Statistics

Measures of Central Tendency:

1. **Mean (Average):** Also called the arithmetic mean or average
   The sum of all the values in the sample divided by the number of values in the sample/population
   μ is the mean of the whole population;
   $\overline{x}$ is the mean of the sample taken from the population

$$\bar{x} := \frac{\sum_{i=1}^{n} x_i}{n}.$$

2. **Median**: The middle value when the data are ordered, so that 50% of the data are above and 50% are below

$$\text{median} := \tilde{x} := \begin{cases} x_{(\frac{n+1}{2})} & \text{n odd} \\ \frac{1}{2}[x_{(\frac{n}{2})} + x_{(\frac{n}{2}+1)}] & \text{n even} \end{cases}.$$

3. **Mode**: the most frequently occurring value

Measures of Dispersion:

1. **Range**: The minimum and maximum values
   $x_{max} - x_{min}$ measures dispersion
2. **Variance**: Measures dispersion around the mean
   Determined by averaging the squared differences of all the values from the mean

   Variance of a population is, $\sigma^2 = \frac{\sum(x-\mu)^2}{n}$

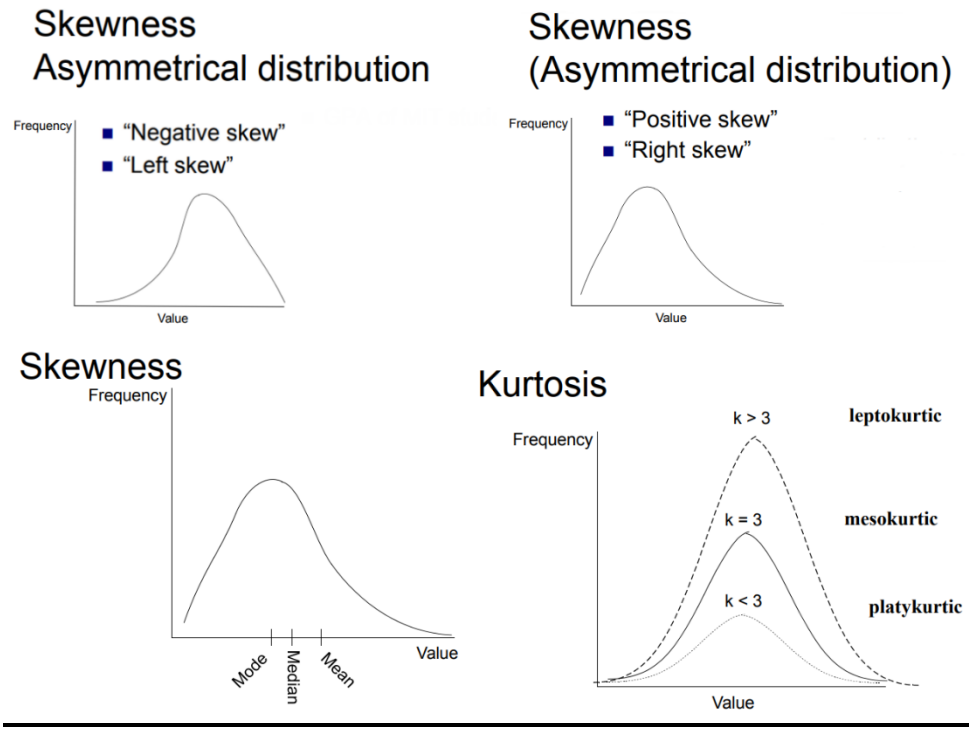
   Variance of a sample is s^2 (note the n-1), $s2 = \frac{\sum(x-\overline{x})^2}{n-1}$
3. **Standard Deviation**: square root of the variance
   Also measures dispersion around the mean but in the same units as the values (instead of square units with variance)
   'σ' is the standard deviation of the population;
   's' is the standard deviation of the sample
4. **Skewness:** Skewness is defined such that if more datapoints lie below the mean of the dataset it is known as negatively skewed. If the distribution is depleted of values below the mean it is positively skewed.

Nature of Skewness

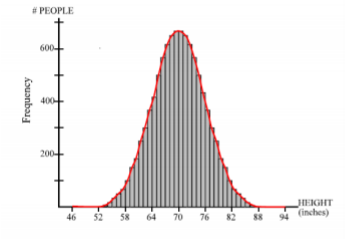
Skewness can be positive or negative or zero.

i) When the values of mean, median and mode are equal, there is no skewness.
ii) When mean > median > mode, skewness will be positive.
iii) When mean < median < mode, skewness will be negative.

5. **Kurtosis**: The second deviation from symmetry is known as kurtosis and compares the population of the tails of the dataset to that of the central region. If a distribution is more peaked than a Gaussian and/or the tails are more populated than a Gaussian then it has a positive kurtosis. If a distribution has tails less populated and/or is less peaked than a Gaussian then it has a negative kurtosis.



