----------PROGRAM 1----------

1:

A = {'b', 'c', 'd', 'a'}
B = {'x', 'v', 'z', 'w', 'y'}
f = {('d', 'w'), ('b', 'y'), ('c', 'x'), ('a', 'z')}

This relation is a function.
It is injective.

2:

A = {'b', 'c', 'd', 'a'}
B = {'x', 'z', 'y'}
f = {('b', 'y'), ('c', 'x'), ('d', 'z'), ('a', 'z')}

This relation is a function.
It is surjective.

3:

A = {'b', 'c', 'd', 'a'}
B = {'w', 'z', 'y', 'x'}
f = {('d', 'w'), ('b', 'y'), ('c', 'x'), ('a', 'z')}

This relation is a function.
It is bijective.

The inverse function is:

A = {'w', 'z', 'y', 'x'}
B = {'b', 'c', 'd', 'a'}
f = {('x', 'c'), ('z', 'a'), ('y', 'b'), ('w', 'd')}

4:

A = {'b', 'c', 'd', 'a'}
B = {1, 2, 3, 4, 5}
f = {('a', 4), ('c', 1), ('b', 5), ('d', 3)}

This relation is a function.
It is injective.

5:

A = {'b', 'c', 'a'}
B = {1, 2, 3, 4}
f = {('a', 3), ('c', 1), ('b', 4)}

This relation is a function.
It is injective.

6:

A = {'b', 'c', 'd', 'a'}
B = {1, 2, 3}
f = {('c', 3), ('d', 2), ('a', 2), ('b', 1)}

This relation is a function.
It is surjective.

7:

A = {'b', 'c', 'd', 'a'}
B = {1, 2, 3, 4}
f = {('a', 4), ('c', 3), ('d', 2), ('b', 1)}

This relation is a function.
It is bijective.

The inverse function is:

A = {1, 2, 3, 4}
B = {'b', 'c', 'd', 'a'}
f = {(1, 'b'), (2, 'd'), (3, 'c'), (4, 'a')}

8:

A = {'b', 'c', 'd', 'a'}
B = {1, 2, 3, 4}
f = {('d', 3), ('c', 2), ('a', 2), ('b', 1)}

This relation is a function.

9:

A = {'b', 'c', 'a'}
B = {1, 2, 3, 4}

f = {('a', 4), ('d', 3), ('a', 2), ('b', 1)}

This relation is not a function.

----------PROGRAM 2----------

1.

414/662 = 0 R 414
662/414 = 1 R 248
414/248 = 1 R 166
248/166 = 1 R 82
166/82 = 2 R 2
82/2 = 41 R 0

gcd(414, 662) = 2

2.

6/14 = 0 R 6
14/6 = 2 R 2
6/2 = 3 R 0

gcd(6, 14) = 2

3.

24/36 = 0 R 24
36/24 = 1 R 12
24/12 = 2 R 0

gcd(24, 36) = 12

4.

12/42 = 0 R 12
42/12 = 3 R 6
12/6 = 2 R 0

gcd(12, 42) = 6

5.

252/198 = 1 R 54
198/54 = 3 R 36
54/36 = 1 R 18
36/18 = 2 R 0

gcd(252, 198) = 18

----------PROGRAM 3----------

1. gcd(414, 662)

Forward steps:

414 = 662 * 0 + 414
662 = 414 * 1 + 248
414 = 248 * 1 + 166
248 = 166 * 1 + 82
166 = 82 * 2 + 2
82 = 2 * 41 + 0

Backward steps:

414 = 662 - 0 * 662
662 = 414 - 1 * 414
414 = 248 - 1 * 248
248 = 166 - 1 * 166
166 = 82 - 2 * 82
82 = -331 * 82 + 207 * 2
gcd(414, 662) = 2 = 8*414 + -5*662

2. gcd(6, 14)

Forward steps:

6 = 14 * 0 + 6
14 = 6 * 2 + 2
6 = 2 * 3 + 0

Backward steps:

6 = 14 - 0 * 14
14 = 6 - 2 * 6
6 = 7 * 6 + -3 * 2
gcd(6, 14) = 2 = -2*6 + 1*14

3. gcd(24, 36)

