Today in Cryptography (5830)

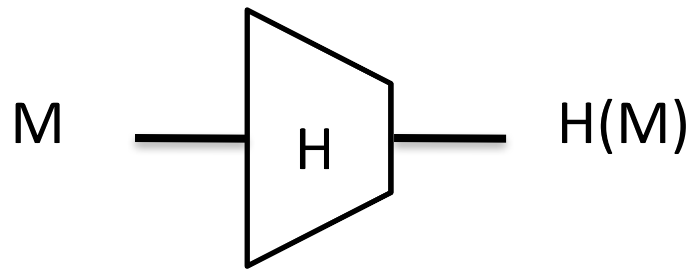
Hash functions
HMAC
Passwords and password-based key derivation

Where we are at

- Authenticated encryption
  - Symmetric encryption providing confidentiality and integrity
  - Security in face of active attackers
  - Uses message authentication codes as cryptographically strong error detection
    - We saw CBC-MAC, built from blockcipher
- Today: cryptographic hash functions
  - Used to build MACs, many other places
  - “Swiss army knife” of cryptography

Cryptographic hash functions

A function H that maps arbitrary bit string to fixed length string of size n



MD5: n = 128 bits
SHA-1: n = 160 bits
SHA-256: n = 256 bits

Many security goals asked of hash functions. Ideally, they behave as if they were a (public) random function.

Applications of hashing

- File comparison
- Digital signatures (coming up later)
- Password hashing
- For message authentication codes

Cryptographic hash functions

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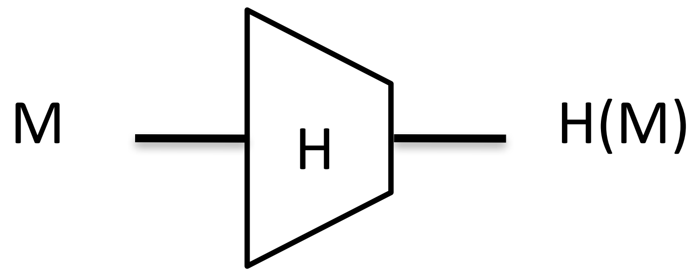

MD5: n = 128 bits
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Collision resistance:

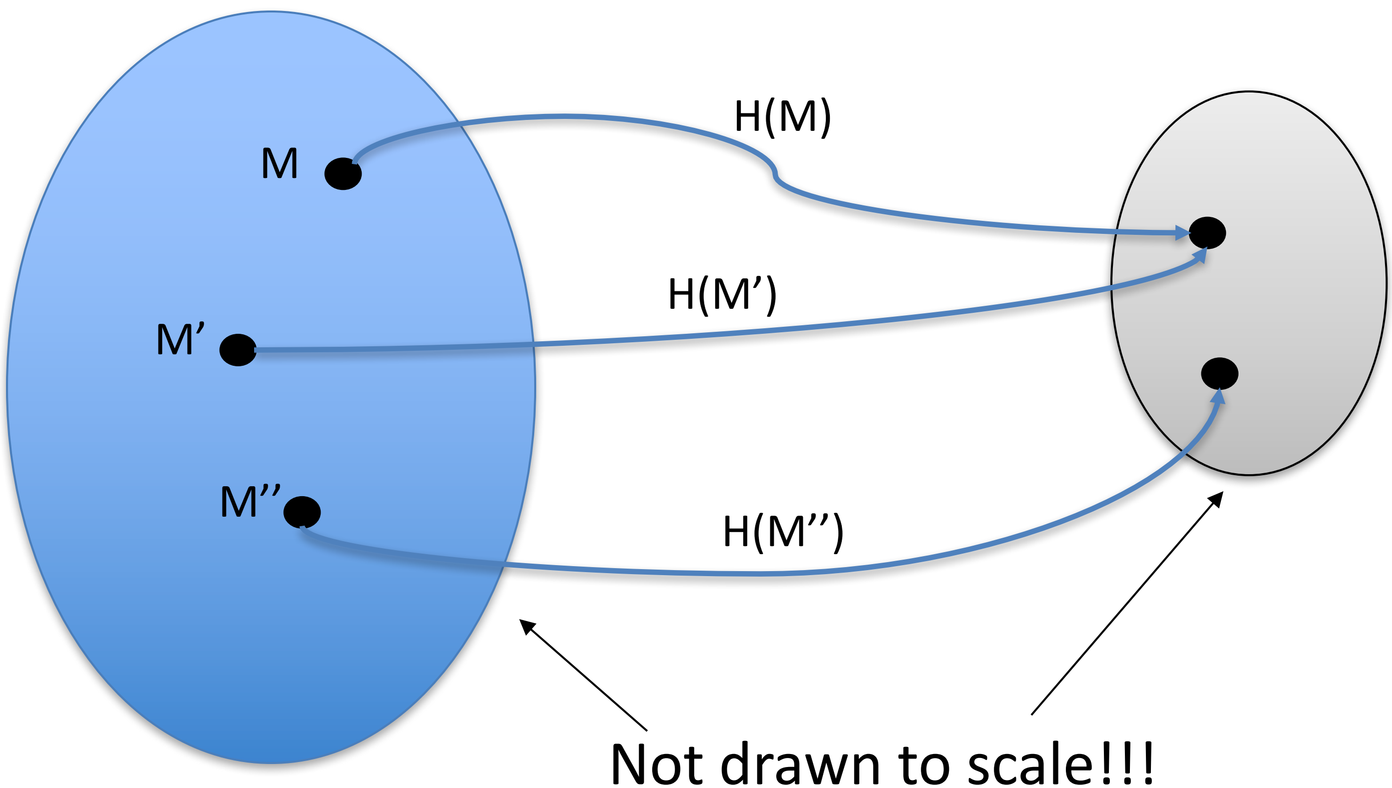
No computationally efficient adversary can find

