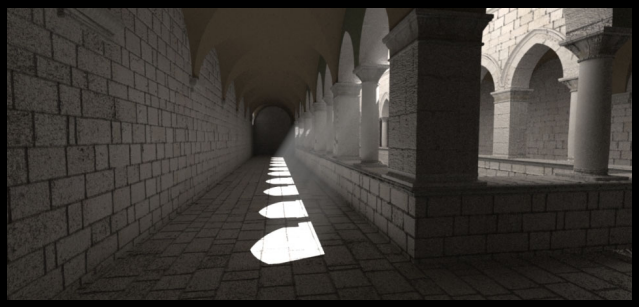
Global Illumination II



70001 – Advanced Computer Graphics: Photographic Image Synthesis

Abhijeet Ghosh

Lecture 15, Feb. 23rd 2024

1

Biased approaches

- Reuse previously computed results
  - Assuming slow change in radiance
- Efficient computation of smooth noise-free images
  - Price: unpredictable behavior!
- Irradiance & radiance caching
- Photon mapping

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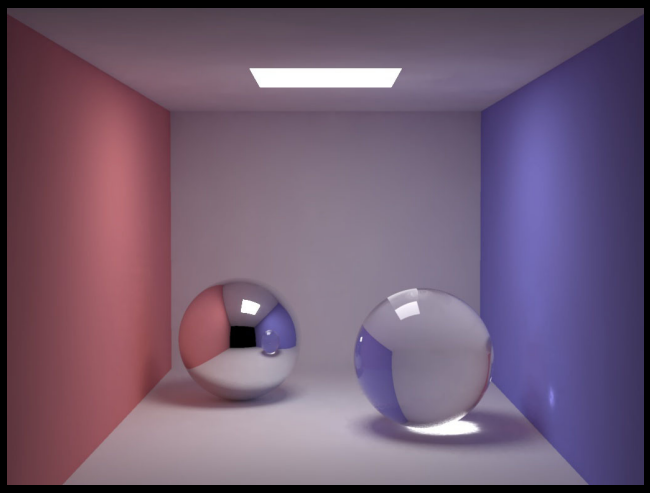
Global Illumination



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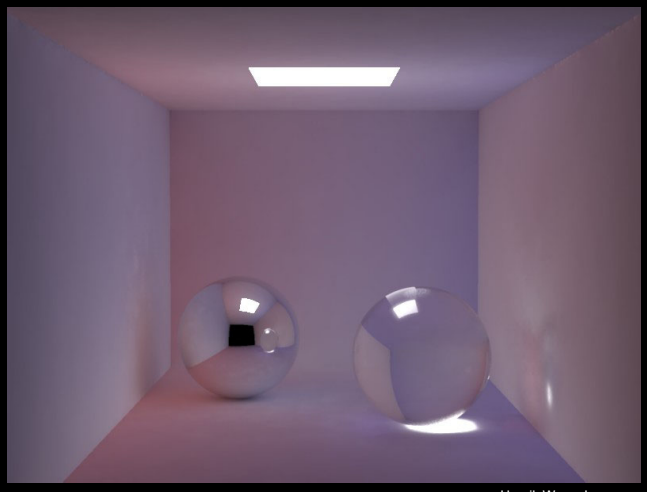
Indirect Irradiance



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Irradiance caching – Ward et al. 98

- Indirect lighting changes slowly compared to direct lighting
  - especially for diffuse surfaces
- Compute at sparse points and interpolate!
- Two issues:
  - How to store an irradiance estimate?
  - When to sample new estimate?

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Irradiance caching

- Irradiance

  $E(x,n) = \int_\Omega L_i(x,\omega_i) \cos\theta_i \, d\omega_i.$

- Radiance

  $L_r(x,\omega_r) = \int_\Omega f_r(x, \omega_r, \omega_i) \, L_i(x,\omega_i) \cos\theta_i \, d\omega_i.$

- Diffuse surface: BRDF constant c

  $L_r(x,\omega_r) = c \int_\Omega L_i(x,\omega_i) \cos\theta_i \, d\omega_i = c \cdot E(x,n)$

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Irradiance caching

- First compute E(x,n) with MC path tracing

$$E(x,n) = \frac{1}{N} \sum_j \frac{L_i(x,\omega_j) \cos\theta_j}{p(\omega_j)}$$

- Sample from cosine weighted distribution $p(\omega_j) = \cos\theta/\pi$

$$E(x,n) = \frac{\pi}{N} \sum_j L_i(x,\omega_j)$$

- Irradiance estimates stored in an octree data-structure

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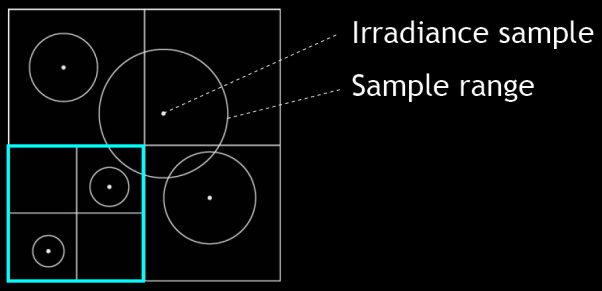
Octree example



- Adaptive subdivision such that each cell contains < 3 samples

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Irradiance sampling

- Each sample is assigned a range for interpolation
- Rate of change of irradiance decides interpolation distance
  - inversely proportional
- Depends on distance to visible surfaces
- Harmonic Mean $\frac{N}{\sum_{i=1}^{N} 1/d_i}$
  - d_i is the distance to first intersection along path i

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Irradiance sample

```
struct irradiance_sample {
  vector3 E  // irradiance
  vector3 n  // normal
  vector3 p  // position
  float r    // range
}
```

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Irradiance interpolation

- Compute the error in estimate when using a cached sample

$$w_j(\mathbf{x}) = \frac{1}{\frac{\|\mathbf{x}-\mathbf{x}_j\|}{r_j} + \sqrt{1 - \mathbf{n}(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x}_j)}} \approx \epsilon_j(\mathbf{x})$$

- If error is acceptable, use a weighted sum of interpolated irradiance

$$E(\mathbf{x}) = \frac{\sum_i w_i(\mathbf{x}) E(\mathbf{x}_i)}{\sum_i w_i(\mathbf{x})}$$

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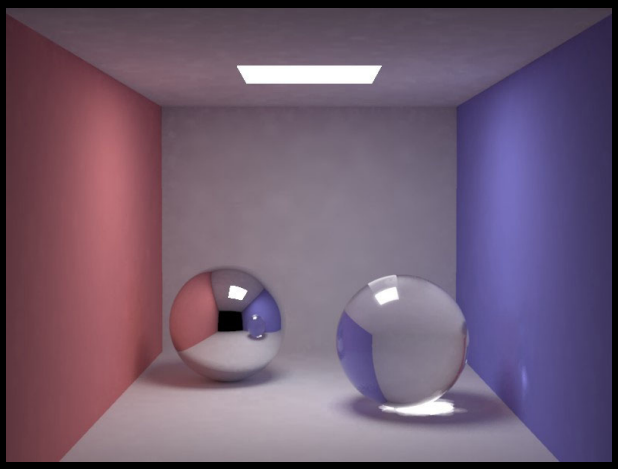
Irradiance gradients



