The XVA of Financial Derivatives: CVA, DVA and FVA Explained

Dongsheng Lu

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Introduction

The 2008 financial crisis, triggered by the Lehman Brothers default, has led to profound changes in derivatives trading and valuations. These changes are partly driven by the banks in realizing the need to treat credit and funding properly; and further by the widespread enforcement of rules and regulations from Basel III, Central Banks to regulatory authorities. On one side, collateralized derivatives and central clearing have become the trend in promoting derivatives trading transparency; and on the other side, bilateral uncollateralized derivatives coexist with little liquidity in market trading. To integrate all these together, we have to dive deep into the discussion of XVA.

An acronym for CVA, DVA, FVA, LVA, RVA, KVA, etc., XVA represents the potential valuation adjustments that one may need to incorporate into the derivative fair valuations and/or economic measurements. The XVA problem touches not only the credit default of the counterparty, but also the practical aspects of funding in derivative trading, as well as banks' balance sheets and capital management. While there are substantial developments among the market participants in agreeing on some of the valuation adjustments, such as collateral values and CVA, there are still considerable controversies over a variety of topics among practitioners, academia, accountants, regulators and financial engineers. These topics range from the sheer existence of FVA, the DVA dilemma, the accounting and relationships of XVAs, and the construction of funding curves, to the risk management of XVAs and optimizing capitals.

In the present book, we try to elaborate the author's view on the above XVA problems in 360 degrees, from the basics of derivative trading, the outset of the XVA problem setup, to the practical aspects of OTC derivative market and the replication of bi-lateral derivative values. We attempt to present a full picture around the credit and funding value adjustments, and many facets of the derivative value replications as well as practical risk management.

In Chapter 1, we introduce the basics of derivative trading, the relevant trading parties, and relationships among them. Customer market derivative trading, through a typical auction process, is examined. Dealers' business models, the broker market, pre-trading and post-trading are also touched briefly. Rules and regulations, as well as their effects on the derivative trading are discussed.

Chapter 2 covers the legal and operational aspects of derivatives trading, which are important to the understanding of valuation adjustments and replication of derivative values are presented. Derivatives accounting and transfer pricing are also touched briefly.

Chapter 3 is focused on credit value adjustments and related concepts. Brief discussion on the DVA dilemma is also covered.

In Chapter 4, we attempt to present a full picture around the funding value adjustment, and many facets of funding value replications. As this is the most controversial topic, we have endeavoured to cover many different topics. We start from collateral value and collateralized transactions, which brings the topic of differential discounting and collateral choice options. This is followed by the replication of derivative values with funding, elaborations on the valuation principles and derivative market liquidity discussions. This is followed by a presentation of the different FVA methodologies and their impacts. To clarify the FVA issues further, we present at the end of the chapter questions and answers for topics that can cause some confusion.

Chapter 5 touches briefly liquidity value adjustment, replacement value adjustment and capital value adjustment. KVA concepts, calculations and capital management issues are discussed. At the end of the chapter, we present a summary of all valuation adjustments, the issues with valuation adjustments and some discussion on this evolving story. Prudent valuation AVAs, capital and their integration with XVAs, as well as the impacts on the derivative users are also discussed.

In Chapter 6, we present some detailed discussions on CVA and FVA modeling, in market factors and credit factors. A dynamic ratings simulation model and an efficient CVA and FVA simulation scheme are presented, with some detailed implementation discussions. A few interesting topics, including CVA/FVA cross, netting set vs. funding set in FVA, and mutual put breaks are discussed at the end of the chapter.

Chapter 7 covers the hedging and risk management of XVAs, including general derivative risk management concepts, risk managing CVA in market and credit risks, as well as the conflict between capital optimization and CVA hedging. We conclude the book with discussions of XVA desk setup and operations.

While XVA is still an evolving story, the author has been actively applying much of the present book in practical trading and risk management. The experience has been mostly successful as well as rewarding, especially the ample direct involvement in designing and executing large portfolio unwinds, derivative intermediations, derivative novations, CSA negotiations, risk management and trading activities.

I INTRODUCTION TO DERIVATIVES TRADING

1 Overview of Derivatives Trading

Financial derivatives are widely used in economic activities by a variety of market participants for the purposes of hedging, investment and speculations, among other things. A derivative is a legal contract agreed between two or more parties, which defines the contingent claims or cash flows "derived" from the underlying price to be paid by the parties in the future. Derivatives can be traded on the exchanges or over the counter (OTC). OTC derivatives and their valuations are the focus of this book.

1.1 The participants in the derivatives market and their interactions

A simple schematic view of the derivatives market participants is shown in Figure 1.1. In the left box is a dealer with different functional departments; the right box shows a number of dealers trading with each other in the broker market (in light gray), and competing in the customer market (in dark gray). At the bottom of the chart, the regulators interact with all market participants, create and enforce rules and regulations, so that the market operates smoothly.

In this chapter, we will look at the market participants, their functions in the market, the market mechanisms, and the various aspects involved in the derivatives trading and market dynamics.

1.1.1 Derivative providers vs. derivative users

Within the derivatives market, there are participants who are "experts" and "providers" of the derivatives products and liquidity. These are what are loosely defined as "dealers". These dealers are expected to be the derivative experts, with expertise in the pricing, trading and risk management of derivative products. They determine the derivatives prices traded in the market through market trading mechanisms. They provide liquidity and two-way prices for products through market making activities and customer trading mechanisms. While there is no general definition of "dealer", within Dodd–Frank rules, "swap dealer" is defined as a swap market maker, with swap dealing activities exceeding the de minimis notional threshold of $8 billion over a 12-month period. This definition is, however, for regulatory purposes.

Market participants taking the other side of the derivative contracts are derivatives users. They could be a variety of business entities, such as insurance companies, regional banks, corporations, hedge funds, municipalities, high net worth individuals, university endowments, debt issuers, and so on. These customers trade derivatives for such purposes as hedging, speculation, or yield enhancements.

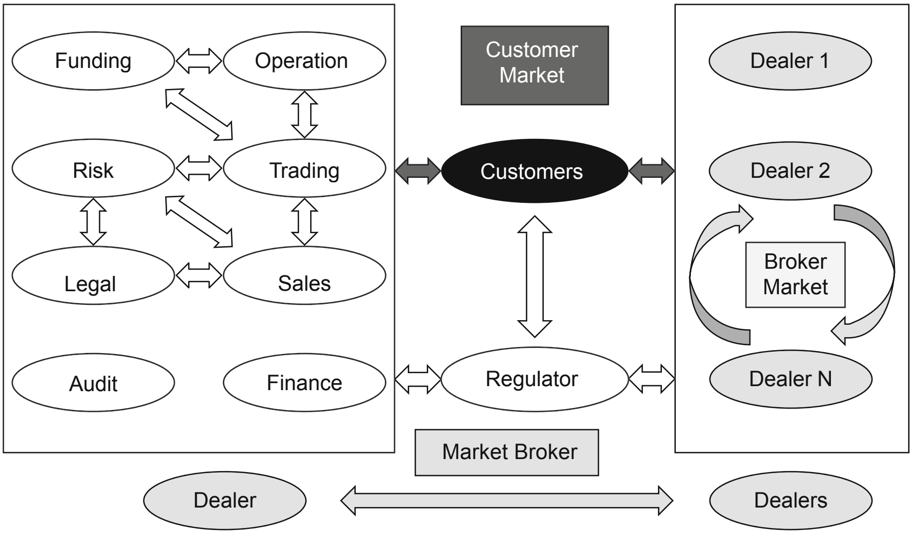

Figure 1.1 The derivative market

For instance, corporations like PepsiCo, having a global business operation, would be looking to hedge their business activities using foreign exchange (FX) spot, FX forward or FX options as well as interest rate derivatives, such as swaps and cross currency swaps. Airline companies would use derivative contracts in oil, such as zero cost collars, to hedge and manage their fuel costs for potential oil price increases;[1] on the opposite side, the shale oil producers using oil derivative contracts to hedge oil price downside exposures: by entering combinations of derivatives at different strikes, some shale oil producers' breakeven prices are effectively increased by $15 per barrel. This creates a significant buffer for these companies from being eliminated by the fast dipping oil price.[2] As derivatives with optionality can provide highly leveraged payouts, hedge funds and professional traders may employ leveraged derivatives to speculate on the market move and profit from it. For investors and asset managers, their targets would be enhancing the returns on their money. Therefore they could be looking for complex structured notes with embedded derivatives so that they may enjoy extra returns and be reasonably protected at the same time.

OTC derivatives are generally traded

- Among dealers, creating a "Broker Market"
- Between dealers and customers, within a "Customer Market".

In the following, we discuss in detail the customer market and the customer market trading mechanisms. We will then explore broker market trading and the role of broker market in OTC derivatives trading.

1.1.2 The dealer-to-customer market and competition

The customer market between dealers and customers may take place through different mechanisms. One common practice is through a market auction mechanism. In this

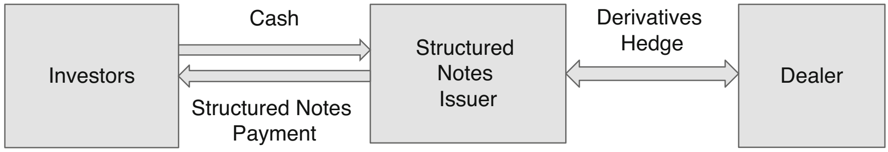

Figure 1.2 Illustration of a structured notes issuance and hedging

process, the customer can ask a few dealers to submit their prices for a derivative trade, where the highest bidder will be awarded the contract.

For example, an issuer of yield enhancing notes with embedded option payoffs would go to dealers to lay off the derivative risks. In Figure 1.2, the structured notes issuer brings in cash by selling the structured notes to the investor and at the same time he promises structured note payments to them. To hedge the derivative risk embedded in the structure notes, the issuer turns to the dealer and enters a derivative transaction, which effectively passes the market risk from the structured notes obligation to the dealer. As a result, the issuer raises cash from the investor and is free of market risk. In providing the derivative pricing to the customers, dealers will price the derivative transactions with credit, funding, capital and various other considerations, which is the focus of this book.

When a customer wants to trade a derivative transaction, it would be advantageous for him/her to approach multiple dealers in a competitive bidding process, as shown in Figure 1.3. Through this process, the customer is able to grab the best price from among the dealers; sometimes, a few top bidders may share a deal when it is too large.

By the nature of the derivatives auction mechanism, dealers compete for the same derivatives contract and try to win the contract with the expectation of making money from the contract. This exposes the dealers to a well-known problem known as "winner's curse": the winner of an auction process with incomplete information tends to overpay. Another way of saying the same thing is that the winner is subject to "adverse selection" from the market. Winners of derivatives auctions can hurt themselves in different ways. For example, they tend to win the trades that they misprice or may not understand completely. If one misprices a certain type of trade

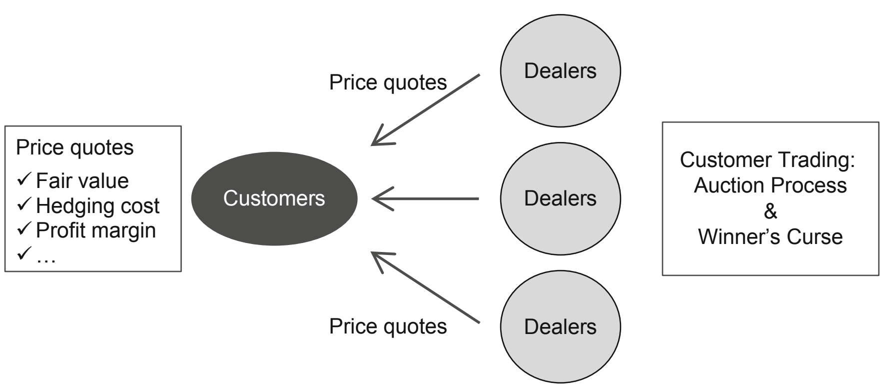

Figure 1.3 Illustration of an auction process in customer derivative trading

for reasons such as pricing model bias, he may win the same type of trade consistently. Similarly, if one misunderstands certain corners of the market in prices or liquidities, such as out-of-the-money skews and smiles, he may be more aggressive than others in quoting the customers. The results of such situations could be potential accumulation of large, less profitable, or even money losing positions, which could show up over time.

"Winner's curse" may show consistency, from pricing model bias, or be transient, due to temporary mispricing in a certain corner of market. It is particularly harmful when consistent pricing bias occurs and at the same time traders become blindly over-confident. That would be a recipe for disaster: they may accumulate into a very large losing position. AIG Financial Product Division's CDS trading position before the 2008 credit crisis is just one of the typical examples.

There are certainly ways to mitigate the "winner's curse" problem among the dealers. One way is to be more careful about the winning percentage of certain types of trade. When a trader is consistently winning one type of trade, it could really become a red flag. Normally dealers may try to back off once a trade is won, as we explain below.

Dealers always strive to understand the auction situation, much like playing a game. To understand how the game works, we can use the simple example shown in Table 1.1. Assume that we have only two auction players A and B, and they can bid with three discrete prices only, indicated below as '+', '0', and '-'. Here '0' means the normal price that one may bid and make money from the winning trade, '+' means a higher price and actually may realize –\$1 eventually, and '-' means a lower bidding price and eventually winning \$2 from the trade.

Table 1.2 shows all the possible situations: if A submits roughly the same price as B, then on average they will be winning the same number of trades. If A submits prices slightly better than B on average by one notch, he will be winning 85% trades on average and B will be winning 15%; while an increase of two notches will lead to a 100% winning percentage for either party. The winning probability would lead to the following profit and loss (P&L) result.

Table 1.1 *Example of discrete pricing*

Winning Price	+	0	−
Real Value	−1	1	2

Table 1.2 *Example of winning probabilities for A and B*

		B		
		+	**0**	−
	+	(50%, 50%)	(85%, 15%)	(100%, 0%)
A	**0**	(15%, 85%)	(50%, 50%)	(85%, 15%)
	−	(0%, 100%)	(15%, 85%)	(50%, 50%)

Table 1.3 *Tabulation of example winnings for A and B*

		B		
		+	**0**	–
	+	(–0.5, –0.5)	(–0.9, 0.2)	(–1.0, 0.0)
A	**0**	(0.2, –0.9)	(0.5, 0.5)	(0.9, 0.3)
	–	(0.0, –1.0)	(0.3, 0.9)	(1.0, 1.0)

There are a few observations from Table 1.3 above:

- It would be favorable for A and B to go to the lower diagonal position, which is the optimal position for both A and B. The lower diagonal represents a multi-party oligopoly situation. In practice this may exist for a short period of time between one, two or more players when a certain product is newly traded in the market. However, it will not be too long before others catch up given the higher profitability that exists.
- It is not desirable to consistently be bidding higher prices to win trades. It would be better for all players to win some trades and lose some by bidding at more realistic levels, so that both would be winning trades and making money at the same time. Therefore, the optimal strategy for the dealers would be staying between '0' and '–' and adjust the winning probability so that both may win the same number of trades, but staying at the lower right corner of the matrix.
- This is only a two-player game. However, the concept can be extended into a multiple-player auction process.

If one dealer becomes aggressive and starts winning more trades, the other dealers may react to that and have to increase their bids in order to win trades. This could end up in a vicious cycle, where the margin becomes thinner and all participants would be making less money. The winner in the game would be the customers conducting the auction. When all dealers are "logical", they would be, in general, winning similar proportions of trades over time. However, when the pie is too small for everyone to get a slice, then even if the dealers are logical, someone has to drop out of the game before the dealers all perform with viable businesses.

If all dealers are logical, and given the business models from all dealers, there could be one optimal configuration for the dealers to take their own shares, forming equilibrium. This equilibrium may be broken by the market, regulations, or market participant behaviors.

1.1.3 The dealer's business model

Dealers' business models play an important role in the process of market pricing and competitions. A properly set-up business model could mean competitive advantages in the market. A business model involving derivatives trading may include, among other things, the following:

- Pricing model accuracy and stability
- Trading and hedging strategy

- Sourcing of inventory of risk
- Risk tolerance/appetite
- Funding, capital, balance sheet and other related cost
- Accounting of derivative value
- Operations and risk management.

Commonly, dealers rely on dynamic replication in realizing the OTC derivative values on a portfolio basis. In doing so, traders perform dynamic hedging on the relevant risk factors and manage the portfolio of derivatives through time. When a derivative transaction gets deeply in or out of the money, or terminated due to expiration or maturity, its value becomes realized.

Clearly, a pricing model is critical to derivatives trading and the dynamic replication of derivative values. Knowing the fair value of the derivatives is essential to trading and risk management. Having a pricing edge over competitors can prove extremely profitable. One good example is the OIS gold rush around the time of the 2007–2008 credit crisis, where Goldman Sachs reportedly made hundreds of millions to a billion dollars by knowing the true value of OIS discounting and the value of different collaterals before other dealers.[3] On the other side, mispricing or underpricing the cost and risk of derivatives can be disastrous, when the true economics of the derivatives are eventually realized after trading. The trading and risk management of power reverse dual currency swaps (PRDC) serves a very good example.[4] This was a very popular leveraged product traded between PRDC notes issuers and dealers, servicing primarily the Japanese investors. The mispricing of volatility skews and FX-interest rate correlations by a number of dealers led to substantial losses for the PRDC trading desks, which amounted to over hundreds of millions of dollars, and some closed the trading desks as the result of losses. AIG financial product division's large hit from PRDC and effective exit from the business provides a good example.

When a dealer executes a derivative trade with a customer, in general they do not go back to the market and hedge the risk on a one-to-one basis. For example, if the customer asks for a bespoke 3¼year into a 6½year swaption straddle, the dealer would hedge the trade using additional liquid swaptions available in the market. Such hedging activities are done usually on a portfolio basis, which is more efficient. Dealers are in the business to build an "inventory of risks" or for "warehousing the risks", and manage on a global portfolio basis, which would clearly be less costly than hedging the individual transactions. In practice traders would only need to manage the residue risks, or "spread" risks from the "risk inventory", as well as Gamma and other higher order Greeks.

This makes sourcing of inventory of risk an important factor in a dealer's business model, because a better sourcing of risk inventory means a more easily manageable and less costly hedging. For example, one firm may have more flow than the other houses and, particularly important, two-way flows. A firm with two-way flows would only need to manage the residue risk left over by long and short positions; while the firm that has access to a one-way flow only would have to find the proper hedge of the risky positions, which will be more costly. Therefore the one-way flow house will not be as competitive and may be priced out of the market. For example, the total return swap market for a large index can be extremely competitive. Without access to the two-way market with substantial offsets between the buying and selling of underlying exposures, it would be very hard to be competitive and profitable at the same time.

Different hedging and risk management strategies could mean very different profitability for dealers' derivative trading desks. Grasping the inherent physics of risk factor dynamics through time is always a difficult task for traders. Misspecification of models and risk management strategies may lead to uneconomic trading decisions, excessive hedging costs, and loss over time. One example is the use of a globally calibrated pricing model with many underlying calibration instruments. If one over-calibrates the model to all market traded instruments with too many parameters, the day-to-day calibration noise from market supply and demand may cause significant over-hedging and under-hedging through time. The cost of managing the derivative product would go up and profitability would go down as a result. On the other hand, good management would mean the ability to capture the dynamic risk behavior properly and calibrate the relevant part of the model parameters to the market. Another example is blindly following dynamic hedging delta without considering the higher order Greeks, such as the behavior of cross gamma terms. As we will show in later chapters, not considering the high order Greeks properly may lead to frequent lock in loss.[5] This is because cross gamma may behave similarly to negative gamma, where you make or lose whichever way the market moves.

Clearly the costs from funding, capital and balance sheet would greatly affect the dealer's competitiveness. While the characteristics around the dealer's business are largely unchanged past years, the newly-minted regulations and capital rules, as well as the increasingly competitive electronic trading environment, have contributed tremendously to the changes in the derivatives trading landscape. With more and more incoming regulations, one can no longer escape the credit, funding and capital costs in replicating the derivative values. If a firm carries a large funding spread, it would be costly to replicate the derivative asset values over time. This means the firm would be less competitive in the market. Capital and return on capital are becoming even more important items pressuring the OTC derivative businesses in making proper trading decisions.[6]

Super-fast computers and electronic trading have also changed how traders are performing their jobs and entertaining customer orders, as well as trading among themselves. With the ever-increasing competition in the electronic trading world, only the smart and strong will be able to survive and make enough money.

1.1.4 The dealer-to-dealer market: the broker market

Dealers trade among themselves, forming a market typically linked by brokers. Brokers are agents that are in constant contact with dealers, trying to line up the supply and demand from the dealer community. This market is, in general, only open to the sell-side derivative dealers, and is closed to outside customers or buy-side community.

Dealers have a number of reasons for wishing to inquire, quote and trade in the broker market.

First of all, dealers like to keep on the top of all market activities, such as anything traded by other dealers, or any inquiries/quotes from other dealers. This allows them to construct a better picture of the overall market, from the relatively small amount of trading/quoting available. For example, traders at dealers' interest rate option desks would want to construct the swaption skew surface, so they frequently ask the broker about the information they need. In the broker market, there are only a limited number of popular skew trades quoted, and an even smaller number of trades are actually executed. However, it is from the limited amount of risk reversals and out of the money swaption quotes that traders attempt to construct a complete skew surface that they feel comfortable with. Certainly the construction would be based on, for example, volatility models as well as some personal experiences. Whenever there is a large transaction traded in the market, it will impact traders' re-evaluation of the market situation, and the traders would readjust the overall market picture accordingly.

Secondly, dealers rely on the more "closed-door" broker market to hedge their risk from customer activities as well as in dynamic hedging of their portfolio risk. Dealers are in the business to warehouse the risk from customer activities, where risk positions tend to accumulate. While they all make money by providing the risk management expertise on the derivative products, proper hedging strategies and hedging practices are critical to the success of the dealers' business. With the presence of the broker market, dealers may be able to find a desirable hedging trade with less cost. For example, a swaption may trade in the customer market with a bid/offer of a half volatility point (percentage point); it could trade in the broker market with a quarter volatility cost or even less after broker negotiations. When the desired hedging trade is large, traders at dealers' trading desks may employ strategies to stage the risks opportunistically and reduce the influence to the market, hence reducing the hedging cost. Through the broker market mechanism, the supply and demand effects from the customers are passed through to the overall market.

At the same time, dealers may also employ the broker market quoting mechanism to express their views about the market and hence influence the direction of the market; and as a result affect the customer market pricing. This may create an issue for financial comptrollers, who are responsible for the integrity of derivative valuation marking on the trading books. Such demand in resolving derivative valuations has led to the emergence of the popular Markit Totem valuation survey service, which has become the standard market data validation service for many different asset classes. In the mostly once-per-month survey, dealers submit their own instrument prices at month ends, such as interest rate curve instrument prices, swaption prices with different maturities and strikes, and so on. The submitted prices are processed by Totem to create consensus prices that are passed back to the dealers. While Totem services have become more and more important for dealers, in order to mark their books at month ends, the influence of broker quotes on the consensus prices is also becoming more evident.

As central clearing and electronic trading has now become the norm for plain vanilla instruments, such as interest rate swaps, inflation swaps and potentially vanilla swaptions,[7] the broker market for these instruments has been completely transformed. Swap pricing has become very transparent with ever-stronger competition among the market participants, evident in the Request For Quote (RFQ) platform as well as exchange trading style mechanisms. With a reduced entry barrier into the electronic trading world, some sophisticated buy-side players may also look to enter such markets, which would intensify the market competition even more.

1.2 The OTC derivatives trading process

The OTC derivatives trading process can be quite involved, depending on the complexity of the product. From the dealers' side, the trading process usually involves sales people, traders, legal, risk, and finance, as well as quants, technologists and back and middle office people. Figure 1.4 shows some of the basics around the pre-trading, trading and post-trading processes.

1.2.1 Pre-trade

Naturally, sales people talk to their customers about their requirements and sometimes suggest products that fit their customers' needs. This process could take under a minute for a simple swap trade or much longer, maybe a few months for a structured deal, or a derivative trade linked to the hedging of a structured notes issuance. Sometimes customers' requirements in return profile would need to be translated into derivatives payoffs, which may involve the structural desk's design of a specific derivative product.

Once the specific product is defined through customer–sales negotiations, the product prices are provided by traders, with indicative quote prices in early trading stages to committed trading prices at final auction stage for trading. During the process, traders study the market to understand the risk involved and they add in a hedging cost, among other things, to the quoted prices.

For a new structured transaction or trading with a new credit entity, legal advisors would need to be involved in order to review the overall transaction from a legal perspective, such as interpreting the specific transaction terms, credit implications, jurisdiction and enforceability, collateral and termination definitions, and so on. For example, some specific CSA terms may not be enforceable under certain jurisdictions, which would affect the trading decision one way or another. Customers may also want to have the capability to terminate a transaction at some future time, which may involve a termination process and settlement procedures.

Risk management is clearly one of the important components within the overall trading process, especially when the transaction affects the overall risk profile of the trading portfolio, or the credit exposure to a specific counterparty. The derivatives risks are managed mainly through two types of risk limits: one for market risk and one for counterparty credit risk. Market risk limit can be risk measure based, such as

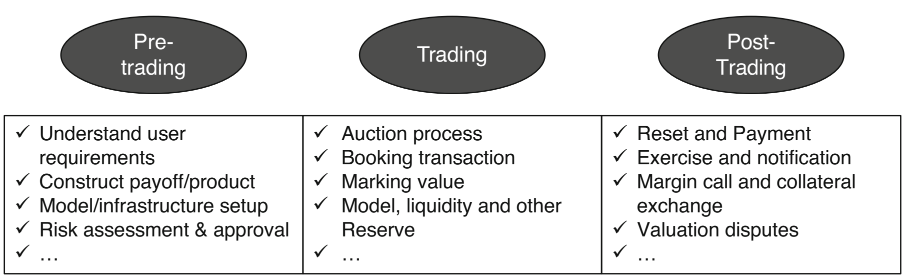

Figure 1.4 The derivatives trading processes

Value at Risk (VAR); or Greeks based, such as delta, Vega; or based on certain stress scenario tests, and so on. Credit risk limits usually are set up based on the counterparty exposures, calculated with a statistical confidence level.

When market risk of a new transaction may lead to a breaching of the risk limit or risk appetite of the firm, the trading desk may have to scale back the transaction, or find a strategy to mitigate the risk, such as to pre-build hedges before the transaction. Otherwise, an approval from market risk management to increase the risk limit would be needed before executing the transaction. More commonly, a large derivative transaction may lead to material increase of credit exposure for a counterparty, which would then require the approval from credit risk officers to extend the credit risk limit, or one may need to find substantial mitigants to make sure credit risk officers are comfortable with the risk.

The modeling and pricing of a complex transaction involves quantitative analysts (quants) developing and vetting the model with various practical considerations, including market, credit, funding and capital considerations in derivatives marking, P&L, attributions, as well as designing hedging strategy and finding the optimal hedges. Within an auction-style customer trading, an understanding of market pricings and competitions are crucial to traders and quants. Feedback from customers, as well as analysis of trading statistics, are critical components in dealers' normal trading activities, especially in finding an optimal point in maintaining a profitable derivative business.

1.2.2 Post-trade

Once a derivative is traded, the derivative contract needs to be serviced until it either expires or is terminated. The servicing of a derivative contract involves:

- Operations of the derivative contract, including marking the derivative contract values, exercise/notifications, reset and payments of cash flows, margin calls and collateral exchanges, valuation dispute resolutions, and so on. Operations are typically done by middle and back office personnel, as well as the collateral management unit.
- Risk managing the derivative contract is undertaken by the trading desk, typically through dynamic hedging, and occasionally from static hedging. Through risk managing the derivatives properly over time, an attempt is made to replicate the derivative values with liquidity and practical considerations.
- Financial comptrollers and risk managers need to ensure the marking of the derivatives are carried out correctly, by establishing proper processes and procedures around the derivatives marking, including market data accuracy, model consistency, model and liquidity reserve adequacy, and so on.

Operations around derivatives will be discussed more in Chapter 2. Below we will touch briefly on the issues around marking the derivatives.

1.2.3 Risk management and model risk

Before a trading desk can trade a particular type of derivative, for example a new derivative product, the modeling and infrastructure for pricing and risk managing the product must be in place. Banks are required by the regulators to go through thorough model validations before a pricing model can be used in trading with customers. Equally important, the risk management infrastructure has to be established before the trading starts, so that the derivative transactions can be marked properly and the appropriate risk sensitivities can be computed and used by the traders.

Derivative valuation models are never perfect and often based on assumptions. For example, a pricing model would assume some stochastic processes for underlying risk factors, sufficient liquidity and efficiency in relevant markets, as well as the validity of replication processes for the derivative values. These assumptions are, at most, good approximations to real life situations, which lead to uncertainties in realizing the derivative values. Improper management of model risk can be far reaching and result in substantial financial losses in the future, as demonstrated historically in many cases, and more evidently in the global financial crisis. Regulators have increased by a tremendous amount the scrutiny on model valuations to ensure that banks maintain proper standards in model risk management. For example, the Office of the Comptroller of the Currency (OCC) published the *Supervisory Guidance on Model Risk Management* and provided a framework that banks must follow in managing model risk.

Tackling model risk is a multi-layered task. First and foremost is the examination of all model assumptions and potential impact on derivative valuations. The second, given the model assumptions, is verification and replication of the specific pricing model. This would include theoretical derivations of formula to actual coding in detailed implementations, covering the model development and implementation in every step. The third layer is the data integrity, calibration and application of models in trading, finance and risk management. On top of all these, model risk governance, data management and audits are all important components of an effective model risk management program.

What is a model?

"Model", in a broad concept under the OCC definition, refers to "a quantitative method, system or approach that applies statistical, economic, financial, or mathematical theories, techniques, and assumptions to process input data into quantitative estimates". In the context of the present book, we are concerned with derivative valuation models.

What is a good derivative valuation model?

As we know, model values are really estimates, or imperfect representations of reality. A good derivative valuation model should give close-to-reality representation for intrinsic risk factors, leading to reasonable realization of derivative values. Deviations from reality, such as market factor dynamics and distributions, significant cost in replication and dynamic hedging, as well as inaccurate parameter estimations, would lead to the model values not being realized. This is exactly why, as we will explain in later chapters, one should account for CVA and FVA in valuations. Missing CVA, one would realize the credit loss stemming from the counterparty default; while missing FVA, one would be realizing losses in funding cost through time.

There are a few indicators for bad model behaviors. One critical indicator is that the model value behaves in a way that bleeds money consistently through time. When this happens, one should really ask the question: What am I missing here? This often is an indicator that the model value is too high, which means some aggressive assumptions have been made in the model in terms of risk factor distribution, model parameter estimations and so on. A simple example is marking the option with too high a volatility. Another example is marking the future asset cash flow with a discounting rate much lower than the real interest rate, or marking the liability cash flow with too high a discounting rate. Similar behaviors can result for incorrect assumptions for any other parameter estimates, including correlations.

When dealers attempt to replicate the derivative values through time, they achieve the replication through dynamic hedging of the underlying risk factors. For complex products, this can be a very involved process, especially when multiple risk factors are correlated. Because of the complexities, dealers often charge a significant risk premium for taking the risk, which covers, for example, hedging cost and capital return. Then the question may arise: What would be the proper value to mark the derivatives on dealers' books, and how much one should one allow the traders to take as profit? On one side, risk managers and financial comptrollers generally take a conservative approach in estimating the derivative values, with conservative model parameters and replication cost included. This would lead to generally conservative pricing model assumptions and conservative parameter estimations for the pricing model. However, this may not be a good trading and risk management model. Instead, trading desks may want to use a model that mimics the reality as close as possible, so that the economic values can be replicated. Overly conservative but prudent valuations may prevent traders from over-marking the derivative values; however, this may lead to uneconomic decision-making. For example, a Bermudan swaption with a conservative model may be uneconomical, because a lower volatility assigned to the future interest rate would underestimate the future option value.

Reconciliation of these two approaches often then leads to the existence of model reserves: on top of the calculated theoretical model values that represent the best estimate of real life outcomes, one would put on a model reserve, representing the estimated potential model uncertainties stemming from model validations. This way, the trading desk would be able to replicate the derivative value as needed, while the model risk is reasonably contained. When the derivative transaction approaches maturity, or the valuation of the derivative converges as it becomes deeply in or out of the money, the model reserve will be released gradually.

1.3 Regulations and controls

Financial derivative contracts are supposed to be used in managing risks, but have long been misused for excessive speculations, leading to losses in banks' books and affecting the health of the financial system. This has prompted more stringent regulations for derivatives trading as well as control processes around it, especially after the 2008 financial crisis.

There are layers of rules and regulations, with which bank dealers have to comply, from different regulators, agencies and authorities. First of all, the banking systems are regulated by central banks, prudential regulators and banking authorities, such as the Federal Reserve Bank (FRB), Office of the Comptroller of the Currency (OCC), European Central Bank (ECB), European Banking Authority (EBA), and the Federal Deposit Insurance Corporation (FDIC). Secondly, they are also regulated by various agencies, such as the Securities and Exchange Commission (SEC) for security based

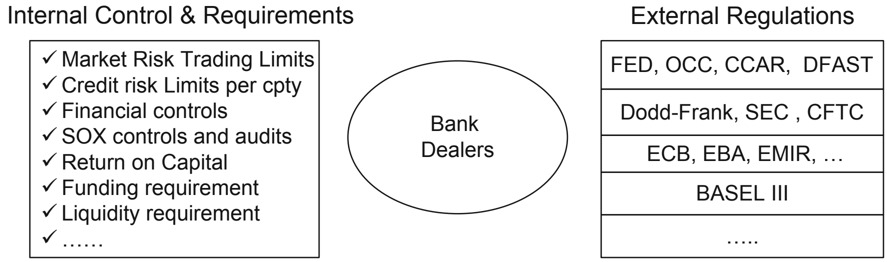

Figure 1.5 External regulations and internal controls for bank dealers

trading, and the Commodities and Futures Trading Commission (CFTC) for futures and swaps trading.

The purpose of the regulations is generally toward ensuring the stability of the financial system as well as providing protection for investors and derivative users. In achieving the goal of having a stable financial system, the regulators require the banks trading derivatives to put in place and maintain adequate processes, control procedures and proper risk management for managing derivatives. At the same time, the banks are also required to have sufficient capital and liquidity in place to keep themselves from going bankrupt, even under stressed situations.

In Figure 1.5, on the right hand side, we show the example external regulators as well as rules and regulations with which bank dealers have to comply. On the left hand side, to adopt these rules and regulations, banks have to implement internal processes and controls, as well as establishing concrete internal policies and requirements for individual businesses. In the following we discuss briefly some of the most important regulations and the profound effects they may have on the derivatives trading and overall market.

1.3.1 External regulations: Dodd–Frank, EMIR and CCAR

In July 2010, the *Dodd–Frank Wall Street Reform and Consumer Protection Act* was passed by US Congress. The goal of the *Dodd–Frank Act* is to improve the transparency of derivative trading, prevent gambling in derivatives by banks and financial institutions, and reduce systemic risk in the derivative markets. The *Dodd–Frank Act* establishes a broad, new regulatory regime, which stands to have profound effects on the market.

Dodd–Frank took several important and necessary steps with respect to improving transparency in the derivatives markets. As mandated by Title VII of *the Dodd–Frank Act*, sufficiently liquid and standardized derivatives transactions, such as interest rate swaps (IRS), inflation swaps, credit default swaps (CDS) and European-style swaptions, will be subject to central clearing requirements, and many of those will be required to be executed through electronic trading platforms. With IRS and CDS already broadly implemented by market participants, inflation swaps, European swaptions and other vanilla derivative products are coming to the market from central counterparties (CCP).

Similarly, across the Atlantic, the European Markets and Infrastructure Regulation (EMIR) was established for European Economic Area (EEA) aiming to reduce the risks posed to the financial system by derivatives transactions. EMIR requires central clearing of standardized derivative contracts above certain thresholds, as well as reporting the derivative transactions to an authorized trade repository. In addition, EMIR also enforces processes targeting risk mitigation, such as dispute resolutions and periodic portfolio reconciliations.

In order to assess the capital adequacy of large banks under economic and financial stress situations, the Federal Reserve started an annual exercise of the Comprehensive Capital Analysis and Review (CCAR), in 2011. The CCAR exercise covers derivatives trading as well as complex bank holding companies, which ensures the large banks maintain sufficient capital with robust and forward-looking capital planning processes. Together with DFAST (Dodd–Frank Act Stress Test), CCAR ensures that the bank holding companies maintain sufficient capital protection under adverse economic conditions.

1.3.2 Basel III

The Basel Committee on Banking Supervision, established by the central bank governors in 1974, is an organization aiming to improve the quality of banking supervision worldwide by establishing standards, guidelines and recommending best practices. The Basel Accords, Basel I, II and III, are the results of the developments from the Basel Committee collaborating with the banking authorities and the financial industry.

Basel I, or the 1988 Basel Accord, was published in 1988 as a set of standard capital requirements for banks, which was enforced by G-10 countries. In 2002, Basel II introduced a quantified market, credit and operational risk framework for capital purposes. While Basel II attempts to deal with capital issues in practice through "three pillars", namely minimum capital standard, supervisory review and market discipline, its weaknesses were clearly demonstrated by the 2008 global financial crisis.[8] In response to these issues, the Basel Committee revised its capital standards in 2010–2011, in what is referred to as Basel III. In addition, they also introduced a leverage ratio for banks, and a new liquidity standard, namely Liquidity Coverage Ratio (LCR) and Net Stable Funding Ratio (NSFR). Basel III's combined capital, liquidity and leverage standards have greatly affected the banks' business practices. For OTC derivatives trading, this has in some way transformed the business landscape completely.

Basel III's new capital standard includes, among other things, enhanced Basel II counterparty credit risk (CCR) capital, as well as a newly-introduced capital on CVA, reflecting the market fluctuation of CVA. With about two-thirds of the counterparty credit losses in the financial crisis suffering deterioration in the creditworthiness of the counterparties, the new capital from CVA market risk fills the gap against the potential mark-to-market losses.

The financial crisis also witnessed the build-up of excessive leverage of the banking system, which forced liquidations and amplified the downward pressure on asset prices. The supplementary leverage ratio (SLR) is designed to have a simple, transparent, non-risk based measure aiming to be complementary to the risk-based capital framework, and ensuring broad and adequate capture of both on- and off-balance sheet leverage of banks. For global systemically important banks (G-SIBs), additional requirements are also set to provide extra protection.

LCR and NSFR are the two critical components in the Basel Committee's reform on liquidity standard in developing a more resilient banking sector. LCR targets the short-term liquidity requirements of the bank under stressed situations, such as substantial credit deterioration; while NSFR aims to ensure the banks would maintain proper liquidity and sustainable maturity structure for assets and liabilities. LCR will be covered in a little more detail in Chapter 5.

1.3.3 Effects of regulations: derivative business changes and unintended consequences

The financial crisis prompted the creation of much more stringent rules and regulations for banks and derivative participants. These rules and regulations have a profound impact on the derivative market participants as well as the overall derivative business landscape. On the one hand, central clearing of standardized derivative contracts leads to a more transparent and stable derivative market by requiring better trading practices, and removing bilateral counterparty risks; on the other hand, the increased capital, liquidity and leverage standards have increased the resilience of the banks under adverse market conditions. Figure 1.6 shows the changes in a firm's balance sheet as regulations are implemented: banks are deleveraged, and more liquidity and capital are reserved to protect the banks from going under.

The substantial increase in capital and liquidity has embedded costs. Coming down to the derivatives, it becomes harder for dealers to keep complex derivative products because they are more costly than before. An increase in market risk capital and model risk reserve is needed; uncollateralized derivatives with significant counterparty credit risk (CCR) capital and CVA capital attaching to it, would become much less attractive. This means dealers would have to charge the customer much more for taking the same trade, which would lead to volume decrease in overall OTC customer market

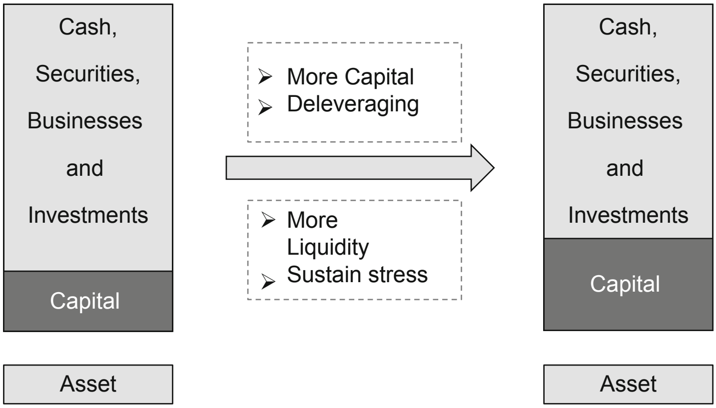

Figure 1.6 Changes in a bank's balance sheet as a result of regulations

activities. Meanwhile the emergence and standardization of electronic trading would make the trading even more competitive and challenging. This only means the weaker ones have to exit the market as it becomes harder to justify the lower return on capital to investors. As a result, the derivative landscape changes.

Regulations always carry unintended consequences. Increased bank capital, intended to protect balance sheets in future downturns, is slowing credit expansion and hence the economic recovery. On one side, increased capital, liquidity and other costs are passed on to the customers, which makes it harder for the derivative users to initiate derivative transactions. For example, if the hedging benefit of a derivative transaction is 100 basis points of the notional, and the cost is increased from 10 basis points to 30 basis points, the derivative user would have much less incentive to hedge through derivatives. This, effectively, pushes the customer away from making unsecured derivative transactions, or the customers would have to consider posting collaterals for the derivative transactions. Either way, the cost of a bilateral derivative transaction for the customers goes up significantly.

On the other hand, regulations have also enforced an enormous amount of documentation, as well as created huge burdens for the banks and derivative users, with an unusual degree of control. In some cases, the requirements of Dodd–Frank are so onerous that it drives the derivative users away from utilizing the derivatives to mitigate the market fluctuations. One example is where some public utility companies effectively withdrew from the utilities swaps market, which they employed in their day-to-day business activities to hold down volatility in natural gas and electricity rates. Similar effects are also observed from the interest rate swap market.

Meanwhile expensive capital requirements for conducting bilateral derivatives transactions have led to business closures among the competing dealers. This also leaves derivative users with fewer choices and increasing cost in trading, and in turn they will make fewer derivative hedges deemed necessary for business activities, and hence be more exposed to the market volatility.

It is not unusual to have cross-border regulation and enforcement differences among different economic and political regions, both in the interpretation of rules and the timing of implementation. The cross-border regulation arbitrage could really drive businesses from one market to another. This can only lead to unnecessary frictions in trading, and create unfairness in competition among banks. One example is that the over-burdening of Dodd–Frank restrictions drives derivative transactions to overseas markets outside the United States. Another example is the CVA capital exemption that European regulators provide for the corporate derivative users. This is inconsistent with the standardized Basel III requirements and will lead to uneven derivative trading costs among European, US and dealers in other regions.

Summary

In this chapter, we have discussed the following:

- We started by introducing the basics of derivatives trading, the derivative market participants, and their relationships.
- The customer market and the auction process are presented in some detail. The competitive auction process is essential in understanding one of the most

important aspects in derivative trading process, which in turn is critical in understanding market quoting and pricing mechanisms.

- Dealer's business model and the broker market are also important in understanding the derivative trading process and the pricing mechanism of "experts" trading with "experts".
- The derivatives pre-trading and post-trading processes, as well as derivatives risk management and model risks, are discussed briefly. They are, however, of particular importance for us to catch the full picture of derivative trading and valuations management.
- Basel III, the Dodd–Frank Act and various new rules and regulations have triggered significant changes in how people behave in derivatives trading and how firms evaluate the derivative business, and the derivative business landscape has been changing as a result. The rules and regulations also carry unintended consequences.

Why are these important?

This is because the economic values of OTC derivatives are not just valuations under theory with stochastic processes and distributions: they are closely linked to how the values are realized in real life. The realization of OTC derivatives cannot be separated from dealers' business models, the risk management aspects, the accounting aspects and the regulation impacts. As we progress through the book, we will see how these are embedded in the replication process of derivative values.

2 Legal Aspects and Operations of Derivatives Trading

An OTC derivative transaction is a legal contract between two trading counterparties. In order for a derivative transaction to be executed, both parties have to agree on all the legal terms. The contract would cover a variety of aspects, including terms relating to:

- Specific derivative transaction payoffs, such as cash flow schedules, payment index, coupon frequency, holiday calendar, etc.
- Options exercise and terminations, including discretionary exercises and mandatory terminations, as well as how to determine the termination values and the settlement procedures.
- Derivatives operations, such as settlement, transfer provision, collaterals types, currencies, thresholds, and margin grace period.
- Counterparty credit and default: what situation would be regarded as default, what would be the legal jurisdiction, and the seniority of the derivative transaction in the payment waterfall, etc.

All these are of critical importance to the existence and operations of OTC derivative transactions.

Once an OTC derivative transaction is executed, operations must be put in place to maintain and service the derivative, so that the legal terms are followed properly, from day one of trading to the termination of the transaction. For example, cash flows have to be agreed upon and paid by both parties; the collaterals required have to be exchanged as expected by both parties; any disputes on the terms or valuations have to be resolved promptly to ensure the derivatives are serviced properly; collaterals have to be borrowed from or funded by the treasury and collateral operations, and so on. As we shall see later, these could have significant impacts on the derivative valuations. With that in mind, Basel III also requires specifically the implementation of a strengthened operational management of collaterals, including monitoring, reporting, analyzing and categorizing collaterals and collateral calls, disputes, rehypothecations, etc.

2.1 Legal aspects of derivatives transactions

Because of the potential complexities involved in the derivatives, common standards are critically important to ensure all market participants are speaking the same language, and hence ambiguity will be reduced to minimum under all possible practical situations. The International Swaps and Dealers Association (ISDA) is the major trade organization that was formed to facilitate standardization of derivatives contract terms. For example, the ISDA Master Agreement is the standard used by dealers and derivatives users to define the basic and general trading terms between the two trading parties.

In general, two or more of the following legal agreements will govern an OTC derivatives transaction:

- ISDA Master Agreement
- Credit Support Annex (CSA)
- Trade Confirmations.

2.1.1 ISDA Master Agreement

The ISDA Master Agreement or simply "ISDA" is generally negotiated between the two counterparties before any trading is initiated between them. It is the single agreement that generally applies to multiple transactions negotiated between the two parties in the future, which does not contain any specific terms of a particular transaction. It covers various terms from defining the default and termination events, netting of trades at close-out, to governing law and jurisdiction. Together with a "Schedule", which contains the amendments and customizations of terms, it provides a framework defining the trade relationship between the counterparties. ISDA is often supplemented by an ISDA Credit Support Annex (CSA), which governs margin collateral posting matters relating to transactions entered into under the ISDA Master and Schedule. Trade confirmation, on the other hand, defines the transaction specific terms. In the following we go over some specifics of these legal agreements.

2.1.2 Credit support annex

CSAs are particularly important for OTC derivatives valuations, due to a variety of terms related to the credit and funding aspects. For the purpose of this book, we are particularly interested in the following terms, which are very important to the XVA of OTC derivatives:

- **Eligible Collateral Currency (Currencies)**
  Collateral currency can be one or multiple currencies. With different currency carrying different funding value, this is an important element of funding calculations.
- **Collateral Thresholds**
  Collateral threshold is the critical component in actual margin calculations. When the netted exposure is above the collateral threshold, the amount that exceeds the threshold will be subject to collateral posting.

Collateral threshold can be a constant, for example zero, or a table defining how much the threshold would be depending on the counterparty ratings. An example is shown in Table 2.1. Collateral threshold can be symmetric, which means both counterparties are following the same schedule, or asymmetric, meaning the collateral schedules are different for the two counterparties. Clearly collateral thresholds have important credit and funding implications.

Table 2.1 *Ratings-based threshold example*

Credit Rating	Collateral Threshold
AAA	50M
AA	20M
A	5M
B	0M

Table 2.2 *Eligible collaterals and haircuts*

Asset	Haircut
Cash	100%
Treasury < 2y	101%
Treasury (2y-5y)	103%
Treasury (5y-10y)	105%
Treasury (>10y)	108%
GSE Passthroughs	115%
Corporate/Municipal	120%

- **Eligible Collateral Assets and Haircuts**
  The CSA defines the collateral assets that the counterparties may post as collateral. For example, one may post cash, government bonds, mortgages, municipal or corporate bonds and so on. Along with the different types of collateral assets, haircuts for each collateral asset would be defined, primarily to protect the counterparties with collateral damage during the margin period. Table 2.2 shows an example for the eligible collaterals with haircuts. Some CSAs also allow assets in multiple currencies, such as G-7 government bonds.
- **Collateral Interest**
  When cash collateral is posted, the receiver of the collateral is obligated to pay interest defined by the CSA; for example, Fed Fund or EONIA with a spread. Collateral interest is fundamental to the funding value calculation, which will be discussed in detail in Chapter 4.

  Collateral interest payment is usually not a problem until the reference rate becomes negative. While the CSA has no specification on the negative interest determination, in May 2014 ISDA published a Collateral Agreement Negative Interest Protocol to clarify the issues around the negative interest payment. In this case, the collateral posting party may have to pay interest to the collateral receiving party when the defined interest payment is negative.
- **Minimum Transfer Amount (MTA)**
  When collaterals are exchanged between the counterparties due to market move, one would want to minimize the cost incurred by the exchange operations. The minimum transfer amount is primarily designed to minimize such operational cost, so that it is only when the expected collateral amount exceeds the MTA that the exchange operation will happen. For example, if MTA is $1 million and the amount to be exchanged from the last margin posting operation is 900K, there will

be no exchange of collateral. When the expected collateral amount accumulates above $1M, the margin posting will happen.

- **Margin Period (MP)**
  The margin period is the grace period defined in CSA/ISDA that either party has to post the variation margin when it is demanded. The margin period may vary from a few hours for foreign exchanges, to a few days for normal derivatives. The standard 2002 ISDA master has shortened the collateral margin period from three business days to one business day.
- **Initial Margin (IM)**
  Derivatives values may move significantly during the margin period due to market moves, which could become an issue for the surviving party. If the derivatives value moves higher for the surviving party and the defaulting party is unable to post collateral, the increase amount during the margin period would be subject to counterparty default. Initial margin is designed to protect one or both parties from such credit situation.

  Initial margin is very common for exchange trading; normally exchange customers are required to post initial margins so that exchanges are fully protected. Clearly, initial margin would depend on the market risk of the trade.

CSA may also cover special credit terms, such as specific downgrade provisions, among other things. We will discuss one special case in Chapter 5 related to replacement value adjustments and liquidity value adjustments.

2.1.3 Confirmations

When an OTC transaction is executed, the two counterparties will need to confirm the agreed trade details so that no ambiguity exists between them. One trading party will create a "confirmation" setting out the specific details of the transaction, which will then be forwarded to the other trading party to confirm the details. When all terms are agreed upon between the parties, the document will be signed by representatives of both parties and kept on file.

In general, a confirmation contains all the financial details around the specific transaction. For example a simple swap confirmation would contain the coupon fixings, day count convention, scheduling, payment information and other relevant economic information. These details are needed to calculate the risk-free valuations without credit and funding considerations.

In addition to the trade-specific financial details, there could be other information included in a confirmation. For example:

- **Trade Specific Break Clause**
  For the purpose of reducing credit risk, mutual put breaks are sometimes included in confirmations, which can be exercised for a specified frequency. These mutual options are discretionary and can be exercised by either party.

  Mutual put breaks are clearly important credit mitigants for both trading parties, and are important components in CVA and Risk Weighted Asset (RWA) calculations. As the banks become more and more sensitive to the capital related requirements, mutual put exercises have become increasingly popular over the past a few years.

While the breaking of transactions would involve exit price evaluations, it is important to have such evaluations also defined clearly in the legal documents. Such definition would generally have funding implications, as we will explain in later chapters.

- **Terminations and Settlement Calculations**
  One of the trading parties may want to exit the transaction at a future point in time, according to their specific business purpose. Such one-sided termination would require a well-defined exiting process, including settlement price. The settlement process and definitions could have important funding implications in termination and exercise situations.

2.1.4 Jurisdiction and enforcement of legal agreements

ISDA, CSA and Confirmations are legal documents that govern the derivatives transaction with specifications covering a variety of legal situations. These legal documents are executed under specific local, sovereign or international laws. The interpretation of law and practice varies from country to country, and from region to region, and in practice the enforcement of these legal contracts can be an issue.

One important practical aspect in the legal contract involves the default process, and the issue related to the enforcement of legal clauses for netting transactions within an ISDA Master agreement. All transactions under a netting agreement are governed by a specific ISDA Master; however, in practice they may not be enforceable in a default court. For example, two transactions with positive and negative values may not be recognized as one single value in the default recovery process. The court may interpret the transactions in favor of one trading entity, in which case it is likely that you may have to pay what you owe, but you may have to go through the default process to get what the counterparty owes you. This is what we called a legal netting opinion.[1] Similarly, obtaining and enforcing a judgment in a foreign country may also encounter issues in practice.

The netting of transactions, allowing the offset of positive and negative exposures among all trades, is fundamental to derivatives trading, as it reduces the credit risk between the two parties. The ability to execute such a netting agreement has profound consequences on the handling of such transactions in terms of credit considerations, as well as in CVA and FVA calculations. Expectation of a netting clause not enforceable within a jurisdiction may lead to complexities in CVA and FVA calculations. For example, one may expect the collateral related operations to be executed normally when the agreement between the trading parties is live and in effect; however, one may have to take a conservative stance in considering the default loss under doubtful jurisdiction. This may mean the funding exposure is normal when the trading parties' agreement is live, while default exposure may need to be calculated differently.

2.2 Collateral and operations

Collateral operations are crucial for derivatives transactions. Collateral exchanges allow trading parties to trade continuously with credit risk limited and under control.

Collateral disruptions may have serious consequences affecting trading and therefore need to be treated with care. In this section we discuss briefly the collateral operations and practical considerations.

Collaterals and Types

The borrower pledges valuable liquid property, known as collateral, to reduce counterparty credit risk in bilateral derivative trading. The amount of collateral required is calculated based on the netted trade values and according the rules defined in the CSA. With the pledge of collateral, the counterparty default risk between the two parties will be reduced by the amount of collateral posted. With the reduction of credit risk, the trading counterparties are able to do more business with each other within a defined credit risk limit.

The type of collateral is specifically defined in the CSA, including the collateral currencies, collateral assets and haircuts. The CSA also defines whether the collateral can be rehypothecated or not. Rehypothecation is the market practice whereby a counterparty can reuse the security pledged as collateral for its own use. Rehypothecation allows the lender to use the pledged collateral in its own funding activities, such as deploying in the repo market, or pledging to others for its own trades and borrowings. Thus collateral offers another important value for the lender: funding value. This will be covered in more detail in later chapters.

Collateral posting process

Market movement drives the change of OTC derivatives values over time. The derivatives value change triggers collateral posting from the owing party when CSA is applied. Such collateral posting is critical in fulfilling the legal requirements from CSAs.

First, the amount of collateral has to be agreed upon between the two trading parties. In practice, it is natural for the two parties to have pricing discrepancies, which must be resolved by certain mechanisms. Once the collateral amount to be posted has been agreed upon, the posting party would then need to post the amount within a margin period of time. ISDA/CSA defines the consequence of such collateral posting failure, which could lead to the declaration of default of the borrower.

For bank dealers, the collaterals posted or received are usually managed by funding desks, where the usage of balance sheet assets or the funding of collaterals would have relevance to the economics of the derivative value. Ideally, the optimal collaterals based on CSA terms would be used in posting, and rehypothecation would be utilized when needed. This will be covered in more details in chapter 4.

Dispute Resolution

It commonly occurs that the valuations between two trading parties may be different. When the difference is small and within a tolerable range, both trading parties are able to operate smoothly. However, significant pricing discrepancies can lead to a dispute situation in collateral operations: one party calls for collateral posting/return, while the other party does not agree and refuses to post/return. Prolonged collateral call disputes could lead to significant problems by creating unnecessary credit risk for the trading parties.

In 2009, ISDA developed a dispute resolution procedure to provide an agreed standard industry approach in dealing with disputed OTC derivative collateral calls. In general, when a dispute happens, the disputing parties would go through the following processes:

- Portfolio reconciliation: This involves complete portfolio deal-by-deal reconciliations between the trading parties, in an effort to find the discrepancies in valuations.
- Consultation and information dispute resolution: Given the valuation discrepancies, it is required that the trading parties should validate the marks through communications.
- Market quotation/polling: This is when the consultation fails and the disputing parties have to resort to market mechanism to find the proper valuations. This involves the dealers providing independent two-way executable quotes.
- Based on the results from dispute resolution, the relevant parties would need to recalculate the delivery amount or return amount, so that the collateral will be transferred accordingly.

The timeline for the dispute resolution is limited to four days, unless an extension is applied. This resolution procedure, implemented in a timely manner, allows the collateral process to operate properly with minimum interruption, and also has some bearing on the counterparty credit risk capital tied to the margin period of risk (MPOR).[2]

2.3 Derivatives accounting and fair value adjustments

Derivatives are carried on the balance sheet as either assets or liabilities at fair value. Fair valuation can be obtained from market valuations with so-called mark-to-market (MTM), when there is liquid market available; or from objectively assessed fair model valuations, when there is no liquid market. Derivative fair value, as adopted by Generally Accepted Accounting Principles (GAAP), is defined as the amount at which the derivative could be bought or sold in a current transaction between willing parties, or transferred to an equivalent party. Fair value accounting requires that any change in fair valuations must be recognized through income statement. The application of derivative fair value covers all situations with exception of hedging instruments within a hedge book. For the purpose of the present book, our discussion will be limited to the trading account, while hedge accounting of derivative under the Financial Accounting Standards Board (FASB) 133 ruling will not be considered.[3]

Fair valuation of a derivative transaction reflects the expected outcome over the lifetime of the derivative. When there is no market price available and fair assessment is needed, one needs to take into account all possible expected outcomes. Such outcomes include credit default of derivative counterparties, expected cost in carrying the derivatives position through maturity, expected loss for inaccurate model assumption etc. This then leads to the assessment of fair value adjustments, namely credit value adjustment, funding value adjustment, valuation model reserve, etc.

Clearly, the marking of simple vanilla instruments, traded as liquid assets in the market, should be market-traded prices. For more complex instruments, valuations would be derived from the simpler vanilla instruments with assumptions, such as sufficient liquidity, no arbitrage, market completeness, specific underlying distributions, and so on. The replication value given these assumptions would depend on the validity of these assumptions, which may hold or break over time. This then prompts the creation of valuation model reserve: one may mark the complex derivatives with the best expectations given reasonable assumptions; however, the expectations may yield different values given reasonably conservative assumptions. Banks are required to keep valuation model reserves for incompleteness in models, in case the best estimate from fair valuation is not realized.