$M \neq M'$ such that $H(M) = H(M')$

Collisions always exist

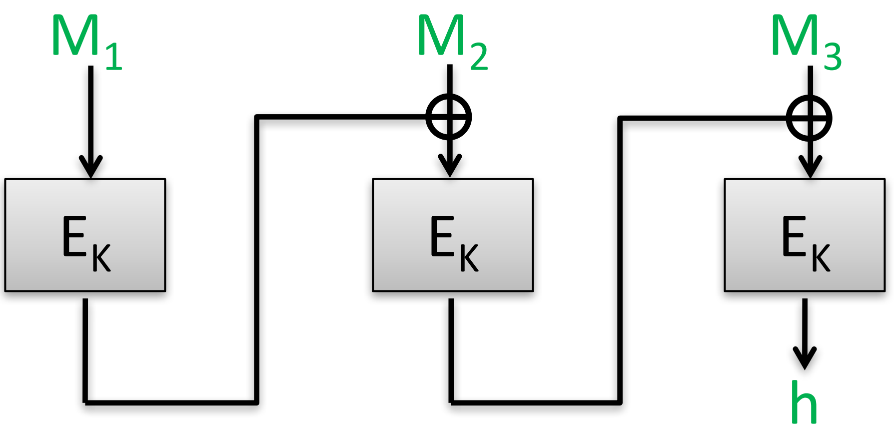
Domain (usually all strings up to some length)
SHA-1: up to length 2^{64}-1

Range $\{0,1\}^n$



Pigeonhole principle:
size of domain larger than size of range implies there must be collisions

CBC-MAC is *not* CR



Key was secret in CBC-MAC.
But hash functions are publicly computable.
One idea is to use a random, public K value known to attacker.

How do we *efficiently* find collisions?

<u>Adversary A(K):</u>
h <- CBC-MAC(0^n)
M_2 <- $D_K(h) \oplus E_K(1^n)$
Return (0^n, $1^n || M_2$)

Birthday attacks

- What is best possible security achievable by hash function with output length n bits?

- <u>Answer</u>: security is only achievable up to $2^{n/2}$ hash computations

The birthday problem

Choose q values $Y_1,\ldots,Y_q$ from $\{0,1\}^n$ at random. What is probability that two are the same?

