1) There are 20 bricks of weight 3.4 Kg each and 30 bricks of weight 3.6 Kg each. Find the Mean, and Standard Deviation of weight for the whole bunch of 50 bricks?

Solution:

Mean (μ) $= \frac{\sum_{i=1}^{i=n} xi}{n} = \frac{20*3.4+30*3.6}{(20+30)} = \frac{68+108}{50} = \frac{176}{50} =$ **3.52**

Variance of a population is, $\sigma^2 = \frac{\sum(x-\mu)^2}{n-1} = \frac{20(3.4-3.52)^2+30(3.6-3.52)^2}{50-1}$

$\sigma^2 = \frac{20*0.0144+30*0.0064}{49} = \frac{0.288+0.192}{49} = \frac{0.48}{49} = 0.0097$

Standard Deviation, **σ = 0.09897**

2) Find the Mean, Standard Deviation of height for the following dataset.

Height (cm)	No of People
150-156	2
157-163	14
164-170	15
171-177	20
178-184	7
185-191	10

Solution:

Height (cm)	No of People (f)	Mid-point of Height (x)	f*x	f*x²
150-156	2	153	306	46818
157-163	14	160	2240	358400
164-170	15	167	2505	418335
171-177	20	174	3480	605520

	178-184	7	181	1267	229327
	185-191	10	188	1880	353440
SUM		**68**	**1023**	**11678**	**2011840**
				μ = 171.735294	

Mean, $\mu = \frac{\sum f * x}{\sum f} = \frac{11678}{68} = \mathbf{171.7352}$

$$SD = \sqrt{\frac{(\sum f)[\sum(fx^2)] - [\sum(fx)]^2}{\sum f(\sum f - 1)}}$$

Where:
SD = Standard Deviation
f = frequency
x = class mark

Standard Deviation, $\sigma = \sqrt{\frac{68 * \mathbf{2011840} - \mathbf{11678} * \mathbf{11678}}{68 * 67}} = \sqrt{\frac{136805120 - \mathbf{136375684}}{68 * 67}} = \sqrt{\frac{429436}{4556}}$

$\sigma = \sqrt{94.2572} = 9.7086$

3) Find the correlation coefficient for the following set of observations.

X	7	14	24	30	45	57
Y	24	34	45	50	61	69

Solution:

Correlation coefficient: $r = \frac{1}{n-1}\sum\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$

	x	y	x*y	x²	y²
	7	24	168	49	576
	14	34	476	196	1156
	24	45	1080	576	2025
	30	50	1500	900	2500
	45	61	2745	2025	3721
	57	69	3933	3249	4761
SUM	**177**	**283**	**9902**	**6995**	**14739**

$\bar{x} = \frac{7+14+24+30+45+57}{6} = \frac{177}{6} = 29.5$

$\overline{y} = \frac{24+34+45+50+61+69}{6} = \frac{283}{6} = 47.16$

Sample Standard Deviation for x, $S_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N-1}}$

$S_x = \sqrt{\frac{(7-29.5)^2+(14-29.5)^2+(24-29.5)^2+(30-29.5)^2+(45-29.5)^2+(57-29.5)^2}{6-1}}$

$S_x = \sqrt{\frac{1773.5}{5}} = \sqrt{354.7} = 18.83$

Sample Standard Deviation for y, S_y

$= \sqrt{\frac{(24-47.16)^2+(34-47.16)^2+(45-47.16)^2+(50-47.16)^2+(61-47.16)^2+(69-47.16)^2}{6-1}}$

$S_y = \sqrt{\frac{1390.83}{5}} = \sqrt{278.16} = 16.67$

Z-Score: How many Standard Deviations (σ) away from the Mean(μ)

$$r = \frac{1}{n-1}\Sigma\left(\frac{x_i - \overline{x}}{s_x}\right)\left(\frac{y_i - \overline{y}}{s_y}\right)$$