Credit valuation adjustment (CVA) and funding valuation adjustment (FVA), as will be discussed in more detail in later chapters, can be viewed as expected losses or gains in the process of replicating the derivative values: CVA is the expected loss from counterparty default, and FVA is the expected funding cost or benefit. These are components of the fair valuations that are based on expectation of future outcomes, and are to be reflected in the daily profit and loss (P&L) for the derivative trading books.

2.4 Treasury, funding channels and transfer pricing

The treasury department within a bank is in the business of managing the assets and liabilities of the bank, as well as the funding and liquidity of the overall firm. When the bank is in need of funds, the treasury department may borrow from the central bank and market lenders; when the bank has excess liquidity, it may attempt to lend in the market, or invest in assets and securities, earning interest returns. The amount that a bank needs to raise will depend on the individual businesses' projected funding and liquidity needs; and on the other hand, treasury's costs incurred in the funding activities will be passed back to the individual businesses. This is so-called transfer pricing.

As underlying market risk factors move up and down in the market, derivative values can swing significantly positive or negative. As a result, derivatives dealers carry a significant balance sheet, with changing assets and liabilities. When derivatives assets and liabilities are collateralized, collaterals have to be obtained and posted to the counterparty, which are usually managed by the funding desk and through collateral operations. Acquiring collateral means the funding desk has to fund the collateral acquisition, and the derivative desk will be charged for the collateral usage on a net basis. When derivative transactions are not collateralized, they are effectively unsecured borrowing and lending with the counterparty. The credit and funding implications of these derivative transactions are the focus of the present book, and will be examined in later chapters.

In achieving the funding goals and fulfilling the liquidity requirements, firms may employ multiple channels in the funding process. The following are just a few of these:

- Retail and commercial deposits
- Issuing short term commercial paper
- Issuing medium- and long-term debt
- Issuing equity and preferred stock
- Convertible bonds

- Operating cash flows such as fees
- Other market funding structures and vehicles.

While each one of these may serve a different purpose and imply different costs to the firm, determining the funding cost of a financial institution can be quite involved. This will be covered in more detail in Chapter 4. Here we use one example to show the funding vehicles and the impact it may have on the market.

One of the popular funding vehicles before the 2008 credit crisis was the Structured Investment Vehicle (SIV). Essentially these vehicles invest in long-term assets, including residential mortgage-backed securities, and loans and securitizations, among other things. Many banks and financial institutions made use of SIVs and term repos to keep their illiquid assets off the balance sheet. Effectively, banks obtain cheap funding through repoeing out balance sheet intensive illiquid assets. This worked well during a good market with the assumption that SIVs may maintain excellent liquidity and credit. However, when the credit crisis hit, the value of illiquid assets, led by sub-prime mortgages, dropped substantially, which triggered the liquidity drain as SIV short-term funding dried up. By October 2008, there were no SIVs left for TARP to rescue, as every one of them was forced into liquidation in a very short time period.

Summary

In this chapter, we have discussed the legal, collateral operation and funding aspects around the OTC derivatives. They are critical components in OTC derivative valuations. The OTC derivative values are realized day over day from operations; their values have to follow the ISDA and CSA terms. As we shall see in later chapters, the legal terms governing collateral, termination and credit default have profound impact on how the OTC derivatives are valued. Unsecured exposure leads to expected default value and CVA; meanwhile, the funding part of unsecured exposure would lead to FVA.

II EXPOSITION OF VARIOUS VALUATION ADJUSTMENTS

Introduction

Traditional derivatives valuations are developed based on the concept of risk neutral and no arbitrage. A risk neutral portfolio should return a risk-free rate, meaning the future cash flows should be discounted using a risk-free rate. Before the 2008 Lehman-induced credit crisis, the prevailing risk-free rate used in the market was the LIBOR rate. If all market participants are discounting cash flows using the same rate, it becomes the market pricing. In the good old days before the crisis this worked well.

Much like the 1987 Black Monday crash taught people the importance of tail risk and volatility skew, the financial turmoil around 2008 has shown people that credit default can be real for very large size financial institutions and entities with superior ratings, and it can come very quickly. The super-senior AAA-rated CDOs, CDO-squares and the like are prone not only to the rating agency flaws, but also the disastrous model problems. The demise of Lehman Brothers and the impact on the financial world could have been much worse if not for the tremendous amount of rescue effort by the US government, the Fed, European Central Bank and government entities throughout the world.

The Lehman default sent the market into a rapid downward spiral, which put all banks and financial institutions into defensive mode. With many firms fighting for their own survival, the liquidity in the market dried up very quickly: firms were extremely cautious when lending to others while many assets became illiquid. Some markets disappeared completely, such as the commercial paper market and the synthetic funding structure market such as structured investment vehicles (SIV). This liquidity squeeze led the skyrocketing of short-term borrowing cost, and showed the market the importance of funding and liquidity. In a stressed market, funding can be very difficult and costly over an extended period of time. Without being prepared, it is easy to have a "run" on the bank and quickly one falls into default.

The funding of uncollateralized derivative trades can lead to significant liquidity issues for the lender. In a bad market, dealers refuse to trade with counterparties who could be viewed as having a liquidity problem, or ask for more collateral from the badly viewed names. Without being prepared with sufficient capital for such a stressed scenario, a financial firm simply cannot survive without external help, such as from government. Regulators quickly came to realize the importance of liquidity and capital – or the lack of liquidity and capital – during the financial crisis. This has formed the basis for the new regulations that came about as a result of trading, risk management, finance, capital and liquidity, among many other things.

The Lehman-triggered credit crisis resulted in some permanent impacts on the market. The split of the degenerate LIBOR curve into multiple LIBOR basis curves

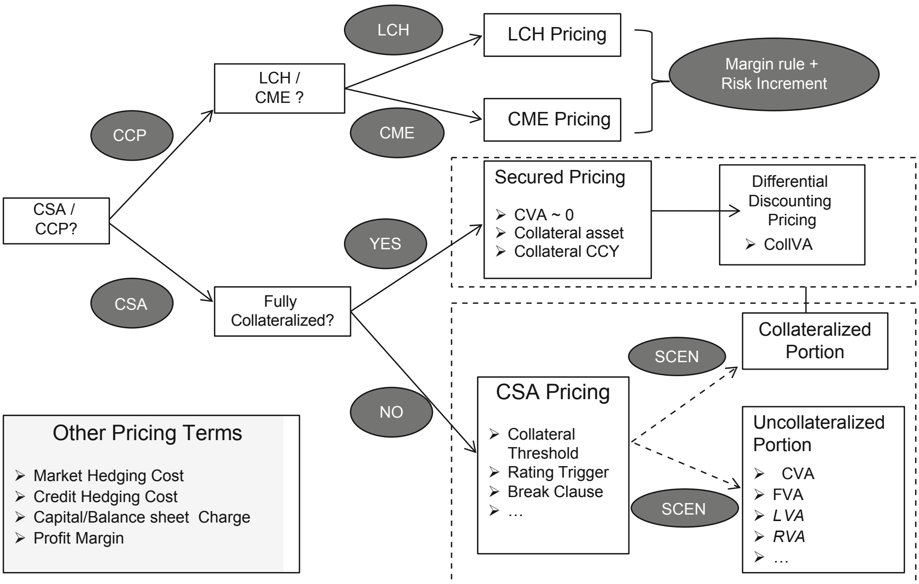

Illustration of XVA calculation logistics for typical client quoting

is one of them. The split of LIBOR basis curves reflects the market perception of different credit risk for different frequency of LIBOR payments. Incorporating credit default risk accurately in derivatives valuations becomes prevalent practice in trading, financial accounting and risk management. With the funding cost being real, more and more banks with centralized funding desks started to include the funding cost as an essential part of the daily profit and loss of each line of business, as well as financial reporting. Moreover, more and more stringent rules and regulations from various authorities and agencies have been implemented and enforced.

While the management of credit risk in banks has quite a long history, the years since 2008 have seen the shift from a passive credit limit-based management to a more active management with more accurate quantitative measures and applications in a much broader scope. Similarly, funding cost management for a derivative desk is not a new concept, either. The practice of calculating funding cost has been in the large investment banks for years; however, it was never really done it on a consistent portfolio basis, but more from an accounting perspective. The 2008 credit crisis changed all that permanently. More and more banks are accounting for funding cost in marking derivatives portfolios; funding and liquidity costs are measured with businesses as well as capital rules in mind. While this process is still evolving, we attempt here to explain all these from a practical perspective.

In the diagram above, we show a simplified logistics used by dealers in typical client quoting activity. When a customer requests a quote, the dealer may ask the following questions sequentially:

1. Is it a bilateral trade with CSA or a central cleared trade? For a central cleared trade going to central counterparties (CCPs), it would go into differential pricing on CCPs given the margin rules and incremental risk from the incoming position.

2. For a bilateral trade, is it fully collateralized? If it is fully collateralized, the derivatives pricing would enter into the secured pricing world, which bears little credit risk and small CVA. However, one would need to differentiate different collateral assets and collateral currencies embedded in CSA. We call this part of the calculation differential discounting pricing. The value of the collateral can be coded as collateral value adjustment, or CollVA.
3. When a bilateral trade is not fully collateralized one normally has to figure out, by way of scenarios, the collateralized portion and uncollateralized portion of counterparty exposures. For the collateralized portion, one can resort to differential discounting; while for the uncollateralized portion, one would have to consider CVA, FVA, LVA, RVA and possibly other XVAs.

In this part of the book, we will cover the basics of valuation adjustments, as being the result of credit defaults, and funding cost, namely Credit Value Adjustment (CVA), Debit or Debt Value Adjustment (DVA), and Funding Value Adjustment (FVA). We will also briefly cover Replacement Value Adjustment (RVA), Liquidity Value Adjustment (LVA) and Capital Value Adjustment (KVA). While some of these have become discussed extensively among financial professionals, others have emerged and not been given the attention they merit.

3 CVA Primer and Credit Default

3.1 Risk-free rate, overnight index swap (OIS) rate and LIBOR curves

The "risk-free" rate is at the heart of derivative valuations in the form of discounting of future cash flows or evaluating investment returns. In the following we discuss the so-called "risk-free" rate, OIS and LIBOR curves.

3.1.1 Risk-free rate and OIS discounting

Before the 2008 financial crisis, LIBOR benchmark rates were treated as "risk-free" rates in discounting future cash flows by the broader market. This worked well until the Lehman default, which triggered the subsequent credit and liquidity meltdown. In Figure 3.1, we show the benchmark 10-year swap rate with the 3-month Fed Fund–LIBOR spreads from 2005 to 2014. In October 2008, the 10-year swap rate dropped from around 4% to below 3% (right scale), the 3-month Fed Fund–LIBOR spread widened to over 350bps (left scale), reflecting the sharply increasing credit and liquidity risk; meanwhile, another credit indicator, the TED spread, peaked at over 450bps. This clearly calls into question the use of LIBOR as the "risk-free" rate in derivative valuations.

The LIBOR as "risk-free" rate was based on the fact that it was close to AA-rated interbank loans. This was expected to be low risk, but as witnessed in times of stress, this risk could be substantial. Instead, the overnight index swap (OIS) rate has become the new benchmark "risk-free" rate; for example, Fed funds (FF) rate in USD, EONIA for Euros and SONIA for Sterling. The Fed funds rate is the interest rate at which depository institutions lend balances at the Fed Reserve to other depository institutions overnight. The Fed fund rate is achieved by the Federal Reserve through open market operations, such as repo and reverse repo of government and agency securities. The weighted average of overnight borrowing and lending among the banks is expected to be the Fed funds rate.

Compared to other interest rate benchmarks, the OIS rate is the closest to a "risk-free" benchmark rate. It is considered safer than unsecured deposits because it occurs in the Federal Reserve System under the oversight of the Federal Reserve. The OIS has also become the standard interest rate that is paid for collateralized derivatives, representing the close to risk-free natural interest rate.

Figure 3.2 shows the derivative desk trading with a counterparty, where the counterparty owes money and posts cash collateral. In return, the derivative desk pays OIS as interest to the counterparty.[1] As a result, the derivative future cash flows will be discounted using OIS, or so-called OIS discounting. This is fair because the money lent will accrue at the same interest rate as the posted cash collateral, so that on a net

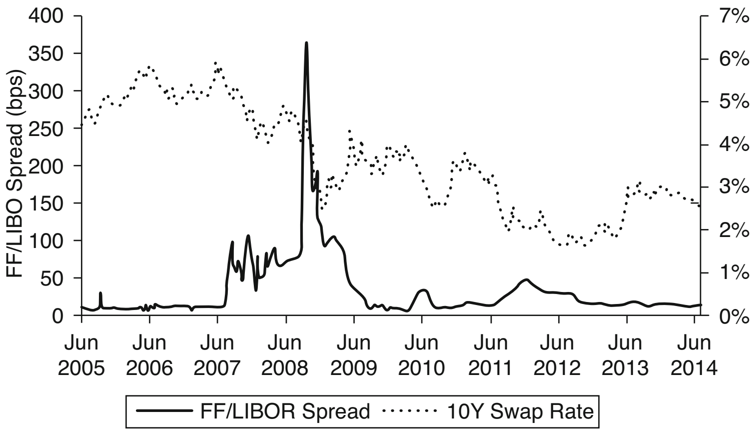

Figure 3.1 FF–Libor spread spikes during the 2007–2008 credit crisis

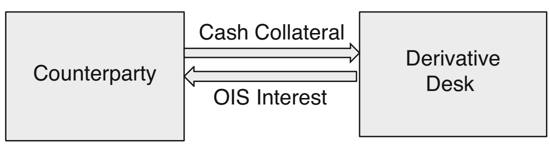

Figure 3.2 Collateralized derivative trading

basis (derivative + collateral), the daily interest carry is zero. This will be discussed extensively in a much broader scope in Chapter 4.

3.1.2 Splitting of LIBOR curves

Post-Lehman crash, the market quickly realized that different LIBOR indices with different payment frequencies imply different credit risks; therefore, a longer term LIBOR index should carry an implied credit spread over the shorter ones. Quickly degenerate LIBOR curves are branched into different curves for different indices. In Figure 3.3, we show the comparison of LIBOR 3-month, 6-month and 12-month curves with OIS index curve. With OIS being the new standardized “risk-free” rate, LIBOR index curves are carrying significant spreads over OIS curves. The graph pattern shown in Figure 3.3 has been one of the permanent impacts on the market, reflecting the valuation changes by market participants given the reality of credit default.

As LIBOR indices are directly referenced by more than 60% of the OTC interest rate derivative market,[2] one should note that the interbank lending market to which LIBOR is referenced is far smaller and carries much less liquidity. This creates a problem in the reliability of LIBOR indices, which are subject to manipulation, as manifested by the LIBOR scandal investigations around 2012.[3] While there has been regulatory and industry work in improving the situation, a better and more reliable set of indices still remains to be seen.

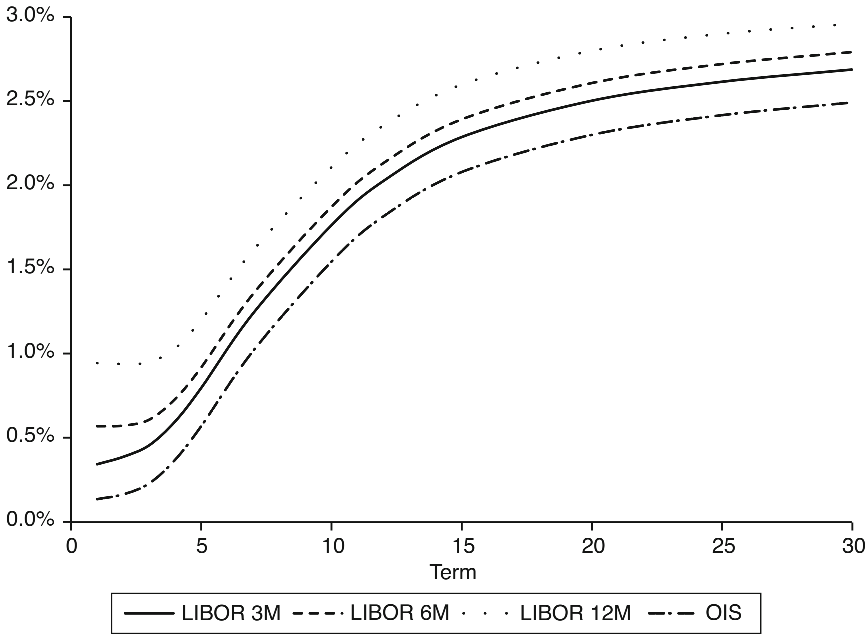

Figure 3.3 Branching of LIBOR curves

3.2 Counterparty credit risk

3.2.1 Counterparty default and recovery

When a counterparty defaults, the derivatives trades with positive values would have to go through the default recovery process. The amount that is to be recovered from the process would depend on, among other things:

- The determination of the default time
- The valuation of the derivatives trades at the default time or exposure at default (EAD)
- The remaining assets of the defaulting party once default is declared
- The seniority of the derivative trade and where it is at the waterfall of payment.

Given all the uncertainties involved in the generally long default recovery process, the exact recovery rate for a specific asset and specific counterparty often will not be known for a long time. In credit risk pricing and modeling, one would make an assumption about the recovery rate for a specific counterparty. The actual loss from the counterparty default would be:

$$Default\ Loss = EAD\,(1-R) = Max\,(V,0)\,(1-R) = V^{+}\,(1-R) = V^{+}LGD$$

where V is the valuation of the portfolio derivative trades facing the counterparty, V^{+} is the positive part of the value, R is the recovery rate and LGD is loss given default, with LGD = 1–R.

The determination of credit default in reality is not necessarily clear-cut. The market may not know the exact time of the occurrence of the default until some time later.

This creates some uncertainty around the potential credit risk, since the valuation of the derivative trades could change in value during the time period. This part of the default timing and EAD uncertainty is called margin period of risk (MPOR) and we will elaborate on this later in specific CVA calculations. Often the default recovery procedure involves the default court, or an arbitration process.

Other factors that may affect the recovery of a defaulted asset include the potential legal risk as well as political risk. For example, the legally documented terms governing the derivatives transactions may not be strictly enforced in a local court, or the court may side with the defaulted party due to particular political environment.

3.2.2 Credit loss and credit exposures

When a counterparty owes money and default occurs, the surviving trading party will suffer a credit loss. The amount that the counterparty owes is the credit exposure that is subject to default loss, or exposure at default. The surviving party, in general, will have to go through the default recovery process to recoup the losses. Meanwhile, it will need to replace the original contracts with similar contracts in the market to avoid exposure to open market risk.

If the portfolio value between the two parties is negative to the surviving party, the surviving party will be able to continue to honor its contract obligations, and there will be no credit loss involved. The surviving party will be able to close the position by making the payment at the market value of the contracts, and enters in the market similar contracts at the same time for market risk purposes. Therefore what matters in credit default risk is the positive exposure (PE) that the defaulting party owes to the non-defaulting party, or

$$PE = Max\left(0, portfolio\, value\, to\, surving\, party\right) = V^{+}(surviving\, party).$$

Often positive exposure is also referred to as replacement cost.

For derivatives, the contract value is really the expectation of future contract values in all future scenarios, meaning different future scenarios having different future exposures. The Expected Exposure (EE) is the average of the exposure distributions at a future point in time, expressed as:

$$EE(t) = \mathbf{E}\left[V^{+}(\mathbf{M}(t))\right] = \int V^{+}(\mathbf{M}(t))\,\mathbf{P}(\mathbf{M}(\mathbf{t}))\,d\boldsymbol{M}$$

where $\boldsymbol{M}(t)$ stands for the market risk variables at time t, $\boldsymbol{P}(\boldsymbol{M}(t))$ is the joint probability density of the market risk variable distributions, and the positive portfolio value PE is the function of $\boldsymbol{M}(t)$.

When credit risk is measured, potential future exposure (PFE) is often used to define the potential risk that may happen in the future. PFE is defined as the stressed positive exposure expected to occur at a future date within a statistical confidence level, for example at 95%. PFE is often used for the purpose of defining trading risk limits in banks so that the credit risk is not overly concentrated on a specific counterparty. A 95% confidence level PFE is a level of potential exposure that is exceeded with less than 5% probability.

Example 1: A 10-year pay fixed 5% and receive floating swap, 5% current swap rate and 20% volatility.

When the interest rate goes up above 5%, the swap contract value will be positive. The future exposures would come from those scenarios with relevant swap rates above 6%. For example, the relevant swap rate at year 5 would be the 5-year swap rate. The future exposure would be the scenarios with 5-year swap rate above 5%. Given constant volatility of 20%, we have expected positive and negative exposures:

$$
\begin{aligned}
EPE(t) &= \int_{-\infty}^{\infty} \max(S(t) - 5\%, 0)\, A(t) P(S)\, dS \\
&= \int_{5\%}^{\infty} (\mathrm{S}(\mathrm{t}) - 5\%)\, A(t) P(S)\, dS = PayerSwaption\left(5\%, 5\%, 20\%, 5y\right) \\
ENE(t) &= \int_{-\infty}^{\infty} \max(5\% - S(t), 0)\, A(t) P(S)\, dS \\
&= \int_{-\infty}^{5\%} (5\% - \mathrm{S}(\mathrm{t}))\mathrm{A}(\mathrm{t}) P(S)\, dS = ReceiverSwaption\left(5\%, 5\%, 20\%, 5y\right)
\end{aligned}
$$

where $P(S)$ is the swap rate distribution, $A(t)$ is the swap accrue, PayerSwaption(S,K, ,T) and Receiver Swaption (S,K,σ,T) refer to the relevant payer and receiver swaption valuations. We have dropped some reference to variable t for convenience.

Figure 3.4 shows the EPE and ENE profile in dotted lines, along with the PFE profiles at 95% confidence. The expected exposure goes up as more swap rate variance kicks in, which reaches a maximum around 3–5 years, and then slowly goes down to zero at maturity.

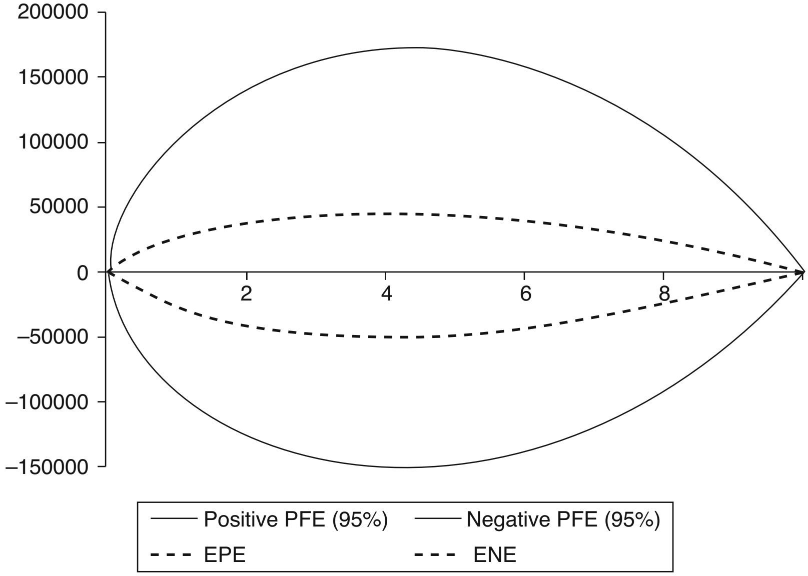

Figure 3.4 Example PFE and EPE, ENE for an interest rate swap

Example 2: A 10-year FX Forward contract with the following: spot X = 1.5, forward delivery strike K = 1.228, domestic interest rate $r_d = 3\%$, foreign interest rate $r_f = 5\%$, FX volatility $\sigma = 15\%$.

The expected positive exposure at 5y would be:

$$EPE(t) = \int_{-\infty}^{\infty} \max\left(X(t) - K, 0\right) P(X)\, dX$$
$$= \int_{K}^{\infty} (\mathrm{X(t)} - \mathrm{K})\, P(X)\, dX = FXOption\left(Call, X, K, \sigma, T, r_d, r_f\right)$$

and negative exposure:

$$ENE(t) = \int_{-\infty}^{\infty} \max\left(K - X(t), 0\right) P(X)\, dX$$
$$= \int_{K}^{\infty} (\mathrm{K} - \mathrm{X(t)})\, P(X)\, dX = FXOption\left(Put, X, K, \sigma, T, r_d, r_f\right)$$

Figure 3.5 shows the EPE, ENE and PFEs for the 10-year forward contract through time. The exposure increases monotonously as time goes to maturity, reaches maximum at maturity. This risk profile is distinctly different from the regular swaps: A regular swap reaches a maximum EPE in the middle before dropping to zero at maturity; while for FX forward transaction, it maxes out right at maturity. Similarly, a traditional cross currency swap with fixed notional exchange at the end would show large terminal credit exposure at the end. Therefore the large CVA, counterparty credit risk capital and CVA capital. These will be discussed later.

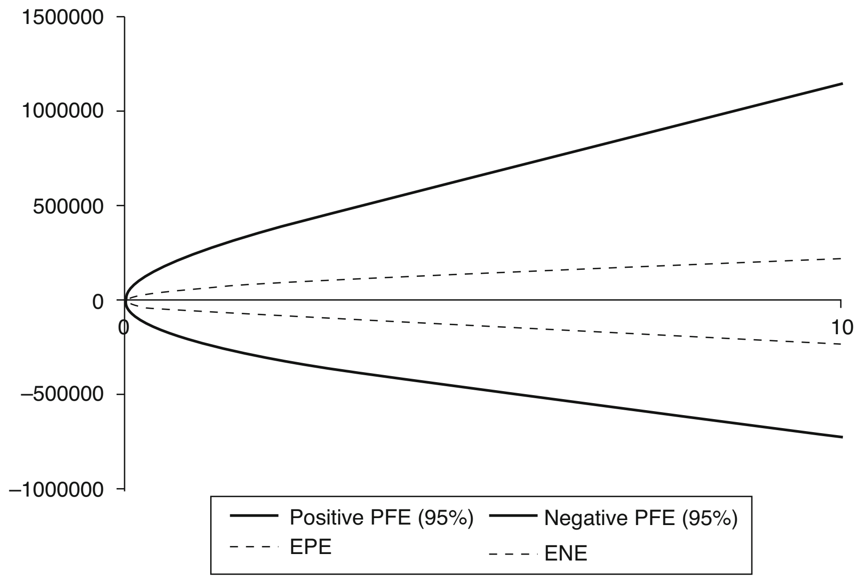

Figure 3.5 Example PFE and EPE, ENE for an FX forward contract

3.3 Credit value adjustment

3.3.1 Introducing CVA

Credit value adjustment (CVA) is the valuation adjustment for derivative contracts due to expected loss from future counterparty default. As CVA is defined within the fair valuation framework, it is regarded as fair market value given counterparty credit risk embedded in the derivative contracts. By definition, CVA is the loss resulting from counterparty default:

$$CVA = \mathbf{E}\left[Counterparty\ Default\ losses\right]$$

where $\mathbf{E}[]$ stands for expectations. Intuitively, one can also view CVA as the difference of portfolio value in a default risk-free setting and the default risky setting, where the counterparty's default is allowed:

$$CVA = Valuation(default - risky)\ Valuation(default\ risk\ free)$$

These two different views should be equivalent.[4]

We denote the firm's credit exposure to the counterparty at time t, with all netting and margin considered, as $V^{+}(\mathbf{M}(t))$. Assuming the counterparty's instantaneous default density is $H(t)$, and the loss given default is LGD, then the CVA would be:

$$CVA = LGD\int_0^T D(t)EE(t)H(t)dt = (1-R)\int_0^T D(t)\mathbf{E}\left[V^{+}\left(\mathbf{M}(t)\right)\right]H(t)dt$$

where D(t) is the discounting factor at time (t). The incremental default probability of a given time slice would be:

$$P_d\,(t \to t+dt) = H\,(t)\,dt$$

and the total default probability from time 0 to T would be:

$$P_d\,(0 \to T) = \int_0^T H\,(t)\,dt.$$

In the above CVA calculation, we have made the assumption that counterparty default is independent of the market risk exposure at default or EAD. When there is correlation between the market risk factors M(t) and the counterparty credit risk, we have a right-way or wrong-way risk exposure. Under such situations, one would have to integrate the credit risk and market risk together, meaning integration through their joint distribution:

$$CVA = (1-R)\int_0^T \left[\iint D(t)1_{\{S(t)<K\}}V^{+}\left(\mathbf{M}(t)\right)P^{J}\left(\mathbf{M}(t), S(t)\right)d\mathbf{M}dS\right]dt$$

where S(t) is the counterparty's credit default variable, $1_{\{S(t)<K\}}$ indicates the condition of counterparty default, where S(t) is less than default threshold K, and

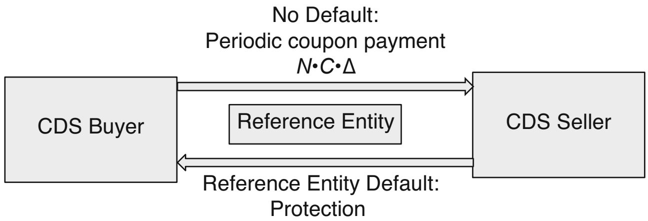

Figure 3.6 Illustration of a CDS transaction

$P^J(\mathbf{M}(t), S(t))$ denotes the joint probability density function for $\mathbf{M}$(t) and S(t). Market factor and credit factor correlation and wrong-way risk are discussed in later sections.

3.3.2 CDS spread, bond spread and credit default probability

To calculate CVA, the credit default probability of the derivative counterparties must be quantified. There are a few ways that firms use to derive probability of default (PD). In general, information can be extracted from the market-traded instruments and possibly hedging CVA exposure using the traded instruments. This leads us to the credit default swap (CDS) market.

A CDS is a credit derivatives contract between two counterparties. CDS started trading in the early 1990s and became very popular in the early 2000s. In a CDS transaction, the protection buyer makes periodic coupon payments, termed as CDS spread, to the seller; in return the seller provides a payoff contingent on the underlying financial instrument default or the occurrence of a credit event of the reference entity.

Figure 3.6 shows the basic mechanism of the CDS trade, where N is the notional amount, C is the CDS spread, Δ is the CDS payment frequency, $N \cdot C \cdot \Delta$ is the coupon payment. At the occurrence of the reference credit event, the CDS seller will fulfill the obligation to provide the credit protection. For example, if the reference entity is a bond, the CDS buyer may deliver the bond to the CDS seller and the CDS seller will pay the par value of the bond.

The size of CDS spread is one of the cleanest indicators for credit entity default probabilities as viewed by the market. Given market traded CDS spreads for different maturities, one can bootstrap to calculate the default probabilities for desired future times, creating a default probability curve. Much like the bootstrapping of any interest rate curves,[5] one can calculate the future default probability given the shorter-term default information and the corresponding CDS spread.

Another type of market-traded instrument is defaultable bond. When a credit entity issues debt in the form of bond, the bond investor will demand an extra premium on top of the risk-free rate:

$$\text{Bond Yield} = \text{Risk-free Rate} + \text{Bond spread}.$$

This extra premium, as indicated by bond spread, represents the credit risk of the credit entity for the specific bond with the specific seniority. The bond spread also contains the bond investor's funding cost, as well as the supply/demand side, or liquidity of the bond.

The extraction of default probability from the bond price is a more involved process than for CDS spreads. This is mainly due to three issues:

- The bond is regarded as funded credit, meaning the bond investor would have to fund the bond purchase. On the other hand, a CDS is unfunded, meaning it reflects the pure credit risk of the reference credit entity with no funding of principals.
- The bond, which has specific seniority attached to it, may not carry the exact same credit risk that corresponds to the derivative contracts. Bond and derivative contracts, with credit exposure to the same credit entity, may share the same default probability, but different recovery rate due to differences in seniorities. Therefore, the estimation of default probability may be skewed by the choice of recovery rates and their accuracy.
- Different bonds may carry different liquidity spread, which indicates the supply and demand side of the equation. When there is a lot of demand, the bond may even carry a negative liquidity spread.

Despite the difficulties above, it is still possible to extract good default information for the credit entity by carefully examining the specific bond.

Often the counterparty one faces in derivative transactions does not carry liquidly traded CDS spreads, or liquid bond offerings, from which to obtain reliable credit default information. Under such situations, one would need to rely on internal resources in arriving at credit default risk. For example, one may rely on internal credit analysts and officers to assign ratings for specific credit entity and arrive at the proper corresponding CDS curves. To make such practice generic and consistent, one may create internal rating CDS curves and pick the closest to use in credit default probability calculations. For example, one could choose to generate generic rating CDS curves by averaging the CDS spreads for entities with the same ratings.

3.3.3 Constructing a default probability curve

The default loss of an obligation depends on three quantities: EAD, PD of the credit entity, and the percentage loss expected at the event of default, or loss given default (LGD):

$$Credit\ Loss = EAD \cdot PD \cdot LGD.$$

The CDS spread defines the coupon payments contingent on the referenced credit entity being live. Therefore before calculating the default probability, one has to define LGD or the recovery rate (RR) for the credit entity, where RR = 1-LGD. There are different ways of modeling PD and RR, including structural models, pioneered by Merton's option model, and reduced form models.[6] This will be discussed in more detail in Chapter 6.

Given recovery rate R of the reference entity, one can bootstrap to construct the default probability curve from the market traded CDS term structure. Assuming we know the CDS spreads are:

$$s_i: \text{ CDS spread for maturity } T_i$$

and we would like to derive the cumulative default probabilities and default intensity:

$$P_d(i): \text{ Cumulative default probability for maturity } T_i$$

with the CDS spread s_i for a T_i maturity swap:

$$s_i = \frac{\mathbf{E}\left[(1-R)\int_0^{T_i} h(t)\, e^{-\int_0^t (r(s)+h(s))ds} dt\right]}{\mathbf{E}\left[\int_0^{T_i} e^{-\int_0^t (r(s)+h(s)ds} dt\right]}$$

The numerator is the protection leg value, which is equal to the discounted losses accumulated through time. $r(t)$ is the discounting interest rate, and $h(t)$ is the instantaneous default intensity. The quantity

$$Q(t) = e^{-\int_0^t h(s)\, ds}$$

gives the survival probability at time t, and

$$V_{premium} = s_i \mathbf{E}\left[\int_0^{T_i} e^{-\int_0^t (r(s)+h(s)ds} dt\right]$$

gives the premium leg value.

The bootstrapping process is usually done from the short term maturity, such as 6 month to 1 year, to a longer term of, for example, 30 years. Given an interpolation scheme, one can solve piece-wise default intensity h(t), which then leads to the cumulative default probabilities:

$$P_d(t) = 1 - Q(t) = 1 - e^{-\int_0^t h(s)\, ds}$$

and the zero coupon default spread would be:

$$s^{zero}(t) = -\frac{1}{t}\left[Q(t) + (1 - Q(t))\, R\right]$$

where $s^{zero}(t)$ is defined as:

$$e^{-s^{zero}(t)t} = Q \ln(t) + (1 - Q(t))\, R$$

$s^{zero}(t)$ can be used in CVA calculations directly through spread discounting. This will be discussed in Chapter 6.

3.3.4 Correlation of market risk exposure and credit default

Intuitively, the correlation between EAD, driven by market risk factors, and credit default of the counterparty can have important effect on the CVA calculations. For example, if EAD is positively correlated with counterparty default, the loss from default could be much higher than the zero correlation situation. On the other hand, if EAD is negatively correlated with counterparty default, CVA could be much less than

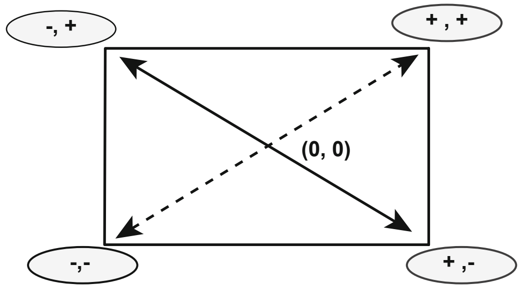

Figure 3.7 Illustration of wrong-way and right-way exposure

zero correlation CVA. The first situation, often termed wrong-way exposure (WWE) is something that one would be very cautious against, while the latter would be very welcome in practice as right-way exposure (RWE).

A classic example of wrong-way exposure is an emerging market (EM) company entering a foreign exchange forward contract with a bank, where the bank benefits when emerging market currency depreciates. This is what usually happens in real life: when the EM company goes into default with the emerging market in turmoil, the EM currency depreciates. Even though the bank's book carries a big profit from the face value of the FX forward contract, it will not be able to realize as the EM counterparty enters into default. On the other hand, the bank's hedge of this FX position would result in a big loss.

Another example is an oil derivative contract is entered into with an airline company, where dealers make money when oil prices goes up. Clearly, spiking oil prices may lead to an increase in the default probability of the airline company, and less probability for the dealers to realize the profit. And hence the wrong-way risk.

Figure 3.7 shows schematically the WWE and RWE situations. The symbols "+" and "–" shows the market factor exposure and counterparty default situation. The top left corner and lower right corner show the wrong-way risk situations, where the market factor exposure and counterparty default are negatively correlated. When the market exposure is low, the counterparty is in good health; while when market exposure increases, the counterparty's credit situation deteriorates, resulting in wrong-way risk. On the other hand, the top right corner and lower left corner indicate the RWE situations.

Another situation where one may be affected by correlations in a default situation is the wrong-way collateral. For example, a company may choose to use a security that is tied to the health of the company or industry as derivative collateral. Or an emerging market counterparty is using native currency in posting collateral. Under such situations, the collateral could be worth much less when the counterparty goes into default, which would then result in additional loss because of collateral damage.

Sometimes, a large derivative portfolio with a counterparty may also lead to the increase in counterparty liability size, which directly impacts the counterparty's default. For example, deterioration of market conditions combined with the substantial increase in liability from the derivative contract may lead to counterparty rating downgrades and the accompanying value drop as well as potential collateral calls may directly drive the counterparty into default. This is arguably a liquidity risk issue coupled with the counterparty credit risk; however, it may also be modeled approximately as an increase of wrong-way correlation in the event of deterioration of market risk condition.

Example 1: Wrong-Way Risk

We use an FX forward transaction between a fictitious currency CCY against USD in our example. The firm enters a 1 billion CCY 1Y sell CCY buy USD FX forward transaction with a CCY native counterparty. When we make money from the FX contract, and CCY economy is in trouble, the counterparty is more likely to default; alternatively, if the counterparty defaults, there is more chance that it is due to the deteriorating CCY market environment and it may also lead to the deterioration market's confidence in CCY, and CCY will devalue.

Assuming an FX exchange rate 1 USD = 10 CCY, CCY 1 year interest rate = 6%, and USD 1 year interest rate = 2%, FX forward rate = 10.62 from interest rate parity. The forward contract would be entered at strike = 10.62 CCY/USD. FX volatility = 20%, counterparty default probability = 2% per year with 50% LGD.

The CVA with no wrong-way risk would be equal to the expected positive exposure multiplied by the default probability and LGD. In the example, EPE would be the FX option value with strike at forward rate.

$$
\begin{aligned}
EPE &= Notional \times Black\text{–}Scholes\left(Call, Spot, Strike, 20\%, 1\,year, 6\%, 2\%\right) \\
&= 736{,}532\ USD \\
CVA &= EPE \times P_d \times LGD = 736{,}532\ USD \times 2\% \times 50\% = 7{,}365\ USD
\end{aligned}
$$

When wrong-way risk is present, the exposure at default would be much higher than EPE. While the modeling of wrong-way risk exposure would in general involve a more complex modeling and simulation of market as well as credit factors, we will simply estimate the impact here.

If the default of the counterparty leads to a 5% devaluation in the CCY currency value, or the devaluation of CCY by 5% leads to the default of the counterparty, the EPE would change from 736K USD to 4.6 million USD, which leads to a CVA around 46K USD. This is a more than sixfold increase in CVA.

Example 2: Wrong-Way Collateral

The firm enters a derivative contract with a counterparty, which is fully collateralized in currency CCY. Under normal situations, the firm would have no credit default risk exposure with the counterparty since the exposure is fully collateralized. Assume the derivative contract value is \$100 million, and it is collateralized by \$100 million equivalent CCY collateral. If the counterparty defaults without affecting the CCY exchange rate, the firm would be able to liquidate the \$100 million collateral and would not have to take any loss. However, if the sudden CCY economic deterioration leads to a 5% devaluation of CCY currency, and also leads to the default of the firm's counterparty, then the \$100 million CCY collateral will be devalued by \$5 million. This means EAD would be \$2.5 million for a 50% LGD.

3.3.5 CSA and credit terms affecting CVA calculations

The CVA calculation methodology and detailed implementations will be covered in Chapter 6. In this section, we will discuss briefly the main CSA and credit terms that affect CVA calculations.

Collateral Threshold and Minimum Transfer Amount (MTA)

Collateral threshold defines the amount below which the borrower would not have to post collateral. When the liability or borrowed amount exceeds the collateral threshold, the borrower would have to post collateral for the amount exceeding the threshold. Therefore, collateral threshold is really the maximum exposure one may be exposed to in the event of counterparty default. The lower the threshold, the lower the CVA.

Often the collateral thresholds defined in a CSA are dependent on the trading entities' credit ratings. The lower the credit rating, the lower the collateral threshold. This intuitively makes sense for practical purposes. A lower threshold is preferable when the counterparty's rating is downgraded, so that the credit default risk is mitigated. However, when the counterparty is maintaining excellent ratings, one would be open to a higher credit exposure.

The ratings-based thresholds were very popular in the bilateral CSAs. While it is practical, it introduces rating dependency and extra complexity in CVA calculations. This will be discussed in more detail in Chapter 6.

MTA may also contribute to CVA valuation when it is significant. While MTA is designed to reduce operational costs by reducing the number of collateral calls, it increases the credit exposure at the time of default. From the expectation point of view, MTA behaves like a collateral threshold approximately half of the MTA size.[7]

Cure period and Margin Period of Risk (MPOR)

In practice, collateral calls are operated on a periodic basis, such as every day. In the event of default, a period of time would be needed to determine the actual default event; afterwards, the surviving party would have to close out the defaulting positions and hedge the resulting market risk or find a replacement. The time it takes to close out the portfolio position and re-hedge the market risk is called the cure period, while the period from most recent collateral exchange to the position close-out is called the margin period of risk. The margin period of risk is commonly assumed to be 10 to 25 business days. For example, the European Banking Authority (EBA)'s standard MPOR floor ranges from 10 to 20 business days depending on the liquidity of the trades and the collateral in the netting set.[8]

The uncertainty in determining the actual default event, and the time it takes to close-out position would potentially affect the final default exposure, thus impacting the CVA calculations. When the exposure is negatively correlated with the counterparty default, this effect can be significant, much like the situation of wrong-way risk and wrong-way collateral. In addition, as the market may move significantly during this period, PFE may increase substantially, which then contributes to the counterparty credit risk capital calculations.

3.3.6 Bilateral CVA: DVA

When two credit entities enter into a trade with each other, there will be an implied borrowing/lending relationship: one entity will owe the other entity. One entity's asset will be another entity's liability. When you carry an asset with potential risk of counterparty default, one would have to adjust the valuation of the asset by CVA. On the other hand, the borrowing party's book would carry a liability. By the argument of symmetry, the liability will need to be adjusted for the borrowing party's own credit risk, in the direction of benefiting decreasing liability. This adjustment, on the opposite side of CVA, is termed debt value adjustment, or DVA.

As a simple example, entity A trades a derivative with entity B, with the effect of A lending to B. The effect of the trade will create an asset on A's book and a liability on B's book. The value of the trade will be, in general, marked at:

$$FairValue = RiskfreeValue + adjustment\,(credit\ risk).$$

Specifically,

$$ValueA = ValueA(credit\ risk\ f\,ree) + CVA(B\text{'s}\ default\ risk)$$
$$ValueB = -Value(A) = ValueB(credit\ risk\ free) + DVA(B\text{'s}\ credit\ risk)$$
$$CVA(B\text{'s}\ credit\ risk) = -DVA(B\text{'s}\ credit\ risk).$$

Trading with the above economics creates an ideal situation and the accounting principal is retained: there is no phantom value created in the trading process. We may call this a traditional "one price" regime, as the market would agree on the same pricing of credit risk and trade on the price. Therefore the "one price" will be the "market price" or "exit price".

Another way to understand the DVA benefit is that, effectively, B has bought a CDS protection against its own default. In the event of default, B does not have the obligation to pay back its debt. A would be able to buy back the CDS in the market from other market participants, who are willing to provide B's default protection. However, B would not be able to sell the CDS that he bought from A to the market. This indeed creates the dilemma in accounting for the DVA's value.

3.3.7 DVA: the dilemma

Ever since its introduction, DVA has always been controversial among professionals in academics and the financial industry. The credit crisis around 2008 also witnessed substantial gains and losses from banks due to DVA accounting, which also brought the regulators into the discussion.

The controversy around DVA comes mainly from one simple practical point: one cannot monetize DVA, the same way as one does with CVAs. DVA can be viewed as owning a CDS protection against its own default. The perfect hedging for DVA would be for the bank to sell its own CDS protection back to the market, which would make DVA as tradable for all market participants. However, this cannot be workable practically. While a firm can buy or sell counterparties' CDS spreads, it cannot sell its own CDS, or provide protection on its own default. The reason is quite obvious: the firm would not be able to provide the protection when the CDS buyer needs it. When the firm is in default, the CDS buyer will not be provided the proper protection for the firm's default loss. This would be the perfect wrong-way exposure with 100% correlation between the protection provider and the reference name.

The root of this issue comes from the fact that DVA originates from the concept of default loss of self-entity. Being the symmetric quantity of CVA from default loss, DVA does not represent the firm's own economics. If one can enjoy the gain from his own default, he would be able to sell the gain to the market. Unfortunately, this is not the case. When a firm is in default, it is broke.

This creates a tremendous problem for the financial institutions, which would not be able to hedge their DVAs properly. Inevitably, the fluctuation of the firm's own CDS spread would introduce substantial P&L from DVA change. In some cases, it could be hundreds of millions to even a few billion dollars for large banks.[9] Without being able to replicate the DVA economics, banks are left with finding proxies to hedge their own default, carrying substantial spread risks afterwards.

One may argue that the firm may use its own bond to hedge the DVA approximately. Because a firm-issued bond represents the firm's credit risk, buying back a previously issued bond would have the same effect as selling self-default protection in the market. This would effectively serve the purpose of hedging DVA. On the other hand, issuing a bond would provide the other side of the equation: buying protection of self-default. Therefore, theoretically buying and selling bonds may serve the purpose in hedging DVA, at least approximately. To make it more accurate, one could consider buying and selling bonds with different seniorities, effectively creating hierarchies of instruments that may continuously hedge the DVA.

This bond-hedging strategy is missing two critical points, among other issues:

1. It is not practical to dynamically buy and sell self-issued bonds in the market. The bond issuance and bond buyback is not a simple process like buying and selling treasury bonds, but DVA is something that fluctuates dynamically with the market on a daily basis. One could plan bond issuance and buyback on a much longer term basis, not on a timescale that one can use in a dynamic hedging process.
2. When the bond price is low, it means the firm's credit is not good. That is exactly the time when the firm would need to beef up its liquidity and capital base, and exactly the time when the firm needs funding. It is not the time for the firm to buy back the bond. On the other hand, when the bond price is high and the firm's credit is excellent, that is the time when the firm may not need extra funding. Bond issuance and buyback are not the same as trading, and cannot be treated like trading with dynamic replication.

Accounting-wise, investment accounting for bond issuance and DVA accounting based on fair value also makes it harder to align the hedges together in practice. For these reasons, one cannot rely on the bond-hedging strategy to dynamically hedge DVA, or replicate the value of DVA. Occasionally one may employ such strategy to mitigate the DVA fluctuations, but definitely not on a scale that is equivalent to dynamic hedging of fair derivative valuations. As a matter of fact, the selective use of self-issued bond strategy is mostly used as a tool in funding management, which advocates the use of funding benefit, rather than DVA as the proper economic term.

DVA represents the value that the firm does not have to pay if self-default happens. In the defaulting process, the firm would not gain anything, meaning it does not have the same real value as cash. Therefore, the firm would not be able to utilize DVA the same way as cash in business activities, such as collateral posting or dividend payment. This is different from the normal derivatives pricing and accounting: the mark to market value of a derivative transaction carried on the balance sheet is regarded as real value to the firm, which can be accounted for in firm valuations and accounting. It can be included in capital, and the dividend payment calculations. DVA will not be.

In the Basel III ruling, we have the following:

> Derecognize in the calculation of Common Equity Tier 1, all unrealized gains and losses that have resulted from changes in the fair value of liabilities that are due to changes in the bank's own credit risk. In addition, with regard to derivative liabilities, derecognize all accounting valuation adjustments arising from the bank's own credit risk. The offsetting between valuation adjustments arising from bank's own credit risk and those arising from its counterparties' credit risk is not allowed.

This clearly states that DVA is not an economic term that is recognized. One should also note that the offsetting of liabilities and assets are allowed if they are coming from the trades with the same counterparty; this is because the asset and liability exposures within the same netting set occurring at the same time point would offset each other: DVA and CVA are essentially the expected losses from the netted liabilities and assets, due to self-entity and counterparty defaults respectively. DVA and CVA from different trades and different counterparties are not allowed to offset for obvious reasons.

More discussions on DVA will be introduced in later chapters when funding benefit is introduced.

3.4 CVA examples

3.4.1 An interest rate swap

Here we use a simple payer interest rate swap example to demonstrate the CVA calculations.

Assuming that interest rate and default behavior are not correlated, CVA for a swap can be calculated semi-analytically through integration. The potential positive exposure (PE) for a paying fixed K and receiving floating swap at time t is a swaption:

$$PE_{pay}(t) = N \int_{-\infty}^{\infty} \max(S(t,T) - K, 0) A(t,T) P(S(t,T)) dS = Payer(K,t,T)$$

where Payer(K,t) is the payer swaption value with expiry t and strike K, N is the notional, S(t,T) is the swap rate for time t with maturity T, K is the fixed rate, P(S) is the probability distribution density function for S, and A(t,T) is the discount accrual for the swap,

$$A(t,T) = \sum_{i=1}^{n} D_i \delta_i$$

with δ_i being the payment period length, D_i being the discounting factor for i-th payment. The CVA for the swap would be:

$$CVA_{pay} = \int_0^T Payer(K,t,T)P_d(t)(1-R(t))dt \approx \sum_i^n Payer_i(K)p_d^i(1-R_i)\Delta t_i$$

where R(t) is the recovery rate at time t, and $P_d(t)$is the marginal default intensity at time t, and the second approximate equality with sum over i to n is for chosen discrete time periods. Certainly the payer swaption value should depend on the swaption volatility, which we have dropped to simplify notation.

One quick observation is that the higher the payer swaption value, the higher the CVA. While the swaption exposure is driven by the interest rate and volatility, CVA for the payer swap will increase with rate and volatility.

One can extend the calculation to the situation with collateral threshold H. Given collateral threshold H, the maximum exposure to counterparty is limited by H, since one would need to post the owed amount above H. Hence the potential exposure is reduced by the part above H:

$$PE_{pay}(H,t,T) = Payer(K,t,T) - Payer(K',t,T)$$

$$K' = K + \frac{H}{A(t,T)N}$$

where H is the collateral threshold, K' is the strike that incremental swap value from K to K' will be equal to the collateral level. The CVA with collateral is then:

$$\begin{aligned} CVA_{pay}(K,H) &= \int_0^T (Payer(K,t,T) - Payer(K',t,T)P_d(t)(1-R(t)))dt \\ &\approx \sum_i^n [payer_i(K) - payer_i(K')]p_d^i(1-R_i)\Delta t_i. \end{aligned}$$

Practically, the sum can be done over the swap period start dates.

3.4.2 An FX forward example

The FX forward is a contract for the promise of the exchange the currency at a specified rate X at future maturity T. Assuming the spot between currencies A and B at current time is X_0:

$$1A = X_0 B$$

The interest rate for A and B are r_A and r_B, the forward FX rate at T, from interest rate parity, would be:

$$X_T = X_0 e^{(r_B - r_A)T}$$

The potential positive exposure for the FX forward at time t would be an FX option:

$$PE_{FWD}(t) = N \int_{-\infty}^{\infty} \max(X(t,T)-K,0)P(X(t,T))dx = \frac{1}{D(t,T)} FXO(k,t,T)$$

where FXO(K,t,T) stands for the FX option value with notional N, strike K, expiry t and for FX forward with maturity T; P(X) is the distribution density for FX forward X, D(t,T) is the discounting from t to T.

The CVA for the FX forward transaction would be:

$$CVA_{FWD} = \int_0^T FXO(K,t,T)P_d(t)(1-R(t))dt \approx \sum_i^n FXO(K,t,T)p_d^i(1-R_i)\Delta t_i.$$

The integration can be done numerically over a discrete set of time points.

Similar to the interest rate swap example, a formula can be derived for the situation when collateral threshold H is present. We will leave to the reader to derive.

4 FVA Primer: Derivatives Pricing with Funding

4.1 The funding market

Borrowing and lending activities are very common in normal business dealings. A business' operation inevitably will involve borrowing: in managing its daily activities, such as purchasing raw materials and equipment, making payments, and expanding business dealings. For a financial institution, the borrowing activity would be even more important, in the sense that financial institutions are generally more leveraged than, for example, industrial companies. On top of borrowing, banks also lend out to customers, trying to make money from the "spreads" that customers pay for their loans and borrowings. In addition, banks also borrow and lend among themselves, forming an interbank market, or wholesale lending market.

Figure 4.1 shows the simplified borrowing/lending market: sitting in the middle is the central bank, which is in control of the short-term borrowing/lending rate by utilizing the interest rate policy tools. It also regulates the banks around it by setting reserve and capital standards. The banks, by lending to a variety of customers, form the general borrowing and lending financial market. In particular, derivative contracts transacted among banks and customers may create a variety of funding situations for them. Because fundings are explicit real economics for the trading entities, banks would have to take them into account properly in derivative valuations.

Trading desks and banks have long recognized funding cost as an off-the-record cost item. It was never brought into the spotlight because funding cost is relatively small compared to the returns from businesses. Funding was relatively easy to obtain as there were different means available for the banks to fund themselves. For example, the existence of structural investment vehicles would also allow the banks to move investments from the balance sheet into some cheap funding. All these funding channels froze after the Lehman Brothers' default. When the market liquidity dried up and the short-term borrowing cost spiked, the funding cost of uncollateralized derivatives became evident. Because funding cost is becoming persistently material to operations of banks, and more stringent regulatory requirements are becoming a reality, adjusting derivative valuations with funding cost has become more important than ever. In this chapter, we will discuss the fundamentals and concepts that lead to the definition and calculations of FVA. However, before discussing the details of FVA, it is necessary to briefly touch upon financial institutions' balance sheet, funding and derivatives operations, as well as the derivative dealer's business model.

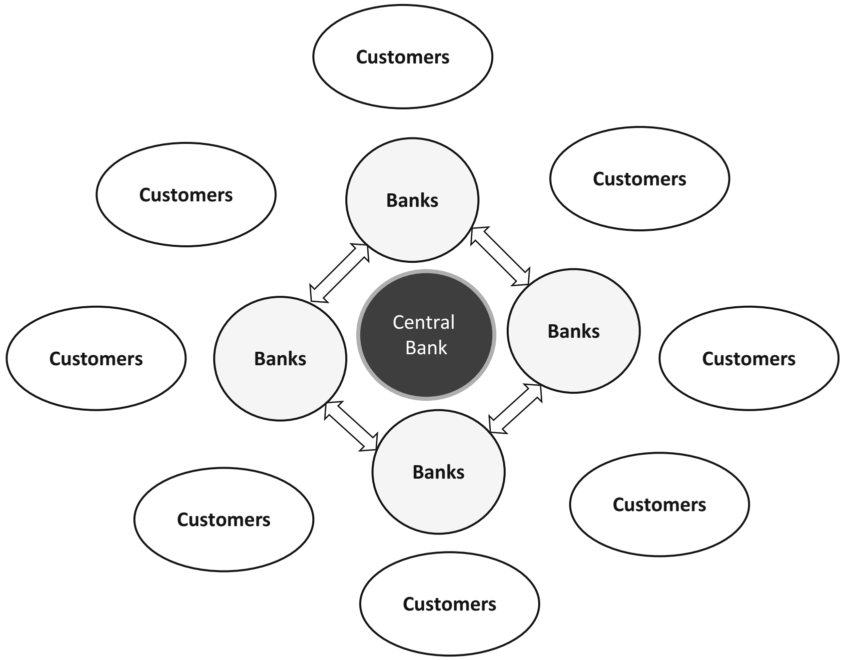

Figure 4.1 The simplified funding market

4.1.1 Financial institutions and the balance sheet

Financial institutions are generally leveraged in that they employ precious capital to purchase assets and investments in the amount that may well exceed its capital. In doing so, they have to borrow money through a variety of funding activities. This can be seen clearly from their balance sheets, which show firms' assets and liabilities.

Figure 4.2 shows a schematic diagram of a financial institution's balance sheet. On the left are the assets, including cash, securities, businesses and investments; on the right are liabilities, including general debt issues and equity. Given the leverage nature of the balance sheet, the servicing of debt issues and the cost associated with it is a significant issue around a bank's overall management. With the leveraging nature of a bank's balance sheets, they are prone to the market's perception of credit evaluations. Under extreme situations, a "run" on the bank may happen, which could quickly lead to a bank's default. This is exactly why regulators are emphasizing the capitalization of banks, to ensure banks are well protected under liquidity stress situations.

The funding management within a firm is generally centralized on a funding desk within the treasury department. Given the funding requirement from all business lines, the treasury department may raise funding through different funding channels. Meanwhile, the cost of the funding process will be passed to the business lines through a centralized funding policy. The funding policy can be a term structure of funding spreads, representing different funding cost for assets with different terms. The presumption of a funding policy is that there is a well-defined equilibrium funding cost related to the funding activity, reflecting the supply and demand of funds, liquidity

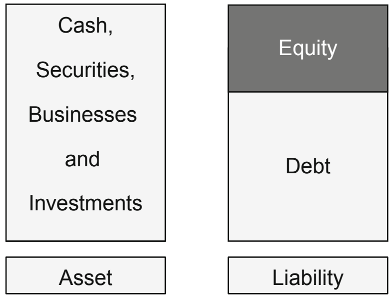

Figure 4.2 A firm's balance sheet: Borrowing and leverage of financial institutions

of the market and credit of the firm. It should be noted that a proper funding curve, which defines the proper funding policy, is needed to ensure proper incentives are enforced in passing the cost/benefits to the line of businesses. Without a proper funding policy, one may generate arbitrage opportunities for individual business lines. This could result in accumulation of trades or business dealings that may result in losses due to funding cost in the future, or as a cost to the whole firm.

4.1.2 Dealer's business model and funding of derivatives

Before discussing FVA, we shall revisit the business model of derivatives dealers.

