Notes_6

Direct Proof of P ⇒ Q

1. Assume LHS is T
2. Argue RHS is T
   $p \to q \equiv \neg q \to \neg p$

Indirect Proof (By Contrapositive)

1. Assume RHS is F
2. Argue LHS is F

Therefore, both (p or q) $\oplus$ ($p \wedge q$) is T $\wedge$ $\neg$ p is T. Then, P is F. Therefore, p ^ q is F. Then p or q is T. Means q is T which means q is T since p is F. So the implication is valid.

Then $p \wedge r$ is T. Therefore both $p \wedge r$ are T. So, p or q is T (since p is T). (p or q) $\oplus$ ($p \wedge r$) is F. So LHS is F. So the implication is valid.

Assume LHS is T

1. p or q T $\wedge$ ($p \wedge r$ | F)
2. p or q | T | $\wedge$ $p \wedge r$ | F

Both p or r is T $\wedge$ $p \oplus q$ is T. Want to show $q \to r$ is T. So assume q is T. Since $p \oplus q$ is T, p is F. Since p or r is T, so r is T which is what we needed to show. So RHS is T.

> *Implication is Valid*

Assume RHS is F. So $q \to r$ is F. means q is T $\wedge$ r is F.

1. p T: Since q is also T $p \oplus q$ is F then LHS is F
2. p F: Since r is also F p or is also F LHS is F