Q-1

For what contour c will it follow Cauchy's Integral theorem that

(i) $\oint_c \frac{1}{z} dz = 0$

(ii) $\oint_c \frac{\cos z}{z^6 - z^2} dz = 0$

(iii) $\oint_c \frac{e^{1/z}}{z^2+9} dz = 0$

Q-2

Integrate $\int_c \operatorname{Re} z \, dz$; c the parabola $y = x^2$ from 0 to $1+i$.

Q-3

Find the upper bound of the absolute value of the integral in Q-2.

Q-4

Are the following sequences bounded? Convergent? Find their limit points.

(a) $z_n = \mathrm{Ln}(2+i)^n$

(b) $z_n = (0.9 + 0.1)^{2n}$

(c) $z_n = (5+5i)^{-n}$

Q-5

Find the radius of convergence from a series of simpler terms.

ⓐ $$\sum_{n=2}^{\infty} \frac{n(n-1)}{3^n}(z-2i)^n$$

ⓑ $$\sum_{n=1}^{\infty} \frac{n}{2^n}(z-2i)^{2n}$$

ⓒ $$\sum_{n=1}^{\infty} \frac{3^n\, n(n+1)}{5^n}(z-1)^{2n}$$

Q-6

Find the Taylor series of the given functions with the given as center and determine the radius of convergence

ⓐ $\sin z, \quad \pi/2$

ⓑ $\dfrac{1}{1-z}, \quad i$

ⓒ $\operatorname{Ln}(1-z), \quad i$

Q-7

Determine the location and kind of the singularities of the following functions. In case of pole also state the order

ⓐ $z^3 \cdot \dfrac{1}{e^{z-i}}$

ⓑ $\dfrac{1}{\cos z - \sin z}$