Figure 4.3 shows the general derivatives business setup for a typical dealer. Dealers transact with the clients and meet their needs by providing various derivatives products; on the other hand, dealers hedge the customer portfolios in the broker and exchange traded markets to make themselves market risk neutral. The customer transactions in the bilateral market are usually uncollateralized or partially-collateralized, including some rating contingent collateralizations; meanwhile the broker and exchange traded markets are mostly fully-collateralized. The asymmetric setup of collateralization in customer and broker/exchange traded markets leads to general funding requirements for the overall derivatives business: if the customer owes money to the dealer, dealer has to fund the money from treasury/funding desk. In this process, the dealer will incur a funding cost.

For example, assume (D) and (C) are trading a swap with bilateral CSA and (C) is not posting collateral. In the swap, (D) pays fixed leg and receives floating leg. The hedging swap (D) enters with (H) will be the opposite direction:

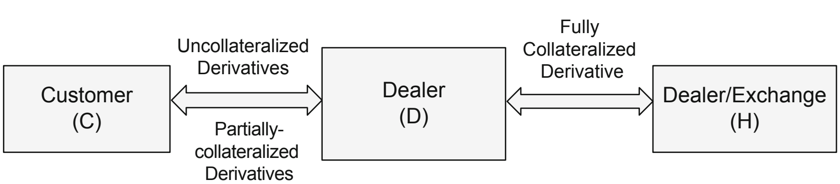

Figure 4.3 Asymmetric funding for dealer's bilateral derivatives business and hedges

- When the interest rate goes up, (C) will owe money to (D), and (D) will owe money to (H). Since (D) and (H) are fully collateralized, (D) will post collateral to (H); however, (C) will not post collateral to (D). Under this situation, (D) will have to fund the amount that is owed by (C).
- When the interest rate goes down, (D) will owe money to (C), and (H) will owe money to (D). In this case, (H) will post collateral to (D), while (D) does not have to post collateral to (C). Therefore (D) is left with some positive funding.

The above shows that different funding scenarios arise when the market moves in different directions. From an overall business point of view, the nature of dealers' customer business would create funding situations that dealers need to manage. This is where FVA comes in. FVA is the proper way to account for the funding of uncollateralized derivatives, which can be done on individual business level, or to much more granularity on a trade-by-trade level.

While regulators have been pushing for more transparency and more standardization in derivatives trading, more and more simple vanilla derivatives are being pushed onto the "exchange" with central counterparties (CCP). This does reduce the funding requirement for dealers in servicing the customer side; however, there are still plenty of derivative transactions that have to be left on the bilateral side. One type of such bilateral trade is the more complex derivatives, including cross currency swaps, some more complex swaps, and various callable option products. Another type of bilateral trade that is not centrally cleared is that where derivative users are exempt from collateral posting. Still, for the foreseeable future to come, FVA will remain a critical issue for dealers in managing the economics of derivative transactions.

4.2 Collateralization, value of money and differential discounting

The collateralizations of derivative contracts are specified in CSAs. The types of collateralization situations may include:

- Fully collateralized
- Uncollateralized
- Partially collateralized.

The partial collateralization occurs when a collateral threshold applies. A minimum transfer amount would also create a partial collateralization situation.

The collaterals that are specified in CSA may include:

- Cash
- Securities and assets
- Choices of cash and assets in one or multiple currencies.

When derivative contracts are collateralized, the valuation of future cash flow can be done through differential discounting methodology, meaning that one may resort to different funding curves for different collaterals. For example, when derivative contracts are collateralized with cash, the discounting of future cash flow is well-defined as an overnight index rate. When securities and assets are used as collateral, the values of securities are linked to cash through the repo market if it

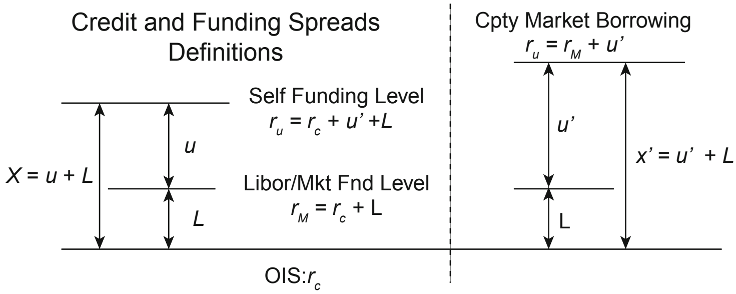

Figure 4.4 Credit and funding spreads definitions

exists, or the funding value realized in the rehypothecation process. When multiple currencies and assets are eligible as collateral, it is more complicated. In general, one may construct a funding curve reflecting the cheapest to deliver collateral curve in discounting future cash flows. To be more accurate, one may attempt to price collateral choice options; however, that consideration has practical limitations.

In the following, we will discuss the borrowing/lending formulations from a simple setup, then we will discuss in more detail the collateralized situations and differential discounting. Afterwards, we will consider uncollateralized situations and funding value adjustment.

4.2.1 Credit spreads, funding spreads and notations

Different components of spreads for a financial institution and its borrowing/lending counterparty are shown in Figure 4.4. A set of definitions for different credit and funding components of spreads are also introduced here. It is worth pointing out that different definitions of spread components exist in the literature. This particular approach is chosen as we feel it provides a way to separate different components in a clean manner and facilitate the following discussion.

Here r_c is the OIS rate, which is selected as the base rate, L is the generic market funding spread defined as the difference between Libor and OIS, and u and u' are the firm's and counterparty's specific funding spread on top of the generic market liquidity premium L. In an ideal world, one would probably expect the specific funding level of a firm be close to its credit spread level. In reality, specific funding spread of a firm can deviate from its credit spread due to the availability of different funding channels.

Before proceeding further, we introduce the following notations that will be used consistently through the remainder of this book to facilitate discussion. For simplicity, quantities with ' are used to indicate the properties of the counterparty. In addition, we use scalar representation for all quantities, which can be easily extended to vectors to represent term structure curves.

SF	Self entity: the financial entity being considered
CP	SF's trading counterparty in derivative trading
r_c	OIS, or "risk-free" rate.
r_u	Funding rate for SF. $r_u = r_c + u + L$.

u	Credit spread for SF
u'	Credit spread for CP
L	Market funding Spread
r_M	Market funding level. $r_M = r_c + L$.
X	Unsecured funding spread for SF. X=u+L
X′	Unsecured funding cost for CP
P_d	Probability of default for SF
$P_d^{'}$	Probability of default for SF
R	Recovery rate in the case of SF default
R′	Recovery rate in the case of CP default
H	Collateral threshold above which SF has to post collateral to CP
H′	Collateral threshold above which CP has to post collateral to SF

4.2.2 The value of collateral

In Figure 4.5, we show the simplified internal funding situation, where the firm's funding desk serves as a pass-through for the line of business (LOB) in funding activities. In a more general sense, the funding desk executes the firm's funding policy established by the internal treasury department, so that it can achieve the overall funding goals for the firm.

When an LOB has funding needs, it will borrow or lend money to the funding desk, where the funding desk will pay or receive OIS+X as interest for the cash borrowed or lent by the LOB. Here, X is the extra funding spread that the LOB has to pay in interest for the funding received. When an LOB has extra cash, the funding desk will also pay the extra spread back to the LOB. On the other hand, the funding desk will go to the market to achieve its overall funding goals by borrowing and lending with the market lenders and borrowers. In the overall funding management process, the funding desk will be borrowing/lending to other LOBs and the market effectively. The funding desk is acting as an agent to transfer the market pricing to the LOBs on a firm-wide basis.

Given the simple model above, it is easy to see the value of collateral in derivative transactions. In Figure 4.6, we have:

- Counterparty owes value Y to LOB, so it posts collateral Y in cash to LOB.
- In return counterparty receives OIS as interest on the cash collateral posted.
- LOB would give the collateral to Funding Desk, for which LOB would receive OIS+X as interest.

On a net basis, the LOB would make extra funding spread X from the cash collateral received from the counterparty. This is exactly the value of collateral.

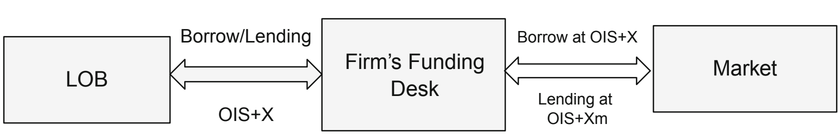

Figure 4.5 Transfer pricing of the funding desk

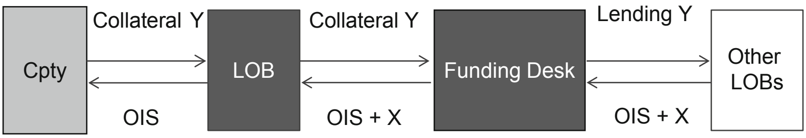

Figure 4.6 The value of collateral

On the opposite side, if the LOB has to post cash collateral to the counterparty, one would need to borrow money from the funding desk, which will charge OIS+X for the cash borrowed. While the LOB receives OIS from the counterparty on the collateral posted, one would be paying extra funding spread X on a net basis. As to the funding desk, it will be borrowing and lending to other LOBs. In the example above, there would be other LOBs borrowing the collateral in the amount Y and paying the funding desk OIS+X.

The above funding example is for cash collaterals. As shown in later sections, the collateral value for non-cash collaterals, such as different collateral assets, would be dependent on the relative funding values of the specific asset, which may be derived from the repo market or cross currency markets.

4.2.3 The value of money: lending to the counterparty

In this section, we show a few cases of borrowing and lending under different situations. Through specific detailed dissection of the borrowing and lending process, we show the costs involved and how proper discounting of the derivatives contract cash flows under different situations.

First we will consider the simple case of lending to counterparty. In all cases below, (Lu & Juan, Credit Value Adjustment and Funding Value Adjustment All Together, 2011) the counterparty will borrow \$1 at t=0 and return money back at t=T. From the replication mechanism, we will derive the value that the counterparty has to pay back at T in order to be fair. Continuous compounding is used as the convention for interest rates.

Case 1

Lending \$1 to counterparty, fully collateralized. We start with 0, borrow \$1 from funding desk, lend to counterparty and receive \$1 as collateral. The following shows all the relevant accounts separately:

	t=0		t=T	
Value of asset:	\$1	————	?	(a)
Collateral to return:	−\$1	————	\$-$e^{r_c T}$	(b)
Put Collateral in funding desk:	\$1	————	\$$e^{r_u T}$	(c)
Borrow \$1 from funding desk:	−\$1	————	\$-$e^{r_u T}$	(d)

Case 1 Continued

where '?' represents the amount that borrower needs to return in order to be fair, (a) represents the \$1 lent to counterparty, (b) is the collateral that we need to return to counterparty along with interest r_c, (c) is that one can put the collateral into work by depositing into funding desk, which would become $e^{r_u T}$ at T, (d) is the money borrowed from funding desk.

From (b), (c), (d), we arrive at the money that counterparty needs to return at maturity:

$$? = e^{r_c T} - e^{r_u T} + e^{r_u T} = e^{r_c T}$$

which means that the interest to be charged is r_c, or the discounting rate for collateralized cash flow is r_c.

Alternatively, one can think of (b) and (c) as the value of owning collateral:

$$V_c^{t=T} = e^{r_u T} - e^{r_c T} \text{ or } V_c^{t=0} = e^{(r_u - r_c)T} - 1$$

and for a small time increment dt:

$$dV_c = (r_u - r_c)dt.$$

This means the received cash collateral would make a net interest spread $X = r_u - r_c$.

Conclusion: Cash collateralized lending should imply a fair interest at the general collateral rate r_c. Equivalently the future cash flow under cash collateralization should be discounted using general collateral rate r_c. In addition, having cash collateral would imply extra funding value in the interest spread $X = r_u - r_c$.

Case 2

Lending \$1 to counterparty, partially collateralized in the amount of x and no credit default considerations. The following shows all the relevant accounts separately:

	t=0		t=T	
Value of asset:	\$1	————	?	(a)
Collateral to return:	$-$\$x	————	\$-x$e^{r_c T}$	(b)
Put Collateral in funding desk:	\$x	————	\$x$e^{r_u T}$	(c)
Borrow \$1 from funding desk:	$-$\$1	————	\$-$e^{r_u T}$	(d)

Under this situation, only part of the borrowing x is collateralized. So the returned collateral would be portion of the collateral with interest and the collateral put in funding desk would also be from the same partial collateral x.

Case 2 Continued

From (b), (c), (d), we arrive at the money that counterparty needs to return at maturity:

$$? = xe^{r_c T} - xe^{r_u T} + e^{r_u T} = xe^{r_c T} + (1-x)e^{r_u T}$$

which means that the growth rate for collateralized part x is r_c, the growth rate for non-collateralized part (1−x) is r_u.

Conclusion: Uncollateralized lending would require a fair interest payment with $r_u = r_c + X$. Comparing to the collateralized lending, one has to pay for lender's funding spread X to compensate for lender's cost.

Case 3

Same as Case 2, but with credit default considerations, i.e. counterparty default. The following shows all the relevant accounts separately:

	t=0		t=T	
Value of asset:	\$1	———	?	(a)
Collateral to return:	−\$x	———	$\$-xe^{r_c T}$	(b)
Put Collateral in funding desk:	\$x	———	$\$xe^{r_u T}$	(c)
Borrow \$1 from funding desk:	−\$1	———	$\$-e^{r_u T}$	(d)
Lost from counterparty default:			$-(1-x)e^{r_u T}(1-R')P'_d$	(e)

where (e) gives the loss of value from counterparty default and P_d' is the probability of default of counterparty. The default can also be treated through extra credit discounting:

$$e^{-u' T} = 1 - (1-R')P'_d \qquad (e')$$

From (b), (c), (d), (e), (e′), we arrive at the money that counterparty needs to return at maturity:

$$? = xe^{r_c T} + (1-x)e^{(r_u + u')T}$$

which means that the growth rate for collateralized part x is r_c, the growth rate for non-collateralized part (1−x) with counterparty credit is $r_u + u'$.

To summarize the above, collateralized value should carry growth rate of r_c and unsecured value should carry growth rate of $r_u + u'$. For a small time increment, \$1 money with x collateral will grow as:

$$dV = xr_c dt + (1-x)(r_u + u')dt.$$

Conclusion: Uncollateralized lending would require two sources of compensation, one for lender's own funding rate r_u, one for borrower's default risk u'.

4.2.4 The value of money: borrowing money

In the case of borrowing money from the counterparty, the same replication mechanism can be used to derive the fair amount that we have to return to the counterparty at maturity. It is easy to confirm that borrowing under full collateral would carry interest of r_c. The non-collateralized value would carry r_u if only funding is considered.

Case 4

In a case where both funding and self-default is included, it is a little bit trickier. It is arguable that the non-collateralized borrowings would carry r_u as shown below in the relevant accounts:

	t=0		t=T	
Value of asset:	\$−1	———	?	(a)
Collateral to return:	\$x	———	\$x$e^{r_c T}$	(b)
Borrow Collateral in funding desk:	\$−x	———	\$−x$e^{r_u T}$	(c)
\$1 premium saved in funding desk:	\$1	———	\$$e^{r_u T}$	(d)
Gain from self-default:			(1−x)$e^{r_u T}$(1−R)P_d or 0	(e)

(e) is listed to compare to Case 3. In reality, one does not gain economically from self-default. Therefore the gain from self-default in our replication should be zero. Then we would have:

$$? = -xe^{r_c T} - (1-x)e^{r_u T} = xe^{r_c T} - (1-x)e^{(r_c+u+L)T}$$

Note that this is the value from the SF's point of view. From the CP's point of view, we know the value that CP is demanding (from Case 3):

$$? = -xe^{r_c T} - (1-x)e^{(r_u+u')T} = -xe^{r_c T} - (1-x)e^{(r_c+u+u'+L)T}$$

which is different from the SF's replication value. This difference also leads to interesting results related to DVA and FVA, as we will discuss in the later sections.

Conclusion: The borrower's funding cost is simply $r_u = X + r_c$.

4.2.5 The value of money: summary

From the above discussions, we summarize the results for all the situations in Table 4.1.

These three situations form the basic foundation for funding value adjustment calculations.

4.2.6 The equilibrium borrowing/lending rate

One interesting question to ask is, under what condition that borrowing and lending would happen. Consider the situation that SF lends to CP. Table 4.2 summarizes the economics from SF's point of view and from CP's point of view. To make it simple, we use simple compounding for interest.

Table 4.1 *Summary of funding situations, costs and benefits*

Situation	Carry on Money	Comments
Secured Lending/Borrowing	Rate of carry $= r_c$	Economically, collateralized rate r_c is paid of full collateral.
Unsecured Lending	Rate of carry $= r_u + u'$	Two sources of cost: 1. Cost of funding 2. Cost of credit
Unsecured Borrowing	Rate of carry $= r_u$	Economically enjoys free funding, not from self default.

Table 4.2 *Economics of lending and borrowing from SF's and CP's point of view.*

SF Lending of CP (SF's point of view)	CP Borrowing from SF (CP's point of view)
$1 + r_u T + u' T$	$1 + r'_u T$

From SF's point of view, the minimum charge that SF will be able to lend to CP is $1 + r_u T + u' T$ from CP's point of view, CP would be willing to pay its own funding rate at $1 + r'_u T$. When these two expectations meet together, we have an equilibrium borrowing/lending between SF and CP:

$$1 + r_u T + u' T = 1 + r'_u T$$

or

$$r_u + u' = r'_u.$$

This is what we call the equilibrium borrowing and lending equation. From this equation, we can deduce some points:

- Since the credit spread u' is always greater than zero, the lender's funding rate is generally lower than the borrower's funding rate. This is a natural behavior in the lending market: only lenders with lower funding cost would be able to make money.
- The better the lender's credit quality, the lower the lender's funding rate, the more competitive that the lender would be in the lending market.
- It is not economical for a lower-rated party to lend to a higher-rated party, since it costs more for the lower-rated party to raise money. And lending to higher-rated party may fetch a lower yield than its borrowing cost. While this naturally would not happen in normal lending activity, this happens very often in the derivatives world, where borrowing and lending do not following an explicit borrowing/lending process. Market movement drives the implicit borrowing/lending relationship for derivatives.

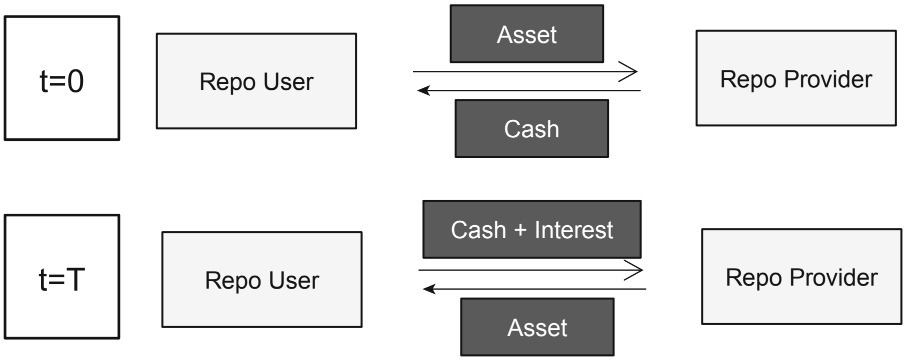

Figure 4.7 Schematic diagram for a repo transaction

The last point about the derivatives is quite important: since the borrowing/lending relationships for derivatives are driven by market movement, the implicit borrowing and lending would not follow the equilibrium borrowing/lending equation. Therefore it is not unusual to see uneconomic borrowing/lending between the trading counterparties. To account for such economics of implied funding, one has to take into account FVA properly in derivatives valuations.

4.2.7 Collateral asset, repo market and rehypothecation

When assets and securities are eligible as collateral under CSA, they carry different funding values from the cash. Often we see the use of US Treasury, government bonds, agency notes, corporate bonds and debt, etc., issued by municipalities. Each of these assets or securities is traded within its own defined market, with different liquidity and pricing mechanisms, which defines the funding value for the user.

One way to link the assets and securities to cash is through the repo market, where it exists. The repo market works in the way shown schematically in the Figure 4.7.

In the repo transaction, a repo user would post the asset to the repo provider, who provides cash to repo user upon receiving the asset. At the same time, the repo user would promise to buy back the asset at a price Cash+Interest at a later date T. Over the transaction, the repo user obtains cash by paying the defined interest, termed as repo rate, while using the asset as collateral in the borrowing activity. Intuitively, the better the quality the asset, the lower the interest.

Through the repo transaction, the linking of asset and cash funding values are defined by the repo rate. Then given the known cash funding value, we can arrive at the funding value for assets. If the repo interest rate is $r_c + p$, then posting the asset would be cheaper than posting cash by a spread p, meaning:[1]

- The funding desk would charge $r_c + X - p$ for the asset used in collateral posting (if one borrows the asset from the funding desk), which is saving a spread p comparing to borrowing cash.

- This can be viewed alternatively as taking the cash and lending in the repo market: one would be able to pocket $r_c + p$ as interest, which is better than the collateral interest paid in cash collateralization by a spread p.
- The discounting of future cash flows, collateralized by the asset, should be discounted at a rate $r_c + p$, comparing to r_c for cash collateral.

The collateralization of assets other than cash can be thought of as cash collateral with collateral interest $r_c + p$. This can be seen clearly from the modification of Case 1 as shown in Case 1a.

Case 1a

Lending \$1 to counterparty, fully collateralized with asset. The asset's repo interest is p. We start with 0, borrow \$1 from funding desk, lend to counterparty and receive \$1 asset as collateral. We repo out the collateral, receiving cash \$1 and has to pay $r_c + p$ as interest in the end. The following shows all the relevant accounts separately:

	t = 0		t = T	
Value of asset:	\$1	————	?	(a)
Payment demanded by repo provider:	−\$1	————	\$$e^{(r_c+p)T}$	(b)
Put Cash in funding desk:	\$1	————	\$$e^{r_u T}$	(c)
Borrow \$1 from funding desk:	−\$1	————	\$-$e^{r_u T}$	(d)

From (b), (c), (d), we arrive at the money that counterparty needs to return at maturity:

$$? = -e^{(r_c+p)T} - e^{r_u T} + e^{r_u T} = e^{(r_c+p)T}$$

This means the future cash flow under asset collateralization should be discounted using $r_c + p$, or the carry of interest for the cashflow V should be $r_c + p$:

$$dV = V(r_c + p)dt.$$

Conclusion: the funding value of collateral asset with repo spread p would imply a discounting rate $r_c + p$.

When $p > r_u - r_c = X$, it is not economical to repo the asset, since it is more costly to repo the asset than to borrow directly from the market. In this case, we regard the collateral asset as having no funding value. Therefore the discounting of the future cash flow will be the same as the situation in Case 2, where there is no credit default risk; however, one would have to incorporate the funding spread in the discounting rate.

When multiple assets are eligible to post at a given time, one may choose to find the optimal asset, or the cheapest to deliver asset in the collateral posting process. Cash is often the last option one anyone would choose, with few exceptions. For example, certain assets may imply negative repo spread, due to the supply/demand issue, such as off-the-run Treasury bonds that are in great demand due to short squeeze.

The above is based on the assumption that there are repo markets existing for the assets. This is generally the case for relatively liquid assets. For illiquid assets that do not have a repo market, the assets are generally of lower quality and often are chosen to be the cheapest to deliver assets. If such an asset is on the balance sheet and one can post the illiquid asset for collateral purposes, excellent value can be created by using the illiquid asset. Otherwise, the asset will be sitting on the balance sheet in any case without generating any extra value.

Sometimes collateral assets posted cannot be rehypothecated as required by CSA. Under such situations, the collateral assets will still provide the credit default risk protection for the lender; however, one would not be able to generate funding value from the posted collateral assets. The posted collaterals will be segregated from other rehypothecatable collaterals. In this case, the lender would still have to fund the asset lending to the full amount, which means the lent asset will be treated as if it is not collateralized when funding value is considered. The carry of interest for this situation would be:

$$dV = r_u V dt = V\left(r_c + X\right) dt$$

which is different from the normal situation by the credit spread.

4.2.8 Collateral currency and cross currency market

When different currencies are eligible in collateral posting, or derivative transactions are collateralized using collateral in a different currency other than the native currency, we need to find out the funding values for collaterals with different currencies. Clearly, the foreign currency collateral may not carry the same funding value. This can be seen from the dissection of the economics of foreign collateralization processes.

We start with two currencies, **A** and **B**. **A** is the native currency for the money that is lent to CP by SF. CP is posting collateral in currency **B** in the amount that is equivalent to the value in **A**. We assume the following:

X_0: Spot exchange rate at t=0, 1**A** = X_0 **B**

r_u^A : SF's funding rate in currency **A**

r_u^B : SF's funding rate in currency **B**

Note the growth rate we used here is generic with no collateral implications defined yet. The dissection of collateralization economics:

Case 5: *Fully Collateralized in Currency B*

	t=0		t=T	
Value of asset in A:	1**A**	————	?	(a)
Collateral to return:	$-X_0$**B**	————	$-X_0 e^{r_c^B T}$ **B**	(b)
Put Collateral in funding desk:	X_0**B**	————	$X_0 e^{r_u^B T}$ **B**	(c)
Borrow 1A from funding desk:	-1**A**	————	\$-$e^{r_u^A T}$ **A**	(d)

Case 5: Continued

Summarizing the accounts, we would have:

$$? = e^{r_u^A T}\mathbf{A} + X_0\left(e^{r_C^B T} - e^{r_u^B T}\right)\mathbf{B} = \left[e^{r_u^A T} + \left(e^{r_C^B T} - e^{r_u^B T}\right)e^{-(r^B - r^A)T}\right]\mathbf{A}$$

For a small time increment dt, the value change would be:

$$dV \approx r_u^A dt - \left(r_u^B - r_C^B\right)dt = \left(r_u^A - r_u^B + r_C^B\right)dt$$

Conclusion: the carry of interest for 1A collateralized using currency B would be: $r_u^A - r_u^B + r_C^B$.

In Case 5, we have also assumed that SF is managing funding in **B** as well, with r_C^B as the general collateral rate in **B**, and r_u^B as the funding rate for SF in currency **B**.

As we know, the carry of interest for value collateralized in currency **B** is r_C^B. This is different from what is derived above for currency **A** collateral: $r_c^{A,B} = r_u^A - r_u^B + r_C^B$. Here we use the superscript to indicate the collateral on the right and the cash flow currency on the left. The difference between the two-currency collateral is driven by the interest differential $r_u^A - r_u^B$. How can we determine the interest differential $r_u^A - r_u^B$? Clearly the two rates r_u^A and r_u^B are the funding rate for SF in two different currencies. Do they have a relationship between them?

The answer is clear when we look at this from the market point of view: posting/receiving collaterals in different currencies can be interchangeable through the FX forward market. In other words, the funding of the two different currencies is linked through the FX forward market: one can fund the collateral in one currency and exchange for another currency for collateral posting, in present and future times. Therefore the equivalency of two-currency funding and collateral posting is determined by the market, providing the market has enough liquidity for it. For example, when one is required to post collateral X_0 in currency **B** and is funding in currency **A**, he may go to the market with 1**A** and exchange for X_0**B** for posting. He may enter an FX forward transaction for time T to swap the currency back. In this way, one can fund in one currency and rely on the FX spot and forward market to manage the collateral posting in another currency. Similarly, the party receiving collateral can manage the currency collateral and exchange the interest accrues from collateral through the FX forward market.

Now we will derive the interest differential $r_u^A - r_u^B$, and the carry interest $r_u^A - r_u^B + r_C^B$. Assuming

X_T: Forward exchange rate at t=T, 1**A**(t=T)=X_T **B**

we know:

- 1**A** will grow into $e^{r_c^A T}$**A** in a risk free money market account;
- 1**B** will grow into $e^{r_c^B T}$ **B** in a risk free money market account in currency **B**;

- 1**A** will grow into $e^{r_u^A T}$**A** in SF's funding desk in currency **A**;
- 1**B** will grow into $e^{r_u^B T}$ **B** in SF's funding desk in currency **B** (presumably);

and assume:

$$X_T = X_0 e^{(r_c^B - r_x^A)T}$$

where r_x^A is what we call the cross currency discounting rate, implied by the FX forward market.

Then we have the two situations equivalent:

- Grow 1**A** in funding desk from 0→T, that leaves us $e^{r_u^A T}\mathbf{A} = X_T e^{r_u^A T}$ **B**;
- Grow X_0**B** in funding desk from 0→T, that leaves us $X_0 e^{r_u^B T}$**B**.

Therefore:

$$X_T e^{r_u^A T} = X_0 e^{r_u^B T} = X_0 e^{(r_c^B - r_x^A)T} e^{r_u^A T} \longrightarrow r_u^A - r_u^B + r_c^B = r_x^A.$$

This is stating that the carry interest of currency A cash flow collateralized in currency B is $r_u^A - r_u^B + r_c^B = r_x^A$, as defined by the FX forward market being the cross currency discounting rate r_x^A.

FX forward market is generally very liquid for short term maturities. However, when long-term collateral value is considered, one would have to resort to the cross currency market to find out the relative funding values of two different currencies. The cross currency swap market implies the concept of cash flow equivalency, similar to the FX forward market, only with a little more complexity.

4.2.9 Cross currency swap market

Cross currency swaps traded under one currency collateral provide a convenient linking between the collateral values of two currencies,[2] effectively the market implied interest differential $r_u^A - r_u^B$. For example, cross currency swaps among dealers based on USD collateral are very common. A USD/EUR cross currency swap could provide the differential between USD collateral compared to EUR collateral.

For a traditional cross currency swap, the trading parties exchange currency notionals at the spot exchange rate; in between the trade date to maturity, they exchange coupon payments (usually every 3 months) for two currencies; at the maturity, notionals for both currencies are exchanged back.

With market supply and demand accounted for, the cross currency swap market gives the future foreign interest rate in a different currency other than the domestic currency. For example, the asset moves by investors and asset managers between currencies would drive the cross currency spread moves, affecting the effective foreign interest rate in the view of domestic players, and as a result affecting the forward exchange rate.

Consider the following cross currency swap traded under USD collateral:

$$\text{USD LIBOR 3M} \longleftarrow\!\!\longrightarrow \text{EUR EIBOR 3M} - \text{X}.$$

Here we have the swaps with an exchange consisting of two legs: one leg paying the USD LIBOR 3M index, the other leg receiving EIBOR 3M index minus a spread. Notionals are exchanged at both start date and maturity.

Clearly the cash flow values for the USD leg should be discounted using USD OIS rate, being USD collateralized cash flows. For the EUR leg, what would be the discounting rate?

This can be solved through equating the two sides of the cross currency swaps by discounting cash flows in each currency to the present time:

$$X_{EUR/USD} \sum_{i=0}^{N} PV_i^{USD} = \sum_{i=0}^{N} PV_i^{EUR}.$$

From the above, we can solve for the discounting rate by calculating the present value of EUR cash flows, so that the cross currency swap leg values are equal for all available maturities. This is known as the bootstrapping the cross currency discounting curve. In general one would combine the short term FX forward market, and the longer term cross currency market to jointly bootstrap the cross currency discounting curve, as the FX forward market is liquid in the shorter terms and the cross currency swap market is more liquid in the longer maturities. The resulting cross currency discounting curve implies how one should be discounting foreign cash flows when domestic collateral is used in posting. We may denote the rate as $r_C^{A,B}$ for domestic collateral currency **A**, foreign cash flow currency **B**, or $r_C^{USD,EUR}$ for the cross currency discounting rate for EUR cashflow collateralized in USD.

Now compare the same leg of cash flow collateralized under EUR collateral: under EUR collateral, the cash flows should be discounted using EONIA, the EUR's version of overnight index rate, or r_C^{EUR}. The difference between r_C^{EUR} and $r_C^{USD,EUR}$ allows us to compare the relative funding value of USD and EUR collaterals.

Combining the collateral asset and collateral currency, one can derive the funding value for a collateral asset in a currency different from cash flow currency. For example, for a cash flow in currency **B**, collateralized by an asset M in currency **A**, with repo interest rate $r_C^A + p_M$. Then the effective discounting rate for currency **B** cash flow would be:

$$r_C^{M,B} = r_u^A - r_u^B + r_C^B + p_M = r_C^{A,B} + p_M.$$

Accordingly, the funding value of any asset in any currency can be derived, providing the existence of a cross currency market for the particular currency, and the available repo market information for the asset.

4.2.10 Multi-collateral choices and choice options

A CSA may allow collateral posting with different currencies and different assets. As we know, different assets and/or different currency collaterals would imply different funding values. This gives the party posting collateral a choice, which could prove to be very valuable. At any given time, there may exist an optimal collateral, or cheapest to deliver collateral for the collateral posting party.

Theoretically the determination of optimal collateral to post, given the market condition, can be done based on the relative funding values of each type of eligible collateral. The relative values for these collaterals will change according to market movement. Consequently, the optimal collateral may change over time. Assume that the relative funding values between two collateral assets a and b is $S_{ab} = r_C^a - r_C^b$, and S_{ab} will change over time. Then the collateral choice value is really an option over the spread S_{ab}, which would dependent on the distribution of S_{ab}.

Naively, the collateral choice option for two assets a and b can be evaluated by assuming a reasonable stochastic process for S_{ab}, which in turn will define the distribution of S_{ab}. For example a normal process with volatility σ_S:

$$dS_{ab} = \sigma_S dW$$

If S_{ab} is not correlated with other risk factors, the discounting effect from the existence of spread S_{ab}, comparing to using a as collateral would be:

$$dC^{ab}(t) \cong \mathbf{E}\left[(1 - e^{-S_{ab}(t)dt})V(t)\right] \cong V(t)\,\mathbf{E}[S_{ab}(t)\,dt] = V(t)\,dt \int S_{ab}(t)\,P(S)\,dS$$

for a small time period from t to t+dt. Here $dC^{ab}(t)$ is the relative value correction for time period dt,V(t) is the value of the cash flow at future time t, P(S) is the probability distribution of S_{ab}, and the expectation is over the distribution of S_{ab}. Integration over time would give the collateral choice option over time:

$$C^{ab}(T) = \int_0^T \int_{-S_{min}}^{S_{max}} V(t)\,S_{ab}(t)\,P(S)\,dS\,dt$$

Alternatively, one may also derive the effective discounting spread that would give the collateral option value:

$$D(T) = \mathbf{E}\left[e^{-\int_0^T \max(0,S(u))du}\right] = \mathbf{E}\left[e^{-\int_0^T S(u)^+ du}\right]$$

where + indicates the positive value. Under normal distribution assumption for S, one could derive an approximation formula for the effective spread at time T.

Practically, there are several issues in monetizing the collateral choice optionality.

First of all, there is, in general, no option market for the funding spread value S_{ab}. Therefore one would be relying on dynamic hedging strategy to replicate such collateral choice option values, which would depend on the accuracy of the model assumptions.

Secondly, when multiple collaterals are present, the complexity in the above optimal collateral decision-making becomes much more involved. In such situations, one may simplify the situation by choosing two optimal collaterals and price the option according to the two cheapest-to-deliver assets and the collateral option between them.

Thirdly, the practical collateral replacement process can be anything but ideal. There would be an operational issue when one would like to replace a 300M asset M with equal value of asset N at one shot, since the collateral asset posting process is most likely more sticky than one would like in an ideal situation.[3]

Lastly, the assumption around the spread process and distribution may not work well. There could also be correlations between the spread and the market risk factors, for example the LIBOR index, that generates the cash flow. This would throw off the collateral choice option values.

Given the difficulty involved in monetizing the collateral choice option, one may choose to simplify the assumptions so that the collateral option is reasonably represented and is practically implementable.

4.2.11 Multi-funding curves, differential discounting and collateral valuation adjustment

Typical dealers would have bilateral CSAs with many different flavors for different counterparties. It is common that one CSA may allow a few or even a dozen assets/currencies, which requires the quantification of optimal collateral values for each CSA case and for different maturity terms. Effectively this means one would need a different funding curve for every different CSA. This is really a quite complex problem to deal with, especially considering the collateral choice options.

Practically, one may achieve this by the following process:

1. Constructing a collateral discounting curve for all eligible currencies and assets
2. Constructing an optimal collateral discounting curve for each CSA
3. Adding collateral choice optionality value based on approximations.

Figure 4.8 shows schematically how the funding curves are constructed.

The first step in constructing different collateral discounting curves for different currencies and assets has been discussed in above sections.

For the second step, we may start by a simple methodology of constructing the cheapest to deliver funding curve for a CSA. Assume we have the following assets and currencies:

$$\text{Cash currencies:} \quad \mathbf{A, B, C, \ldots}$$
$$\text{Eligible assets:} \quad \mathrm{M, N, P, \ldots}$$

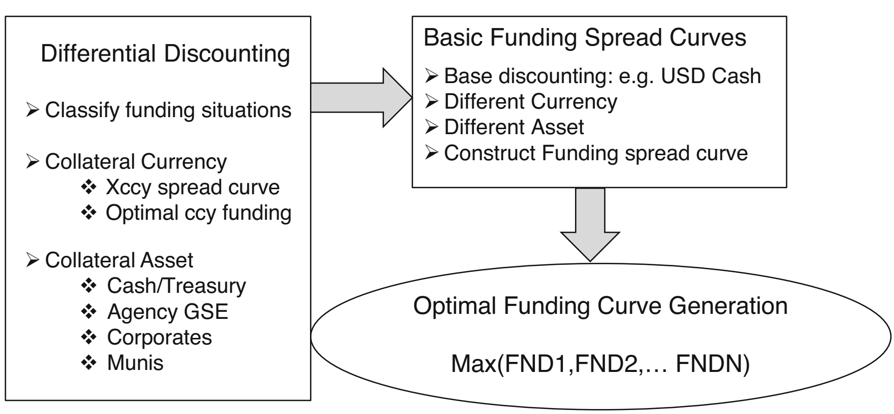

Figure 4.8 Constructing funding curves

meaning A, B, C . . . are the currency cash, and M, N, P, . . . are the assets that are allowed by the CSA. Their respective funding with respect to chosen base currency A cash are:

$$S_{A,B} = r_u^B - r_u^A \quad \cdots \quad \text{and so on}$$
$$S_{A,M} = r_C^M - r_u^A \quad \cdots \quad \text{and so on}$$

then the optimal collateral could be:

$$r_C^{Opt} = r_C^A + \max\left(S_{A,B}, S_{A,C}, \ldots, S_{A,M}, S_{A,N}, S_{A,P}, \ldots\right)$$

This represents the static situation given the relative funding values.

Without worrying about the values of the collateral choice optionality, the static optimal collateral funding curve can be derived using the above construction: for any given time period in the future, one can derive the relative funding values among the currencies and assets, where the currency funding values can be obtained from cross currency market spreads, and the assets can be derived by future repo rate projections, or combinations of the two. This will provide us with the level zero optimal discounting curve for cash flows governed by the CSA.

In practice, optionality can be added for collateral choices approximately, even for multiple assets. As the first order approximation, one may choose the two optimal collateral assets for a given time period:

$$r_C^{Opt}\left(t_i \rightarrow t_{i+1}\right) \quad \text{and} \quad r_C^{Next}\left(t_i \rightarrow t_{i+1}\right).$$

Then the collateral choice option can be estimated for that time period given approximations from model derivations, such as the one we discussed in previous section. Essentially we will be modeling the spread:

$$S^{Opt}\left(t_i \rightarrow t_{i+1}\right) = r_C^{Opt}\left(t_i \rightarrow t_{i+1}\right) - r_C^{Next}\left(t_i \rightarrow t_{i+1}\right).$$

To model the stickiness of the collateral in the posting process, one may add a barrier in calculating the option value. The barrier would effectively add cost to the collateral replacement process, which would also reduce the digital effect in the collateral change boundary by smoothing the boundary.

Given the above-derived optimal discounting curves for different CSAs, we are ready to calculate the derivative values. This is what we call differential discounting for collateralized derivatives. Figure 4.9 shows schematically the process from constructing CSA curves to differential discounting in derivatives valuations: the CSAs are first classified into different categories, from which the CSA curves are constructed, based on the eligible collaterals. Then the mapping between the counterparty portfolios and the CSA curves are done so that each trade under the same portfolio will be discounted using the same CSA discounting curve.

In practice, one may define the base valuation given a choice of base cash collateral. On top of the base valuation, differential discounting gives a collateral valuation adjustment (CollVA), defined as:

$$CollVA = V\left(r_C^{Opt}\right) - V\left(r_C^{base}\right).$$

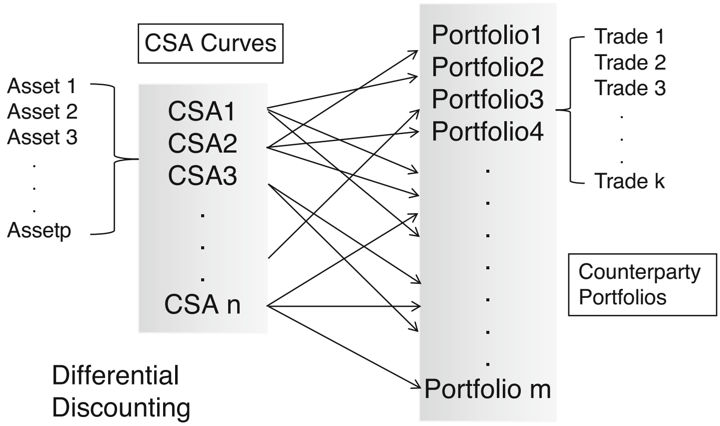

Figure 4.9 Differential discounting for collateralized derivatives

Here we use $V(r_C^{Opt})$ to represent the valuation under optimal funding curve, and $V(r_C^{base})$ as the valuation under base collateral curve.

4.2.12 Collateral operations and replication of collateral values

The above derivations of collateral funding values and construction of collateral funding curves are based on several key assumptions, each of which have an impact on the realization of funding values in practice.

First of all, collateral operation is a very complex process. Optimal collateral posting, given the existing available collateral pool, collaterals expected to be received from counterparties, and the requirements from increased liabilities to be posted, is extremely challenging. The challenge is not only mathematical, but also operational. To put this problem into concrete context, we use a simple collateral modeling process. Assume:

c_iA_i : collaterals available in the pool, we have c_i units of asset A_i
r_iA_i : collaterals expected to be received within the collateral posting window, r_i units of asset A_i
V_j : collaterals required to be posted to counterparty j
h_{ij} : collaterals required as increased liabilities that is posted to counterparty j using asset A_i
X_i : funding spread for asset A_i.

The simple mathematical problem would be to optimize the funding value in collateral posting:

$$\max\left(funding\ value\right) = \max\left(\sum_{i,j} h_{ij}A_iX_i\right)$$

subject to at least the following constraints:

$$\sum_i h_{ij} A_i = V_j$$

$$\sum_j h_{ij} \leq c_i + r_i$$

$$h_{ij} = 0, \quad \textit{if asset } i \textit{ is not allowed in } CSA_j.$$

The first constraint is that one has to fulfill the obligation to post the collateral equal to the increased liabilities. The second constraint is that the total collateral posted for a particular asset type has to be limited by the available asset pool. The third constraint is basically the CSA asset eligibility.

Practically, other constraints may include the minimum units of moveable collateral assets, the collateral operational cost, stickiness of the collateral, rehypothecation, collateral timing, collateral disputes and any other collateral related issues. The second constraint can also be relaxed by accessing the asset repo market for certain collaterals, which could make the optimization process more complicated as liquidity enters the equation. Overall one would need to account for the sub-optimality and inefficiency in the collateral allocation and posting process. Therefore, the theoretical option value from collateral choice needs to be discounted in practice.

In the above simple example, we only considered the optimization of funding value for a certain collateral call period, ignoring any other considerations, such as sub-optimal collateral posting from previous periods, which could be optimized in the current period.

Secondly, the management of cross currency collaterals would require the existence of the cross currency swap market and the hedging of cross currency basis, in additional to the OIS basis in different currencies. While it is well recognized this may be difficult for minor currencies, it happens that even the major currency may face significant liquidity issues, especially for the longer-term maturities. Similarly, the evaluation of asset funding values based on the repo market would also depend on the liquidity of the repo market. A wide bid-offer spread and the availability of the collateral within the market may prove to be problematic, especially under market stress situations.

Certainly managing collaterals in different currencies would also require the collateral operations in different currencies, and may require the management of money centers in different funding currencies. While major banks have operations worldwide, they may fund their operations in a specific currency. This may create inconsistency and sub-optimality in realizing and replicating the collateral funding value. For example, a CSA may allow USD, EUR, and JPY as collaterals, and we know JPY is the cheapest-to-deliver collateral. If the derivatives borrower in the derivative transaction does not have JPY and posts USD instead, then the collateral choice value will be lost. The derivatives lender would be happy as the collateral value adjustment would be gaining in value over time through daily accrual of collateral interest saving. In this case, the borrower may choose to swap the USD currency into JPY in an FX forward or cross currency swap market, and then post JPY. This way one would realize the collateral choice value; however, this may involve rolling and rebalancing of the

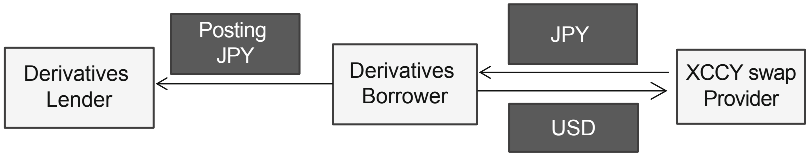

Figure 4.10 Cross currency collateral posting

FX forward/cross currency swap position as the derivative valuation and collateral obligation changes. This rebalancing could turn out to be costly if it is done frequently. The process is shown in schematically in Figure 4.10.

The same situation could happen with collateral assets. If the CSA allows multiple collateral assets, and the derivatives borrower is not posting the cheapest collateral asset for the reason of not having the collateral asset, collateral choice value would be lost. In this situation, the borrower could go to the repo market to lend out the cash, get the cheapest-to-deliver asset and make extra repo interest. The cheapest-to-deliver asset could be posted to the derivatives lender as collateral. This is called reverse repo of the collateral asset. Through this reverse repo, the collateral choice value would be realized through the received repo interest. The process is shown in Figure 4.11.

The stickiness of the collateral in operations is a tricky issue as well. The change of cheapest to deliver collateral due to market movement may not be realized instantly in the collateral operations. If it was optimal to post collateral asset/currency A at one time point, and suddenly the market movement drives the cheapest-to-deliver collateral to be another asset/currency B, one may not be able to substitute the B collateral instantly. The collateral substitution process cannot be an instant process because one has to go through a period of time in order to substitute all A collaterals with the new CTD asset/currency B. For example, SF owes CP \$500M and posts an equivalent amount of EUR cash as collateral. As GBP/EUR cross currency basis moves, suddenly GBP becomes the CTD currency. In order to substitute the collateral, SF has to:

- Raise the equivalent amount of GBP cash to post as collateral
- Recall all \$500M equivalent EUR cash collateral
- Substitute the EUR collateral with GBP cash.

In practice, none of the above processes can be instant. It may involve a process to find the proper funding source of GBP, for example. In addition, one has to consider the friction from the operations that one may have to go through: if GBP/EUR cross currency basis swings back, the substitution has to be reversed. It is clearly not

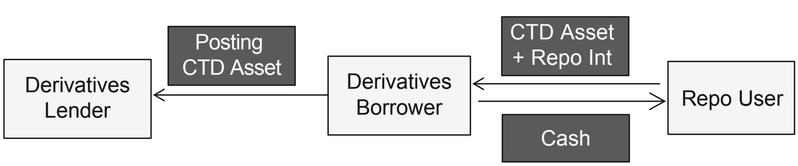

Figure 4.11 Reverse repo in collateral operations

desirable to flip the collaterals frequently. Thus in practice, the collateral substitution process, especially involving a large obligation amount, would have to go through a time period, thus showing stickiness of CTD collateral posting. This stickiness in collateral substitution inevitably would affect the realization of CTD choice value.

4.2.13 Derivative valuations with collateral

In this section, we derive formally the derivative valuations under collateralization.[4] We follow the standard derivatives replication approach below.

Let $S(t)$ be the underlying for the derivatives with value $V(t)$ at time t. $S(t)$ follows a standard Brownian motion process with drift $\mu_s(t)$ and volatility $\sigma_s(t)$:

$$\frac{dS(t)}{S(t)} = \mu_s(t)\,dt + \sigma_s(t)dW(t)$$

where $dW(t)$ is the standard Wiener process. From Ito's Lemma we have:

$$dV = \left[\frac{dV}{dt} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 V}{\partial S^2}\right]dt + \frac{\partial V}{\partial S}dS = \left[\theta + \frac{\sigma^2 S^2}{2}\gamma\right]dt + \delta dS.$$

For simplicity, we have dropped dependence for the variables. In the second equals sign, we have replaced the quantities with Greeks letter: θ is for option theta or time decay, δ is for option delta and γ is for option gamma following the standard convention.

In deriving the derivative value, we consider the following:

- First we transact the derivative with the counterparty, borrow V from treasury, make payment V to the counterparty and receive collateral in the amount V. In this process, money borrowed from funding desk and the collateral received are matching in value. As we have derived in previous sections, the carry of interest for value V would be $r_c + p$ with p being the collateral repo interest.

$$dC = r_C^{Opt}\,Vdt = \left(r_c + p\right)V\,dt.$$

- We need to hedge the delta risk and form a risk-free portfolio:

$$\Pi = V - \delta S.$$

The change of the portfolio in value for a small time increment dt would be:

$$d\Pi = dV - \delta dS = \left[\theta + \frac{\sigma^2 S^2}{2}\gamma\right]dt$$

which should be default risk-free, and equal to the carry of interest of two terms V and δdS:

$$\left[\theta + \frac{\sigma^2 S^2}{2}\gamma\right]dt = \left(r_c + p\right)Vdt - \mu_s\delta S\,dt$$

or

$$\theta + \mu_s S\delta + \frac{\sigma^2 S^2}{2}\gamma = \left(r_c + p\right) V.$$

The solution to the above equation would lead to the derivative valuations. The Black–Scholes formula for simple European options can be modified so that the discounting rate for the cash flow value would be the optimal collateral discounting rate $r_c + p$. When optimal collateral is considered, p is a function of multiple collateral funding spreads.

4.2.14 Standardized CSA (SCSA) and tri-party optimization

SCSA

Traditional bilateral CSAs can be very complex, with embedded collateral options, collateral thresholds, rating triggers and so on. These create complexities in valuations and operations for the derivative market participants. The process of trading bilateral derivatives with complex CSAs can be very involved, because there is not enough transparency as to how the trading parties are evaluating the derivatives. And, as a result, there is not much liquidity in the bilateral derivative novation/intermediation market.

In order to standardize the practice and eliminate the ambiguities in derivatives pricing, in 2013 the International Swaps and Derivatives Association (ISDA) published the Standard Credit Support Annex (SCSA) after a few years of research. The SCSA seeks to standardize the market practice in collateral management in, amongst other things, the following areas:

- SCSA removes embedded optionality in the existing CSA. Collateral is limited to cash in the native currency of the underlying swap, which accrues at OIS interest rate. As a result, the derivative valuations naturally fall into the OIS discounting, which removes the ambiguity in discounting the future cash flows.
- SCSA specifies the settlement on a net basis across all currencies based on a chosen G7 Transport Currency, which removes the cross currency settlement risk. The SCSA embeds an Implied Swap Adjustment (ISA) methodology to synthesize the economics of posting cash collateral in the silo currencies, with the daily set of SCSA rates published by the Bloomberg SCSA page.
- SCSA removes ATEs as a specified condition, except for limited circumstances. This reduces inconsistent valuations caused by rating triggers.

The implementation of SCSA would reduce the disputes in valuations among the dealers and customers, and therefore it would be much easier to "sell" derivatives through novation. With SCSA, it would also be much easier for dealers to reduce their book by performing tri-party netting. As the valuation discrepancies decrease, the cross netting of bilateral derivative risks can be standardized so that all parties involved can achieve better management in operations, risk and margining. In fact, tri-party netting services among the dealers have become very popular and are now a standardized practice. This allows the dealers to reduce the number of trades, operations and, potentially, the margin requirements.

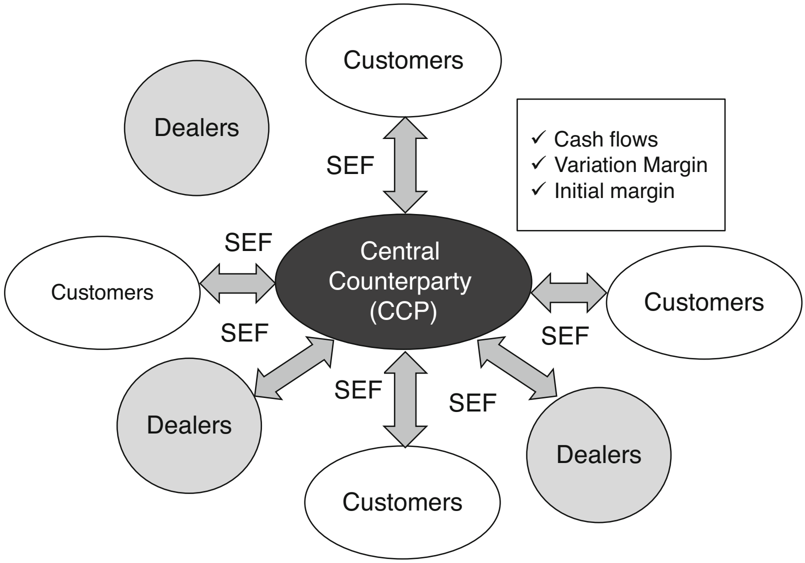

Figure 4.12 CCP and central clearing

Dodd–Frank, EMIR and Central Clearing

In July 2010, the *Dodd–Frank Wall Street Reform and Consumer Protection Act* was passed in US Congress. One of the key mandates of Dodd–Frank Act is to standardize derivatives trading and promote transparency. For OTC interest rate swaps (IRS) and Index CDS, market participants are required to clear their swaps through a central counterparty (CCP), such as LCH or CME, unless exemptions are given. Under the new mandatory clearing regime, the counterparty that dealers and customers face would be a central clearinghouse, to which they post variation and initial margins. Under this regime, the customers will no longer have credit risk exposure to the dealers that they trade with. Instead everyone will be facing the CCP, with customers transacting their swaps through Swap Execution Facilities (SEF). Through advanced initial margin calculations and posting, CCPs attempt to minimize the credit risk of the system for historically would-be bilateral derivative transactions. As long as the CCP is well managed and well capitalized, the "chain reaction" following a major dealer defaulting will not destabilize the financial system through the derivative transactions. Figure 4.12 shows a very simplified picture of central clearing of derivative transactions for dealers and customers.

Similarly, the European Securities and Markets Authority (ESMA) also published in the European Markets and Infrastructure Regulation (EMIR) the mandate for OTC derivative market participants to clear IRS in a number of currencies through CCPs.

There are a few potential issues of the CCP setup, which we highlight below:

- The product coverage for central clearing is still limited. While IRS, CDS and some inflation swaps are being standardized to clear through CCPs, many customized swaps, cross currency swaps, and other option products are currently not being cleared by CCPs. This leads to an inefficient risk and capital allocation: option

products and their hedges are treated differently, which may be subject to initial margin at the same time.[5] The bilateral risks for non-cleared products still remain.
- CCPs are becoming systematic important entities that cannot default. If a CCP defaults, all derivative transactions cleared through the CCP would be affected. This would have much broader impact on the market than the legacy bilateral world.
- Multiple CCPs competing at the same time could potentially result in a situation that demands more initial margins for the market participants, which would be higher than the bilateral situations. This potentially creates a drain on market liquidity. For example, if you long delta on one CCP and short delta on another as the result of customer transaction and hedging, then you would have to post twice as much initial margin due to the naked position on both CCPs. The prime example is the persistent spreads between LCH and CME swaps, which could be around 2 basis points for a 10-year vanilla swap. This reflects the customer preference on once CCP and the cost from the dealers to post margins on both CCPs.

As central clearing has become mandatory for most market participants in the derivatives market, more and more constructive work has been done to optimize the initial margin requirements and to reduce the capital requirement in derivatives trading. However, problems remain for non-cleared products, as well as the tail risk for CCP default.

4.2.15 Example of fully collateralized derivatives

Swaption with Mixture Discounting

Historically, swaption valuation has been relatively simple; given LIBOR curve, strike and volatility, one can apply the Black formula to arrive at the swaption premium easily:

$$V\left(swaption\right) = swapDiscAcc \cdot BlackFormula(payrec, f, K, \sigma, T)$$

where f, K, σ, T stands for forward, strike, volatility and term respectively. This was changed with the co-existence of bilateral CSA, central clearing and differential discounting. There could be at least three layers of complexities:

1. Forward premium vs. spot premium: To mitigate the credit risk and collateral requirements, forward premium swaption has become the prevailing type that is traded in the market. In a forward premium swaption trade, the premium is paid at the time of the swaption expiry. Because of the delayed payment in premium, there is less credit risk for the swaption buyer.
2. The underlying swap can be bilateral or exchange traded (LCH, CME, Eurex etc): The reference swap that the swaption will be settled into can be a bilateral swap, or an exchanged traded swap. In Figure 4.13, a mixture discounting example is shown. The underlying swap that the swaption will be settled into after option expiry is an "exchange traded" LCH swap, which will be based on OIS discounting. However, the trade itself is done based on a bilateral CSA, which means the collateral calculation and posting will be based on bilateral CSA. Therefore the discounting for the option values should be a bilateral CSA discounting.

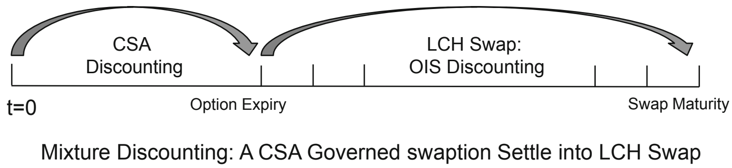

Figure 4.13 Mixture discounting example

3. Differential discounting on collateral: swaption valuation is dependent on the collateral that is used. Native currency collateral swaption would trade differently from the foreign currency collateral swaptions.

Adding to the above is the physical settlement vs. cash settlement issues for swaptions. All these require dealers to interpret the swaption quotes and related conventions properly: in the broker market, the swaption forward premiums based on certain currency cash collateral are quoted with assumption of centrally cleared swaps. These quotes are processed to generate the proper volatility surface, which are then used in the customized swaptions valuations.

In the following we show an example of a swaption straddle and a payer swaption valued under different situations.

Example: Mixture discounting.
Buying a 1-year into 10-year swaption, with strike = 2%, volatility = 20%. Trading entity's funding rate = OIS + 100 bps.

