

# Attenuation-based Light Field Displays

## **Bachelorarbeit**

der Philosophisch-naturwissenschaftlichen Fakultät  
der Universität Bern

vorgelegt von

Adrian Wälchli

2015

Leiter der Arbeit:  
Prof. Dr. Matthias Zwicker  
Institut für Informatik und angewandte Mathematik

## Abstract

Abstract goes here

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Related Work . . . . .	1
<b>2</b>	<b>Capturing a Light Field</b>	<b>2</b>
2.1	The Plenoptic Function and the Light Field . . . . .	2
2.2	Light Field Acquisition . . . . .	3
2.3	Visualization . . . . .	4
2.4	Light Field Tomography . . . . .	5
2.5	Spectral Analysis . . . . .	5
<b>A</b>	<b>ap1</b>	<b>6</b>
A.1	apsec1 . . . . .	6
	<b>List of Tables</b>	<b>7</b>
	<b>List of Figures</b>	<b>7</b>
	<b>Bibliography</b>	<b>8</b>

## Chapter 1

# Introduction

### 1.1 Related Work

## Chapter 2

# Capturing a Light Field

### 2.1 The Plenoptic Function and the Light Field

The plenoptic function, as introduced by [AB91], is a 7D function that describes the intensity of light for every frequency, along every light ray in space, at any time. It is defined as

$$P: \mathbb{R}^3 \times [0, 2\pi) \times [0, \pi] \times \mathbb{R}^2 \rightarrow \mathbb{R}^+ \\ (x, y, z, \theta, \phi, t, \lambda) \mapsto P(x, y, z, \theta, \phi, t, \lambda),$$

where the parameters  $(x, y, z)$  are the coordinates of a point in 3D space and the angles  $(\theta, \phi)$  describe the direction of an incoming light ray at time  $t$ . The light's intensity is given for every wavelength  $\lambda$  and thus, the plenoptic function not only captures the visible frequency spectrum but all electromagnetic waves. A commonly used measure for light is the radiance, which is obtained from  $P$  by integrating over all wavelengths:  $R(x, y, z, \theta, \phi, t) = \int_{\mathbb{R}} P(x, y, z, \theta, \phi, t, \lambda) d\lambda$ .

In practice, it is impossible to acquire all the data needed to model the 7D plenoptic function and hence it is reasonable to consider only a subset of the parameters. Dropping the time parameter  $t$  in  $R(x, y, z, \theta, \phi, t)$  yields a 5D function for the radiance in a static scene. As described by [LH96], this five dimensional representation can further be reduced to four dimensions in the following way. The radiance along a line is constant in free space and so, the 5D plenoptic function holds redundant information for the points on this line. Ignoring this redundancy leads to the equivalent 4D parameterization of the ray space. [LH96] propose a parameterization by two parallel planes, as seen in figure 2.1, where the coordinates of the lines (rays) are given by the intersections with the two planes. The **4D light field**  $L(u, v, s, t)$  is therefore defined as the radiance along the line intersecting the two planes at coordinates  $(u, v)$  and  $(s, t)$ . This two plane parameterization of the light field is the most common one seen in literature, but there are many ways to choose a parameterization. For instance, one can use a plane and two angles to define each ray passing this plane, which would result in a light field  $L(u, v, \theta, \phi)$  where  $\theta, \phi \in (0, \pi)$ .

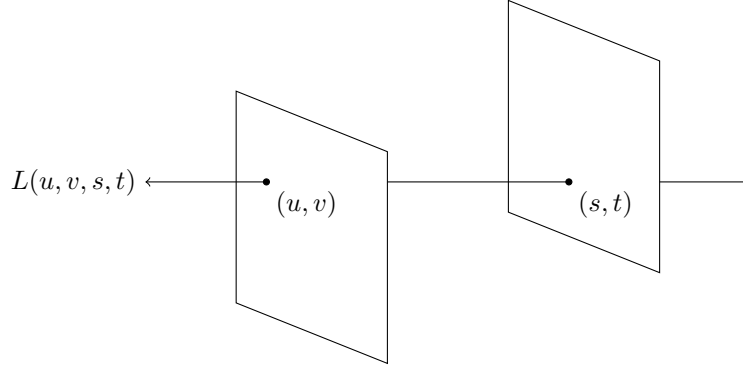


Figure 2.1: Parametrization of the light field with two planes.

## 2.2 Light Field Acquisition

For practical applications, the light field must be discretized and so an appropriate sampling method needs to be chosen. One way is to capture the light field with a grid of optical systems, e.g. cameras. Typically, the  $(u, v)$ -plane is sampled on a grid  $G_{uv} = \{(u_i, v_j) \mid i = 1, \dots, n, j = 1, \dots, m\}$  on the  $(u, v)$ -plane with a resolution  $n \times m$ . The extent in horizontal (vertical) direction is called the horizontal (vertical) **baseline**. This means that only a slice of the actual light field can be captured and the two planes are clipped to form a rectangle.

