UNIVERSITY OF SHEFFIELD

System Identification

Assignment

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Module Code: ACS 6129

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Relevant code can be extracted via the QR Code

Part A – Non – Parametric Methods

1.1. Single Realization of the PRBS Signal

The characteristic equation shown in Equation 1 was implemented in MATLAB Simulink to generate the autocorrelation result and periodic signal shown in Figure 1 and Figure 2, respectively.

$$(D^7 \oplus D)x = x \qquad Eq.1$$

$$Period\ of\ the\ signal = 2^n - 1 = 127 \qquad Eq.2$$

The autocorrelation of the PRBS signal shown in Figure 1 does have an overall similar outlook to the white noise autocorrelation however it does not provide a perfect approximation since the white noise has a value of zero at all time lag except at zero. This suggests that the autocorrelation of the PRBS was a bit noisy therefore, the averaging process was used to counteract the noise and to obtain a deterministic signal from the stochastic signal.

Figure 2 shows the periodic signal of the PRBS signal with the period of 127 as evident from the plot and supported by Equation 2.

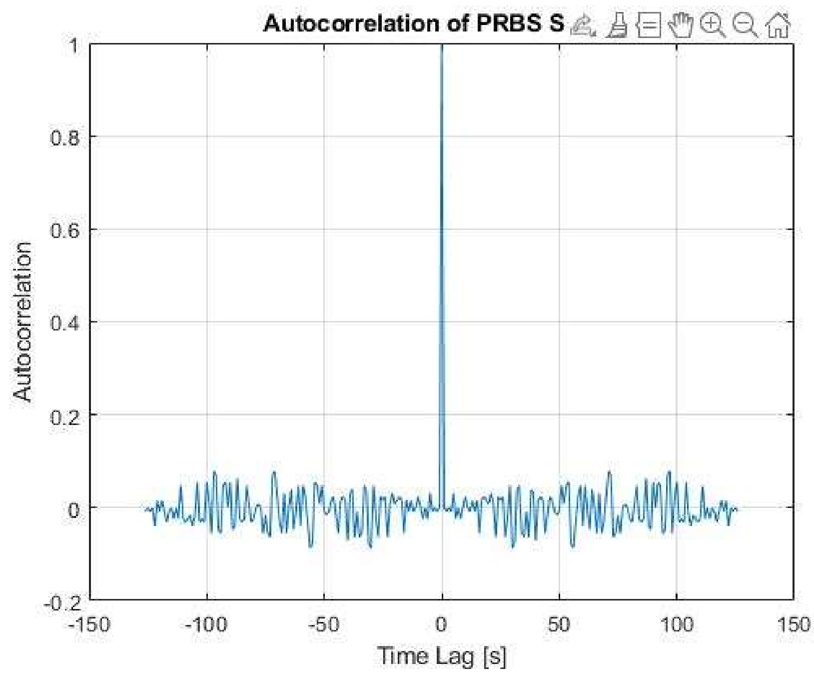

Figure 1 – Autocorrelation of single realization of the PRBS signal generated via MATLAB Simulink using Equation 1.

