CHAPTER 14

ATTENUATION AND ATTENUATORS

EXPRESSION OF ATTENUATION IN DECIBELS AND IN NEPERS

The *decibel* is fundamentally a unit of power ratio, but, as has been shown in Chapter 5, it can be used to express current ratios when the resistive components of the impedances through which the current flows are equal, and voltage ratios when the conductive components of these impedances are equal. The *neper* is fundamentally a unit of current ratio, but it can be used to express power ratios when the resistive components of the impedances are equal.

The loss of power in a transmission line or electrical network is known as " attenuation ". Attenuation may be measured using either the decibel or the neper notation.

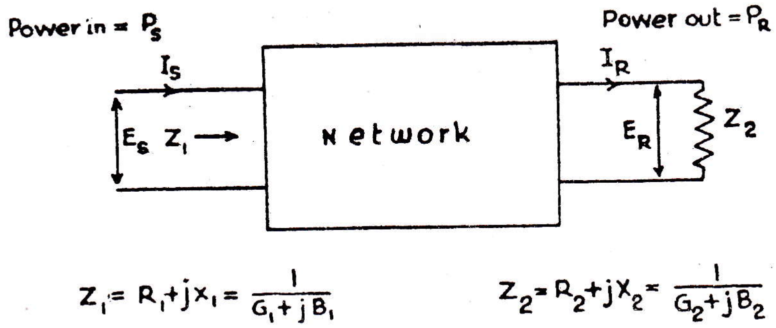


FIG. 610.—Attenuation measured using decibel and neper notations.

If the power entering a network is P_S and the power leaving it is P_R (*see* Fig. 610), then the attenuation in decibels is defined as :—

$$\text{Attenuation in decibels} = 10 \,.\, \log_{10}\left|\frac{P_S}{P_R}\right| \qquad (1)$$

If the current entering a network is I_S and the current leaving it is I_R, then the attenuation in nepers is defined as :—

$$\text{Attenuation in nepers} = \log_e\left|\frac{I_S}{I_R}\right| \qquad (2)$$

Because of its derivation from the exponential e, the neper is the most convenient unit for expressing attenuation in theoretical work. The decibel, on the other hand, being defined in terms of logarithms to base 10, is a more convenient unit in practical calculations using the decimal system of reckoning. The conditions under which the two units may be used can be summarised in the following equations, the notation of which is indicated in Fig. 610 :—

$$\text{Attenuation in db} = 10 \,.\, \log_{10} \left| \frac{P_S}{P_R} \right| \tag{3}$$

$$= 20 \,.\, \log_{10} \left| \frac{I_S}{I_R} \right| \text{ (provided that } R_1 = R_2\text{)} \tag{4}$$

$$= 20 \,.\, \log_{10} \left| \frac{E_S}{E_R} \right| \text{ (provided that } G_1 = G_2\text{)} \tag{5}$$

$$\text{Attenuation in nepers} = \log_e \left| \frac{I_S}{I_R} \right| \tag{6}$$

$$= \log_e \left| \frac{E_S}{E_R} \right| \text{ (provided that } |Z_1| = |Z_2|\text{)} \tag{7}$$

$$= \tfrac{1}{2} \log_e \left| \frac{P_S}{P_R} \right| \text{ (provided that } R_1 = R_2\text{)} \tag{8}$$

If the resistive components of the impedances at the input and output of the network are equal, then the attenuation may be readily converted from one notation to the other, for :—

$$\begin{aligned}
(\text{Attenuation in db}) &= 20 \,.\, \log_{10} \left| \frac{I_S}{I_R} \right| \\
&= 20 \,.\, \log_e \left| \frac{I_S}{I_R} \right| \times \log_{10} e \\
&= 8{\cdot}686 \,.\, \log_e \left| \frac{I_S}{I_R} \right| \\
&= 8{\cdot}686 \times (\text{attenuation in nepers})
\end{aligned}$$

Thus :—

$$\text{Attenuation in db} = 8{\cdot}686 \times \text{attenuation in nepers (provided that } R_1 = R_2\text{)} \tag{9}$$

$$\text{Attenuation in nepers} = 0{\cdot}1151 \times \text{attenuation in db (provided that } R_1 = R_2\text{)} \tag{10}$$

ATTENUATING NETWORKS

In transmission equipment, it is frequently desired to attenuate the currents and voltages at certain stages. Attenuators and pads are networks designed to meet this requirement, and since, to prevent attenuation distortion, all frequencies must be attenuated to the same degree, the networks must consist of purely resistive components. No phase-shift will be introduced by such networks ; thus, for each network, the phase constant (β) will be zero, and the propagation constant (γ) will simply be equal to the attenuation constant (α)*. A fixed attenuator is sometimes known as a " pad ".

* Bearing these facts in mind, the designs and properties of attenuating networks may be deduced from the equations of Chapter 13. In this chapter, however, the results will, in many cases, be obtained in a simple manner from first principles for the benefit of the readers less familiar with the subject.

These networks, by choice of suitable resistances, may have any required value of attenuation. They may be designed to have any resistive value of characteristic impedance, if symmetrical, or of image impedances if asymmetrical. One of these networks may therefore be used in place of a transformer for matching between circuits of different resistive impedance, thus avoiding, particularly in carrier-frequency circuits, the attenuation distortion introduced by a transformer. The attenuation introduced will be of little consequence if valve amplification is included in the circuit.

There are three conditions that the attenuating network must fulfil. It must give :—

(1) the correct input impedance ;
(2) the correct output impedance ;
(3) the specified attenuation.

This attenuation is usually quoted in decibels :—

Attenuation in decibels $$D = 10 \log_{10} \frac{P_S}{P_R}$$

where P_S is power input and P_R power output.

In the following considerations the symbol N will be used for $\sqrt{\frac{P_S}{P_R}}$, *i.e.* :—

$$\text{Attenuation } D = 20 \log_{10} \sqrt{\frac{P_S}{P_R}} = 20 \log_{10} N \qquad (11)$$

If both pairs of terminals of the network are matched to the same impedance, then :—

$$\frac{P_S}{P_R} = \frac{I_S^2}{I_R^2}$$

Therefore for a pad in a symmetrical circuit, $N = \frac{I_S}{I_R}$, but in an asymmetrical circuit the value $N = \sqrt{\frac{P_S}{P_R}}$ must always be used.

Symmetrical T type

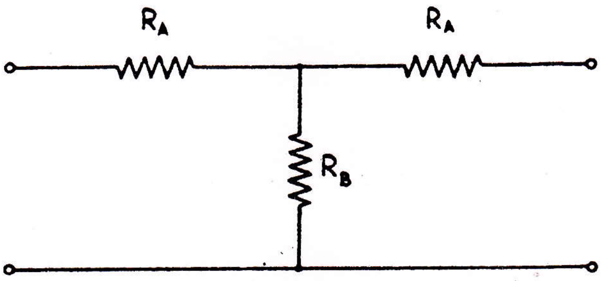


FIG. 611.—Symmetrical T network.

