A Simple Method of Forecasting Option Prices Based on Neural Networks

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Abstract. Options are an important financial derivative for the investors to control their investment risks in the security markets. The forecasting activity should realistically identify the option price in the future without knowing underlying asset price in advance. In this paper, a simple method of forecasting option prices based on neural networks is presented. We modify the traditional option pricing methods, enabling them to be eligible for forecasting the option prices. Then we employ the neural networks to further decrease the forecasting errors of the modified conventional methods. Finally, the experimental studies are conducted on the data of the Hong Kong option market, and the results demonstrate that the neural networks are able to improve the forecasting performance considerably. Conclusively, our neural network methods on option forecasting are fairly effectual in practice.

Keywords: Option prices, forecasting, Hong Kong option market, neural networks.

1 Introduction

In recent years, options have gained a conspicuous popularity in the security markets and have attracted numerous investors and speculators to accommodate the options in their portfolios. However, the investors' portfolios might be severely subject to jeopardy if they are unable to manage their options in a scientific way. It is therefore an indispensable step for the investors to establish justifiable and effective strategies while trading options.

Among the option pricing methods, the Black-Scholes model is mostly influential. The model prices the options based on the hypothesis that the underlying asset price follows a lognormal distribution [1]. In the last several years, many scholars also turned to the non-parametric methods such as the neural network (NN) methods. Intuitively, the simplest approach to use the NN is to estimate the whole option pricing function directly from the data [2]. However, the experiments demonstrated that the standard NNs do not performed satisfactorily because of a combination of factors, such as nonstationarity and noise of data [2]. To achieve a better performance, some scholars combined the conventional pricing model and the NN, and introduced a bootstrap approach to predict the difference between conventional parametric models and observed option prices. Also, [2] improved his method and presented a hybrid approach to enhance the performance including option boundaries including the

B.-C. Chien et al. (Eds.): IEA/AIE 2009, LNAI 5579, pp. 586–593, 2009.

scenario of deep-in-the-money. In [3], as opposed to learning the complicated option market straightforwardly, to acquire a more accurate result in the option market, the NNs are implemented upon the results of the well-known successful traditional methods. Since the platform of the traditional methods has already largely managed the nonstationarity and noise of the data, as well as the skew case of the deep-in-the money options, the burdens of NN approaches are greatly alleviated. [3] finally implemented the experiments on the option pricing for the Hong Kong option market, and a prominent improvement on pricing precisions over any of the conventional methods was archived.

The reminder of the paper is organized as follows. Section 2 modifies the four conventional option pricing approaches, enabling them to be qualified for forecasting the option prices. Section 3 presents two forecasting approaches by constructing the NNs on the platform supported by the conventional methods. Section 4 covers the empirical studies using data from the Hong Kong market. Finally, section 5 concludes the paper.

2 Option Price Forecasting by Modifying Conventional Approaches

The successful conventional option pricing methods used in this paper include the binomial tree (BT) method, the Black-Scholes (BS) method, the finite difference (FD) method, and the Monte Carlo (MC) method. Since all these methods are for determining the present theoretical option prices given the present underlying asset prices, for the purpose of forecasting the option prices at the end of next period, some modifications should be made.

In the modified convention models, the following notations are used, the underlying asset price S, the option's exercise price X, the current risk-free interest rate r, the asset price volatility σ, the time interval (length of a time period) Δt, the expiration time T, the option price C, and the forecasted option price $\hat{C}$.

2.1 A Modified Binomial Tree (BT) Method to Forecast Option Prices

The BT method assumes that within Δt, the underlying asset price $S_{t,\,i}$ (t = 0, 1, …, n-1) either goes up to $S_{t+1,\,i}$ (t = 1, …, n) according to a certain rate $u = e^{\sqrt{\sigma^2 \Delta t + (r\Delta t)^2}}$, or goes down to $S_{t+1,\,i+1}$ (t = 1, …, n) according to another rate $d = e^{-\sqrt{\sigma^2 \Delta t + (r\Delta t)^2}}$, where σ can be obtained from the asset's historical prices, i = 0, 1, …, n.

The BT contains t periods and (n+1) possible asset prices $S_{n,\,0}$, $S_{n,\,1}$, …, $S_{n,\,n}$ at the expiration time. There are two scenarios when the option is expired at the end of the last period. In the first scenario, its price at that moment is the difference between the asset price $S_{n,\,i}$ and the exercise price X if the option is in-the-money. In addition, noticing that option is a right other than an obligation, for an example of call option, when the asset price is lower than the exercise price, the investor has the right to choose not to exercise the option, thus making the option price 0. In summary of the above two scenarios, the option price is, $C_{n,\,i} = \max\{0, S_{n,\,i} - X\}$, i = 0, 1, …, n,

$C_{t,i} = \max\left\{S_{t,i} - X,\ e^{-r\Delta t}\left[q\,C_{t+1,\,i} + (1-q)\,C_{t+1,\,i+1}\right]\right\}$, where q is the risk-neutral probability.