Table 4.3 shows example market data:

Table 4.3 *Example swaption and interest rate details*

1y10y swaption	Fwd Swap rate	1 y discounting rate	fwd swap discount accure
OIS Discounting	2.065%	0.34%	9.334
OIS+100bp discounting	2.042%	1.34%	8.787

The forward swap rates are different underlying two discounting situations: OIS discounting and OIS+100bps discounting, which is trading entity's funding rate. The swaption straddle and a payer swaption valuation under different collateral and forward swap rate situations is shown in Table 4.4:

Table 4.4 *Example mixture discounting calculations*

Forward swap calculation	collateralization	Straddle value	Payer swaption
Central Clear Swap (OIS disc)	Full in cash	3,060,502	1,832,691
Central Clear Swap (OIS disc)	Uncollateralized	3,030,050	1,814,455
Bilateral swap	Uncollateralized	2,844,281	1,606,898
Vega		148,744	74,372

Example Continued

The difference between the collateralization with the same forward OIS swap gives 30K difference for the straddle and 18K for the payer swaption. The difference between the central cleared swap and bilateral swap gives 186K difference for straddle, which amounts to over 1 vega; while for payer swaption, the difference is 208K, or close to 3 vol. The significant difference for the payer swaption comes from two factors: a lowered swap rate for bilateral swap, and a smaller forward swap discount accrues given the higher discounting rate. In this example, we have ignored the smaller convexity terms.

4.3 Funding value adjustment

4.3.1 Inseparable principle: Derivative valuation and funding of derivatives

Derivative valuations are determined by discounting the future contingent cash flow payoffs to the present time. The question is, how should one discount the future cash flows? This brings us to the general principle of derivative valuations and the funding of derivatives: the valuation of the derivatives and the funding situation of the derivatives cannot be separated. This covers not only the collateralized derivatives, but also the uncollateralized derivatives.

The question should really be: What is the interest accrued for extra cash on hand, and what is the interest charge for borrowed money?

As we know, OTC derivative values can be replicated by dynamic hedging of underlying risk factors through forming a "risk-free" portfolio.[6] In this replication process, one would need to specify a money market account to accrue interest for extra cash, and an account to pay interest for borrowed money. Extending this to a more sophisticated real-life situation, one may specify an account for deposited collaterals and another account for posted collaterals, which carries funding value for collaterals. Therefore we may have four different accounts in replicating the derivative values through time:

- Money market account for extra cash: accrues interest
- Debit account for borrowed money: pays interest
- Collaterals received: accrues collateral funding values
- Collaterals posted: pays based on collateral funding values.

The specification of these accounts essentially mimics real-life funding situations, and forms the basis for discounting future cash flows embedded in derivatives. This also implies that the derivative values should not be separated from the funding of the derivatives. In real life, all four accounts exist for the derivative trading business at the funding desk, where internal borrowing, lending and collateral economics are executed.

The essence of the differential discounting methodology, as we discussed in the previous section, is that the future cash flow needs to be discounted at a rate consistent with collateral values. If the collateral provides a funding value of OIS+10bps, then the derivatives contract should be discounted at OIS+10bps, so that the total value

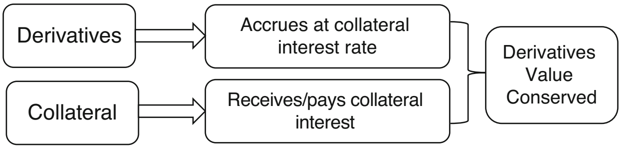

Figure 4.14 Collateralized valuation

for "Derivative + Collateral" together is conserved. This is shown in Figure 4.14: derivatives accrue at a collateral interest rate while the collateral received/posted would pay/receive the same collateral interest. The result is that we have a fair value of derivatives/collateral combinations conserved: the positive/negative interest accrual between derivatives and collaterals is zero. If the derivative is accruing at a different rate from collateral interest, then there will be a constant decay over time, either bleeding money or making money over time. The funding value of the collateral is reflected in the discounting curve used in the valuation of derivative cash flows.

From the collateral discounting methodology, we show that the derivative valuation is not separable from the funding situation of the derivative: we have to figure out the proper discounting rate for future cash flows, or equivalently the carry of interest for the asset value.

Similarly, the discounting of cash flow from uncollateralized lending is not separable from the funding situation of the asset. Given the funding activity around the derivatives lending, one would be able to apply the proper discounting rate in valuations. In other words, the funding situation around derivatives lending defines how the derivatives asset should be discounted. On the other hand, equally and symmetrically, the funding situation also defines how the derivatives liability should be discounted. They are tied together.

As we will discuss in more detail, the bilateral uncollateralized lending and borrowing, would be closely linked to the derivatives funding situation of the specific firm. This is mostly due to the fact that bilateral uncollateralized derivatives are not liquidly traded, and therefore have to be carried on a dealer's book for a long time. Thus the cost of carrying the trade including funding cost needs to be marked through proper discounting, as is expected from the derivative value replication process. We will elaborate on this in much more detail in later sections.

Unless there is a liquid trading market for these derivatives, where the funding component of the derivatives is determined by the market and market funding, the valuation of the derivatives cannot be separated from the firm's specific funding situations of the derivative, as derivatives age day over day. The consideration of specific funding situations leads to the discussion as to how FVA should be evaluated. As we will show in later sections, the origin of specific funding and FVA in uncollateralized derivatives is the inefficiency of this particular market.

4.3.2 Funding for uncollateralized derivatives – From the firm's economic point of view

Derivatives are contingent claims with payoffs driven by market risk variables. The funding of derivatives can be decomposed into every instantaneous funding of

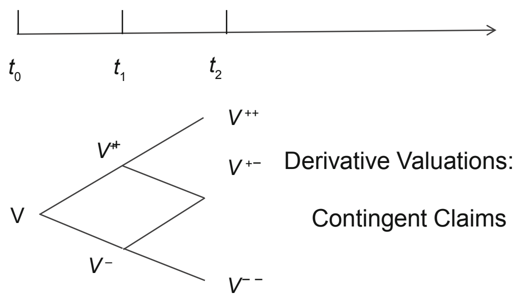

Figure 4.15 Derivative valuation with contingent claims

derivative values in different scenarios and times, meaning implied lending and borrowing activity at any moment. The total funding for the derivative would be the aggregated funding economics for all situations. As shown in Figure 4.14, one would have to consider all the funding situations for $V, V^{+}, V^{-}, V^{++}, V^{+-}, V^{--}, \cdots,$ at $t_0, t_1, t_2, \cdots$.

In the following we will look at the funding cost aspect from uncollateralized lending.

Recall from previous sections that we derived the carry of interest for assets from lending activities. In order to be fair, one would have to charge two costs in uncollateralized lending:

- Credit default risk of borrower: u'. This is taken into account by CVA.
- Funding cost of the lender: $r_u = r_c + X$, where X is the funding spread for lender.

As a result, the asset from lending activity should accrue with interest rate: $r_u + u' = r_c + X + u'$; or alternatively, the asset's future cash flows should be discounted at a rate equal to $r_c + X + u'$. The funding spread X part of the economic impact on the derivative values would be the funding cost adjustment (FCA):

$$FCA = \boldsymbol{E}\left[funding\, impact(V^{+}, \boldsymbol{X})\right]$$

V^{+} is for the positive asset value from lending activity. With X being positive, FCA is always a negative term as it reduces the asset from extra discounting.

On the other side of the funding activity, what would be the discounting rate for a liability from a borrowing activity? From the previous section, we have derived that the discounting rate for any borrowing should be our own funding rate $r_u = r_c + X$. As compared with the cash collateralized situation, we have a funding benefit adjustment (FBA) here, symmetric to the funding cost:

$$FBA = \boldsymbol{E}\left[funding\, impact(V^{-}, \boldsymbol{X})\right].$$

V^{-} is for the negative liability value from borrowing activity. FBA is always a positive term as it reduces the liability from extra discounting.

Notice that the funding cost and funding benefit is symmetric, meaning when an asset and a liability are of equal value, their FCA and FBA will offset each other. This is

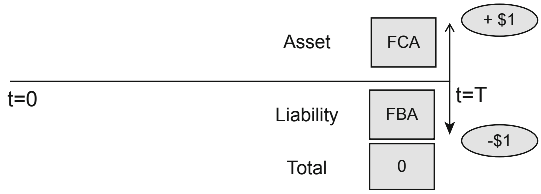

Figure 4.16 Asset and liability funding symmetry

the correct expectation from the firm's point of view, and is shown in Figure 4.16. If one expects to have offsetting positive cash flow and negative cash flow for the same time point in the future, there should be no net funding effect: one does not need to raise funds for future cash flows. This is the essence of a centralized funding management.

We can extend this to the firm's overall funding situation: the funding desk would be managing the net of funding cost and funding benefit from different businesses, and maintaining an equilibrium funding state. From the big picture point of view, the funding cost and benefit should be allowed to be netted to make funding management efficient throughout the firm.[7]

4.3.3 FBA and DVA: Linking, overlap and double-counting

FBA is the funding benefit that a borrower enjoys for the funding received. The present value of the future liability should be discounted at risk-free rate plus a funding spread: $r_u = r_c + X$. Compared with the risk-free discounting, the reduction of liability is achieved through the funding spread X. For a small time slice, FBA is:

$$FBA\,(t \to t + dt) = V^{-} X dt.$$

As we discussed in the previous chapter, DVA is equal to the cost that the lender suffers when the DVA owner is in default. It is a reduction in the liability of the borrower, in the amount of the borrower's own default risk indicated by the CDS spread u. For a small time increment dt:

$$DVA\,(t \to t + dt) = V^{-} u\, dt.$$

In a simplified situation as we show in the Figure 4.17, the relationship between FBA and DVA is:

$$FBA\,(t \to t + dt) = DVA\,(t \to t + dt) + V^{-} L\, dt$$

where L is the market funding spread. Intuitively, DVA is only the credit component, while FBA would include both credit and market funding spread. Even though this is an overly simplified situation, it depicts a clear picture conceptually. In practice, there are multiple levels of complexities, and we will come back and discuss these in later sections.

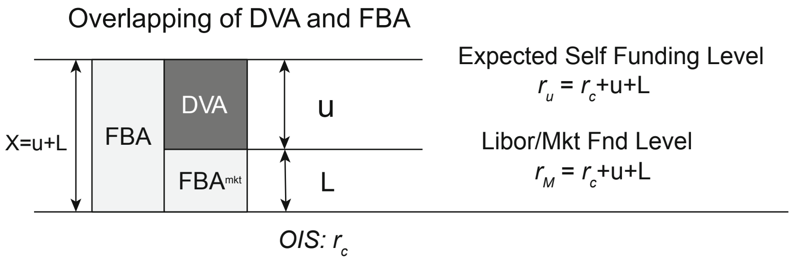

Figure 4.17 Overlapping of DVA and FBA

In this simplified situation, one can decompose the FBA into two terms, DVA and market funding benefit FBA^{mkt}

$$FBA = DVA + FBA^{mkt}.$$

This relationship is shown clearly in Figure 4.17.

Because most firms are still reporting DVA in their accounting due to historical reasons, the funding benefit should then be limited to the market funding spread. If both FBA and DVA are included in the valuations, there would be an overlap and one would be double counting the credit spread component of the benefit. Therefore only one should be chosen in marking the derivative valuations, not both. In practice, the funding curve of the firm may not be as simple as $r_u = r_c + X = r_c + L + u$. In this case, one has to be careful in accounting for DVA or FBA in derivatives marking.

4.3.4 FBA or DVA: Which one to use?

FBA and DVA: Pros and cons

FBA and DVA are similar as they are both benefits from the borrowing activity, where the liability will be reduced. For the simple situation we discussed above, FBA and DVA differs by a market funding spread, where DVA represents only the credit default component, while FBA includes an extra market funding component. This difference is similar to the comparison of CDS spread and bond spread, where CDS is “unfunded”, and bond spread is “funded”: the bond buyer has to take real money, including the principal, to buy the bond, and the CDS buyer only needs to pay a small CDS spread interest.[8] So we can think of DVA as unfunded and FBA as funded. The question is then, which one should we use and how should we use it?

As we discussed in the previous chapter, the use of DVA faces some critical issues:

- DVA cannot be replicated or hedged properly by the firm.
- DVA is not economical to the firm and cannot be used to pay dividend.

This makes DVA management very difficult in practice. As a result of these issues, one may ask further questions, for example:

- Should one give DVA benefit in derivatives trading? In quoting the customers, marking the transactions and calculating P&L?

- If DVA is not economical, how much should one give to the business for a new derivative transaction if it results in significant DVA?
- If DVA is not given to the business making the derivatives transaction, and the firm has to mark to it due to accounting, then there will be no incentive for the business to compete for these trades in the market. Would this be the desirable behavior?
- If DVA is not given and one has to mark to DVA, how would one deal with the economics of unwinding the customer transactions?

In reality, there is really no right answer to these questions. Resolving one problem would lead to a new problem. If we simply apply DVA to all new transactions then, at the end of the day, we may accumulate a lot of transactions containing DVAs. As we know they will not be treated as "real money" for the firm, this is clearly undesirable.

If we do not apply the DVA benefit to derivative transactions in trading, then we cannot compete for any DVA-related transaction. Besides, we know DVA could be a real benefit in some situations. For example, when DVA transactions can be offset against CVA transactions for the same counterparty, and providing that the asset and liability exposures offsetting with each other, DVA is real: the future assets and liabilities offset against each other. If there is a general two-way flow with a same counterparty in terms of credit exposure, then DVA could be truly realized in the sense that it reduces or offsets the CVA.[9]

While CVA is linked to counterparty credit and DVA is linked to the self-default, credit exposure change from market movement still poses a great challenge to the DVA management: offsetting CVA and DVA exposures with the same counterparty can suddenly become DVA exposure only. Then what would you do with these "not real money" DVAs?

All these issues originate from the inconsistency between DVA marking and the real economics of the firm. There is really no perfect solution to this.

On the other hand, FBA does not have the similar issue. Why?

- First of all, FBA and FCA coexist together. FBA and FCA are symmetrical, meaning the positive asset and negative liability offset against each other in funding calculations and management. The balancing between FBA and FCA reflects the funding requirement of the business: it reflects the overall funding of all transactions. As the market moves, the potential future funding of derivative transactions changes.
- On the aggregate of all businesses, the firm will manage the overall funding requirements by setting up proper funding policies. When funding is plentiful, it might be desirable to encourage the use of funding to achieve a better return; when funding is scarce, it might be desirable to encourage bringing funding into the firm, and discourage taking funding-intensive transactions.

FBA management is embedded in the funding management along with FCA, and is a natural component of a firm's business management. Going back to the dilemma that DVA is not of real value to the firm economically by being default benefit, FBA *is* economical to the firm: it offsets against FCA across different counterparties. By looking at the FBA and FCA together within a business and with a changing market, one can effectively hedge FBA and FCA together. When funding exposure changes,

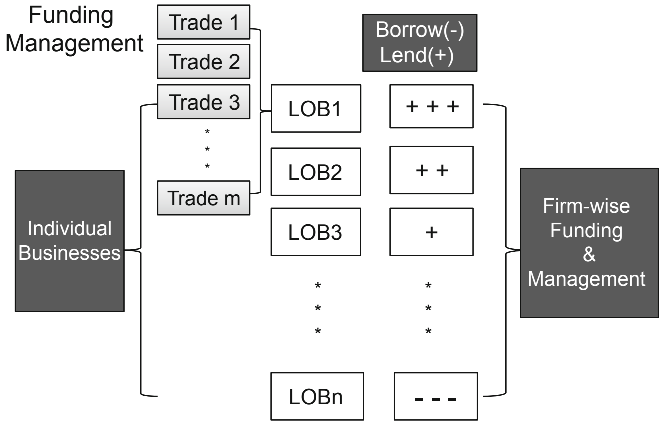

Figure 4.18 Centralized funding management

one can make business and strategy decisions to manage the funding cost and benefits, given a funding policy.

Funding management and optimal funding

Figure 4.18 shows schematically the funding management within a firm: the funding requirements come from the aggregate of individual businesses, which may have funding costs or benefits at a specific time. For individual businesses, the funding requirement comes from the aggregate of all transactions. Overall, the funding desk of the firm will be managing the balance of funding from all line of businesses.

When there is more FBA than FCA for an LOB, the business would be effectively borrowing more money than it is lending. Over time, FBA will decay away, showing up as bleeding through time with liability increasing. This LOB's expected borrowed funds will be paid interest with the firm's funding spread X, which offsets the increasing liability. The interest payment by the funding desk could be simply transfer pricing from another LOB, which borrows funds for its own business activity. Within the overall picture, the firm's funding policy provides the allocation methodology of funding cost and benefits among individual businesses. The firm's funding desk is simply executing the policy through transfer pricing. In addition, the firm's treasury would also balance the funding requirements in both the short term and long term through repo transactions, commercial paper and debt issuances. Through all these, the firm realizes FCA and FBA among all businesses.

In reality, firms generally have different ways of funding themselves. In Chapter 1, we discussed some of the available funding channels that firms may utilize. A firm may be able to achieve funding levels that are better than what would have been implied in the credit market by optimizing the funding activities:

$$r_u = r_c + X = r_c + L^{opt} + u < r_c + L^{mkt} + u$$

where L^{opt} indicates the liquidity spread from optimizing the funding channels, and L^{mkt} is the market expected liquidity spread or market funding spread. $r_c + L^{mkt} + u$

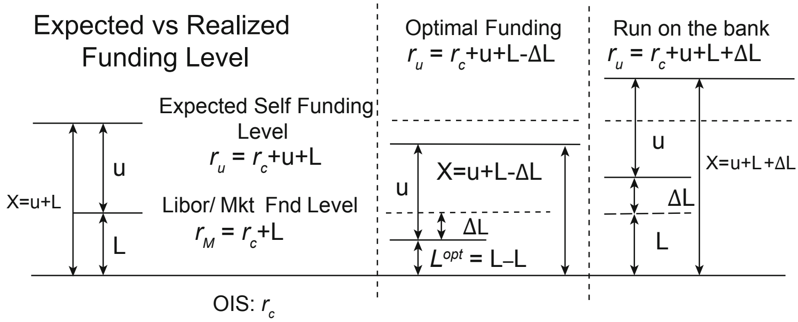

Figure 4.19 Expected and realized funding levels

represents the market expected equilibrium funding spread level. Here we may say that the liquidity channels available to the firm allows it to optimize the liquidity access, and effectively achieve a smaller liquidity spread than the market funding spread. Certainly, in a stressed situation, where the firm is facing a "run" on the bank, and no one is willing to take the credit risk, one would have

$$L^{opt} \gg L^{mkt}$$

which means it will cost the firm a large liquidity spread, in addition to the market CDS spread. In a more general case, we may express as:

$$r_u = r_c + X = r_c + L^{mkt} + u \pm \Delta L$$

where ΔL represents the difference between the firm's specific funding spread from expected equilibrium funding spread. Figure 4.19 shows the situations that arise when a credit crisis happens; the actual funding level of the firm will have an extra funding spread that it has to pay in the market, while the optimizing funding channel may lead to smaller liquidity spreads $L^{opt} = L^* = L - \Delta L$.

In this setup, we could extend the DVA/FBA relationship in a more general sense: if one uses bilateral CVA in accounting, then one should only use FBA^{opt} to account for the funding benefit portion with spread L^{opt}; if one does not use DVA and use FBA completely, then the full spread $X^{opt} = u + L^{opt}$ will be used in both FCA and FBA calculations. This is shown in Figure 4.20.

Alternatively, if one takes the market funding level L as being given, one may think of the optimal funding level as:

$$r_u = r_c + X = r_c + L + u^{mkt} \pm \Delta u = r_c + L + u^*$$

where u^* is the optimal specific funding level. This can be thought of as the possibility of raising funding without paying the full market CDS spread: this could be true in that market implied CDS spread often includes a significant risk premium. Optimizing the funding channel could lead to a saving in the payment of the full credit risk spread. Figure 4.21 shows an alternative way to understand the relationship between FBA and

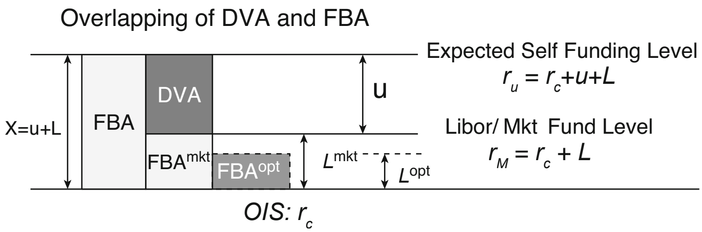

Figure 4.20 Overlapping of DVA and FBA with optimized liquidity spread

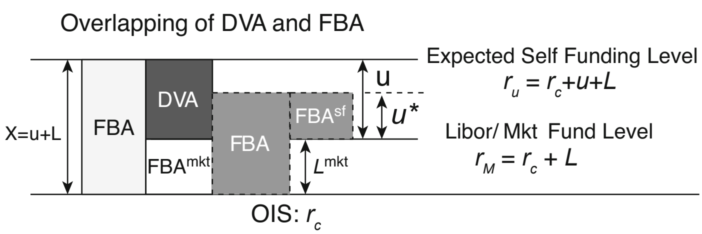

Figure 4.21 Overlapping of DVA and FBA with optimized funding spread

DVA: the full funding benefit would be lower with $u^* < u$, and $FBA = FBA^{mkt} + FBA^{SF}$, where SF indicates the specific part of the funding spread, given the market funding spread is known.

Using FBA, symmetric to FCA, would be ideal to reflect the derivative borrower's economics. One has to be cautious, however, that FBA should not exceed FCA in significant size for the firm overall. Otherwise, the additional FBA would show as a decaying value overtime. This is, however, the overall funding management issue, where the firm ends up with too much borrowing.

In practice, DVA are still prevalent in banks' financial statements, mostly as the result of the existing accounting standards. With DVA as given, one would need to pay attention to the overlapping FBA/DVA, where only the FBA^{mkt} part should be calculated.

4.3.5 Specific funding spread and market funding spread: Which one?

So far we have been discussing the funding cost from the point of view of firm's economics: unsecured lending to counterparty would incur the funding cost experienced by the firm. Therefore it is fair for the lender to charge the borrower the lender's specific funding cost, $r_u = r_c + X$, along with the borrower's credit default component u'. This is consistent with the lender's economics, where the lender is happy to make the trade.

However, one may argue that the firm's valuation based on the firm's specific funding cost would not reflect the market's valuation. If another firm in the market is valuing the same transaction, a different funding curve may be used. If we refer valuations from different firms in the market being the real "market", then the "market

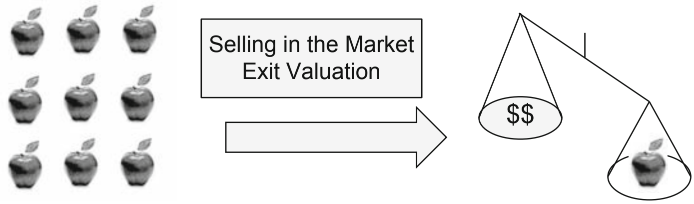

Figure 4.22 Market mechanism: Exit valuation in the market

price" for the transaction should be using the "market" funding curve. Putting aside the issue of determining the "market" funding curve level, intuitively the "market" transaction price should be based on "market" funding curve $r_M = r_c + L$.

This touches a difficult point in FVA discussions. Before exploring the answer to the market-funding question, we will visit two important points.

Point 1: Accounting with market mechanism vs. internal model

We start first by discussing the accounting principles for derivative transactions.

In general there are two mechanisms that one may follow to account for fair derivative values in financial accounting. The first is what we call "market mechanism", or "market valuations". This happens when the derivatives have a liquid trading market. When the bid/offer spread is low, the derivative transaction should follow the "market mechanism" with MTM set to the average market price (see Figure 4.22). This is also called the "exit price".

This is exactly where the "market price" and "market" funding curve argument comes from. If a "market" exists for the derivative transaction priced using "market" funding curve, then one should be marking FVA based on "market" funding curve. There are two prerequisites for the use of "market mechanism":

- Existing market trading with low bid/offer spread.
- Market liquidity is reasonably deep enough for the marking of the relevant derivative transactions.

This is illustrated by the simple example of selling apples (Figure 4.23). If we have a warehouse of apples in City A and we need to mark the price to these apples, what price should we mark it to? If there is enough market to sell the apples to in City B, then one can mark the apples to the market price, after frictions involved the selling process, like the transportation costs. If there is not enough liquidity in the market, or the friction to sell the apple is prohibitively high from City A to the market in City B, then one would not be able to market the apple prices to the market prices in City B.

This leads to the second method in marking derivative transactions: when there is no liquid market for the derivatives, or the market carries a large bid/offer spread. Under this situation, marking to bid would indicate a "fire-sale" price and is clearly not desirable. Marking to mid is also not possible, since one cannot reasonably exit the trade at mid-price, at least not in a way acceptable by accounting standards. In this case, one may resort to the internal model (IM) approach or "private pricing" mechanism, where the intrinsic value of the asset/derivative will be calculated along with the carrying cost over time. Aside from the model validation aspect, the real

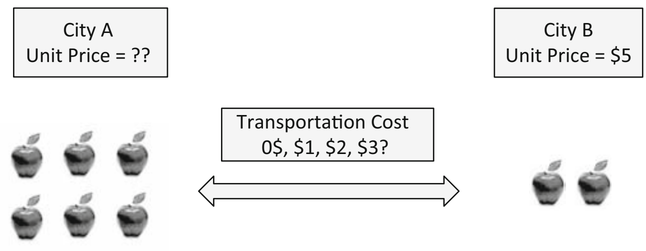

Figure 4.23 Illustration of market liquidity and friction

economic value of the asset/derivative may be marked at the calculated model value plus the expected carrying cost:

$$V^{IM} = V^{M} + C$$

where IM stands for internal model, V^{M} is the model price, and C is the expected carrying cost.

We use the simple case of marking a bond as an example. For a bond traded in a liquid market, where there is a clear market value for the bond, one should be marking the bond to the market-traded price. However, if there is no market for the specific bond, or the market carries a large bid/offer spread, how can we mark the bond price? Here one may try to calculate the bond price based on the IM approach. V^{M} would be the cash flow value V^{CF}, taking into account the bond issuer's credit default:

$$V^{M} = V^{CF} + CVA.$$

Ignoring the operational and other costs, the cost of carrying the bond would include the funding cost of the bond over the expected liquidation time horizon T_L:

$$C = FCA = -\int_0^{T_L} V(t)\,X(t)\,dt.$$

Given these, one may want to mark the bond holding value as:

$$V = V^{IM} = V^{CF} + CVA + FCA.$$

Considering the existing market bid, one may say

$$V = \max(V^{bid}, V^{IM}) = \max(V^{bid}, V^{CF} + CVA + FCA).$$

Here the fair value of the bond is marked as the maximum between the fire-sale bid price and the internal model price. Notice we have used liquidation time horizon T_L, which means that the cost associated in carrying the bond should only be calculated up to the expected exiting time.

The above discussions can be generalized with the definition of the expected liquidation time horizon T_L:

- Market mechanism is only the special case that the funding cost is close to zero, as $T_L \to 0$.
- For the general case that stays between market mechanism and internal model approach, one may use the T_L definition to calculate the carrying cost and arrive at a proper fair value.

Similar thinking can be extended from the bond to the derivatives. An existing liquid market for derivatives would justify the "market mechanism" accounting; without that, derivatives will be carried forward for a long time period. In this case, a "private pricing" or an "internal model" approach would be the proper accounting method for the derivative valuations.

Why is Liquidation Time Horizon important?

Derivative valuations are usually based on theories with ideal assumptions attached. One big assumption is the liquidity of the market. However, real life is never black and white; rather, it is somewhere in between. The normal derivative market is not perfectly liquid; neither does it have no liquidity at all: the market lies somewhere between the two. It is therefore important to use the correct concept of liquidity for the trading of particular derivative products or transactions.

Liquidation time horizon is a general concept, which can help with the marking of derivatives in a more general sense. It can be used in dealing with liquidity issues around the exit price of an asset. As we all know, the selling and marking of 1 million shares MSFT cannot be done at the current market bid; the final exit price would depend heavily on our specification of the speed at which we would like to sell it. Generally, the longer the timeframe allowed for exiting the position, the less it costs. If an immediate sale is required, it will have to sweep many levels in the market, and there will be also significant impacts on the market supply of liquidity, which may prove to be extremely costly.

Similarly for an unsecured derivative transaction: if the goal is to replicate the value of the derivative through dynamic hedging, then the derivative value should reflect the fair valuation with future replication costs. It may be cheaper to sell directly in the market, as other dealers would put on a relatively wide bid price. If the goal is to sell the derivative right away or as soon as possible, then one should mark the derivative close to the market bid price. Certainly, we have the middle ground of selling within a time horizon, which may mean a different exit price. This then provides us one way of marking the derivative values:

$$\textit{True Derivative Value} = \textit{Ideal Fair Valuation} + \textit{Replication Cost within Liquidation Time Horizon}$$

Point 2: Selling bilateral derivatives: Novation and intermediation

It is useful to examine the selling process of bilateral derivatives. There are two mechanisms that one may utilize either to sell the bilateral derivatives completely, or unload the portion of credit and funding liabilities to someone else. The former is called novation and is shown in Figure 4.24.

The latter is called intermediation as shown in Figure 4.25.

In a novation situation, the seller of the derivative will tear up the contract with the counterparty, and the buyer will step in to fulfill any obligations left by the seller. In the process, the buyer will take over any market and credit risk of the trade, for which a premium will be charged. In order for this to occur, seller, buyer and counterparty all have to agree on the terms. Therefore, the economics of such a trade can only work when a "win–win" situation happens. For example, when the derivative transaction costs the seller in capital or CVA or funding in a way that is significantly higher than the buyer, there may be a room for buyer to step in to face the counterparty and make money from the process; at the same time that seller is relieved from the costs. When a better credit quality dealer is novating derivatives transactions to a lesser credit quality dealer for pricing or market risk reasons, novations also have to be agreed by the counterparties. Counterparties may object to such novations if they do not feel comfortable with the worse credit risk they are facing. On the other hand, if the buyer has a better credit quality, counterparties are usually happy to accept the change.

In the intermediation situation, a seller will not be out of the transaction completely. Instead, the buyer will be sitting in between the seller and the counterparty to pass through any cash flows. As long as seller and counterparty are both alive, the cash flow for the buyer will be back-to-back. Therefore the buyer is not exposed to the primary market risk. The buyer will be exposed to credit risk though, to the counterparty, and possibly funding liability. Again in order for this to happen, all three parties have to agree on the trade terms, and on a "win–win" situation.

There is a limited amount of trades and inquiries in the market among the dealers to novate and intermediate the bilateral derivative transactions, which could be triggered by credit, market or capital issues. The process can be quite long and complex, since

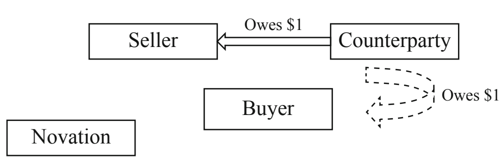

Figure 4.24 Novation of bilateral derivatives

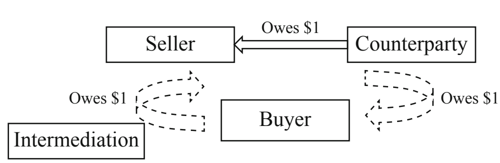

Figure 4.25 Intermediation of bilateral derivatives

legal issues, credit issues and operational issues could be involved other than the economics of the trade. Then the tri-party agreement would need to be negotiated and signed into effect. Normally for sophisticated derivatives transactions, it may take the buyer a significant amount of time to figure out all the legality and potential issues involved in the transaction. This also limits the liquidity of the bilateral derivatives novation/intermediation market.

Question: Specific Funding Spread or Market Funding Spread?

Now we are ready to go back and re-visit the question: Should specific funding spread be used in marking the derivative asset, or should one be using market funding spread instead?

The real question to ask is: Should we use “market mechanism” with “market” funding spread, or “private pricing” mechanism with “specific” funding spread?

In order to use the “market” funding spread within “market mechanism” accounting, we need the existence of a liquid market with “market” funding spread traded. The two questions that one may ask are:

- Is there such a market?
- Is the market liquid enough for banks to comfortably mark the derivative valuations to it?

First of all, as we have discussed, there are some market activities in derivative novation/intermediation. These markets are really illiquid. Clearly, if we are talking about marking the derivative asset sitting on bank’s book in the amount of billions of dollars, these activities would not be sufficient to justify derivative asset marking, given the strict accounting standard.

Secondly, if there exists some market trading of the funding component itself, one would be able to link the derivatives funding to the general funding market. Or, one could say that the funding component of the derivative can be marked with “market mechanism”. There have been efforts in the market to bring a “consensus” on the market funding level. For example, Mark-it Totem service provides an FVA-included pricing survey, in an attempt to bring “consensus” to the use of funding level in derivative pricing. However, this may prove to be too much of a stretch: there is really no liquidity in such markets, and the linking between the so-called “consensus” and the true market will be far-fetched unlikely to create “prudent” valuations. The real issue is that the liquidation time horizon for the derivative assets could be very long, so you are carrying them on the balance sheet for a long time. The costs are unavoidable.

This leaves us with the internal model pricing approach. For bilateral derivatives that are hard to sell in the market, one would have to carry the position through time, mostly through maturity. The valuation of the trade would then have to take the expected cost into account, including credit and funding:

$$C = CVA + FVA + \dots$$

The funding cost would have to reflect the firm’s specific funding cost.

4.3.6 "Market price" and "one price" regime

Traditionally, when a deal is struck between parties A and B, both parties generally agree on the same risk-free rate for discounting. In addition, party A's CVA would be equal to the negative of party B's DVA:

- Risk Free Value (A) = Risk Free Value (B) = Discounted (risk free rate)
- CVA(A) = -DVA(B)
- DVA(A) = -CVA(B).

Therefore we have the traditional accounting symmetry:

- MTM(A) = -MTM(B)
- MTM(A) + MTM(B) = 0.

The accounting principal is retained: there is no phantom value created in the trading process. We can call this a traditional "one price" regime, as the market would agree on the same pricing of credit risk and trade on the price. Therefore the "one price" will be the "market price" or "exit price". This presumption changed in the market following the credit crisis.

When a derivatives contract is collateralized, there is a clear market valuation agreed by different market participants, at which time derivative valuations can be marked to the market value. This is as close as we can get to a "one price" regime in the traditional world. However, the notion that a valuation should be based on some sort of exit valuation in the absence of collateral is actually not a consistent concept, unless the derivative contract is to be terminated at that moment. Since liquidation is not a simple thing for uncollateralized derivatives in general, it would be prudent to have funding cost included in accounting valuations to reflect the true economics of the trade.

When a deal is struck in the market, for example, a derivatives user goes to a few dealers and finds the best price in an auction process. When providing requested quotes, dealers will price in different components, including, among other things:

- Theoretical model price for fully collateralized situation
- Credit default risk
- Funding cost/benefit
- Liquidity reqirement
- Required return for capital
- Hedging cost (market and credit)
- Other terms (model risk, reserve, etc.).

If we assume that no one wants to enter into a money-losing deal, one has to incorporate all costs into the pricing. Each of these terms can be important to a viable competitive trading business, and gaining an edge in one of them can prove to be advantageous in trading. For example, a business model with access to two-way flow would have much less hedging cost, which could enhance the profit margin and squeeze other dealers with access to only one-way flow. Similarly, funding cost is a significant component of the economic value for a derivative contract and cannot be ignored. If the contract is not priced with the proper funding cost, one may win a deal and lose later through funding interest payment. As we all know, "winner's curse" could prove to be disastrous if one keeps accumulating bad deals in trading.

Clearly the party with an advantageous position in funding is more competitive, but eventually the winner will be based on the balance of all these terms together. And, for the above reasons, for bilateral uncollateralized derivative transactions, the traditional "one price" regime with an agreeable price among all participants simply does not exist anymore. This is an inefficient market where different market participants see different valuations for the same derivative contract.

4.3.7 FVA asymmetry: Is the no-arbitrage rule broken?

From the borrower's point of view, the funding value of the same transaction may not be the same as the lender. As we discussed in section 4.2.6, only when the lending/borrowing follows the equilibrium borrowing and lending equation would be fair for both parties: $r_u + u' = r'_u$. The left side is the equation is the lender's cost: funding cost + CVA; the right hand side is the borrower's funding benefit. The lender's cost will be paid by the borrower, which will be offset by the borrower's gain in benefit:

$$CVA\,(Lender) + FVA_{Asset}\,(Lender) = -FVA_{Liability}(Borrower)$$

where CVA(lender) indicates the CVA calculation from the lender's point of view on the borrower's credit, $FVA_{Asset}(Lender)$ is the funding cost by the lender holding the asset from the lending activity, $FVA_{Liability}(Borrower)$ is borrower's funding benefit from the liability. Or we also have:

$$V\,(Lender) = -V(Borrower)$$

with

$$\begin{aligned} V\,(Lender) &= V^{RiskFree}(Lender) + CVA\,(Lender) + FVA_{Asset}\,(Lender) \\ V\,(Borrower) &= V^{RiskFree}(Borrower) + FVA_{Liability}(Borrower) \\ V^{RiskFree}\,(Lender) &= -V^{RiskFree}(Borrower). \end{aligned}$$

So on day one of the transaction, it is fair for both the borrower and lender. We call the above situation symmetric valuation between the lending and borrowing. This is a happy story.

However, if the equation is broken for one reason or another, either one party would be making extra money, or one party would be upset from the trade:

- If $r_u + u' > r'_u$ and the lender is charging more than the borrower's funding cost, then the borrower will not be happy. On day one of the transaction, the borrower may be booking a loss. If on the other hand, the lender is charging r'_u, then the lender will be booking a loss from the transaction, which will not be desirable and certainly the lending activity will not be sustainable. Actually it is likely that both parties may book a loss on day one, if the transaction is done between $r_u + u'$ and r'_u.
- If $r_u + u' < r'_u$ and the lender is charging less than the borrower's funding cost, it is likely we have a win–win situation for both. This happens when the lender has a superior rating and very low funding cost.

Under the above situations, the pricing of the funding between the two parties will be asymmetric, meaning that the valuations of the two parties will not be offsetting against each other:

$$CVA\,(Lender) + FVA_{Asset}\,(Lender) \neq -FVA_{Liability}(Borrower)$$

and

$$V\,(Lender) \neq -V(Borrower)$$
$$V\,(Lender) + V(Borrower) \neq 0.$$

For bilateral derivatives transactions, the equilibrium equation will be broken most of the time for the following reasons:

- First of all, derivatives are contingent claims and their valuations fluctuate constantly with the market movement. Implicit in a derivative transaction is that the lender could become the borrower as the value flips sign where an asset becomes a liability. With the equilibrium equation depending rather on the delicate balance of FVA terms, any asset/liability amount change would tip the balance of the equation. This inevitably will lead to the asymmetry in the two counterparties' valuations.
- Secondly, when a deal is struck between the derivative counterparties, it is the total price that needs to be agreed by the two parties. Within the total derivative price, there are many different terms embedded. The funding cost/benefit is only one of them. Therefore it is very likely that the lending/borrowing equation is violated on the first day of the trade.

These generate situations where both trading parties could be making a profit or loss at the same time. In other words, positive or negative values could have been created purely from trading activities.

The question is then, if both parties can generate profits by trading with each other, is this creating an arbitrage opportunity that will break the rules of derivative pricing? Not really. The value is created through the fact that the party with better credit would have less funding cost. For example, a better credit entity would be able to make profit by trading at market level while incurring less than market expected cost. This is natural behavior since the better credit entity has to put aside more capital to maintain the better credit. So it has to make a better return than the lesser credit entity. In addition, this value is not unlimited and the funding gain/loss can affect the overall funding for the firm as well as the balance sheet situation. Unsecured funding is always limited and would create pressure in the credit assessment of the firm. The more you lend the less credit you may have. More importantly, return on capital is a critical parameter that the investor community and firm management will always watch out for. Therefore, the "arbitrage" opportunity is really competitive advantage due to credit quality and is not to be extended in general. The opposite side of gaining from funding value adjustment is the loss in intangible credit. Such lending advantage through better credit has a business scope that the firm has to live within. Beyond the business scope with excessive lending would lead to deteriorating credit and loss of advantage. Hence there should be no worry about the no-arbitrage pricing rule being broken due to the seemingly "arbitrage" opportunity. This is simply the natural market competition dynamics.

Furthermore, when the "one price" regime is broken, one also loses the traditional single risk neutral measure that every market participant shares (Kenyon & Green, 2014).[10] When the regulatory requirements, such as capital, leverage ratio and liquidity standards, are taken into account, the differential pricing can be so large that it is way beyond the traditional bid–offer spread. Even though everyone sees the same derivative contract with different value, there is no arbitrage opportunity in it: the different cost of carry prohibits one from making riskless profit. The market trading will be determined by the best bid and ask prices, which represent the lowest cost for either side of the market. None of the market participants would really be able to explore arbitrage opportunity if competition remains. In an equilibrium market, when a firm has advantage over other market participants, the advantage would be reflected in the firm having a bigger share of the market; the disadvantaged firms will have much smaller share or are simply priced out of the market. In fact, for unsecured derivatives, not only does there exist significant asymmetry in firms' specific fundings, but also asymmetries due to idiosyncrasy in firm-specific capital and liquidity costs. All this means the "one price" regime only happens in an ideal world, not something one can really achieve in real life. The theory of a "two-price" or "multi-price" and yet arbitrage-free economy is beyond the scope of this book.[11]

4.3.8 Close-out valuation and funding implications

There are multiple situations that may terminate a derivative contract, given the break clause specified in the CSA. The two most common break clauses are:

- Additional Termination Event (ATE): these are automatic terminations triggered by rating downgrade of the trading counterparty.
- Mutual Terminations or Mutual Puts: these are discretionary terminations.

ATEs are generally applied at the counterparty level, while mutual puts are at the trade level. When the trades are subject to termination, the trading counterparties have to agree on an exit valuation price so they can settle the contract termination(s).

Other than the fact that default risk is excluded from the exit valuation, it is not entirely clear at what value the derivative contract will be settled upon termination.[12] From an ideal accounting point of view, the derivative contract should be settled at the "market" exit price and not subject to a specific firm's valuation. However, there is no liquid market for bilateral derivatives; therefore one has to resort to some sort of "market" mechanism in pricing.

In a case where there is no agreed price between the trading counterparties, ISDA defines methodology on the close out valuations. Under ISDA 1992, one can choose from two methods, the "market quotation" or the "loss" method. "Market quotation" is a process to obtain market price from, for example, at least three dealers and arrives at an average "market" price. Normally, when this is not available, one can resort to the "loss" method for close-out valuations, where the replacement value is found in closing the position or realizing the value. The modified ISDA 2002 further defines the close-out valuation as being the finding of "replacement value" using commercially reasonable procedures.

Given the rules from ISDA, one may reasonably agree that the close-out valuations can be approximated as being from the "market quotation" process for anything that is not extremely illiquid. If a "market quotation" is not available, then one can attempt to find a "replacement value" from commercially reasonable procedures, including liquidation.

In the "market quotation" process, one obtains at least three prices from quoting dealers. How the dealers price the derivative transaction determines the eventual outcome of the settlement amount in the termination process. Although there is no clear definition as to how each dealer may price the derivative transaction, a dealer would need to price according to "replacement value", just as if they were taking over the transaction. Under this situation, dealers would most likely include real funding values into the quoted price, meaning they would include FVA into the pricing. In the quoting process, CSA terms, including collateral information, would be supplied to the quoting dealers. Therefore one may reasonably expect that the dealers would provide CSA enabled pricing with FVA included.

While there is no concrete information about the specific funding curve used in the "market quotation" process, one may reasonably expect that the average funding used would be around the market funding level. Given this assumption, we can calculate the FVA conditional on the ATE and MP situations based on market funding level:

$$FVA^{scen}_{|ATE,MP}(t) = FVA^{scen}(t, \boldsymbol{r_M})$$

where we suggest the FVA under ATE and MP conditions be valuated using the market funding curve $\boldsymbol{r_M}$. This also applies to the termination situation when default happens. When the counterparty defaults, the valuation of the derivative transaction, subject to recovery process, will most likely go to a reasonable market channel. The FVA calculation for that scenario would be:

$$FVA^{scen}_{|CptyDefault}(t) = R' \cdot FVA^{scen}(t, \boldsymbol{r_M})$$

where R' is the counterparty recovery rate. The defaulted portion $(1 - R')$ will go to CVA calculation. Similarly for self-default,

$$FVA^{scen}_{|SelfDefault}(t) = R \cdot FVA^{scen}(t, \boldsymbol{r_M})$$

4.3.9 Funding policy and construction of funding curve

So far we have assumed there exists a funding curve reflecting a firm's equilibrium funding cost. For the simple cases, a firm's funding curve will be:

$$r_u = r_c + X = r_c + L + u$$

where L is "market's funding spread", or the spread reflecting the liquidity of the market that the firm has to pay in raising the funding. The term u is simply the credit component accounting for the firm's default risk. This is quite simplistic in concept; however, it is rather complex in reality.

Table 4.5 *Example funding spreads implied by S&P index option market*

Term	Funding Spread
3M	0.07%
6M	0.20%
1Y	0.29%
2Y	0.33%
5Y	0.44%
10Y	0.44%

Market funding curve?

There have been efforts among the market participants, banks and/or dealers, to come up with some sort of consensus on the funding curve, or market-funding curve. With a consensus market funding curve, one would be able use to mark the bilateral uncollateralized derivatives in a way consistent with the market mechanism. While in certain special cases, a market funding curve may work; this practice is flawed in general. Here is why.

There are some special cases, where market participants trade funding, or instruments with funding implications, for relatively short maturities, and with limited liquidity. For example, the index total return swap market, where the future index return reflects the return provider's funding cost. If one can fund cheaply in the market, he would be more competitive in the market. This can also be seen clearly from the market traded index option as well. The index combo[13] prices reflect the market view on the funding of index replication. In these markets, the return provider or the seller of the index would require the index return buyer to pay for the cost of funding. In Table 4.5, a sample market implied funding spread above LIBOR is shown for S&P 500 index. This market has limited liquidity in the short term, and for longer terms above 2Y, the liquidity is even less.

Major banks and dealers, who have adopted FVA reporting in recent years, mostly carry a large bilateral book with uncollateralized assets. These assets are generally in billions or tens of billions of dollars. The maturities of these assets can be quite long, such as 30 years into the future. Since there is no conceivable funding market deep enough for such a large size asset, it would not make sense to mark these super-size assets to the "market funding curve". With the banks having to carry these assets for a long time and without foreseeable liquidation horizon in place, the funding of these assets and the cost of it would be realized through assets' maturities. Therefore, it is more realistic to mark these bilateral derivative assets to a more firm-specific funding curve.

This situation may change, however, if the future derivative market is developed into a state where it has fewer bilateral uncollateralized future obligations. In that case, it may be easier to raise funding in a more developed market with enough liquidity to mark the uncollateralized assets under market mechanism. While we cannot exclude the possibility that a deeper market may develop for funding derivatives, the present market condition does not offer a case that major banks can rely on to mark the large size derivative asset book. The possibility of having such a deep liquidity funding market is rather remote at this point.

Requirements for a funding policy

The worst situation to get into is to have the wrong trades piled up in the books, and end up losing money through time. With the wrong incentive, individual businesses may take the excessive loose funding policy to win funding cost-intensive trades; or in the other extreme, the individual businesses will be shut out from participating in viable business because of excessive funding cost. It is critically important to set up funding policy properly through a well-constructed funding curve.

In general, the construction of a funding curve should:

- provide proper incentive for the businesses, so that no business would employ funding arbitrage in taking excessive funding risk;
- reflect not only the short-term liquidity requirement for operations and regulatory requirements, but also the proper cost/benefit of long term asset/liabilities';
- reflect the proper funding management incentive in the overall firm's funding management and expectations; and
- be directly linked to the firm's credit, or CDS spread, as well as the market liquidity.

The proper methodology is for the firm to evaluate the funding needs, in the short term and long term, and construct a funding curve that fits the firm's funding profile and also takes future business expectations into consideration. One has to realize that there is no perfect scientific way of choosing the proper funding curve. We will discuss a few important points here.

One significant issue in funding curve construction is the disconnection between short-term and long-term fundings. Banks generally do not manage funding on a matched term basis, meaning they generally raise relatively short-term liquidity while investing in long-term assets. In this process, banks would keep the spread between short-term liability and long-term assets as profit. For example, retail deposits are short-term liquidity; however, banks lend out the deposit to mortgage borrowers for much longer terms. Banks are required by the Fed and regulators to maintain adequate capital levels so they can remain solvent in unexpected events; however, they are not required to raise long-term funding right now to match the long-term liabilities. This disconnection leads to an issue in the funding curve construction.

In the short term, the funding cost is clear to a bank: the Fed utilizes policy tools to tie the short-term interest rate to certain level. Banks' short-term funding costs will be tied to that, in addition to their short-term credit. This is actually an easier term to determine, since banks' short term lending/borrowing activities are managed mostly in sync with the market: they manage the short-term funding requirements to a particular time horizon, such as one month, to ensure there is sufficient liquidity to cover all normal business activities, as well as any regulatory requirements, such as liquidity stress tests. Basel III has specified a clear measure of short-term liquidity, namely liquidity coverage ratio, which is calculated to include short-term liquidity stress. For example, the collateral requirement as a result of 3-notch credit rating downgrade is stipulated by the LCR rule. This short-term funding management leads to a relatively clear visible level of funding cost for the firm.

The longer term is much less evident. Banks issue debt to manage some medium- to long-term funding and liquidity needs from businesses; however, historically there has not been much requirement for banks to match the terms of assets and their fundings. This is, as many have pointed out, one of the issues that led to the liquidity meltdown in 2008. In an attempt to address the term liquidity problem, Basel III has specified a critical measure that extends the short-term liquidity requirement to more reliable long-term funding management, namely the Net Stable Funding Ratio (NSFR). This requirement pushes the banks to maintain proper funding liquidity and with the implementation of NSFR, banks would have a clearer long-term funding cost beyond a one-year time horizon.

In summary, there are three types of funding and liquidity requirements that would become the ingredients leading to the funding curve construction:

1. Funding requirements from running the normal business dealings. This would be the net requirements from all businesses. The requirement would be from an expected incremental business level, and would be sufficient for the firm to run operations smoothly.
2. Short-term liquidity requirements from, for example, Basel III LCR. This, together with the short-term funding requirement from businesses, largely defines the firm's short-term funding cost curve.
3. Long-term funding requirements from for example Basel III NSFR. This, together with the long-term funding requirement from businesses, defines the firm's long-term funding cost curve.

The construction of a firm's funding curve is constructed based on the reasonable expected operations of individual businesses, as well as extra capital protection to keep the firm from going down. Marking a 20B derivative asset on the book is not saying that one would need to be able to fund 20B with the given funding spread. If one taps the market for 20B funding for urgent needs, it will have significant liquidity impact. The funding curve is the expected funding cost for a net asset/liability on a firm-wide basis. The expected net funding and liquidity requirement from all businesses in the near future defines an expected funding amount and the cost level that the funding desk will target.

Practically, the funding curve may be constructed asymmetrically, meaning the funding cost spread is set to be greater than the funding benefit spread. This would discourage individual businesses to pile up net funding benefit trades, and the firm would then be generally more conservative from an FVA point of view. This funding curve asymmetry brings up the concept of funding set, which has significant implications in FVA calculations. This topic will be covered in Chapter 6 when CVA/FVA calculations are discussed.

Another question one may ask in funding curve construction would be: Should equity be included in the funding level decision? The answer to this question may become clearer after we differentiate the equity and funding in Chapter 5, when we discuss capital related adjustments.

4.3.10 FVA and funding curve dependency on derivative transactions

So far our discussion of FVA has been limited to a setup, where the firm's funding curve is known and is expected to be stable. Under such a setup, one can calculate the funding cost as an integration of all funding cost over time given a defined funding spread X. In most of the practical situations, fundings are managed over expectations of present and future business activities with costs and benefits offsetting each other. As such, a stable funding curve is expected over a reasonable time horizon.

However, the funding spread X may change as a result of a specific funding activity. For example, if one raises funding in the market by issuing more debt and invests in a risky business or derivative transaction, effectively the firm will carry more debt on the balance sheet, become more leveraged, and the probability of default may increase. In this case, the firm's CDS spread and funding spread will change as a result. Four things may happen:

1. A borrowing cost would be paid in issuing the debt.
2. A CVA and FCA would be charged from the business investment.
3. If the business investment or derivative transaction is riskier than firm's existing investments, the firm's credit may effectively worsen and this would affect the overall firm asset/liability valuations. The overall impact on the firm valuations would be negative.
4. The potential liquidity and capital requirements from the derivative transaction could add more liquidity risk in the short or long term for the firm, and therefore leads to an increase in the funding spread. This would lead to the change in overall funding and other adjustments for the firm.

The real question we are concerned with here is: What is the FVA for this situation and what do we need for dealing with uncollateralized derivative valuations?

For normal derivative transactions, the incremental funding requirements would be small when looking at the funding requirement of the firm. If there is a significant offset between the supply and demand of funding among different businesses, then one would only have to worry about the net funding required to run the overall firm. At an individual transaction level, the transaction-by-transaction impact on funding curve should not be considered. This is because the funding curve construction already takes these into consideration.

However, if there were an unexpected transaction large enough to affect the firm's credit or funding profile, one would then have to consider the overall impact to the firm, including credit, funding, liquidity and capital. Practically this is a very difficult problem, because a large firm's credit evaluation is quite a complex modeling issue, and its credit rating normally remains at one level for a long time until some critical point, which makes the quantification of such impact a problem. Credit agencies and market participants would not have access to, or really pay attention to every single transaction that banks conduct. Therefore the realization of credit and funding impact would have a significant latency. It is similarly true for the firm's liquidity situation. In order to have a proper quantification on the transaction dependency of funding spread, one would need to have a sound credit evaluation model as well as funding and liquidity evaluation model. While one may attempt to establish such a framework theoretically, it is beyond the scope of simple and practical funding value adjustment considerations.

4.4 Derivatives accounting

4.4.1 Derivatives accounting with collateralization, CVA and FVA

For bilateral derivatives, inevitably one has to deal with CollVA, CVA and FVA. These three terms are the result of credit and funding considerations. Intuitively, the fair value of a derivative transaction would be:

$$V^{Fair} = V^{M} + CollVA + CVA + FVA = V^{Coll} + CVA + FVA$$

where V^{M} is the model value based on cash collateral in a chosen base currency, and V^{Coll} represents the fully collateralized valuations with collateral value adjustment *CollVA* included. Here CVA can be one-way or two-way credit value adjustments, and FVA could be one-way or two-way funding value adjustments, depending on the methodologies.

In the Table 4.6, we show three different methodologies that one may use in accounting for CVA and FVA in derivative valuations. We use M1, M2, and M3 to refer to the three methodologies in later discussions.

In M1, we have traditional bilateral CVA accounting for credit risk. There is also a symmetric funding cost and benefit based on market funding spreads. This methodology is based on "market mechanism". M2 differs from M1 in that there is a specific funding cost additional to the market funding cost. M3 differs from M2 in the treatment of DVA. Instead of DVA, M3 uses a full bilateral FVA approach with symmetric FCA/FBA calculations, including specific funding spread. Here we use u^{*} for the more general specific funding spread, which could be different from the full CDS spreads.

The question is, which methodology is better?

As we have discussed in previous sections, there is no liquid market funding available, which would allow banks to mark the significant derivative assets sitting on the trading book. This means specific funding cost should be taken into account. M2 and M3 would be of more practical application than M1.

Recall that DVA is not economical for the firm's equity holder, and FBA may be a term that is conceptually better than DVA. This means M3 would be a better choice than M2, if it is allowed by the regulators and accountants.

The March 2014 ruling by the European Banking Authority on prudent valuation clarified, in Article 13, how the investing and funding costs in AVA (additional value adjustment) should be calculated:

1. Institutions shall calculate the investing and funding costs AVA to reflect the valuation uncertainty in the funding costs used when assessing the exit price according to the applicable accounting framework.
2. Institutions shall include the element of the AVA relating to market price uncertainty within the market price uncertainty AVA category. The element of the AVA relating to close-out cost uncertainty shall be included within the

Table 4.6 *Comparison of funding methodologies*

		Base	Credit	Market Funding	Specific Funding
Methodology 1	Liability	OIS(rc) + CollVA	DVA(u)	MktFund Benefit (+L)	None
	Asset	OIS(rc) + CollVA	CVA(u')	MktFund Cost (−L)	None
Methodology 2	Liability	OIS(rc) + CollVA	DVA(u)	MktFund Benefit (+L)	None
	Asset	OIS(rc) + CollVA	CVA(u')	MktFund Cost (−L)	Specific Fund cost ($-u^*$)
Methodology 3	Liability	OIS(rc) + CollVA	None	MktFund Benefit (+L)	Specific Fund Benefit ($+u^*$)
	Asset	OIS(rc) + CollVA	CVA(u')	MktFund Cost (−L)	Specific Fund cost ($-u^*$)

close-out costs AVA category. The element of the AVA relating to model risk shall be included within the model risk AVA category.

This follows from the more increasingly adopted FVA calculations among the banks over the years.

4.4.2 Derivatives valuations, risk neutral measure, replications and deviations

Traditional derivative pricing theory usually starts with assumptions of no arbitrage, complete market and the existence of a risk neutral measure. Under the risk neutral measure, risk-free assets will earn the same return of risk-free rate. The consequence of this is that the valuations of derivatives are independent of the risk preference of the derivative users. This leads to the universal pricing of derivatives, given the same risk neutral measure for every market participant.

The risk neutral measure has been fundamental to the derivative pricing theory. However, the reality is never perfect. The deviations of the real world from the ideal world may come from various factors, such as market frictions, incomplete market, credit, funding, capital considerations and total illiquidity in OTC derivatives.

One may approach these deviations in two different ways. One approach is to start from the ideal world valuations, and make valuation adjustments based on the impact of imperfections to the derivative valuations. These adjustments serve as perturbations to the ideal world assumptions. Another approach is to price the derivative directly based on expected real life replication values. In the context of credit and funding, one may price the derivatives based on expectations of future counterparty defaults and real carrying funding cost through time. These two approaches would yield similar final valuations.

One interesting question to ask is: Is there a risk neutral measure when the ideal world assumptions are broken?

When perturbations are market-driven and not idiosyncratic to the specific firm, the risk neutral assumption would still hold. For example, the market friction in underlying market trading. However, if the perturbation has idiosyncratic component, then the risk neutral assumption would be broken. This is because the idiosyncratic risk preference would lead to different prices from different firms, meaning there would be no universal risk measure for "the market", and there would be no well-defined market prices.

Going back to the discussion of FVA, the origin of FVA comes from the illiquidity of the unsecured derivatives. Without the ability to buy and sell these derivatives, a firm is left with no option but to keep them on the books and dynamically replicate the derivative values through time. Under this situation, the risk neutral assumption would not hold and the specific firm valuation would become the true derivative valuation.

This, however, does not mean that the risk neutral measure is not useful. The risk neutral measure would be still useful, as it serves a fundamental base scenario value to start from. This enables one to value and risk manage the same way as in an ideal world; the only difference is that deviations from the risk neutral measure and ideal assumptions may need to be managed separately, just as we discussed in the first perturbation approach.