$r = \frac{1}{6-1}\left\{\frac{7-29.5}{18.83} * \frac{24-47.16}{16.67} + \frac{14-29.5}{18.83} * \frac{34-47.16}{16.67} + \frac{24-29.5}{18.83} * \frac{45-47.16}{16.67} + \frac{30-29.5}{18.83} * \frac{50-47.16}{16.67} + \frac{45-29.5}{18.83} * \frac{61-47.16}{16.67} + \frac{57-29.5}{18.83} * \frac{69-47.16}{16.67}\right\}$

$r = \frac{1}{5}\{(-1.19) * (-1.38) + (-0.82) * (-0.78) + (-0.29) * (-0.12) + (0.02) * (0.17) + (0.82) * (0.83) + (1.46) * (1.31)\}$

$r = \frac{1}{5}\{1.64 + 0.64 + 0.03 + 0.003 + 0.68 + 1.91\} = \frac{4.903}{5}$

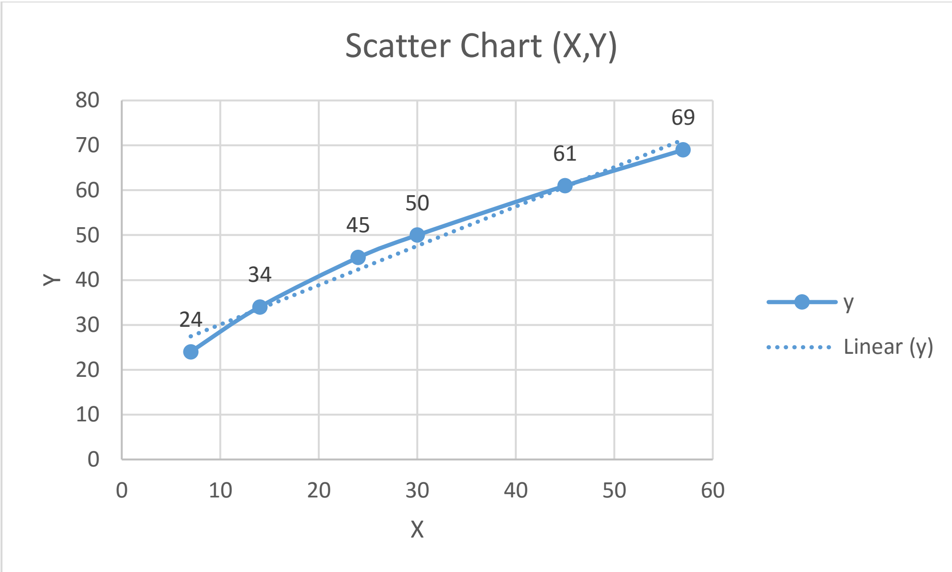
r = 0.98

or, we can calculate the correlation coefficient as below

$$r = r_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\ \sqrt{n\sum y_i^2 - (\sum y_i)^2}}.$$

$r = \frac{6*9902-177*283}{\sqrt{6*6995-177*177}\sqrt{6*14739-283*283}} = \frac{59412-50091}{\sqrt{41970-31329}\sqrt{88434-80089}}$

$= \frac{9321}{\sqrt{10641}\sqrt{8345}} = \frac{9321}{103.15*91.35} = \frac{9321}{9423.2296} = \mathbf{0.9891}$



Since ‘r’ is close to 1 (r = 0.98) and the curve is almost linear, they are strongly related and positive.

4) Find the correlation coefficient for the following data set and interpret the results.