This is one of the most common types of pad, and consists of a divided series arm and one central shunt arm. The pad used between equal impedances will be symmetrical, *i.e.*, the series arm is divided into two equal parts (*see* Fig. 611). The values of the series and shunt arms for a given value of impedance and attenuation will now be determined.

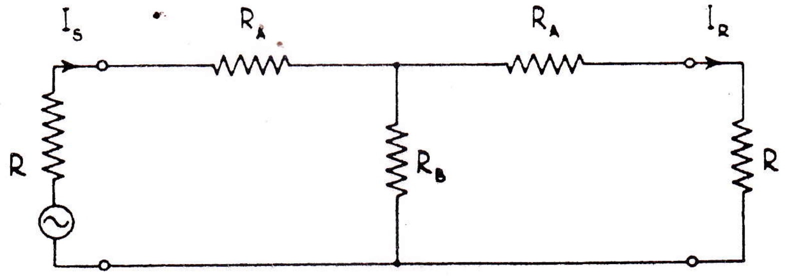


FIG. 612.—Symmetrical T network.

Consider the input current I_s (Fig. 612). At the shunt arm it divides in proportion to the conductances.

$$\text{Hence} \qquad I_R = \frac{R_B}{R_B + R_A + R} \cdot I_s \tag{12}$$

$$\therefore \qquad N \equiv \frac{I_s}{I_R} = \frac{R_B + R_A + R}{R_B} \tag{13}$$

But the impedance looking into the attenuator is required to be R.

$$\text{Hence} \qquad R = R_A + \frac{R_B(R_A + R)}{R_B + R_A + R}$$

$$= R_A + \frac{R_A + R}{N}$$

$$\therefore \qquad R(N - 1) = R_A(N + 1)$$

$$\therefore \qquad R_A = R\left(\frac{N - 1}{N + 1}\right) \tag{14}$$

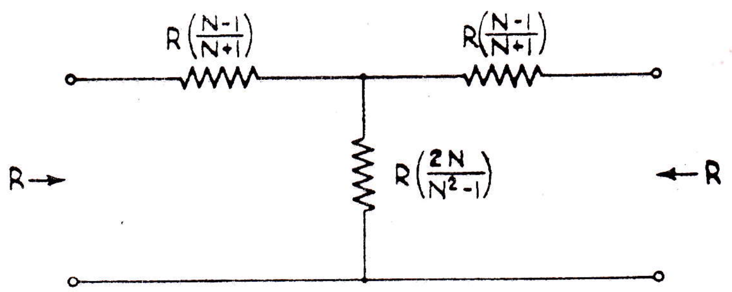


FIG. 613.—T network having input and output impedance equal to R.

$$\text{But} \qquad N = \frac{R_B + R_A + R}{R_B}$$

$$\therefore \qquad R_B(N - 1) = R_A + R$$

$$= R\left(\frac{2N}{N + 1}\right)$$

$$\therefore \qquad R_B = R\left(\frac{2N}{N^2 - 1}\right) \tag{15}$$

Using these formulae, therefore, an attenuator can be designed to give the specified attenuation, and to be properly matched to the circuit.

The resultant T section is shown in Fig. 613.

It may be noted that the results obtained also follow directly from Fig. 574 (Chapter 13, page 575), bearing in mind that in this case the characteristic impedance of the required T section is R and the propagation constant is α.

$$\text{i.e.}\quad R_A = R\tanh\frac{\alpha}{2} = R\frac{e^{\frac{\alpha}{2}} - e^{-\frac{\alpha}{2}}}{e^{\frac{\alpha}{2}} + e^{-\frac{\alpha}{2}}} = R\frac{e^{\alpha} - 1}{e^{\alpha} + 1} = R\left(\frac{N-1}{N+1}\right)$$

$$\text{and}\quad R_B = \frac{R}{\sinh \alpha} = \frac{2R}{e^{\alpha} - e^{-\alpha}} = \frac{2Re^{\alpha}}{e^{2\alpha} - 1} = R\left(\frac{2N}{N^2 - 1}\right)$$

$$\text{where } e^{\alpha} = \frac{I_S}{I_R} = N$$

Example.—Design a T type pad to give 25 db attenuation and to have a characteristic impedance of 600 ohms.

$$N = \text{antilog}_{10}\frac{D}{20} = \text{antilog}_{10}\frac{25}{20}$$

$$= 17{\cdot}8$$

$$R_A = R\left(\frac{N-1}{N+1}\right)$$

$$= 600 \times \frac{16{\cdot}8}{18{\cdot}8}$$

$$= 536 \text{ ohms.}$$

$$R_B = 2R\left(\frac{N}{N^2 - 1}\right)$$

$$= 1200 \times \frac{17{\cdot}8}{316}$$

$$= 67{\cdot}6 \text{ ohms.} \quad \textit{Ans.}$$

Asymmetrical T type

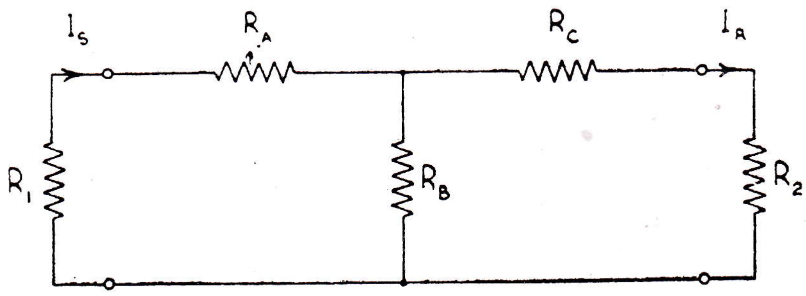


Fig. 614.—Asymmetrical T section.

Fig. 614 shows a pad that is not symmetrical.

Here $$N = \sqrt{\frac{P_S}{P_B}} = \sqrt{\frac{I_S{}^2R_1}{I_B{}^2R_2}} \tag{16}$$

Using this and formulae for the input and output impedances, it can be shown that :—

$$R_A = R_1\left(\frac{N^2+1}{N^2-1}\right) - 2\sqrt{R_1R_2}\left(\frac{N}{N^2-1}\right) \tag{17}$$

$$R_B = 2\sqrt{R_1R_2}\left(\frac{N}{N^2-1}\right) \tag{18}$$

$$R_C = R_2\left(\frac{N^2+1}{N^2-1}\right) - 2\sqrt{R_1R_2}\left(\frac{N}{N^2-1}\right) \tag{19}$$

A network consisting of these components will have image impedances R_1 and R_2. If $R_1 > R_2$ then it will be found that $R_A > R_C$.

L type

If a pad is required for matching purposes only, then the design will be such as to give minimum attenuation. Examining the T section it will be seen that this condition will be reached when R_C has been reduced to zero.

This then forms the L type pad (*see* Fig. 615).

