

Exam_Review_2

Recursion

Requirements: be able to write simple recursive functions

Definition: defining a problem in terms of itself

A function that calls upon itself

Example:

```
// power function C++
unsigned int pow(int x, int n){
    if(n==0){return 1;}
    else if(n<0){
        return 1/(x * pow(x, n-1));
    }
    else {
        return (x * pow(x, n-1));
    }
}

// factorial function
int fact(int x){
    if(n==1){return 1;}
    else return x*(fact(x-1));
}
```

Math Review

Induction: We need to prove $n = 1$, as well as the LHS implies the RHS when $n = n+1$. Prove $P(n) \Rightarrow P(n+1)$.

Example: For all natural numbers n , the sum of the first n natural numbers is equal to $(n \cdot (n + 1)) / 2$.

Basis Step: $n = 1$

LHS = 1

$$\text{RHS} = (1 \cdot (1 + 1)) / 2 = 2 / 2 = 1$$

Inductive Hypothesis: Assume that the statement is true for some arbitrary positive integer n .

$$1 + 2 + 3 + \dots + n = (n \cdot (n + 1)) / 2$$

Inductive Step: Prove that the statement is also true for $k + 1$.

$$1. \quad 1 + 2 + 3 + \dots + k + (k + 1)$$

$$2. \quad (k \cdot (k + 1) + 2(k + 1)) / 2$$

$$3. \quad = (k^2 + k + 2k + 2) / 2$$

$$4. \quad = (k^2 + 3k + 2) / 2$$

$$5. \quad = ((k + 1) \cdot (k + 2)) / 2$$

C++ Basic Concepts

Member 's

| Name | Definition |
|-----------|--|
| Public | Accessible from anywhere, assuming the object is within scope |
| Private | Accessible only from within the class scope |
| Protected | The same protections as private, however the this data is available within derived classes as well |

Parameter Passing Methods

| Name | Desc | Modify | Issue | Benefit |
|---------------------|--|-----------------------------------|------------------|--------------------------------------|
| Pass by value | A copy is passed to function | Does not affect original argument | Overhead of Copy | |
| Pass by reference | Passes reference to argument | Directly affects original | | Avoids copy |
| Pass by Pointer | Passes pointer to actual argument | Directly affects original | | Doesn't copy and allows for null ptr |
| Pass by Const Ref | Passes const ref to argument | Cannot modify Value | | Avoid Copy |
| Pass by Const Value | Similar to pass by value | Cannot modify argument | | Ensures unchanged |
| Pass by Rvalue | Transfer of ownership or modification of temp object | Efficient operations | | |

Examples

```
void passByValue(int x) {  
    x = 10; // Changes to x won't affect the original value  
}  
  
void passByReference(int& x) {  
    x = 10; // Changes to x affect the original value  
}  
  
void passByPointer(int* ptr) {  
    *ptr = 10; // Changes to the value pointed to by ptr affect the  
original value  
}  
  
void passByConstReference(const int& x) {  
    // Cannot modify x, but avoids copying for large objects  
}  
  
void passByConstValue(const int x) {  
    // Cannot modify x  
}  
  
void moveSemantics(int&& x) {  
    // x is an rvalue reference  
}
```

Return Passing

| Name | Desc | Use |
|----------------------------|--|--|
| Return by Value | returns a copy of value or object | Returning small simple datatypes/object |
| Return by Reference | returns a reference to value or object | Modify value outside of function |
| Return by Pointer | returns a pointer to value or object | Return pointer/indicate potential failure |
| Return by Const Reference | returns const reference to value or object | Returning values that should/cannot be modified |
| Return by Rvalue Reference | returns a r value reference to value or object | Transferring ownership, or efficiently returning temporary objects |
| Return by struct | Allows you to return multiple values | touple / struct |