Vehicle model	Mileage (m/g)	Price $’000
1	19	14.94
2	19	14.8
3	20	24.76
4	20	14.93
5	20	13.95
6	21	17.88
7	21	11.65
8	22	17.9
9	23	21.5
10	24	13.25
11	25	9.6
12	17	13.95
13	28	13.07
14	32	6.6
15	33	9.41
16	34	5.87
17	35	6.49

Solution:

Let's consider that based on mileage and other factors, price of the vehicle is decided.
So, let's assume that, Mileage = x (independent) and Price = y (dependent)

$$\text{Correlation Coefficient, } r = r_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\ \sqrt{n\sum y_i^2 - (\sum y_i)^2}}.$$

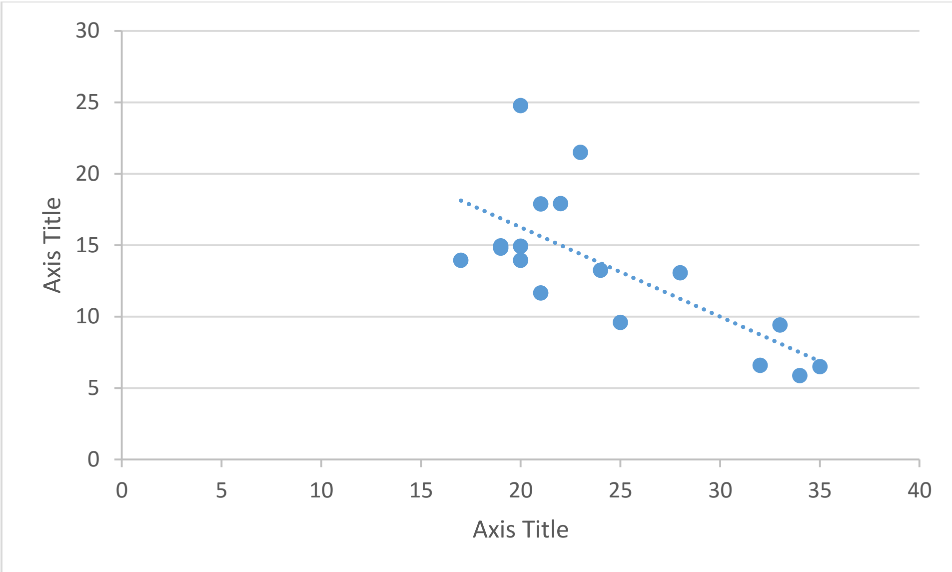
Vehicle model	Mileage (x)	Price (y)	xy	x^2	y^2
1	19	14.94	283.86	361	223.2036
2	19	14.8	281.2	361	219.04
3	20	24.76	495.2	400	613.0576
4	20	14.93	298.6	400	222.9049
5	20	13.95	279	400	194.6025
6	21	17.88	375.48	441	319.6944
7	21	11.65	244.65	441	135.7225
8	22	17.9	393.8	484	320.41
9	23	21.5	494.5	529	462.25
10	24	13.25	318	576	175.5625
11	25	9.6	240	625	92.16
12	17	13.95	237.15	289	194.6025
13	28	13.07	365.96	784	170.8249
14	32	6.6	211.2	1024	43.56
15	33	9.41	310.53	1089	88.5481
16	34	5.87	199.58	1156	34.4569
17	35	6.49	227.15	1225	42.1201
SUM	**413**	**230.55**	**5255.86**	**10585**	**3552.721**

$$r = \frac{17*5255.86-413*230.55}{\sqrt{17*10585-413*413}\sqrt{17*3552.721-230.55*230.55}}$$

$$= \frac{89349.62-95217.15}{\sqrt{179945-170569}\sqrt{60396.257-53153.3025}}$$

$$= \frac{-5867.53}{\sqrt{9376}\sqrt{7242.9545}} = \frac{-5867.53}{96.82*85.1} = \frac{-5867.53}{8240.2115} = \mathbf{-0.712}$$

$$\text{Threshold} = \frac{1.96}{\sqrt{n}} = \frac{1.96}{\sqrt{17}} = \frac{1.96}{4.123} = 0.475$$



Observations: -

i) r is negative, hence its Inverse correlation or negative correlation.

ii) Magnitude of ‘r’ is 0.712, hence it’s a medium correlation coefficient

5) Local ice cream shop keeps track of how much ice cream sell versus the noon temperature on that day. Here are their figures for the last 12 days.

a. Identify if there is a linear or otherwise relationship between Ice Cream Sales and Temperature at noon of the day.

b. Predict the Ice Cream sales if the noon temperature is 26.5 degree centigrade.

