

Introduction to Statistics

Mean: It is sum of all values divided by the number of values. for dataset $X = \{x_1, x_2, x_3, \dots, x_n\}$:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

example let $X = \{10, 15, 20, 25, 30\}$

$$\bar{x} = \frac{10+15+20+25+30}{5}$$

$$\bar{x} = \frac{100}{5} = 20$$

Median: The median is the middle value in a sorted, ascending or descending, list of numbers. In case of even number of observation it's the average of two middle numbers.

example: $Y = \{8, 12, 15, 22, 30, 35\}$

$$\text{median} = \frac{15+22}{2} = 18.5$$

Mode: It is a number that appears the most frequently in a dataset. A set of data may have one, more than one or no mode at all.

example:

$$Z = \{12, 15, 12, 18, 22, 12\}$$

$$\text{mode} = 12.$$

Gaussian Distribution

shape: It is bell shaped and symmetric around its mean.

parameters: Characterized by its mean μ and standard deviation σ .

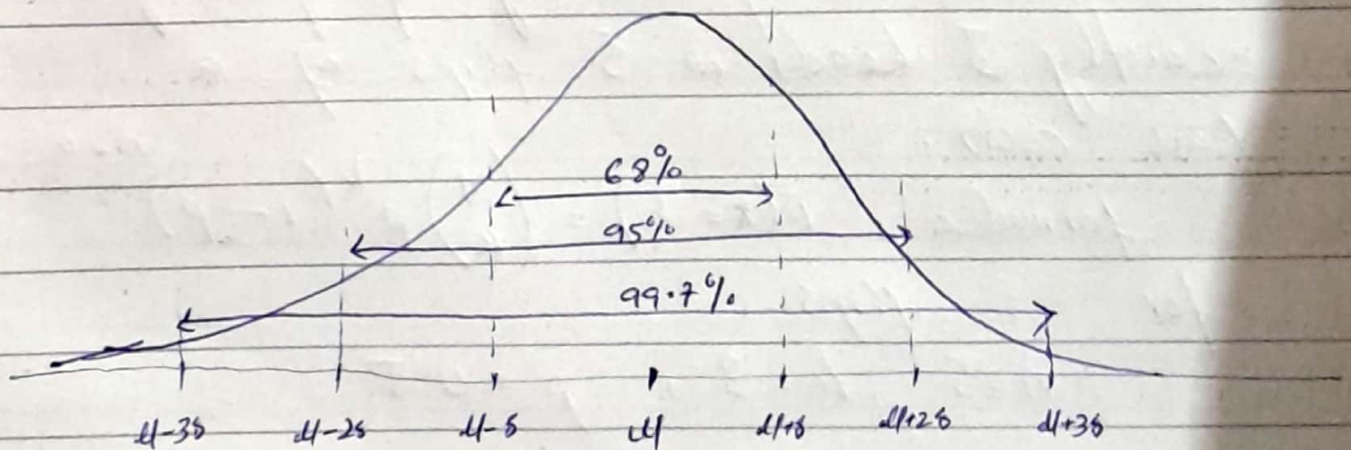
Probability Density function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

68-95-99.7 rule :

- ⇒ About 68% of the data falls within one standard deviation of the mean. ($\mu \pm \sigma$)
- ⇒ About 95% falls within two σ . ($\mu \pm 2\sigma$)
- ⇒ About 99.7% falls within 3 standard devs.

Applications: Widely used in various fields for modeling continuous data due to its prevalence in natural phenomena.



Normal Distribution:

A gaussian distribution with a mean of zero and a standard deviation of 1 is called a standard normal distribution.

Binomial Distribution:

Describes the number of successes in a fixed number of independent Bernoulli trials.

~~Q~~ Calculate the probability of getting exactly 3 heads in 5 flips of a fair coin.

formula: $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$
for n flips,
 $n=5$, $k=3$, $p=0.5$

$$P(X=3) = \binom{5}{3} (0.5)^3 (0.5)^2 = \underline{\underline{0.3125}}$$

Poisson Distribution

Models the number of events occurring in a fixed interval of time or space.

~~eg~~

calculate the probability of exactly 2 customers arriving in a poisson process with an average rate of 3 cust per hour.

Poisson Probability formula:

$$P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

for $\lambda = 3, k = 2$:

$$P(X=2) = \frac{3^2 e^{-3}}{2!} = \frac{9(0.0498)}{2} = \underline{\underline{0.224}}$$

Probability:

calculate the probability of rolling
a 6 on a fair six-sided die!

total outcomes = 6

favorable outcome = 1

$$\text{Probability} = \frac{1}{6} \quad \underline{\underline{\text{Ans}}}$$