2.2 A Modified Black-Scholes (BS) Method to Forecast Option Prices

The BS method [1] first assumes that the movement of the asset price exhibits the lognormal distribution, and then utilizes the Ito's Lemma to obtain the option price as $C = f_{BS}(S, X, t, r, \sigma)$. When the BS method is employed to predict the option price at the end of the next period, it is required to first estimate the corresponding asset price S_1. Here we choose a simple approach to achieve the estimation by $S_1 = e^{r\Delta t}S$. Subsequently, the forecasted option price at the end of the next period based on the BS method can be obtained as $\hat{C} = f_{BS}(S_1, X, t\text{-}\Delta t, r, \sigma)$.

2.3 A Modified Finite Difference (FD) Method to Forecast Option Prices

The FD method employs a numeric approach to compute the option price, which is carried out via solving some differential equations with respect to the option. The essence of this approach lies in the transformation from differential equations into difference equations, which thereafter are solved in an iterative fashion by computers.

The conventional FD method segments T into a few time periods with the identical size Δt, and also segments the asset price, whose minimum is 0 and maximum $S_{\max}$, with the interval as ΔS. The computing process can be illustrated by a rectangular grid. The horizontal line represents the number of periods, while the vertical one the prices. Each node contains the underlying asset price and the option price.

In order to apply the FD method into option forecasting, we first establish an option price grid based on the FD pricing method. Second, we estimate the asset price in the next period in a similar way as we do with the BS method, namely $S_1 = e^{r\Delta t}S$. Third, we find in the second column of grid for the nodes closest to S_1. We use the interpolation method and locate the first sampling point, whose coordinate is formulated as $i = \left\lfloor \frac{S_1}{\Delta S} \right\rfloor$, where ΔS involves the granularity for calibrating the option price, $\lfloor \bullet \rfloor$ denotes the floor function. Note that the coordinate of the second sampling point is i+1. The proportion of interpolation can be calculated as $\chi = \frac{S_1}{\Delta S} - i$. We can therefore forecast the option price in the second period based on the FD method as $\hat{C} = (1 - \chi)C_i + \chi C_{i+1}$.

2.4 A Modified Monte Carlo (MC) Method to Forecast Option Prices

The MC method assumes that the asset price exhibits a lognormal distribution, which enables that the asset prices at the expiration time can be found by "averaging" a large of number of stochastic sampling trial paths. By discounting the average result back to the present, the corresponding option price is obtained.

After we have acquired the present option price C using the standard MC approach, the forecast counterpart at the end of next period can be calculated by $\hat{C} = e^{r\Delta t}C$.

3 Improving Forecasting Precisions Based on Neural Networks

The linear neural network (LNN) is adopted to learn the forecasting errors resulted from the four modified approaches compared to the real option prices.

The input vector of the LNN consists of the forecasted prices produced by the four aforementioned methods, $\left(\hat{C}_1(t),\ \hat{C}_2(t),\ \hat{C}_3(t),\ \hat{C}_4(t)\right)^\tau$ where $\hat{C}_1(t)$, $\hat{C}_2(t)$, $\hat{C}_3(t)$, and $\hat{C}_4(t)$ (t = -L, …, 0) are the forecast option prices at the end of period t, and $(\bullet)^\tau$ representing the transpose of (•). The subscript 1 denotes the BT method, 2 the BS method, 3 the FD method, and 4 the MC method, respectively.

The target output of the LNN is $C(t)$, where $C(t)$ (t = -L, …, 0) is the real option price at the end of period t. The activation function in the output layer is linear, and the training algorithm is the back propagation. The relation can therefore be depicted as $\hat{C}(t)=\sum_{i=1}^{4} w_i\hat{C}_i(t)+b$. Noticing that the option prices are most likely influenced by their recent historical prices, a dynamic sliding window of length L is designed to accommodate the option's historical prices and forecasted prices. In training, t is sequentially assigned the values from -L to 0, namely, the training set is in a dynamic fashion.

After the network has been trained, the forecasted prices for the next period, $\hat{U}'$ are input into the network in order to implement a new forecast $\hat{V} = \hat{C}(1)$ for the next period.

Too, the multilayer perceptron (MLP) can also be utilized in forecasting option prices based on the modified the four conventional forecast results. The training and the forecasting procedures in the MLP resemble those in the LNN, whereas the difference in between lies in that the MLP includes a hidden layer and nonlinear activation functions, which enables it to learn the nonlinear features which are not "absorbed" by the modified conventional methods. Since it is well-known that the option market is a complex nonlinear system, the option price, a typical financial index of the option market, is supposed to exhibit the nonlinear features as well. In theory, the MLP is therefore preferable to the LNN.