4.4.3 Derivatives valuations under collateralization, CVA and FVA

Continuing from section 4.2.13, we derive the partial differential equation for derivatives valuations under the general collateralization condition: the derivative contract may be non-collateralized, collateralized or partially collateralized.

In deriving the derivative value, we consider the following steps:

- First we transact the derivative with counterparty, borrow V from treasury, make payment V to counterparty and receive collateral in the amount C. Here C will depend on the CSA and collateral threshold. As we have derived in previous sections, the carry of interest for value C would be $r_c + p$ with p being the collateral repo interest:

$$dX_{Coll} = r_C^{Opt} Cdt = \left(r_c + p\right) Cdt.$$

  - The uncollateralized portion V-C, will be carrying with firm's specific funding cost:

$$dX_{Uncoll} = r_u(V - C)dt = (r_c + X)(V - C)dt.$$

  - Hence the total derivative carry value change would be:

$$dX = dX_{Coll} + dX_{Uncoll} = \left[\left(r_c + p\right) C + (r_c + X)(V - C)\right] dt.$$

  - Collapsing the terms:

$$dX = \left[r_c V + pC + X(V - C)\right] dt.$$

- We need to hedge the delta risk and form a risk-free portfolio:

$$\Pi = V - \delta S.$$

The change of the portfolio in value for a small time increment dt would be:

$$d\Pi = dV - \delta dS = \left[\theta + \frac{\sigma^2 S^2}{2}\gamma\right] dt$$

which should be market risk-free, and equal to the carry of interest of two terms V and δdS:

$$\left[\theta + \mu_s S\delta + \frac{\sigma^2 S^2}{2}\gamma\right] dt = \left[r_c V + pC + X(V - C)\right] dt$$

and

$$\theta + \mu_s S\delta + \frac{\sigma^2 S^2}{2}\gamma = r_c V + pC + X(V - C).$$

The carry of derivative value over time is decomposed into three parts:

- Risk free interest r_c on the total value: $r_c V$
- Extra collateral cost comparing to cash on the collateral amount C: pC
- The funding cost on the uncollateralized amount: $X(V - C)$.

This applies to both positive and negative values. When applying to negative values, we have used the symmetric FCA/FBA in term $X(V - C)$.

In the above derivation, we have not considered the default risk, which would become a cost that is linked to the default probability of the counterparty:

$$dCVA = -u'(V - C)^+.$$

Therefore, when u' is not stochastic, we have:

$$\theta + \mu_s S\delta + \frac{\sigma^2 S^2}{2}\gamma = r_c V + pC + X(V - C) - u'(V - C)^+.$$

On the right hand side, this equation includes the following terms:

- Drift for base derivative value and funding value for partial collateral: $r_c V + pC$
- Symmetric funding cost/benefit for uncollateralized portion: $X(V - C)$
- One-way CVA credit cost: $-u'(V - C)^+$.

More rigorously, we can have a hedging strategy with both underlying S and counterparty CDS u' hedged at the same time:

$$\begin{aligned}\theta + \mu_s S\delta + \mu_u u'\delta_u + \frac{\sigma^2 S^2}{2}\gamma + \frac{\sigma_u^2 u'^2}{2}\gamma_u + \lambda_{Su} S u' \\ = r_c V + pC + X(V - C) - u'(V - C)^+\end{aligned}$$

where $\delta_u = \frac{\partial V}{\partial u'}$ is the CDS delta hedge, $\gamma_u = \frac{\partial^2 V}{\partial u'^2}$ is the gamma term, and $\lambda_{Su} = \frac{\partial^2 V}{\partial S \partial u'}$ is the cross gamma for S and u'.

Given a constant collateral threshold H, we would have:

$$C = \max\left(0, V^{+} - H\right) - \max(0, -V^{-} - H)$$

Clearly the above derivation would correspond to M3 methodology, with $X = L + u^{*}$. With the symmetric FVA assumption, there would be no DVA. Applying CSA, calculating C and solving the equation would lead to proper valuations with CVA and FVA included.

For M1, and M2, one would need to include the DVA benefit term. Note, however, the hedging argument would not work for DVA since one cannot hedge or replicate DVA properly.

4.5 FVA questions and debate

FVA is a very controversial issue in concept, accounting, modeling and implementations.[14] In the following, we will go through a number of questions or dilemmas in more detail to try to clarify the problems around FVA through examples. For each discussion point, there will be a question to answer, and the extensive discussion around the topic will be provided. While many of the problems are potentially very deep, the discussions here may be limited. However, we hope to present a picture that is clear and consistent throughout.

4.5.1 Debate point: Existence of FVA 1

Question: Theoretically the discounting rate for a project should be determined by the risk of the project.[15] Why is the firm's funding cost relevant? There should no FVA apart from the market defined discounting rate.

The question looks quite reasonable. How can a risk project's discounting rate depend on a firm's specific funding cost?

Well, recall our discussion about the two accounting methods for derivatives, namely market mechanism and private pricing approach. The question is clearly coming from the sole perspective of market mechanism: the market's price and the market's discount rate would determine how a risk project should be discounted. Therefore the firm's specific funding cost should be irrelevant here. This is, however, not accurate and misses the broader picture by ignoring the specifics of the risk projects and any practical liquidity of the risk projects.

The risk projects we are considering here are bilateral uncollateralized derivatives. They are governed by ISDA and bilateral CSA. Given there is little liquidity in the novation/intermediation market for bilateral derivatives, one cannot simply put a market price on their valuations. Instead, the derivative transactions would have to be carried forward for a long period of time. Such a situation leaves the firm with no other choice but to take the internal model approach to account for the derivative transaction values. In doing so, the cost in carrying such a position would have to be properly calculated with the relevant funding curves, which have to be linked to the firm's specific funding curve. Hence the existence of FVA is justified.

Another relevant question to ask on the opposite side would be:
For the billions of dollars of derivatives assets sitting on banks' trading books, how would you discount the future cash flows from these assets, given the limited liquidity in the market?

Certainly for derivatives with reasonably liquid trading, the discounting rate should be determined by the market. This is because the market would provide a clear exit price with the market funding curve.

4.5.2 Debate point: Existence of FVA 2

Question: The Modigliani-Miller (M-M) theorem states the valuation of a risky project would not depend on how it is financed, which means the value of the firm should not depend on its funding rate. This fundamental theorem in capital structure really says there should be no FVA. Is this right?

To explore this question, we need to look at the M-M theorem in a little more detail.

Let us consider two firms that are identical except for their capital structure. One is financed purely by equity; the other is leveraged, meaning that part of the firm is financed by equity; and the other part is financed by borrowing or debt issuance. The M-M theorem states that the value of the two firms should be exactly the same. If we apply this to derivative valuations, we would conclude that the same derivative should carry identical valuations across firms, and therefore there should be no FVA linking to specific firm's funding.

Is there anything wrong in the above logic?

The M-M theorem on risky project valuations carries some assumptions:

- The market needs to be efficient and information needs to be symmetric between investors and corporation; and
- Investors and corporations borrow at the same rate.

If we examine these assumptions in the uncollateralized derivative market, it is not difficult to see the flaws in the application of the M-M theorem to this market. First of all, this uncollateralized derivative market is inefficient. One simply cannot liquidate the uncollateralized derivative freely. The novation process is complex and potentially costly. Secondly, it is quite clear that investors and corporations do not borrow at the same funding rate. Without the same funding rate, they would not be able to communicate the valuation at the same level.

Given the breakdown of the above assumptions, as precisely what we discussed in previous sections, M-M theorem does not hold in the uncollateralized derivative market, and therefore the "no FVA" conclusion is incorrect.

4.5.3 Debate point: FVA as private adjustment

Question: Funding cost is just like other costs, like hedging cost from market friction, operational cost and other costs, so shouldn't we account for FVA the same way as for private adjustments, just like other cost items?

The question looks reasonable from its appearance that funding costs are the same as other costs, such as the bid/offer spread that one has to pay when hedging derivative transactions. However, there are intrinsic differences and this is why it is better to treat funding costs within fair valuations.

Hedging costs, operational costs, technology and compliance, as well other business related costs, are more associated with the business model of the relevant derivative trading desk or line of business. Although they may be different across firms, they are generally of similar magnitude for different dealers. For example, dealers would execute hedging transactions in the same market with same market frictions. For example, hedging is usually done on a portfolio basis; dealers warehouse risks and carry certain dynamic hedging costs constantly. While there may be a day one hedging cost for a specific transaction, this cost is passed through P&L instantly. There would be no need to make extra adjustment. From a global portfolio basis, the expected cost per transaction is not a well-defined number. It is better to account for them on a portfolio basis, more as a business cost. Similarly, other costs, such as operational costs, are really passed on within the business model. In some situations, banks do adjust the derivative portfolio valuation by expected frictions such as valuation reserves; however, this happens only on a portfolio basis, not on an individual transaction level.[16]

Funding cost, however, could be very different. Derivative values are replicated through dynamic hedging, including borrowing and lending. This is embedded deeply in the derivative trader's mind and is the foundation of derivative trading. Funding cost is the well-defined cost that is to be incurred given the known condition from the firm, such as the firm's funding curve. This cost is well-defined for funding a set of transactions, which mimics the real-life situation. In addition, the funding cost is well-defined not only for funding a set of transactions, but also at the individual transaction level: the marginal funding cost for a transaction within a funding set of transactions can be calculated specifically. Another reason funding costs are different from other business costs is that the funding cost is usually much larger than any other cost items and is relevant enough to affect the economics of derivatives trading. Although they may be different across firms, they are generally of similar magnitude for different dealers. For example, dealers would execute hedging transactions in the same market with the same market frictions.

4.5.4 Dilemma: Banking book and trading book consistency

Question: Should banking book and trading book markings be consistent with each other?

Consider two identical assets on A's book as a result of two different activities. The assets are both \$1 future cash flow payments by counterparty B at time T. One of the assets is simply a loan to B, which will be sitting on the banking book. The other asset is a result of a derivative trade with B, which is carried on the trading book(see Figure 4.26). The question is: should the present values of these two trades be marked exactly the same?

Figure 4.26 One asset cash flow in the future

It is very tempting to conclude immediately that these two trades should be marked exactly the same way, since they are exactly the same. The cash flow and risk are exactly the same! How can the same cash flow be marked inconsistently?

The answer is not as obvious as it looks. The issues related here are much deeper.

One of the issues is related to the difference in banking book marking and trading book marking. In a "held for maturity" accounting for a loan, one would expect the lender to hold the asset to maturity, and in this case, the marking of the asset will follow accrual accounting. This means the marking of the cash flow $1 at T may be linked to the interest spread that B pays on day one of borrowing. The expected interest income will be accrued evenly through maturity T. This means that the value of the banking book cash flow may be linked to the "past" state. The marking of this banking book asset is not real "present value", its value is tied to the past history, and is less sensitive to the daily market fluctuations.

On the other hand, the trading book asset will be marked based on trading book or fair value accounting. The only way to mark this asset is through cash flow discounting to present time, using a discounting rate, either from market implied rate or a reasonably constructed funding curve. This present valuing of future cash flow is clearly different from the banking book valuation. If the market has moved away from the original loan spread, the two valuations would be remarkably different.

How can these two accounting values be consistent?

The two values, as explained above, can be self-consistent within the accounting regime. Although their marks may be ostensibly different, they are consistent in the sense that they are looked at differently from two distinct views. Banking books, such as loans, are constructed based on largely "real" or "physical" measures. Because of the nature of long-term holding to maturity, the value of the asset should not be affected by short-term trading activities, where prices reflect the short-term market view on risk premiums. Throughout the maturity, risk premiums would go up and down all the time. Marking long-term assets to such short-term market moves would not make much sense unless the probability of default in "real" measure changes, which would then be reflected in loan loss reserves and the likes.

Banks make money by lending to counterparties, from small mortgage borrowers, individual and large business owners, to industrial players, and so on. For these borrowers, there may be really no reliable market trading for the credit of individual borrowers. Banks make money when they charge extra spreads, which are expected to exceed the actual default rate of these borrowers. The realized profit of loss of the bank would depend on the "actual" realized loss from these borrowers. If on day one, the loan office made a wrong decision and charged a too small spread from the borrower, it will be realized as a loss in the future if the borrower defaults. Only the "real" loss should affect the banking book asset price, not the short-term risk premiums. The sub-prime crisis, as prelude to the more giant financial meltdown, was caused by the irresponsible lending practice that led to a situation with too little spread for too much risk. In this case, it caused hefty losses and massive loan loss reserves in many banks. On the other hand, when the market recovers and the lending practices are tightened, banks return to profit again as loan loss reserves are released. In summary, the banking book is viewed in the "real" measure.

The trading book is viewed differently. Credit defaults in the trading book are largely viewed and marked in a risk neutral measure. If the counterparty borrowing

Table 4.7 *Comparison of implied, realized and non-default spreads*

S&P Credit Rating	Implied PD	Avg Spread (bp)	Realized Default (bp)	Non Default Spread (bp)
AAA	0.29%	49	7	42
AA	0.49%	81	7.5	73
A	0.65%	109	13	96
BBB	1.04%	173	48	125
BB	2.34%	391	182	209
B	4.91%	818	428	390
CCC	7.67%	1278	908	370

the money carries a CDS spread, the default of this counterparty has to be marked consistently with the CDS spread. For the counterparties with no market CDS spread, reasonable effort should be devoted to constructing a risk-neutral CDS pricing curve, where the market traded risk premium will be included. This way the asset in a trading book will be marked in “risk neutral” measure. In this measure, the value of the asset will be changing constantly as market moves.

In Table 4.7, we show the implied probability of default from CDS spread compared to the realized default spread. The probability of default data is taken from year end 2013, where the market implied credit spread has shrunk significantly following the credit crisis. The historical realized default data included the defaults in the credit crisis. The comparison shows that the majority of the CDS spread charge is really the risk premium linked to the market fluctuation of the CDS spread, not the actual default. In the “real” or “physical” measure, the default spread is much smaller than the market implied spread in a “risk neutral” measure. This means the trading book asset marking could be very different from the banking book marking, even for the same entity: in the banking book, one does not have to worry about the market fluctuation of the counterparty CDS, while in the trading book, one would have to charge the extra “market uncertainty” risk premium. The comparison is even more extreme when CDS spreads were wide during the credit crisis, where the implied CDS spread would double what is shown in the table, and therefore the non-default spread would be much higher compared to realized default.

From the above discussions, we see that the banking book accounting and trading book accounting for assets with exactly the same cash flow with exactly the same counterparty may not be marked the same way. However, separately they are consistent within the two different accounting methodologies.

Is this wrong?

Not really. This is related to the business model differences between the banking and trading books. The market price of risk concepts and risk premiums are constructed differently between the books. For trading books, one has to follow the market, at least with reasonable effort to link to the market risk premium; however, for banking books, “real” measure or “real” risk premiums are adopted. There is no obligation for banking book prices to follow the market prices actively. Under stressed situations, a distressed asset market would reflect fire-sale price, driven mostly from one-way selling pressure. However, banks should be allowed to mark the distressed asset to have a more reasonable “real” measure expectation given the changing market, other than marking

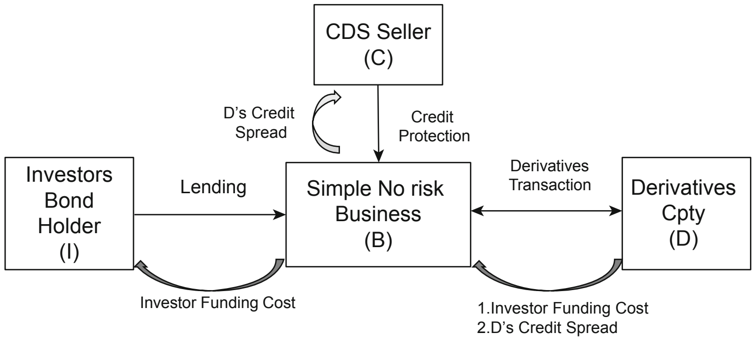

Figure 4.27 Funding cost for a default risk-free business

to the "fire-sale" prices. This was a hotly discussed issue in the credit crisis, and the banks were allowed to mark more reasonably for these distressed assets.

4.5.5 Discussion: Funding cost for a default risk-free business

For a derivatives business, I can hedge market risk away, hedge credit risk away, then I am left with a risk-free business, so my funding should be the risk free-rate. Is that correct?

In Figure 4.27 we show a simple construct of business (B) with no risk: Investors (I) lends money to the business (B), whose only business with the derivatives counterparty (D) is a derivatives transaction. Let us make it simple in that the derivatives transaction is effectively a lending to D. To hedge the credit default exposure to D, B goes to the CDS seller (C) to buy credit protection. This way B is effectively a riskless operation.

Traditionally, the fair return from a riskless business should be at a risk-free rate. Immediately, we have questions:

1) What is the risk-free rate here?
   Well, we know the real risk-free rate is the overnight index rate, or general collateral rate r_c, which is based on the convention of collateralized lending. Clearly, this is not the case in our example, because (B) is not posting collateral to (I). Therefore the traditional "risk-free" rate will not work here: (I) has to charge higher than OIS rate to make sense.
2) What is the difference between (I) investing in (B) vs. the collateralized lending?
   The difference here is the missing collateral value. Not having the collateral, (I) has to come up with the equivalent of the missing collateral. The cost of doing that is really (I)'s funding cost: r_c+ (I)'s funding spread, which is the cost that (I) has to pay in raising money.

From the above analysis, in order to be fair, investors (I) will have to charge (B) "investor's funding cost". Unless however, investors have "free" money, where (I)'s funding spread would be regarded as being close to zero. In such a case, the return required from (B) could be close to general collateral rate r_c . Generally such "free" money does not exist in reality. (Note: When the Fed executed the QE program, there were occasions when money was close to “free” for investors and banks, even with negative interest rates.)

Another possibility of obtaining cheaper funding is to have the ability to repo the “riskless” construct. If one can use the “riskless” construct as collateral, one may borrow at close to OIS rate from the investor. However, this is really a big “if”. Generally this is not an option when we deal with derivatives valuations. Practically, repo of derivatives would not work because of various complexities involved in the derivatives repo process.

In conclusion, even if (B) is riskless, it has to pay “investor’s funding cost” to the lender. If (B) is borrowing from the general market, then this “investor’s funding cost” can be interpreted as market’s average funding cost $r_c + L$.

Now we consider the fair cost that (D) needs to pay (B). In order to be even, (B) needs to be paid on the cost from (I), and the (D)’s charge on credit protection. Therefore, (D) will be paying (B) the two components, which are passed through to (C) and (I). In the whole process, starting with nothing, (B) has no risk and makes no money.

4.5.6 Discussion: Funding cost for a bond

When I purchase a bond and mark the bond value on my book, do I need to add my funding cost to the bond price?

This clearly touches on a confusing point in funding value adjustment.

First of all, a bond price has the embedded funding cost of the investor. Assuming the OIS rate is the risk-free rate, the bond price will imply a funding spread reflecting the cost for the investors buying the bond, in addition to the credit and liquidity spread tied to it. This funding cost is really the market implied funding and liquidity cost, which reflects the market supply and demand for the bond as well.

Secondly, if the bond is liquidly traded in the market, there will be a well-defined market price. So the bond value has to be marked to the market price, which does not include the buyer’s specific funding. This is “market mechanism” accounting.

This is a typical case where we have no ambiguity when the price of the instrument is available from the market: the exit price is well-defined in the market, therefore it should be marked with market implied funding, which is already embedded in the market traded bond price. We have the following:

$$V_{LB} = V_{mid}$$

where *LB* stands for liquid bond. We can interpret the valuation of liquid bond this way: When there is a well-defined liquid exit price, the *liquidation time horizon* for the instrument is really immediate, which means zero or extremely small funding cost.

However, if the bond itself is not liquidly traded, one cannot sell the bond easily in the market. This implies the bid/ask spread is significant. How should we mark the bond in this case? Clearly, mid-price may not be sufficient: the exit price of the bond will not be mid-price. Adding specific funding spread for carrying the bond on top of the mid-price would make sense: The *liquidation time horizon* for the bond is long. Before the bond can be liquidated, the proper cost for carrying the instrument on the book should be included in the bond pricing. Certainly the funding cost adjusted price

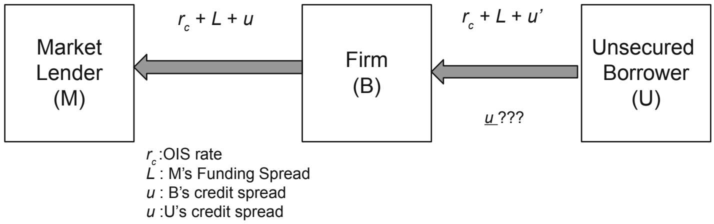

Figure 4.28 Passing funding cost to customers

cannot exceed the bid/ask range. In another words, ignoring any other cost in carrying the bond, the bond price of illiquid bond may be marked as:

$$V_{IB} = \max(V_{bid}, \ V_{mid} + FVA)$$

where *IB* stands for illiquid bond, and FVA is calculated based on the specific funding spread of the bond holder and within the liquidation time horizon of the bond or the holding period. So if the bond's bid/ask spread is wide and the carrying funding cost for the bond is small, then it is more accurate to mark the bond with FVA.

In summary, the answer to the question would depend on the liquidity, or the liquidation time horizon of the bond. For a liquidly traded bond, there should be no funding cost as the liquidation time horizon is short; while for an illiquidly traded bond, one may need to add the funding cost for carrying the bond through the liquidation time horizon.

Question: Passing the cost to the customer

From market trading point of view, can the funding cost be transferred to the borrower? If you charge the customer your own funding cost in addition to customer's credit, you may not be as competitive and may not win any business.

This is really a fundamental question. The best way of looking at the economics of unsecured lending is to examine the cash flow on a daily basis. In Figure 4.28, we show an example:

- Firm (B) is borrowing from market lender (M) while lending to unsecured borrower (U).
- In borrowing the money from (M), (B) has to pay the interest to cover
  - ✓ OIS rate r_c
  - ✓ M's funding spread
  - ✓ B's credit default
- On the other hand, (U) has to pay (B)
  - ✓ the interest to cover OIS rate
  - ✓ M's funding spread (pass through cost)
  - ✓ (U)'s credit default.

The question is now: Is it right to charge (B)'s own funding spread $\underline{u}$? Would the charge of $\underline{u}$ make one much less competitive? Here we use under bar to indicate that it may be different from (B)'s credit spread u.

Table 4.8 *Listing of interest cost terms charged by the lender*

Pay	Rec	Comment
r_c	r_c	OIS rate
L	L	Market funding Spread
u	u'	Credit defaults
	u???	Funding cost?

First, we know (B)'s interest payments r_c and L cancels with (U)'s r_c and L payments. We also know (U)'s interest payment u' is to cover (U)'s credit default: if (U) defaults, the loss to (B) is real. Then what is left is (B)'s interest payment u to (M), and the payment in question $\underline{u}$.

One of the effective ways to judge whether a model is working well or not, is to examine and see if the model has bleeding over time. Here we see a bleeding problem if we do not charge the term u: every day passing by, (B) will incur a loss in the amount of $u \cdot dt$ that is paid to (M). This leads to our conclusion that (U) has to pay $\underline{u}$ to (B), otherwise (B)'s business model will not work.

Now would this funding charge u make (B) much less competitive in the market?

First we have to realize that the lower credit party will not be competitive in the lending market. This is a natural behavior of the market: You have to carry very good credit to be able to compete in the lending market. The better the credit, the lower the funding cost. This is a general and fair rule in market competition.

Secondly, when participants in general all charge the self-funding cost u, then the market mechanism will work its way through competition. Indeed, more and more dealers are pricing FVA in the market. If a market participant does not do that, he may win a lot of funding cost intensive trades and accumulate losses through time. This loss will show up eventually in the form of write-downs, a problem which many banks have gone through in introducing FVAs.

With banks increasingly recognizing FVA, more and more market participants are pricing FVAs in derivatives trading. FVA has become one critical component among various competitive pricing measures in the bilateral derivative market, and it is the proper measure for the real economic cost and is fair. Indeed, banks with high funding costs often engage in intermediation discussions in order to get relief from the funding cost embedded in derivative asset trades.

So the answer to the question is, if FVA makes one less competitive in the market, it really means that he should think twice about the derivative transactions that he is entertaining. FVA reflects the natural market competition in borrowing and lending.

4.5.7 Question: Fair value marking in borrowing and funding cost charge in asset lending

When a business borrows \$1 from the market investor, paying for example $r_c + L + u$, the borrowing will be marked at \$1 at the end of the day through DVA. So since you gain from DVA, there is no cost to the business at day 1: you borrow \$1 and mark at \$1, so why on *the other side, should the business charge the borrower an additional term u even if the borrower is already paying for its own default u' ?*

The argument in the question can be deceiving in that it is true that one marks the borrowing at \$1 with DVA. However, it leads to the wrong conclusion. This question clearly shows confusion between static fair valuation on the balance sheet vs. the cost from payment of cash flow. For the discussion below, we will use bilateral CVA, FCA and FBA^M.

It is true that the debt would be marked at its fair valuation with DVA included in it. Let us use a simple example below: business is borrowing \$1 from market investor, liable to pay interest $r_c + L + u$. All the future liability cash flows including interest payments and principal would be discounted using the same spread (as shown in DVA and FBA^M) so that the current value would be \$1: for each period

$$V = -\frac{1 + (r_c + L + u)\, dt}{1 + (r_c + L + u)\, dt} = -1.$$

We can also regard the fair valuation as being the result of three things: OIS discounted price, market funding benefit term, and DVA. We use approximate equal because calculating separately may not get exactly \$1 because of cross terms between L and u.

$$V \approx V_{OIS} + FBA(L) + DVA(u).$$

On the business' balance sheet, we have an asset of \$1 (we have not invested/lent yet), and a liability of \$1 to the investor. These two offset each other. All is good.

Now let us take a look at what happens when time moves forward.

Conceivably, the liability can be marked as \$1 if there is no market movement: the future interest payment with principal will be discounted using the same spread, so the present value is still \$1. However, there is one thing that has changed while moving forward in time: the business has to pay interest to the market investor as time passes by, including r_c, L, and u. This is where the cost is coming from: every day that passes, the business has to pay back the interest amount to the investors. This dynamic operating cost of interest is irrelevant to the marking of the liability.

So to make the business worthwhile, one has to invest/lend the \$1 asset wisely to the borrower. The minimum fair interest charge should include u. Without charging this cost from the asset side investment, one will end up bleeding through time.

4.5.8 Question: Repo of derivatives

Can derivatives be repoed in the market?

The repo market provides one important funding source for businesses and operations. When cash is needed, a repo user can enter a repo transaction with a repo provider, where the repo provider will give cash to the repo users. In return, the repo user will hand over securities with some haircuts, and at the same time promise an interest when the securities are bought back. (See Figure 4.29.)

If one can repo derivatives in the same way as securities, it will reduce the uncertainty around the funding of derivatives. The funding of derivatives will then

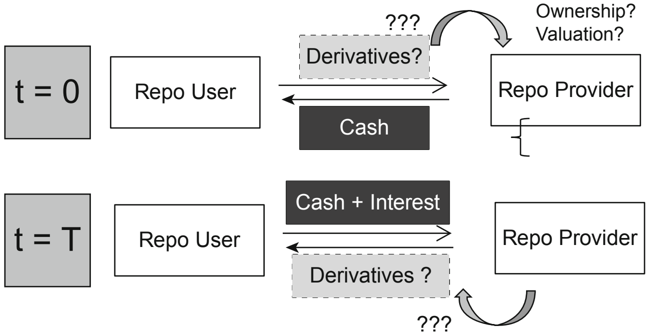

Figure 4.29 Repo of derivatives

be determined by the repo market. Unfortunately, this is almost impossible in practice for a number of reasons, which we discuss below.

One critical component of the repo transaction is that the repo provider needs to own the securities that are handed over, along with the cash sent out. As we all know, derivatives are legal transactions that are based on bilateral agreements such as CSA, which is difficult to transfer among different parties. This is almost as the same as the novation process discussed earlier. Therefore without a plausible way for the repo provider to own the derivative transaction, the repo of the derivatives will be very difficult, if impossible, to implement. In addition, the servicing of derivatives would require dynamic posting of variation margins and initial margins, which makes ownership by the repo provider unlikely in reality.

Furthermore, pricing of derivatives can be sophisticated and derivative values can vary greatly over time, which would lead to very large haircuts. This would defeat the purpose of relying on derivatives repo market for funding the transaction.

4.5.9 Question: Supra bond marking and funding spread

When you take a look at supranational names like the World Bank (IBRD), European Investment Bank (EIB), etc., their bonds are traded with close to zero or sometimes even negative liquidity spreads. Does this mean one should not charge the market funding spread when lending to the supra names in a derivatives trade?

It is true that the supra names all have superior ratings due to the nature of their structural formation. Their ratings are often better than even many of the sovereigns, which attract demands for their bonds. Due to the special demands of supra-issued bonds, the yields are often sub-LIBOR, which means these supras' funding cost is lower than LIBOR, meaning smaller liquidity spread L. Sometimes, the liquidity spread could even be negative.

However, this has nothing to do with how much you, the business, would charge the supra names in a derivatives transaction. The funding cost you may incur comes from your own funding activity, so your investment or embedded lending to supra names would not be affected by this. As long as the market is charging you for liquidity spread (market funding spread), you would be bearing the cost and should be charging the derivatives counterparty for it.

Note however, this behavior could create difference in comparing the value of market traded supra name debt and the value you are marking the derivative. Market traded supra name debt may imply the special demand of supra-issued bonds, which shows small liquidity spread L. On the other hand, in marking the derivative asset that the supra is borrowing from you, you may be marking using your own funding cost.

One good question: If I own supra debt and derivatives at the same time, would it be ok for me to mark differently the cash flows owed by the same supra name at the same time in the future in debt and derivatives?

This is a truly core FVA question. The answer would be yes, one may mark the same cash flow differently. The reason for having the different marking mechanism is the exchangeability of cash flow between the supra-issued bond and the derivatives transaction. We will use the following analogy.

In truth, this is not much different from what we discussed in differentiating the liquidity involved in marking uncollateralized derivatives: you cannot easily liquidate/novate the derivatives contract you executed with the supra names. Therefore you will bear the cost of keeping the position for a long liquidation time horizon. The funding cost incurred during this effective duration would be the funding cost that we are discussing here. On the other hand, the supra-name bonds have a relatively short liquidation time horizon, which would be marked differently from the derivatives.

Indeed, people with lower credit would seek funding/credit relief in the market. In the following we assume that we will look to better credit third party novation/intermediation for a relief in the funding cost incurred. Let us say:

V_- : derivative value without CVA, FVA
FVA_L : expected market funding cost for keeping the trade through maturity
FVA_u : expected specific funding cost for keeping the trade through maturity
C: novation/intermediation cost for the derivative, on top of CVA and FVA_L

Here C is the cost that the third party would charge to take ownership of credit and funding cost of the derivatives; effectively C represents the cost in exiting the market. For simplicity, we assume all parties agrees on the CVA and FVA_L calculations. Then the fair value of the derivatives would be:

$$V = \min\left(V_- + CVA + FVA_L + FVA_u,\ V_- + CVA + FVA_L + C\right)$$

This value would be different from the marking of supra-named debt.

4.5.10 Question: Differential funding levels with a firm

Within a firm, there could be different lines of businesses, which carry different risks. This would mean that these different business lines should carry different funding cost. Would it be correct or better to use specific funding spreads for different business lines to mark their books differently?

This question touches a subtle point about funding management.

The funding cost of a firm depends on many different aspects of the firm's businesses. In a simple way, we may take the funding cost of the firm as being closely linked to the balance sheet of the business, the leverage of the firm and the risk of the business.

Different businesses may carry different risks; however, they may be employed to achieve some common business goal from the overall firm perspective. For example, the retail deposit side of the bank may serve in the overall bank as a low cost funding channel, whose funding will be used in other different business lines. The derivatives business would be one of the business users of this funding.

An individual business may not be big enough to raise funding efficiently in the market and therefore requires the firm to manage the funding cost/benefit on a firm-wide basis.

The firm's funding policy can be constructed so that it may provide incentives for businesses to realign their individual goals to firm's common goal.

However, if the firm has business lines with very different risks and individual funding capabilities, it may be optimal for the firm to split the firm-wide funding into individual fundings. This way, businesses with really good credit quality can take advantage of lower funding cost to operate at a more efficient level; while the much lower credit quality business has to bear the individual funding cost and be viable for itself.

Certainly, splitting can be employed within a firm to manage the different funding requirements and funding levels. This clearly depends on the firm's funding management, especially in providing proper incentive for the individual business lines to operate within its own means.

Summary

In this chapter, we have discussed the following:

- Derivative valuation and the funding of derivatives are not separable. This is the general principle of derivative valuation. Within the traditional derivatives pricing framework, funding is simply the necessary borrowing and lending activities involved in the replication of derivative values. While other market risk factors can be replicated exactly the same way in a risk-neutral setting, the evaluation of derivative funding value would depend on the collateralization of derivatives.
- Collateralized derivatives will have their valuations linked to the funding value of the collateral. Different collateral currencies and assets carry different funding values. Collateralized derivatives will be valued based on differential discounting. Realization of the collateral values may be limited by practical constraints in operations.
- Uncollateralized derivatives have to incorporate the funding cost of the derivatives into the valuations, namely FVA, with FCA for funding cost and FBA for funding benefit.
- DVA and FVA have overlaps in that they both carry the benefits for liabilities linked to the firm's credit risk. Symmetrically to FCA, FBA is a better economic term, while DVA is not economical and cannot be replicated by the firm.
- There are two types of accounting for derivatives within a trading book, "market mechanism" and "private pricing approach". Market mechanism does not apply to the unsecured derivatives, because novation/intermediation market does not carry much liquidity. Therefore firms have to calculate funding costs in carrying the positions.

- As evidenced by market novation activities of unsecured derivatives, the "market" quoted prices provided by individual banks would incorporate various pricing charges including FVA. This shows that there is no unique price agreed upon by the "market". Rather, because of the liquidity of such market, it is prudent for firms to carry the expected "replication" funding value for the unsecured derivative transactions.
- FVA reflects the relevant economics incurred during the traditional derivative value replication process for the unsecured derivatives. The embedded borrowing and lending accounts in traditional derivative pricing theory need to be replaced by the firm-specific economics, leading to the expected FVA calculations.
- The two accounting methodologies can be generalized using the expected liquidation horizon for the derivative. Market mechanism is a special case with $T_L \to 0$; while for very illiquid derivatives, one has to include in the fair valuation the funding cost through T_L:

$$FCA = -\int_0^{T_L} V(t)\, X(t)\, dt.$$

- Potential market "consensus" funding curve, or market funding source does not carry sufficient liquidity for banks to mark their funding exposure. Banks would have to carry the derivative asset on the book for a long period of time, funded by banks' specific funding spreads.
- A firm's funding policy is implemented through a proper funding curve, which provides proper incentive for businesses in capturing trades with the proper funding implications.

5 Other Valuation Adjustments

5.1 Liquidity value adjustment (LVA)

5.1.1 Introduction

Embedded within the funding management, every firm also has its liquidity management. Liquidity management serves two purposes. First of all, the firm has to ensure it has enough liquid assets to cover its operations, such as purchases, interest payments, payrolls and collateral posting. These are critically important to the firm's operations. When the firm fails to fulfill any of the obligations, it essentially declares bankruptcy. Secondly it has to fulfill regulatory requirements, which ensure the firm remains viable under fluctuating market conditions. For example, the Basel III liquidity risk measurement and standards, introduced in December 2010, provides a framework that financial institutions need to adopt along with the capital rules. These rules require the firms to put aside additional liquid assets as a precaution against uncertain market liquidity stress situations, such as the events of December 2008.

Under the new Basel III rules, liquidity coverage ratio (LCR) and net stable funding ratio (NSFR) are two stepwise critical requirements that come into effect from 2015 to 2019. These rules will generate additional liquidity requirements for the financial institutions. For example, a stress scenario of public debt credit downgrade up to and including a few notches has to be applied in assessing the potential collateral required for derivative transactions. This is viewed as a necessary requirement in such a situation, as demonstrated by the 2008 financial crisis. When AIG was downgraded by rating agencies because of losses in mortgage-backed securities, AIG's counterparty collateral call for the CDS positions amounted to $100 billion.

This requirement means that when a derivatives transaction is executed, a liquidity related cost would be incurred: the firm has to put aside additional short-term liquidity to cover the potential stress scenario, which hampers the firm's ability to make extra returns on this short-term liquidity. That is a cost to the firm, and therefore a cost for the derivatives transaction, which bears the consequences of the additional liquidity requirement. Similarly, NSFR imposes requirements on the term funding of various types of derivative related transactions. This means some of the short-term fundings that banks utilized before the rule came into effect have to be replaced with term funding greater than one year. For example, repo and stock-borrow transactions may have term funding implications. The cost from NSFR requirement on these transactions could have significant cost implications on derivative trading.

This leads to what we may call Liquidity Value Adjustment, or LVA.[1]

In the following and as an example, we will explain in detail the requirement and calculations of LVA as of the result of LCR rules.

5.1.2 LCR and LVA calculations

LCR liquidity requirements

LCR requires the firm to manage sufficient liquidity for a time horizon of 30 days even under stressed situations. To satisfy the LCR rule, financial institutions would need to calculate, among other things:

- A 30-day inflow and outflow for high quality liquid asset (HQLA) and non-HQLA assets.
- The inflow and outflow should cover stress situations, for example adverse deposit run rate, rating agency downgrades, and so on.

The stock of HQLA should exceed the total net cash outflows over the 30-day period:

$$\frac{Stock\ of\ HQLA}{Total\ Net\ Cash\ Outflows\ over\ 30\ Calendar\ days} \geq 100\%$$

As to the derivatives, there could be a number of extra liquidity requirements that must be satisfied for a 30-calendar-day time horizon, including the liquidity requirements from running business as usual (BAU), such as cash flow payments, valuation changes, collateral moves, collateral replacement, terminations, mutual put exercises, etc. These can be computed from the existing derivative positions and existing collaterals. One example is that even if the counterparty is posting cash as collateral, as long as CSA allows the counterparty to post a non-HQLA asset, one has to calculate the collateral replacement requirement and put aside the HQLA for LCR purposes.

The second type of LCR requirement beyond BAU is the liquidity requirement under adverse stress situations. This includes potential valuation and collateral changes under market movement as well as credit downgrade of the firm. For example, as a result of rating agency downgrades, one would expect at least three different types of potential liquidity flows as summarized below:

- In addition to the business as usual 30-calendar-day liquidity requirements, firms are required to calculate the potential derivative valuation changes over 30 calendar days. This may be computed from examining the historical rolling 30-day periods and MTM fluctuations. There would also be collateral changes as a result of the valuation changes.
- For ratings-based threshold CSA, firms would be required to post additional collateral for liabilities because of the downgrade. For example, a firm with A+ rating has a liability of $50 million with a counterparty given a current collateral threshold of $20 million (see Table 5.1). So the firm will be posting $30 million collateral. When the firm gets downgraded to BBB+ or below, the collateral threshold becomes 0. Under this situation, the firm would be required to post an additional $20 million collateral.
- For CSA with ATEs, the transactions with the counterparty could be terminated automatically as a result of potential rating downgrade. When this happens, the transactions would be terminated with cash flow value: the firm will be receiving cash for assets and making cash payment for liabilities. At the same time, the collaterals posted by either party will be returned. For the example above, if ATE is

Table 5.1 *Example CSA collateral thresholds*

Credit Rating	Collateral Threshold
A+ and above	20M
A/A−	10M
BBB+ and below	0M

triggered as the result of rating downgrade, the firm would have to pay $50 million in cash to the counterparty and get $30 million collateral back. This generates a net outflow of $20 million HQLA asset if the collateral posted is HQLA; if collateral is non-HQLA asset, then the HQLA outflow would be $50 million.

- For some special transactions, rating downgrade provisions would trigger collateral flow, which becomes a component within the LCR liquidity requirement. One example will be discussed in the next section along with a discussion on RVA.

When both ratings-based threshold and ATEs are effective, one may need to account for these two situations properly to avoid double-counting. For the example discussed above, if $20 million is accounted for in the LCR downgrade calculations, one would expect that the $20 million extra collateral will be returned upon ATE trigger: at the time of trigger, the firm is posting $50 million collateral, and should expect to get all $50 million collateral back. Separate calculations of ratings-based threshold and ATE would double-count the $20 million liquidity cost.

LVA calculations from LCR

There are two implications due to the regulatory liquidity requirements. First of all, when the firm does not have enough liquid assets to fulfill the liquidity requirement, it would need to raise liquidity by, for example, issuing debt in the market. This inevitably would affect the firm's overall funding cost. As a result, the firm would have a higher funding spread curve, which would increase the FCA for the firm. Therefore the extra LCR liquidity requirement would lead to an increase in firm's FVA.

Secondly, when liquidity assets are put aside due to the LCR requirements, the firm would not be able to lend out the money or employ the assets to make extra returns. The opportunity cost could be significant, which we would indicate as a spread r_o, and for HQLA the cost spread would be r_o^{HQ} and for non-HQLA: r_o^{LQ}. Certainly, one can differentiate the cost for different assets practically, similar to differential discounting.

If the extra liquidity that is required for an individual transaction is Q with an expected life of T years, then the liquidity cost for the transaction would be: $LVA = -\sum_i Q_i^{HQ} r_o^i T$, where the index i is for different HQLA assets. More generically, one would need to calculate the liquidity cost for transactions in simulations as an integral:

$$LVA = -\int_0^T \sum_i Q_i^{HQ}(t) r_o^i(t)\, dt.$$

The calculation of Q(t) can be implemented as a step in FVA aggregations: for each scenario and time slice, one would be able to apply the CSA and expected liquidity requirement and evaluate the liquidity cost, similar to the FVA calculations.

5.2 RVA: Replacement value adjustment

5.2.1 RVA: Special CSA features

Securitization vehicles, or special purpose vehicles (SPV) are a special group of derivative users. We will use SPV as a general name for the class of vehicles throughout our discussions. Apart from their investment activities, SPVs also hedge their interest rate, currency and other market risks with bank dealers. A schematic diagram of SPV is shown in Figure 5.1. It is critically important for these vehicles to maintain their credit ratings as required by the investors. While these entities are generally rated by rating agencies, such as Moody's, S&P and Fitch, they are also required by these rating agencies to trade only with highly rated banks, or SPVs may lose their credit rating. In order to satisfy the rating agency requirements, SPVs execute derivative trades with banks under CSAs with special rating based trigger requirements.

There are different variations of special CSAs between SPVs and banks. Here we will only discuss some of the more common features and the implications. Generally there are a few special features for the CSAs between SPVs and banks:

- SPVs are generally backed by investment assets without much cash in hand. Therefore, the CSA between SPVs and banks are generally of a one-way nature: SPVs will never post collateral. This implies that bank dealers would have to bear the burden of potential funding implications from the transaction.
- There are two types of trigger events embedded in the CSA. The first is the so-called collateral event. When the counterparty bank is downgraded below certain trigger level L_1, the bank would have to post collateral that is equal to mark to market of the derivative transaction, plus a buffer amount. The buffer, which is similar in effect to an initial margin, is generally very punitive. The buffer amount also depends on the type of derivative transaction, the asset type, instrument type and maturity.

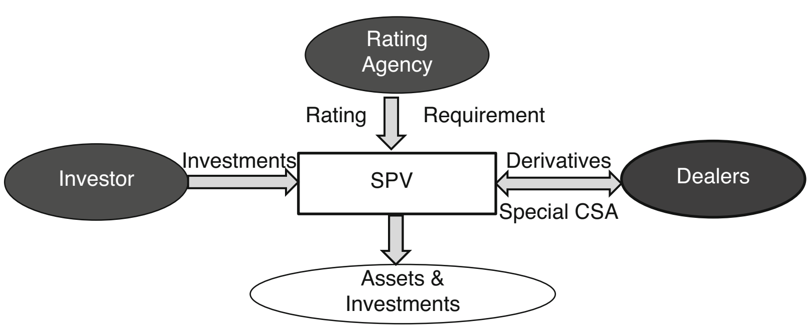

Figure 5.1 Illustration of an SPV transaction

For example, a 10-year cross currency swap with major currency pairs may require 15% of outstanding notional amount as a buffer; for a 10-year fix-float interest rate swap, it may be 10% of the average swap notional; while for an interest rate cap, the buffer may be 2–3% of the notional.

- The second trigger event is a replacement event, which happens when the dealer bank is downgraded further to another trigger credit rating L_2. When this replacement event happens, the dealer bank would have to find a replacement for itself, or face more severe consequences. For example, the derivative transaction may lose its seniority from the SPV's payment waterfall schedule. Losing waterfall seniority is as though the bank is trading with a high yield name, instead of the super senior rating like AAA or AA+. The loss from waterfall seniority change would be very serious.

These special CSA features create a very challenging situation for banks dealing with derivative transactions with SPVs, from CVA, FVA, LVA to RVA, as we will discuss below.

5.2.2 Collateral event, FVA and LVA

The collateral event would cause potential funding cost to the dealer bank, which is one of the conditional funding costs for FVA calculations. While this may be very costly for the dealer who is close to or at the trigger rating level, the calculation of this funding cost is generally straightforward given the buffer amount and the probability of downgrade. The FVA of the collateral event can be decomposed into two FVA terms as below:

$$FVA_{CollEvent} \cong FVA_{RBT=L_1} - \int_0^T 1_{\{r(t)\leq L_1\}} B(t)\, X(r(t), t)\, dt.$$

The first term is the FVA for the posting of transaction MTM upon the trigger of downgrade. This is equivalent to have a rating-based threshold CSA with infinity threshold above trigger rating L_1 and zero for ratings at or below L_1. This term includes both FCA and FBA: when bank owes money to SPV and collateral event is not triggered, there will be funding benefit for the bank. On the other hand, when MTM is positive or a collateral event is triggered, FCA will result.

The second term accounts for the extra buffer amount, where $X(r(t), t)$ is the funding spread for the dealer bank, B(t) is the buffer amount, r(t) is the rating for the dealer bank, and $1_{\{r(t)\leq L_1\}}$ is the probability of the collateral event being triggered at time t. When the collateral event is triggered, the funding cost due to posting the collateral buffer is equal to the buffer amount multiplied by the funding spread. Notice that $X(r(t), t)$ will also depend on the rating of the bank: when the bank gets downgraded to L_1 or below, the funding spread will not be the same as current funding spread, and generally will be rating dependent. The specification of function $X(r(t), t)$ can be done based on historical studies.

Here the approximate equals sign is used as the FVA calculation for collateral event is not exact in this case. One would also need to subtract the funding cost upon replacement event happens, under which the bank will be finding a replacement counterparty and the funding cost will be replaced with RVA. Or more strictly, we have:

$$FVA_{CollEvent} = FVA_{RBT=L_1,L_2} - \int_0^T 1_{\{L_2 < r(t) \le L_1\}} B(t)\, X(t)\, dt$$

where we use $FVA_{RBT=L_1,L_2}$ to indicate the ratings-based threshold as being a very odd one:

- Threshold = Infinity, if $r(t) > L_1$
- Threshold = 0, if $L_2 < r(t) \le L_1$
- Threshold = Infinity, if $r(t) \le L_2$.

As a consequence of a potential collateral event, the firm would also have to put aside additional liquidity to cover the potential liquidity requirement. If the LCR credit stress situation would trigger the collateral event, one would need to put aside the buffer amount B(t) as liquidity protection, which would result in the liquidity cost:

$$LVA = -\int_0^T B(t)\, r_o\, dt$$

where r_o is the opportunity cost spread discussed in the previous section.

The most difficult part is dealing with the replacement event: calculating the cost associated with replacing yourself in the event of further downgrade. This replacement cost is what we call replacement value adjustment (RVA).

Including the CVA that the bank dealer will face because of SPV default, the total valuation adjustment for the derivative transaction would be:

$$VA_{Total} = CVA + FVA + LVA + RVA.$$

5.2.3 RVA: The difficulty

Figure 5.2 shows schematically the replacement situation. When a bank dealer is downgraded to a certain level, a highly-rated counterparty needs to be found to step into the bank's shoes to replace its derivative obligations. By doing so, the replacement counterparty will charge the original bank a premium to take the deal and to face the structure deal or SPV.

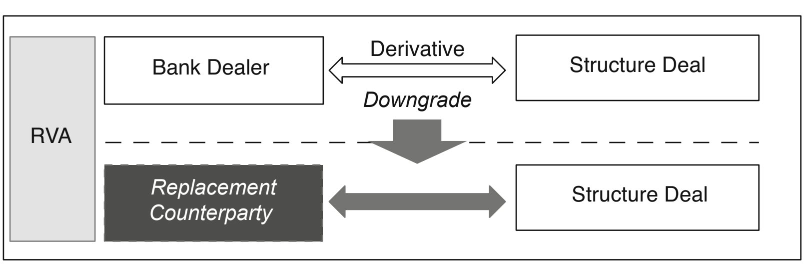

Figure 5.2 Illustration of replacement process

There are a few reasons why RVA is very difficult to calculate:

- Clearly, the replacement counterparty (RC) needs to be willing to step into the derivative transaction to face the structure deal. The only information we have is that the replacement counterparty needs to be highly rated. We have no information as to who the counterparty would be, or the counterparty's actual rating.
- Once the RC steps into the transaction, the RC will have to face any potential future collateral event and replacement event as well; therefore RC would have to take these costs into consideration.
- The RC may charge a significant risk premium on top of the expected cost, including the cost of putting aside liquidity and capital, which would be an uncertain factor in the RVA calculation.
- There may be potential liquidity squeeze and risk appetite change in the market, and some situation similar to wrong-way risk. For example, downgrade of the original bank dealer may have impact on the risk appetite of the high-rated bank dealers, who may charge an additional risk premium at the time of downgrade.

For the above reasons, one has to make some assumptions in calculating the RVA, including:

- The RC's rating: one may make an assumption that the bank dealer willing to consider taking the replacement position will be high rated, for example AA-/A. Given the assumed rating, one may calculate the required CVA, FVA and LVA.
- If the exposure that SPV owes is $100, how much capital would the RC have to put aside, and how much return would the RC demand in order to make its investment worthwhile?

While the items in the first bullet point may be valuated given RC's rating, assumed funding curve and liquidity cost, the second bullet point leads us to the consideration of capital and its effect on market derivative valuations.

5.3 KVA: Capital value adjustment?

5.3.1 KVA introduction

One of the significant regulations after the 2008 financial crisis is the much-increased requirement for banks to beef up their capital base in market risk, credit risk, operational risk and others, such as model risk. The central point of the regulations is to increase the cost for the banks to take leveraged positions, so that the banks will be much less risky and have sufficient funds to stay alive even under stressed situations. Under Basel III, for example, additional capital is required to cover the CVA market risk, as CVA fluctuates on a daily basis. All these have forced the banks to take a deeper look at their capitals for all business lines and evaluate the profitability of each of them. It is quite often that we see banks close a particular business due to the fact that return on capital is not enough. This also leads to the discussion and emergence of so-called capital valuation adjustment, or KVA.

There are distinct differences between capital cost and funding cost that one may need to understand before accounting for it as a derivative valuation adjustment. As we know, funding cost is the cost of borrowing from the market lenders in order to operate properly. If we recall the balance sheet of a bank, the funding cost is the cost involved in the servicing of debt or funding vehicles. It is realized cost on a daily basis; failing to pay funding cost to the lender would effectively mean default. For example, the extra funding spread paid to the debt holder as interest has to be paid in full.

Capital, although also a required large pool of funds, is different from the normal business funding cost. Capital in general is the pool of money that the equity shareholder has to put aside or invest upfront. To make more money, the firm borrows additionally from debt investors to buy assets or to run the desired business. In the equity shareholder's mind, the business must achieve a certain required target return on the capital. However, this return does not have to occur if the firm is not making enough money.

The ratio of income to the capital size, or return on capital, reflects the profitability of the firm in the eye of its shareholders. This is different from the funding cost in FVA calculations: the capital is the money that belongs to the equity holders, while the funding cost is the cost involved in the leveraged business operations and borrowing activity. The cost of capital is not a day-to-day cost of the business that has to be paid out, while funding cost is linked to the actual payments to the lenders. The result of not achieving the target return on capital could be a shutdown of the business; or with the increasing regulatory capital requirement, the shareholders may have to come to the reality of accepting the lower return.

This difference is shown in Figure 5.3. The equity holder is the owner of the firm and receives a dividend payment, which actually comes from income distribution. When income is not available, or there is not enough income to distribute dividends, the firm will still be alive. This contingency is shown in the figure as dotted lines. It is not the same as interest payments to the lender, which could lead to default if the firm fails to deliver. Funding cost involved in the derivative value replication process is truly a

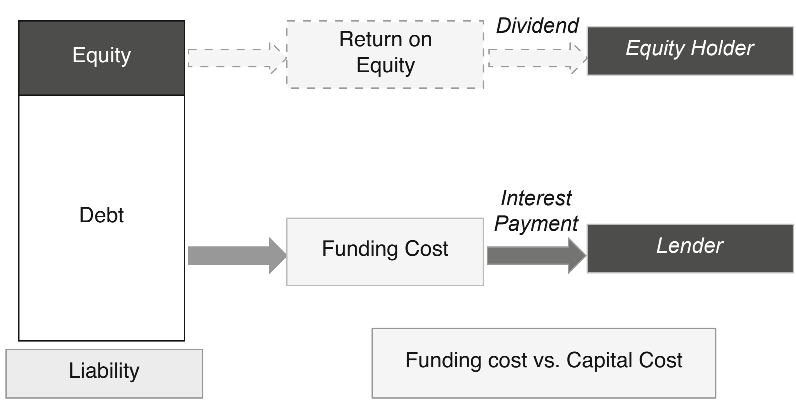

Figure 5.3 Illustration of funding and capital cost difference

fair valuation adjustment, while capital return is not a real cost in the derivative value replication process.

As the result of this difference between capital return and funding cost, the treatment of capital cost in accounting has to be different from FVA. FVA is a real valuation adjustment that reflects the actual cost involved in running the business or carrying the derivatives position through time. It is a true cost in the derivative value replication process. Effectively, one may take funding cost as affecting the future cash flow discounting for derivative transactions. Therefore, FVA needs to be incorporated in the fair valuations. On the other hand, capital cost does not involve any real external transactional cost, it only reflects the required return from the equity holder internally. Accounting-wise, any profit from the business transactions will be recorded as direct income, the size of which, comparing to capital size, will be measurement of business profitability. It is not an actual cost paid to someone in the derivative value replication process. Therefore capital cost would not affect the intrinsic derivative valuations. For this reason, KVA should not be included in the fair valuation adjustment, the same way as CVA and FVA. Rather, KVA should be regarded as a target return measure that businesses and transactions should be achieving:

$$Income_i = Value_i - Cost_i \geq KVA = Capital_i \cdot ROC$$

where ROC is the return on capital, and $Capital_i$ is the required capital for individual transaction i. The above formula simply states that the income from a transaction should be greater or equal to the required capital return. In practice, this is included as the premium that one charges the customer in trading.

As the capital requirements from Basel III are getting implemented among the banks, the return on capital may become the dominant term in the charges that the bank dealers demand from the derivative users, which would push the derivative users away from trading unsecured derivatives, or consider posting collaterals to reduce the bank dealer's capital charge. We will discuss this more in section 5.4.

5.3.2 Accounting, exit pricing and the KVA

When talking about derivative pricing, regulators and accountants often resort to indications of market exit valuations. For unsecured derivatives, the market is often illiquid with a wide bid and offer. When such a derivative transaction is novated in the market, the "buyer" would include in the price all expected costs, such as credit, funding and capital. Now the question comes: If the exit valuation as we discuss here would include the capital cost of the buyer, wouldn't it be better to include KVA it in the fair valuations? Credit and funding are real costs and should be reflected in the fair valuations; capital cost is not real cost, and is really the income that dealers make from the trading. We show an example in Figure 5.4. In this example, a dealer enters a derivative contract with a customer at \$82. The theoretical value of the derivative contract before valuation adjustment is \$100. The CVA and FVA are \$4 and \$2 respectively, which makes the replication value for the derivative contract at \$94. Barring any other cost, the dealer would be making \$12 as the return on capital. Assuming other dealers would be willing to enter the contract at \$82, this leaves the market exit value at \$82.

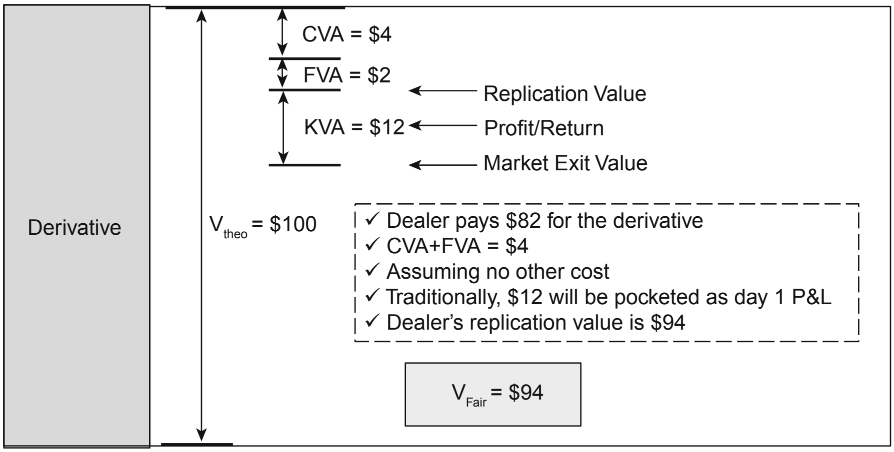

Figure 5.4 Example for KVA, replication value vs. market exit value

What is the fair value of the derivative contract? If we were to liquidate the derivative immediately, the derivative value should be $82; however, if the dealer plans to manage the derivative contract through its maturity, the expected replication value would be $94.

Question is then, where should the dealer mark the derivative? There are three choices:

- Choice 1: Marking at $100. Clearly this is improper behavior, representing some aggressive dealer markings before the credit crisis;
- Choice 2: Marking at $94 and dynamically replicating the derivatives values through time. This would make sense given dealer's normal business model. $12 will be booked as P&L.
- Choice 3: Marking at $82 from market exit pricing, and keep a reserve of $12 as KVA, which would drip into income as the derivative gets closer to maturity, or when the risk of the derivative contract goes away. $82 can also be taken as the market bid price, or fire-sale price.

Both choice 2 and 3 may happen in practice. In choice 2, the marking is consistent with the derivative fair valuations; $12 of profit will be used in return on capital calculations. If a derivative business is running with an equilibrium capital cost from a dynamic portfolio turning over through time, choice 2 is the correct methodology. Choice 3 applies for a more conservative setting, adopting an immediate liquidation time horizon. While this may appeal to some conservative risk managers and accountants, this approach may not be consistent with the given business model. Since the future replication cost is already taken into account at $94 price, reserving profit without reasons for potential future P&L loss may not be consistent with the fair value accounting practices.