1000 sample rays, w > 10

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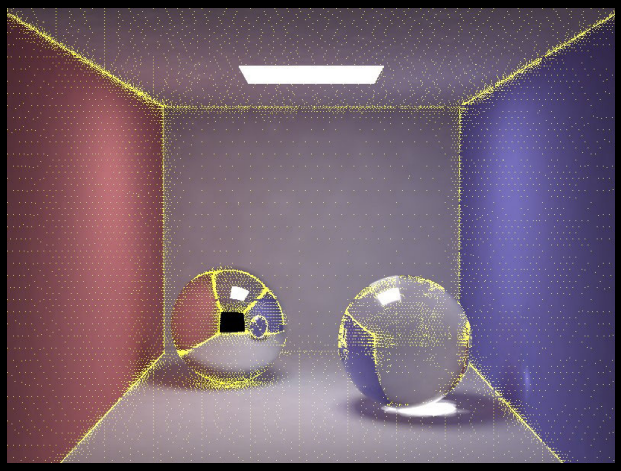
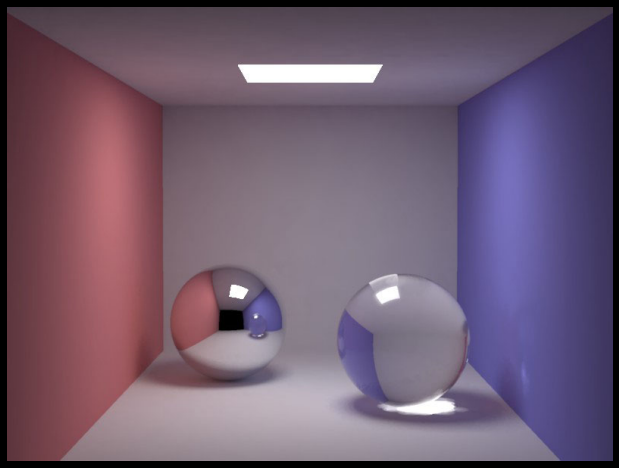
Irradiance cache positions



1000 sample rays, w > 10

13

Irradiance gradients



5000 sample rays, w > 10

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Glossy surfaces?

- Irradiance caching works well for diffuse surfaces
- For glossy surfaces we need to account for the directional dependence of the BRDF!

$$L_r(x,\omega_r) \approx \left(\int_\Omega f_r(x, \omega_r, \omega_i)\, d\omega_i\right)\left(\int_\Omega L_i(x,\omega_i) \cos\theta_i\, d\omega_i\right)$$

$$\approx \rho_{hd}(\omega_r) \cdot E(x,n)$$

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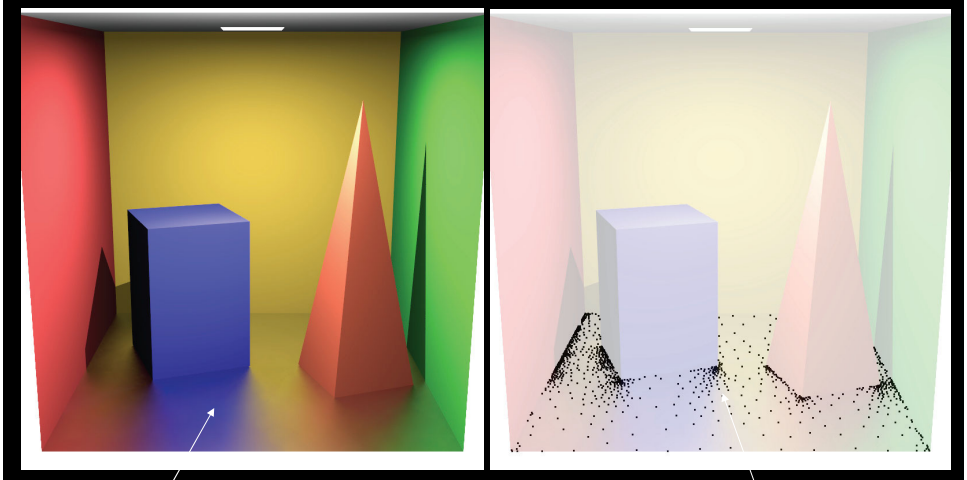
Radiance caching – Křivánek et al. 05



smooth indirect term

sparse computation and interpolation

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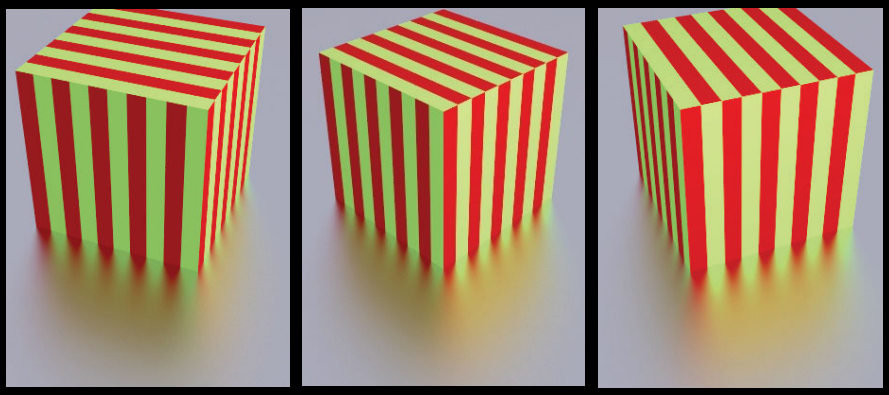
View dependence



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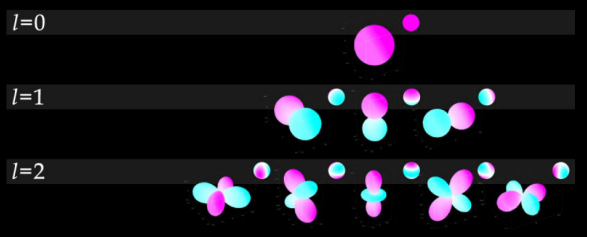
Incoming radiance representation

- Spherical Harmonics
  - basis functions on a sphere
- Approximate $\rho_{hd}(\omega_r)$ in the SH basis
  - 3D rotations of SH coefficients for change in viewpoint
- Dot with SH irradiance coefficient



[Green 2003]