Temperature (centigrade)	Ice Cream Sales ($)
14.2	215
16.4	325
11.9	185
15.2	332
18.5	406
22.1	522
19.4	412
25.1	614
23.4	544
18.1	421
22.6	445
17.2	408

Solution:

$$\text{Correlation Coefficient, } r = r_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\ \sqrt{n\sum y_i^2 - (\sum y_i)^2}}.$$

	Temperature (centigrade) (x)	Ice Cream Sales ($) (y)	x*y	x²	y²
	14.2	215	3053	201.64	46225
	16.4	325	5330	268.96	105625
	11.9	185	2201.5	141.61	34225
	15.2	332	5046.4	231.04	110224
	18.5	406	7511	342.25	164836
	22.1	522	11536.2	488.41	272484
	19.4	412	7992.8	376.36	169744
	25.1	614	15411.4	630.01	376996
	23.4	544	12729.6	547.56	295936
	18.1	421	7620.1	327.61	177241
	22.6	445	10057	510.76	198025
	17.2	408	7017.6	295.84	166464
SUM	**224.1**	**4829**	**95506.6**	**4362.05**	**2118025**
Average (Mean)	**18.675**	**402.4167**	**7958.883**	**363.5042**	**176502.1**

$$r = \frac{12*95506.6-224.1*4829}{\sqrt{12*4362.05-224.1*224.1}\sqrt{12*2118025-4829*4829}}$$

$$= \frac{1146079.2-1082178.9}{\sqrt{52344.6-50220.81}\sqrt{25416300-23319241}}$$

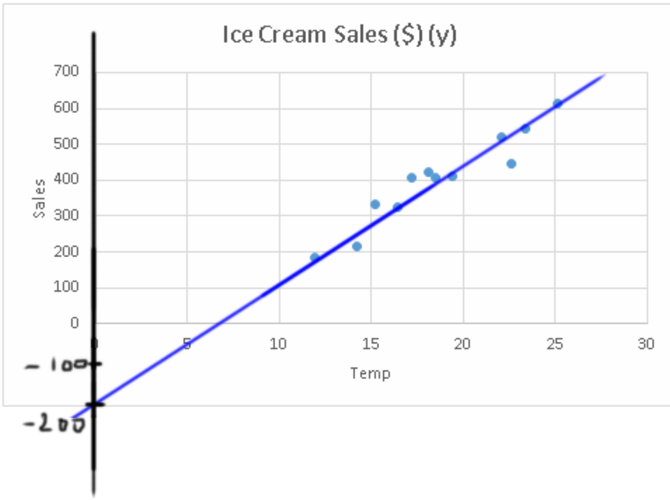
$$= \frac{63900.3}{\sqrt{2123.79}\sqrt{2097059}} = \frac{63900.3}{46.08*1448.12} = \frac{63900.3}{66729.3696} =$$

r = 0.9576

$$\text{Threshold} = \frac{1.96}{\sqrt{n}} = \frac{1.96}{\sqrt{12}} = \frac{1.96}{3.46} = 0.566$$

Correlation coefficient is greater than threshold and r is close to 1, hence it’s a very strong correlation.

Scatter plot for the given values looks like the below graph



By looking at the scatter-plot, we can conclude that there is a linear relationship between temperature and ice-cream sales and they are directly proportional to each other.