The argument against taking $100 as fair valuation is clear: it does not represent the replication value or fair valuation of the trade. One is simply putting the profit or return on capital in the valuation. This is against any of the other methodologies.

5.3.3 Calculating KVA

KVA represents the cost of putting aside the required capital for keeping OTC derivatives on banks' books. While the capital requirement and its calculation, such as from Basel III, may be quite involved, KVA is even more complex.

There are at least two levels of sophistication here.

First of all, KVA is an integration of cost over time. If we carry a bilateral unsecured 30-year swap with a single counterparty, the required capital would change over time. To represent the carrying capital cost through time, one would have to calculate the capital cost over time:

$$KVA = ROC \int Capital\,(t)\,dt.$$

It is only through integration of the capital cost that one would be able to arrive at such a cost from a fair value point of view. For the 30-year swap, assuming we are concerned about the counterparty credit capital and CVA capital, and we also assume the counterparty's credit rating does not move, then the capital cost would depend on the maturity. The capital requirement for this position would change over time. Clearly the capital calculation over time would be very challenging.

This integration of KVA on an individual transaction or one single counterparty netting set level may be too punitive, and this leads to our second point. Arguably, KVA is a business model-dependent quantity: capital can be managed, or "hedged". If one is able to hedge KVA, then the future capital requirement may really depend on how this management is done. If one buys an individually named CDS to hedge the capital, one may assume that this hedge will continue on a rolling basis. For example, a counterparty may use the most liquid 5-year CDS to hedge the 30-year exposure on a rolling basis. Under such situations, the capital requirement over time is critically dependent on the decision-making in capital "hedge". This is precisely why capital management is needed and why KVA depends on the specific firm's capital decision.

5.3.4 KVA management

KVA, if calculated properly, provides a good quantitative measure for a bank to manage business profitability systematically. For individual business lines, there would be a clear quantitative measure about the capital usage and returns, which would lead to proper business-decision making as well as resource optimization.

Firms may also employ hedging instruments to reduce the capital requirements when return is not enough. Assume KVA is a function of all market tradable risk factors $\mathbf{r_M}$:

$$KVA = KVA(\mathbf{r_M})$$

$$\delta_i = \frac{\partial KVA}{\partial r_M^i}.$$

One would be able to hedge the tradable risks based on δ_i; certainly one may also attempt to buy options to hedge the large KVA swings.

Unfortunately hedging KVA is a complex management decision, and could be controversial: without KVA included in the accounting valuations, the hedge of KVA may leave outsized risk positions that are so large that they could dominate the bank's market risk position. Without approval from the regulators in permitting the capital hedge, the bank would be left in the position where the market risk is left unhedged. For CVA capital, one may attempt to hedge the credit portion of the capital with regulator-allowed CDS. However, this creates a similar problem to the market risk hedge: one can never hedge the original risky positions and their capital at the same time; this is simply because these two things are for different purposes and are originated from different sources. One is from extreme tails; one is from expectations. This leads to a dilemma in hedging P&L and capital. This issue will be discussed further in Chapter 7.

5.4 Summarizing XVAs

5.4.1 XVA summary

Dealers realize derivative values in two ways. One way is to sell the derivative in the market; the other way is to dynamically replicate the derivative value through time. When the derivative is liquidly traded, there is no doubt as to the market value of the derivative; however, when the liquidity is not available, one has to resort to the fundamentals and dynamic replication. Credit, funding and liquidity are three different origins of costs that are closely linked to the derivative replication process, resulting in CVA, FVA and LVA calculations. We use a simple example below to summarize and show these concepts.

In Figure 5.5, we show the simplest equation, ignoring other high order terms and representing the replication of a derivative value in an ideal world.[2] The left hand side is the change of the constructed delta neutral, and risk-free portfolio value, and the right hand side shows the decomposition of the portfolio P&L:

- The first term $(V - \delta S) r\, dt$ is the borrowing and lending of the risk-free portfolio;
- The second term represents the offset between option gamma $\frac{1}{2}\gamma\, dS^2$ and theta term $\frac{1}{2}\gamma\, \sigma^2 dt$.

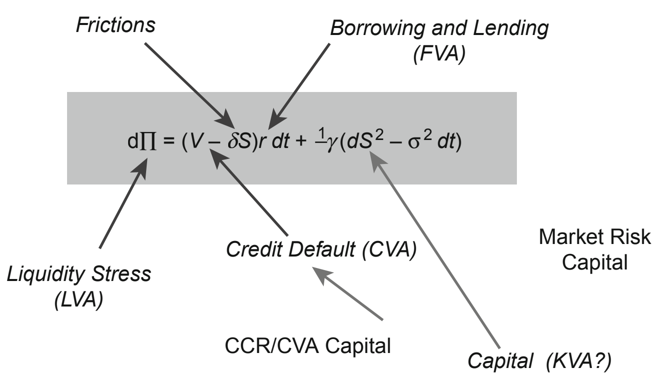

Figure 5.5 Dynamic replication of derivative value and valuation adjustments

In the real world, however, frictions can kick in while dynamically hedging δS; counterparty credit default of V would result in CVA, which comes from expected loss in V; borrowing and lending of $(V - \delta S)$ would lead to funding cost and benefit or FVA, where real funding rates different from a risk-free rate r should be used. Meanwhile, extra liquidity may be required to be put aside in maintaining the derivative over time; and the opportunity cost of this liquidity requirement is LVA. All these are true costs in replicating the derivative values.

In maintaining the derivative through time, capital would be required to keep the derivative on the book. The potential market fluctuations from dS^2 would lead to market risk capital; the potential counterparty credit risk (CCR) and CVA fluctuations would lead to CCR and CVA capitals. To make the trade worthwhile, a bank's equity investors would require specific returns on the risks taken. The quantification of this capital return is termed KVA. While KVA is not a true cost to the firm, and is not a true "valuation adjustment" from fair value point of view, it is a good measure of profitability from a derivative transaction level, and portfolio levels.

These valuation adjustments represent perturbations in derivative fair value calculations, deviating from the ideal theoretical assumptions, such as perfect liquidity, no arbitrage, risk neutrality, perfect credit, complete market, and so on. With market liquidity unavailable, unsecured derivatives valuations would have to incorporate practical considerations. Some of these considerations are linked to the market and have market consistent valuations, such as market friction in trading delta hedges; collaterals and collateral values may also be included in this category as they are mostly agreeable among the dealers within reasonable range. Some other considerations have to be linked to firm specificities, such as a firm's borrowing and lending rates. As market consistent considerations are less contentious within the derivative community, the treatment of a multi-price economy within a no-arbitrage setting is still an evolving story.

5.4.2 Prudent valuations and additional valuation adjustments (AVA)

One relevant and important development in recent years is the prudent valuation standard from the EBA. The goal of prudent valuation is to adjust the fair derivative valuations on the balance sheet based on relative conservative assumptions, so that they are close to exit values. The conservative valuation standard is defined by the EBA and implemented on a national level; for example, FSA in UK.

There are three main sources of costs in the prudent valuation standard for AVAs:

- Potential uncertainties arising from market price, valuation parameters and model risk;
- Any expected future costs, such as funding, administrative, early termination and liquidity;
- Any other adjustments arising from, for example, operational risks, etc.

The standard also provides guidance as to how these valuation adjustments may be evaluated, and how aggregations may be done across different risk factors. The final draft of the Regulatory Technical Standard (RTS)[3] defines a quantitative statistical measure around the conservativeness of the valuation: the level of the prudent valuation after all AVAs should lead to a 90% confidence in exiting the derivative in the market. This is a significant and challenging requirement for OTC derivatives.

The AVA adjustments are subtracted from Tier 1 capital, meaning this part of derivative valuation would not be treated as true value in capital terms; alternatively it may be thought of as additional amount of AVA capital that has to be raised.

Given what we have discussed so far, the question is, how do we reconcile the AVAs and the valuation adjustments we have discussed in the book, namely CVA, FVA, RVA, LVA and lastly, return on capital or KVA?

5.4.3 Integrating XVAs, AVAs and capital

Prudent valuation takes its approach from the fundamentals of realizing the derivative values and to the extent that one would be confident enough to exit in the market; however, the application of the standard along with other valuation adjustments and existing Basel III capital may carry issues.

Let us take the valuations and adjustments for illiquid unsecured derivatives one term at a time and we may see a clearer picture. First we start with a valuation with perfect collateralization and proceed with the following adjustments:

1. Credit default will lead to CVA. For AVA purposes, we take CVA adjusted value as a base valuation.
2. Funding and FVA would be included as one of AVA adjustment; our definition of LVA would fit into this category as well.
3. In addition, for illiquid derivatives, one would have to include administrative costs, and take market illiquidity in hedging into consideration. These are termed as future expected costs in derivative replication.
4. Prudent valuation also requires the evaluation of model risk uncertainties as well as valuation parameter uncertainties.
5. Prudent valuation also requires the evaluation of expected termination or close-out costs; our definition of RVA may be included in this category as a replacement event is deemed to be a complex termination or close-out.

The costs in (1), (2) and (3) above are associated with maintaining and replicating the derivatives through time. For item (4) model risk and valuation parameter uncertainties, banks have various existing forms in place, including model risk reserves and valuation parameter reserves. So this is nothing new. Item (5) is really for derivatives that have a defined bid–offer market or potential future forced terminations, so the close-out costs would matter.

The valuation adjustments, termed XVAs in this book, are for derivative fair valuations. While some of them, such as FVA, LVA and RVA, may be similar to the terms under prudent valuation, they are implemented differently and may overlap with each other. AVAs are implemented as a reduction in tier 1 capital, while XVAs and accounting fair valuations would automatically be reflected in banks' books and records, as well as in balance sheet and capital reduction. AVAs will not be reflected in firms' daily P&L, while FVA, LVA and RVA will. In practical implementation, we would need to avoid the double-counting of XVAs and AVAs.

The central question is, are all XVAs, or AVAs additive and with capital?

Capital is about potential uncertainties that firms need to cover under stressed situations; and return on capital means the profit that the derivative would have to return. One may regard AVA adjusted valuation as the base value in capital calculations; the potential uncertainties under stress situations, leading to much lower than the base values, are what banks need to prepare as capital. Adding prudent valuation AVAs on top of Basel III capital would certainly ensure the banks are well-capitalized, taking into account the special nature of OTC derivatives. This is conservative in nature; however, there could be situations where it is overly conservative. For example, model risk and parameter uncertainties under normal market conditions may be much less under stressed market conditions. This is because the derivative values may well converge to intrinsic values under extreme market conditions used in capital calculation, where the model risk would be lower. While the capital, XVAs and AVAs are usually calculated independently, the integration of these terms together may become a very challenging process for banks. Often we have to let conservativeness dominate the day.

Another issue in integrating the valuation adjustments is the dilemma of risk management. Should AVA and KVA be dynamically hedged? We have touched briefly on the model risk part in Chapter 1; however, this is indeed a difficult issue in practice. We will discuss this further in Chapter 7.

5.4.4 Derivative users: Problems and hopes

In the years following the 2008 financial crisis, we have witnessed the emergence of CVA, FVA and other valuation adjustments in the framework of derivative fair valuations; capital requirements and the return on capital have become more and more important issues to banks' normal profitability management; and prudent valuation and AVAs have also become integral components of the capital framework that have seen implementation among some banks. All these changes on the dealer side in turn have profound implications to the derivative user community: because of the sum of different valuation adjustments and capital costs it could become very expensive for derivative users to enter an unsecured derivative.

While the institutional users have become accustomed to the collateralization and exchange style trading in derivatives, there are some real derivative users that have seen their costs of doing derivatives sky-rocketing. These users, who cannot afford to post collaterals, would have to swallow the rising costs passed on from the banks, or stop hedging. For example, municipal debt issuers, who issue fixed rate debt to raise money, cannot afford to post collaterals and would have to pay high premiums for an unsecured derivative.

Without any doubt, these derivative users are pushed back from the derivative markets as a result: compared to the good old days, it becomes very costly for derivative users to go into the market to hedge their risks, and they will have second thoughts when contemplating hedging the issuance with dealers. The real question is then, if the real derivative users are pushed back from this market, isn't this defeating one of the most important purposes of the OTC derivative market?

The picture is not pretty from the banks' side either. The shrinking of overall derivative trading has hit the banks' derivative trading business significantly. With banks complaining about the profitability of, for example, interest rate derivatives,

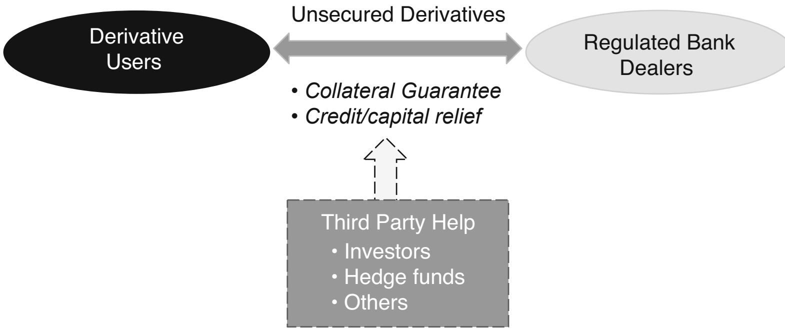

Figure 5.6 Helping the derivative users

they are also struggling to find a solution.[4] This is because from the capital return point of view, interest rate derivative businesses have been underperforming: a long-term unsecured interest rate derivative would be costly in capital and it would be very hard to earn enough returns to satisfy profitability requirements.

This is indeed a difficult issue. It would be hard to ask the dealers to take less than what they need to feed themselves but on the other hand, asking the derivative users to pay a much larger risk premium would definitely push them away from trading derivatives.

Could there be any help to alleviate these situations, for both dealers and derivative users?

One option would be for the regulators to come in and provide help to the derivative users. However, anything a regulator provides would create asymmetric situations in the market, with some derivative users being treated differently than others. It may be easy to allow them not to post collaterals; however, it would be hard to ask dealers not to charge as much.

The better solution may have to come from the market. Conceivably, some third party white knight player can come in to provide collateral guarantee for the derivative users. We show a conjectured situation in the Figure 5.6. Here a third party would come in to provide help between the derivative users and the regulated bank dealers. If we have different derivative users trading derivatives in both directions, for example receive fix and pay fix swaps, then a big enough third party would be able to come in to provide relief to both the derivative users and the regulated bank dealers. In return, the third party investors would be able to make returns from being exposed to derivative users' default, providing funding of the collateral, and capital relief to the banks. As the return could be significant, the investors would be happy as well.

While XVAs, AVAs and Capitals are still evolving, we have yet to see how the market will evolve in the coming years.

Summary

In this chapter, we have discussed two different valuation adjustments, namely LVA and RVA. We have limited our discussion of LVA to the scope of banks' liquidity management and liquidity requirements under the new Basel rules, such as LCR and NSFR. RVA is a special situation that bank dealers face when dealing with some SPVs.

While these two valuation adjustments are considered part of derivative valuations, KVA should be treated differently. Capital value adjustment provides a good quantitative measure for business profitability; however, it does not represent true cost to the business. Rather it is more in the sense of profit size. Calculating KVA is rather a complex process, as it is linked closely to how capital is managed within a particular firm. Managing and hedging KVA would encounter issues around the treatment of hedging instrument risk, as well as mismatches between capital optimization and P&L fluctuations.

We summarized the origin of XVAs using a simple derivative value replication example. XVAs and capital returns are essential components that one encounters in dynamic replication of derivative values through time. For illiquid OTC derivatives, this is the proper way to obtain fair derivative valuations, other than fire-sale in the market.

XVAs, prudent valuations and capital are three important developments in the market. The integration of the three pieces together consistently is a very challenging issue. Often regulators and banks have let conservativeness dominate the day. This also leaves the question unanswered about the derivatives and real derivative users: Who will be coming to the rescue?

III XVA MODELING AND IMPLEMENTATION

6 CVA and FVA Modeling and Implementation

6.1 General CVA/FVA modeling

6.1.1 CVA/FVA Modeling and computations: Complexity

CVA/FVA modeling is complex, mainly because:

- The potential credit/funding exposures are generally non-linear. For a linear fixed-for-floating interest rate swap, one may value the swap based on discounting of cash flows; however, the CVA/FVA exposures would depend on the distribution of the interest rate, and it is really a swaption.
- CVA/FVA must be calculated on a net basis. If you have done a thousand trades across different asset classes with one counterparty, you have to net all the future exposures while calculating the default loss or funding cost/benefit. This means you would need to have a consistent simulation framework for all the risk factors involved across different asset classes.
- CVA/FVA calculations have to deal with complex CSAs, such as ratings-based threshold, and automatic/discretionary triggers/terminations, among other things. This means the rating migrations must be incorporated in the CVA/FVA calculations.
- There may exist significant correlations between credit and market risk factors, which need to be accounted for in CVA/FVA calculations. This means the market risk factors and credit risk factors have to be consistently simulated.
- For FVA, funding and collaterals are involved, which means multi-currency/asset collateral curves and collateral choice options must be dealt with, as well as the firm's own funding curves. When asymmetric funding curve is used, one may have to deal with funding set and netting set issues.
- There are other legal issues surrounding the enforceability of CSAs in different jurisdictions and authorities, which could introduce extra dimension of issues in calculating CVA properly.

The implementation of a good CVA/FVA calculation system is also very demanding, simply because of the huge amount of market data, trading instruments and potential computations involved. We can perform an estimate on how much computation is needed for a trading portfolio with 100,000 instruments:

- 100,000 instruments
- 10,000 simulation paths for market risk factors
- 100,000 simulation paths for credit risk factors.

In order to aggregate the instrument values within a netting set, one has to find a common time grid where the instrument values are simulated/computed. To arrive at a desired accuracy, a simulation time grid of 50–100 or more points is often necessary.

Given all the above information, a brute-force computation would involve 100,000 instruments x 10,000 simulation paths x 100 time points, or 100 billion instrument values. The default/credit information of 100,000 simulation paths x 100 time points = 10 million would be needed. While a derivatives valuation itself can be very complicated and computationally intensive, it is a daunting task to calculate CVA/FVA for a large derivatives portfolio. In addition, the risks and Greeks for CVA/FVA would have to be computed, so that one can risk manage CVA/FVA properly. This can easily add hundreds of market scenarios depending on the number of risk factors involved and the granularity of the Greeks needed. That makes instrument valuations in the 10 trillion scale. All these require smart and efficient implementation of CVA/FVA calculations, as well as finding advanced technology solutions to solve the problem, such as distributed computing, GPU technology, Adjoint Algorithmic Differentiation etc.

In this chapter we will visit the modeling of CVA/FVA calculations and system implementations.

6.1.2 General CVA/FVA calculations

Portfolio values within a netting agreement and bilateral credit ratings are two essential components of the CVA/FVA calculations.[1] Given portfolio value and bilateral credit ratings, one can apply CSA and legal terms to calculate EAD or funding exposure for CVA/FVA calculations as shown in Figure 6.1:

While there are different methodologies available for CVA/FVA calculations, in general the following intuitive steps are involved:

- Simulation of market risk variables.
- Evaluation of derivative contract values.
- Aggregation of contract values consistently for same time slices and scenarios.
- Application of CSA given credit rating simulations.
- Calculation of CVA based on collection of defaulted values or collection of funding cost/benefit based on funding exposure and funding curve.

To obtain the market values of a portfolio, one has to evaluate all the instruments within the portfolio. This would involve simulation of market risk variables and performing contract valuations, similar to regular derivative valuations. However, the

Figure 6.1 Aggregation of default/funding exposure

requirement of CVA market valuations can be very different from the model used in trading and mark-to-market models. For a trading model for a given instrument, one would price the instrument within risk neutral and no arbitrage framework with, amongst others, the following considerations:

1. Accuracy: the trading model has to be accurate.
2. Performance: the trading model must be generally high performance.
3. Greek stability: one has to manage market risk dynamically and this requires accurate and stable Greeks, including P&L attributions and performance.
4. Long-term performance and dynamic model behavior: the trading model has to be consistent with dynamics of market risk variables so that derivative value can be replicated properly and efficiently.

Given these criteria, one may choose the models that are best fit for individual products, without considering the cross-product differences. For example, one may choose to use short rate models, Markov-functional or quasi-Gaussian models for Bermudan swaptions, and use the Libor market model to price other structured interest rate derivative products, such as snowballs or callable range accruals. Each of the models chosen will be calibrated to the market within the risk neutral and no-arbitrage setup. As long as one understands the pros and cons of the model used for each specific product, one can effectively manage the products well. This is different for CVA/FVA calculations.

The netting requirement in CVA/FVA calculations creates a quite challenging situation for market risk variable simulations and contract valuations, since one would need to evaluate all the instruments consistently to be netted together: the instrument values have to be aggregated, conditional on the same values for market risk variables, and at the same time. Figure 6.2 shows the concept intuitively: from t=0 to t=T, all market risk variables are simulated schematically. In scenario k at time t_j, the portfolio value would be the aggregate of all instrument values V_{ijk} for the same time slice j and scenario k. The total value from the netting set $V_k(t_j)$ will be used in default/funding exposure calculations given CSA terms.

From the above discussion, we have some observations. First, it is clear that the sum of individual instrument values requires consistency of model valuations. One may argue that using different pricing models for different instruments, as

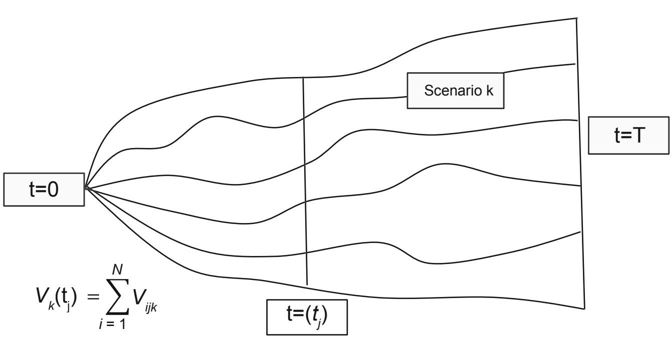

Figure 6.2 Netting of portfolio exposure in Monte Carlo scenarios

in the trading models, may not be consistent: the same interest rate curves do not carry the same dynamics within different models and is therefore inherently inconsistent. This is probably less of an issue if the pricing models are accurate enough. Secondly, the above computations could be massive. Given the practical computational constraints, one may not be able to afford the same accuracy that one uses for trading. One may have to sacrifice accuracy for performance. Even after tricks are employed, CVA/FVA computations can still be immensely expensive compared to normal derivative valuations. For this reason, it is an absolute necessity to have more efficient methodology in obtaining market factor exposures, as well as obtaining credit information in scenarios.

Deriving consistent credit ratings information, including default, from credit simulations is equally challenging. There are different ways of performing credit risk factor simulations. CDS spreads, containing credit default information, can be simulated based on risk neutral or historical/physical measures. Alternatively, one may perform credit simulations in a structural model, which calibrates to market traded CDS spreads. A ratings transition matrix can be utilized to derive rating migrations in simulations as well. To make market factors and credit factors consistent, one would need to employ a correlation matrix so that the risk factors are simulated together. For example below, we show a correlation matrix schematically: $\rho_{\mathbf{MM}}$ represents the correlation matrix among market factors, $\rho_{\mathbf{CC}}$ are correlations for credit factors, and $\rho_{\mathbf{MC}}$ are for cross correlations between market and credit factors. When wrong-way risk is involved, the $\rho_{\mathbf{MC}}$ may prove to be critically important.

$$\rho = \begin{bmatrix} \rho_{\mathbf{MM}} & \rho_{\mathbf{MC}} \\ \rho_{\mathbf{CM}} & \rho_{\mathbf{CC}} \end{bmatrix}.$$

One tricky issue is the convergence. While it may be easier for market factors to converge, credit defaults and rating migrations with small probabilities may be much harder to converge. This would then require techniques to combine the two factors together efficiently and consistently.

Figure 6.3 shows the general CVA/FVA calculations schematically.

When market factors and credit factors are not correlated, one would have more choices in performing CVA and FVA calculations. Market factors can be simulated independently to arrive at market factor exposures $V_k(t_j)$. On the other hand,

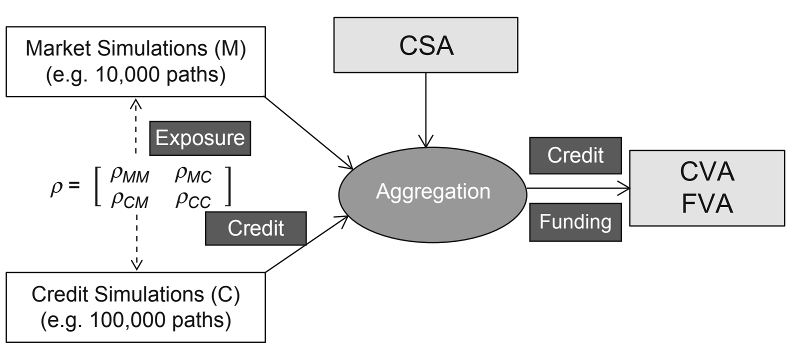

Figure 6.3 General CVA/FVA calculations

credit factors will be derived independently, for example, ratings distribution for the counterparties at different time points $L_k(t_j)$, and discrete default probabilities

$PDK = \mathbf{1}_{\tau_k^{'} < T}{}^2$ with $\tau_k^{'}$ being the default time for scenario $_k$. If the CSA does not have rating dependency, then CVA could be simply:

$$CVA = LGD \sum_k \min\left(H^{'}, V_k^{+}(\tau_k^{'})\right) D\left(\tau_k^{'}\right) PD_k$$

where H' is the collateral threshold, D_k represents the discounting factor at default time.

6.2 Market risk modeling

6.2.1 Market risk modeling and instrument valuations

When the derivatives instruments within the same netting set cover a broad range of products and assets, valuations of market risk exposure can be quite involved. One brute-force way of performing such valuations is to apply derivative valuations for each scenario/time nodes for each instrument. Assume the valuation function of instrument i is $P_i(\boldsymbol{M})$, where $\boldsymbol{M}$ represents the market risk variables. Then the portfolio value can be calculated as:

$$V_k\left(t_j\right) = \sum\nolimits_{i=1}^{N} V_{ijk} = \sum\nolimits_{i=1}^{N} P_i(\boldsymbol{M}_{\mathbf{jk}})$$

where V_{ijk} is the value for instrument i, time t_j, and scenario k. Note here $P_i(\boldsymbol{M})$ could be using any model, including an accurate trading model. Therefore this brute-force calculation would be an ideal way to apply different pricing models for different instruments in a CVA calculation. However, we realize immediately that the application of a valuation function for each scenario and time slice would be prohibitively expensive. Practically one would have to sacrifice the accuracy requirements significantly by using small number of scenarios, reducing the number of time slices, and/or reducing the accuracy of valuation function. This compromise in accuracy however, may not be desirable.

There are two different schemes that one may use to calculate CVA/FVA efficiently, one is the backward discounting scheme, and the other is the forward simulation scheme. We will discuss these two schemes in a more general sense, and later discuss a more efficient simulation methodology in implementing CVA/FVA calculations.

6.2.2 Risk neutral measure vs. historical measure

In CVA/FVA calculations, one may choose to generate market risk and credit risk scenarios based on historical measure or risk neutral measure. Historical measure, or “real” measure, or “physical” measure, uses the historical realized probability distributions, or processes calibrated to the historically realized returns, in generating the market and credit scenarios; risk neutral measure is to calibrate the processes to market traded instrument prices using the risk-neutral distributions for market and credit factors implied in the market traded instrument prices.

The logic of choosing a risk neutral measure is quite clear: CVA and FVA are valuation adjustments embedded within the derivatives fair valuations. If there are instruments available in the market place from which to hedge and infer information, one should be pricing the CVA/FVA in a risk neutral measure based on the no-arbitrage principle. For example, one can hedge the volatility of interest rate using swaptions, which means the interest rate exposure should be calculated based on the interest rate volatility implied in the swaption market. Indeed, the exposure for a simple swap is equivalent to that of a swaption. Similarly, for a counterparty carrying a market traded CDS, one can hedge the CVA exposure by buying CDSs in the market. Therefore counterparty default scenarios generated in CVA calculations should be consistent with the market traded CDS prices in a risk neutral setting.

One consequence of risk neutral pricing is that risk premiums are included for all market factors; therefore deviations from real life may result due to the existence of significant risk premiums. Another issue with risk neutral pricing is availability of efficient market information for the market factors. As we discussed before, different business models and accounting schemes may choose to use "historical" measure instead. For example, for a bank's loan portfolio, a CVA calculation may follow expectations based on "historical" evidence, rather than including within it large market risk premiums. This is because the loan portfolio construction and the relevant business model are not based on risk-neutral measure; and effectively, there is really no risk-neutral measure to rely on. What the banks could do is to look at the broad history and derive the historical implied probability of default for generally not traded commercial names and retail mortgage borrowers. For the bank, there is normally no way to hedge the underlying risk, and therefore one may choose the historical measure for underlying credit in the CVA calculations.

A good successful model should mimic the reality as close as possible. This is especially so in a risk management setting: a chosen measure should give good representation of risk in real life. Using risk neutral measure has two meanings. One is the ability to hedge using market traded instruments; the other is the reliance on market information to construct the possible future outcomes, as we may say, the "market is right". However, the market is not necessary always right, and sometimes full of distortions and "risk premiums". Under such situations, physical measure, constructed properly, may offer advantages, especially in having desirable model behavior in dynamic hedging risks. In addition, one also has the flexibility to use risk neutral measure for some risk factors and physical measure for other risk factors, for example a joint measure model.[3]

6.2.3 Netting and generic time grid

CVA and FVA calculations all require proper netting of portfolio values in all future scenarios. Only after the netting of portfolio instrument values would one be able to apply CSA terms properly. In order to aggregate the instrument values, one would have to make sure that all instrument valuations are consistently valued with the same market conditions, and at the same time point. Instrument pricings under different market conditions, or at different time points, are clearly not net-able. This inevitably requires consistency in scenario generations, and the existence of a generic time grid

Table 6.1 *Generic time grid points selection*

Short Term	1, 2, 3, 5, 7, 14, 30 Days
Monthly	1M – 12M
Quarterly	12M – 7Y
Semi-annually	7Y – 30Y
Annually	30Y – 40Y
Every 5 years	40Y – 60Y

that one may use to collect all instrument valuations within the same netting set. In this section we use an example to illustrate the generic time grid idea.

In generating the generic time grid for netting, there are physical limitations: one would not be able to generate a time grid with every day in it, which will be computationally prohibitive. The choice of such grid would depend on the portfolio under consideration and the accuracy that one is attempting to achieve. Clearly the finer the grid points, the more accuracy of the valuations, but also the more expensive the computations would be.

In Table 6.1, we generate a generic time grid in a more practical way. The last time grid point is selected based on the longest maturity of these instruments, say 60 years. The grids are distributed with more time points in the front and less points in the back. In the short term, the time points are much denser than the longer terms. We have dense points from daily to weekly to monthly within 1 year, and quarterly points after 1 year and up to 7 years, semi-annual points up to 20 years, and so on. In total we have 102 points.

Choosing this type of generic time grid is practically reasonable as being the compromise of accuracy, consistency and practical computational cost. Given that cash flow at 10 years 1 week is barely distinguishable from cash flow at 10 years looking at the present time, fewer time grid points are required as the term point goes further into the future. The impact of discreteness of this 1-week time would become more evident as time passes, which by design, would become more accurate as it approaches the 10-year mark. As it gets near to the 10-year time, the discreteness of 1-week time becomes naturally close to daily frequency. This natural migration from coarser grid to denser points provides a practical balance to computational cost without sacrificing much accuracy. Depending on the user choice of accuracy, composition of portfolio and the computational resources at disposal, one may choose a denser or coarser grid than the above example.

Based on the generic time grid established, one could add all relevant trade specific time points to the grid, such as cash flow, exercise, or trigger schedules, to generate a grand time grid for derivative valuations. Here a general risk neutral tree/lattice or scenario set can be produced as in regular derivative pricing. For interest rate derivatives, models such as a short rate tree or Libor market model may serve the present purpose for CVA/FVA calculations. However, for the structured derivatives desk with exotic instruments priced with various different pricing models, a consistent model should be selected for the CVA calculations. For example, a globally calibrated cross currency LMM seems to be a good choice as most fixed income derivatives can be priced reasonably well with this model.

6.2.4 Backward Discounting Scheme

When a default-free future cash flow is valued, one discounts the cash flow (see Figure 6.4) using risk-free rate to present time:

$$V_0 = e^{-r_c T} \tag{6.1}$$

where r_c is the risk free rate, and T is the cash flow time.

Credit default leads to the valuation of an asset to be less than risk-free discounted value:

$$V_d = e^{-(r_c+u)T} = (1 - PD \cdot LGD)e^{-r_c T} \tag{6.2}$$

where V_d is the defaultable value, u is the credit spread and PD is the probability of default from t= 0 to T. So defaultable value V_d can be calculated in two equivalent ways:

- Credit-discounted value with credit spread u included
- From future cash flow, subtract the expected default loss value $PD \cdot LGD$, then discount it back using risk-free rate.

More generally for continuous time:

$$V_d = \int_0^T V_{nd}(t)e^{-u(t)dt} = V_{nd}(0) - LGD\int_0^T V_{nd}(t)P_d(t)dt \tag{6.3}$$

where $V_{nd}(t)$ is the no-default valuation at time t, and $P_d(t)$ is the probability density of default.

CVA is defined as:

$$CVA = V_d - V_{nd} = -LGD\int_0^T V_{nd}(t)P_d(t)dt \tag{6.4}$$

So one can calculate CVA using two different methodologies, mathematically equivalent:

- Credit discounting methodology with spread u, and take the difference $V_d - V_{nd}$;
- Calculate actual loss given PD and LGD, effectively collecting default losses $LGD \cdot V_{nd}(t)P_d(t)dt$ through time.

The first methodology leads to the backward discounting scheme:

$$CVA = Value\left(risky\ discount\right) - value(risk\,free\ discount)$$

Figure 6.4 Discounting of cash flow payment $1 expected at maturity T

The second methodology, a forward simulation scheme, will be discussed in later sections.

Similarly, for cash flow with funding implications, there are also two different ways:

- Funding spread enabled discounting
- Directly subtract the funding cost/benefit from cash flow based on funding payment schedule, then discount using risk-free rate.

Or

$$V_f = e^{-(r_c+X)T} = (1 - FC)e^{-r_c T}$$

where X is the funding spread and FC is the actual funding cost incurred from 0→T due to funding spread. More generally, FVA would be:

$$FVA = V_f - V_{nf} = -\int_0^T V_{nf}(t)X(t)dt.$$

In a backward discounting scheme, one can calculate the FVA as:[4]

$$FVA = Value\left(discount\ with\ funding\right) - Value(discount\ without\ funding).$$

This backward discounting methodology can be implemented with any models, whether it is PDE approach, Trinomial trees, or Monte Carlo simulations. Two valuations will be performed in the valuation of CVA or FVA, one for risky/funded discounting, and one for riskless/non-funded discounting. The two valuations would be exactly the same, except spreads used in discounting. The difference between the two valuations would be CVA/FVA. A simple CVA/FVA calculation for a single cash flow expected in a 1-year time period is shown in the following example:

Example 6.1: A single cash flow $1 at 1 year, assuming all rates are simple compounded annual rate (Table 6.2).

Table 6.2 *Example CVA/FVA calculation*

Cash flow	$100	Risk-free rate	5%
Default Prob.	2%	Expected Loss(@1y)	0.80
LGD	40%	Present Value	94.48

Table 6.3 shows the CVA/FVA calculation:

Table 6.3 *Example CVA/FVA calculation*

Discounting Spread	0.847%	Funding Spread	1.00%
Risk-free valuation	95.24	No Funding spread	95.24
Risky valuation	94.48	Funded valuation	94.34
CVA	−0.76	FVA	−0.90

One may also realize immediately that there would be a difference in calculating, for example:

$$FVA = Value\left(Risky,\ no\ funding\ spread\right) - Value(Risky,\ funded).$$

Comparing to simple risk-free FVA calculations:

$$FVA = Value\left(Risk\,free,\ no\ funding\ spread\right) - Value(Risk\,free,\ funded).$$

Or for CVA calculations, one may calculate with funding spread included in both valuations:

$$CVA = Value\left(Risky,\ funded\right) - Value(Risk\,free,\ funded).$$

In another words, the order of CVA vs. FVA calculations could matter in the actual CVA/FVA values. This may turn out to be a tricky issue, as we will discuss in more detail later. Here we will simply say that the cross term between CVA and FVA is rather small compared to the actual CVA and FVA itself, so we could leave this issue on the side for now.

6.2.5 Backward discounting scheme: Pros and cons

For one single deal or a few deals, backward discounting can be the most efficient way in calculating CVA/FVA. This is because one can perform valuations based on any chosen model with relative ease, including trading quality models: the only change in pricing CVA/FVA compared to the standard derivatives pricing is simply the application of extra credit/funding spreads in discounting. This has a comparative advantage over globally calibrated simulation models and forward simulation methodology for its simplicity and accuracy, especially under simple portfolio situations. Practically it is quite convenient for trading desks to use this type of CVA/FVA pricing, whenever it is appropriate.

For callable products, it is also convenient to incorporate CVA/FVA into the exercise decision-making when an exercise boundary is needed. Intuitively, exercise boundary will be affected by the credit condition and expected CVA in the future:

- In exercising an option, one generally is cancelling the future loss of underlying payments, or future potential and expected liabilities to the counterparty. However, expected liabilities should be discounted given the benefit from DVA or FBA, which means it will marginally reduce the option exercise incentive.
- On the other hand, one would tend to keep the option when the future expected cash flow is positive or is an asset. Taking CVA and FCA into account, the asset should be discounted, which means CVA and FCA would marginally reduce the option owner's incentive to keep the options.
- The change in exercise boundary and exercise decision will in turn affect the CVA and FVA calculations. While this effect is in general small in nature, one may want to take into account for certain types of callable products.

In a backward discounting scheme, the exercise boundary is naturally obtained by comparing the exercise strike and properly discounted cash flow value with credit and/or funding spreads included. Intuitively the decision at time t for scenario k would be

$$Exercise_k(t) = \begin{cases} Yes, \; if \; V_k^{X,u,u'}(t) \geq K(t) \\ No, \; if \; V_k^{X,u,u'}(t) < K(t) \end{cases}$$

where X, u, u' are used to indicate the extra discounting effect from credit and funding spreads, and K(t) is the exercise strike. For a forward simulation scheme, it is much more difficult to incorporate the CVA/FVA adjustments in an option exercise decision-making.

It is quite tricky when dealing with rating triggers in a backward discounting scheme. For example, CSAs with rating based threshold would require proper credit rating migrations. Without specific credit rating simulations, a simple methodology may be used to calculate expected credit migration probabilities, and discount the exposures based on different rating spread curves, or different funding curves. Here we use FVA calculation to illustrate the method; the CVA calculation can be done the similar way.

- Derive the credit rating distribution for each of the time slices based on, for example, propagating the rating transition matrix, or simulating the CDS spreads or hazard rate. This would lead to something like this: there is a defined probability for each rating, which may also have a rating specific funding spread $X(t, m)$. For the CVA calculation it would be a rating specific credit spread $u'(t, m)$. Figure 6.5 shows an example of probability distribution for different rating m at time t as P(t, m).
- From the pricing function, one would know, at each node or each scenario point, the derivative valuation. For simplicity, we use $V_k(t)$ to represent the value at t for scenario k. Now we would need to discount the value back to t-1.

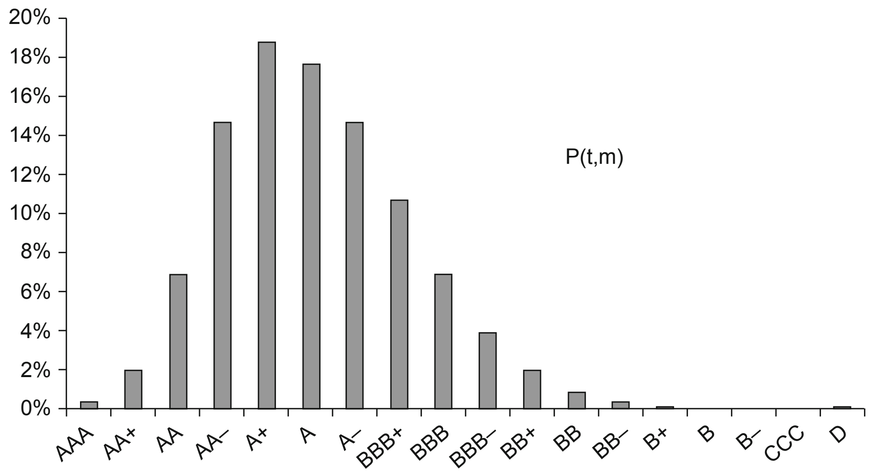

Figure 6.5 Example ratings distribution P(t,m) for rating m and time t.

- Given the credit rating distribution, we have the probability of the counterparty at each rating $P(t, m)$, where m is the index for credit rating. Assuming the funding spread for rating m at time t is $X(t, m)$, and the collateral threshold for self and counterparty at rating m are $H(m)$ and $H^{'}(m)$ respectively, then we have:

$$V_k(t-1) = \sum_{m=1}^{M} P(t,m) \begin{bmatrix} \min\left(H'(m), V^{+}(t,k)\right) e^{-(r_c(t)+X(t,m))\Delta t} \\ +\max\left(0, V^{+}(t,k) - H^{'}(m)\right) e^{-r_c(t)\Delta t} \\ -\min\left(H(m), -V^{-}(t,k)\right) e^{-(r_c(t)+X(t,m))\Delta t} \\ -\max\left(0, -V^{-}(t,k) - H(m)\right) e^{-r_c(t)\Delta t} \end{bmatrix}$$

$$+P_d(t) \begin{bmatrix} R^{'} V^{+}(t,k) \\ + \\ RV^{-}(t,k) \end{bmatrix} e^{-r_d(t)\Delta t}.$$

Here the first sum is over the non-default ratings, within which the first term is the discounting of the uncollateralized portion of the positive value discounted using rating specific spread $X(t, m)$; the second term is the collateralized portion of the positive value discounted using collateral rate; the third term is for the uncollateralized portion of the negative value discounted using specific spread; and the fourth term is for the collateralized negative values. For a counterparty default scenario with probability $P_d(t)$, the value of the scenario will be replaced with the recovery value R for self-default recovery and R' for counterparty default recovery. Under this default scenario, the discounting rate is not clear and we use $r_d(t)$ instead.[5]

- The above discounting scheme can be applied in any pricing methodology, which is simple to implement. For CVA calculations with rating triggers, one could replace the funding spread with credit spread.

The above simple methodology is approximate in the sense that the ratings migration and the exposure calculations are completely detached, whereby every scenario would be using the same rating distribution $P(t, m)$. In addition, there is no dynamic rating migration and only static distributions. This shortcoming can be corrected in a global simulation scheme with credit and market factors correlated together.

For a more complex netting portfolio with multi-assets and multi-products, the backward discounting scheme can be much more involved, and may only work with very limited scope. This is because the discounting would require netted exposure at a given point: the application of CSA would split the netted exposure into a collateralized portion and uncollateralized portion. These two portions would then carry different discounting curves. For the same reason, it is also difficult to apply more complex CSA terms, and it is very challenging to calculate wrong-way risk, and incremental CVA/FVA within a backward discounting methodology.

6.2.6 Forward simulation scheme

In a forward simulation scheme, computations are done along all simulated paths going forward in time. In this framework a large number of Monte Carlo scenarios are generated for market risk factors that impact the valuation of the derivative portfolio at discrete time steps from current time to the time when all instruments within the portfolio expire. Each individual trade within the portfolio is valued at every time step along every simulated path. From this, we calculate the exposure profile of the portfolio at each time step. Netting agreements, collateral thresholds, and other legal clauses are applied during the calculation of exposure profile. The time steps are:

1. Generate consistent simulation paths for market factors, in a chosen measure (risk neutral or historical measure).
2. Generate credit information from simulating credit risk factors dynamically.
3. Evaluate all instruments on all generic time slices and all scenarios.
4. Aggregate netted portfolio exposure with the same counterparty.
5. Apply CSA terms based on exposure and credit information.
6. Collect default loss or calculation funding cost/benefits for each path.

Assuming defaults are independent of market risk factors, the CVA can be calculated as:

$$CVA = -(1-R')\sum_k \mathbf{1}_{\tau_k' < T} D_k\left(\tau_k'\right) CSA(V_k^+\left(\tau_k'\right)) \tag{6.5}$$

where

τ_k'	The counterparty default time for scenario k;
$\mathbf{1}_{\tau_k' < T}$	Default indicator: 1 if counterparty defaults before maturity, otherwise 0;
T	The longest maturity of the portfolio;
$D_k(\tau_k')$	The discounting rate from default point to the present time;
R	The recovery for self-default;
R'	The recovery for counterparty default;
$V_k^+(\tau_k')$	The netted positive exposure at default time τ_k' for scenario k;
$CSA(\ldots)$	Indicates the application of CSA terms on the netted exposure.

Similarly, DVA can be calculated as:

$$DVA = (1-R)\sum_k \mathbf{1}_{\tau_k < T} D_k(\tau_k)\, CSA(V_k^-(\tau_k)).$$

For FVA calculations,

$$\begin{aligned} FCA &= -\sum_k \int_{t=0}^{T} D_k(t)\, CSA\left(V_k^+(t)\right) X(t)\, dt \\ &= -\sum_k \sum_j D_k\left(t_j^G\right) CSA\left(V_k^+\left(t_j^G\right)\right) X\left(t_j^G\right) \Delta t_j \end{aligned}$$

where we have collected the funding cost for each scenario k and for the netted uncollateralized exposures, at all generic time grid points j. This assumes the base values are fully collateralized valuations. Similarly for funding benefit:

$$FBA = \sum_k \sum_j D_k\left(t_j^G\right) CSA\left(V_k^-\left(t_j^G\right)\right) X\left(t_j^G\right) \Delta t_j.$$

If CSAs are relatively simple without rating triggers, the above calculations may be much simplified in that one does not have to simulate the credit risk factors dynamically. If one assumes zero correlation between the market and credit risk factors, the CVA calculation becomes:

$$CVA = -(1-R')\int P_d(t)D(t)E\left[CSA(V_k^+\left(\tau_k'\right)\right]dt.$$

The computation requirement involved in a naïve forward simulation is very intensive: we need valuations for all instruments, all scenarios and all time slices. While this might still be manageable for linear instruments with the powerful computing machines routinely employed by the top financial institutions, the brute-force approach quickly becomes uneconomical for more complex derivatives and exotic instruments. In order to reduce the required computation time, pricing accuracy must be compromised by making approximations in pricing function, or one has to find a much more efficient methodology in calculating CVA/FVA.

6.2.7 Forward simulation scheme: CSA implementation

To implement the naïve brute-force forward simulation and net instruments values across the portfolio, one way is to value the instruments for each scenario on all generic time points t_j^G. Using the following notation:

$V_{i,k}(t_j^G)$: i-th instrument value for k-th scenario at t_j^G
$V_k(t_j^G)$: portfolio value for k-th scenario at t_j^G
$V_k^{exp}\left(t_j^G\right)$: Uncollateralized/default exposure for k-th scenario at t_j^G

where $V_{i,k}(t_j^G) = \boldsymbol{P}_i(t_j^G, \boldsymbol{M}_{jk})$. Here $\boldsymbol{P}_i()$ is the pricing function for instrument i, and $\boldsymbol{M}_{jk}$ represents the market data for scenario k and at t_j^G.

Assuming we have credit rating information from simulating credit risk factors together:

$L_k(t_j^G)$: credit rating level for self at t_j^G and in k-th scenario
$L'_k(t_j^G)$: credit rating level for counterparty at t_j^G and in k-th scenario.

The default indicators for scenario k would be:

$$\mathbf{1}_{\tau_k<T} \cong \mathbf{1}_{\min_j\left(L_k(t_j^G)\right)=D}$$

$$\mathbf{1}_{\tau_k'<T} \cong \mathbf{1}_{\min_j\left(L_k'(t_j^G)\right)=D}$$

$\min_j(L_k(t_j^G)) = D$ means the minimum credit rating for all time slices is equal to default (D).

Then the netted counterparty exposure at the default time would be:

$$V_k^{exp}\left(\tau'_k\right) \approx V_k^{exp}\left(\tau'^{-}_k\right) = CSA\left(V_k\left(\tau'^{-}_k\right), L_k'\left(\tau'^{-}_k\right)\right) = CSA\left(\sum_i V_{i,k}(\tau'^{-}_k), L_k'\left(\tau'^{-}_k\right)\right)$$

with ${\tau'}_k^-$ being the generic time slice right before the default time. We use $\approx$ to indicate that the exposure at ${\tau'}_k^-$ would be approximate. One could correct this by adding bridging between ${\tau'}_k^-$ and ${\tau'}_k^+$ in implementations.

Given the portfolio value, the application of CSA is straightforward. For example:

- For a rating based threshold $H'(L)$, default exposure at t_j^G would be:

$$V_k^{exp}\left(t_j^G\right) = \min\left(\max\left[0, \sum_i V_{i,k}(t_j^G)\right] H'\left({L_k}'\left(t_j^G\right)\right)\right).$$

CVA for this scenario would be

$$CVA_k = -(1-R')\mathbf{1}_{\left({L_k}'(t_j^G)\right)=D} D_k\left({\tau'}_k^-\right)\min\left(\max\left[0, \sum_i V_{i,k}(t_j^G)\right], H'\left({L_k}'\left({\tau'}_k^-\right)\right)\right).$$

- For ATE, assuming the arrival time of breaching ATE level is ${\alpha'}_k$, CVA would be zero if ${\alpha_k}' < {\tau_k}'$, meaning the trade will be terminated due to ATE before counterparty default. Therefore the CVA for scenario k would be:

$$CVA_k = -(1-R')\mathbf{1}_{\left({L_k}'(t_j^G)\right)=D\&\&{\alpha_k}'>{\tau_k}'} D_k\left({\tau'}_k^-\right) V_k^{exp}\left({\tau'}_k^-\right).$$

  CVA only exists if the counterparty credit rating touches default before ATE level.
- For minimum transfer amounts MTA, the adjusted default exposure would be:

$$V_k^{exp}\left(t_j^G\right) = V_k^{exp}\left(t_j^G\right) + mod\left(\max\left[0, V_k^+\left(t_j^G\right) - H'\left({L_k}'\left(t_j^G\right)\right)\right], MTA\right)$$

where the mod() function is returning the remainder of numerator and denominator.

6.2.8 Forward simulation scheme: Pros and cons

A forward simulation scheme allows the easy implementation of portfolio netting, as well as direct application of various complex credit terms specified in CSA. This makes the forward simulation scheme very appealing in dealing with CVA/FVA calculations for large portfolios with complex bilateral CSAs, in cases where the backward discounting scheme may have a hard time.

Often one would face situations that incremental CVA/FVA calculations are needed. For example:

- A new deal is added to an existing portfolio, or to be traded;
- An existing deal or a sub-portfolio of trades are to be unwound.

Under both situations, one would need to evaluate the impact of the new deal or the existing deal given the portfolio. This can be done quite easily within a forward simulation scheme. We use the new deal calculation as an example below:

- From existing portfolio valuations, all $V_k(t_j^G)$ can be pre-computed for generated market factor scenarios and archived.

- The new deal valuations are performed for the same market factor scenarios as: $A_k(t_j^G)$.
- The total portfolio valuation with the new deal included would be: $V_k^{new}(t_j^G) = V_k(t_j^G) + A_k(t_j^G)$.
- The new default exposure would be:

$$V_k^{exp}\left(\tau'^{-}_{k}\right) = CSA\left(V_k^{new}\left(\tau'^{-}_{k}\right), L_k'\left(\tau'^{-}_{k}\right)\right).$$

- Given the new default exposure, one can arrive at the new CVA/FVA. The incremental CVA/FVA would be: $CVA^{inc} = CVA^{new} - CVA^{old}$.

Similarly the marginal CVA/FVA for an existing deal or a sub-portfolio of trades can be computed. The only difference is that the new portfolio is the one without the terminating deal.

As market factors are simulated directly together with credit factors, a forward simulation scheme offers an easy and efficient implementation of wrong-way/right-way risk calculations. As the market factor exposure is the most expensive part of the job, one can keep everything the same for market factor exposure calculations. The change of correlation between market and credit factors $\rho_{\mathbf{MC}}$ can be achieved by recalculating the credit factors. This is shown in Figure 6.6.

Here we use $\boldsymbol{w_1, w_2, \ldots, w_{nm}}$ to indicate the market factor random numbers that are used in simulations, and $\boldsymbol{x_1, x_2, \ldots, x_{nc}}$ to represent the credit factor random numbers. They satisfy the following:

$$\rho\left(\boldsymbol{x_i},\boldsymbol{x_j}\right) = \rho_{\boldsymbol{ij}}^{\boldsymbol{MM}}\,;\, \rho\left(\boldsymbol{x_i},\boldsymbol{x_j}\right) = \rho_{\boldsymbol{ij}}^{\boldsymbol{MC}};\, \rho\left(\boldsymbol{x_i},\boldsymbol{x_j}\right) = \rho_{\boldsymbol{ij}}^{\boldsymbol{CC}}.$$

As the market and credit factors correlation changes from $\rho_{\mathbf{MC}}$ to $\rho'_{\mathbf{MC}}$, the market factor random numbers $\boldsymbol{w_i}$ will be kept the same, which means the market factor exposure will be the same. To satisfy the correlation change, one will need to solve for the new credit factor random numbers, $x_1', x_2', \ldots, x_{nc}'$ so that the new correlation

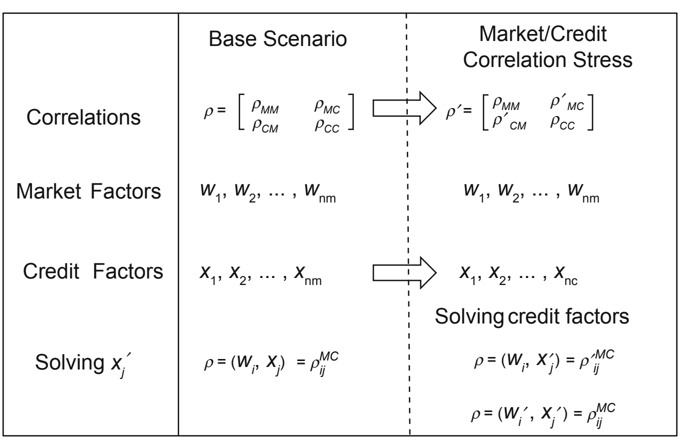

Figure 6.6 Schematic diagram for wrong-way risk calculations

matrix is observed:

$$\rho\left(w_i, x_j'\right) = \rho'^{MC}_{ij} \text{ and } \rho\left(x'_i, x_j'\right) = \rho^{CC}_{ij}.$$

These two sets of equation can be solved incrementally.

Once the credit factor random numbers are solved, new credit scenarios can be obtained by simulating the credit factors. The same credit aggregation steps can be followed to calculate new CVA and FVA given the new correlation matrix.

While forward simulation is attractive in dealing with complex CSAs and derivative products, there are some significant challenges:

- The computations can be prohibitively expensive for a naïve brute-force implementation. An efficient methodology is needed for practical implementations. In the next section, we will discuss such an efficient simulation methodology.
- Incorporating the CVA/FVA in the exercise boundary decision is not as straightforward as in a backward discounting methodology. While one can approximate such impact by estimating the expected conditional future CVA/FVA at exercise time, this is of less significance in practice.

In later sections, we will discuss a very efficient forward simulation methodology that makes the CVA/FVA computations practical. Before discussing that, we will go over the modeling of credit factors.

6.3 Credit risk factor modeling

There are two critical components in CVA calculations: counterparty default and the portfolio exposure at default. The calculation of both components may require knowledge of the counterparty credit information, and hence the modeling of credit risk factors. For bilateral CSA with ratings-based clauses, counterparty ratings are needed in CVA calculations, in addition to default probabilities. Similarly, for FVA calculations, the funding exposures would be ratings-dependent as well.

6.3.1 Modeling counterparty credit: Structural and reduced form models

In general there are two classes of models that one may use in modeling the credit defaults. One is called structural models; the other is called reduced-form models.

Structural model

In a structural model, after Merton's pioneer work in 1974 (Merton, 1974), the asset of the reference entity A is modeled stochastically, for example as a lognormal process:

$$dA = A\mu\, dt + A\sigma_A dW.$$

The reference entity defaults when its asset level is below the default level X:

$$P_d = \mathbf{E}\left[\mathbf{1}_{A<X}\right].$$

By modeling the assets and the default barrier, both from a firm's capital structure, one may derive information about the future counterparty default. Extending the model further, one may model the reference entity ratings as well.

Because the modeling of a structural model involves primitive quantities of the firm with less observability, the calibration of structural model to market observables, such as CDS spread, is not a straightforward process. However, if the reference entity has equity traded in the market, one may be able to relate the asset volatility to the market equity:

$$A = D + E.$$

For example, from the simple model, a firm's equity could be approximately given by Black–Scholes–Merton formula:

$$E = A\Phi(d_1) - Xe^{-rT}\Phi(d_2)$$

with X being the default barrier, and

$$d_1 = \frac{\ln\left(\frac{A}{X}\right) + \left(\mu + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}, \quad d_2 = d_1 - \sigma_A\sqrt{T}.$$

More accurately, one may regard the survival probability of the reference entity as not hitting the barrier X, similar to the standard American binary option pricing:

$$Q(t) = N(d_1) - \frac{A}{X}N(d_2).$$

This survival probability can be linked to the market traded CDS default probabilities.

Reduced form model

A reduced form model intends to model risk factors that are closely linked to market observables in a risk neutral world.[6] For example, one may model the hazard rate $h(t)$, which is the instantaneous default intensity of the reference entity. $h(t)\Delta t$ would give the default probability for the time interval t to $t + \Delta t$, providing the reference entity has no default in the past. Since $h(t)$ can be related to the market traded CDS spreads easily, there is no need to model the structural entities such as a firm's asset.

The default probability from t to $t + \mathrm{d}t$ would be:

$$dP_d(t) = h(t)e^{\int_0^t -h(\tau)d\tau}\,dt$$

and the cumulative default probability to time T would be:

$$P_d(T) = \int_0^T dP_d(t) = \int_0^T h(t)e^{\int_0^t -h(\tau)d\tau}\,dt = 1 - Q(T)$$

where Q(t) is the survival probability at time t. With given recovery rate R for the reference entity, one can relate h(t) to the CDS spread s easily:

$$V_{Premium\,Leg} = s\int_0^T D(t)\,Q(t)\,dt = V_{Default\,Leg} = (1-R)\int_0^T D(t)P_d(t)dt.$$

To model the hazard rate h(t) stochastically, one may choose a process for it. For example, a CIR process would be:

$$dh(t) = k(c - h(t))\,dt + \sigma_h \sqrt{h(t)}dW$$

where k is the mean reversion speed and the c is the mean reversion level. One can correlate dW with the market factors to incorporate the wrong way and right-way risks into simulations.