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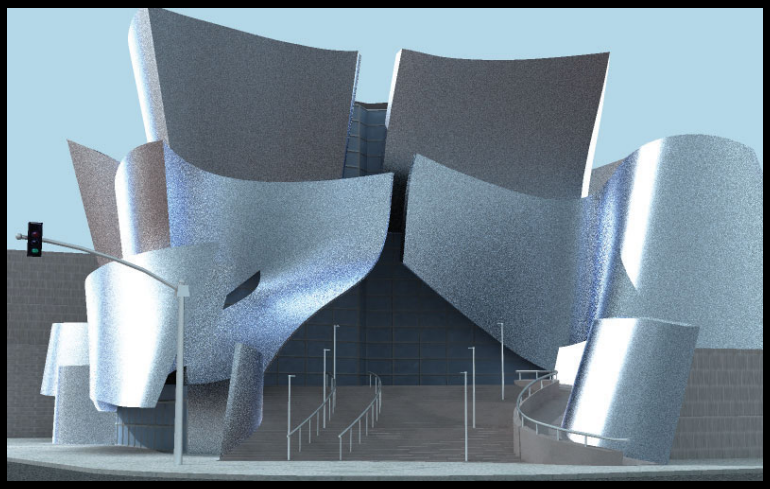
MC Importance Sampling



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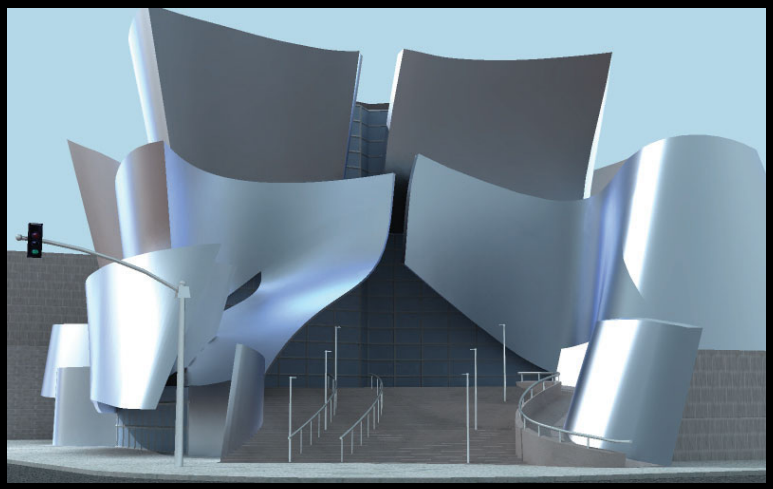
Radiance caching



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Indirect specular?

- Caustics are still hard to simulate with radiance caching ☹
- Solution: Photon Mapping [Jensen 2001]
- 2 pass algorithm
  - trace photon into the scene from light source
  - gather photon density from camera

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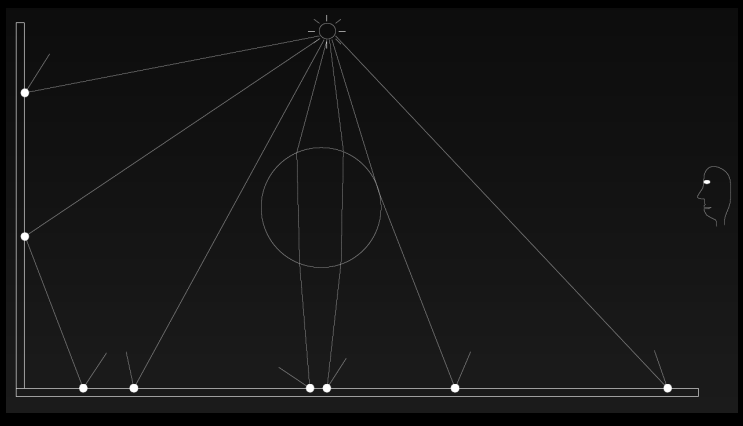
Photon tracing



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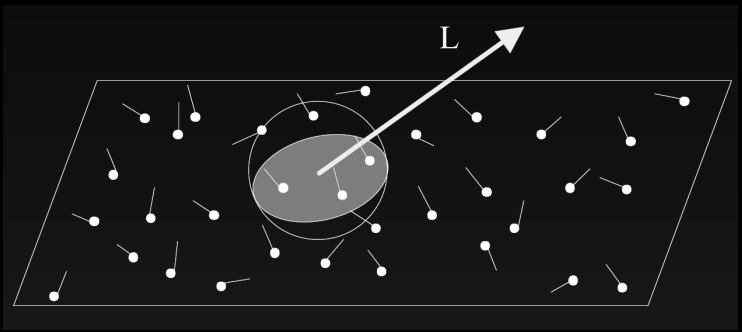
Photon density



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Radiance estimate

$$
\begin{aligned}
L(x,\vec{\omega}) &= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta' \, d\omega \\
&= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{\cos\theta' \, d\omega \, dA} \cos\theta' d\omega \\
&= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi\ (x,\vec{\omega}')}{dA} \\
&\approx \sum_{p=1}^{n} f_r(x,\vec{\omega}'_p,\vec{\omega}) \frac{\Delta\Phi_p(x,\vec{\omega}'_p)}{\pi r^2}
\end{aligned}
$$

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Photon map data-structure

- Balanced kd-tree

```
struct photon = {
  float position[3];
  rgbe power;         // power packed as 4 bytes
  char phi, theta;    // incoming direction
  short flags;
}
```

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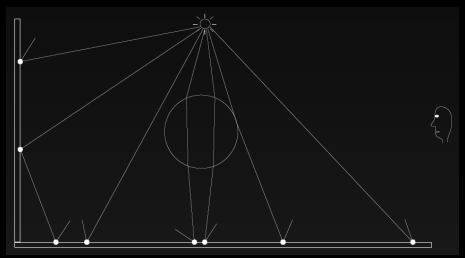
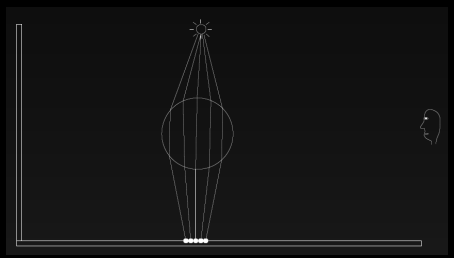
Two photon maps



Global photon map



Caustic photon map

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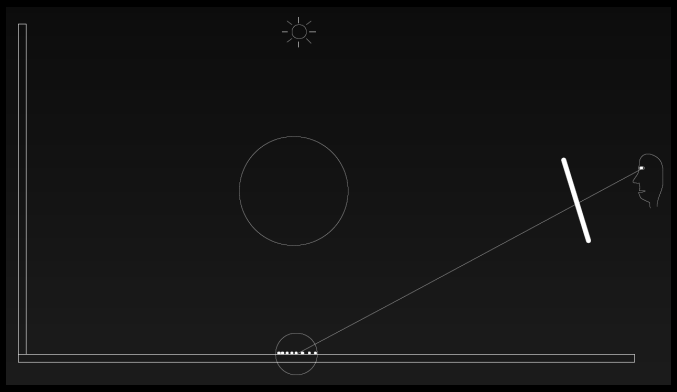
Rendering caustics



1st diffuse bounce

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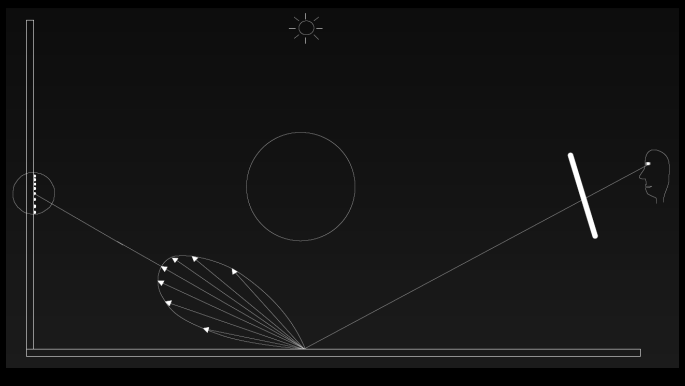
Rendering indirect illumination



2nd diffuse bounce

- Also known as final gather!

