

## Question 1

If we were to multiply normally one by one, the runtime would be  $O(n)$ .

Using the fact that  $m^n = (m^2)^{n/2}$ , we can continuously halve the exponent to compute in only  $\log n$  multiplications.

If the power is even, we reduce to  $m^n = (m^2)^{\frac{n}{2}}$

If the power is odd, we reduce to  $m^n = (m^2)^{\frac{n}{2}} \times m$

For example

$$2^8 = 2^{2^4} = 4^4 = 4^{2^2} = 16^2 = 256$$

$$2^9 = 2 \times 2^{2^4} = 2 \times 4^4 = 2 \times 4^{2^2} = 2 \times 16^2 = 2 \times 256 = 512$$