_{2,L}^{linear}$ is less than $\kappa q_{1,L} k_{1,L}^{linear}$, then we find a solution since in this case the amount of borrowing made by firms with linear technology is less than a possible debt limit, so there is no need to solve a model with binding financial constraint.

(g) If $d_{2,L}^{linear}$ is higher than $\kappa q_{1,L} k_{1,L}^{linear}$, then it means that the amount of borrowing made by firms with linear technology is higher than a possible debt limit. It indicates that a financial constraint should bind, so we resolve the model with binding financial constraint.

(h) As output, this function gives the value of agents utility (1), a vector of endogenous variables, and an indicator, which take 0 is for a given vectors of parameters if financial constraint does not bind, and takes value of 1 if financial constraint binds.

The next step is to find optimal exogenous policy variables, namely, $\theta_1, i_1, i_{2,L}, i_{2,H}$. To do it we minimize the non-linear solver function varying policy variable to find the combination with minimize a utility function:

2. Minimize function

(a) As an input, it takes an initial values of policy variables: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$, and a vector of parameter values: β, α, ϵ, n_0^{linear}, k_{-1}, ϕ, κ and ρ_L.

(b) It takes a negative value of the utility (from the non-linear solver) and applies a minimize function varying a vector of policy variables: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$ and taking parameter values: β, α, ϵ, n_0^{linear}, k_{-1}, ϕ, κ and ρ_L as given

(c) As output, it gives the vector of policy variables: θ_1, i_1, $i_{2,L}$ and $i_{2,H}$ which gives the minimum value of negative utility. Minimizing the negative utility is the same as maximizing the utility.