### Oblique Projection

Oblique projection, as shown in figure 2.2(a), is a special case of orthographic projection: The parallel rays do not need to be perpendicular to the image plane of the camera. The advantage is that there is a one-to-one correspondence between camera position and ray angle, since all rays in one camera are parallel. This means that the angular resolution is simply the number of cameras, and the spatial resolution is the number of pixels in the image plane. Given a light field  $L(u, v, s, t)$  and the distance  $d$  between the two planes, a re-parameterization  $L'(\theta, \phi, s, t)$  can be obtained according to figure 2.2(b) by the transformation

$$\theta = \arctan\left(\frac{u-s}{d}\right), \quad \phi = \arctan\left(\frac{v-t}{d}\right).$$

However, this type of projection is only applicable for synthetic scenes that are rendered with a computer.

### Perspective Projection

The angles of the rays in a light field captured by perspective projections are determined by the focal length and the sensor resolution of the camera. For a camera light field, typically it is expected that

- All cameras are placed at grid positions in  $G_{uv}$  on the same plane, called the  $(u, v)$ -plane,
- The optical axes of the cameras are parallel to the  $(u, v)$ -plane,

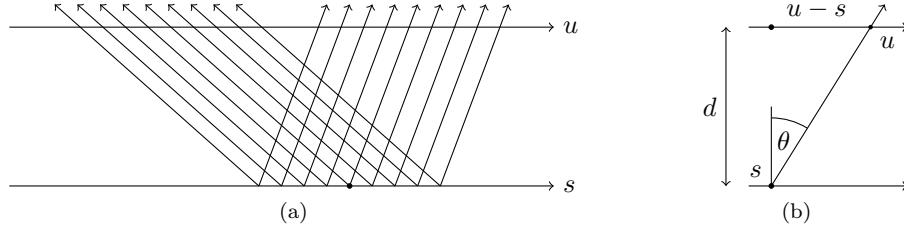


Figure 2.2: (a) Light field acquisition using oblique projection. (b) Re-parameterization of the two-plane representation to angular coordinates.

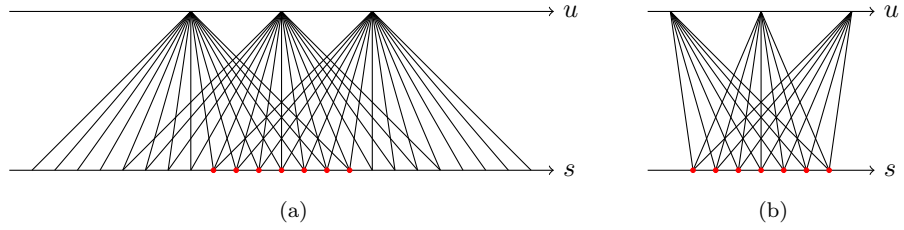


Figure 2.3: Perspective projections of a scene. (a) Projections with three pin-hole cameras. (b) Discarding unused rays corresponds to cropping the camera images.

- All cameras have the same intrinsic parameters (e.g. focal length).

In this case, the focal planes of all cameras coincide with a common focal plane, the  $(s, t)$ -plane. Figure 2.3(a) shows this scenario for three cameras in two dimensions. Each camera captures sample points on the  $(s, t)$ -plane, but not every point on the  $(s, t)$ -plane is captured by ever camera. As demonstrated in figure 2.3(b), the camera images need to be rectified such that all discrete coordinates  $(u, v, s, t)$  correspond to valid rays.

## 2.3 Visualization

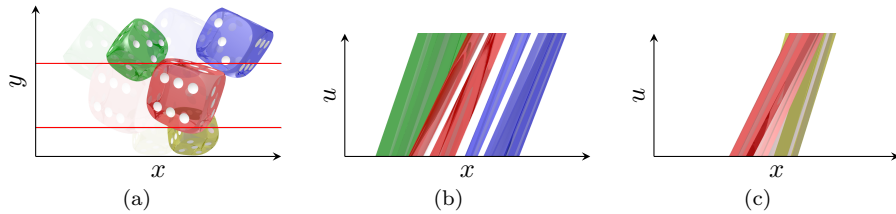


Figure 2.4: (a) Raw 3D light field rendered from 500 positions along a horizontal baseline. Two scanlines are extracted from every image. (b) The slope of the epipolar lines from blue and green dice are larger than those from the red die. (c) Epipolar lines of the yellow die have an even larger slope, indicating greater depth.

