Digital Image Processing - CS 663
Group - 23
Assignment 2 - Question 2

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1 Overview

In this question, we performed edge-preserving smoothing using Bilateral Filtering. The basic algorithm involves adjusting pixel value for a particular pixel by weighing in all the surrounding pixels based on the distance from this particular pixel in space as well as in terms of intensity. A gaussian kernel is used to define the weights, hence we have 2 tunable parameters, one for distance and one for intensity.
To speed up the algorithm, data collection for a value of standard deviation was performed only upto a distance of 1 standard deviation from the original pixel. Hence we have a variable window size which depends on the standard deviation of distance. The values of the two standard deviations were tuned on the basis of Root Mean Square Distance(RMSD), defined as:

$$RMSD(A,B) = \sqrt{\frac{\Sigma_p(A(p) - B(p))}{N}}$$

where A and B are the Original image and the bilateral filter output image, N is the number of pixels and A(p) refers to a pixel in A.
The parameters were tuned manually using binary search, by first indexing over all the values in [1, 2, 4, 8, 16, 32, 64, 128, 256] for intensity and similar for distance. Then we zoomed onto the area having the minimum RMSD and increased the resolution, until we had a resolution of 1 decimal place. After this, the RMSD for various standard deviations(0.9 and 1.1 times the original) were computed as a proof to show that we had a minima.(NOTE: We haven't scaled the image to [0,1] range here in this question)

2 Bilateral Filter Operation on images

2.1 Grass.png

2.1.1 Tuning the standard deviations

Let σ_d be the standard deviation for density and σ_i be the standard deviation for intensity.First a binary search was performed as explained above We found a minima at $\sigma_d = 1$ and $\sigma_i = 32$

Standard deviation of distance varies along x axis------>								
	1	2	4	8	16	32	64	128
1	11.75812	11.75402	11.74611	11.74436	11.74694	11.7497	11.7521	11.7532
2	11.73729	11.70971	11.68303	11.68186	11.69944	11.71561	11.72451	11.72897
4	11.59213	11.45599	11.39479	11.43204	11.52094	11.58722	11.6211	11.63798
8	10.85805	10.44335	10.41536	10.6582	10.99665	11.21393	11.32185	11.37156
16	8.92793	8.463166	8.904575	9.747583	10.6294	11.14131	11.36273	11.46703
32	7.366782	8.159093	10.06756	12.54542	14.92071	16.09447	16.60316	16.85542
64	7.663639	10.50085	14.80123	19.92637	23.78696	25.64589	26.58269	27.13159
128	8.322653	12.50576	18.49145	25.06327	29.19002	31.24393	32.43877	33.21219

Figure 1: Initial Binary Search Run

Then we performed a linear search for integer σ_i in [16, 64] and σ_d in [0.9,1,1.1]. The cropped image is shown here(First column is σ_i)

30	7.488911	7.427829	7.515579
31	7.448399	7.394347	7.515384
32	7.413828	7.366782	7.521344
33	7.384664	7.344573	7.532791
34	7.360412	7.327202	7.549111
35	7.340614	7.314194	7.569743
36	7.324848	7.305109	7.594171
37	7.312724	7.299546	7.621925
38	7.303884	7.297136	7.652575
39	7.297999	7.297542	7.685732
40	7.294765	7.300456	7.721041
41	7.293908	7.305596	7.758182
42	7.295172	7.312708	7.796862
43	7.298329	7.321558	7.83682

(a) Initial linear search

37.7	7.306212	7.297548	7.6431
37.8	7.305406	7.297382	7.646232
37.9	7.30463	7.297245	7.649391
38	7.303884	7.297136	7.652575
38.1	7.303168	7.297055	7.655784
38.2	7.302481	7.297002	7.659018
38.3	7.301823	7.296976	7.662276
38.4	7.301193	7.296978	7.665558
38.5	7.300592	7.297007	7.668863
38.6	7.300018	7.297062	7.672192
38.7	7.299473	7.297143	7.675544
38.8	7.298954	7.29725	7.678918
38.9	7.298463	7.297383	7.682314
39	7.297999	7.297542	7.685732

(b) Final linear search

We then found the minima at $\sigma_d = 1$ and $\sigma_i = 38$. We then checked for 21 values of σ_i in [37,39] and σ_d in [0.9, 1 1.1]. The final minima was found for $\sigma_d = 1$ and $\sigma_i = 38.3$. The optimum RMSD value is 7.296976