E ADDITIONAL RESULTS

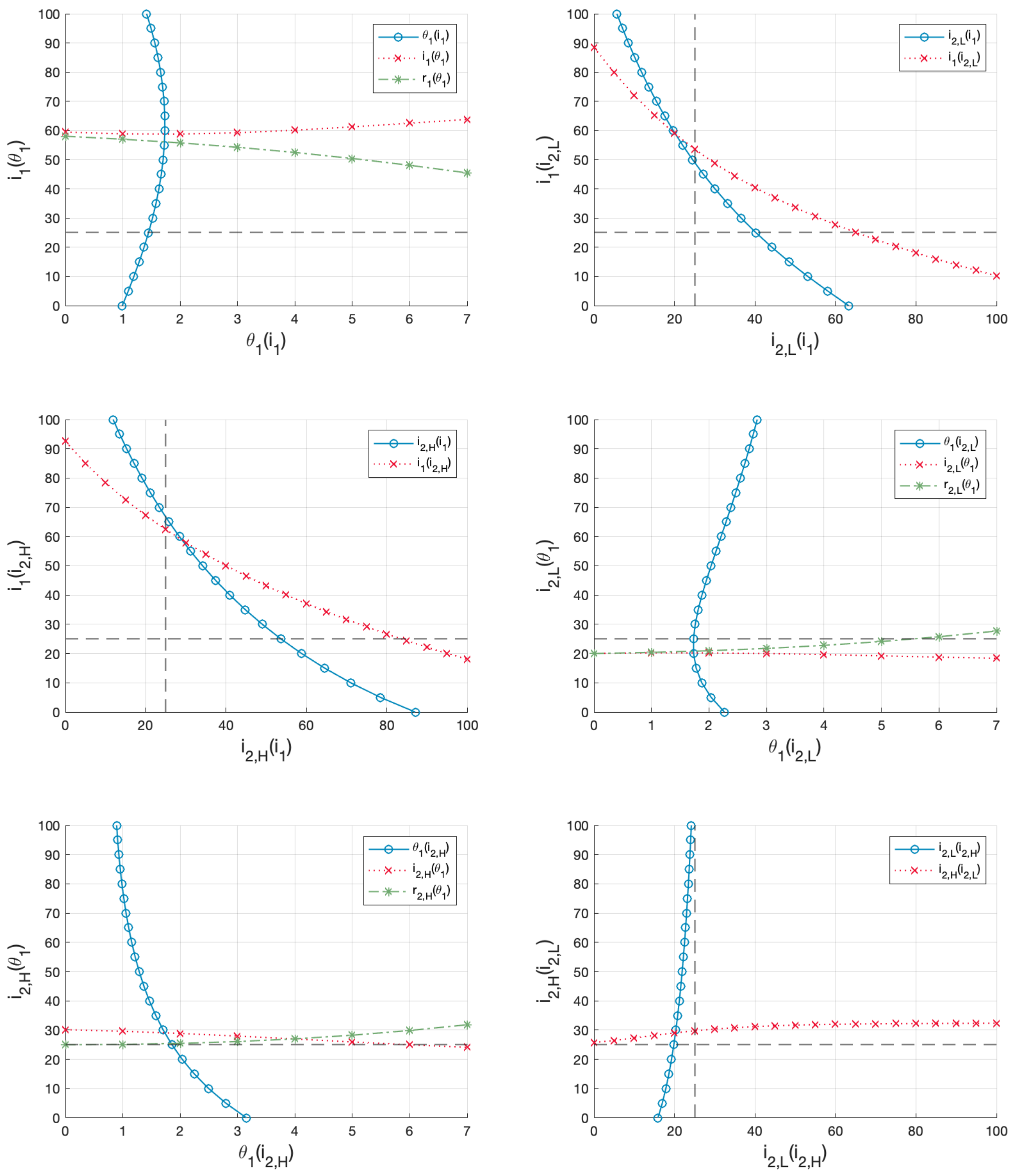

Figure 9: Policy functions

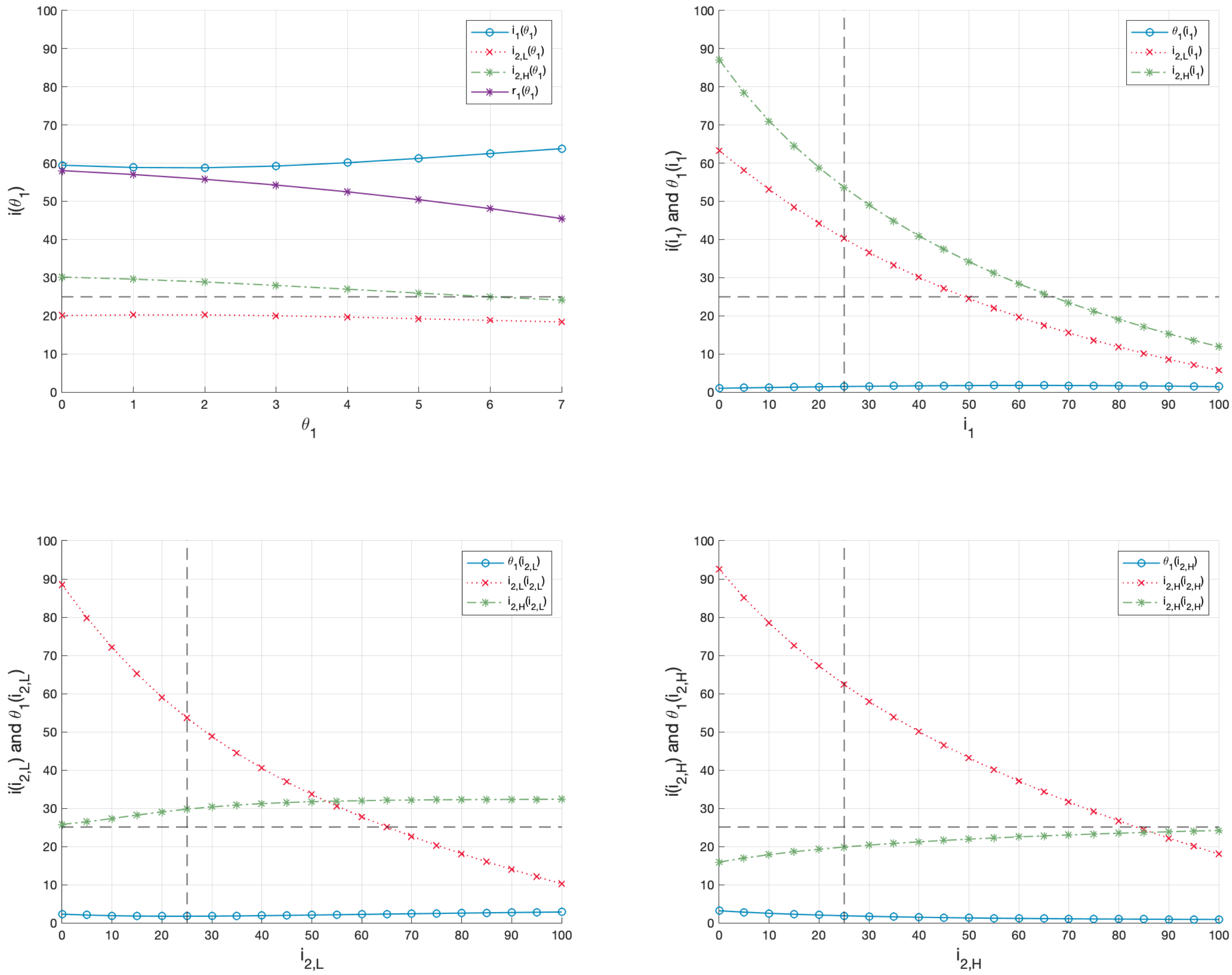

Figure 10: Policy functions