Examples

```
int returnByValue() {  
    return 42; // Returns a copy of the integer 42  
}  
  
int& returnByReference(int& x) {  
    return x; // Returns a reference to the integer x  
}  
  
int* returnByPointer(int* ptr) {  
    return ptr; // Returns a pointer to the integer pointed to by  
ptr  
}  
  
const int& returnByConstReference(int& x) {  
    return x; // Returns a constant reference to the integer x  
}  
  
int&& returnByRvalueReference() {  
    return std::move(someTempObject); // Returns an rvalue reference  
}  
  
std::tuple<int, double> returnByTuple() {  
    return std::make_tuple(42, 3.14); // Returns a tuple with two  
values  
}
```

Generic Programming

Template Functions: represents a type parameter that is filled in with a specific data type when you use the function.

```
template <typename T>  
T maximum(T a, T b) {  
    return (a > b) ? a : b;  
}
```

The above function creates an template T and allows for passing of:

1. Objects

2. Primitive Types (int, char, etc)
 3. Pointers
 4. References
 5. Enums
 6. Arrays
 7. Function Pointers
 8. Member Function Pointers
 9. ETC
-

Algorithm Analysis

Definition: Algorithm analysis is the process of evaluating the efficiency and performance of algorithms. It helps determine how an algorithm's execution time and resource usage (e.g., memory) grow as the size of the input data increases. It provides insights into the algorithm's scalability and helps make informed choices about which algorithm to use for a particular problem.

| Type | Description | Represents |
|-----------|---|--------------------------------|
| Big O | Stating that the algorithms running time is at most proportional to a function of the input size | Upper bounds |
| Big Omega | Stating that the algorithms running time is at least proportional to a function of the input size | Lower bounds |
| Big Theta | Stating that the algorithms running time grows at the same rate as a function of the input size | Average (Both Upper and Lower) |

Order of Important Functions

| Order | Time Complexity | Name |
|-------|-----------------|-------------------|
| 1 | $O(1)$ | Constant time |
| 2 | $O(\log n)$ | Logarithmic time |
| 3 | $O(n)$ | Linear time |
| 4 | $O(n \log n)$ | Linearithmic time |
| 5 | $O(n^2)$ | Quadratic time |
| 6 | $O(2^n)$ | Exponential time |
| 7 | $O(n!)$ | Factorial time |

Determining Complexity of Functions/Algorithms

1. Count the basic operation in the algorithm

2. Express the count as a function of the input size (use n)
3. Simplify the function if plausible
4. Analyze worst case (Big O), average case (Big Θ), and the best case (Big Ω).

A Few Data Structures

Prototypes: refer to the high-level, abstract descriptions or blueprints of data structures.

| Structure Name | Operations | Properties | Use Cases | Complexity |
|----------------|-------------------------------------|---------------------------------------|--|--|
| Stack | push, pop, top, is_empty | LIFO (Last-In-First-Out) | Function call tracking, expression evaluation | Typically $O(1)$ |
| Vector | push_back, pop_back, size, capacity | Resizable, order collection | Dynamic lists, array like behavior | Push and Pop back $O(1)$ |
| Tree | insert, delete, search, traversal | Hierarchical, organized structure | Representing hierarchical data, searching, sorting | Depends on the specific type of tree (e.g., $O(\log n)$ for balanced binary search trees). |
| Array | Indexing, insertion, deletion | Ordered collection of elements | Sequential access data storage | Indexing is $O(1)$, but insertion/deletion can be $O(n)$ in the worst case. |
| Queue | enqueue, dequeue, is_empty | FIFO (First-In-First-Out) order | Task scheduling, breadth-first search | $O(1)$ for enqueue and dequeue. |
| Linked List | insert, delete, search, traversal | Dynamic, node-based structure | Efficient insertion/deletion, memory management | Depends on the specific type, but can be $O(1)$ for some operations. |
| Hash Set | insert, erase, find | Unordered collection with fast access | Fast lookup, eliminating duplicates | $O(1)$ average case for most operations, but can be slower in case of collisions. |