Normal distribution



- Skewness = 0
- Kurtosis = 3

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)/2\sigma^2}$$

Measures of Association:

1. **Covariance:** Degree with which y depends on x. Covariance is a measure of how much two random variables vary together.

   Suppose X and Y are random variables with means μ_X and μ_Y. The covariance of X and Y is defined as

   $$Cov(X, Y) = E((X - \mu_X)(Y - \mu_Y))$$

2. **Correlation Coefficient:** This is also called Pearson Correlation Coefficient

Measures the strength of a linear relationship between two variables

Sample correlation coefficient, $r = \frac{S_{xy}}{S_x S_y}$

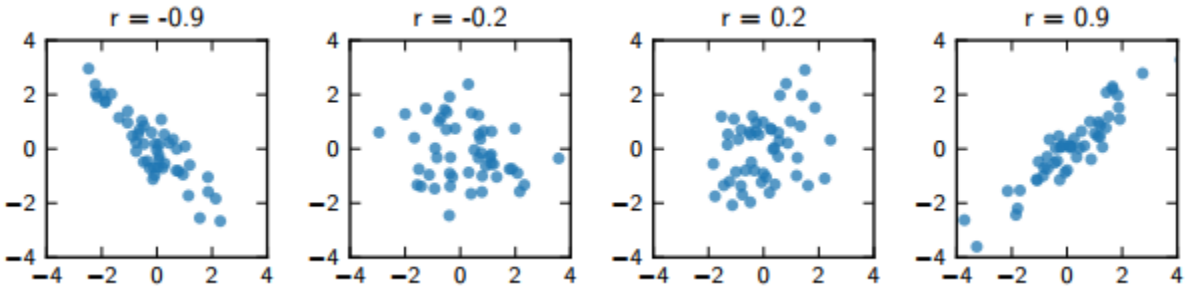
where

$$S_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$S_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

We can rewrite r as

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\frac{(x_i - \bar{x})}{S_x}\frac{(y_i - \bar{y})}{S_y},$$



- r ∈ [−1, 1] and is ±1 only when data fall along a straight line
- sign(r) indicates the slope of the line
- If r is -1, means oppositely related
- If r is 0, means there is no dependency

3. **Linear relationship: Regression** is a set of technique for estimating relationships. One of the simplest type of relationship is **linear regression**.

Let's take equation of the line, **y = mx + c**

x – independent variable or predictor variable

y – dependent variable or response variable

m – slope of the line

If 'm' is close to 0 indicates little to no relationship

Value of m with large positive or negative values indicate large positive or negative relationships respectively

c – constant, intercept of the line

Problem: Find whether the Stock x and y are linearly correlated?

Year	Stock (x)	Stock (y)	x - $\bar{x}$	$(x - \bar{x})^2$	y-$\bar{y}$	$(y-\bar{y})^2$	$(x - \bar{x})(y-\bar{y})$
2001	5	-2	3	9	-5	25	-15
2002	3	1	1	1	-2	4	-2
2003	-2	6	-4	16	3	9	-12
2004	2	7	0	0	4	16	0
Sum	8	12		26		54	-29
	$\bar{x}$ =2	$\bar{y}$=3		$S_x^2 = \frac{26}{3}$ = 8.67		$S_y^2 = \frac{54}{3}$ = 18	$S_{xy} = \frac{-29}{3}$ = -9.66
				S_x=2.94		S_y=4.24	

$$r_{xy} = \frac{S_{xy}}{S_x S_y} = \frac{-9.66}{(2.94)(4.24)} = -0.77$$

$$r_{xy} < \frac{1.96}{\sqrt{n}} = \frac{1.96}{\sqrt{4}} = 0.98 \text{ i.e. } r_{xy} < 0.98$$

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	**0.773952743**	**Pearson Coefficient, r = 0.773952743**						
R Square	0.599002849							
Adjusted R Square	0.398504274							
Standard Error	3.290429011							
Observations	4							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	32.34615385	32.34615385	2.987566607	0.226047257			
Residual	2	21.65384615	10.82692308					
Total	3	54						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	5.230769231	2.091031143	2.501526219	0.129482995	-3.766211626	14.22775009	-3.766211626	14.22775009
Stock (x)	-1.115384615	0.645306221	-1.728457870	0.226047257	-3.891913187	1.661143956	-3.891913187	1.661143956

Problem: Annual revenues and profits of an IT company for the years 2004 to 2014 are given. Find the correlation coefficient.

