Constrained Model Predictive Control for Isolated Power System

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***Abstract*— A constrained dual-mode Linear Quadratic Model Predictive Control (LQ-MPC) was designed to adjust the reference power, Δp^{ref} in response to frequency deviation and meet the desired specification. Relevant MPC parameters were evaluated to ensure guaranteed stability and recursive feasibility.**

I. Introduction

This report aims to design, implement and critically evaluate a Linear Quadratic Model Predictive Control, LQ-MPC using Finite-Receding Horizon dual-mode approach for a linear SISO isolated power system shown in Figure 1. The designed MPC regulates the reference power, Δp^{ref} in response to frequency deviations. Additionally, the report briefly discusses the effects of the tuning parameters including horizon length, *N* and cost matrices, *Q* & *R* with respect to the performance and guaranteed stability and recursive feasibility of the MPC. MATLAB is used to simulate results.

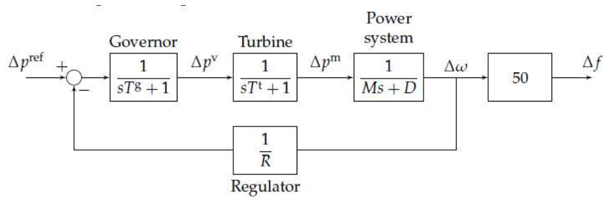

Figure 1 - Isolated power system under frequency control [1]

II. State-Space Model

The continuous-time space model was transformed to Discrete Linear Time-Invariant (LTI) representation shown in Equation 1 using '*zoh*' and MATLAB function, *c2d*.

$$x(k+1) = A\,x(k) + B\,u(k)$$
$$y(k) = C\,x(k) + D(k) \quad (1)$$

Where $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ and $x \in \mathbb{R}^m$ are the input, output and states, respectively. The state-space equation with Dynamic matrix, *A*, Control (Input) matrix, *B* and Output matrix, *C* shown in Equation 2 provides an insight into the internal interaction of the dynamic system.

$$A = \begin{bmatrix} -0.08 & 0.1 & 0 \\ 0 & -2 & 2 \\ -50 & 0 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \quad C = \begin{bmatrix} 50 & 0 & 0 \end{bmatrix} \quad (2)$$

Following assumptions were made while considering the system and designing the MPC:

- system matrices, A, B and C are perfectly known i.e., model is perfectly accurate with no uncertainty.
- No disturbances, noise, model errors and delays.
- Purely deterministic control problem.

The rank of the matrix, (A, B) and (A, C) being three with state, x_3 and x_1 having an input and output signal respectively, indicates the system is both, reachable and observable.

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III. Designing the LQ-MPC Controller

A. Problem Statement

The designed LQ-MPC controller aims to minimize the cost function shown in Equation 3.

$$\mathbb{P}_N(x(k)) = \min_{u(k)} \sum_{i=0}^{N-1} [\mathbf{x}^T(k+i|k)Q\mathbf{x}(k+i|k) + \mathbf{u}^T(k+i|k)Ru(k+i|k)] + \mathbf{x}^T(k+N|k)P\mathbf{x}(k+N|k) \quad (3)$$

The optimal solution is indicated by the convergence of $x \to x_{ss}$ and $u \to u_{ss}$. This is subjected to, for $i = 0, \dots N-1$

$$P_x x(k+i|k) \le q_x - P_x x_{ss}$$
$$P_u u(k+i|k) \le q_u - P_u u_{ss}$$
$$P_{x_N}\, x(k+N|k) \le \tilde{q}_{x_N} \quad (4)$$

Cost matrices, *Q* and *R* in conjunction with *N,* the horizon length was tuned to achieve the performance of the controller while *P* indicates the terminal cost.

Equation 5 and Equation 6 shows the inequality constraints implemented on output, $|\Delta f|$ and input, $|\Delta p^{ref}|$, respectively.

$$\begin{bmatrix} C \\ -C \end{bmatrix} x(k+i|k) \le \begin{bmatrix} y_{max} \\ -y_{min} \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} I \\ -I \end{bmatrix} u(k+i|k) \le \begin{bmatrix} u_{max} \\ -u_{min} \end{bmatrix} \quad (6)$$

B. LQ-MPC Design

For stable and feasible MPC with uniquely optimal and convergent solution, matrix *Q* and *R* must be positive semi-definite ($Q \gtrsim 0$) and positive definite ($R \succcurlyeq 0$), respectively, with $(Q^{1/2}, A) \to observable$ and the system being reachable.