Let $Coll_i$ be event that $Y_i = Y_j$ for some $j < i$

$$
\begin{aligned}
\Pr[Coll] &= \Pr[Coll_1 \vee \ \ldots \vee Coll_q\,] \\
&\leq \Pr[Coll_1] + \ldots + \Pr[Coll_q] \\
&= \frac{0}{2^n} + \frac{1}{2^n} + \frac{2}{2^n} + \ldots + \frac{q}{2^n} \\
&= \frac{q(q-1)}{2^n}
\end{aligned}
$$

Another proof shows that if $q \leq 2^{(n+1)/2}$

$$\Pr[Coll] \geq \frac{0.3 \cdot q(q-1)}{2^n}$$

The birthday attack

Let m be some length in domain of hash function H

<u>Adversary A:</u>

For i = 1 to q do:

X_i <-\$ $\{0,1\}^m$

h_i <- $H(X_i)$

If exists i,j s.t. $X_i \neq X_j$ and $h_i = h_j$ then

Return (X_i, X_j)

Return fail

Same # of domain points map to each range point

If H is *regular* then probability of success is exactly birthday probability

$$\Pr[\text{A finds collision}] \geq \frac{0.3 \cdot q(q-1)}{2^n}$$

Birthday attack run times

MD5:	n = 128 bits	2^{64}	MD5 computations
SHA-1:	n = 160 bits	2^{80}	SHA-1 computations
SHA-256:	n = 256 bits	2^{128}	SHA-2 computations

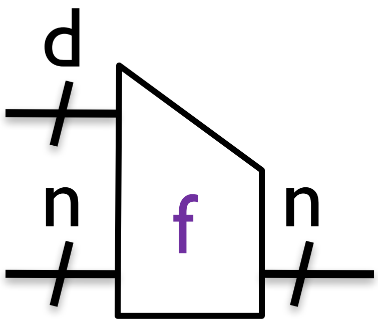
2^{64} too small by today's standards!

Bitcoin network computes about 2^{61} SHA-256 hashes ***per second***

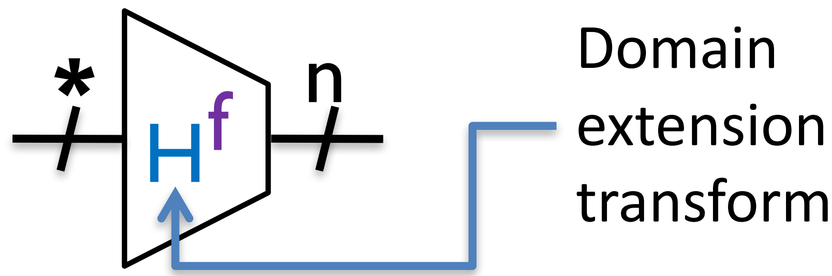
https://blockchain.info/charts/hash-rate

Two-step design for hash functions

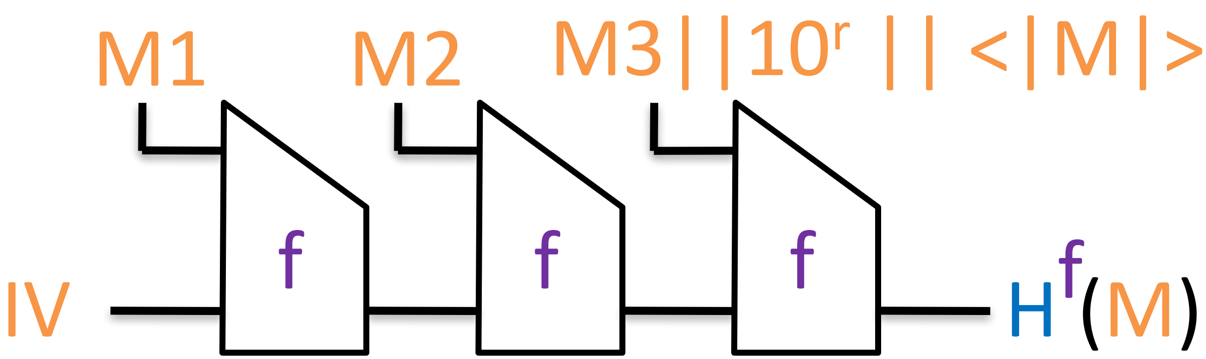
Compression Function



Hash Function



Domain extension called “Merkle-Damgard with strengthening”



Used in MD-x, SHA-1, SHA-256, ...

IV is a fixed constant. Not randomly chosen.

Building compression functions

- Can build compression functions from suitable block ciphers

$$f(z,m) = E(m,z) \oplus z$$

Called Davies-Meyer construction

- Can use AES, but security too low. Why?