2.1.2 Output Image

Figure 3: Original Image

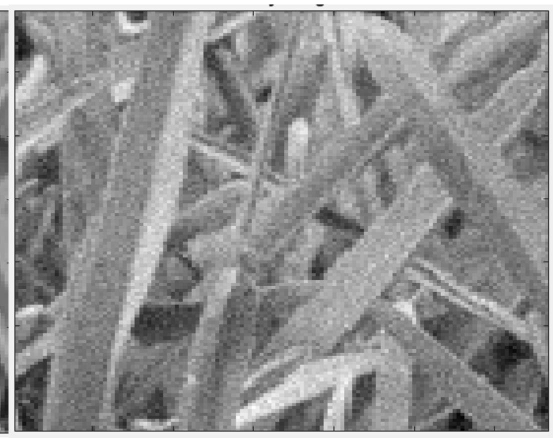

Figure 4: Noisy Image

Figure 5: Bilateral Filter Output

2.1.3 RMSD Values

σ_d	σ_i	RMSD
1	38.3	7.296976
0.9	38.3	7.301823
1.1	38.3	7.662276
1	34.47	7.320573
1	42.13	7.313765

Table 1: RMSD Values

2.1.4 Gaussian Mask

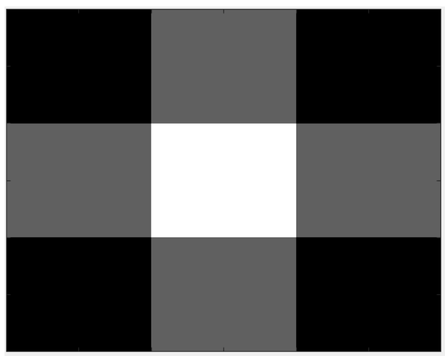

Figure 6: Gaussian Mask

2.2 HoneyCombReal.png

2.2.1 Tuning the standard deviations

First a binary search was performed as was explained above. We found a minima at $\sigma_d = 1$ and $\sigma_i = 32$

	1	2	4	8	16	32
1	12.64983247	12.64440093	12.63704934	12.63552643	12.63613388	12.63728543
2	12.62608378	12.59247268	12.56501841	12.56913898	12.58003913	12.58789384
4	12.47241972	12.3147937	12.24691273	12.30096943	12.36369803	12.40044457
8	11.69446005	11.19678525	11.14515621	11.43750708	11.69900238	11.82276883
16	9.519132994	8.802573693	9.130175793	10.10461461	10.88937004	11.04768434
32	7.479684096	7.605335399	9.215241123	11.94199137	13.87616049	13.65527889
64	7.556841088	9.696274134	13.91604611	19.51884168	22.58815855	22.50232481
128	8.630226414	12.47795159	18.38366579	25.38132443	28.88945883	29.48823451
256	9.196369279	13.69560095	20.10337102	27.43379811	31.06048794	31.97987266

Figure 7: Initial Binary Search Run

Then we performed a linear search for integer σ_i in [16, 64] and σ_d in [0.8,0.9,1,1.1,1.2].