4 Experiments

In this section, the time period is assumed to be one day, namely, the time granularity Δt equals one day, $\Delta t = 1$. In reality, for forecasting the options' prices in the next trading day, we always have enough time in the overnight to train the NN.

In the experiments, the option data are chosen from the Hong Kong option market, exemplified as shown in Table 1. The number of options amounts to 320. The data gathered span from date 2005-08-01 to date 2005-10-28 (64 trading days in total). For each day, the data include the stock price, the stock price volatility, and the real option price on the next day. In the calculation, the risk-free interest rate r is assumed 0.025.

It is worth-noting that the data also include the scenario for the deep-in-the money options, exhibited as the very high option prices.

We first simulate the four modified conventional methods (BT, BS, FD, and MC) using the Java APIs on the platform. For the NN forecasting, the Matlab NN toolbox is used, with the Bayesian regularization back-propagation (*trainbr*) adopted as the training algorithm. The reason that we use back-propagation is that it is a proved successful method in forecasting [2]. The number of maximal epochs in training is 100 and the performance goal for the error tolerance is 0.1. For the LNN approach, there is only one node in the output layer with a linear activation function, specifically the *pureline* function. For the multilayer perceptron, a three-layer nonlinear NN is established with *tansig* as the nonlinear activation function.

As aforementioned, the NN acquires the "knowledge" of the current option market via a learning process carried out on the historical data, and then applies this knowledge into further forecasting, in order to gain a better forecasting performance. In appearance, the NN's capability to master the current market's features increases with the size of the training samples, namely a larger L is preferred. However, the empirical studies in the GARCH model imply the length of sliding window should not be very large. In addition, the current research accomplishments in the financial society have demonstrated that the financial data can be considered stationary only during a short span of time. As a consequence, we only experiment with values of L as 3, 5, 8 and 12 in our studies, respectively.

It is well-known that the three-layer feedforward NNs are able to approximate any continuous functions arbitrarily well [4], or the three-layer NN is capable of meeting our needs. Since the Bayesian regularization back-propagation training algorithm (*trainbr*) in the Matlab NN toolbox not only minimizes the errors incurred by training samples, but also those by the magnitudes of weights and biases, which is equivalent to incorporating a penalty function to keep the number of weights to the minimum, the number of the hidden nodes is set to 12 without worrying about overfitting.

In order to make a comparison among different methods regarding the forecast performance, the assessing criterions for the forecasting errors should be given. There are two approaches to define the assessing criterions, the absolute assessing criterions and the relative assessing criterions.

For the absolute assessing criterions, there are a couple of ways to define the average absolute forecasting error. In this paper, we simply use $e=\frac{1}{K}\sum_{t=-K+1}^{0}\left|\hat{C}(t)-C(t)\right|$ for a certain option where K is the number of days that we are concerned. Thus the total average absolute forecasting error for N options is $\bar{e}=\frac{\sum_{j=1}^{N}e_j}{N}$, where e_j represents the forecasting error for the option j. The relative assessing criterion defines the average relative forecasting error in the past T days as $\alpha=\frac{1}{K}\sum_{t=-K+1}^{0}\frac{\left|\hat{C}(t)-C(t)\right|}{C(t-1)}$, for a certain option. Thus the total average relative forecasting error for N options is $\bar{\alpha}=\frac{\sum_{j=1}^{N}\alpha_j}{N}$, where α_j represents the error for the option j. If α or $\bar{\alpha}$ is less than a certain value, say, 10%, the forecast can be considered as an effective one.

In the experiments, we choose $K = 52$ and $N = 320$.

We use the modified conventional methods and the NN approaches to forecast the prices for 320 options, each of whose data spans across three months from 2005-08-01 to 2005-10-28, or 64 trading days. Since some data have to be used in finding the parameters in the modified models, e.g., the volatilities σ, the forecasting work begins from the 12th trading day, and lasts for 52 days.

Fig. 1 shows the total average relative forecasting errors for all the 320 options given by different methods. It can be observed that the NN approaches achieve the errors around 11% which are much better than those around 22% and 23% given by the conventional methods.

It is suggested three points. First, the MLP is likely to be more capable to seize the complex nonlinear features in the option markets. Second, the nonlinearity of the option prices have been largely assimilated by the platform of the four conventional methods, and the NN is able to focus on decreasing the errors. Third, there are still some residual errors on the platform, which can be coped with by the MLP.

