Quiz 3

1)

p	q	r	$(p \otimes q) \oplus r$	$p \to q$	LHS	RHS	LHS $\Rightarrow$ RHS
T	T	T	F	T	F	T	T
T	T	F	F	T	F	T	T
T	F	T	F	F	F	F	T
T	F	F	T	F	F	F	T
F	T	T	F	T	F	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	F	F
F	F	F	F	T	F	F	T

No, this is not a valid implication because for values pF, qF, ~~q~~T the implication is False

2) prove by verbal argument that $p \wedge (q \oplus r) \Rightarrow (p \wedge q) \vee (p \wedge r)$

If we begin w/ the assumption $p \wedge (q \oplus r)$, then distribute $\wedge$ (and) for $\oplus$ (xor), we are left w/ $(p \wedge q) \oplus (p \wedge r)$. We may then substitute $\oplus$ for $(\wedge \neg)$ resulting in $(p \wedge \neg q) \vee (\neg(p \wedge r))$. Then by applying Demorgans law ~~[illegible]~~ to $\to$ $\neg(p \wedge r)$ we are left with,

$(p \wedge q)(\neg p \vee \neg r)$, then distributing over the proposition again, $(q \vee (\neg p \wedge \neg r))$, simplifying to, $q \vee (\neg p \wedge \neg r)$. This is equivelent to the statement, $(p \wedge q) \vee (p \wedge r)$, meaning the implication is true.

<u>Reversal</u>

Assuming $(p \wedge q) \vee (p \wedge r)$, considering $p \wedge q$ is equivelent to $p \wedge (q \oplus r)$ ~~[illegible]~~, we can see that both $p \wedge (q \oplus r)$ are results of $(p \wedge q) \vee (p \wedge r) \Rightarrow p \wedge (q \oplus r)$. We can then state that they are logically equivelent.

Quiz 3 Aiden Allen

3) are these two sets equal or not equal: $\emptyset, \{\emptyset\}$

no, $\emptyset$ represents an empty set containing no elements, compared to $\{\emptyset\}$ which is a set containing an empty set.

4) prove $\sum_{i=1}^{n} (2i - 1) = n^2$ via induction.

Basis Step

$(2(\emptyset) - 1) = 1^2$ ✓
$= 2 - 1 = 1$

A) The value I should be used to prove the basis step.

b) that the base cases are equivelent

Inductive Step $n+\cancel{1}$, proof

$(n+1)^2 = (n^2 + n + n + 1) = 2n + 1 + n^2$ ✓

$(2(n+1) - 1) + \sum (2i - 1)$

$(2n + 2 - 1) + n^2$

$2n + 1 + n^2$ ✓

we can see that both the LHS and RHS are equal.

c) $\sum_{i=1}^{K} (2i - 1) = K^2 \rightarrow \sum_{i=1}^{K+1} (2i - 1) = (K+1)^2$

D) The proof has been confirmed seeing as both the basis steps result in 0, and the $K \rightarrow K+1$. $\square$