Year	Revenue (x)	Profit (y)	$x - \bar{x}$	$(x - \bar{x})^2$	$y - \bar{y}$	$(y - \bar{y})^2$	$(x - \bar{x})(y - \bar{y})$	x^2	xy
2004	225	42	-39.0909091	1528.09917	-13	169	508.1818182	50625	9450
2005	237	43	-27.0909091	733.917355	-12	144	325.0909091	56169	10191
2006	245	48	-19.0909091	364.46281	-7	49	133.6363636	60025	11760
2007	222	40	-42.0909091	1771.64463	-15	225	631.3636364	49284	8880
2008	265	60	0.909090909	0.82644628	5	25	4.545454545	70225	15900
2009	270	56	5.909090909	34.9173554	1	1	5.909090909	72900	15120
2010	254	53	-10.0909091	101.826446	-2	4	20.18181818	64516	13462
2011	280	60	15.90909091	253.099174	5	25	79.54545455	78400	16800
2012	290	62	25.90909091	671.280992	7	49	181.3636364	84100	17980
2013	305	65	40.90909091	1673.55372	10	100	409.0909091	93025	19825
2014	312	76	47.90909091	2295.28099	21	441	1006.090909	97344	23712
Sum	2905	605		9428.90909		1232	3305	776613	163080
	264.0909091	55							

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.969695512	**Pearson Coefficient, r = 0.969695512159858**						
R Square	0.940309386							
Adjusted R Square	0.933677096							
Standard Error	2.858492922							
Observations	11							
ANOVA								
	df	*SS*	*MS*	*F*	*Significance F*			
Regression	1	1158.461	1158.461164	141.777475	8.23E-07			
Residual	9	73.53884	8.170981786					
Total	10	1232						
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-37.56855126	7.821902	-4.80299424	0.00096967	-55.2629	-19.8742	-55.26292339	-19.8742
Revenue (x)	0.35051775	0.029438	11.90703468	8.227E-07	0.283925	0.417111	0.283924659	0.417111

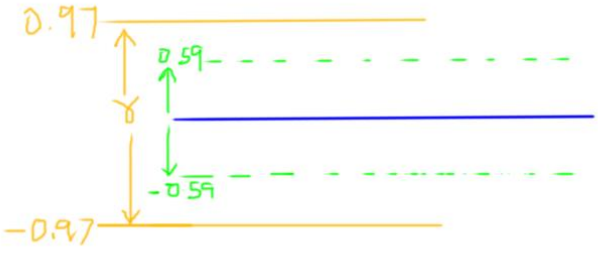
$S_x = \sqrt{\frac{\sum(x-\overline{x})^2}{n-1}} = \sqrt{\frac{9429}{10}} = \sqrt{942.9} = 30.69$

$S_y = \sqrt{\frac{\sum(y-\overline{y})^2}{n-1}} = \sqrt{\frac{1232}{10}} = 11.09$

$S_{xy} = \frac{\sum(x-\overline{x})(y-\overline{y})}{n-1} = \frac{3305}{10} = 330.5$

$r_{xy} = \frac{Sxy}{SxSy} = \frac{330.5}{30.69*11.09} = 0.971$

Threshold $= \frac{1.96}{\sqrt{n}} = \frac{1.96}{\sqrt{11}} = \frac{1.96}{3.31} = 0.5921$



Problem: In the above problem, if the revenue for the year 2015 is $350 million. Forecast the profit for 2015 in terms of millions of dollars.

776613 m + 2905 c = 163080

2905 m + 11 c = 605

m = 0.35

c = -37.57

If x = 350, then y = $ 84.9 million

Problem: In the above problem, find the profit for the year 2004

Y = mx + c

= 0.35 * 225 – 37.57

= 78.75 – 37.57

= 41.18

Steps to follow in solving a problem:

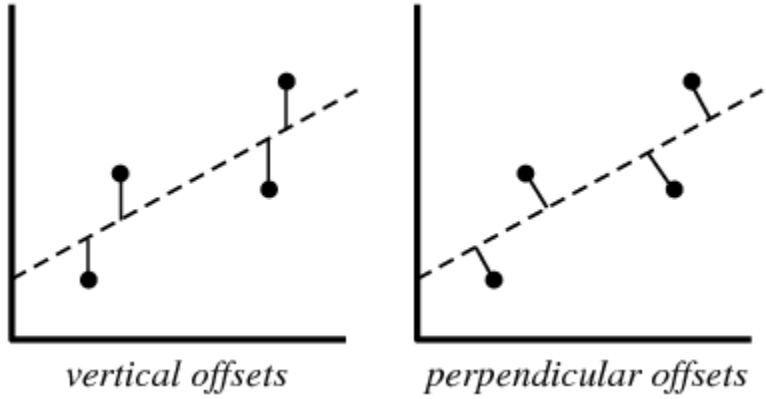
1. Scatter Plot
2. Identify Trend and Outliers
3. Transform data (if required)
4. Validate the Trend
5. Fit a curve
6. Forecast
7. Verify the forecasted value