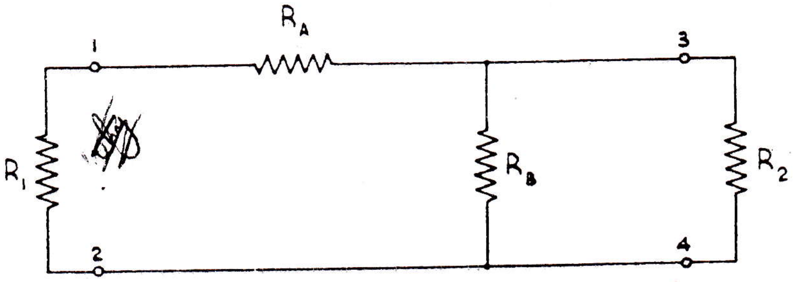


FIG. 615.—L type network.

To obtain values for R_A and R_B, consider input impedances. Looking in at terminals 1 and 2 :—

$$R_1 = R_A + \frac{R_2R_B}{R_2 + R_B}$$

$$\therefore\ R_1R_B + R_1R_2 = R_AR_B + R_2R_A + R_2R_B \tag{20}$$

and looking in at terminals 3 and 4 :—

$$R_2 = \frac{R_B(R_1 + R_A)}{R_B + R_1 + R_A}$$

$$R_2R_B + R_2R_A + R_1R_2 = R_BR_A + R_1R_B \tag{21}$$

Adding equations 20 and 21 :—

$$2R_1R_2 = 2R_AR_B$$

or $$R_A = \frac{R_1R_2}{R_B}$$

Substituting in (20) :—

$$R_B = \sqrt{\frac{R_1 R_2^2}{R_1 - R_2}} \tag{22}$$

Hence
$$R_A = \sqrt{R_1(R_1 - R_2)} \tag{23}$$

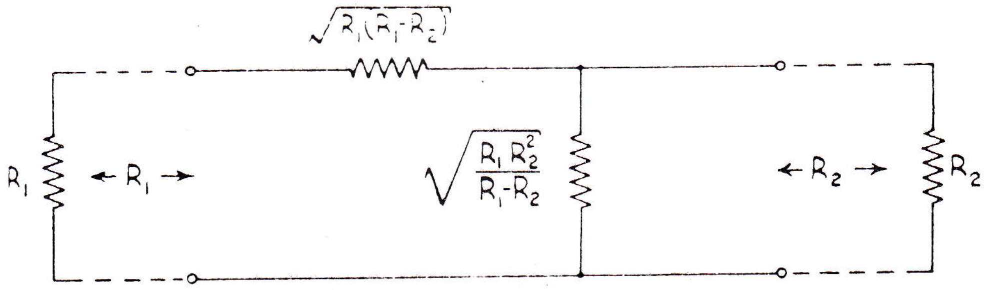


FIG. 616.—L network having image impedances R_1 and R_2.

Fig. 616 shows the resultant L type network. It is a network having image impedances R_1 and R_2.

π type

This attenuator is another common type, consisting of one series arm and two shunt arms. When used purely as an attenuator between equal impedances, symmetry demands that these two shunt arms shall be equal.

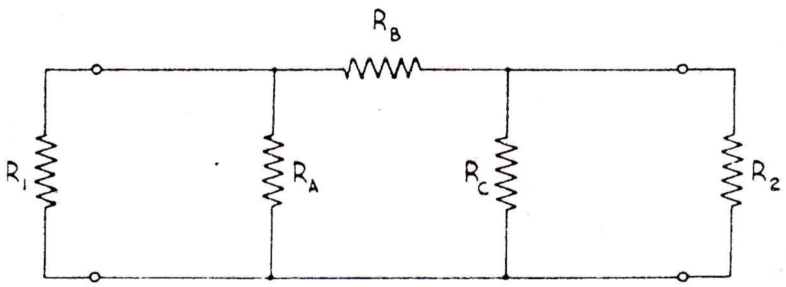


FIG. 617.—π network.

By calculation similar to that used for the T section it may be shown that :—

$$R_A = R_C = R\left(\frac{N+1}{N-1}\right) \tag{24}$$

$$R_B = R\left(\frac{N^2-1}{2N}\right) \tag{25}$$

In a pad used for matching, $R_A \neq R_C$, and the following formulae apply :—

$$R_A = R_1\left(\frac{N^2-1}{N^2-2NS+1}\right) \tag{26}$$

$$R_B = \frac{\sqrt{R_1 R_2}}{2}\left(\frac{N^2-1}{N}\right) \tag{27}$$

$$R_O = R_2\left(\frac{N^2 - 1}{N^2 - 2\frac{N}{S} + 1}\right) \tag{28}$$

where $S^2 = \frac{R_1}{R_2}$

There is no difference in the performance of the T and π type pads and each one will suit any requirement, but one will probably be found to have more suitable or standard components than the other. It may be noted that a deviation of 5 per cent. from the calculated values of the resistances will mismatch the impedances by no more than the same amount, and vary the attenuation by as little as 0·5 db.

Balanced T, L and π types

When it is required to balance the two legs of the circuit, as is frequently the case in transmission equipment, then the preceding pads must be modified by dividing the series arm into two equal halves and inserting one half in each leg (*see* Fig. 618).

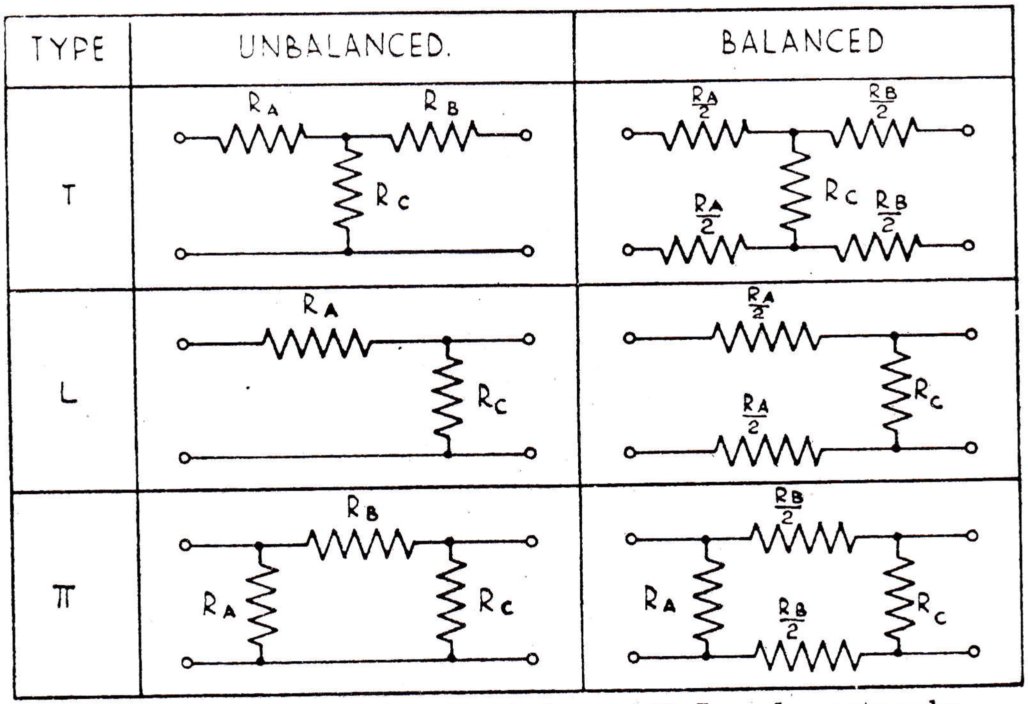


FIG. 618.—Balanced and unbalanced T, L and π networks.