	0.8	0.9	1	1.1	1.2
16	9.902701775	9.675355028	9.519132994	9.164189731	9.060710072
17	9.680959047	9.446888414	9.287727201	8.930907317	8.82986655
18	9.471585801	9.232817263	9.072181772	8.716638014	8.619131561
19	9.27516609	9.033553592	8.872755326	8.52131224	8.428285124
20	9.091920035	8.849131898	8.68932454	8.344498851	8.256757401
21	8.921780612	8.679295803	8.52147775	8.185506301	8.103732169
22	8.764461894	8.523573874	8.368595905	8.043466862	7.968231985
23	8.619517865	8.381343894	8.229920273	7.917404616	7.849186061
24	8.48639169	8.25188569	8.104607236	7.80628854	7.745482437
25	8.364455872	8.134423308	7.991771269	7.709072531	7.656006571
26	8.253044159	8.028157741	7.890517623	7.624724462	7.579668703
27	8.15147624	7.932291573	7.799966345	7.552246365	7.515422314
28	8.05907632	7.846046915	7.719269226	7.490687639	7.462275696
29	7.975186604	7.76867786	7.647621069	7.439152934	7.419298398
30	7.899176643	7.69947858	7.584266524	7.396806054	7.385623921
31	7.830449371	7.637788009	7.528503508	7.362871001	7.360449776
32	7.768444552	7.58299193	7.479684096	7.336631	7.343035751
33	7.712640276	7.534523144	7.437213578	7.317426189	7.332701022
34	7.662552984	7.491860271	7.400548264	7.304650471	7.328820568
35	7.617736472	7.454525625	7.369192485	7.297747884	7.330821234
36	7.577780194	7.422082515	7.342695134	7.296208783	7.338177674
37	7.542307123	7.394132237	7.320646003	7.29956599	7.350408331
38	7.510971381	7.370310933	7.302672099	7.307391057	7.367071573
39	7.483455775	7.350286486	7.288434073	7.319290721	7.387762045
40	7.459469337	7.333755514	7.277622839	7.334903598	7.412107289
41	7.438744968	7.320440549	7.269956438	7.353897143	7.439764641
42	7.421037193	7.31008742	7.26517717	7.375964898	7.470418416
43	7.406120093	7.302462867	7.263049004	7.400824001	7.503777378
44	7.3937854	7.29735239	7.263355266	7.42821297	7.539572478
45	7.383840779	7.294558319	7.26589658	7.457889729	7.577554839
46	7.376108274	7.293898104	7.270489061	7.489629862	7.617493972
47	7.370422932	7.295202806	7.276962729	7.523225077	7.659176207
48	7.36663157	7.298315764	7.285160119	7.558481848	7.702403294
49	7.364591695	7.303091439	7.294935082	7.595220226	7.74699118
50	7.364170545	7.309394389	7.30615173	7.633272781	7.792768921
51	7.365244249	7.31709839	7.318683529	7.672483681	7.839577716
52	7.367697086	7.326085655	7.332412509	7.712707861	7.887270055
53	7.371420834	7.33624616	7.347228573	7.753810295	7.935708944
54	7.376314194	7.347477046	7.363028892	7.795665336	7.984767221
55	7.382282287	7.359682101	7.379717382	7.838156124	8.034326926
56	7.389236201	7.372771293	7.397204236	7.881174045	8.084278732
57	7.397092589	7.386660363	7.415405505	7.924618243	8.134521424
58	7.405773311	7.401270455	7.434242733	7.96839516	8.184961411
59	7.415205113	7.416527783	7.453642622	8.012418123	8.235512279
60	7.425319326	7.432363334	7.473536722	8.056606941	8.286094375
61	7.436051605	7.448712587	7.493861161	8.100887546	8.336634403
62	7.447341678	7.465515262	7.51455638	8.14519164	8.387065061
63	7.459133118	7.482715086	7.535566899	8.189456363	8.437324683
64	7.471373133	7.500259567	7.556841088	8.233623988	8.48735691

Figure 8: Linear Search Run

We then found the minima at $\sigma_d = 1$ and $\sigma_i = 43$. We then checked for 21 values of σ_i in [42,44] and σ_d in [0.9, 1 1.1].

	0.9	1	1.1
42	7.31008742	7.26517717	7.375964898
42.1	7.309205858	7.264848773	7.378329673
42.2	7.308351344	7.264546647	7.380722073
42.3	7.307523658	7.264270568	7.38314183
42.4	7.30672258	7.264020315	7.385588677
42.5	7.305947894	7.26379567	7.388062351
42.6	7.305199384	7.263596414	7.390562588
42.7	7.304476837	7.263422332	7.393089129
42.8	7.303780041	7.263273211	7.395641715
42.9	7.303108788	7.263148839	7.398220091
43	7.302462867	7.263049004	7.400824001
43.1	7.301842073	7.2629735	7.403453195
43.2	7.301246202	7.26292212	7.40610742
43.3	7.300675049	7.262894657	7.40878643
43.4	7.300128414	7.26289091	7.411489976
43.5	7.299606097	7.262910676	7.414217816
43.6	7.299107899	7.262953756	7.416969705
43.7	7.298633623	7.26301995	7.419745404
43.8	7.298183075	7.263109063	7.422544672
43.9	7.297756061	7.2632209	7.425367273
44	7.29735239	7.263355266	7.42821297

Figure 9: Linear Search Run

The final minima was found for $\sigma_d = 1$ and $\sigma_i = 43.4$. The optimum RMSD value is 7.26289091.