When only default information is needed, one may calibrate static h(t) term structure to the CDS spreads and use a Poisson process to simulate the credit default:

$$dP_d(t) = \begin{cases} 1, & \textit{with probability}\, h(t)\,dt \\ 0, & \textit{with probability}\, 1 - h(t)dt. \end{cases}$$

At any given time t, the non-default scenarios will follow a Poisson distribution with jump intensity of h(t) and jump probability h(t)dt.

Obtaining counterparty rating information

Rating information for the counterparty is critical for CSAs with rating triggers, such as the ratings-based threshold and ATEs. One may extend the structural and reduced form models to incorporate rating information in simulations.

Extending the concept of default probability in a structural model, one may attempt to calibrate the rating boundaries in a credit simulation:

$$P(m,t) = \mathbf{E}\left[\mathbf{1}_{X_{min}<A<X_{max}}\right] = \sum_k \mathbf{1}_{X_{\min}(m,t)<A_k(t)<X_{max}(m.t)}$$

where $X_{\min}(m,t)$ and $X_{max}(m,t)$ are the asset boundaries that the reference entities may fall into the rating m. Given the known transition probabilities at different time point, one may calibrate the rating boundaries and use later in credit simulations. (See Figure 6.7) In the calibration, one may calibrate the default boundary first given known risk neutral default probabilities, and then calibrate the rating boundaries.

For a reduced form model, one may need to design a mapping function $G(h,m)$, which maps the instantaneous default intensity h to the rating m. Such mapping can

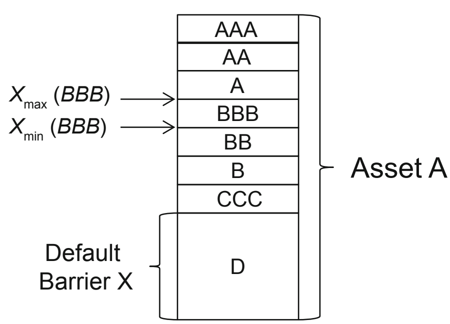

Figure 6.7 Asset boundaries for different credit ratings

be derived from historical data, similar to that which rating agencies use in generating market-based ratings from credit spreads. An example of such is shown in Table 6.4:

Table 6.4 *Example mapping function for ratings vs. CDS spreads G(h,m)*

	min	max
AAA	0.00%	0.49%
AA	0.49%	0.69%
A	0.69%	1.02%
BBB	1.02%	2.10%
BB	2.10%	4.20%
B	4.20%	7.00%
C	7.00%	35.00%
D	35.00%	100.00%

The above table shows the minimum and maximum annual default intensity levels for each rating. The ranges need to be calibrated to the traded default probabilities so they are lined up properly with market conditions. One can also tune the mapping table $G(h, m)$ to the historical or risk neutral transition matrix.

There are some real practical issues with above mapping approach. For example, the choice and calibration of the mapping function can be a really difficult exercise. We also note that the rating simulation is not dynamic, meaning that the downgrade of a reference entity would not have any effect on the default intensity simulations.

6.3.2 Transition matrix and dynamic ratings migration

A credit entity can default in two different ways: jump to default, and transition to default. In Figure 6.8 below, we show that an AA-rated entity can default through any ratings: jump to default directly to D, or transition to BBB and then default, or transition to A to BBB and then default, and so on. Ratings migration is important to CVA/FVA calculations, and therefore one should attempt to derive the transition probabilities in a reasonable way.

A ratings transition matrix provides an effective way to describe the future distribution of ratings given a starting point. In Table 6.5, we show an example historical transition matrix, which is modified based on S&P's one-year corporate transition rates from 1981 to 2011:[7]

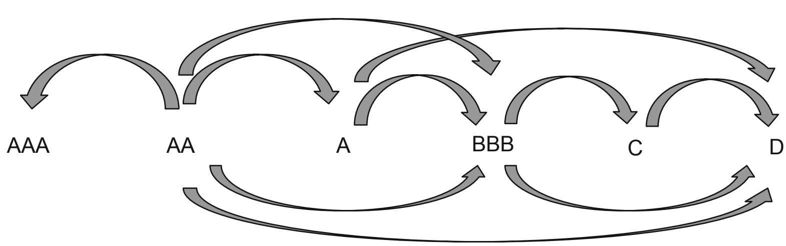

Figure 6.8 Illustration of transitional defaults

Table 6.5 *S&P's 1-year corporate transition rates from 1981 to 2011*

From/to	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	90.44%	8.71%	0.59%	0.04%	0.12%	0.04%	0.04%	0.01%
AA	0.59%	90.02%	8.40%	0.68%	0.14%	0.09%	0.04%	0.04%
A	0.05%	1.88%	91.37%	5.90%	0.48%	0.20%	0.03%	0.08%
BBB	0.01%	0.15%	3.80%	90.35%	4.49%	0.77%	0.14%	0.29%
BB	0.03%	0.07%	0.22%	5.64%	83.32%	8.92%	0.75%	1.05%
B	0.00%	0.06%	0.17%	0.27%	5.63%	83.53%	5.21%	5.14%
CCC/C	0.00%	0.00%	0.27%	0.41%	1.01%	14.19%	52.19%	31.94%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100%

Table 6.6 *Propagated rating distribution at 10 years*

10 Year	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	37.69%	35.89%	18.21%	4.99%	1.41%	0.85%	0.17%	0.79%
AA	2.53%	39.37%	37.84%	13.76%	3.11%	1.67%	0.26%	1.45%
A	0.50%	8.66%	49.16%	27.99%	6.91%	3.53%	0.49%	2.77%
BBB	0.16%	2.29%	18.08%	45.86%	15.97%	9.02%	1.23%	7.39%
BB	0.14%	0.80%	5.17%	19.47%	25.97%	22.63%	3.11%	22.71%
B	0.04%	0.39%	1.83%	6.06%	14.01%	26.54%	3.92%	47.21%
CCC/C	0.02%	0.17%	0.96%	2.44%	5.15%	10.60%	1.68%	78.99%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100%

For example, starting from rating AA, the probabilities of the reference entity ending in AAA, AA, A, BBB, BB, B, C and D after one year are in the second row of the table, as 0.59%, 90.02%, 8.4% and so on. Assuming the transition matrix does not change over time, the propagation of ratings can be easily derived by propagating the transition matrix through time. In 10 years, the ratings distribution would be as in Table 6.6, obtained by multiplying the matrix 10 times.

One immediate observation is that the default probability of high rated entities after 10 years is much greater than 10 times of the one-year default probability. For example, the A-rating's one year default probability is 0.08%, but the 10-year default probability is 2.77% »0.8%. The extra default probability of 1.97% has to come from the transition default, where the A-rating is downgraded to the lower rating and then default. If the historical transition matrix is true, then almost 71% of the default is from transition default, not jump to default.

This point can be important to the ratings-based threshold application in CVA/FVA calculations. If there is a significant chance of transitional default, there would be a substantial probability that the counterparty may transition into a rating with lower thresholds or zero thresholds before defaulting. This would reduce the CVA exposure significantly, since lower to zero thresholds would translate into lower to zero exposures at default.

As a result of the above observation, one would also expect the average annual default probability for a high rated entity to increase over time. For example, the average annual default from AA would increase from 4 basis points for 1 year to 14.5

basis points at 10 years. This is consistent with the market observed CDS spread term structures: transitional default is the major factor that drives the CDS term structure for a high rated entity to be upward sloping. On the other hand, for a lower rated entity, the transition into a higher rating would lead to the opposite position: the upgrade of a lower rating to higher rating, leading to less default probability and possibly a downward sloping CDS term structure.

6.3.3 Transitional default calculations

From a given transition matrix, one may calculate the transitional default probability easily. For example, one may calculate the transitional default probability of an AA rated entity through a BBB rating, meaning the probability of an AA transition to a BBB before defaulting. In Table 6.7, we modify the transition matrix so that from AAA to C, only BBB is allowed to default: all the default probabilities for non-BBB ratings are set to zero. This way, all cumulative default probabilities through time are through BBB transitions.

Table 6.7 *1-year transition matrix with default through BBB*

From/to	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	90.44%	8.71%	0.59%	0.04%	0.12%	0.04%	0.04%	**0.00%**
AA	0.59%	90.02%	8.40%	0.68%	0.14%	0.09%	0.04%	**0.00%**
A	0.05%	1.88%	91.37%	5.90%	0.48%	0.20%	0.03%	**0.00%**
BBB	0.01%	0.15%	3.80%	90.35%	4.49%	0.77%	0.14%	0.29%
BB	0.03%	0.07%	0.22%	5.64%	83.32%	8.92%	0.75%	**0.00%**
B	0.00%	0.06%	0.17%	0.27%	5.63%	83.53%	5.21%	**0.00%**
CCC/C	0.00%	0.00%	0.27%	0.41%	1.01%	14.19%	52.19%	**0.00%**
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100%

The resulting 10-year distribution would be:

Table 6.8 *Propagated 10-year ratings distribution with default through BBB*

10 Year	AAA	AA	A	BBB	BB	B	CCC/C	D
AAA	37.68%	35.89%	18.21%	4.98%	1.39%	0.84%	0.17%	0.04%
AA	2.51%	39.37%	37.85%	13.78%	3.10%	1.67%	0.26%	0.16%
A	0.48%	8.63%	49.14%	28.01%	6.90%	3.54%	0.49%	0.50%
BBB	0.15%	2.29%	18.06%	45.85%	15.96%	9.04%	1.23%	2.01%
BB	0.13%	0.82%	5.18%	19.48%	25.97%	22.64%	3.12%	0.39%
B	0.04%	0.41%	1.85%	6.07%	14.02%	26.56%	3.93%	0.08%
CCC/C	0.01%	0.17%	0.97%	2.46%	5.16%	10.61%	1.68%	0.04%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100%

The transitional default of all ratings AAA to C through BBB is 0.04%, 0.16%, 0.5%, 2.01%, 0.39%, 0.08%, 0.04%. Comparing to the original 10-year distribution, we can see, for example, out of the total default probability of an A-rated entity 2.77%, 0.5%

is through a BBB transition. For a AA-rated entity, 0.16% out of 1.45% total default is through BBB. The same exercise can be done for any transitional defaults through any levels. Similarly, one may make changes to transition matrix to achieve different transitional default calculations, such as the probability of touching BBB or BB or B or C before defaulting. There are many different ways of manipulating the transition matrix to calculate desired transitional defaults.

One easy application of the above simple transition calculations is to calculate all possible transitional default probabilities and apply in the CVA calculations directly. A transitional default table for different maturities can be constructed, so that one would be able to evaluate the impact of transitional defaults, as well as the impact of ratings-based thresholds.

6.3.4 Risk neutralization of transition matrix

A historical transition matrix, propagating in time, can provide time dependent rating migrations information. However, the defaults generated from the propagation in general do not match the risk neutral default probabilities derived from the CDS market. The reason for this discrepancy is obvious: the historical default behavior is different from what is predicted in the market place for the future, and does not reflect the current credit environment. And more importantly, market traded defaults also include a risk premium.

There are different ways in bridging the gap between a historical transition matrix and risk neutral default probabilities. In developing the term structure model of credit spreads, Jarrow, Lando, & Turnbull (1997) used a transition matrix approach with matrix parameters calibrated to the market traded spreads. The calibration is done by applying a risk premium to each rating and for each time period so that the market default is matched. This process is done year over year with a different risk premium at different times, implying that the transition matrix will be different over time. The resulting rating process becomes a non-homogeneous Markov chain. The following shows the transition matrix from t to t+1:

$$\tilde{Q}_{t,t+1} = \begin{pmatrix} \tilde{q}_{11}t,t+1 & \tilde{q}_{12}t,t+1 & \cdots & \tilde{q}_{1n}t,t+1 \\ \tilde{q}_{21}t,t+1 & \tilde{q}_{22}t,t+1 & \cdots & \tilde{q}_{2n}t,t+1 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \tag{6.6}$$

where the tildes indicate the risk neutral world. The linking between the historical world and the risk neutral world is through:

$$\tilde{q}(t,t+1) = \pi_i(t)q_{ij}(t,t+1) \tag{6.7}$$

with $\pi_i(t)$ being the risk premium applied to the historical transitions. Starting from a historical transition matrix and applying a risk premium as necessary, we arrive at a transition matrix with default probability matching the market over time. Here the focus of the risk neutralization process is to make sure the risk neutral default probability is obtained.

Another way to risk neutralizing the transition matrix is to risk-neutralize the transition matrix completely based on market traded CDS term structure (Lu & Juan, An Efficient, Distributable, Risk Neutral Framework for CVA Calculation, 2010). The idea comes from two observations:

- Historical default probabilities are largely much smaller than the market implied default probabilities; implying that the market is charging a large risk premium for potential market fluctuations.
- Market traded default probabilities come mostly from the potential rating transitions (downgrades) and transitional defaults.

If one does not start from the historical transition matrix, it is possible to optimize the transition matrix to arrive at a best estimate of market implied transition probabilities and market implied default probabilities at the same time. Below is a simple example to illustrate the derivation of the risk neutralized transition matrix (RNTM). Assume you have a transition matrix of 4x4 with A, B, C, D rating and the market traded default probability (based on recovery assumption) at different points in time as shown in Table 6.9,

Table 6.9 *Probability of default term structure for an example 3-rating system*

Pd	A	B	C
1	0.31%	1.72%	6.28%
2	0.72%	4.27%	11.80%
5	2.60%	13.00%	25.60%
10	7.00%	30.00%	48.00%

where the default probabilities for ratings A, B, C at listed for terms 1, 2, 5, and 10 yrs. Then the 3x3 matrix excluding the default row and column can be used as parameters in fitting to the default matrix. The optimized transition matrix shown in Table 6.10

Table 6.10 *Optimized transition matrix for the example 3-rating system*

TM	A	B	C	D
A	**96.00%**	**2.50%**	**1.19%**	0.31%
B	**0.40%**	**83.00%**	**14.87%**	1.73%
C	**0.41%**	**1.00%**	**92.30%**	6.29%
D	0.00%	0.00%	0.00%	100.00%

should recover all the market traded default probabilities within reasonable range, and therefore it can be considered to be risk neutral. In this case, the transition process is almost perfectly time-homogeneous Markovian. In practical implementations, one may use a multi-dimensional optimizer, such as Levenberg–Marquardt algorithm, to optimize the transition matrix parameters in a multi-dimensional space.

In general we may consider the market traded defaults as:

Market Implied Defaults = Transition Matrix Migrations + Perturbations from Average Long-Term View.

The perturbations from the long-term view includes the specific risk of the particular company from the average rating view and noises in the market place due to liquidity. They could be real, or simply noise, which may be used in making trading decisions.

The following shows a more realistic example with 8 ratings. The market implied default probabilities are shown in Table 6.11.

Table 6.11 *Probability of default term structure for an example 8-rating system*

P(Default)	AAA	AA	A	BAA	BA	B	C
1	0.51%	0.81%	0.92%	1.25%	2.87%	6.67%	12.50%
2	1.21%	1.93%	2.24%	3.06%	7.12%	13.85%	24.52%
3	2.13%	3.36%	3.94%	5.38%	12.36%	24.19%	37.30%
5	**4.33%**	**6.75%**	**7.89%**	**10.74%**	**21.94%**	**41.23%**	**56.80%**
7	6.37%	9.97%	11.69%	15.80%	31.00%	55.46%	70.68%
10	**9.57%**	**14.96%**	**17.83%**	**23.79%**	**44.54%**	**72.65%**	**87.80%**
15	14.37%	23.74%	27.89%	36.21%	58.11%	83.77%	97.28%
20	20.11%	32.38%	37.46%	48.07%	72.34%	97.30%	99.00%
25	25.05%	39.91%	46.12%	58.85%	84.17%	99.00%	99.00%
30	28.24%	46.17%	53.70%	68.20%	93.18%	99.00%	99.00%

In the transition matrix optimization, one may also use a weighted scheme, where the weightings of short/long term defaults are different: there is virtually no long-term default trading in the market; therefore, much less weight may be put on the long-term points. The optimization parameters are the transition matrix parameters from AAA to C, which totals 49 parameters. Additionally, one may put in extra constraints, such as continuity of transition probabilities, monotonicity, and so on.

Table 6.12 gives one optimized transition matrix, which does a reasonably good job in replicating most of the default information:

Table 6.12 *The optimized 1-year transition matrix for the 8-rating system*

1 Year TM	AAA	AA	A	BAA	BA	B	C	D
AAA	**89.60%**	9.21%	0.13%	0.15%	0.15%	0.10%	0.10%	0.56%
AA	1.80%	**84.84%**	9.77%	1.59%	0.35%	0.25%	0.30%	1.10%
A	2.40%	3.76%	**83.88%**	6.83%	0.69%	0.50%	0.65%	1.28%
BAA	2.72%	2.90%	3.81%	**81.89%**	5.43%	0.73%	0.85%	1.66%
BA	1.06%	1.61%	2.34%	2.80%	**77.56%**	6.13%	5.08%	3.43%
B	0.01%	0.01%	0.01%	1.46%	3.28%	**63.26%**	26.85%	5.11%
C	0.01%	0.01%	0.01%	0.01%	1.87%	6.00%	**72.00%**	20.09%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

This matrix gives the following fitting residues from 1 to 20 years (Table 6.13).

Table 6.13 *Fitting errors in the 8-rating example case*

Error	AAA	AA	A	BAA	BA	B	C
1	0.05%	0.29%	0.36%	0.41%	0.56%	−1.56%	7.59%
2	−0.01%	0.35%	0.47%	0.46%	0.40%	0.02%	10.42%
3	−0.22%	0.18%	0.32%	0.17%	−0.27%	−0.63%	8.90%
5	−0.77%	−0.45%	−0.23%	−0.66%	−0.12%	0.13%	5.18%
7	−0.88%	−0.64%	−0.33%	−0.80%	0.25%	−0.28%	1.51%
10	−0.73%	−0.72%	−0.64%	−1.26%	−1.14%	−3.39%	−6.10%
15	1.02%	−0.80%	−0.92%	−2.07%	−0.32%	−1.95%	−7.55%
20	2.58%	−0.80%	−1.39%	−4.26%	−5.54%	−9.52%	−5.61%

While this is not perfect fitting to the market traded defaults, it does reflect a reasonable transition probability viewed by the market. This risk-neutralized transition matrix can be used as a starting transition matrix for credit factor simulations, in which a perfect fitting to CDS spread can be achieved.

6.3.5 A Simple dynamic credit simulation model

In this section, we present a simple credit simulation model, which involves dynamic ratings transitions and can be calibrated to market traded CDS spread easily. There are two ingredients in this credit simulation model:

1. It starts from a structural model setting with an artificial asset price of the credit entity. The entity would fall into default when the asset price drops below the defined default boundary. Similarly, one may find the boundaries that correspond to the specific credit ratings.
2. In the present model, a rating transition matrix, governing the dynamic evolution of ratings distribution, is used for each time period; at each period the default probabilities are also calibrated to CDS spread exactly in simulations.

In this model, the credit rating migration is assumed to follow a non-homogenous finite state Markov process with the transition matrices calibrated to the current market data. Mathematically, for a counterparty, its rating transition probability at time t_k, with rating R_{t_k}, is given by the equation below:

$$p_{t_k,t_{k+1};i,j} = p\left(R_{t_{k+1}} = j \mid R_{t_k} = i\right)$$

where the rating transition matrix $[p_{t_k,t_{k+1};i,j}]$ can be user-specified or calibrated through a risk neutralization process. The complete matrix is:

$$\tilde{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & & p_{2n} \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Assume the credit entity starts at rating i, and it evolves into different ratings in one year with probability p_{ij}. The default probability from the transition matrix is p_{in}. The

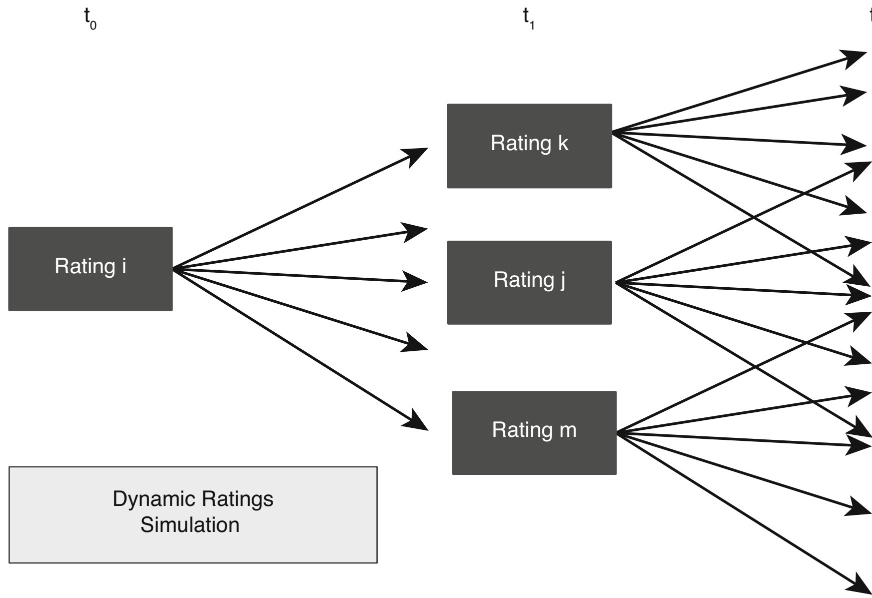

Figure 6.9 Dynamic simulations of ratings

dynamic evolution is shown in Figure 6.8. From the starting time t_0 to the next time period t_1, the credit entity with rating i will evolve into rating k, j, m and so on. The distribution of k, j, m ratings are determined by the i-th row in the transition matrix. In Monte Carlo simulations, random numbers are generated and ratings boundaries, including default, are determined by the distributions (as shown in Figure 6.9):

$$X_d = N^{-1}(p_{1i})$$

$$X_j^{min} = X_{j-1}^{max} = N^{-1}\left(\sum\nolimits_{l=d}^{j-1} p_{il}\right)$$

where $N^{-1}()$ represents the inversion function of normal random variate. Given each scenario's random number and asset level, each scenario's rating will be determined.

Migrating into next time period $t_1 \to t_2$, the ratings, such as k, j, m, will follow a new transition matrix with distinct ratings distributions for each individual rating levels. Rating k will follow the distribution from row k in the transition matrix, rating j will follow the j-th row, and so on. This way, the ratings will be simulated dynamically.

In between each time period, the transition matrix will be calibrated exactly to the market traded CDS spreads for the relevant entity. In the calibration, the default probabilities for each rating, or the last column of the transition matrix, will be calibrated to the market default probabilities.

$$\tilde{P}_{t_i} \cdot \begin{pmatrix} p_{1n}^{i+1} \\ p_{2n}^{i+1} \\ \vdots \\ p_{nn}^{i+1} \end{pmatrix} = \begin{pmatrix} p_{d1}^{i+1} \\ p_{d2}^{i+1} \\ \vdots \\ p_{dn}^{i+1} \end{pmatrix}$$

where $\tilde{P}_{t_i}$ is the cumulative probability matrix at time t_i, quantities p_{kn}^{i+1} are the default column items to be calibrated within the new transition matrix, the right hand side p_{dk}^{i+1} are the known cumulative default probabilities for rating k. In real implementations, the above equation can be solved directly from matrix inversion, or optimized with constraints. The practical constraints can include positivity of incremental default, and the monotonicity of the default probability with ratings. These may be necessary for a longer-term cumulative probability matrix, which may become ill-conditioned at very long terms. Once the default components are solved, the probability of the other ratings may be adjusted to ensure that the sum of probabilities for all ratings will be 1. This step-wise risk neutralization of the transition matrix represents reasonable efforts in linking the market default information to ratings transition; however, in practice, one is always limited by the availability of market information as well as the market liquidity considerations.

In calibrating to the market default within simulations, boundaries may be shifted so that the exact default probabilities are matched. Alternatively, one can randomly re-rate the scenario from different ratings based on conditional probabilities, so that a default scenario can become non-default, and a non-default scenario can become default. If the simulated scenarios give a probability of default than exact default, one can pick a non-default scenario to become default, so that the default probability perfectly matches the market. Based on the different default probabilities for each rating, one can randomly choose the scenario from non-default scenarios to become default. On the other hand, when the default probability from scenarios is more than the expected default probability, one can choose the default scenario and reset into non-default ratings according to the conditional default probabilities. To match the default probability precisely, one can also assign a partial default for a specific scenario. This in general does not have a material impact on the CVA/FVA results.

6.4 Efficient CVA/FVA calculations

In this section we discuss efficient CVA/FVA calculations, applicable to multiple asset classes.

6.4.1 Efficient exposure calculations

Portfolio-wise, exposure at default and funding exposure is the most costly part of CVA/FVA calculations, due to CSA netting requirements. Following the previous discussions, we attempt to value individual instrument i at all required time points, such as generic time grid t_j^G, for all scenarios: V_{ijk}, so that we can calculate the netted portfolio value: $V_k(t_j^G)$.

To arrive at the individual instrument value, one may use a pricing function $\boldsymbol{P}()$ for the given market condition: $V_{ijk} = \boldsymbol{P}_i(t_j^G, \boldsymbol{M}_\mathbf{k})$. However, this would be very expensive. For complex instruments, it would be prohibitive in practice. There are two different ways to make the computations more efficient.

Approach 1: Deriving approximate valuation function $\tilde{P}()$

The first approach is to arrive at a more approximate valuation function $\tilde{P}()$**.** This approximate valuation function can be obtained in different ways. One of them is to make use of regressions on market variables:

$$\{\boldsymbol{P}_i\left(t_j^G,\boldsymbol{M}_{\boldsymbol{k}}\right) \xrightarrow{Regression} \tilde{P}_{\boldsymbol{i}}(t_j^G\boldsymbol{M})$$

where {}represent the set of valuations. This can be achieved by minimizing the utility function, for example:

$$\sum_k w_k\left[\tilde{P}_{\mathbf{i}}\left(t_j^G,\boldsymbol{M}_{\boldsymbol{k}}\right)-\mathbf{P}_i\left(t_j^G,\boldsymbol{M}_{\boldsymbol{k}}\right)\right]^2$$

where $\tilde{P}_{\boldsymbol{i}}()$ represents the approximate regressed valuation as a function of market variables, and w_k is the weighting for the specific scenario with $\sum_k w_k = 1$. The regression process would be performed for all required time slices t_j^G, so that the instrument valuations $\tilde{P}_{\mathbf{i}}()$ can be applied later for consistent market/credit factor simulations, where netting across a single counterparty is achieved. This same regression methodology is commonly used in time-dependent callable product valuations.

The derivation of the approximate valuation function $\tilde{P}_{\mathbf{i}}()$ can be done based on any pricing approach, such as finite difference PDE, trinomial trees, or Monte Carlo simulations. So the trading quality pricing model can be used here. Usually the regression process is performed before CVA calculations, with regression function saved, or with $\tilde{P}_{\mathbf{i}}()$ tabulated for later use. The process would be:

1. Selection Market Variables → Valuate instruments for X scenarios for all time slices t_j^G
   → Archive regression results/tabulations of $\tilde{P}_{\mathbf{i}}()$ **.**
2. CVA/FVA simulations → Apply $\tilde{P}_{\mathbf{i}}()$ for all t_j^G and all Y scenarios $\mathbf{M_k}$
   → Netting of V_{ijk} to get $V_k(t_j^G)$.
3. The $\tilde{P}_{\mathbf{i}}()$ can be applied to all CVA/FVA Greeks, risk and stress calculations.

The scenarios X and Y are clearly different. The success of this approach would be dependent on the stability of the regression, the explanatory power of the market variables, and the complexity of the payoffs. In addition, when the number of instruments becomes large, the required regression and tabulation could be significant.

Approach 2: Brownian bridging Monte Carlo simulations

Another efficient approach in exposure calculations is to use a trick in deriving consistent market scenarios based on a Brownian Bridge, and is applicable to a global Monte Carlo simulation based methodology.

We assume the following convention:

N_M : Number of market risk factors
N_C : Number of credit risk factors
N_T : Number of generic time grid periods
N_S : Number of Monte Carlo paths

For Monte Carlo simulations, one would need to generate $L = N_T \times (N_M + N_C)$ random number vectors $\boldsymbol{R}_l$. Each of the random number vectors is N_S long, and all the random vectors are independent: $\rho(\boldsymbol{R}_l, \boldsymbol{R}_m) =$ for $l \neq m$.

In the present approach, one would price the instruments on the same pricing grid the same way as in normal derivative valuations. The pricing grid would include all instrument-specific schedules, such as cash flow dates, exercise dates and notifications. Figure 6.10 shows the situation schematically: each specific deal A and B, would have a different pricing grid from the generic time grid. This allows accurate valuations with all necessary pricing time points without sacrificing the valuation accuracy.

There is one problem here though: the random numbers are generate for generic time grid periods t_j^G, and not for t_j^A or t_j^B. Therefore, the random numbers $\boldsymbol{R}_l$ cannot be applied in direct instrument valuations.

To resolve this problem, we can make use of Brownian bridge method to construct random numbers are consistent with the original set of $\boldsymbol{R}_l$ random numbers. In Figure 6.11 we show this schematically: The generic time grid is shown in black, with the time slice intervals dt, dt and so on. The random numbers $\boldsymbol{R}_l$ for the intervals are dz and dz' respectively. The trade specific grid is shown in gray. Our goal is to derive the random number dz'' which is for the trade specific time slice $dt_2 + dt_3$. In order to derive dz'', we attempt to derive the random numbers dz_1, dz_2, dz_3, dz_4.

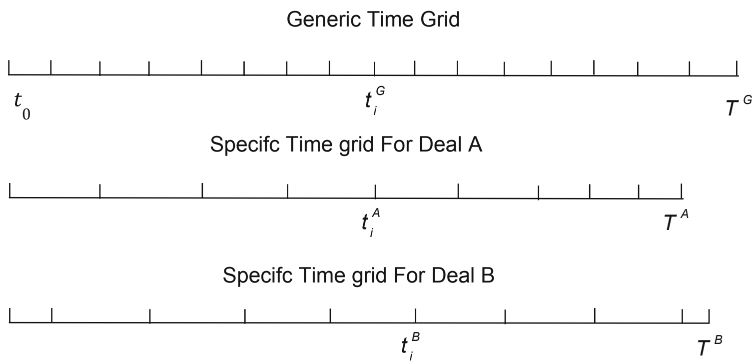

Figure 6.10 Illustration of specific trading time grid and generic time grid

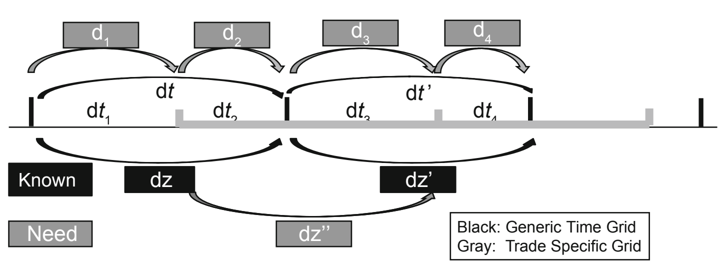

Figure 6.11 Illustration of Brownian bridging of random numbers

From the Brownian bridge formulation, we know:

$$dz_1 = \sqrt{\frac{dt_1}{dt}}dz + \sqrt{\frac{dt_2}{dt}}dw$$

where dw is a random number vector that is independent of dz, with $\rho(dz, dw) = 0$. Similarly:

$$dz_2 = \sqrt{\frac{dt_2}{dt}}dz - \sqrt{\frac{dt_1}{dt}}dw$$

$$dz_3 = \sqrt{\frac{dt_3}{dt'}}dz' + \sqrt{\frac{dt_4}{dt'}}dw'$$

Given dz_2 and dz_3, we can arrive at dz'' as:

$$dz'' = add\left(dz_2, dz_3\right) = \frac{1}{\sqrt{dt_2 + dt_3}}\left(\sqrt{dt_2}dz_2 + \sqrt{dt_3}dz_3\right)$$

where the $add()$ operator indicates combining the two random numbers together. Through the above process, we have obtained the random sequence dz'' with the desired property:

$$\rho\left(dz'', dz\right) = \frac{dt_2}{\sqrt{dt(dt_2 + dt_3)}}$$

$$\rho\left(dz'', dz'\right) = \frac{dt_3}{\sqrt{dt'(dt_2 + dt_3)}}$$

where the correlations are conserved. It is also easy to see that the random sequences for trade specific time intervals are also independent of each other.

The above-derived random sequences for the trade specific time grid would have the proper correlations with all the $N_T \times (N_M + N_C)$ original random sequences, which would guarantee the valuations from the specific trade valuations in scenarios are net-able across the portfolio.

The process of this approach works in the following steps:

1. Generations of all $N_T \times (N_M + N_C)$ independent random vectors, where $N_T \times N_C$ will be used for credit factor simulations.
2. For each trade, derive the random vectors $\mathbf{R_i}'$, which are consistent with the original random vectors $\mathbf{R_l}$.
3. Perform valuations of specific transactions within the same netting set using the newly derived random vectors $\mathbf{R_i}'$, where the market exposures are evaluated for each transaction for all scenarios and every trade specific time slices.
4. One may also include the generic time grid in the above simulations to obtain the scenario exposures. Alternatively, one may also derive the exposures at generic time points based on scenario discounting, and proper interpolations.
5. Aggregate the exposure on all scenarios and generic time points, and apply CSA to calculate the credit and funding exposures, which would be used in CVA/FVA calculations.

To make the valuations more efficient, one may generate $N_T \times (N_M + N_C)$ random vectors and save beforehand. Then the Brownian bridge approach can be applied dynamically in each trade valuation and market exposure calculations. We note that the valuations can be done exactly the same as a normal derivative valuation with only the replacement of Brownian bridge random numbers. In the whole process, only one Monte Carlo simulation is needed for every transaction, where the complete set of market exposures is derived.

One practically good approximation that one may make in a market exposure calculation is to use realized scenario values to replace the conditional expected market exposure for the specific scenario in step 3:

$$V_{ijk} = V_{ijk}^{expected} \cong V_{ijk}^{realized}$$

One can prove that within the limit of a large number of scenarios, using a realized scenario values would be essentially the same as expected scenario values in CVA: the expectation of all realized scenario values for a specific market scenario $\boldsymbol{M_k}$ at t_j would be the same as conditional expected scenario values.

6.4.2 Cash flow compression for linear instruments

For linear instruments, such as interest rate swaps, the payment index resets and cash flows are known and are generally additive. This leads to a more efficient CVA/FVA calculation for linear swaps: one can distribute and compress the expected resets and cash flows for the portfolio swaps together. For example, the cash flow expectations can be distributed on the generic time grid points. (See Figure 6.12 below).

For fixed leg cash flows, the values are known; for the floating index payments, one can accumulate the notionals on different time points for the same index, which are then distributed on the generic time grid points:

$$\sum_i^{Exact} V_{ik}(t_j^G) = V_k^{Compress}\left(t_j^G\right) = \sum_i w_{ij} V_i^{fixed}(t_j^G) + \sum_i w_{ij} V_i^{floating}(\mathbf{M_k}, t_j^G)$$

where the first term on the right hand side is the fixed leg payments, and the second term is the floating leg payments in scenario k. The weighting w_{ij} is used to distribute the cash flows for instrument i onto the generic time point t_j^G. The fixed leg cash flow does not depend on the market scenarios, and the floating leg $V_i^{floating}(\boldsymbol{M_k} t_j^G)$ is a function of the market factors $\boldsymbol{M_k}$ in scenario k.

Clearly the distribution of cash flows on a discrete time grid would lead to approximations. This is practically accurate as CVA/FVA would not be sensitive to the

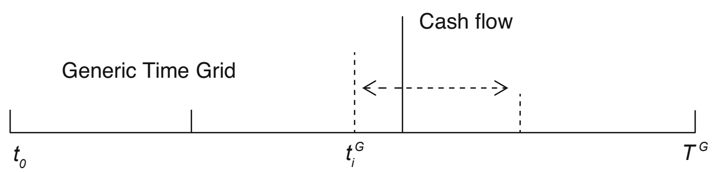

Figure 6.12 Illustration of cash flow compression

timings of distant future cash flows. Given a good choice of generic time grid points, with denser grid points in the shorter term and sparser in the longer terms, this would not be an issue at all.

Cash flow compression can be a very useful technique when a large number of linear instruments are needed for netting in a large counterparty portfolio. One may create one single instrument for all fixed cash flows for each currency, and create one instrument for each type of payment index. For example, one for LIBOR 3 month index, one for 1 month index, one for OIS index, one for BMA index, one for EIBOR 6 month index, and so on.[8] When you can replace thousands of linear swap transactions with a few trades only, this can be a tremendous saving in computation time.

6.4.3 Efficient incremental and marginal CVA/FVA calculations

When a new trade is executed under a bilateral CSA with an existing portfolio, it generates a CVA/FVA impact that may or may not be the same as individually calculated CVA/FVA. This is what we call an incremental CVA/FVA calculation. For example, we have the following:

- SF and CP have very different credit ratings, say SF = AAA with 30bps default probability, and CP = BB with 300bps default probability.
- We have one existing trade with asset of 100 million, which translates into 2.4 million of CVA, if assuming 40% recovery.
- Now we have an exact offsetting trade with liability of 100 million. We know the total portfolio would have CVA=0, since all exposures are netted to be zero. So the incremental impact would be positive 2.4 million, which is the cancellation of original negative 2.4 million CVA.
- Individually, however, this new trade will have a DVA of 0.24 million outside the context of portfolio, which is vastly different from the incremental CVA of 2.4 million.

Here we see rating differentials between the trading counterparties as being one of the important factors that may lead to disparities in individual and incremental CVA calculations. Other factors are the CSA terms, such as collateral thresholds and exposure maturity distributions.

Incremental CVA/FVA can be critically important to live trading decision-making, and its accuracy may impact the competitiveness of the trade. Efficient incremental CVA/FVA calculation for a new derivative trade (see Figure 6.13) as described in section 6.2.8, can be done in the following steps:

1. The portfolio exposures V_{jk}^{Port} are known for existing trades, and are pre-computed in portfolio CVA/FVA calculations through CSA aggregation with credit simulations. These exposures are archived for all generic time grid point j and scenario k. For each counterparty, this could be 10,000 scenario x 100 time grid = 1 million valuations.
2. Calculate the market exposure for the new trade V_{jk}^{New} with the same market data and consistent random numbers, as prescribed in previous sections: pre-generated generic random numbers can be used to generate the consistent deal-specific random numbers through a Brownian bridge.

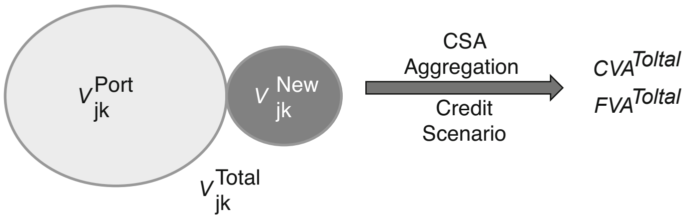

Figure 6.13 Illustration of incremental CVA/FVA calculation for a new trade

3. The total exposure for the new portfolio would be $V_{jk}^{Total} = V_{jk}^{Port} + V_{jk}^{New}$, which can be aggregated with existing credit scenarios. The new CVA/FVA can be calculated in the process.
4. Incremental $CVA^{Inc}(FVA^{Inc}) = CVA^{Toltal}(FVA^{Toltal}) - CVA^{Port}(FVA^{Port})$. For the example above, $CVA^{Inc} = 0 - (-2.4\,million) = 2.4\,million.$

The only new calculation really involved above is step 2 in deriving the market exposure for the new trade; other quantities needed for the credit aggregation are already known from existing computations. As the credit aggregation step is a quick process, the incremental CVA/FVA calculation is very efficient.

Similarly the marginal CVA/FVA effect from an existing trade can be calculated by subtracting the total exposures from the surviving portfolio exposure. This may be required when a trade is being terminated, or one needs to allocate CVA/FVA among the existing portfolio trades.

6.4.4 Greeks computations and distributed computing

So far we have considered single valuations of CVA/FVA and have focused on how to improve the efficiency of such valuations from a modeling perspective. As we know, single CVA/FVA valuation is only the starting point for the management of portfolio risks. In order to manage the credit and funding risks, one would need to be able to access all Greeks, including first order delta, vega Greeks, to second order gamma, and theta, as well as cross Greeks, such as Vanna and correlation sensitivity. Greeks computation is the most time consuming part of the required massive CVA/FVA computations.

The most naïve Greek computations would be brute-force bump and re-compute. For all the risk factors, where the Greeks are needed, the risk factors will be bumped with CVA/FVA and re-computed exactly the same as base CVA/FVA calculations. This methodology would be easy to implement, especially for a distributed computing platform.

In Figure 6.14 we show schematically a simple distributed computing architecture. The simplest computing unit would be a CVA/FVA computing given a market data scenario with trade, legal and counterparty information. The computing units will be distributed as individual jobs to each of the computing nodes within a farm of computers. The results of the computation, such as deal valuations for a specific simulation path and future time points, can be fed into a database, or messaged back to the CVA/FVA engine for aggregation.

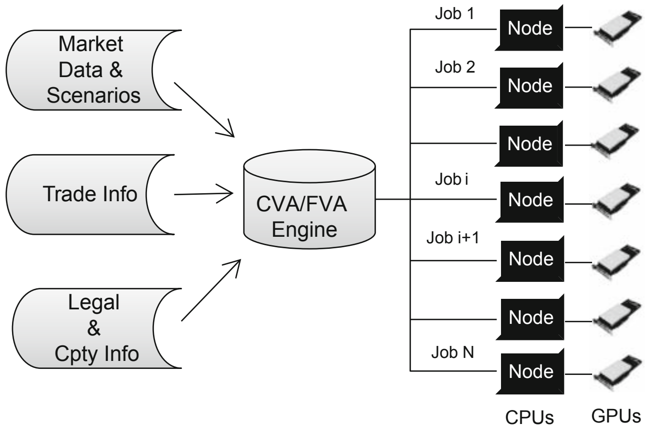

Figure 6.14 Illustration of distributed computing with CPUs and GPUs

There can be all kinds of flavors in designing the computation units and the architecture of distributing the jobs. In general, the smaller the computation jobs are, the more efficiency one may enjoy from the distributed computing. This is, of course, assuming the overheads of distributing jobs are much smaller compared to the computing time for each job. When the job-distributing overhead is significant, one would then need to reconsider the distribution of jobs so that the optimal computational time is achieved.

Another technology available is the graphics processing unit (GPU), traditionally used for real-time rendering and gaming. GPU is mostly suited for speeding up a part of the computations that is embedded in the most time consuming loops, such as 10,000 simulation paths. For example, the valuations for all 10,000 market factor simulation paths can be distributed through parallel computations on the GPUs very efficiently, providing the memory requirement for each path is smaller than the GPU device memory, and the valuations for paths are independent. One can also embed multi-threading into the GPU programming to add one more layers of parallelism with shared memory access of data among threads. When the distributed computing nodes are equipped with GPU units, one may layer the GPU parallelism on top of the distributed CPU computing, which would add more computational power and enable fast computation of CVA/FVA and relevant Greeks.

6.4.5 Adjoint algorithmic differentiation

Adjoint algorithmic differentiation (AAD) has been successfully applied in financial engineering in recent years, especially in speeding up complex Greeks computations requiring extensive derivative valuations. AAD is a set of programming techniques, which helps the numerical evaluation of derivatives of a function. In doing so, AAD decomposes the computer program into elementary arithmetic operations, such as addition, subtraction, multiplication, division, etc., and elementary functions, such as exp, log, etc. By repeated applications of the chain rule to these operations, derivatives can be obtained automatically with limited increase in computational expenses. While AAD can be very useful in speeding up the Greeks computations, it requires significant programming work to re-write the existing pricing routines completely. The readers are referred to Capriotti & Giles, Algorithmic Differentiation: Adjoint Greeks Made Easy (2012) for more details.

6.5 Other considerations

6.5.1 CVA/FVA cross: Default on funded exposure or funding on non-default asset?

In CVA calculations, what is the real exposure at default? Is it fully collateralized valuation or fully unsecured valuation, or funded exposure? The answer is not as clear as it may seem. Even though the impact of using different funding assumptions in a CVA calculation for EAD is relatively minor, it is worthwhile to discuss here from theoretical point of view.

In section 4.3.8, we discussed the close-out valuations when a derivative trade is terminated based on the "market quotation" mechanism and the ISDA ruling on the settlement process. Based on what we have discussed on the "market quotation", the termination settlement value of a derivative trade is really, however, funded exposure, based on market funding level. So it is not simple funded exposure with specific funding spread. This means one may use market funding in CVA exposure calculations. While there is no perfection in determining the exact exposure at default, this market funding exposure would represent the best estimate from valuation point of view.

From the funding perspective, the funding effect under credit default is not exactly continuous, meaning the funding spread will switch from specific funding to market funding. For non-default time periods, the funding cost/benefit would be based on specific funding spread, whereas for default time and afterwards, the funding spread should be based on market funding spread.

Summarizing the above discussion, the consistent CVA/FVA calculation logic would be:

- When calculating CVA, the exposure at default should be based on market funding spread, meaning the future cash flows and contingent future cash flows should be discounted based on the market average funding curve.
- When calculating FVA, the funding for a non-default scenario should be the same as using the firm's specific funding curve to calculate funding cost and benefit; however, when default happens, the average market funding spread should be used.

This is represented intuitively as:

$$CVA = Value\left(risky,\ marketFunding\right) - value(riskfree,\ marketFunding)$$

in a backward calculation scheme, and in a forward simulation scheme:

$$CVA = -(1 - R')\int_0^T V^+(t, \boldsymbol{r_M}) D(t, \boldsymbol{r_u})\, P_d(t)\, dt$$

where $V^+(t, \boldsymbol{r_M})$ represents the positive exposure calculated based on market funding level $\boldsymbol{r_M}$. Note the discounting of this default loss to present time would depend on the collateral situation and specific funding spread theoretically.

Similarly, the FVA calculation in a forward calculation scheme would be:

$$FCA = -\int_0^T \left[(1 - P_d(t))\, X + R' P_d(t)\, X_M\right] V^+(t)\, D(t)\, dt$$

meaning the funding cost for the non-default portion $1 - P_d(t)$ would be using specific funding spread X, while the default portion $P_d(t)$ would be using market funding spread X_M. Similarly formula can be specified for FBA.

In a backward discounting scheme, one could incorporate different funding spreads in the discounting rate depending on default probability to represent the funding curve discontinuity at default.

6.5.2 Netting set and funding set in FVA calculations

CVAs are generally calculated based on a netting set, where all transactions are netted together in calculating the credit exposure subject to default. Funding and collateral operations are defined, generally, within the same netting set. The credit exposure, not covered by collateral, would be the same as the funding exposure that needs to be funded for the same bilateral counterparty.

Funding value, calculated based on firm specific funding curves, however, could be symmetric or asymmetric between funding cost and benefit. When it is symmetric, one recognizes the same amount of funding benefit as the funding cost for same amount of liability and assets. Under this situation, one can value FVA quite readily given the similar CVA exposures.

When the funding values are asymmetric, for example when one applies a different funding benefit spread curve compared to the funding cost curve, the FVA calculation may be more complicated: when assets and liabilities are offsetting each other, they are really symmetric; however, when assets and liabilities are alone by themselves, the funding spreads are different.

The existence of a different funding benefit curve compared to a funding cost curve may occur due to some practical considerations. First of all, a firm's funding can be done through different channels, some are cheap fundings and some are relatively expensive. A firm may not want to offer the wrong funding incentives to traders and have them piling up too much funding benefit. When a firm has net cash in hand, the short-term return on this cash will be less than what they pay in the debt market. Secondly, from conservative point of view, a firm may not want to have much outright funding benefit linked to its own credit. Making money with worsening credit does not appear well to the regulators.

This then brings up the concept of the funding set as well as an FVA calculation issue: A firm's funding is generally operated on a funding set basis, not on a counterparty netting basis. Here the funding set covers the overall set of funding requirements of the entire firm. (See Figure 6.15) For example, the firm may deal with a thousand counterparties, all with different collateral requirements. For CSAs allowing rehypothecation, the collaterals received from one counterparty can be used to generate funding value, such as posting to other counterparties or realizing the

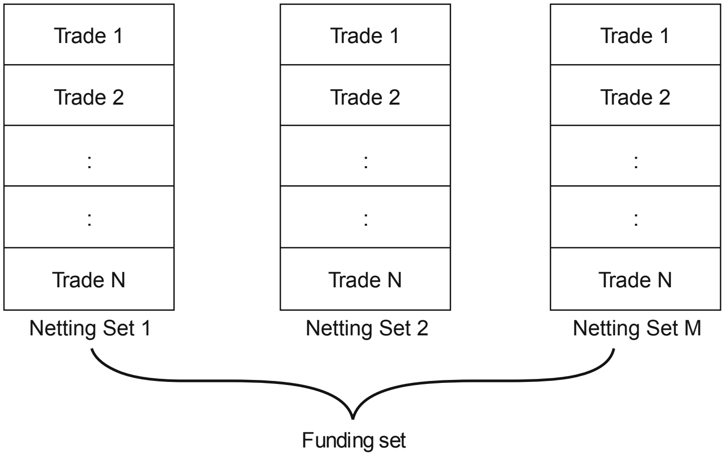

Figure 6.15 Illustration of netting set vs. funding set

funding value in the repo market. For expected payables and receivables happening at the same time, the firm would only have to deal with the net of the payables and receivables. So effectively, the asset and liability exposures would net with each other before the funding operations are done. This clearly reduces the funding cost calculations based on the sum of each individual netting set basis.

To see this more clearly, we assume the funding cost and benefit spread to be X^+ and X^-, then the total FVA would be as the following for netting set:

$$FVA_{NettingSet} = \sum_i FVA_i = -X^+ \sum_i \sum_{j,k} Asset_{i,j,k} + X^- \sum_i \sum_k Liability_{i,j,k}$$

where index i indicates i-th counterparty, and j, k indicate the k-th scenario at j-th time. Here the funding cost for assets and benefit for liabilities are essentially calculated individually and summed together. For a funding set FVA calculation, it would be like the following:

$$FVA_{FundingSet} = -X^+ \sum_{j,k} Asset_{j,k} + X^- \sum_{j,k} Liability_{j,k}$$

where the j-th time k-th scenario $Asset_{j,k}$ would be netted asset against liabilities among all counterparties, and similarly for liabilities. The funding set asset, summed over all netting set exposures, would be less than the sum of netting set assets:

$$Asset_{j,k} = \max\left(\sum_i NEP_{i,j,k}, 0\right) \leq \sum_i Asset_{i,j,k}$$

where NEP is the netted exposure for a single netting set. The above condition leads to the result that the funding set FVA would always less equal than the netting set FVA. When $X^+ = X^-$, we would have $FVA_{NettingSet} = FVA_{FundingSet}$.

Reflecting the asymmetric funding cost and benefit in the FVA calculation would require a much larger funding set in FVA calculations. Schematically, this may be done through the following process:

1. Determine the exposures for all netting sets within the large funding set;
2. Net all the netting set exposures to become funding set exposure;
3. Based on funding set exposure and the given funding cost/benefit curves to calculate the asymmetric funding costs and benefits.

This methodology will allow the enjoyment of more funding benefit than would be calculated based on individual netting sets, due to the ability to net the cost side more effectively using funding benefits. This is especially the case when benefit curve is very different from the cost curve.

6.5.3 Mutual put breaks

Mutual Put breaks or Mutual Puts (MP), usually defined in trade confirmations, give both counterparties options to terminate individual transactions at the specific dates with certain frequency, such as every 3 years, 5 years or 10 years. Mutual put breaks give the trading counterparties an option to break from the transaction, therefore reducing the counterparty default risk, especially for very long-term transactions. Barring any transaction cost and operational considerations, in theory, MPs should always be exercised by one of the parties. This is because at any one moment, there would be one party that may gain from exercising the MP.[9] So for a 30-year transaction with an every-5-year mutual put break, the credit default risk is really only at most 5-year, theoretically. As we know, CVA comes mostly from expected long-term default risks; a mutual put break option can be handy for the trading counterparties to mitigate counterparty credit risks and reduce CVA cost. In fact, most bank dealers would incorporate MPs into potential exposure calculations as well as in their credit risk limit considerations.

Practically, however, trading counterparties may choose not to exercise the MP options at the MP dates, especially if there is little or no change in the credit standing of either counterparty. This was often the case historically before the 2008 credit crisis. From the bank dealer's point of view, termination of the transaction may have customer relationship consequences, especially for a customer with good credit standing: "Why are you terminating a transaction with me if I have good outstanding credit?" Termination of a transaction means that the customer would have to replace the transaction in the market, which leads to extra work and cost for the customer.

This obviously creates a problem: bank dealers were using MPs as credit mitigants in their credit risk evaluations and decision-making. If these credit mitigants are not realized, should they be accounted for in CVA/FVA calculations, and in credit risk decision-making?

One way to look at the decision-making from the bank dealer's point of view is to treat the MP decision as a fresh credit risk decision: At the time of MP exercise, does the counterparty have good outstanding credit or not? Are we willing to take another 5 year credit with the counterparty? Clearly, if the counterparty is downgraded, the exercise decision may be easier. If the counterparty has good outstanding credit, the credit risk officer would be willing to extend the credit risk for another 5 years. From a credit risk management point of view, which is mostly based on potential future exposure, MP is a good risk mitigation tool and its value is realized.

This, however, could be a problem if MP is taken into account for CVA/FVA calculations. If at the MP date, the counterparty's expected future exposure creates a CVA charge, it would be economical to exercise the MP. Otherwise, the consequence of extending for another 5 years would be a loss in P&L: the day after MP will see a sudden increase in CVA. Given this, one would argue that one might not want to take MP into the expected loss, or CVA calculations, if MPs are not exercised practically. Then the error will be on the conservative side: if the exercise does not happen on the MP date, CVA will be continuous; on the other hand, if the MP is exercised, there would be a CVA gain because the future credit loss is terminated by the exercise decision. However, to make this a conservative valuation, one may need to take MP into account for DVA calculations.[10] This leads to a situation of asymmetric use of MPs: accounting for MPs in DVA and not for CVA.

One modification of the above conservative approach is to take MPs into account with a modified MP. Consider a 30-year trade with a break every 5 years as an example. One may use 2 MP periods in calculating the expected future exposure in CVA calculations. So one would be computing the CVA based on the MP date at 10 years in CVA calculations. This would reduce the size of P&L discontinuity at MP dates: the difference between the 10-year exposure and 5-year exposure is the discontinuity that may happen at the MP date. Another variation of this is to take MPs fully into account, but reserve the amount for the next MP period for CVA. So the CVA/DVA will be calculated based on full MP exercises; however, the difference between 10-year MP CVA and 5-year MP CVA will be reserved.

After the 2008 credit crisis, more and more firms are exercising MPs and realizing the values economically (Cameron, 2012). This is also driven by two facts:

- Firms are more sensitive to the credit risk as well as the capital requirements accompanied by the credit exposure. Exercising MPs makes a significant difference in saving capital in addition to the P&L effect.
- More and more market participants, including the customers, are coming to realize the true value of MPs and their impacts. Exercising MPs with significant P&L and capital impacts have become common practice in the market, much like the fact that FVA is a reality and one has to take it into account in valuations.

In view of the market trend and perception changes, as long as one establishes a proper procedure in exercising the MPs and making the proper economic and credit decisions, MPs may be incorporated in CVA/FVA, credit risk management and capital management decisions.

Summary

In this chapter, we have discussed:

- The complexities involved in CVA and FVA modeling and computations. One of the more tricky issues is that one has to deal with the netting of many trades within one netting set. While CVA and FVA computations are very expensive and challenging, a better and efficient methodology is needed.

- A backward discounting scheme and forward simulation scheme may be used in CVA and FVA calculations. A backward discounting scheme can be very efficient in calculating a small number of trades with single market risk factors; however, it lacks of the necessary sophistication and consistency in dealing with more complex CSA and netting issues for a more generic derivative portfolio. Forward simulation can handle very complex CSA situations and for multiple assets, it requires efficient algorithms as well as resolving consistency in netting portfolio of trades.
- Credit risk factors can be modeled in a structural model or a reduced model framework. While a structural model may be difficult to calibrate, a reduced form model simulates credit risk factors directly linked to market traded CDS prices.
- To deal with CSAs with ratings-based thresholds and rating triggers, one may use a transition matrix-based model to derive rating transition information. The rating transition matrix can be optimized to calibrate to the market implied default probability term structure, and be made close to "risk-neutral". A dynamic credit rating simulation model is presented.
- Two efficient CVA/FVA calculation schemes are discussed, one deriving fast and approximate valuation functions for all instruments; the other is Brownian bridging random numbers. The latter is a more general methodology that can be applied across asset classes. Other tricks in improving computational performances are discussed briefly.
- CVA and FVA cross term is an interesting subject. Default on funded exposure has discontinuity at the default time. Mutual put break and its implementation is another interesting subject that is covered.
- Asymmetric funding leads to the concept of funding set, comparing to netting set. Funding set implementation would represent closer to truly optimized funding operations with rehypothecations.

IV XVA RISK MANAGEMENT AND HEDGING

7 CVA and FVA Risk Management

7.1 Derivatives risk management primer

CVA and FVA for derivative transactions are complex in that they can be driven by many different market risk factors in addition to the credit risk factors. Before discussing the risk management of CVA and FVA, it is worthwhile first looking at the basics of derivative valuation and risk management.

7.1.1 Derivatives valuations and replication

Derivatives are contingent claims with payoffs depending on one or more risk factors. Due to the stochastic nature of the underlying risk factors, the risk management of derivatives is always very involved.

Taking the simple European stock option with constant volatility as an example. One may consider different ways of pricing the option. We start with the simple assumption of a Geometric Brownian motion for the underlying S:

$$dS = \mu S dt + \sigma_S S dz \tag{7.1}$$

where dz is the standard Wiener process, μ is the drift. With the risk-neutral argument that one has no risk-preference in a risk-neutral world, one can set $\mu = r - q$, where r is the traditional risk free rate, and q represents the dividend payment rate. Then given the stochastic process, one can derive the forward distribution for the stock at maturity T:

$$P\left(\ln \frac{S_T}{S}\right) = \phi((r - q - \frac{\sigma^2}{2})T, \frac{\sigma}{\sqrt{T}})$$

where $\phi(0,1)$ is the standard normal distribution. With the above known distribution at maturity T, one can calculate the derivatives value in a risk-neutral world:

$$V\left(c, S, X, \sigma_S, t, T, r, q\right) = ce^{-r(T-t)}\left[SN(cd_1) - XN(cd_2)\right] \tag{7.2}$$

with

$$c = \begin{cases} 1 & for\, call \\ -1 & for\, put \end{cases}$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + (r + \frac{\sigma_S^2}{2})(T-t)}{\sigma_S\sqrt{T-t}}, \quad d_2 = d_1 - \sigma_S\sqrt{T-t}$$

X is the strike, $N()$ is cumulative probability distribution function for a normal distribution.