When designing these balanced pads the components of the unbalanced type should be calculated using the formulae already quoted, and the series arm divided between the two legs. The characteristics of this derived pad—that is, the impedance and the attenuation—will be identical to those of the unbalanced pad.

Bridged-T type

Fig. 619 shows a symmetrical bridged-T tyde section used between equal impedances.

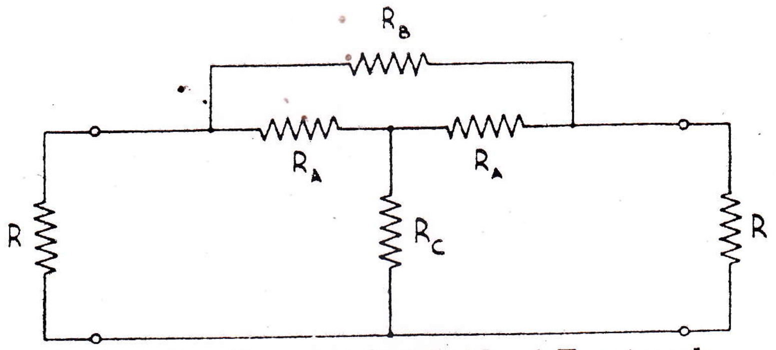


FIG. 619.—Symmetrical bridged-T network.

The network may be designed to have a constant impedance R, but any desired attenuation, by making :—

$$R_B R_C = R_A{}^2 = R^2$$

Thus, to vary the attenuation, without changing the design impedance, only two resistances have to be varied, *viz.*, R_B and R_C. It should be noted that, in the case of a symmetrical T or π section attenuator, three resistances have to be varied to change the attenuation without altering the impedance.

The design formulae for the bridged-T section are :—

$$R_A = R \tag{29}$$
$$R_B = R(N - 1) \tag{30}$$
$$R_C = \frac{R_A{}^2}{R_B} = \frac{R}{N-1} \tag{31}$$

Example.—

Design a bridged-T attenuator having an attenuation of 40 db when working between two 600 ohms impedances.

To give 40 db attenuation, $N = 100$.

$R_A = 600$ ohms. *Ans.*

$R_B = 600\ (100 - 1)$
$= 59{,}400$ ohms. *Ans.*

$$R_C = \frac{600}{(100 - 1)}$$
$= 6{\cdot}06$ ohms. *Ans.*

The network is shown in Fig. 620.

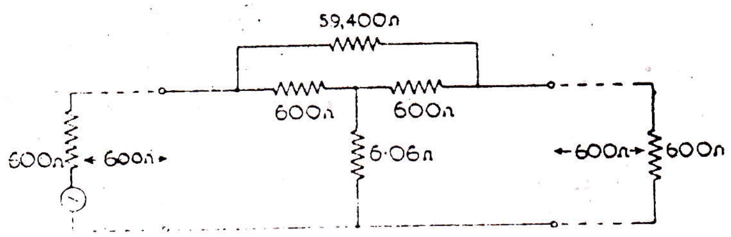


FIG. 620.—Bridged-T network having an attenuation of 40 db.

Lattice type

This type, shown in Fig. 621, is occasionally used.

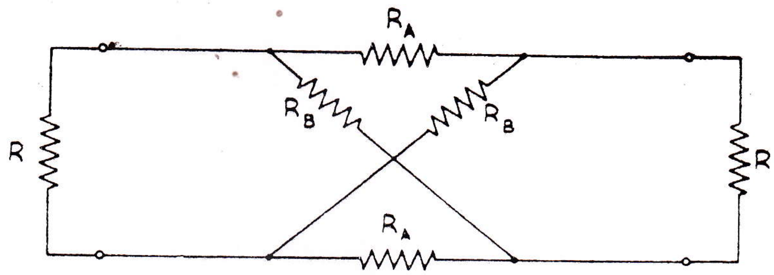


FIG. 621.—Lattice network.

The characteristic impedance R of the network can best be determined by consideration of its open-circuit and short-circuit impedances.

$$R_{OC} = \frac{R_A + R_B}{2} \tag{32}$$

$$R_{SC} = 2\,\frac{R_A R_B}{R_A + R_B} \tag{33}$$

$$\therefore \quad R = \sqrt{R_{OC} R_{SC}}$$
$$= \sqrt{R_A R_B} \tag{34}$$

This gives the condition for matching the network to its adjacent circuit. It can, of course, be used only in a symmetrical case.

It may be shown that :—

$$N = \frac{I_S}{I_R} = \frac{R_A + R_B + 2R}{R_B - R_A}$$

But, from equation 34, $R_B = \dfrac{R^2}{R_A}$

$$\therefore \quad N = \frac{R_A{}^2 + R^2 + 2R\,R_A}{R^2 - R_A{}^2}$$
$$= \frac{R + R_A}{R - R_A} \tag{35}$$

Hence

$$R_A = R\left(\frac{N-1}{N+1}\right) \tag{36}$$

$$R_B = R\left(\frac{N+1}{N-1}\right) \tag{37}$$

DESIGN OF ATTENUATORS AND PADS

The steps in the design of any pad may be summarised as follows :—

(1) Determine the type of pad to be used. In some cases alternative networks will be possible, and that one should be chosen which gives the most convenient component values.

(2) Change the required decibel attenuation D to ratio N by use of Table XVII or the graph in Fig. 622.

(3) If a T or π network is to be used for matching between unequal impedances R_1 and R_2, verify that the value of attenuation chosen is greater than the minimum permissible attenuation given by Fig. 623, otherwise one arm of the network will work out to be negative. With the L type pad this is unnecessary.

(4) With the information so obtained and a knowledge of the design impedance, evaluate the component values by use of the equations already stated above, noting that when it is necessary to insert a loss of more than 40 db, it is usually more convenient to use two smaller pads in series.