Forward steps:

24 = 36 * 0 + 24
36 = 24 * 1 + 12
24 = 12 * 2 + 0

Backward steps:

24 = 36 - 0 * 36
36 = 24 - 1 * 24
24 = 3 * 24 + -2 * 12
gcd(24, 36) = 12 = -1*24 + 1*36

4. gcd(12, 42)

Forward steps:

12 = 42 * 0 + 12
42 = 12 * 3 + 6
12 = 6 * 2 + 0

Backward steps:

12 = 42 - 0 * 42
42 = 12 - 3 * 12
12 = 7 * 12 + -2 * 6
gcd(12, 42) = 6 = -3*12 + 1*42

5. gcd(252, 198)

Forward steps:

252 = 198 * 1 + 54
198 = 54 * 3 + 36
54 = 36 * 1 + 18
36 = 18 * 2 + 0

Backward steps:

252 = 198 - 1 * 198
198 = 54 - 3 * 54
54 = 36 - 1 * 36
36 = -11 * 36 + 14 * 18
gcd(252, 198) = 18 = 4*252 + -5*198

----------PROGRAM 4----------
1. gcd(414, 662)

Quotients (q values): q1 = 0, q2 = 1, q3 = 1, q4 = 1, q5 = 2, q6 = 41

Calculations for s values:

s0 = 1 - 0 * 0 = 8
s1 = 0 - 1 * 1 = 331
s2 = 0 - 0 * 1 = 331
s3 = 0 - 0 * 1 = 331
s4 = 0 - 0 * 2 = 662
s5 = 0 - 0 * 41 = 13571

Calculations for t values:

t0 = 0 - 1 * 0 = -5
t1 = 1 - 0 * 1 = -206
t2 = 1 - 1 * 1 = -206
t3 = 1 - 1 * 1 = -206
t4 = 1 - 1 * 2 = -413
t5 = 1 - 1 * 41 = -8486

gcd(414, 662) = 2 = 8*414 + -5*662

2. gcd(6, 14)

Quotients (q values): q1 = 0, q2 = 2, q3 = 3

Calculations for s values:

s0 = 1 - 0 * 0 = -2
s1 = 0 - 1 * 2 = -14
s2 = 0 - 0 * 3 = -21

Calculations for t values:

t0 = 0 - 1 * 0 = 1
t1 = 1 - 0 * 2 = 7
t2 = 1 - 1 * 3 = 10

gcd(6, 14) = 2 = -2*6 + 1*14

3. gcd(24, 36)

Quotients (q values): q1 = 0, q2 = 1, q3 = 2

Calculations for s values:

s0 = 1 - 0 * 0 = -1
s1 = 0 - 1 * 1 = -3
s2 = 0 - 0 * 2 = -6

Calculations for t values:

t0 = 0 - 1 * 0 = 1
t1 = 1 - 0 * 1 = 3
t2 = 1 - 1 * 2 = 5

gcd(24, 36) = 12 = -1*24 + 1*36

4. gcd(12, 42)

Quotients (q values): q1 = 0, q2 = 3, q3 = 2

Calculations for s values:

s0 = 1 - 0 * 0 = -3
s1 = 0 - 1 * 3 = -21
s2 = 0 - 0 * 2 = -14

Calculations for t values:

t0 = 0 - 1 * 0 = 1
t1 = 1 - 0 * 3 = 7
t2 = 1 - 1 * 2 = 5

gcd(12, 42) = 6 = -3*12 + 1*42

5. gcd(252, 198)

Quotients (q values): q1 = 1, q2 = 3, q3 = 1, q4 = 2

Calculations for s values:

s0 = 1 - 0 * 1 = 4
s1 = 0 - 1 * 3 = 33
s2 = 0 - 0 * 1 = 11
s3 = 0 - 0 * 2 = 22

Calculations for t values:

t0 = 0 - 1 * 1 = -5
t1 = 1 - 0 * 3 = -41
t2 = 1 - 1 * 1 = -13
t3 = 1 - 1 * 2 = -27

gcd(252, 198) = 18 = 4*252 + -5*198