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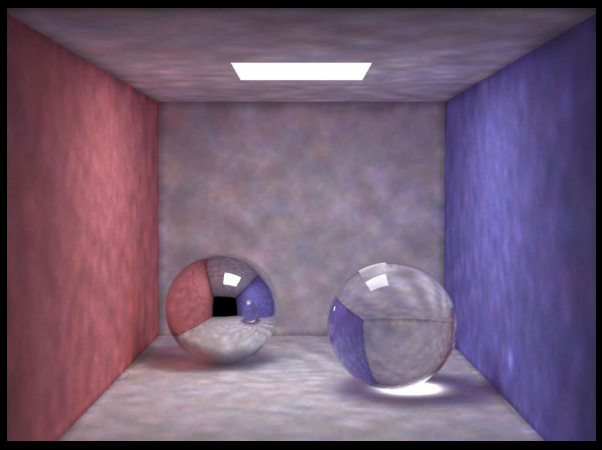
Photon map visualization



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100,000 photons, 50 photons for density estimate

(1st bounce density evaluation)

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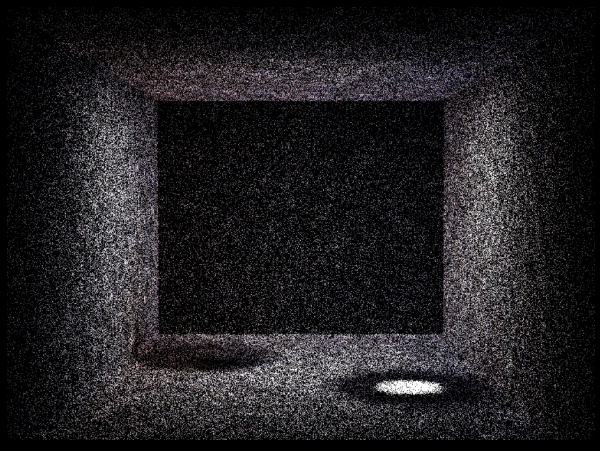
Photon map visualization



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100,000 photons, 50 photons for density estimate

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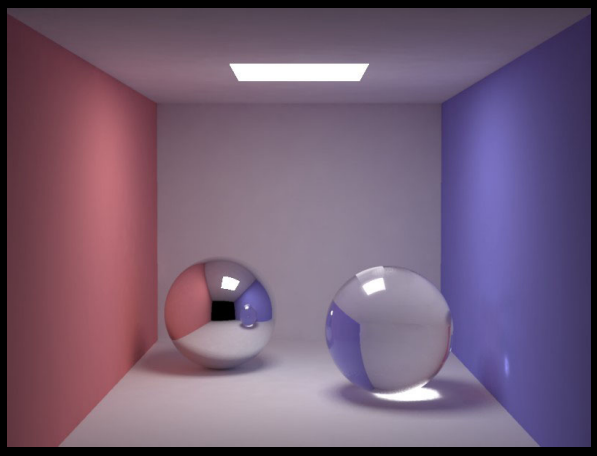
Rendering with final gather



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- 2nd bounce density evaluation for indirect illumination

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Photon Mapping - Cognac glass

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Photon Mapping – Mies house

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Photon Mapping – subsurface scattering

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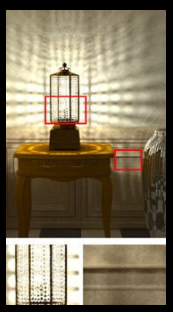
Caustics dominated scene



- Photon mapping still too expensive
  - too much memory consumption
- Solution: Progressive Photon Mapping [Hachisuka et al. 08]
  - resolve SDS paths with arbitrary accuracy with finite memory!
- Multiple photon tracing steps for incremental estimate
  - no storage of previously traced photons

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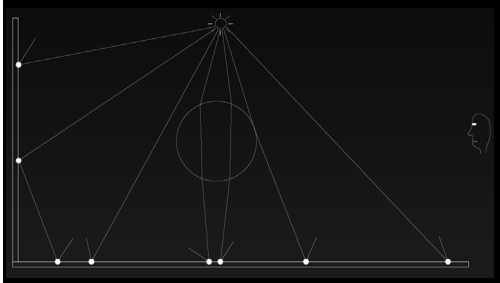
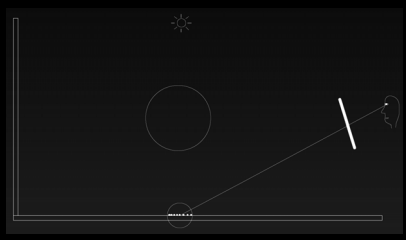
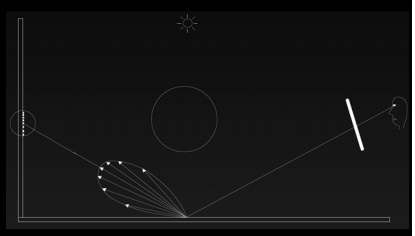
Regular Photon Mapping







- 1st pass photon tracing from light source to diffuse surfaces
- 2nd pass of ray tracing from camera to scene hit points for evaluation of photon density estimate

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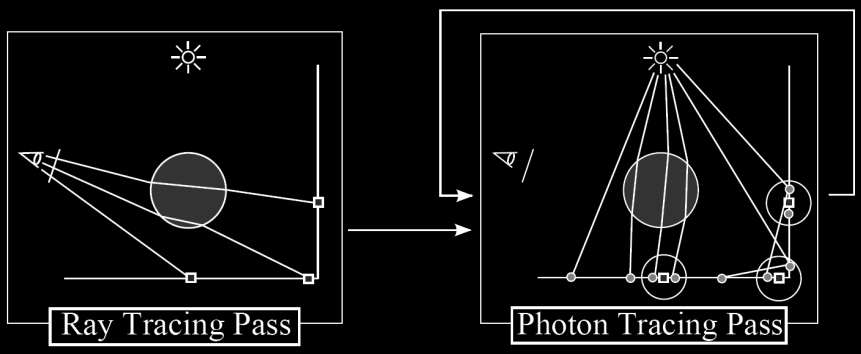
Progressive Photon Mapping



- 1st pass ray trace from camera to scene hit points
- 2nd pass (multiple) of photon tracing from light source to evaluate photon density at camera hit points. No need to store older photons! Update of density estimate after each pass.