TABLE XVII

N Values $\left(N = \sqrt{\frac{P_S}{P_R}}\right)$

D (db)	N	D (db)	N	D (db)	N	D (db)	N
1·0	1·122	18·0	7·943	35·0	56·234	52·0	398·11
2·0	1·259	19·0	8·912	36·0	63·096	53·0	446·68
3·0	1·412	20·0	10·000	37·0	70·795	54·0	501·19
4·0	1·585	21·0	11·220	38·0	79·433	55·0	562·34
5·0	1·778	22·0	12·590	39·0	89·125	56·0	630·96
6·0	1·995	23·0	14·125	40·0	100·000	57·0	707·95
7·0	2·239	24·0	15·849	41·0	112·20	58·0	794·33
8·0	2·512	25·0	17·783	42·0	125·89	59·0	891·25
9·0	2·818	26·0	19·953	43·0	141·25	60·0	1000·0
10·0	3·162	27·0	22·387	44·0	158·49	65·0	1778·3
11·0	3·548	28·0	25·119	45·0	177·83	70·0	3162·3
12·0	3·981	29·0	28·184	46·0	199·53	75·0	5623·4
13·0	4·467	30·0	31·623	47·0	223·87	80·0	10000·0
14·0	5·012	31·0	35·481	48·0	251·19	85·0	17783
15·0	5·623	32·0	39·811	49·0	281·84	90·0	31623
16·0	6·310	33·0	44·668	50·0	316·23	95·0	56234
17·0	7·079	34·0	50·119	51·0	354·81	100·0	10^5

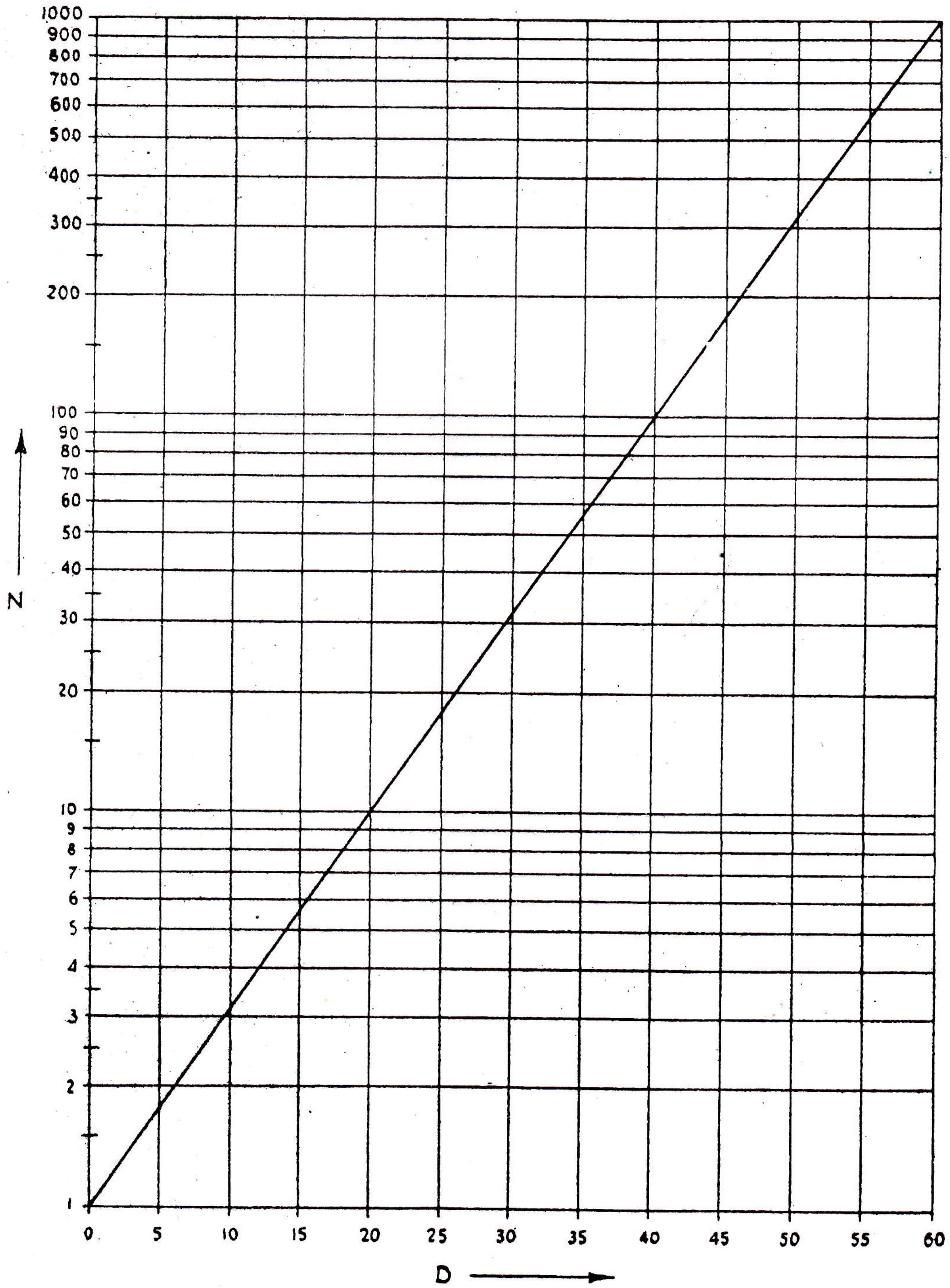


Fig. 622.—Graph for converting given decibel loss D to ratio N.

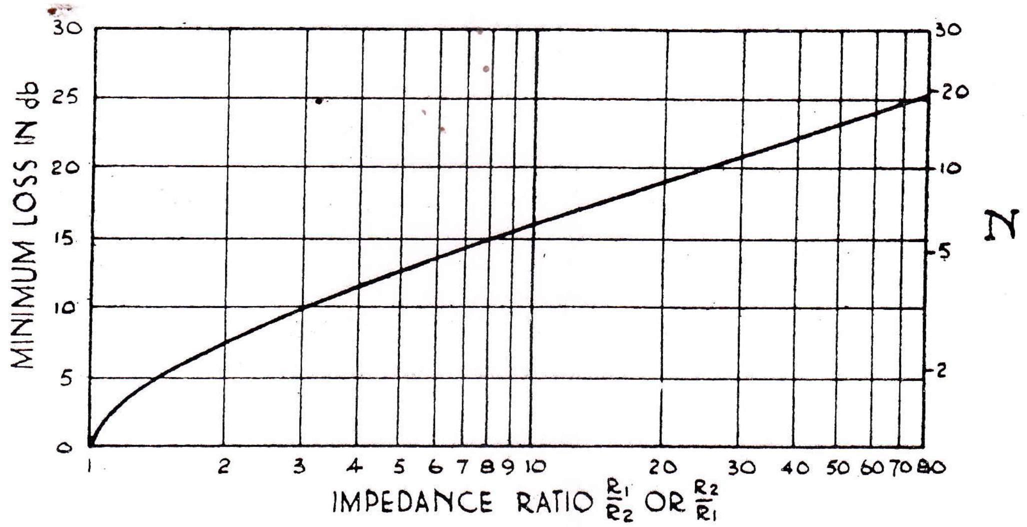


FIG. 623.—Minimum loss in T or π networks for given ratios of input and output impedances.

Example.—

Design an attenuating network to match between 400 and 800 ohms, and to give an attenuation of 15 db.

Following the steps indicated, the pad used would be either a T or π type. From Fig. 623, $\dfrac{R_2}{R_1} = 2$ allows a minimum of 7 db attenuation. From the table on p. 615, for 15 db attenuation $N = 5{\cdot}623$.