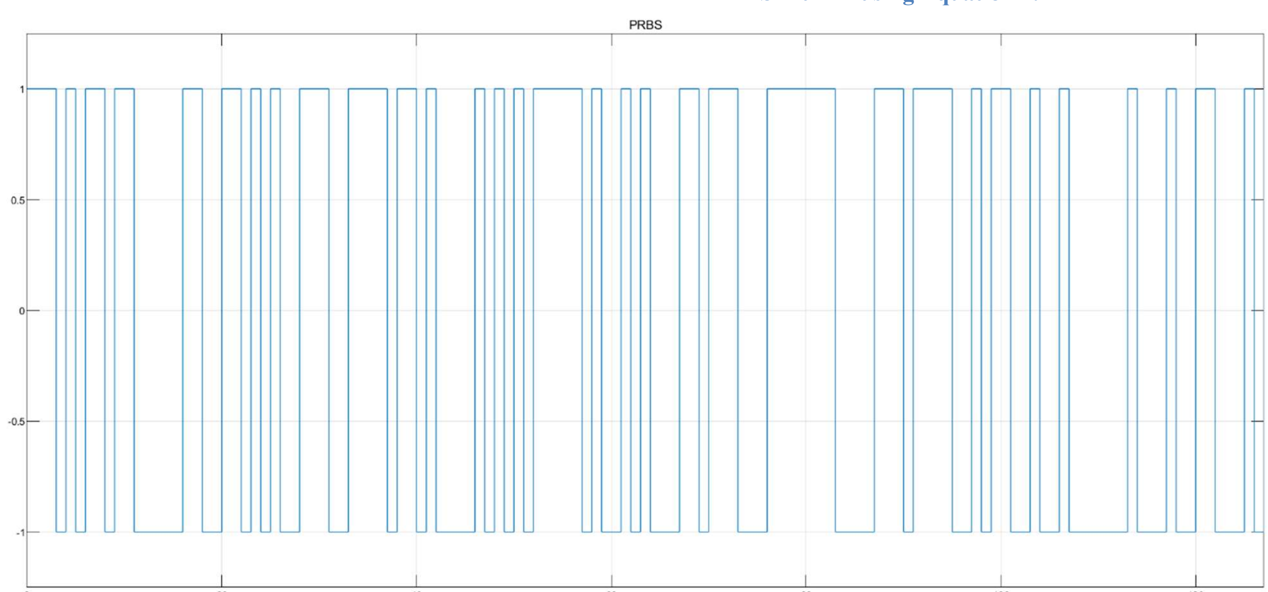

Figure 2 - Periodic PRBS Signal

1.2. Average Correlation of 50 realizations

The average correlation of 50 realizations shown in Figure 3 provides a far better approximation to the white noise than the single realization shown in Figure 1 since it almost perfectly resembles with the white noise. Figure 3 clearly shows that the autocorrelation of the PRBS signal has an autocorrelation value of zero at all time lag except at zero. Hence, it better resembles white noise.

It was observed that each realization in the ensemble was different due to the randomness in the process. Therefore, the averaging process allowed to remove the random effect/noise effect in the measured output signal.

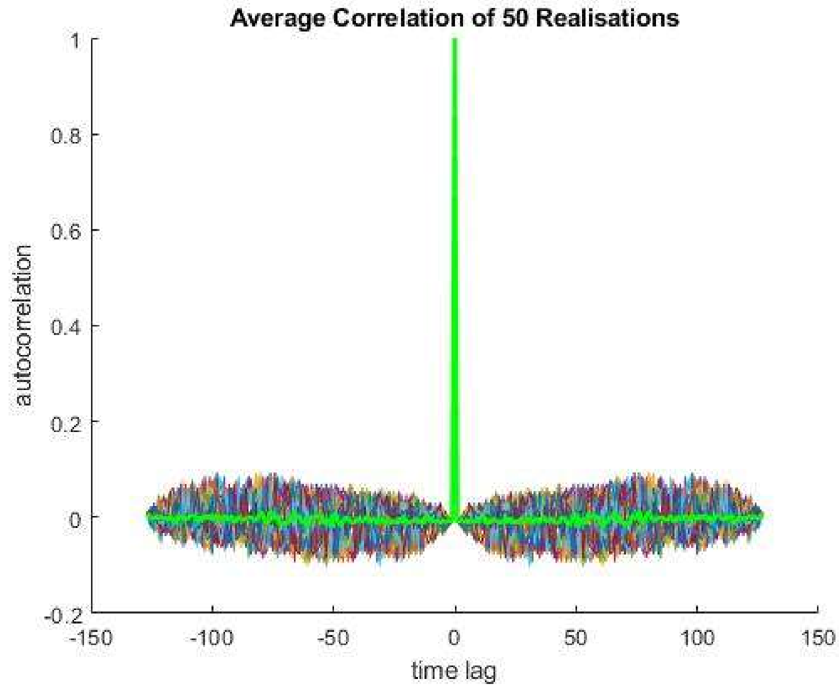

Figure 3 - Average correlation of 50 realizations shown by thick green line.

Assumptions were made during the simulation process and are listed below:

- The conditions remained the same in every realization of the process.
- The process was linear, time-invariant, stationary, and ergodic.

1.3. Step Testing

The noise was added to the system in the form of Auto-Regressive (AR) noise model which resulted in Equation 3.

$$y(k) = \frac{z^{-1}}{1-0.9z^{-1}}u(k) + \frac{1}{1-0.9z^{-1}}\epsilon(k) Eq.3$$

A different result was observed each time the system was simulated due to the stochastic properties of the system and the randomness in the noise.