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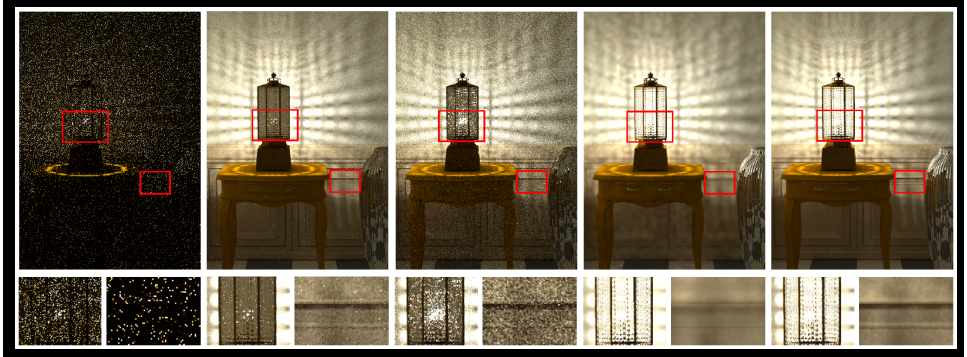
Progressive photon mapping



[Hachisuka et al. 08]

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Photon density estimation

- Source of bias:

$$L(x,\vec{\omega}) \approx \sum_{p=1}^{n} \frac{f_r(x,\vec{\omega},\vec{\omega}_p)\phi_p(x_p,\vec{\omega}_p)}{\pi r^2},$$

  - radiance estimate converges with increase in density
  - infinite density in the limit

- Trace N photons, estimate with N^{β} photons, β in [0,1]:

$$L(x,\vec{\omega}) = \lim_{N\to\infty} \sum_{p=1}^{\lfloor N^{\beta} \rfloor} \frac{f_r(x,\vec{\omega},\vec{\omega}_p)\phi_p(x_p,\vec{\omega}_p)}{\pi r^2},$$

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Progressive density estimate

- Density $d(x) = n/\pi r^2$
- If radius r remains constant, then density estimate is averaged
  - suboptimal
- Key idea is to progressively reduce r with each photon tracing pass
  - while increasing photon accumulation!

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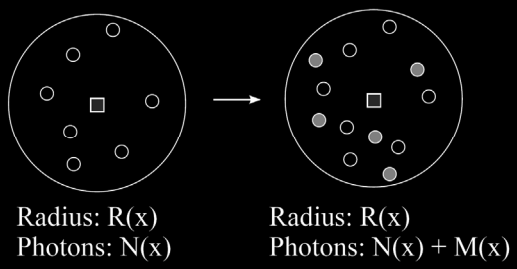
Radius reduction



- After second round of photon tracing, local density with constant radius R(x)

  $d'(x) = (N(x) + M(x))/(\pi * R(x)^2)$

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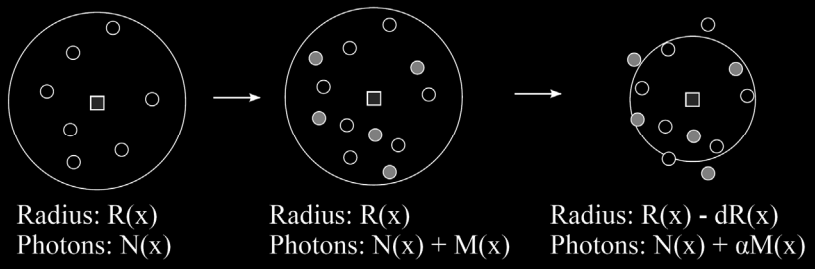
Radius reduction



$$dR(x) = R(x) - R(x)\sqrt{\frac{N(x) + \alpha M(x)}{N(x) + M(x)}}.$$

- $\alpha \rightarrow [0, 1]$

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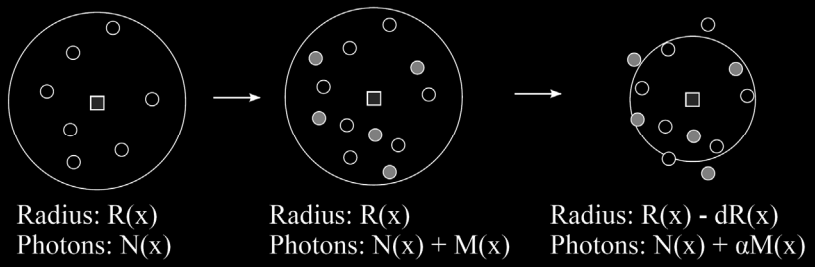
Radius reduction



$$dR(x) = R(x) - R(x)\sqrt{\frac{N(x) + \alpha M(x)}{N(x) + M(x)}}.$$

- d'(x) = (N(x) + αM(x))/(π * (R(x) - dR(x))^2)

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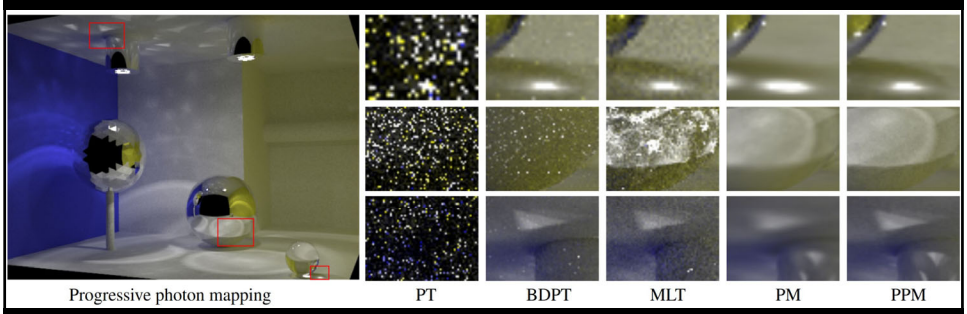
Progressive PM Results



[Hachisuka et al. 08]

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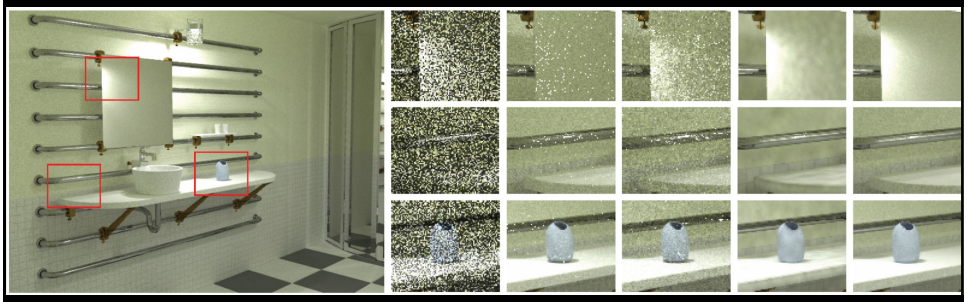
Progressive PM Results