We know the general equation of line, y = mx+c,
where m is slope and c is y-intercept

Let's derive the slope of the line, m = $\frac{y2-y1}{x2-x1} = \frac{614-325}{25.1-16.4} = \frac{289}{8.7}$ = **33.21**

y-intercept (from the graph), **c = -200**

so, the equation of the curve is,

y = 33.21x – 200

When x = 26.5°c, y (Ice Cream Sales) = 33.21*26.5-200 = 880.065 – 200 = **680.065**

6) Weight loss of a person is assumed to depend on the number of hours of exercise in gym. Observed values of these for 8 people are given in the table below. Validate if the assumption is right. Predict the weight loss for 70 hours of exercise in the gym.

Hours spent in gym	Weight loss in Kg
100	15
75	11
80	15
90	14
60	8
50	9
25	2
40	5

Solution:

$$\text{Correlation Coefficient, } r = r_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\,\sqrt{n\sum y_i^2 - (\sum y_i)^2}}.$$

	Hours spent in gym (x)	Weight loss in Kg (y)	x*y	x^2	y^2
	100	15	1500	10000	225
	75	11	825	5625	121
	80	15	1200	6400	225
	90	14	1260	8100	196
	60	8	480	3600	64
	50	9	450	2500	81
	25	2	50	625	4
	40	5	200	1600	25
SUM	**520**	**79**	**5965**	**38450**	**941**
Average (Mean)	**65**	**9.875**	**745.625**	**4806.25**	**117.625**

$$r = \frac{8*\mathbf{5965}-\mathbf{520}*\mathbf{79}}{\sqrt{8*\mathbf{38450}-\mathbf{520}*\mathbf{520}}\sqrt{8*941-79*79}}$$

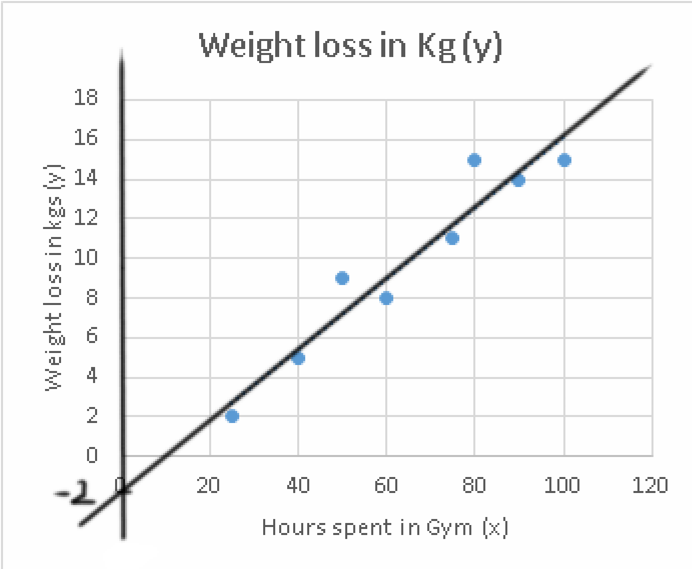
$$= \frac{47720-41080}{\sqrt{307600-270400}\sqrt{7528-6241}}$$

$$= \frac{6640}{\sqrt{37200}\sqrt{1287}} = \frac{6640}{192.87*35.87} = \frac{6640}{6918.2469}$$

r = 0.9597

$$\text{Threshold} = \frac{1.96}{\sqrt{n}} = \frac{1.96}{\sqrt{8}} = \frac{1.96}{2.82} = 0.695$$

Correlation coefficient is greater than threshold and r is close to 1, hence it’s a very strong correlation.



We know the general equation of line, y = mx+c,
where m is slope and c is y-intercept

Let's derive the slope of the line, m = $\frac{y2-y1}{x2-x1}$ = $\frac{14-5}{90-40}$ = $\frac{9}{50}$ = **0.18**

y-intercept, **c = -2**

so, the equation of the curve is,
y = 0.18x – 2
It's evident from the curve that as the number of hours in gym is increasing, weight loss is increasing. They are positively correlated and the correlation is strong.

When x = 70, y =0.18*70 - 2 = 12.6 – 2 = **10.6**