



Probability Methods in Engineering

Lecture 3

Prepared by

Dr. Safdar Nawaz Khan Marwat
DCSE, UET Peshawar



Sample Space

- Same experimental procedure but different sample spaces
 - ❑ Toss a coin three times and note the outcomes
 - ❑ Toss a coin three times and note the number of heads

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$S = \{0, 1, 2, 3\}$$

- Same sample space but different representations
 - ❑ Pick a number at random between zero and one
 - ❑ Measure the time between page requests in a Web server

$$S = \{x : 0 \leq x \leq 1\}$$

$$S = \{x : x \geq 0\}$$

$$[0, 1]$$

$$[0, \infty)$$





Sample Space (cont.)

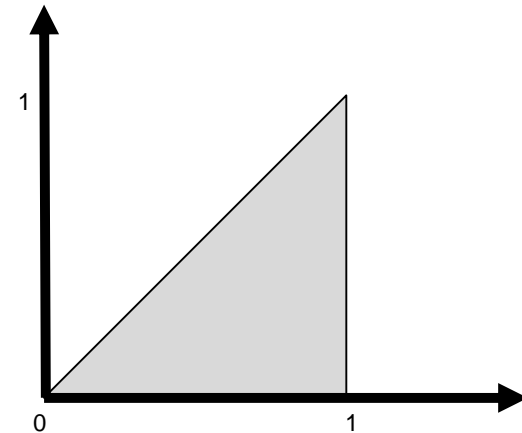
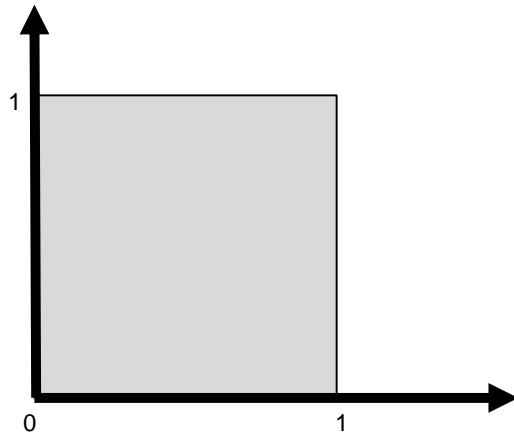
➤ Multidimensional sample spaces

- ❑ Pick two numbers at random between zero and one
- ❑ Pick a number x at random between zero and one, then pick a number y at random between zero and y

$$S = \{(x, y) : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$$

$$S = \{(x, y) : 0 \leq x \leq 1 \text{ and } 0 \leq y \leq x\}$$

$$S = \{(x, y) : 0 \leq y \leq x \leq 1\}$$





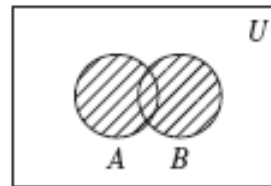
Set Theory

- Representation of events by sets
- Capital letters for names S, A, B, \dots
- Small letters for elements a, b, x, y, \dots
- Venn diagram illustrates sets and their interrelationship

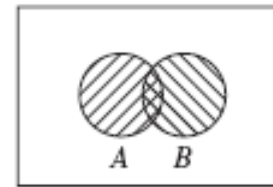
Source: C. Görg, *Communication Networks II*, University of Bremen, Germany



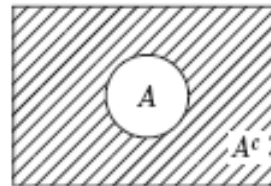
Set Theory (cont.)



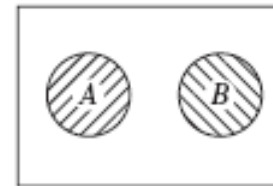
(a) $A \cup B$



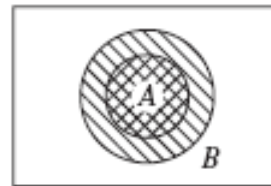
(b) $A \cap B$



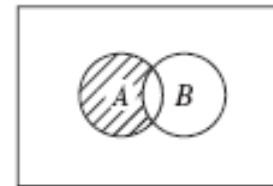
(c) A^c



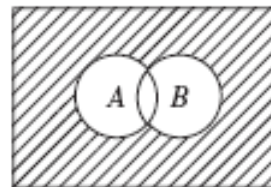
(d) $A \cap B = \emptyset$



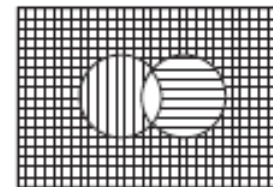
(e) $A \subset B$



(f) $A - B$



(g) $(A \cup B)^c$



(h) $A^c \cap B^c$



Axioms of Probability

$$0 \leq P[A] \leq 1$$

$$P[S] = 1$$

➤ If $A \cap B = \emptyset$

$$P[A \cup B] = P[A] + P[B]$$

➤ If $A_i \cap A_j = \emptyset$ for all $i \neq j$

$$P\left[\bigcup_{k=1}^{\infty} A_k\right] = \sum_{k=1}^{\infty} P[A_k]$$



Corollaries

- Rules or propositions based on or derived from axiom(s)

$$P[A^c] = 1 - P[A]$$

$$P[\emptyset] = 0$$

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

$$P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[B \cap C] - P[A \cap C] + P[A \cap B \cap C]$$



Examples

Discrete countably finite sample space

- An urn contains 10 identical balls numbered 0, 1, ... , 9. A random experiment involves selecting a ball from the urn and noting the number of the ball. Find the probability of the following events:
- ☐ $A = \text{"number of ball selected is odd,"}$
 - ☐ $B = \text{"number of ball selected is a multiple of 3,"}$
 - ☐ $C = \text{"number of ball selected is less than 5,"}$
 - ☐ $D = A \cup B$
 - ☐ $E = A \cup B \cup C$



Examples (cont.)

Discrete countably infinite sample space

- A fair coin is tossed repeatedly until the first heads shows up; the outcome of the experiment is the number of tosses required until the first heads occurs. Find a probability law for this experiment.



Examples (cont.)

Continuous uncountably infinite sample space

- Consider the random experiment “pick a number x at random between zero and one.” Let the probability that the outcome falls in a subinterval of S be proportional to the length of the subinterval.
 - ❑ What is the sample space of this random experiment?
 - ❑ What is the probability that the outcome falls in the interval $[0, 0.5]$, $[0.5, 1]$?
 - ❑ What is the probability that the outcome is 0.5?
 - ❑ What is the probability that the outcome falls in either $[0, 0.2]$ or $[0.8, 1]$?
 - ❑ What is the probability that the outcome falls in either $[0.3, 0.6]$ or $[0.5, 0.8]$?