This is a static way looking at the valuation of the simple European option: it is an integration of payoff from a static forward probability distribution of S which, in turn, depends on the volatility of S. So the value of the derivatives can be viewed as a function of spot S, volatility σ_S, and maturity T.

It should be noted that this method can be extended when the underlying stochastic process does not give a simple lognormal distribution. When the log-return is not normally distributed, one can simply integrate the payoff with a different distribution, for example, a skewed-distribution. An example is the Corrado–Su skew model based on skewness and kurtosis expansion.[1]

Another way of valuing the option is to dynamically hedge the option, so that it becomes delta neutral at any given time. One can form a portfolio with a self-financing strategy, including the option, delta hedge and money market account in it:

$$\Pi = V - \frac{dV}{dS}S = V - \delta S \tag{7.3}$$

By construction, the portfolio is delta-neutral with underlying S, and has no stochastic risk in it. Therefore it should earn a return at the risk-free rate. For the simplicity of discussion, we will use r as risk-free rate here. In the limit of infinitely small dt for dynamic hedging, one can derive the value of the option using the partial differential equation from Black–Scholes derivations:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = \Pi r$$

and re-arranging the delta term, we have:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = Vr \tag{7.4}$$

This second way of valuing the option offers one a way to replicate the option value dynamically: if you dynamically hedge the delta of the option, and by the nature of the self-financing strategy, you would be able to replicate the option value through the maturity. This also offers a way to risk manage the option risk.

7.1.2 Delta hedging and replication

What does it really mean when delta is dynamically hedged?

From the Black–Scholes equation, we have

$$\frac{\partial V}{\partial t} + \frac{1}{2}\gamma \sigma_S{}^2 = Vr - \delta S r \text{ or } \theta = (V - \delta S)\, r - \frac{1}{2}\gamma \sigma_S{}^2 \tag{7.5}$$

This equation shows when delta is hedged, the theta of the option is equal to the carry interest term of the portfolio value $(V - \delta S)$ minus the gamma term. When the interest rate term is small, the theta bleeding term of the option is equal to the expected gamma term.

Now if we look at the P&L for the option during the dt period, assuming constant volatility:

$$dV = \frac{\partial V}{\partial t}dt + \delta dS + \frac{1}{2}\gamma\, dS^2 + \varepsilon\left(dS^3\right) = (V - \delta S)\, rdt + \delta dS + \frac{1}{2}\gamma\,(dS^2 - \sigma_S^2\, dt) + \varepsilon\left(dS^3\right) \tag{7.6}$$

and for the delta hedged portfolio from Equation 7.3:

$$d\Pi = (V - \delta S)\, rdt + \frac{1}{2}\gamma\left(dS^2 - \sigma_S^2\, dt\right) + \varepsilon(dS^3)$$

This equation shows that, apart from the carry interest term, the portfolio gain or loss depends on the size of two terms:

$\frac{1}{2}\gamma\, dS^2$: realized gamma term

$\frac{1}{2}\gamma\, S^2\sigma_s^2\, dt$: expected gamma term

since dS is the realized return for S, and $\sigma_S\sqrt{dt}$ is the "expected" return given volatility σ_S:

$$E\left[dS^2\right] = S^2\sigma_s^2\, dt.$$

With the interest carry term usually being small, this means the P&L of the dynamic hedging strategy would depend largely on the realized vs. expected returns, or realized vs. expected volatilities. Another way of looking at this is the difference between the theta and gamma returns:

- Theta is the cost that one pays to long the optionality or the non-linear return of the option. Theta impact is what is implied: $\frac{1}{2}\gamma\, S^2\sigma_s^2\, dt$.
- Gamma is the positive non-linear return when the market moves. The P&L impact from gamma is realized one: $\frac{1}{2}\gamma\, dS^2$.

If the realized non-linear return is greater than the theta bleeding, one would make money from longing the option. On the other hand, if the realized non-linear return is less than the theta bleeding, one would lose money from it. In practical derivatives trading, one of the most important tasks is to manage theta and gamma:

- Long gamma would make money from outsized market move; however, this could be painful when the market is not moving and the option value keeps bleeding away from theta.
- Short gamma would be making money from theta every single day; however, it could be losing a lot from outsized market moves.

The process of dynamic delta hedging can be regarded as a process of realizing the value of gamma, and sometimes referred in trading as "scalping". In the "scalping" process, the derivative value is replicated.

7.1.3 "Scalping" with transaction cost

The above simple example has one stochastic risk factor in it, as we assume the volatility of the stock being constant. In this example, we also see the importance of implied parameter choice, in this case σ_S. In reality, the realized volatility can be larger or smaller than implied volatility, creating P&L after dynamic hedging. If you are wrong about the implied volatility, eventually it will show up in the P&L over time, when the option expires or when the option becomes deeply in- or out-of-the-money. In real life trading is not frictionless. Practical delta "scalping" will also have to take into account the market liquidity and transaction cost in delta hedging. Intuitively, one would want to hedge less frequently when liquidity is low and transaction cost is high. This introduces the concept of hedging frequency. The simplest strategy to reduce hedging cost is to hedge at a longer time interval. This achieves the goal of saving on transaction cost; however, there is a risk that the market may move too fast during the hedging interval, which would result in significant market loss.

A better way to reduce hedging needs is to adopt a hedging band defined by hedging thresholds. In this strategy, the scalping will only occur when the delta is above the hedging thresholds; effectively the hedging thresholds define a hedging and no-hedging zone. Clearly the larger the no-hedging zone, the less frequent the scalping activity, and the less hedging cost there would be; however, a large no-hedging zone would mean a large delta risk close to the zone boundary and effectively one would be "running" a significant position potentially leading to loss in trading. The balance between the two impacts would form an optimal point for hedging strategy.

Intuitively, the hedging threshold would depend on at least two things: the size of the transaction cost and how fast the delta position flips with the underlying move. One would expect that the higher the cost, the wider the no-hedging zone. On the other hand, if the position flips fast with the underlying move, one would expect a larger hedging zone to reduce the hedging cost. If the position is sticky when the underlying moves, one could reduce the hedging zone size and so reduce the market risk exposure. One critical measure of this stickiness of position is the option gamma, as it defines exactly how delta moves with underlying price. An example of a hedging band is shown in Figure 7.1 for a call option.

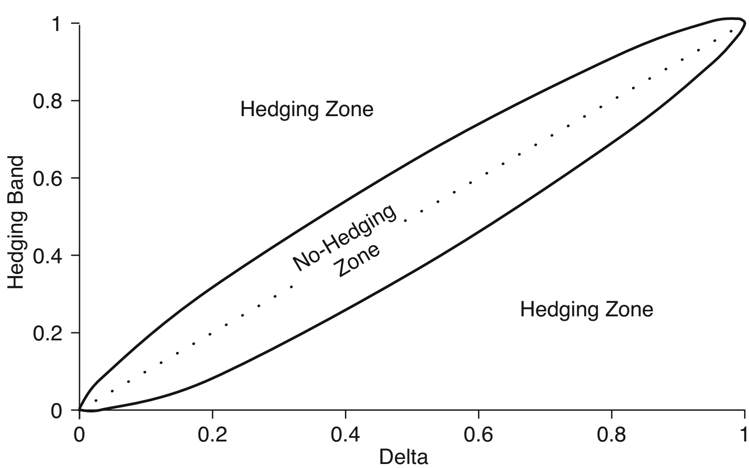

Figure 7.1 An example delta hedging band for a call option

A classical delta-flipping example is the "pinning" option[2] close to expiry carrying huge gamma, similar to a digital option. This huge gamma would result in fast scalping back and forth as the market fluctuates. While this phenomenon may be resolved with boundary smoothing techniques,[3] traders often take these "pinning" options out of the portfolio pool to manage separately, effectively creating a large no-hedging zone. In the following we discuss briefly the formalized approach in incorporating these two quantities in designing a good hedging strategy in the presence of transaction cost.

Practically one could construct a utility to optimize the two impacts. There are two types of utility functions in common use. One is a traditional utility function based on the terminal wealth of the trader; the other is the local utility such as local P&L over quadratic variations of P&L (Martin, 2014).[4] One popular utility function takes an exponential form:

$$U(W) = -e^{-\xi W}$$

where W stands for terminal wealth and ξ represents risk aversion. The other local utility function could be simply Sharpe ratio with risk defined as P&L variations. In some cases, one may use the Sortino ratio or tail risk in the utility as well. In general, maximization of utility functions is a very involved process; here we will state some of the general results.

First of all, a universal scaling law is obtained in all situations, independent of the utility function:

$$\Delta H \propto \left(\frac{3\varepsilon\gamma^2}{2\xi}\right)^{1/3}$$

where ΔH is the half-width of the hedging band, γ is the option gamma for the position. This formula describes the mathematic relationships between the hedging zone parameter ΔH, the transaction cost ε and the delta flipping rate with spot γ, as we discussed above. In addition, the more risk averse the trader is, the larger ξ, the narrower the hedging band.

One additional interesting result is that the long option trader will behave differently from the short option trader. In general, the long option delta band will be wider than the short option band. This is consistent with the practice that long option traders let the delta "run", and so reap much larger gains as the market keeps moving. On the other hand, the short option trader will be more risk-averse, since continued delta move in one direction hurts badly for short gamma positions. In general, traders under a short gamma position will be more defensive. Effectively, the long option position will experience a slightly lower volatility (Leland, 1985):[5]

$$\sigma_{Long} \approx \sigma\sqrt{1 - \frac{\varepsilon}{\sigma}\sqrt{\frac{8}{\pi\,\Delta t}}}$$

where ε is the transaction cost, Δt is the hedging frequency, and for short options, the volatility will be slightly higher:

$$\sigma_{Short} \approx \sigma\sqrt{1 + \frac{\varepsilon}{\sigma}\sqrt{\frac{8}{\pi\,\Delta t}}}.$$

Although this result is approximate, in practice it gives us a good approximate measure of the transaction cost impact.

7.1.4 Multiple risk factors

When two or more risk factors are involved in derivatives prices, risk management becomes much more involved. This is due to the fact that one has to deal with cross gamma and correlation as we will show below.

We start with a derivative depending on two risk factors R and X:

$$V = V(X,R).$$

The change in derivative value would be like the following, with simple Taylor expansion to second order:

$$dV = \theta\, dt + \delta_X dX + \delta_R dR + \frac{1}{2}\gamma_X dX^2 + \frac{1}{2}\gamma_R dR^2 + \lambda_{XR} dXdR + \varepsilon((X|R)^3)$$

where θ is theta or decay in time and $\lambda_{XR} = \frac{\partial^2 V}{\partial X \partial R}$ is the cross gamma or "Vanna" between X and R.

Assume X and R follow geometric Brownian motion with correlation:

$$dX = \mu_X X dt + \sigma_X X dz_X$$

$$dR = \mu_R R dt + \sigma_R R dz_R$$

$\rho_{XR} = \rho(dz_X dz_R)$, or $dz_X \cdot dz_R = \rho\, dt$

Similar to Black–Scholes derivations above, we can form a risk-neutral portfolio by eliminating the delta risk of R and X, and we arrive at a familiar relationship:

$$\theta\, dt + \frac{1}{2}\gamma_X dX^2 + \frac{1}{2}\gamma_R dR^2 + \lambda_{XR} dXdR = (V - \delta_X X - \delta_R R)\, r dt$$

and

$$-\theta = \frac{1}{2}\gamma_X X^2 \sigma_X^2 + \frac{1}{2}\gamma_R R^2 \sigma_R^2 + \rho_{XR}\lambda_{XR} XR\sigma_X\sigma_R - (V - \delta_X X - \delta_R R)\, r. \tag{7.7}$$

This equation tells us that theta loss would be compensated by the expected gamma gain in X and R, and the expected correlated cross gamma term between X and R, subtracting an interest carry term. Clearly this is much more involved when X and R are risk-managed together: one does not only need to care about the dynamic delta hedging of the X and R, the choice of implied volatility parameters σ_X and σ_R, but also the correlation term between X and R.

Looking at the P&L for an interval dt:

$$\begin{aligned} dV = {} & \frac{1}{2}\gamma_X \left(dX^2 - X^2\sigma_X^2 dt\right) + \frac{1}{2}\gamma_R \left(dR^2 - R^2\sigma_R^2 dt\right) + \lambda_{XR}\left(dRdX - \rho_{XR} XR\sigma_X\sigma_R\right) \\ & + (V - \delta_X X - \delta_R R)\, r. \end{aligned} \tag{7.8}$$

This can be interpreted as the realized versus expected gamma terms for X and R, and the realized versus expected correlation cross gamma term, plus an interest accrual.

Clearly the accumulated cross gamma term P&L through time would depend on the realized correlation between X and R, in addition to the realized return for X and R. If realized correlation is much greater than the expected correlation used in the stochastic process, the cross gamma term will have significant contribution to the P&L.

The cross gamma terms are very important in managing the complex derivative products. Mis-specification of correlation and mis-pricing of the derivatives structure could lead to substantial losses. Historically there have been quite a number of such cases where large trading losses resulted. Among them, PRDC (power reverse dual currency swap) is one of the most prominent. This product is very sensitive to the correlations between JPY/USD FX spot and the USD interest rate. With the correlations being realized around 0% for the years before the 2008 credit crisis, people have largely used small correlations in pricing and hedging PRDCs. This correlation moved from 0%∼15% to 50%∼70% in the credit crisis, which prompted large losses in PRDC books for firms carrying large positions, such as AIG Financial Products, where hundreds of millions dollars evaporated very quickly. While many financial firms with substantial positions lost money in the 2008–2009 period, a number of the PRDC desks closed as a result.

The mis-pricing of the cross gamma term can affect the risk management of derivatives and valuations primarily in two different ways.

First, when market moves, the change of deltas of X and R would be affected by the cross gamma terms:

$$d\delta_R = \gamma_R dR + \lambda_{XR} dX$$

$$d\delta_X = \gamma_X dX + \lambda_{XR} dR. \tag{7.9}$$

In Figure 7.2, we show the cross gamma effect schematically. The four corners represent the 4 scenarios: (0,0) represents the base scenario where X and R are not changed. (0,dR) represents the scenario where X is not moving while R is moved by dR. Similarly, (dX,0) represents the scenario where R is not moving and X is moved by dX, while (dX,dR) is for both X and R moving by dX and dR respectively. Dash-dotted

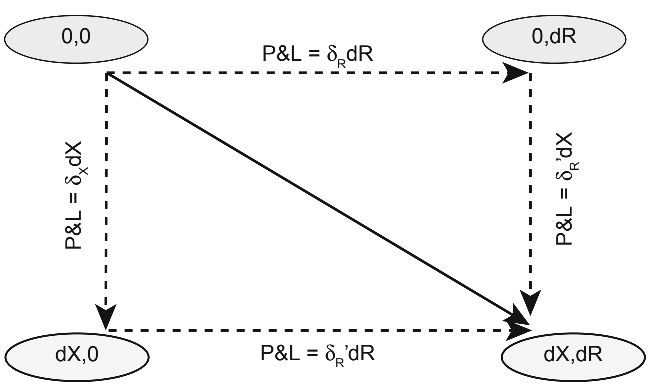

Figure 7.2 Illustration of cross gamma effect

arrows show the scenario path changing dX first, then dR; dotted path shows the scenario path changing dR first and then dX; while solid-line path shows the scenario changing from base to (dX,dR) directly.

First we look at the deltas:

- Base scenario (0,0): δ_X , δ_R
- Scenario (dX,0) : $\delta_R^{'} = \delta_R + d\delta_R = \delta_R + \lambda_{XR} dX$
- Scenario (0, dR): $\delta_X^{'} = \delta_X + d\delta_X = \delta_X + \lambda_{XR} dR$.

The P&L for three paths (here for simplicity, we are not showing any other high order Greeks, including gamma):

- Dash-dotted Path: $P\&L = \delta_X dX + \delta_R^{'} dR = \delta_X dX + \delta_R dR + \lambda_{XR} dX dR$
- Dotted-line Path: $P\&L = \delta_R dR + \delta_X^{'} dX = \delta_R dR + \delta_X dX + \lambda_{XR} dX dR$
- Solid-line Path: $P\&L = \delta_R dR + \delta_X dX + \lambda_{XR} dX dR$.

As expected, the three paths will have the same total P&L, as the starting and ending scenarios are the same. In practice, the realization of the scenario path is relevant as the hedging activity would be path-dependent. Intuitively, when X and R are not correlated, either dotted or dash-dotted paths will result in majority of the situations. Under both situations, the hedger would be able to hedge the cross-gamma-ed delta away to become neutral in position: at base scenario, one hedges away both delta; when dX or dR moves individually, one can hedge away the change in both deltas; the second move in dR and dX are then neutral in risk and therefore no P&L would result.

However, when X and R are correlated, the solid-line scenarios would show up statistically in a lot of practical scenarios. In such situations, we will make or lose money depending on the sign of cross gamma λ_{XR} and the correlation between dX and dR: let's assume λ_{XR} is negative and the correlation is positive. The P&L term in dX and dR would be cancelled with hedged X and R positions; however, the cross term would be left as a negative one. In this case, the hedger has no chance to hedge the change in delta until it happens. Another way to look at this is: the average of δ_X and δ_R for the solid-line path should have the cross-gamma term in it. The P&L from these delta deviations lead to the loss in P&L.

Notice that if one hedges at scenario (dX, dR), and now the market turns around completely, meaning it goes from (dX, dR) to (0,0). With perfect delta hedging at (dX, dR), one would lose again when the path is reversed, much like the behavior of negative gamma. The direction of the market does not matter, as long as the dX and dR moves are correlated, one would be losing money from the cross gamma term. Dynamic hedging under this situation is effectively locking in loss.

In the case of PRDC, what is important is the JPY/USD FX Spot X and USD interest rate R correlations. When the interest rate moves significantly higher, it generates a large short JPY FX delta position. However, FX has been moving higher along with the interest rate, which means the hedge of the extra FX delta position would mean a loss. On the other hand, the FX move also generates a large interest rate delta position in R, whose neutralization will lead to loss.

Effectively, it is the cross gamma term loss under dynamic delta hedging. This can be seen by looking at it in an alternative way. The P&L of the cross gamma term (X,R) over a small interval dt would be:

$$\lambda_{XR}(dRdX - \rho_{XR}\sigma_X\sigma_R dt)$$

with the first part being realized correlation cross gamma term, and the second being the implied cross gamma term included in the daily theta bleeding. The net of the two terms tells us that if one marks the correlation low, the result of daily P&L will be largely losses over time. This is exactly what happened in the case of PRDC: trading desks with substantial positions are forced to mark correlations higher so that the daily bleeding from cross term would be small or zero.

The expected cross gamma term can also be interconverted into X or R from correlations. If we express dz_X as:

$$dz_X = \rho_{XR}dz_R + \sqrt{1-\rho_{XR}^2}dz \tag{7.10}$$

and ignoring the boring drift term, we have:

$$dX \sim \rho_{XR}\frac{\sigma_X}{\sigma_R}dR + \sqrt{1-\rho_{XR}^2}\sigma_X dX$$

$$dX^2 \sim \rho_{XR}{}^2\left[\frac{\sigma_X}{\sigma_R}\right]^2 dR^2 + (1-\rho_{XR}^2)\sigma_X^2 dX^2$$

$$dXdR \sim \rho_{XR}\frac{\sigma_X}{\sigma_R}dR^2. \tag{7.11}$$

From this, we could translate the gamma term:

$$\frac{1}{2}\gamma_X dX^2 \sim \frac{1}{2}\gamma_X\rho_{XR}^2\left[\frac{\sigma_X}{\sigma_R}\right]^2 dR^2 + \gamma_X(1-\rho_{XR}^2)\sigma_X^2 dX^2 \tag{7.12}$$

and the cross gamma term:

$$\lambda_{XR}dRdX \sim \lambda_{XR}\rho_{XR}\frac{\sigma_X}{\sigma_R}dR^2. \tag{7.13}$$

When equation (7.10) holds, the cross gamma can be transformed into individual risk factor gamma and hedge accordingly. In practice, managing the correlation and cross gamma based on (7.13) is a tricky issue. Sometimes, the gamma from an illiquid market factor may be offset partially by using gamma from a liquid factor, as shown in (7.12).

Equation (7.13) shows when the cross gamma is negative and correlation is positive, as in the case of PRDC, the cross gamma term behaves like a negative gamma term. To protect against the “bad” cross gamma term, one has to buy “good” gamma from the market. The consequences of not having protection from “good” gamma would be losses over time in the form of bleeding, or hedging loss from rebalancing of cross-gamma-ed positions.

Correlation is usually scarcely traded with little liquidity; therefore the hedging of cross correlation between risk factors would be very challenging. A more prudent approach could be to take a fair correlation that matches the market realized correlation, and hedge accordingly. Sometimes, one marks the book using one conservative correlation, while managing the book using an expected fair correlation. The difference would be kept as model reserve. The result of marking to a conservative correlation is that one will realize some positive decay over time, in the form of correlation model reserve release.

7.1.5 Mean variance hedging

When there are two or more risk factors presenting, mean variance hedging is another methodology in finding the proper delta in managing the options risk. As the name suggests, mean variance hedging is achieved by minimizing the instantaneous variance of a delta-hedged portfolio.[6] We start by assuming that we will find the optimal hedge of X in the two risk factor (X,R) situation.

Defining δ_{mv}^X as the amount of X that one optimally hedges the portfolio:

$$\Pi = V - \delta_{mv}^X X$$

and the instantaneous change of the portfolio

$$d\Pi = \theta\, dt + \left(\delta_X - \delta_{mv}^X\right) dX + \delta_R dR + \frac{1}{2}\gamma_X dX^2 + \frac{1}{2}\gamma_R dR^2 + \lambda_{XR} dRdX.$$

The minimization of the variance of $d\Pi$ is equivalent to find δ_{mv}^X to make the instantaneous covariance of $d\Pi$ with X to be zero. If we use <,>to indicate the instantaneous covariance between two random variables:

$$< d\Pi dX > = < dV - \delta_{mv}^X dXdX >$$

then we would like to have the above covariance to be zero:

$$< d\Pi dX > = \delta_X < dXdX > + \delta_R < dRdX > - \delta_{mv}^X < dXdX > = 0.$$

Here we have used the fact that second order terms are processes of order dt. So we have:

$$\delta_{mv}^X = \delta_X + \delta_R \frac{< dR, dX >}{< dX, dX >} = \delta_X + \delta_R \frac{dR}{dX}.$$

This is a natural intuitive result: the mv delta hedge comes from a simple delta term δ_X plus an implicit delta term from risk factor R from correlation between R and X. One could also interpret the second term as a term in the chain rule of partial derivatives.

The above derivation can be extended to multiple risk factor case when situations can be more complex. One may decide on the optimal hedge given the liquidity choice of different risk factors and order of hedging.

7.1.6 Specification of model parameters and hedging

In constructing a model and in derivative risk management, one would like to have the process and dynamics for relevant risk factors to be mimicking reality as much as possible, so that the derivative value can be replicated. Deviations from real life situations would eventually lead to consequences in P&L through time.

In the above example with a simple European option, one could choose the volatility parameter in valuating and risk managing the option. Misspecification of the volatility parameter would lead to P&Ls through maturity with dynamic delta hedging. If the volatility parameter is greater than the realized volatility, the long option player will be bleeding through time: theta loss will be more than the gamma gain. On the other hand, the long option player will make money day over day from decay with a volatility specified too low. An alternative way to understand this is that the misspecified volatility will generate delta hedges that are further apart from "true" volatility and "true" hedges, which will lead to errors in the replication process, and hence the P&L.

A similar discussion can be extended to other model parameters, such as correlations. From Equation (7.8), the P&L from realized covariance compared to implied covariance would be: $\lambda_{XR}\,(dRdX - \rho_{XR}XR\sigma_X\sigma_R)$, which contains the combined effect of correlation between R and X, and volatility effects of R and X. When the volatility of R and X are realized at about the same as implied volatilities σ_X and σ_R, this P&L term reflects mostly the difference between realized and implied correlations.

As we extend a similar concept to the volatility itself, we see some issues immediately. When the option is traded in the market, the price of the option will be available and the implied volatility will be known. This implied volatility would fluctuate over time. As we know, delta will change when volatility moves, which would prompt a new delta hedge from the option trader. One may ask some questions:

- Are such delta hedges triggered by the volatility move creating unnecessary hedges?
- The implied volatility parameter would reflect the market view and supply demand, which would carry implied risk premium from the market. This may not be the optimal choice for option value replication. Could one choose a different volatility parameter to replicate the option values, disregarding the fluctuations of implied volatility?
- Furthermore, what would be the optimal hedging strategy if volatility is stochastic?

There are different ways of risk managing the option and replicating/realizing the value of the option. Clearly, the constantly changing volatility implied from the market would introduce extra hedges from Vanna. Such behavior could introduce unnecessary hedges. If one knows what the "true" volatility would be, one may use that volatility in risk managing the position, even though the official book would be marked to the implied volatility in the market. In practice, traders may choose to use a different volatility in calculating their option delta in managing the option risk, such as removing the risk premium from the market implied volatility, or removing the noise from market temporary fluctuations in implied volatility.

As long as the underlying return dynamics gives a distribution close to "true" volatility representation, this could be an optimal risk management choice. However, when a portfolio of options with different maturities and strikes are present, it becomes hard to choose a "true" volatility in risk management: different options may have a different Black–Scholes volatility due to the presence of skew, and volatility term structure. For options with given maturity and different strikes, we could say that we are dealing with a skewed distribution. However, the simple "true" volatility would not work in this case. A simple remedy in this case is to find a distribution that mimics the "true" skew distribution, such as shifted lognormal distribution,[7] or a little more complicated Corrado-Su expansion model (Corrado & Su, 1996). In such a skew model, one may be able to price multiple options with one given set of skew parameters.

This cannot, however, extend much further beyond when we think about the origin of volatility skew and the need of a proper volatility dynamics in risk managing a portfolio of options. For a more general situation, one would really need a volatility model. On one side, one may expect Black–Scholes lognormal volatility to increase when underlying spot drops, giving a "negative" skew. The existence of a downside protection demand will also drive a steeper "negative" skew. For example, the S&P 500 index volatility surface always shows a stable "negative" skew through time. On the other side, one may expect the volatility to be clustered; there would be low volatility and high volatility periods. A good volatility model should account for at least these two effects. This leads us to the progression of volatility models: local volatility (LV) model, stochastic volatility (SV) model, stochastic local volatility (SLV) and so on. As the model becomes more sophisticated, the model parameter choices will become more complicated. However, the principle is the same, one would need to find the proper model with the parameterizations chosen as close to real life as possible. Similarly, this is also applicable to the hedging and replication of derivative values in CVA and FVA.

7.2 Risk managing CVA

7.2.1 CVA risk management

CVA is a complex derivative by definition. CVA risk would involve at least two classes of factors. The first is market risk factors, such as FX, interest rate, equity and their volatilities, which affect the derivative valuations and market risk exposure; the other class is credit risk factors covering counterparties' default risk, such as CDS spreads, as well as model parameters driving counterparties' credit rating migrations.

Naturally, anything that affects the derivatives valuations would affect the exposure at default calculations. To manage CVA risk, one would have to calculate the sensitivity of CVA on these risk factors. As the EAD distribution is affected by the volatilities of the underlying market risk factor, the volatilities of underlying market risk factors are also among the market risks that one would need to manage. For example, CVA for a simple interest swap would be driven by swap rates as well as volatilities of swap rates implied from the the swaption market. A CVA for an FX forward would be affected by an FX spot, both currency interest rates as well as the FX volatility, and to lesser extent the interest rate volatilities.

The counterparty's CDS affects the eventual default probabilities through time, which would affect the size of default losses and CVA. For bilateral derivative transactions governed by CSAs with rating triggers, such as ratings-based threshold and ATEs, the rating migrations would also affect the EAD. Therefore the parameters for ratings migration would be important as well. If these parameters are affected by the market-traded quantities, such as CDS term structures, then the CVA could be affected by the CDS term structure in a more complex way.

When CDS fluctuates, CVA would increase or decrease, which in turn would also affect the market risk sensitivity change. For example, for a bilateral interest rate swap, the swap delta would increase in magnitude when CDS spread widens. This is because the CVA would be increasing given widening CDS spread, which also means the CVA would be more sensitive to the interest rate move. On the other hand, when the market risk factor moves, the exposure from the trade would change, which also affects the credit risk size or the amount of CDS one needs to buy or sell in hedging. So if the change of swap rate leads to increasing CVA, it would lead to increasing CDS sensitivity as well. This cross interaction is the cross gamma between market and credit risk factors.

Other than CDS spread move, the ratings change from rating agency upgrades and downgrades would also change the CVA as well. This is in addition to the CDS spread impact. If the CSA has rating triggers in it, the upgrades/downgrades, including jumps in rating, may have significant impact on the CVA. Depending on the rating migration model, the impact of upgrade/downgrade may show significant differences.

In the following we will examine the market, credit risk, cross risks and their management.

7.2.2 CVA market risk: Delta, gamma and theta

Assume the CVA for a counterparty can be expressed as a function of market risk factors **M,** credit risk factor **C** and parameters G, where **M** and **C** can be vectors of individual factors:

$$CVA = CVA(\mathbf{M}, \mathbf{C}, G)$$

As one may have many derivative transactions across different asset classes with one single counterparty, the market factors **M** would cover a broad range of underlying risk factors, as well as their volatilities. We calculate the individual delta for a specific i-th market factor as:

$$\delta_{M_i} = \frac{\partial CVA}{\partial M_i}.$$

Here for simplicity, we would include the first order volatility risk in δ_{M_i}. The corresponding second order gamma would be:

$$\gamma_{M_i} = \frac{\partial^2 CVA}{\partial {M_i}^2}$$

as the CVA being the expectation of losses from a specific part of the underlying market factor distribution, for example, the part of swap rate distribution above strike for a payer swap, one naturally has short gamma for CVA. Given the same credit default probability, the wider the tail distribution for the underlying market factor, the larger the expectation of CVA loss would be. Similarly, one would long gamma for DVA.

Naturally there would be cross gamma between the market factors when a product is sensitive to multiple risk factors, which would include the common vanna term between underlying spot and volatility:

$$\lambda_{ij} = \frac{\partial^2 CVA}{\partial M_i \partial M_j}.$$

The theta for CVA:

$$\theta_{CVA} = \frac{dCVA}{dt}$$

CVA theta θ_{CVA} has two components in it:

- Option theta that is similar to any option product: compensation for the gamma and cross gamma.
- Time decay in interest accrues, and more importantly time decay in credit risk. Every day passing by and with the counterparty surviving, the default probability would decrease by one day assuming the same CDS spread. Hence one gains in daily credit risk decay.

For a CVA desk, the trader would be managing δ_{M_i} dynamically on a daily basis, which also includes the hedge of option vega. When volatility skew is involved, one would manage the skew risk the same way as the normal option products. For example, one may choose an expected interest rate and volatility dynamics and hedge accordingly. These are really no different from the normal option risk management.

Similarly the option theta and the gamma, cross gamma would be managed together. In practice, one may need to determine the comfort level of gamma that one may tolerate, with the balance of option theta and gamma in mind.

The positive credit risk theta decay in CVA leads us to the credit risk management in CVA.

7.2.3 Risk managing credit risk

Of more concern in CVA risk management is credit risk hedging. There are multiple levels of complexities in practice. We will briefly discuss each one of them.

Hedging credit names

The first question one may ask in hedging CVA credit risk would be: What do we use in hedging credit risk? In certain situations, it is more a case of: Should we hedge the CVA credit risk?

Before making the hedging decision, one would need to clarify the purpose of hedging. The common of credit risk hedging goals would be:

- CVA P&L fluctuations
- Counterparty credit default risk
- Reduction of the regulatory capital requirements.

Counterparty default risk and reduction of capital usage are more of a tail risk that happens with a very small probability; CVA P&L fluctuation is what happens dynamically on a daily basis. Here we will discuss the dynamic hedging of CVA P&L

fluctuations first. The complexity of dual considerations in both CVA P&L and tail risk for capitals at the same time will be discussed in the next section.

Contingent CDS (CCDS), tailored to the credit risk profile of a specific derivative trade, is probably the best hedging instrument that one may be able to utilize in hedging counterparty credit risk. Like traditional CDS contracts, CCDS would provide payment triggered by a credit event on a reference entity. The payment amount would be determined by the value of the derivative transaction at the time of the credit event occurring. For example, a contingent CDS for a plain vanilla swap would cover the exact credit default risk with the payment amount equal to the swap value when the counterparty defaults. Besides very small mismatches, CCDS serves as a perfect product for CVA hedging.

One important advantage of CCDS offers over traditional CDS hedges is the unconditional protection of CVA credit risk, including the correlations between market and credit risk factors; and hence protection against wrong-way risk. This is especially attractive as Basel III emphasizes the importance of wrong-way risk while permitting CCDS as eligible hedges. However, the one critical issue with CCDS is the lack of liquidity. There has been an absence of protection sellers, which makes it hard for banks to materialize the advantages of CCDS as CVA hedges. Another reason that CCDS is not attracting liquidity is because of the intrinsic complexity of CVA netting set. Normally a dealer's book with a counterparty contains a lot of transactions, which makes the construction of a matching CCDS difficult. While one may attempt to design a derivative transaction with exposures mimicking the CVA portfolio, it may not work well for a more complex CSA with market changes.

Then the next candidate comes down to single name CDS hedges. Clearly when individual name CDS has liquid trading in the market, one may resort to the liquid CDS to hedge the CVA credit risk exposure. The most liquid traded CDS is usually 5-year. Based on calculated CS01, defined as the change in value for 1 basis point of CDS shift, one may derive the proper hedge needed:

$$CS01_{CVA} = \frac{\partial CVA}{\partial S_{5Y}} \text{ and } CS01_{CDS} = \frac{\partial V_{CDS}}{\partial S_{5Y}}$$

where S_{5Y} is the 5-year CDS spread. In computing the $CS01_{CVA}$, one can assume some relationship between the 5-year CDS spread and other term CDS spreads, from which the hedge ratio can then be obtained.

The CDS notional needed to hedge the CVA would be:

$$N = N_{5Y} \frac{CS01_{CVA}}{CS01_{CDS}}$$

where N_{5Y} is the unit 5 year CDS notional.

There are some issues that one needs to recognize in this type of CDS hedging.

First, one should note that CDS hedges may not be able to match the CVA terms exactly. One may be able to hedge the CVA P&L approximately; however, the mismatch of CDS maturities and CVA terms will naturally lead to mismatch of theta decays. For example, a 5-year CDS hedging against a 20-year CVA exposure will lead to mismatch in daily decay. Intuitively, 5-year CDS protection will decay faster than the 20-year

CVA, given that the 5-year CDS hedge provides the same CDS sensitivity as a 20-year CVA exposure. It should also be noted that, for the same reason, capital mitigation in this case would not be as effective because of the mismatches in maturity terms. This then brings us to the discussion of CVA bucket sensitivity term structure and hedging CVA using CDS with multiple maturities.

The second issue of CDS hedging is its inability to cover downgrade situations well. When rating triggers are present, such as RBT and ATEs, the CVA exposure would not be replicated by the CDS protection. The CVA exposure is really dependent on counterparty downgrades, while the CDS protection in general only depends on the counterparty default event. This mismatch would generally lead to mismatch in hedging and basis risk: when the counterparty is downgraded, the EAD may be reduced as a result of RBT. Even though the counterparty default probability is increased, CVA may decrease as a result. Under extreme situations, the collateral threshold may become zero, which means the derivative transactions would be fully collateralized and CVA would be close to zero. This also means the original CDS hedges need to be unwound.

To hedge the CVA exposure term structure more accurately, one may choose to use CDS with different maturities. This requires the bucket sensitivity calculations:

$$CS01_{CVA}(T_i) = \frac{\partial CVA}{\partial S_i}$$

where S_i is the maturity T_i CDS spread. This bucket sensitivity can be calculated in different ways. While one can never hedge CVA perfectly given limited liquidity of CDS, one recommended way is to compute the bucket sensitivity based on a defined dense CDS bucket, from which the effective hedge ratios for selected liquid CDS hedges can be optimized.

In calculating the derivative of $\frac{\partial CVA}{\partial S_i}$, one may choose to first compute the derivative with respect to the zero default spread curve r_j bootstrapped from the CDS spreads $\frac{\partial CVA}{\partial r_j}$, and then convert them into the derivative of S_i based on Jacobian transformation $J_{ij} = \frac{\partial S_i}{\partial r_j}$:

$$\frac{\partial CVA}{\partial S_i} = \sum_j J_{ij}^{-1} \frac{\partial CVA}{\partial r_j}.$$

To hedge the CVA P&L fluctuations for a portfolio of credit names, one may also resort to index CDS, such as CDX indices. In this situation, one would need to compute the hedge ratio properly by examining the relationship between the index CDS and portfolio CDS names, which could be obtained by examining the historical performances or a more complicated factor decomposition. In this case, however, one would have to manage significant basis risk between individual names and index CDS.

Proxy CDS hedge and basis risk

Dealers trade with a variety of customers with different industries, different credit entities, and different credit ratings. A lot of these customer names do not have liquid bonds or CDS traded in the market. Often dealers have to rely on the internal credit assessments to determine the credit situation of the counterparty and find a proxy in constructing the credit default probability curves, from which CVA can be calculated.

This is reasonable given the practical situation; however, it leads to a problem in hedging these illiquid name exposures.

Banks may either leave the credit risk of these names naked, or hedge them by using proxies. For example, one may use CDX or iTraxx to hedge individual names, or use MCDX to hedge municipal names, or in some cases, use equity and equity options to hedge the CDS fluctuations. Hedging illiquid names using proxies means one has to manage the basis risk between the illiquid name credit and the proxy hedges. The tricky decision in using proxy hedges is to understand when the specific proxy hedge works and when it fails.

While the proxy and the single name it hedges often carry some common underlying driving factors, they have idiosyncratic components by themselves. Under extreme situations, the proxy hedge and the underlying CDS may even diverge in opposite directions. For example, one may choose to use the equity market to hedge the CDS market fluctuations. In the period of financial crisis, this may work very well if the proper hedge ratio is used. If the equity market goes down, the CDS spread will widen as the perception of economic health and creditworthiness of the company deteriorates. However, when the financial market turns around, the equity market performance may start to deviate from the CDS moves. In the down market, banks, institutions and companies have to deleverage as the asset market goes down; however, when the equity market goes up, the company may start to engage more leverage in their operations again. The leveraging activity may lead to the appreciation of company stock, since the prospect of the company making more money in the future. However, the creditworthiness of the company may actually go down because of more debt incurred in the leveraging activity. Under this situation, the equity performance and the CDS performance may become even negatively correlated; and one could really get hurt in equity hedge and CVA credit exposure mismatches.

Managing market, credit cross gamma and wrong-way risk

There is always a significant cross gamma term between the market and credit risk factors in CVA hedging considerations. Naturally there would be cross term between the market and credit factors, even if there is no correlation between them:

$$\lambda_{iS} = \frac{\partial^2 CVA}{\partial M_i \partial S}$$

where S is the CDS spread of the credit name, λ_{iS} is the cross gamma between the ith market factor and the counterparty CDS spread. In general, when CDS spread widens, CVA increases; meanwhile CVA will be more sensitive to the market factor, meaning $\delta_{M_i} = \frac{\partial CVA}{\partial M_i}$ would increase in magnitude as a result. As we discussed in previous sections, when correlation is close to zero, the management of cross gamma is relatively easy: dynamic hedging individual risk factor delta would be sufficient and there is little cost in doing that. The expected cross gamma P&L will be close to zero when correlation ρ_{iS} is close to zero:

$$E[PL_{iS}] = E[\lambda_{iS} dM_i dS] = \rho_{iS}\lambda_{iS}\sigma_i\sigma_S dt \approx 0.$$

When correlation ρ_{iS} is not zero, the expected cross P&L will not be zero. One would then need to pay special attention to the hedging of cross gamma. If significant negative cross gamma effect is expected, one may resort to buy gamma from the market to offset the negative cross gammas, similar to what we discussed in previous sections. This then touches the wrong-way risk management. When the derivative value is becoming more positive, the counterparty has more chance to default; or when the counterparty defaults, the size of the expected default exposure is larger than the expected exposure without correlations. While there is generally no good way in hedging wrong-way correlations, one could employ option strategies to mitigate the tail risk with wrong-way exposure. For example, one may purchase deeply out-of-money options to protect the event when FX or interest rate jumps to the wrong-way direction significantly.

7.2.4 Hedging CVA risk and optimizing capital

In practice, banks have to deal with CVA, FVA, capital, leverage ratio and liquidity standards at the same time, which is highly dependent on the rules and regulations. While the Basel III regulations are very comprehensive, the management of it is even more complex considering the complexities in individual business and each practical situation. The rules and regulations have changed the business world tremendously, not only in how one should be approaching business planning and execution, but also, in many situations, how then can lead to difficult decisions such as exiting specific businesses that are not making enough returns on capital. As a result, the business landscape changes so that only the strongest survive.

One of the most daunting tasks for banks is to optimize the use of capital. For uncollateralized derivatives, it is CVA market risk capital and counterparty credit risk (CCR) capital. CCR capital is the amount of money that a bank has to put on the side to cover losses in the event of counterparty defaults. This default-induced charge is to ensure the bank has sufficient funds to cover itself under the actual default situation. CVA market risk capital is introduced in Basel III to account for the potential CVA P&L fluctuations as the market moves. With CVA and CCR capital being very large for uncollateralized derivatives, the banks are in constant need to optimize and reduce the capital usage. This leads to the consideration of using CDS and other hedging instruments to reduce the capital.

Banks are permitted under Basel III rules to reduce their CVA exposures by entering into eligible CDS hedges. Specifically, banks may enter into single name CDS, contingent CDS, other equivalent hedging instruments, which references the counterparty directly and index CDS. Tranched or nth-to-default CDS are not allowed as CVA capital hedge. Proxies are not permitted unless they are proved to be effective to the satisfaction of relevant authorities.

There is, however, a mismatch between regulatory capital mitigation and CVA P&L hedging using CDS. One can optimize the capital reduction using CDS as allowed by the regulators, but the result could be more P&L fluctuations. This is because the capital optimization using CDS may lead to over-hedging of CVA P&L, meaning some of the CDS and CVA would be naked because of mismatch in expected exposure profile, and hence mismatch in CDS spread sensitivity. The reason is intuitive: regulatory capitals are meant to protect one from extreme stress situations based on small probability event, and accounting CVA P&L is really based on expected exposure looking at present time. These two exposure differences naturally would

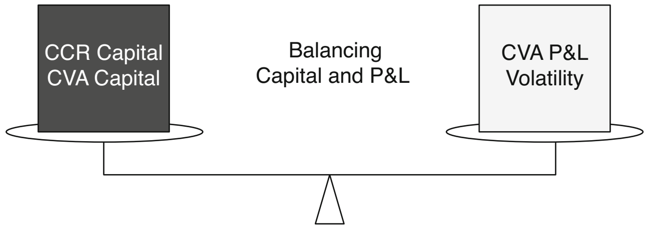

Figure 7.3 Balancing between CVA P&L hedging and capital optimization

lead to different sensitivities to the underlying risk factors. In addition, capitals are designed to be conservative and punitive, which would require much higher hedging ratio comparing to accounting CVA P&L hedging. Figure 7.3 shows the delicacy in balancing the decision-making of CVA P&L volatility and the cost of capital: if the firm is more focused on capital, it will be exposed to more CVA P&L fluctuations, and vice versa.

When rating triggers are present, this issue could be more severe: regulatory capital calculations do not allow ratings-based credit risk mitigations, while CVA fair valuation may incorporate them into the calculations. CVA fair valuation would reflect what might happen in reality: as a counterparty gets downgraded, the credit risk exposure is mitigated by the fact that banks may receive more collateral from the counterparty. This potential change of collateral in future scenarios, however, is not permitted in capital calculations. This mismatch in rating triggers treatment leads to more mismatches in CDS sensitivities, and hence problems in balancing the two effects.

When an index CDS is used in mitigating the capital usage, only a fraction, for example 50%, of the index CDS will be allowed in the capital reduction calculation. This means the effect of capital mitigation using index CDS would be more limited. Combined with the fact that one would need to manage the basis risk between hedging individual name credit and index CDS, it is even more challenging to achieve the desirable capital savings and CVA P&L hedging at the same time. Proxy hedges, not permissible under the regulatory hedging rule, are even worse.

It is, then, a tricky management decision to find a desirable point that balances out the need of capital optimization and CVA P&L fluctuations as shown in Figure 7.3. Without more effective hedging tools and better matching between capital and P&L sensitivities, banks will always find themselves in difficult situations with undesirable earnings volatility or excessive capital usage.[8]

7.3 XVA desk setup and operations

As the complexities and skillsets involved in dealing with credit and funding are quite different from the regular derivatives trading, it would be very onerous for each individual business and trading desk to manage their own XVA exposures. Ideally one would need some specialized desks to extract the XVA part of economics, as well as operations, out of the individual businesses and manage in a centralized manner. This leads to the discussion of XVA desk setup.

Ideally one would need the XVA desk to achieve the following goals:

- Alleviating the pain involved in the XVA part of pricing and risk management for individual businesses. During the live trading process with XVA implications, the XVA desk will provide the official pricing and XVA charges to individual desks; afterwards, individual desks would not have to worry about the XVA economics. This way the individual desks will be able to focus on their own businesses.
- Managing XVA risks, including managing market risks and credit risks, as well as downgrades and defaults. This would include the hedging of market and credit risks, replicating and realizing the collateral values as well as funding values of the derivatives.
- Managing XVA operations, including collateral operations, monitoring of CSA related activities, CSA negotiations, trigger and exercise operations as well as decision making, and so on. When a counterparty gets downgraded or enters into default, the XVA desk would need to manage the corresponding operations along with trading desks, credit officers and legal departments.
- The XVA desk will also be engaged in seeking opportunities in the market in optimizing the CVA and FVA, as well as capital usage, such as looking for intermediation and novation opportunities for XVA intensive trades, moving bilateral trades to central clearing, exploring market pricing differences that may result in win–win for trading counterparties.

In Figure 7.4, we show schematically the setup of the XVA desk. The XVA desk would provide CVA/FVA as well as other XVAs to the LOBs and trading desks; the XVA desk would make decisions in terms of hedging market and credit risks by entering CDS protection as well as market risk hedges; in managing gamma and cross gamma, the XVA desk may enter option hedges with option desks or other dealers.

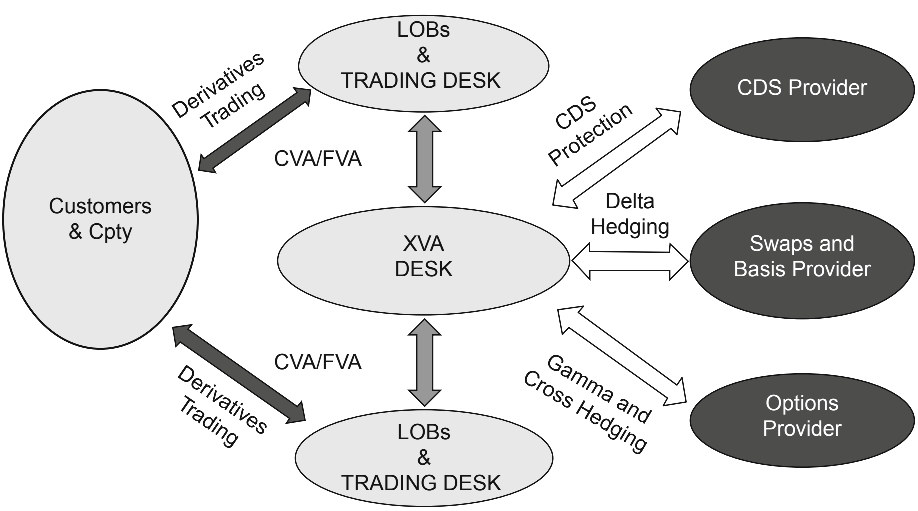

Figure 7.4 XVA Desk setup and operations

Here we use mutual put decision-making as an example to show XVA desk management. Mutual put breaks are deal-specific terms that allows both trading counterparties to exit the transaction at certain dates with specific frequency. Mutual puts are good credit risk mitigants because, legally, it stops the credit risk exposure to the counterparty at the next mutual put date. As a mutual put exercise may have significant economic effect as well as credit and capital effects, one would need to make the proper decision before the mutual put date passes. Missing the right decision may lead to economic losses as well as increase in capital usage as the derivative transaction gets extended.

When the mutual put dates get close, the XVA desk would need to make a decision about whether to exercise the derivative transaction. If it is optimal economically to exercise the mutual put, the XVA desk would need to involve the relevant parties in the process and inform the counterparty to find the best way to execute the mutual put break. In practice, there may be different ways that dealers could handle the mutual put breaks:

- It is very likely that customers would be willing to post collaterals for the derivative transaction subject to mutual put. In this case, dealers would be happy to take the collateral, which reduces the CVA and capital charge to minimum while retaining the transaction for customer.
- If termination is to be executed, the two trading counterparties would have to agree on the termination price for the derivative transaction. When there is significant difference between the two, one has to resort to market settlement process and resolution procedures to nail down the termination value.
- In practice, there are cases where banks value the customer relationships and would reconsider the exercise of mutual put. However, that may be regarded as a fresh credit and economic decision for management in credit risks and individual businesses. Ideally the XVA desk would need to be compensated for the decision by businesses, because the re-evaluation and relationship decision would lead to XVA desk economic loss.

To achieve the above, the XVA desk needs to establish internal policies and procedures in monitoring the incoming mutual put exercises and the effects on CVA, FVA, credit risk and capital, which would allow the banks to make objective decisions smoothly. As mutual put exercises are becoming more normal in trading activities, it is naturally the XVA desk's responsibility to execute them on a daily basis.

Another example of what the XVA desk may do is to find opportunities in the market to optimize funding, liquidity and capital usages. For example, the XVA desk may attempt to negotiate CSAs with other dealers and customers, so that capital usage can be saved. If both dealers would have to face capital charges due to the existence of a collateral threshold, it may be optimal for both parties to set the collateral threshold to zero. For a transaction that has RVA implications, the XVA desk might attempt to find banks with better credit ratings to take over the transaction with less cost than what is carried on the book. This is because the RVA calculation is really credit-rating dependent: the worse the rating, the higher the probability to touch the replacement event level, the higher the RVA. Similarly the better rating bank would have less charge on the liquidity, because of the lower probability to touch the collateral event level and the lower funding spread.

Summary

In this chapter, we have discussed the following:

- General derivative risk management concepts, including dynamic hedging and replication, scalping with and without transaction cost, and proper specification of model parameters in risk managing derivatives. Two- and multi-risk factor risk management with correlation and cross gamma are also discussed.
- Risk management of CVA market and credit risks, including market risk hedging, and credit risk hedging using CCDS, single name CDS, index and proxy. One specific problem in hedging CVA risk and optimizing Basel III CVA capital is touched briefly.
- XVA desk setup and operations. The XVA desk allows all lines of businesses to lay off any XVA-related exposure, cost and management. We also discussed the XVA desk operation with mutual put exercise as an example.

Notes

Chapter 1

1. For example, Southwest and Delta both have significant oil hedging programs in place. For Delta, every $1 of Brent crude price is worth around $40 million.
2. See for example, "Oil Crash Exposes New Risks for U.S. Shale Drillers", Bloomberg, December 19, 2014.
3. See for example, "Goldman and the OIS gold rush: How fortunes were made from a discounting change", *Risk* magazine, May 2013.
4. See for example, "Banks pull out of PRDC market", *Risk* magazine, January 2010.
5. This will be discussed in more detail in chapter 7.
6. This will be discussed in chapter 5 and chapter 7.
7. Vanilla swaption clearing has been in development with CME's swaption clearing projected to be in effect November 2014.
8. For example, see "Systemically Important Banks and Capital Regulation Challenges". OECD Publishing, December 2011.

Chapter 2

1. Examples can be found in default situations involving municipal entities as well as in foreign jurisdictions. In the market, there are legal services available to provide standard color-coded reports on netting opinions for different jurisdictions.
2. This will be discussed in briefly in Chapter 3.
3. Further information on the FAS 133 rule on derivative accounting can be found in most up-to-date publications on hedging and on the website: www.fasb.org.

Chapter 3

1. With the use of CSAs, in practice, there could be a spread on top of OIS rate as collateral interest. We use this simplified OIS flat standard rate to define the risk-free rate here; the specifics of different CSA collateral interests will be discussed in Chapter 4 with general collateral value adjustment.
2. For USD OTC interest rate derivatives, 65% of around $171 trillion outstanding notional is estimated to be linked to LIBOR.
3. See for example, "Behind the Libor Scandal", *The New York Times*, July 10, 2012.
4. There are some subtle differences related to default value determination, which may lead to some deviations between the two due to approximations. Some situations are discussed later in the book.

5. See for example, Piterbarg & Andersen, Interest Rate Derivatives Modeling (2011).
6. See for example, Merton (1974), and Hull & White, Valuing Credit Default Swaps I: No Counterparty Default Risk (2000).
7. This is because, given threshold + ½MTA, roughly half scenarios will be aggressive and half will be conservative, and the average will be close to zero.
8. See EBA Final draft regulatory technical standards on the margin period of risk used for the clearing members' exposure to clients under Article 304(5) of the European Regulation (EU) No 575/2013, July 2014.
9. For example, JP Morgan's DVA gain in third quarter 2011 was $1.9 billion; for the same period, Bank of America reported $1.7 billion gain and Citigroup's DVA gain was $1.9 billion.

Chapter 4

1. For repo transactions and collateral value discussions, we have ignored haircut for clarity purposes. For a general funding and repo consideration, one would need to update the formula with haircut, where the asset used in the collateral posting and repo transactions would be $1+h instead of $1. However, this does not affect the outcome.
2. See for example, Fujii & Takahashi (2011).
3. See for example Piterbarg, Optimal Posting of Sticky Collateral (2013) for more discussion on sticky collateral and modeling.
4. Similar derivations see for example, Piterbarg, Funding Beyond Discounting: Collateral Agreements and Derivative Pricing (2011).
5. Bilateral derivatives will be subject to initial margin rules, such as Standardized Initial Margin Model (SIMM) published by ISDA on December 2013. See reference: ISDA, December 2013 for details.
6. For example, traditional Black–Scholes–Merton derivation of option values, as shown in 4.2.13 for collateralized valuations and later sections for more general situations.
7. Asymmetric funding cost and benefit considerations are discussed in the funding curve construction later in this chapter, and also covered in Chapter 6 when funding set is introduced. Symmetric funding cost and benefit are simplified version of funding policy in practice. Readers are also referred to, for example, Albanese, Andersen, & Labichino (October 2013) for more discussions.
8. One should note that, other than the "funding" component, bond spread also contains the effect from supply and demand.
9. Here we are focused on the CVA/DVA exposures, which are assets and liabilities with the same counterparty. They are allowed to offset; this is not to be confused with the CVA and DVA for different counterparties.
10. See Kenyon, C. and Green, A., "Regulatory Costs Break Risk Neutrality", *Risk*, August, 2014; "Risk-Neutral Pricing – Hull and White Debate Kenyon and Green", *Risk*, October, 2014.
11. See Madan (2014) for a two-price economy asset pricing theory.
12. See for example, M. Cameron (2012), 'Banks Tout Break Clauses as Capital Mitigant', *Risk*, March, 2012.

13. The combo option involves a sell call and buy put on the same strike, whose price depends on the forward of the option underlier. Traders use combo option prices to invert the forward price in the index option market.
14. There are many references, for example, Burgard & Kjaer (2013), Pallavicini, Perini, & Brigo (2012), Morini & Prampolini (2011), Hull & White, The FVA Debate (2012), Hull, White, Kenyon, & Green (2014) and reference therein.
15. See for example, Hull & White, The FVA Debate (2012).
16. The new prudent valuation regulation actually is attempting to take these costs into account in equity Tier 1 capital. This will be discussed in Chapter 5.

Chapter 5

1. Liquidity may mean trading liquidity, market funding liquidity and banking liquidity, etc. The trading liquidity is related to market friction considerations; and market funding liquidity may show up in an investment asset's liquidity spread. Here we use liquidity as in LVA to specifically account for the regulatory required liquidity related cost. This is an increasingly more important term in unsecured derivative valuations.
2. The derivation can be found in Chapter 7.1.
3. See (EBA FINAL Draft Regulatory Technial Standards on Prudent Valuation Under Article 105(14) of Regulation (EU), 2014)
4. See for example, "Deutsche Bank CRO: Derivatives becoming loss leader", *Risk* magazine, October, 2014.

Chapter 6

1. See for example, Pykhtin & Zhu (2007).
2. We use throughout the book, notation $\mathbf{1}_{condition}$ to indicate when condition is met, the term is equal to 1; otherwise, it equals to 0.
3. See for example a joint measure model in Hull, Sokol, & White (2014) Modeling the Short Rate: The Real and Risk Neutral Worlds.
4. We will ignore any convexity related effects due to different discounting measures, as they are small enough comparing to CVA/FVA term themselves.
5. See discussion at the end of this chapter for more details.
6. See Hull & White, Valuing Credit Default Swaps I: No Counterparty Default Risk (2000), Canabarro & Duffie (2003) and references therein.
7. See Standard and Poors (2012).
8. When basis spreads are approximated to be constant, one may compress the floating instruments even further.
9. This assumption is accurate when two trading parties agree on the valuation and there is no friction. If transaction and operational costs cannot be ignored, this assumption is approximate; however, still reasonable for CVA calculations.
10. Assuming one is calculating DVA based on accounting requirement.

Chapter 7

1. See Corrado, C. and Su, T. (1996) "Skewness and Kurtosis in S&P 500 Index Returns Implied by Option Prices", *Journal of Financial Research* 19:175–192.
2. "Pinning" option is an option close to expiry with underlying spot sitting close to strike. In the limiting case of Black–Scholes world, the gamma is infinite as delta flips from 1 to 0 for a small move in spot.
3. Effectively, one may regard the market price of the underlying as a band instead of one single point in deriving the option price, which creates stickiness in option delta and therefore smoothing the delta behavior.
4. For a summary, see for example Martin, R. (2014) "Optimal Trading under Proportional Transaction Costs", *Risk*, August 2014, pp. 54–59, and references therein.
5. See Leland, H. (1985) "Option Pricing and Replication with Transaction Cost", *Journal of Finance*, 40:1283–1302.
6. See for example, Schweizer, M. (1991) "Option Hedging for Semimartingales". *Stochastic Processes and their Applications* 37: 339–367; Alexander, C. and Nogueira, L. (2007) "Model-free Hedge Ratios and Scale-Invariant Models", *Journal of Banking and Finance* 31: 1839–1861.
7. See for example Piterbarg & Andersen, Interest Rate Derivatives Modeling (2011) for a variety of models in interest rate derivatives.
8. For example, see Carver (2013).

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