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Quiz 1

1) $\sum_{k=1}^{n} 2k+1 = n(n+2)$

Basis step: $p(1)$ – $\sum_{k=1}^{1} 2(1)+1 = 1(1+2)$

$$2+1 = 1(3)$$

LHS = RHS $\quad 3 = 3$

Inductive step: Assume $p(n)$, and prove $p(n+1)$.

$$\sum_{k=1}^{n+1} 2k+1 = (n+1)(\underbrace{n+1+2}_{(n+1)+2})$$

LHS

$$(2k+1) + 2(n+1)+1$$

$$n(n+2) + (2(n+1)+1)$$

$$= n^2 + 2n + 2n + 3$$

$$(n+1)((n+1)+2) = (n+1)(n+3)$$

$$= n^2 + 3n + n + 3$$

$$= n^2 + 4n + 3$$

LHS and RHS are thus equal.

We have proven that $p(1)$ is True, as well as $p(n) \rightarrow p(n+1)$ so $p(n+1)$ is True $\forall n$.

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Quiz $\emptyset$

$$\sum_{k=1}^{n} k(k-1) = \frac{(n-1)n(n+1)}{3}$$

Base case:

$$\underbrace{1(1-1) = 1(0) = 0}_{LHS} \qquad \frac{(1-1)1(1+1)}{3} = \frac{0(1)(2)}{3} = \underbrace{\emptyset}_{RHS}$$

Induction:

$$\sum_{k=1}^{n} k(k+1) = \frac{(n+1)(n)(n+1)}{3}$$

$$\text{LHS: } \sum_{k=1}^{n+1} k(k+1) = \sum_{k=1}^{n} k(k+1) + (n+1)(n+2)$$

substitute for $\sum_{k=1}^{n} k(k+1)$, $\frac{(n-1)n(n+1)}{3} + (n+1)(n+2)$

$$\frac{(n+1)(n)(n+1) + 3(n+1)(n+2)}{3} = \frac{(n+1)(n-1)(n+3)(n+2)}{3}$$

$$\frac{(n+1)(n^2+2n+6)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$$

$$\frac{n(n+1)(n+2)}{3} = RHS \quad \square$$

we have proven via mathmatical induction that the RHS is equal to the LHS, thus proving $p(n) \to p(n+1)$, making this True $\forall n$.