(*a*) T type (*see* page 609) :—

$$R_A = R_1\left(\frac{N^2+1}{N^2-1}\right) - 2\sqrt{R_1R_2}\left(\frac{N}{N^2-1}\right)$$

$= 218$ ohms. *Ans.*

$$R_B = 2\sqrt{R_1R_2}\left(\frac{N}{N^2-1}\right)$$

$= 208$ ohms. *Ans.*

$$R_C = R_2\left(\frac{N^2+1}{N^2-1}\right) - 2\sqrt{R_1R_2}\left(\frac{N}{N^2-1}\right)$$

$= 644$ ohms. *Ans.*

The complete T section is illustrated in Fig. 624*b*.

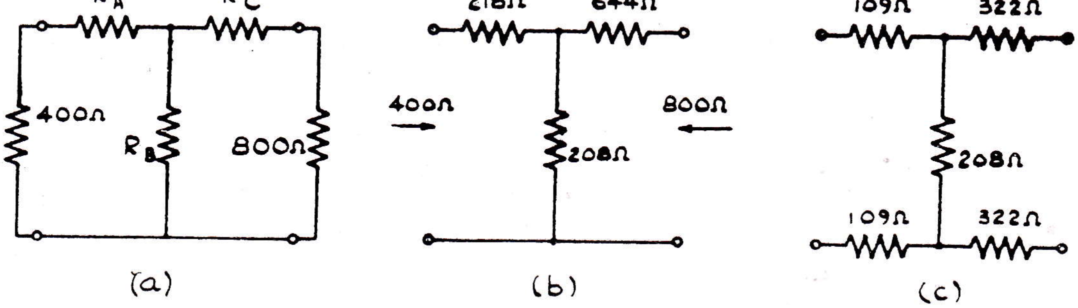


FIG. 624.—T network having input impedance of 400Ω and output impedance of 800Ω.

The values are quite suitable from a practical viewpoint, and the pad could now be constructed, dividing R_A and R_C between the two arms, as in Fig. 624*c*, if a balanced pad is required.

(*b*) π type (*see* page 611) :—

$$R_A = R_1\left(\frac{N^2 - 1}{N^2 - 2NS + 1}\right)$$
$$= 497 \text{ ohms.} \quad \textit{Ans.}$$

$$R_B = \frac{\sqrt{R_1R_2}}{2}\left(\frac{N^2 - 1}{N}\right)$$
$$= 1540 \text{ ohms.} \quad \textit{Ans.}$$

$$R_C = R_2\left(\frac{N^2 - 1}{N^2 - 2\frac{N}{S} + 1}\right)$$
$$= 1463 \text{ ohms.} \quad \textit{Ans.}$$

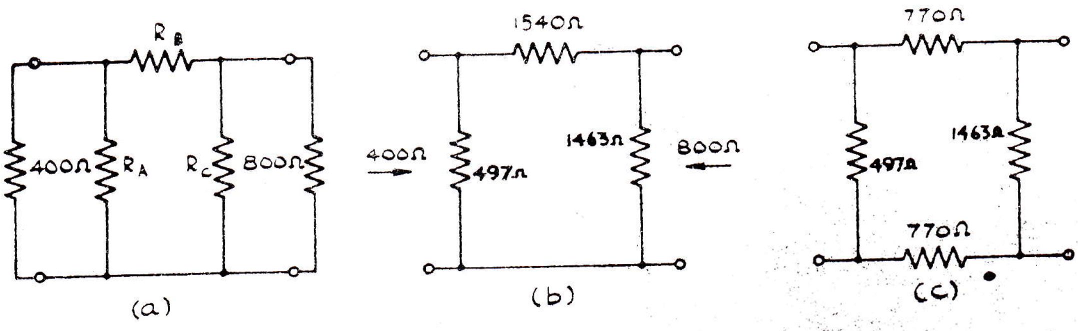


FIG. 625.—π network having input impedance of 400 Ω and output impedance of 800 Ω.

The complete π section is shown in Fig. 625*b*. Fig. 625*c* shows the corresponding balanced form. It will be noted that there is a considerable difference in the resistance values for the T and π types, and the more convenient may be used.

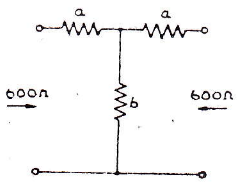
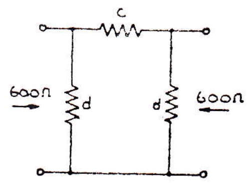
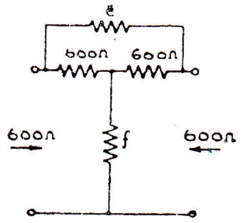
Components for attenuators in 600 ohm circuits

As 600 ohms is the characteristic impedance of most line communication circuits, the majority of attenuators and pads will come under this heading. Figs. 626 and 627 give graphs showing component values for T and π networks respectively for use in such circuits. Table XVIII gives the component values for T, π and bridged-T networks.

If the characteristic impedance is not 600 ohms, but R, the values for the components must all be multiplied by $\frac{R}{600}$.

TABLE XVIII

Pads designed for 600 ohms characteristic impedance.

Loss D in db	T pad		π pad		Bridged-T pad	
						
	a	*b*	*c*	*d*	*e*	*f*
1	34·50	5201	69·2	10,436	73·2	4918
2	68·79	2583	139·4	5233	155	2317
3	102·5	1703	211·1	3512	247	1456
4	135·8	1258	286·2	2651	351	1025
5	168·0	987·1	365·0	2142	467	771
6	199·4	803·4	448·1	1806	597	603
7	229·4	670·0	537·3	1569	743	485
8	258·3	567·5	634·1	1394	907	397
9	285·7	487·1	738·9	1260	1091	330
10	311·7	421·9	853·1	1155	1297	278
11	336·3	367·2	980·3	1071	1530	235
12	359·1	321·7	1119	1003	1789	201
13	380·5	282·7	1273	946·1	2080	173
14	400·4	249·3	1444	899·1	2407	149
15	418·8	220·1	1633	859·5	2773	130
20	490·9	121·2	2970	733·3	5400	66·7
25	536·1	67·61	5324	671·4	10,070	35·8
30	563·2	37·99	9486	639·0	18,370	19·6
35	579·0	21·35	16,864	621·6	33,140	10·9
40	589·1	12·00	30,000	612·1	59,400	6·06
45	593·3	6·748	53,350	606·8	106,100	3·40
50	596·2	3·795	94,860	603·8	189,100	1·90

