

# Basic Probability Theory

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## Basic terms and concepts:

- ❖ **Random Experiment:** If it is not possible to predict the outcome of an experiment with certainty before the experiment, then such experiments are called random experiment.
- ❖ **Sample Space:** The totality of all the possible outcomes of a random experiment is called the sample space of the experiment and it is generally denoted by the letter S.

Example: - The sample space (S) for tossing a fair coin

$S = \{\text{Head}, \text{Tail}\}$

- **Finite sample space:** If the set of all possible outcomes is finite then sample space is called a finite sample space.  
 $S = \{1, 2, 3, 4\}$
- **Countably Infinite sample space:** When the outcomes of the experiment can be put in correspondence with natural numbers, it is said to be countably infinite.  
 $S = \{1, 2, 3, 4, 5, 6, \dots\}$
- **Uncountable sample space:** When the outcomes of the experiment cannot be put in correspondence with natural numbers, it is said to be uncountably infinite.  
 $S = x \mid \{x : (1, 2)\}$

❖ **Event:** An event can be defined as the outcome or a set of outcomes of a random experiment.

- Mutually exclusive events: A list of events can be regarded as mutually exclusive if and only if:

$$A_1 \cap A_2 = \begin{cases} A_1 & | A_1 = A_2 \\ \text{Null} & | A_1 \neq A_2 \end{cases}$$

- Equiprobable events: When events have equal chance of occurring. Example: In case of tossing a fair coin the probability of getting a head or tail = 0.5
- Collectively exhaustive events: The events are called collectively exhaustive when the union of events covers the whole sample space.

$$(A_1 \cup A_2 \cup A_3 \dots) = S$$

❖ **Axioms of probability:** If A & B are events in space S

- $1 \geq P(A) \geq 0$
- $P(S) = 1$
- $P(A \cup B) = P(A) + P(B)$  where  $A \cap B = \text{null}$

❖ **Constant:** An entity whose value is fixed.

❖ **Variable:** An entity whose value may or may not change.

- Deterministic: If the outcome of a variable is fixed, i.e. if a variable will always have the exact same value, we call this a deterministic variable.
- Random Variable: A random variable X on a sample space S is a function  $X : S \rightarrow R$  that assigns a real number X(s) to each sample point  $s \in S$ .
  - Discrete Random Variable: If the values of random variable are finite or countable. A random variable defined on a discrete sample space will be discrete, while it is possible to define a discrete random variable on a continuous sample space.
  - Continuous Random Variable: If a random variable takes all values within a range. The domain can be discrete or continuous.

- ❖ Probability Density Function (PDF): A function of a continuous random variable, whose integral across an interval gives the probability that the value of the variable lies within the same interval.

$$P(a \leq x \leq b) = \int_a^b f(x) dx$$