- SHA-1 uses custom E with k = d = 512 and n = 160
  - Message block length of SHA-1 is 512 bits

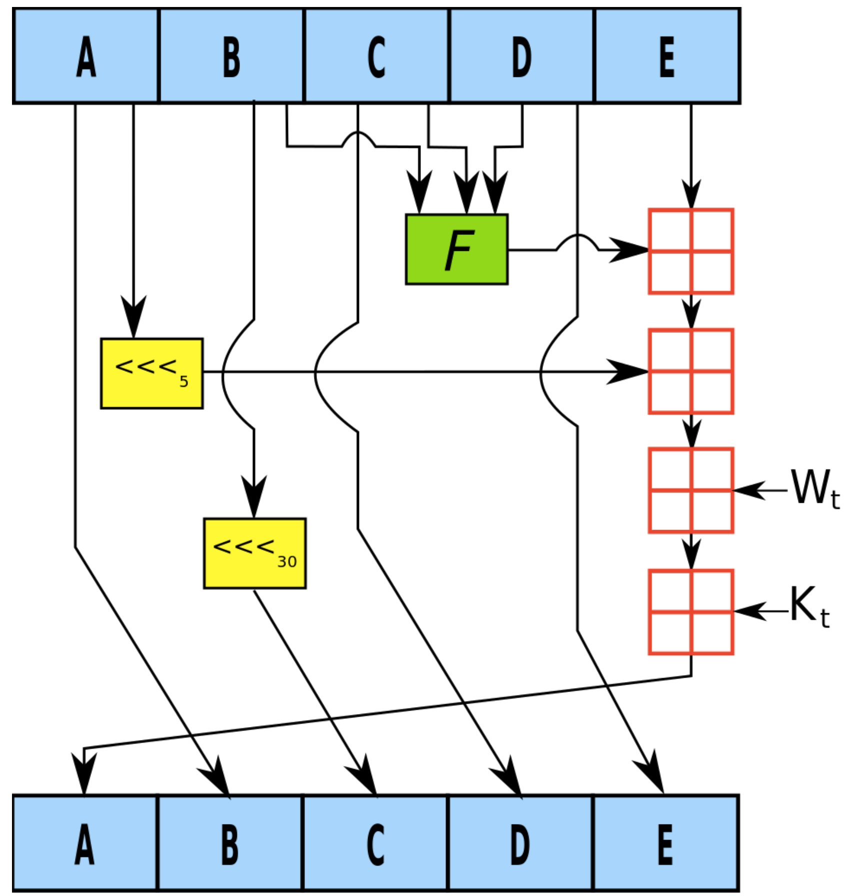
SHA-1 compression function

Expand 512-bit message into
$W_1,\ldots,W_{80}$ strings of length 32 bit values
(Think of this as "key schedule")

Chaining variable is 160 bits, 5 32-bit values
A , B , C , D , E

F(B,C,D) function that changes over rounds:
0-19: (B **and** C) **or** ((**not** B) **and** D)
20-39: B **xor** C **xor** D
40-59: (B **and** C) **or** (B **and** C) **or** (C **and** D)
60-79: B **xor** C **xor** D

Constants K_1 , … , K_{80} differ across rounds



Faster attacks than birthday?

- 2004: Full break of MD5 announced by Xiaoyun Wang and co-authors
  - MD5 is easy to break now. You can download programs to do it on your laptop
- 2005: Announced faster than 2^{80} attack against SHA-1 by Wang et al.
  - Not practical to run (2^{69} estimated cost)
- 2017: CWI and Google announce first demonstrated collision

SHAttered attack

Chosen prefix P. Find two pairs of message blocks M_1 , M_2 and M_1' , M_2' such that for any suffix S:

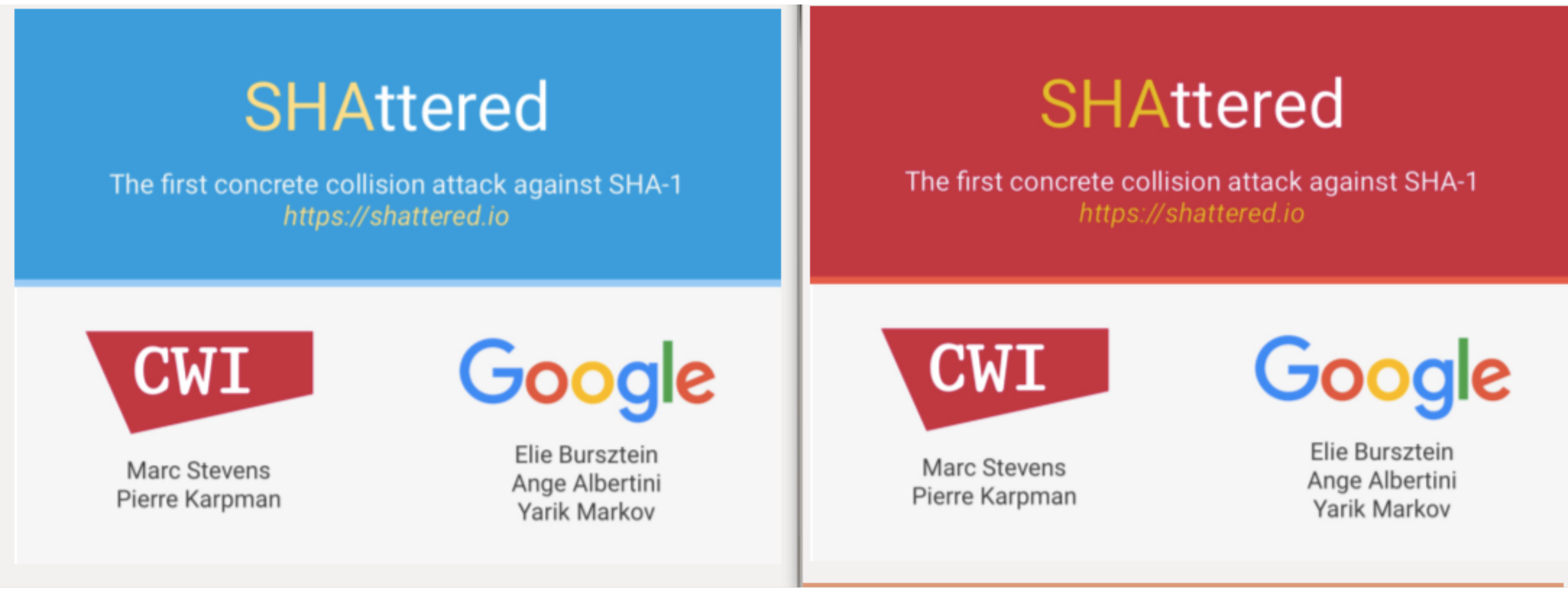
$$SHA\text{-}1(P \,||\, M_1 \,||\, M_2 \,||\, S) = SHA\text{-}1(P \,||\, M_1' \,||\, M_2' \,||\, S)$$

Referred to as a *identical-prefix collision attack*

How? Pick P, find M_1 and M_1' that form near-collision on chaining variable. Then complete collision by finding M_2 and M_2'

They show how to extend to build colliding PDF files

SHAttered attack



Required $2^{63.1}$ SHA-1 compression function applications
100,000x faster than birthday attack (2^{80})

Fallout of attack

SVN repositories can be broken (DoS attack)

- Checking in the two SHAttered PDFs corrupts repo

Linus Torvalds misunderstands security…

(to paraphrase) GIT’s ok because we can trust everyone

https://plus.google.com/+LinusTorvalds/posts/7tp2gYWQugL

Marc Stevens & Dan Shumow (Microsoft) developed *counter-cryptanalysis tool*

Way to detect if a particular file is one half of colliding pair

Deployed at several large companies

Ongoing migration away from SHA-1 to SHA-256 / SHA-3

Applications of hashing

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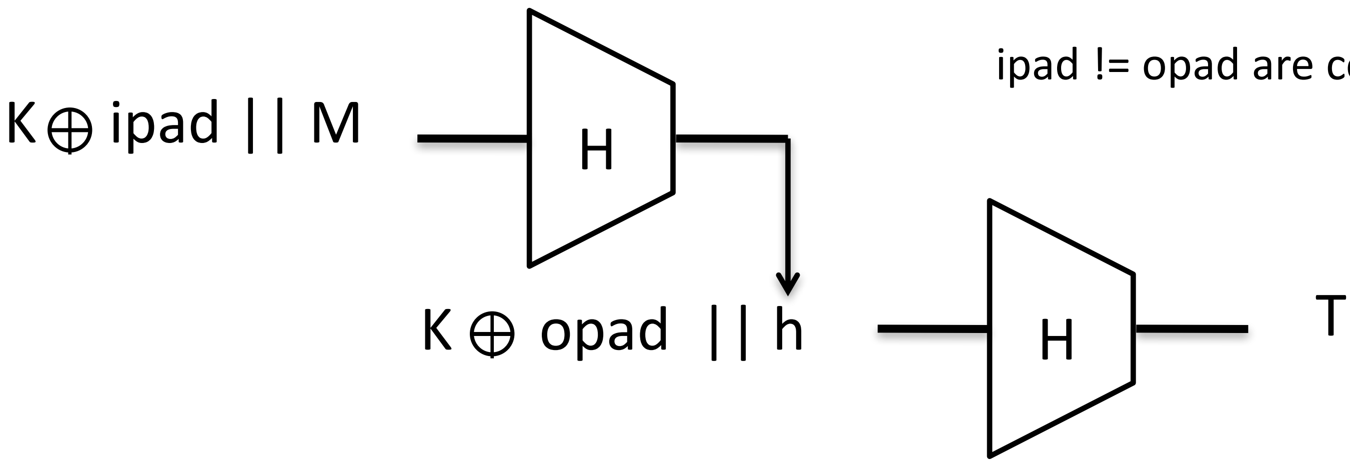
- Digital signatures (coming up later)
- Password hashing
- For message authentication codes