[Hachisuka et al. 08]

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Many light methods

- Virtual point lights (VPLs) [Keller 97]
  - Approximate GI as contribution from a few virtual point lights
- Light Cuts [Walter et al. 05]
  - Hierarchical culling of point light contributions for efficient approximation

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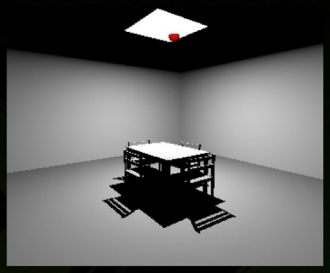
Instant Radiosity with VPLs [Keller 97]

- Approximate GI as contribution from a few virtual point lights
- Hardware (OpenGL) implementation using the accumulation buffer!
  - Soft shadowing computed as average of many images rendered with a point light placed at different positions on the area light

$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1}$  $=$

i

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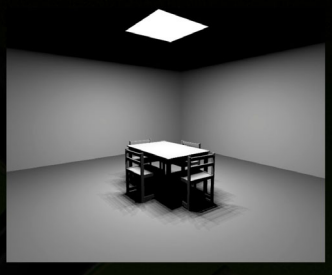
Instant Radiosity with VPLs [Keller 97]

- Approximate GI as contribution from a few virtual point lights
- Hardware (OpenGL) implementation using the accumulation buffer!
  - Soft shadowing computed as average of many images rendered with a point light placed at different positions on the area light

$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1}$  $=$ 

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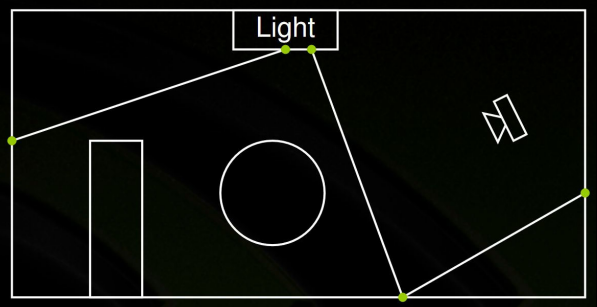
Instant Radiosity with VPLs [Keller 97]

- Light path vertices stored as point lights
  - similar to a small photon map!



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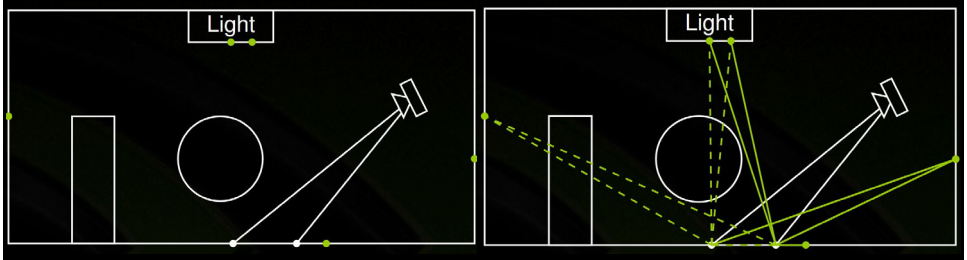
Instant Radiosity with VPLs [Keller 97]

- Light path vertices stored as point lights
  - trace camera rays and then connect to VPLs with shadow rays



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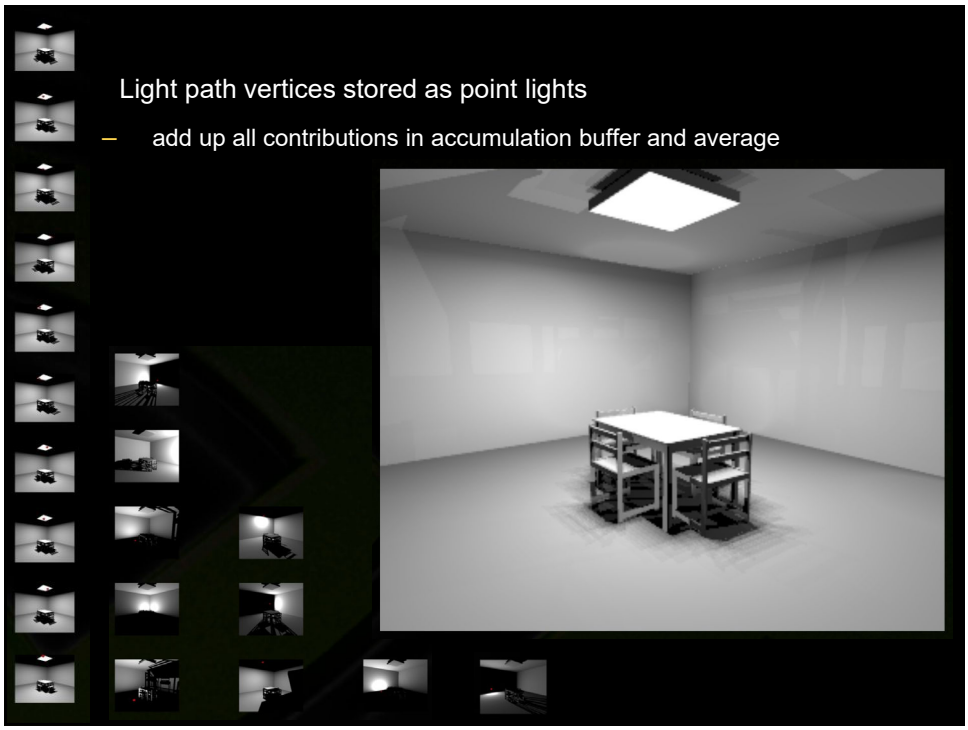
Light path vertices stored as point lights
– add up all contributions in accumulation buffer and average