For estimating the whole picture of improvement levels across the 320 options, we use the BS method as a base for comparison (it is apparent from the above experiments that the four conventional methods commit the similar levels of errors) and implement a statistic for the distribution of the error decreasing percentages contributed by the MLP, a slightly better NN in empirical studies. In particular, for each option, label e_{MLP} the average absolute forecasting error of the MLP, e_{BS} that of the BS. Define the error decreasing percentage as $\eta = 1 - \frac{e_{\text{MLP}}}{e_{\text{BS}}}$. The distribution of η is shown in Fig. 2. Clearly, $\eta < 1$. If $\eta < 0$, $e_{\text{BS}} < e_{\text{MLP}}$. The larger is η, the more improvement percentage is.

From Fig. 2, it is found that for some 12.5% of options, the MLP does not enhance the forecasting performance, while the average absolute forecasting errors for the other 87.5% of options, the MLP approach is superior to the conventional methods. Additionally, around 10% of the options have achieved an increase in performance more than 0.8. This is equivalent to the fact that for these options, the errors made by the MLP approach are only around 1/5 those of the conventional methods.

An NN can have different lengths of sliding windows. Fig. 3 shows the total average absolute forecasting errors by the LNN and the MLP with different sliding window lengths. We also conduct various experiments based on other values of L, and it is perceived that the adequate lengths of sliding windows, e.g., $L = 12$, produce the best performance (see Fig. 3).

Table 1. The examples of options chosen from the Hong Kong option market (the source of the options' prices is available on the website of the Hong Kong Exchange and Clearing Limited with http://www.hkex.com.hk/tod/markinfo/setdata.asp while the underlying assets' the website of Yahoo finance with http://finance.yahoo.com/q/hp)

option category	option code	option name
BEA	0023.HK	Bank of East Asia
CKH	0001.HK	Cheung Kong
HEH	0006.HK	HK Electric
HWL	0013.HK	Hutchison

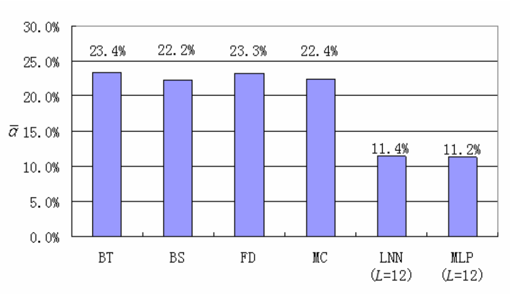

Fig. 1. The average forecasting errors for all options given by the different methods

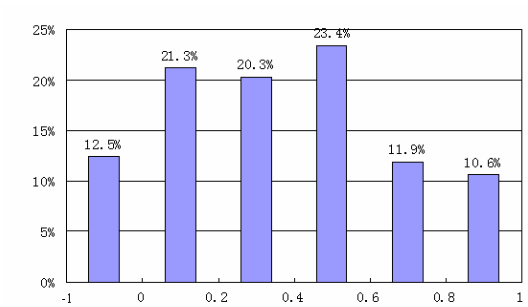

Fig. 2. The distribution of error decreasing percentage η

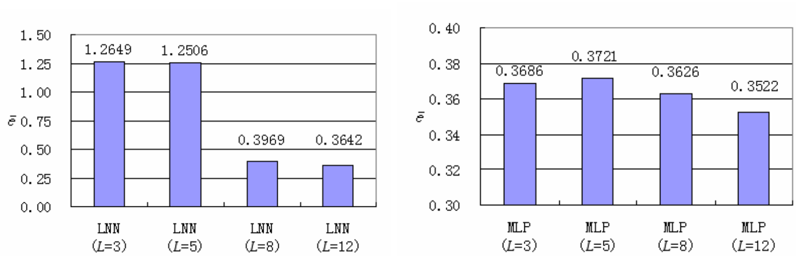

Fig. 3. The total average absolute forecasting errors by the LNN and MLP approaches using different sliding window lengths

5 Conclusions

In this paper, the experimental studies substantiate that in general the NN approaches significantly decrease the forecasted errors compared to the conventional ones, and improve the forecast accuracy accordingly. In addition, the MLP approach outplays the LNN approach slightly. It is positive to believe that the MLP is slightly preferable to the LNN, and the platform upheld by the conventional methods works with a function for shielding the nonlinearity of the option price movements and automatically managing some particular scenarios in option prices. Compared to the conventional option price forecasting methods, the NNs are characteristic of higher forecast accuracy, which enables it a preferable alternative for investors who seek a more effectual approach to seize the option prices.

Apparently, while the option pricing provides the theoretical result and is only instructive to the market practice indirectly, the less-error-incurring NN-based forecasting commits more profits directly for the market practitioners, and enables them to gain the wealth in the option markets realistically.

Future work includes conducting experimental studies with comparisons to other neural network methods as well as other machine learning techniques.

Acknowledgment. The project is sponsored by the NSF of China under grant numbers 70571003 and 70871001.

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