Doubly Linked List Implementation

C++

```
#include <iostream>

template <typename T>
class DoublyLinkedList {
private:
    struct Node {
        T data;
        Node* next;
        Node* prev;
        Node(const T& value) : data(value), next(nullptr),
prev(nullptr) {}
    };

    Node* head;
    Node* tail;

public:
    DoublyLinkedList() : head(nullptr), tail(nullptr) {}

    // Insert an element at the end of the list
    void insert(const T& value) {
        Node* newNode = new Node(value);
        if (head == nullptr) {
            head = newNode;
            tail = newNode;
        } else {
            tail->next = newNode;
            newNode->prev = tail;
            tail = newNode;
        }
    }

    // Erase an element with the specified value
    void erase(const T& value) {
        Node* current = head;
        while (current) {
            if (current->data == value) {
                if (current->prev) {
                    current->prev->next = current->next;
                } else {

```

```

        head = current->next;
    }
    if (current->next) {
        current->next->prev = current->prev;
    } else {
        tail = current->prev;
    }
    delete current;
    return;
}
current = current->next;
}
}

// Iterator class for the doubly-linked list
class Iterator {
private:
    Node* current;

public:
    Iterator(Node* node) : current(node) {}

    T& operator*() {
        return current->data;
    }

    Iterator& operator++() {
        current = current->next;
        return *this;
    }

    Iterator& operator--() {
        current = current->prev;
        return *this;
    }

    bool operator!=(const Iterator& other) {
        return current != other.current;
    }
};

```



```

Iterator begin() {
    return Iterator(head);
}

Iterator end() {
    return Iterator(nullptr);
}
};

```

Double-ended Queue

```

#include <iostream>
#include <stdexcept>

template <typename T>
class Deque {
private:
    T* elements;
    int capacity;
    int size;
    int front;
    int back;

public:
    Deque(int initial_capacity = 10) : capacity(initial_capacity),
size(0), front(0), back(0) {
        elements = new T[capacity];
    }

    ~Deque() {
        delete[] elements;
    }

    bool empty() const {
        return size == 0;
    }

    int count() const {
        return size;
    }

    void push_front(const T& value) {

```

```

        if (size == capacity) {
            resize();
        }
        front = (front - 1 + capacity) % capacity;
        elements[front] = value;
        size++;
    }

    void push_back(const T& value) {
        if (size == capacity) {
            resize();
        }
        elements[back] = value;
        back = (back + 1) % capacity;
        size++;
    }

    void pop_front() {
        if (empty()) {
            throw std::out_of_range("Deque is empty.");
        }
        front = (front + 1) % capacity;
        size--;
    }

    void pop_back() {
        if (empty()) {
            throw std::out_of_range("Deque is empty.");
        }
        back = (back - 1 + capacity) % capacity;
        size--;
    }

    T& operator[](int index) {
        if (index < 0 || index >= size) {
            throw std::out_of_range("Index out of bounds.");
        }
        return elements[(front + index) % capacity];
    }

    T& front_element() {

```

```

    if (empty()) {
        throw std::out_of_range("Deque is empty.");
    }
    return elements[front];
}

T& back_element() {
    if (empty()) {
        throw std::out_of_range("Deque is empty.");
    }
    return elements[(back - 1 + capacity) % capacity];
}

class Iterator {
private:
    Deque& deque;
    int index;

public:
    Iterator(D deque& d, int i) : deque(d), index(i) {}

    T& operator*() {
        return deque[index];
    }

    Iterator& operator++() {
        index = (index + 1) % deque.size;
        return *this;
    }

    bool operator!=(const Iterator& other) {
        return index != other.index;
    }
};

Iterator begin() {
    return Iterator(*this, 0);
}

Iterator end() {
    return Iterator(*this, size);
}

```

```

    }

private:
    void resize() {
        int new_capacity = capacity * 2;
        T* new_elements = new T[new_capacity];
        for (int i = 0; i < size; i++) {
            new_elements[i] = elements[(front + i) % capacity];
        }
        front = 0;
        back = size;
        capacity = new_capacity;
        delete[] elements;
        elements = new_elements;
    }
};

```

Complexity of key methods:

- Accessing elements by index (`[]`): $O(1)$
- Accessing front or back elements (`front_element` and `back_element`): $O(1)$
- Pushing and popping elements from the front or back (`push_front` , `pop_front` , `push_back` , and `pop_back`): $O(1)$ amortized (