Building PRFs with hash functions: HMAC

Use a hash function H to build a MAC. K is a secret key

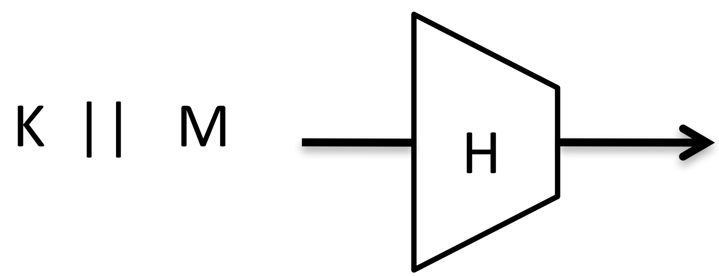


ipad != opad are constants

This is slight simplification, assuming |K| less than block length of H

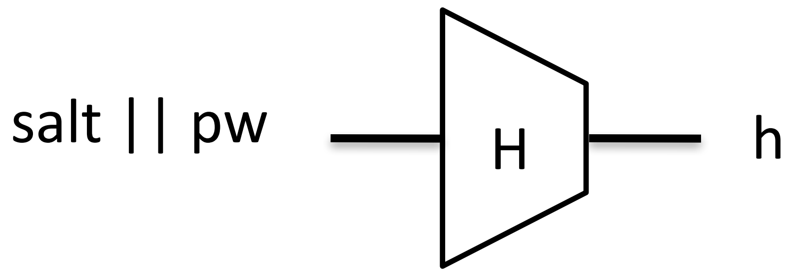
HMAC-SHA-1, HMAC-SHA-256, etc.

What's wrong with this PRF construction?



Password hashing

Password hashing. Choose random salt and store (salt,h) where:



The idea: Attacker, given (salt,h), should not be able to recover pw

Or can they?

For each guess pw':
    If H(salt||pw') = h then
        Ret pw'

Rainbow tables speed this up in practice by way of precomption. Large salts make rainbow tables impractical

```
rist@seclab-laptop1:~/work/teaching/642-fall-2011/slides$ openssl speed sha1
Doing sha1 for 3s on 16 size blocks: 4109047 sha1's in 3.00s
Doing sha1 for 3s on 64 size blocks: 3108267 sha1's in 2.99s
Doing sha1 for 3s on 256 size blocks: 1755265 sha1's in 3.00s
Doing sha1 for 3s on 1024 size blocks: 636540 sha1's in 3.00s
Doing sha1 for 3s on 8192 size blocks: 93850 sha1's in 3.00s
OpenSSL 1.0.0d 8 Feb 2011
```

```
rist@seclab-laptop1:~/work/teaching/642-fall-2011/slides$ openssl speed aes-128-
cbc
Doing aes-128 cbc for 3s on 16 size blocks: 27022606 aes-128 cbc's in 3.00s
Doing aes-128 cbc for 3s on 64 size blocks: 6828856 aes-128 cbc's in 2.99s
Doing aes-128 cbc for 3s on 256 size blocks: 1653364 aes-128 cbc's in 3.00s
Doing aes-128 cbc for 3s on 1024 size blocks: 438909 aes-128 cbc's in 2.99s
Doing aes-128 cbc for 3s on 8192 size blocks: 54108 aes-128 cbc's in 3.00s
OpenSSL 1.0.0d 8 Feb 2011
```