The true and the estimate step response of the system with a single realization were extracted from MATLAB Simulink and are shown in Figure 4. It is apparent from the graph that with a single realization, the estimate step response did not provide a satisfactory result since it did not track the true step response. Therefore, the averaging process with 100 realizations was considered with a 95% confidence level.

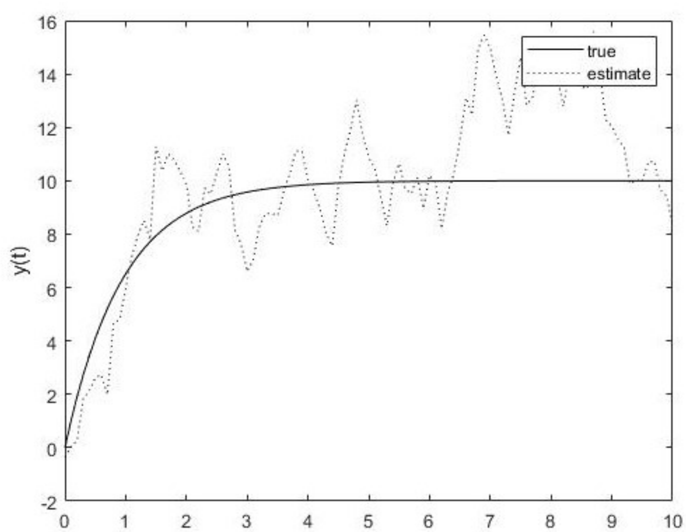

Figure 4 - True step response and the estimate step response with a single realization

Figure 5 shows the result obtained after the averaging process was used with 100 realizations and with a 95% confidence level.

The estimate step response with 100 realizations almost perfectly tracked the true step response and remained within the 95% confidence level.

The implementation of the averaging process with 100 realizations resulted in a better estimated when compared with the single run. This is because the averaging operations allowed to obtain deterministic signal/quantities from the stochastic signal.

From the observation of the simulation process, it was deduced that the averaging operation removed the random effects since the system was stationary and ergodic which improved the response.

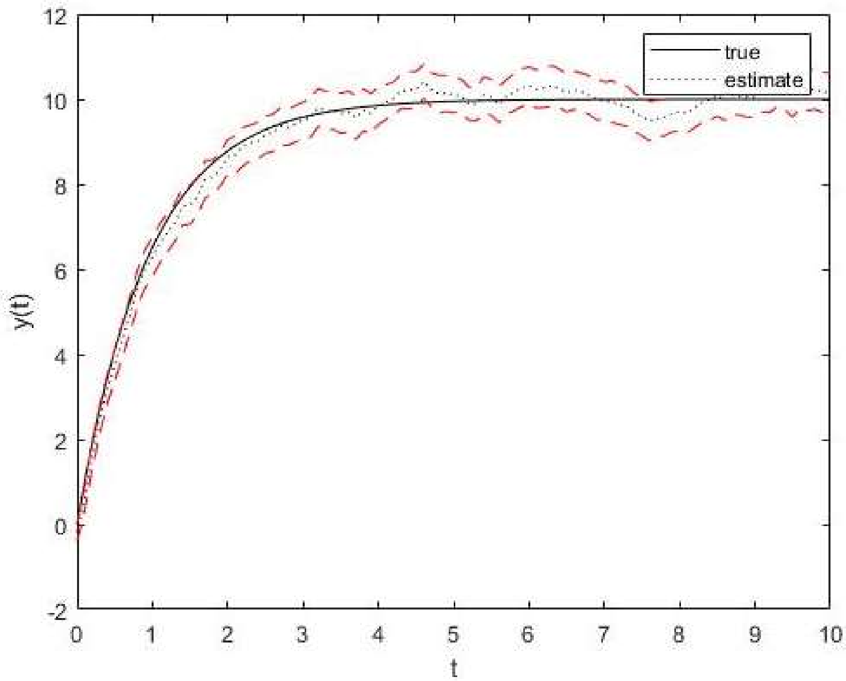

Figure 5 - Step response of the system with 100 realizations and a 95% confidence level (dashed red line)

Part B – Parametric Methods: Least Square Est.