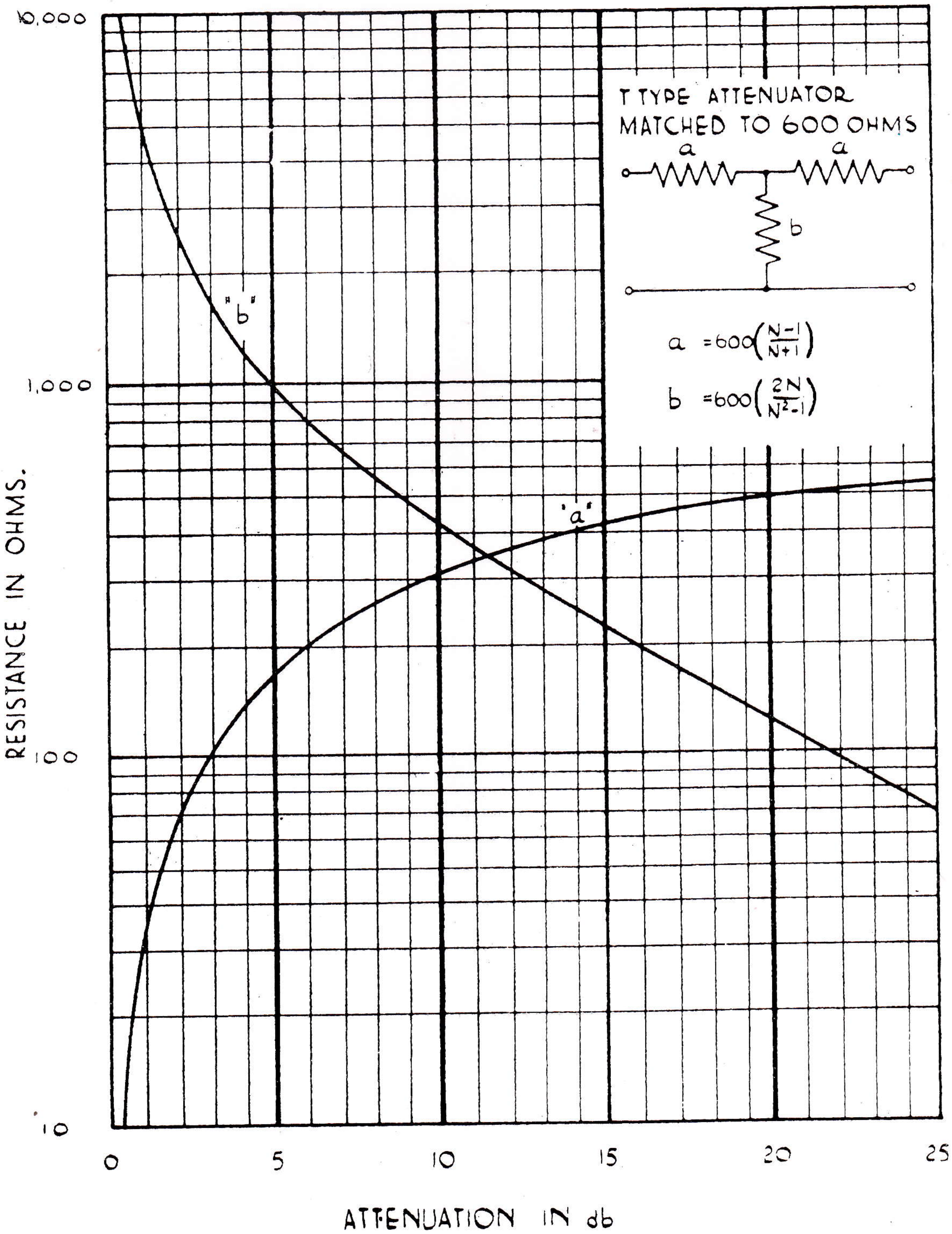


FIG. 626.—Graph giving component values for T network (characteristic impedance 600 Ω).

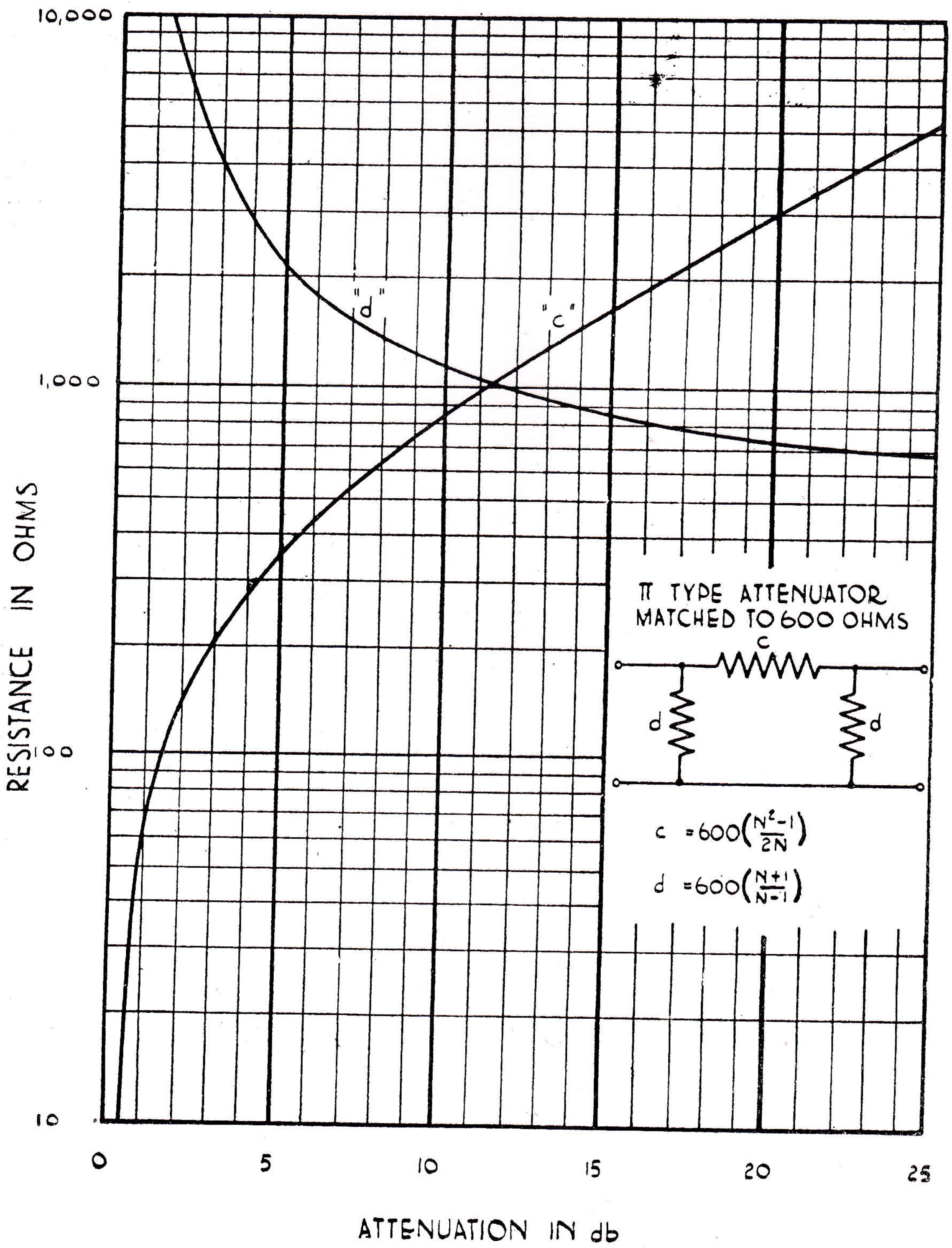


FIG. 627.—Graph giving component values for π network (characteristic impedance 600 Ω).