The cost function shown in Equation 3 was processed further to obtain a *Vectorized* form illustrated in Equation 7 with $x(k)$ eliminated ($\boldsymbol{x} = x - x_{ss}$ & $\boldsymbol{u} = u - u_{ss}$).

$$\min_{u(k)} \frac{1}{2}\boldsymbol{u}^T(k)Hu(k) + \boldsymbol{x}^T(k)L^T\boldsymbol{u}(k) + \boldsymbol{x}^T(k)M\boldsymbol{x}(k) \quad (7)$$

subject to $P_c\,\boldsymbol{u}(k) \le q_c + S_c\boldsymbol{x}(k)$

P_c, q_c, S_c depicts the stacked inequality constraints with the prediction matrix, $x = F\,x(k) + G\,u(k)$.

$$F = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} \quad G = \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1} & A^{N-1} & \cdots & B \end{bmatrix} \quad (8)$$

Equation 9 depicts the form of the optimal control sequence to the cost function stated in Equation 3, solved via MATLAB.

$$\{u(k|k), \dots, u(k+N-1|k)\} \;\&\; \{x(k+1|k), , x(k+N|k)\} \quad (9)$$

The control law of mode-1 was non-linear time-invariant making eigenvalues analysis needless to suggest stability and required the need to satisfy the Lyapunov function, Equation 11 to guarantee stability.

$$u(k) = u^0(k) \qquad (10)$$

Non-linear control, u^0 was obtained from the first row of the MATLAB function, *'quadprog'* since MPC utilizes the receding-horizon principle.

Deadbeat mode-2 control gain, K was implemented using MATLAB function, *acker* with the desired eigenvalues within the unit circle, $|\lambda| < 1$. Desired poles (eigenvalues) of [0 0 0] was designated to ensure a dead-beat response and better convergence of the states to the origin. This also guaranteed, the eigenvalues of the closed-loop system, $(A + BK)$ to be within a unit circle thus achieving stability.

Furthermore, it was ensured that the terminal cost, P satisfied the Lyapunov function shown in Equation 11 and (A, B) was stabilizable for the controller to be stable and the solution to the problem statement (Equation 3) to be feasible.

$$(A + BK)^T P(A + BK) - P + (Q + K^T RK) = 0 \qquad (11)$$

C. Brief LQ-MPC Implementation procedure

Following procedure was followed to implement the LQ-MPC while ensuring stability and feasibility within a certain region.

<u>Offline</u> [Initial Condition x = [0 0 0.29]

1. $(A, B) \rightarrow stabilizable, Q = C^T C, R = 100$
2. Continuous to Discrete-Time domain via '*c2d'*.
3. Implement the LQ-MPC illustrated in Equation 7.
4. Obtain the deadbeat mode-2 gain, $K \rightarrow$ *'acker'*.
5. Compute terminal cost, P via Lyapunov Equation
6. Extend mode-1 by n, *(size(A,1))*
7. Deduce the prediction matrices, F and G
8. Calculate the cost matrices, H, L & M
9. Check the stability of the closed-loop system.
10. Compute the constraint matrices, P_c , q_c , & S_c

<u>On-Line</u>

11. Calc $\boldsymbol{x}(k)$ & $\boldsymbol{u}(k)$
12. Compute constraint using new $\boldsymbol{x}(k)$ & $\boldsymbol{u}(k)$.
13. Compute Equation 7
14. Deduce the solution to the LQ-MPC *Problem Statement,* Equation 3 via *quadprog.*
15. Check the feasibility via the third output argument of *quadprog* < 1.
16. Apply the control input, u to the closed-loop system, $x = Ax + B\boldsymbol{u}$ & $ys(:, k) = C * x_{ss}$
17. Repeat from step 11.