2.1. Model parameter estimate, $\widehat{\beta}$ via Least Square Estimate of ARX Model

$$\beta = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -0.9 \\ 1.0 \end{bmatrix} \qquad Eq.4$$

$$\hat{\beta} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} -0.0707 \\ 0.6205 \end{bmatrix} \; for, N = 10 \qquad Eq.5$$

Equation 5 shows the model parameter estimate, $\hat{\beta}$ when the number of samples, N was set to 10. It is apparent that the amount of point used to obtain $\hat{\beta}$ was not enough since the estimate does not converge to the model parameter, β shown in Equation 4. Therefore, the number of points was increased to 100.

2.2. Increasing No. of points, N to improve the result.

Number of points, N was increased to 100 which allowed convergence and resulted in obtaining better model estimate parameters, $\hat{\beta}$ shown in Equation 6 that converged to the model parameter, β shown in Equation 4.

$$\widehat{\beta} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} -0.9127 \\ 1.1332 \end{bmatrix} \; for, N = 100 \qquad Eq.6$$

Table 1 - First seven residuals when N = 10 & N =100

$Residual(Y - \hat{Y}), N = 10$	$Residual(Y - \hat{Y}), N = 100$
-0.1242	0.5730
-1.0326	-0.262
0.6037	-0.224
0.0150	0.6464
0.9070	0.5875
0.1576	2.5721
-2.15	0.52

Table 1 and Figure 7 shows the residuals and autocorrelation plot, respectively of the system when N was changed to 100. The residual became closer to zero while the autocorrelation plot depicts a better approximation to white noise when N = 100. Figure 7 suggest that increasing the number of points resulted in a reduction in the noise effects and improved the variance. This is because more samples were collected in each realization.

NOTE: Only the first seven residuals are shown in Table 1, though there were nine and 99 residuals when N = 10 & 100

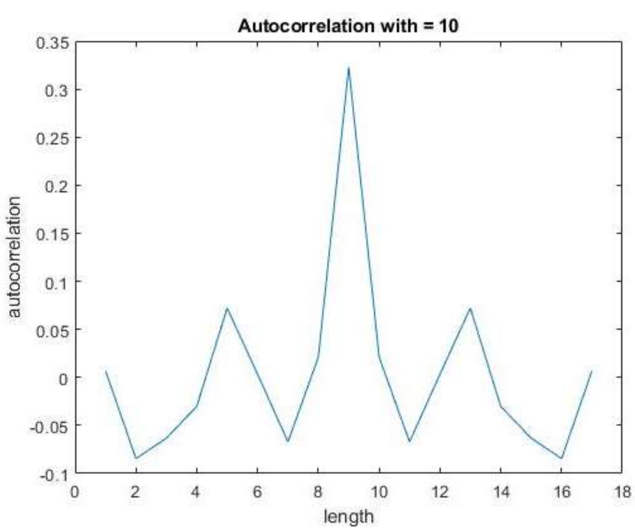

Figure 6 – Autocorrelation plot with N =10

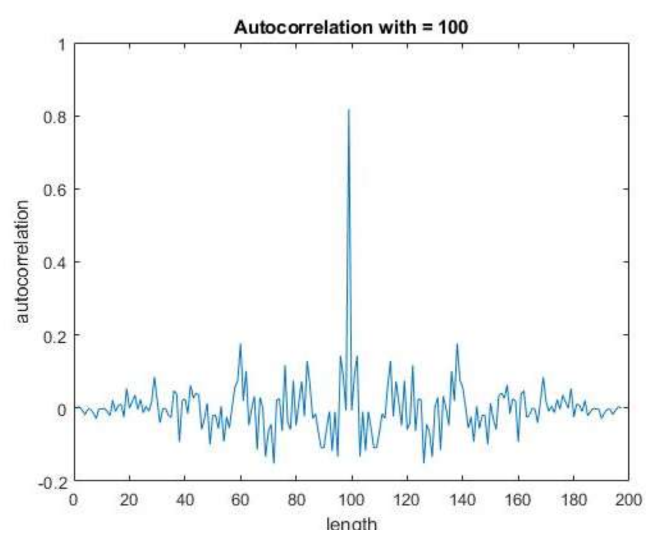

Figure 7 - Autocorrelation plot with N = 100

2.3. Simulation of Output Error Model

$$\hat{\beta} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} -0.6698 \\ 0.8029 \end{bmatrix} \; for, N = 100 \qquad Eq.7$$

$$\hat{\beta} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} -0.8214 \\ 1.3383 \end{bmatrix} \; for, N = 100 \; \& \; m = 100 \qquad Eq.8$$

$Where: m = Number\ of\ runs\ of\ the\ system$

Model estimate parameter, $\hat{\beta}$ calculated using only a single run is shown in Equation 7 which illustrates that it did not converge to the model parameter, β shown in Equation 4 and hence considered biased. Therefore, 100 number of iterations were used to calculate, $\hat{\beta}$ shown in Equation 8 which converged to model parameters.