## 2.4 Light Field Tomography

The light field display is modeled by a volumetric attenuator  $\mu(x, y, z)$  that attenuates the light traveling through its material. According to the Beer-Lambert law, the intensity of a light ray  $\mathcal{R} \subset \mathbb{R}^3$  passing through the material decreases exponentially over distance:

$$I = I_0 e^{-\int_{\mathcal{R}} \mu(r) dr}. \quad (2.1)$$

The incident intensity  $I_0$  is the intensity of the ray before it enters the attenuator. Equation 2.1 can be rewritten into

$$\bar{I} := \log \left( \frac{I}{I_0} \right) = - \int_{\mathcal{R}} \mu(r) dr. \quad (2.2)$$

Now, let the attenuator  $\mu(x, y, z)$  be a cubic slab of height  $d$  in Z-direction and let  $L(u, v, s, t)$  be the two-plane parameterization of the light field such that the  $(s, t)$ -plane coincides with the  $(x, y)$ -plane of the attenuator and the  $(u, v)$ -plane is at distance  $d$ . The set of points describing the ray defined by the coordinates  $(u, v, s, t)$  is

$$\mathcal{R} = \left\{ \lambda a + b \mid a = \begin{pmatrix} u-s \\ v-t \\ d \end{pmatrix}, b = \begin{pmatrix} s \\ t \\ 0 \end{pmatrix}, \lambda \in \mathbb{R} \right\}. \quad (2.3)$$

A point  $p = (x, y, z)^T$  is part of the ray  $\mathcal{R}$  if and only if

$$\exists \lambda \in \mathbb{R} : p = \lambda a + b \quad \iff \quad a \times (p - b) = 0, \quad (2.4)$$

where  $\times$  denotes the cross product. Now,  $I$  can be replaced with the light field  $L$  and the right hand side of equation 2.2 can be written as an integral over  $\mathbb{R}^3$ :

$$\bar{L}(u, v, s, t) = - \int_{\mathbb{R}^3} \mu(p) \delta(a \times (p - b)) dp. \quad (2.5)$$

Here,  $\delta$  denotes the Dirac delta function on  $\mathbb{R}^3$  and  $\mu$  is zero outside the boundaries of the slab. This means that integrand is only non-zero for points on the ray with coordinates  $(u, v, s, t)$ .

Combining equation 2.1 and 2.5 gives the light field emitted by the attenuator. The goal is to produce such an attenuation display that emits a given target light field.

In computed tomography, the radon transform

## 2.5 Spectral Analysis



# Appendix A

## ap1

### A.1 apsec1

## List of Tables

# List of Figures

2.1	Parametrization of the light field with two planes. . . . .	3
2.2	(a) Light field acquisition using oblique projection. (b) Re-parameterization of the two-plane representation to angular coordinates. . . . .	4
2.3	Perspective projections of a scene. (a) Projections with three pin-hole cameras. (b) Discarding unused rays corresponds to cropping the camera images. . . . .	4
2.4	(a) Raw 3D light field rendered from 500 positions along a horizontal baseline. Two scanlines are extracted from every image. (b) The slope of the epipolar lines from blue and green dice are larger than those from the red die. (c) Epipolar lines of the yellow die have an even larger slope, indicating greater depth. . . . .	4

# Bibliography

- [AB91] ADELSON, E. H. ; BERGEN, J.: The Plenoptic Function and the Elements of Early Vision. In: *Computational Models of Visual Processing* (1991), S. 3–20
- [LH96] LEVOY, M. ; HANRAHAN, P.: Light Field Rendering. (1996), S. 1–12
- [WLHR11] WETZSTEIN, G. ; LANMAN, D. ; HEIDRICH, W. ; RASKAR, R.: Layered 3D: Tomographic Image Synthesis for Attenuation-based Light Field and High Dynamic Range Displays. In: *ACM Trans. Graph.* 30 (2011), Nr. 4
- [WLHR12] WETZSTEIN, G. ; LANMAN, D. ; HIRSCH, M. ; RASKAR, R.: Tensor Displays: Compressive Light Field Synthesis using Multilayer Displays with Directional Backlighting. In: *ACM Trans. Graph. (Proc. SIGGRAPH)* 31 (2012), Nr. 4, S. 1–11
- [Yan10] YAN, Ming: Convergence Analysis of SART by Bregman Iteration and Dual Gradient Descent. (2010), S. 1–15

# **Erklärung**

gemäss Art. 28 Abs. 2 RSL 05

Name/Vorname: .....

Matrikelnummer: .....

Studiengang: .....

Bachelor ☐      Master ☐      Dissertation ☐

Titel der Arbeit: .....

.....

.....

LeiterIn der Arbeit: .....

.....

Ich erkläre hiermit, dass ich diese Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen benutzt habe. Alle Stellen, die wörtlich oder sinngemäss aus Quellen entnommen wurden, habe ich als solche gekennzeichnet. Mir ist bekannt, dass andernfalls der Senat gemäss Artikel 36 Absatz 1 Buchstabe o des Gesetzes vom 5. September 1996 über die Universität zum Entzug des auf Grund dieser Arbeit verliehenen Titels berechtigt ist.

.....

Ort/Datum

.....

Unterschrift