D. Desired Specification of LQ-MPC

- Guaranteed stability
- Satisfy constraints $|\Delta p^{ref}| \leq 0.5$ & $|\Delta f| \leq 0.5$
- Large Operating region for Δf
- Frequency settle to $|\Delta f| \leq 0.01\ Hz\ in\ 2\ seconds$

NOTE: Large *Cond(H),* matrix singular, λ approaches 0.

IV. RESULTS

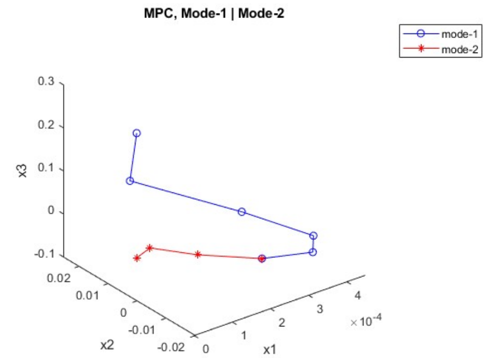

Figure 2 - Mode 1 and Mode 2 tracking: converges to the origin but never really reaches it.

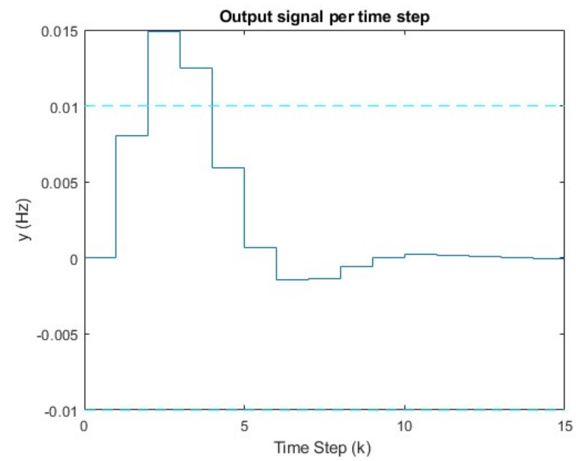

Figure 3 - Output Signal Change per time step: within hard constraint

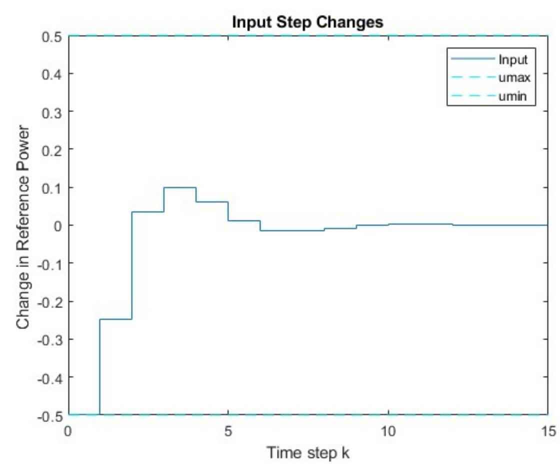

Figure 4 - Input Signal Change per time step: within hard constraint

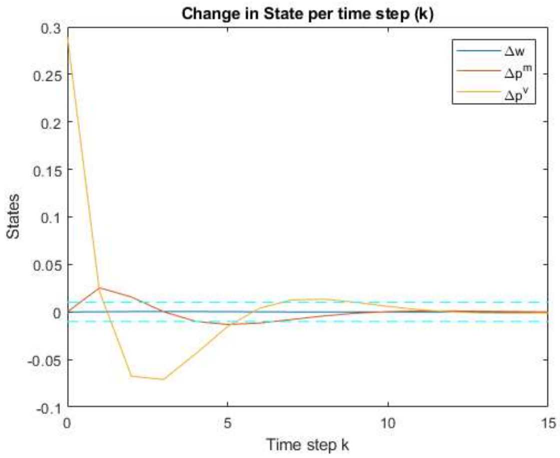

Figure 5 - Change in States per time step (k)

