Notes_5

P logically implies Q is written as P ⇒ Q, which means any of the following

1. Every assignment of truth values to the atomic proposition p, q, r that makes P T also makes Q T
2. P → Q is always T, regardless of the truth values assigned to the atomic propositions
3. P → Q is a tautology
4. From a truth table

p	q	r	.....	P	Q	P→Q
T	T	T		F	T	T
T	T	F		T	T	T
T	F	T		F	F	T
T	F	F		F	F	T
F	T	T		F	F	T
F	T	F		T	T	T
F	F	F		T	T	T

P is logically equivalent to Q written P ≡ Q ∨ P <=⇒ Q
means P ⇒ Q ∧ Q ⇒ P

1. Every assignment of truth values on atomic proposition p, q, r, ... makes both P ∧ Q T ∨ F
2. P <-→ Q is a tautology
3. P & Q Truth table is identical

De Margon's Law

1) LHS = ¬(p ∧ q) ≡ (¬ p) ∨ (¬ q) = RHS

p	q	¬ p	¬ q	p ∧ q	¬(p ∧ q)	(¬ p) ∨ (¬ q)	LHS <-> RHS
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	T	F	T	T	T
F	F	T	T	F	T	T	T

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Columns Are Identical
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2. ¬ (p ∨ q) ≡ (¬ p) ∧ (¬ q)

> *"Direct Proof of LHS ⇒ RHS"*

Assume LHS: ¬(p ∨ q) is T then p ∨ q is F, meaning both p is F ∧ so is q. Then both ¬ q is T ∧ ¬ p T. Theref∨e, ¬ p ∧ ¬ q is T. Hence RHS is T.

> *So, LHS ⇒ RHS*

Distributive Laws

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Algebraically:
$a \cdot (b + c) = ab + ac$
$a + (b \cdot c)! = (a + b)(a + c)$

Direct Proof:
$p \wedge (\neg q \vee r) \Rightarrow (p \wedge q) \vee (p \wedge r)$

- Assume LHS is T
- Therefore both p is T ∧ q ∨ r is T
- Then either q is T ∨ r is T
- In case (1), p ∧ q is T which means (p ∧ q) ∨ (p ∧ r) is T so RHS is T
- In case (2), p ∧ r is T which means (p ∧ r) is T so RHS is T

> *So, LHS ⇒ RHS*