VARIABLE ATTENUATORS

Variable attenuators are so designed as to have a constant input and output impedance, but a variable attenuation. They may be divided into several classes depending on the method of achieving the result.

The elementary type has the simple construction of a T or π section and the resistors are variable. All are ganged together so that at different positions the pad impedance is unaltered although the attenuation is varied (*see* Fig. 628*a* and *b*).

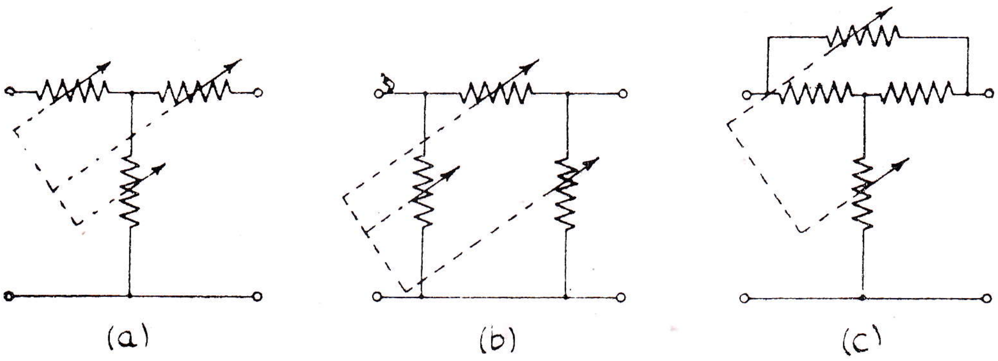


FIG. 628.—Variable attenuator.

The bridged-T type (*see* Fig. 628*c*) has already been discussed. This has the advantage compared with those mentioned above that only two resistors have to be varied, as compared with three when T or π sections are used.

A further type, simple in construction and design, consists of a number of pads, of equal impedance but different attenuation, connected in series. Each pad may be switched in or out as required.

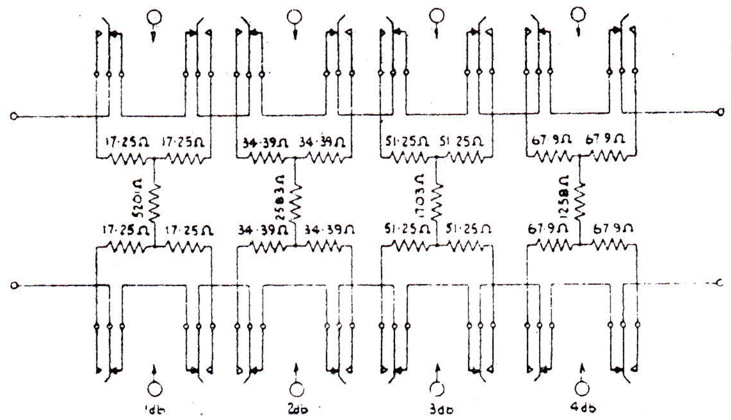


FIG. 629.—Non-reactive adjustable attenuator (characteristic impedance $600\,\Omega$).

Fig. 629 shows such an attenuator.

Each pad is of the balanced T type and may be brought into circuit by operation of the appropriate key. The resistances are shown in ohms, and the characteristic impedance of each pad is 600 ohms.

Fig. 630 shows how the principle may be extended so that, with appropriate switching, only three pads, namely 5, 10 and 20 db, need be employed to form a variable attenuator covering the range from 0 to 30 db in 5 db steps.

*S*1 Posn.	Att. db.
1	0
2	5
3	10
4	15
5	20
6	25
7	30

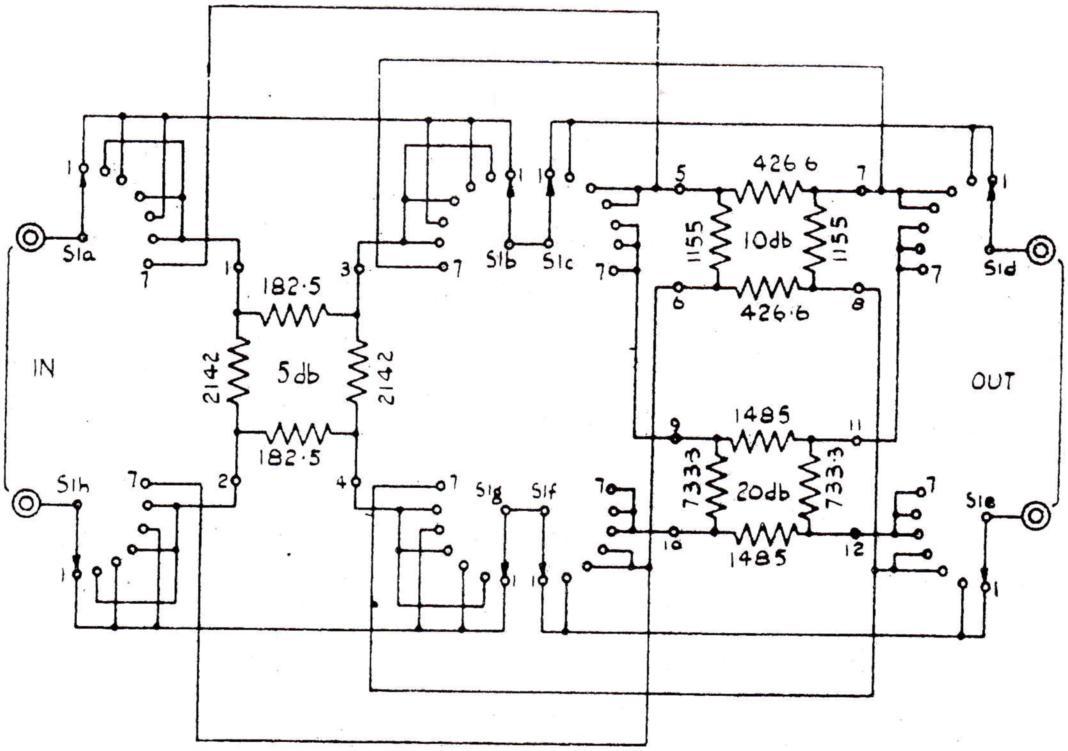


FIG. 630.—Non-reactive adjustable attenuator : Range 0–30 db.

TRANSMISSION MEASUREMENTS

It has been seen (Chap. 5) that the power level in a circuit may be expressed using the decibel notation provided that a reference power is stated; in line communication this reference power is taken as 1mW. A decibel-meter, which is the name given to an instrument measuring power levels when the decibel notation is used, should be a specially calibrated wattmeter. However, it is found impossible in practice to design a wattmeter that is sufficiently accurate at all frequencies over the required range. On the other hand if all measurements are taken at points having the same standard impedance, the voltage will give a direct