2.2.2 Gaussian Mask

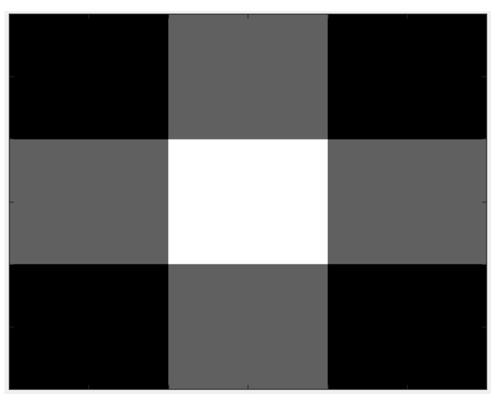

Figure 10: Gaussian Mask

2.2.3 Output Images

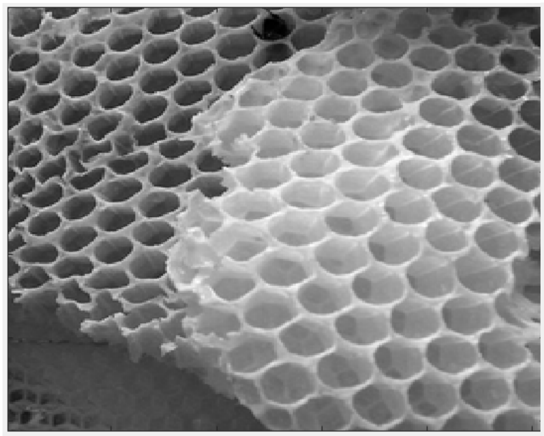

Figure 11: Original Image

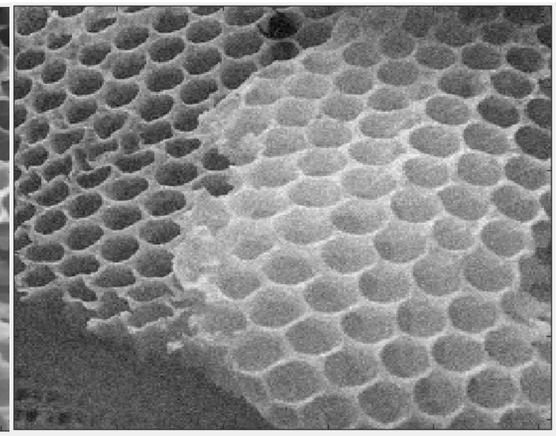

Figure 12: Noisy Image

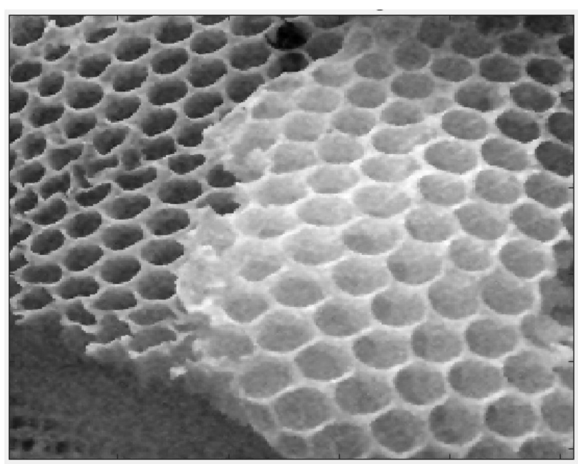

Figure 13: Bilateral Filter Output

2.2.4 RMSD Values

σ_d	σ_i	RMSD
1	43.4	7.26289091
0.9	43.4	7.300128414
1.1	43.4	7.411489976
1	39.06	7.287691698
1	47.74	7.282871318

Table 2: RMSD Values

2.3 Barbara.mat

2.3.1 Tuning the standard deviation

As is done in all the images above, first a binary search was performed, in which the least RMDS was observed for $\sigma_d = 2$ and $_i = 8$. After this, a linear search was performed for σ_i in [4,12] and $_i$ in [1,4]. The optimum value now was observed at $\sigma_d = 2$ and $\sigma_i = 10$. After some more linear searches, we found the optimum value of $\sigma_i = 10.4$ and $\sigma_d = 1.3$. The optimum RMSD value is 3.294366.

	1	2	4	8	16	space
1	4.95635	4.932984	4.910325	4.903449	4.91064	
2	4.856115	4.755639	4.691205	4.690539	4.730724	
4	4.42409	4.156881	4.060977	4.121469	4.26339	
8	3.60275	3.363793	3.379089	3.580238	3.876748	
16	3.426746	3.858029	4.102984	4.539165	5.112402	
32	4.334618	5.605029	6.079255	7.029652	8.450298	
64	5.05561	6.642645	7.291181	8.69978	10.89048	
128	5.300323	6.964592	7.681595	9.263764	11.735	
intensity						

Figure 14: Initial Binary Search Run

	1	2	3	4
4	4.42409	4.156881	4.079555	4.060977
5	4.172796	3.866633	3.794389	3.785868
6	3.944005	3.633825	3.576239	3.580045
7	3.752767	3.467228	3.43008	3.446723
8	3.60275	3.363793	3.349874	3.379089
9	3.49168	3.315669	3.325694	3.366909
10	3.41499	3.31387	3.347193	3.399718
11	3.367712	3.34981	3.404948	3.46809
12	3.3452	3.415776	3.490809	3.563962