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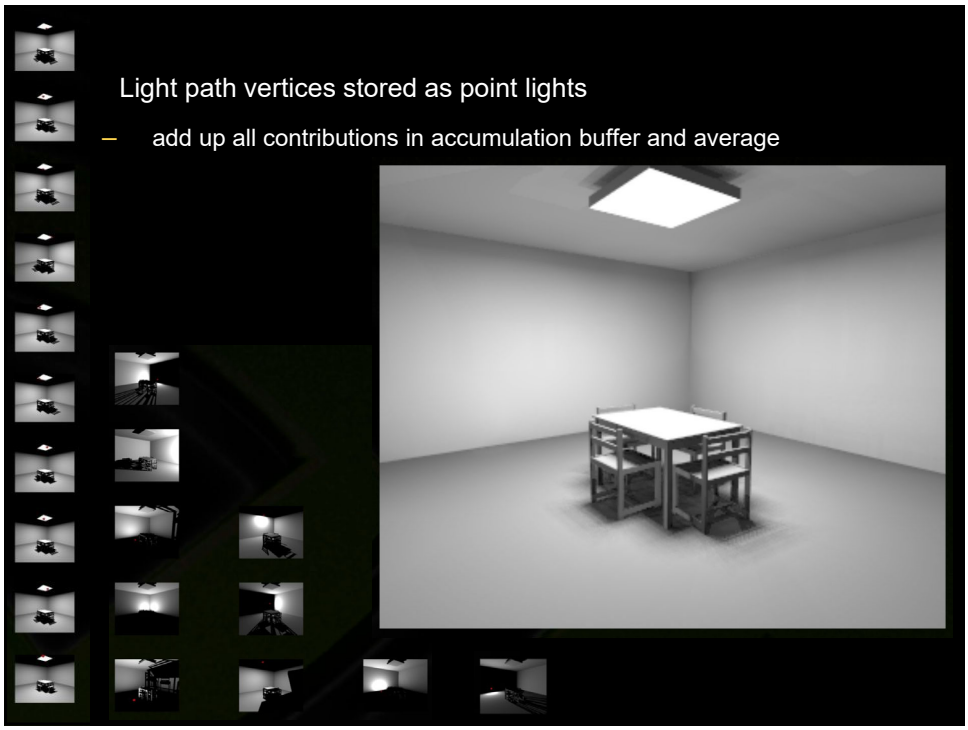
Light path vertices stored as point lights
– add up all contributions in accumulation buffer and average

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Scalability with many lights – Lightcuts
[Walter et al. 05]

- Simulate complex illumination using point lights
  - Area lights
  - HDR environment maps
  - Sun & sky light
  - Indirect illumination
- More lights ➔ more accurate
  - And more expensive
  - Naive cost linear in lights

Area lights + Sun/sky + Indirect

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Accurate approximation with few lights
[Walter et al. 05]

- Do we really need to evaluate all lights?
  - Thousands to millions of lights
  - Average contribution miniscule
  - Below human perceptual limits
    - Weber's Law (roughly 2%)
- Evaluate a small subset
  - Chosen adaptively
  - Perceptually accurate approximation

Environment lighting + Indirect

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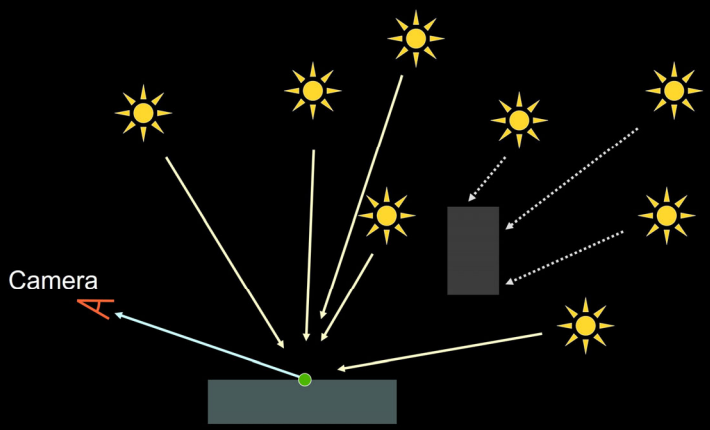
Lightcuts problem

- All lights do not contribute equally to a surface
  - Some may be blocked or affected by BRDF



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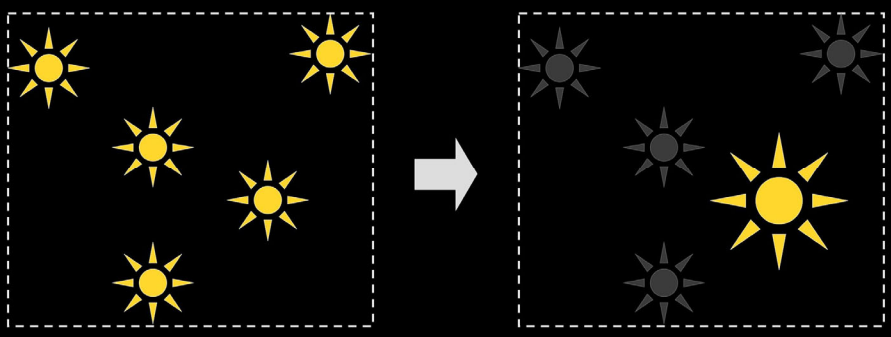
Lightcuts key idea - clustering

- Cluster a group of lights and represent them with a representative light



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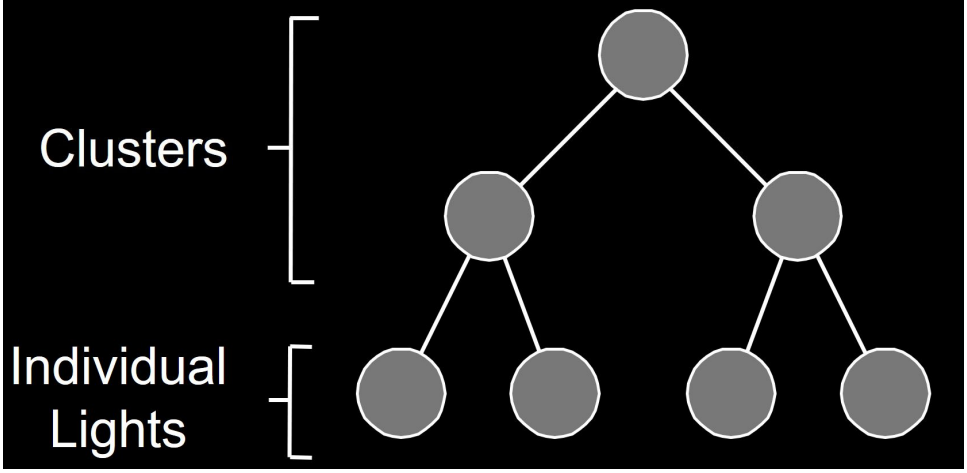
Light tree