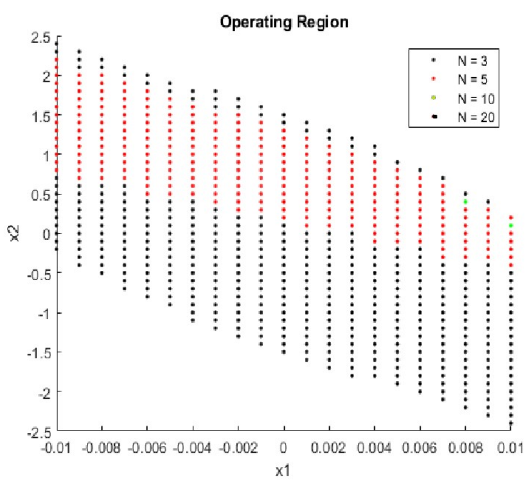

Figure 6 - Operation / Feasibility Region N = 3 | 5 |10 |20

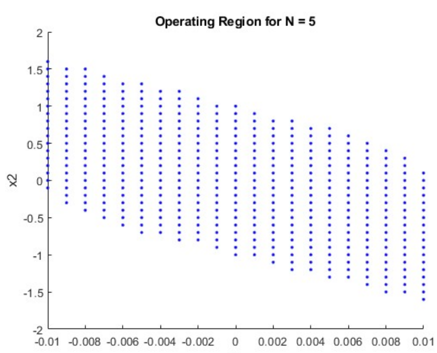

Figure 7 - Operating Region of N = 5

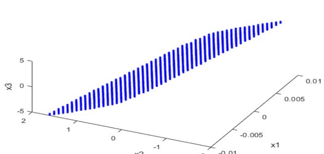

Figure 8 - 3D view of the operating region of N=5

IV. OBSERVATIONS / ANALYSIS

By implementing the trial-and-error approach, a horizon length, N of five was selected which provided a relatively smoother state trajectory shown in Figure 2 while required reasonable computation to power the simulation on a budget PC. It was observed that increasing the Horizon length, N prompted a smoother trajectory with states converging closer to the origin and improved the overall closed-loop performance of the system. A horizon of five with mode-2 implemented, generated high confidence prediction and ensured state convergence. Having a large horizon will aid in a slight improvement in convergence to the origin however, the prediction data obtained from the simulation will then lack confidence. Additionally, horizon length, N of less than three was not considered to ensure $N \geq n$, where $n = size(A, 1)$.

The operating region of states, x_1, x_2, and x_3 increased with the Horizon length, N as supported by Figure 6 due to more prediction data available at each time step however, the result generated via simulation at $N = 20$ may not be rational. Hence, the horizon of five was selected with the feasibility region shown in Figure 8. It was observed during the simulation that the boundary limits were only implemented on state, x_1 since the hard output constraint, $y = |Cx|$ where $C = [50\ 0\ 0]$ had no output signal in second and third argument. Therefore, states, x_2 and x_3 were not directly affected by the output constraint.

Matrix, Q and R were tuned to achieve the desired transient performance particularly the settling time of the output frequency being $\leq 0.01\,Hz$ within 2 seconds as depicted in Figure 3. Increasing the weighting of the matrix, R resulted in increased settling time / slow response. Hence, Q and R matrices were tuned according to the desired specification while ensuring Q and R remained positive-semidefinite and positive definite, respectively, confirming stability. Moreover, the Lyapunov function shown in Equation 11 was satisfied to guarantee stability with closed-loop eigenvalues of the system within a unit circle.

The designed MPC met the hard input and output constraint, $u = |\Delta p^{ref}| \leq 0.5$ and $y = |\Delta f| \leq 0.5$ as illustrated by Figure 3 and Figure 4, respectively.

TABLE I. TUNING PARAMETERS

Table Column Head	
Parameter	***Value***
k	[0 0 0]
R	0.25
$Q = C'C * Q\_des$	$Q\_des = 100$
N	5
Lyapunouv	Satisfied – Guaranteed Stability

V. CONCLUSION

The designed dual-mode LQ-MPC converged to the origin and met the desired specification however, the tuning can be further improved by running on HPC to obtain results quicker, allowing extra iterations. MPC is state-of-the-art controller which is most commonly used in industries after PID.

VI. REFERENCES

[1] D. P. Trodden, "ACS 6116 Advanced Control: Assignment," University of Sheffield . [Online].