Least square technique: -

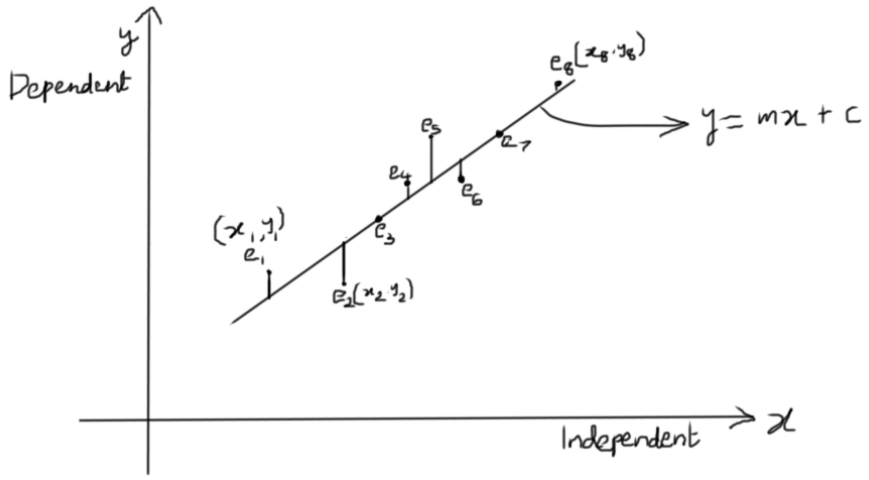
The least squares method is a form of mathematical regression analysis that finds the *line of best fit* for a dataset, providing a visual demonstration of the relationship between the data points. Each point of data is representative of the relationship between a known independent variable and an unknown dependent variable.

In regression analysis, dependent variables are designated on the vertical Y axis and independent variables are designated on the horizontal X axis. These designations will form the equation for the line of best fit, which is determined from the least squares

method.



In practice, the ***vertical offsets*** from a line (polynomial, surface, hyperplane, etc.) are almost always minimized instead of the ***perpendicular offsets***. In addition, the fitting technique can be easily generalized from a best-fit *line* to a best-fit *polynomial* when sums of vertical distances are used. In any case, for a reasonable number of noisy data points, the difference between vertical and perpendicular fits is quite small.



The method of least squares gives a way to find the best estimate, assuming that the errors (i.e. the differences from the true value) are random and unbiased.

Let's us assume the equation of the line as

$$y = mx + c$$

Error, $e_1^2 = [y_1 - (mx_1 + c)]^2$

$e_2^2 = [y_2 - (mx_2 + c)]^2$

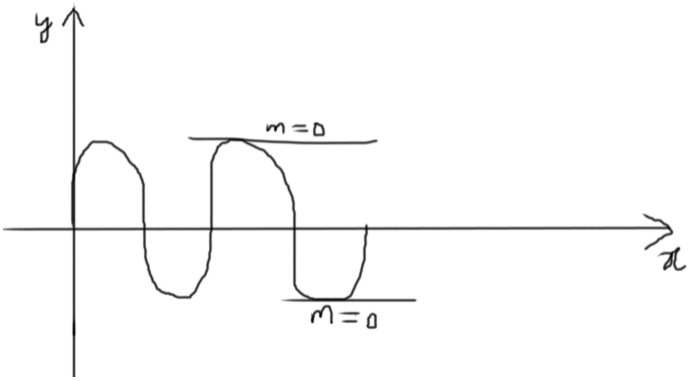
We need to find the sum of squares of the deviation is minimal.

i.e., $e_1^2 + e_2^2 + \cdots + e_8^2$ should be minimum

which means $\sum e_i^2$ should be minimum

$\sum e_i^2 = \sum[y_i - (mx_i + c)]^2$ is minimum

At maximum point or minimum point of wave curve, the slope should be zero



$\frac{\partial e_i^2}{\partial x}$ = 0

$\frac{\partial e_i^2}{\partial x}$ = 2 $\sum[y_i - (mx_i + c)](-x)$ = 0

$\frac{\partial e_i^2}{\partial c}$ = 2 $\sum[y_i - (mx_i + c)](-1)$ = 0

$\frac{\partial e_i^2}{\partial c}$ = -$\sum(x_i y_i) + m \sum x_i^2 + c \sum x$ = 0

$\frac{\partial e_i^2}{\partial c}$ -$\sum y_i + m \sum x_i + c \sum$ = 0

m $\sum x_i^2$ + c $\sum x = \sum x_i y_i$

m $\sum x_i$ + c. n = $\sum y_i$

We have 2 equations and 2 unknowns, so we would be able to solve.