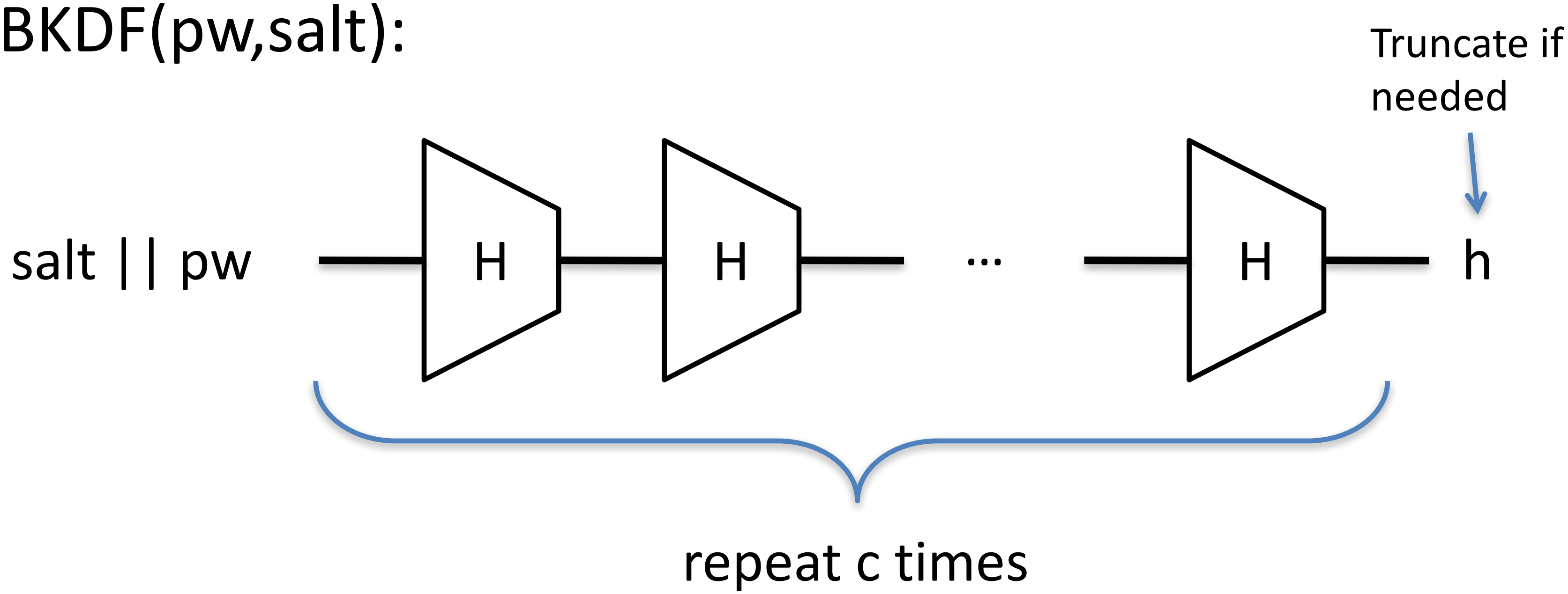
Say c = 4096. Generous back of envelope* suggests that in 1 second, can test 252 passwords and so a naïve brute-force:

6 numerical digits	10^6 = 1,000,000	~ 3968 seconds
6 lower case alphanumeric digits	36^6 = 2,176,782,336	~ 99 days
8 alphanumeric + 10 special symbols	72^8 = 722,204,136,308,736	~ 33million days

* I did the arithmetic…

Password-based Key Derivation (PBKDF)

PBKDF(pw,salt):



PKCS#5 standardizes PBKDF1 and PBKDF2, which are both hash-chain based.

Only slows down by a factor of c

scrypt, argon2:   memory-hard hashing functions

Another application of PBKDFs: PW-based encryption

<u>Enc(pw,M):</u>
salt
K <- PBKDF(pw,salt)
C <- AEnc(K,M)
Return (salt,C)

<u>Dec(pw,salt||C):</u>
K <- PBKDF(pw,salt)
M <- ADec(K,C)
Return M

Here Enc is an AE scheme (e.g., CBC + HMAC)

Summary

- Hash functions
  - Used in a variety of applications
  - Core requirement collision resistance
- Birthday attacks break them in time $2^{n/2}$ for range size n bits
- Built from compression functions, which in turn can be viewed as block-cipher-based function
- Recent demonstration of SHA-1 collision