

Hash Table

Hash Tables support insert, remove and search in $O(1)$ time.

🔥 Important

Idea Behind Hash Table: $O(1)$ is the worst case for insert, remove, search

Issues that need to be solved

- Key may or may not be an integer
 - Need to use a function to map the key into an integer. This part of the functionality of a hash function. To hash the key into an index.
- Key space can be infinite
 - **Examples:** The key is a string of any length
 - Need to restrict the hash function to return a small value in order to index into the hash table
- Hash typically map a fairly large number
- **Use a hash function to map key:** integer or non integer to a relatively small integer as the index to the hash table
- To get good performance we must also use the hash function to evenly map keys into different indices of the hash table

Hashing

- Data items stored in an **array** of some fixed size
 - Hash table
- Search performed using some part of the data item
 - key
- Used for performing insertions, deletions, and finds in constant average time $O(1)$
- Operations requiring ordering information not supported efficiently

0	
1	
2	
3	john 25000
4	phil 31250
5	
6	dave 27500
7	mary 28200
8	
9	

- The array has many unused entries.
- The array does not start from index 0.
- The index of an data item is computed from the data item.

Applications of Hash Tables

- Comparing search efficiency of different data structures:
 - Vector, list: $O(N)$
 - Binary search tree: $O(\log(N))$
 - Hash table: $O(1)$
- C++ STL: `std::unordered_map`, `std::unordered_set`
- Compilers to keep track of declared variables
 - Symbol tables
- Game programs to keep track of positions visited
 - Transportation table
- On-line spelling checkers

Hashing Functions

- Map keys to integers
- `Hash(key) = Integer`
- Evenly distributed index values
 - Even if the data is not evenly distributed
- **Assumptions:**
 - K: an unsigned 32 bit int
 - M: the number of buckets (the number of entries in a hash table)

- **Goal:**
 - If a bit is changed in K all bits are equally likely to change for Hash(K)
 - So that items evenly distributed in hash table

Simple Function

- `Hash(K) = K % M`
- Where M is of any integer value
- Values of K may not be **evenly distributed**, however `Hash(k)` must be **evenly distributed**.
- **If** M = 10, K = 10, 20, 30, 40
- **Then** `K % M = 0, 0, 0, 0, 0 ...`

Another Example

- `Hash(K) = K % P`
- Where P is Prime
- **Suppose** then P = 11, K = 10, 20, 30, 40
- `Then K % P = 10, 9, 8, 7`
- **A well designed hash table always has a prime number of entries**

Hashing a Sequence of Keys

- $K = \{K_1, K_2, \dots, K_n\}$
- `Hash("test") = 98157`
- Design Principles
 - Use the entire key
 - Use the ordering information

Use the Entire Key

```

unsigned int Hash(const string& Key){
    unsigned int hash = 0;
    for(string::size_type k = 0; k != K.size(); ++k)
    {
        hash = hash ^ Key[k]; // Xor
    }
    return hash;
}
// Problem: Hash("ab") == Hash("ba")

```

C++

Use the Ordering Information

```

unsigned int Hash(const string &Key){
    unsigned int hash = 0;
    for(size_type j = 0; j < Key.size(); ++j)
    {
        hash = hash ^ Key[j];
        hash = hash * (j % 32);
    }
    return hash;
}

```

C++

Better Hash Function

```

unsigned int Hash(const string& S){
    size_type i;
    long unsigned int bigval = S[0];
    for(i = 1; i < S.size(); ++i){
        // low16 * magicNumber
        bigval = ((bigval & 65535) * 18000)
            + (bigval >> 16) // high16
            + S[i];
    }
    bigval = ((bigval & 65535) * 18000) + (bigval >> 16);
    // bigval = low16 * magicNumber + high16

    // return low16
    return bigval & 65535;
}

```

C++

Even if the function just returned 0 it would still be a legit hash function. Replacing the hash function in any hash table with this would still work but the $O(1)$ complexity may not be maintained.