Question 2

Substituting $y = x^{100}$, we have instead $P(x) = A_0 + A_1 y + A_2 y^2$

The degree of $P(x)^2$ is four which means we will need five different equations to compute a unique answer for each coefficient. We choose these points to be {-2, -1, 0, 1, 2}.

Thus, we have:

$$P(-2) = A_0 - 2A_1 + 4A_2$$

$$P(-1) = A_0 - A_1 + A_2$$

$$P(0) = A_0$$

$$P(1) = A_0 + A_1 + A_2$$

$$P(2) = A_0 + 2A_1 + 4A_2$$

Therefore, with five large integer multiplications (RHS square), we can get these expressions

$$Let\ P(x)^2 = a + by + cy^2 + dy^3 + ey^4$$

$$P(-2)^2 = a - 2b + 4c - 8d + 16e = (A_0 - 2A_1 + 4A_2)^2$$

$$P(-1)^2 = a - b + c - d + e = (A_0 - A_1 + A_2)^2$$

$$P(0)^2 = a = {A_0}^2$$

$$P(1)^2 = a + b + c + d + e = (A_0 + A_1 + A_2)^2$$

$$P(2)^2 = a + 2b + 4c + 8d + 16e = (A_0 + 2A_1 + 4A_2)^2$$

With these equations, we can calculate $P(x)^2$ via back substitution and replace our initial $y = x^{100}$ to get $P(x)^2$ as a variable of x. Solving these equations requires multiplication BUT the multiplication will be between a large number and a smaller constant which does not count as a multiplication between two arbitrarily large numbers.