- A binary tree of lights and light clusters



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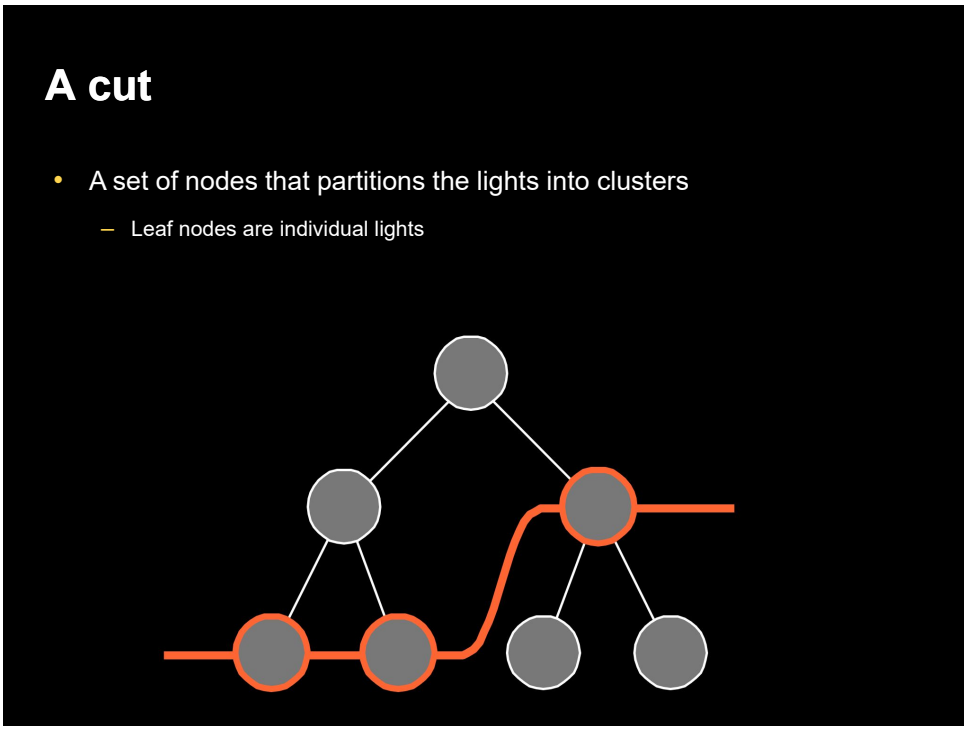
A cut

- A set of nodes that partitions the lights into clusters
  - Leaf nodes are individual lights



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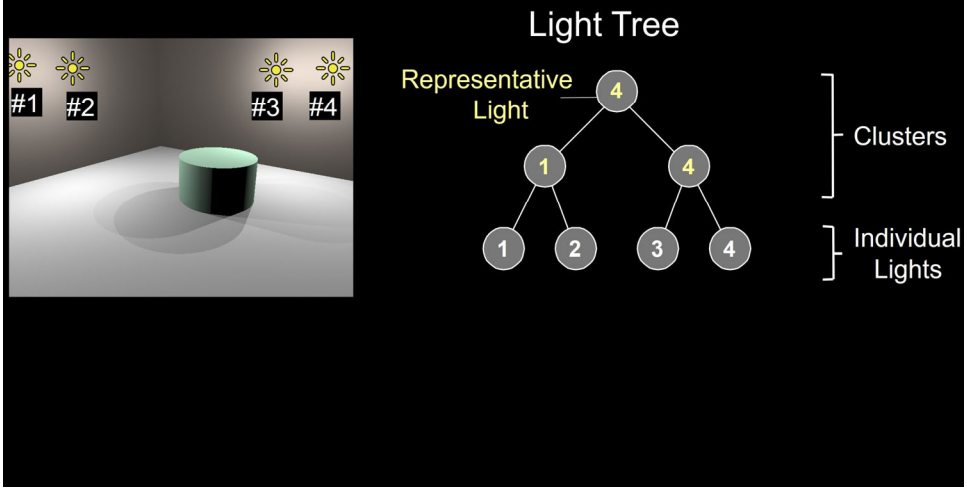
An example

- A scene with 4 point lights



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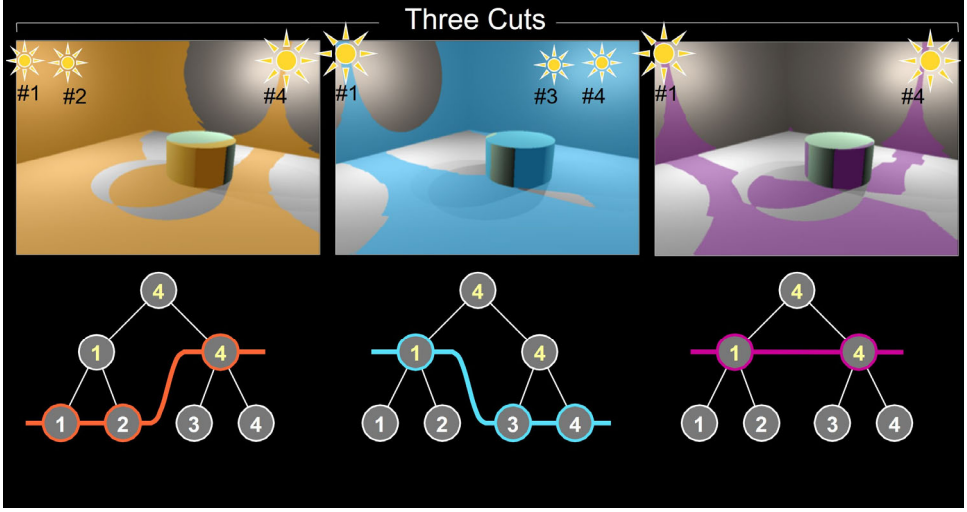
An example

- Three possible cuts of 3 lights



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Lightcuts algorithm

- Pre-process
  - Convert illumination to point lights
  - Build light tree
- For each visible point
  - Choose a cut to approximate the local illumination
    - Bound maximum error of cluster approximation
    - Refine cluster if error bound is too large
- Ensure each cluster's error < perceptual threshold
  - Webber's Law 2% of signal

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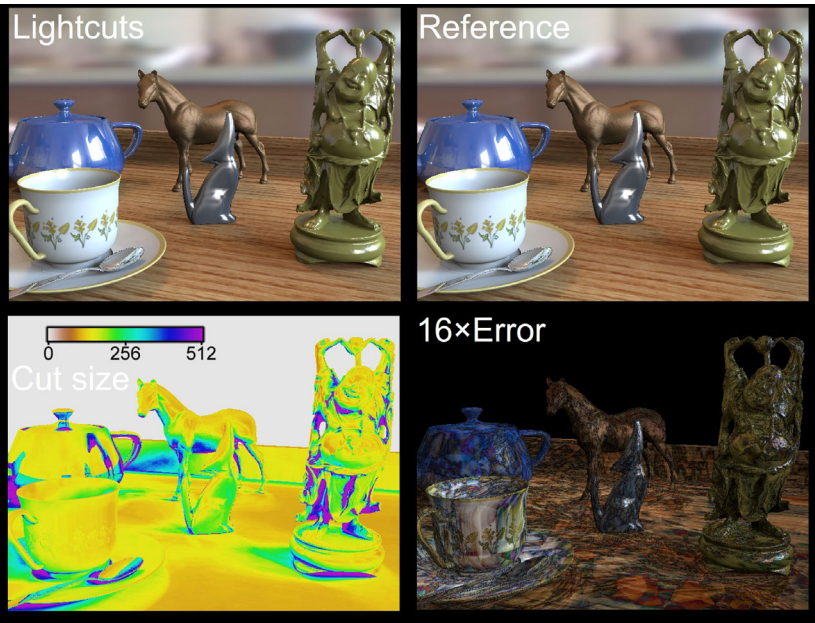

- 13000 VPLs for scene lit with environmental illumination + indirect
  - Cut uses only up to 512 lights for each shaded surface point

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