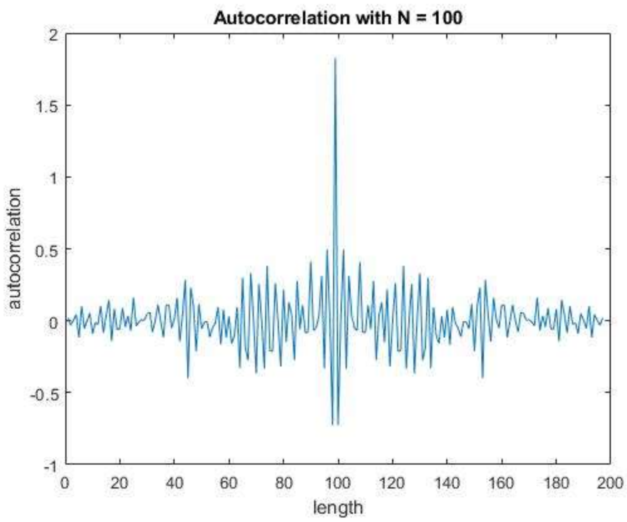

Figure 9 - Autocorrelation with N = 100

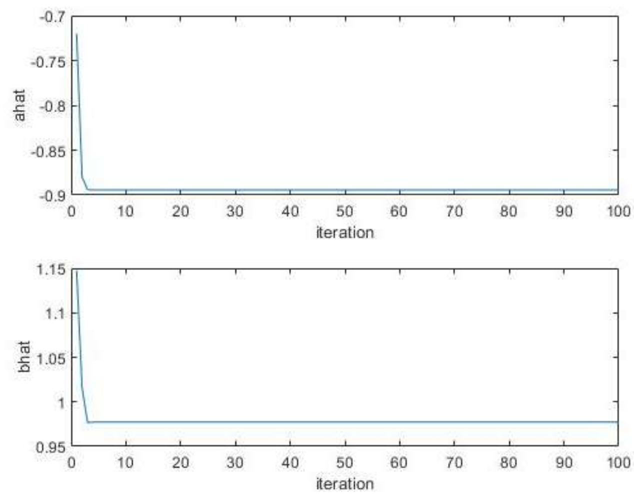

Figure 8 - Histogram of $\hat{a}$ *and* $\hat{b}$

The histogram of $\hat{a}$ and $\hat{b}$ shown in Figure 8 suggest that the model estimate parameters almost converged to -0.9 and 1, respectively. However, there was a slight difference between the model parameter, $[-0.9\ 1]$ and the model estimate parameter, $[-0.8\ 1.3]$. This can be further reduced by improving/increasing the number of point/samples, N.

It was deduced from observing the simulation procedure, that the convergence was greatly affected by the number of points, N and number of iterations/realizations, m. In other words, increasing N and m will enable further improvement in the convergence.

2.4. Comparison of Non-Parametric (Part A) and Parametric (Part B)

Parametric identification assumes ‘finite set of parameters’ with future predictions independent of the observed data while non-parametric identification assumes ‘infinite-dimensional set of parameters’, therefore making non-parametric much more flexible. (Tangirala, 2015). Comparing the result obtained in Non-parametric (Part A) and Parametric (Part B), it was deduced that the estimate step response of the non-parametric model was close to true response (within 95% confidence level) for the number of points, $N = 100$ as shown in Figure 5. While the autocorrelation of the parametric was not an ideal approximation to the white noise when $N = 100$ as supported by Figure 7. Therefore, it was concluded that non-parametric was the better model when $N = 100$.

Bibliography

Tangirala, A. K. (2015). *PRINCIPLES OF SYSTEM IDENTIFICATION Theory and Practice.* CRC Press, Taylor & Francis Group.