Figure 15: Initial Linear Search Run

	1.9	2	2.1
9	3.314757	3.315669	3.303044
9.1	3.31238	3.313552	3.301964
9.2	3.310456	3.311888	3.301338
9.3	3.308977	3.31067	3.301154
9.4	3.307933	3.309888	3.301405
9.5	3.307316	3.309533	3.30208
9.6	3.307117	3.309597	3.30317
9.7	3.307328	3.310071	3.304667
9.8	3.30794	3.310947	3.306561
9.9	3.308946	3.312216	3.308844
10	3.310337	3.31387	3.311506
10.1	3.312104	3.3159	3.314539
10.2	3.31424	3.318299	3.317935
10.3	3.316736	3.321059	3.321685
10.4	3.319585	3.324171	3.325781
10.5	3.322779	3.327627	3.330214
10.6	3.326311	3.33142	3.334976
10.7	3.330172	3.335543	3.340061
10.8	3.334355	3.339987	3.345458
10.9	3.338853	3.344745	3.351162
11	3.343659	3.34981	3.357164

	1.5	1.6	1.7	1.8	1.9
9.4	3.304474	3.304017	3.304743	3.306153	3.307933
9.5	3.302256	3.302306	3.303459	3.305229	3.307316
9.6	3.30045	3.30101	3.302591	3.304723	3.307117
9.7	3.299048	3.300121	3.302132	3.304626	3.307328
9.8	3.298043	3.299631	3.302073	3.30493	3.30794

	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
9.5	3.449351	3.337597	3.319166	3.309056	3.304088	3.302256	3.302306	3.303459	3.305229	3.307316	3.309533
9.6	3.441861	3.332554	3.315149	3.30591	3.301672	3.30045	3.30101	3.302591	3.304723	3.307117	3.309597
9.7	3.434686	3.327886	3.31152	3.303159	3.299657	3.299048	3.300121	3.302132	3.304626	3.307328	3.310071
9.8	3.427818	3.323585	3.30827	3.300796	3.298035	3.298043	3.299631	3.302073	3.30493	3.30794	3.310947
9.9	3.421255	3.319646	3.305393	3.298814	3.296798	3.297427	3.299531	3.302406	3.305627	3.308946	3.312216
10	3.41499	3.316062	3.302882	3.297205	3.29594	3.297192	3.299815	3.303123	3.306709	3.310337	3.31387
10.1	3.409019	3.312828	3.300731	3.295963	3.295453	3.29733	3.300473	3.304216	3.308167	3.312104	3.3159
10.2	3.403337	3.309937	3.298933	3.295081	3.295329	3.297835	3.301499	3.305677	3.309994	3.31424	3.318299

	1.1	1.2	1.3	1.4
10	3.316062	3.302882	3.297205	3.29594
10.1	3.312828	3.300731	3.295963	3.295453
10.2	3.309937	3.298933	3.295081	3.295329
10.3	3.307383	3.297482	3.29455	3.295562
10.4	3.305159	3.296371	3.294366	3.296144
10.5	3.303261	3.295593	3.294521	3.297068
10.6	3.301682	3.295143	3.295008	3.298327

Figure 16: Linear Search Runs

2.3.2 Gaussian Mask

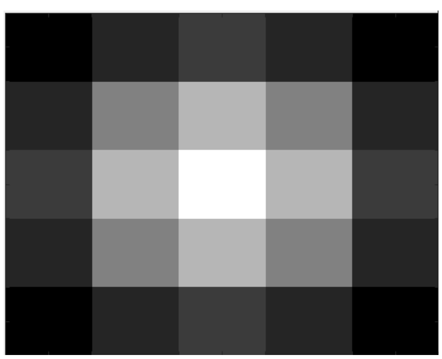

Figure 17: Initial Binary Search Run

2.3.3 Output Images

Figure 18: Original Image

Figure 19: Noisy Image

Figure 20: Bilateral Filter Output

2.3.4 RMSD Values

σ_d	σ_i	RMSD
1.3	10.4	3.294366147
1.17	10.4	3.298116177
1.43	10.4	3.297123959
1.3	9.36	3.314137427
1.3	11.44	3.31131997

Table 3: RMSD Values

3 Results

- Barbara.mat: $\sigma_{distance} = 1.3$, $\sigma_{intensity} = 10.4$, RMSD = 3.294
- Grass.png: $\sigma_{distance} = 1$, $\sigma_{intensity} = 38.3$, RMSD = 7.296
- HoneyCombReal.png: $\sigma_{distance} = 1$, $\sigma_{intensity} = 43.4$, RMSD = 7.263