Probability Methods in Engineering

Prepared By
Dr. Safdar Nawaz Khan Marwat
DCSE, UET Peshawar

Lecture 4

Counting Methods

- Counting methods for determination of probability
  - Experiments with finite sample spaces
  - Equiprobable outcomes
- Probability of an event is the ratio of:
  - Number of outcomes in the event of interest
  - Total number of outcomes in the sample space

Counting Methods (Cont.)

- Sampling with replacement
  - With ordering
  - Without ordering
- Sampling without replacement
  - With ordering
  - Without ordering

Counting Methods (Cont.)

- Sampling with replacement with ordering
  - k draws from n objects
  - Number of distinct ordered k-tuples = n^k
- Sampling without replacement with ordering
  - k draws from n objects
  - Number of distinct ordered k-tuples = $n\,(n-1)\,...\,(n-k+1)$

Examples

Sampling with replacement with ordering

- An urn contains five balls numbered 1 to 5. Suppose we select two balls from the urn with replacement. How many distinct ordered pairs are possible? What is the probability that the two draws yield the same number?

Examples (cont.)

Sampling without replacement with ordering

- An urn contains five balls numbered 1 to 5. Suppose we select two balls in succession without replacement. How many distinct ordered pairs are possible? What is the probability that the first ball has a number larger than that of the second ball?

Examples (cont.)

Sampling with replacement with ordering

- An urn contains five balls numbered 1, 2, ... , 5. Suppose we draw three balls with replacement. What is the probability that all three balls are different?

Permutations

- The permutation is the number of different arrangement which can be made by picking "k" number of things from the available "n" things.
- Arrangement of things

without replacement with order

- Sampling without replacement with ordering
  - E.g. form a number of 3 digits from 1, 2, 3, 4
- Number of all possible permutations

$$\overset{n}{\underset{k}{P}} = \frac{n!}{(n-k)!} = n(n-1)\ldots(n-k+1)$$

- If all objects are drawn, $(n = k)$
  - Number of all possible permutations is n factorial or $n!$

Combinations

without rep

no O

terms

~~all~~

- The different ways of selecting a group, by taking some or all the members of a set, without the following order.
- Selection of things

- Sampling without replacement and without ordering
  - E. g. make a team of 5 players from a total of 9
- Record the result without considering the order
- k objects are drawn and termed as a combination

- Number of combinations for n objects and k draws

- Also called "n choose k"

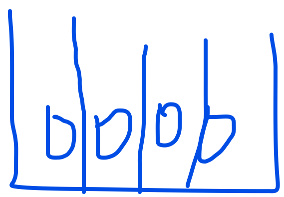
$$\boxed{{}^{n}_{k}C = \binom{n}{k} = \frac{n!}{k!(n-k)!}}$$

Examples (cont.)

- Find the number of permutations of three distinct objects {1, 2, 3} while
  - Drawing all objects
  - Drawing 2 objects
- Also find the number of combinations of three distinct objects {1, 2, 3} while
  - Drawing all objects
  - Drawing 2 objects

Examples (cont.)

- Suppose that 4 balls are placed at random into 4 cells, where more than 1 ball is allowed to occupy a cell. What is the probability that all cells are occupied?



w rep

w b

$n = 4$

$k = 4$

$n^k = ?$

$\boxed{4^4 = 256}$

Examples (cont.)

- A fair coin is tossed 5 times. Coin tosses are independent events.
  1. Find P(first 3 tosses are heads) .
  2. Find P(first 4 are heads),
  3. Find P(at least one head in 4 tosses).

$T_1 \quad T_2 \quad T_3$

$P_1(H) = 1/2$

$P_2(H) = 1/2$

$P_3 = 1/2$

$= 1/8$

$1/2 * 1/2 * 1/2 * 1/2 = 1/16$

$P(no\ H) = \frac{1}{16}$

$P_r = 1 - 1/16$

$= 15/16$

Examples (cont.)

- In a jar with 5 red, 6 blue and 2 white marbles. Two marbles are selected, find the probability that both are red if:
  - a) If two marbles are selected with replacement.
  - b) If two marbles are selected without replacement.

Examples (cont.)

- Assume that 10% of adults in the Russia are right handed. Find the probability that three selected adults all are right handed.

Examples (cont.)

- The Computer society has 18 members. An election is held to choose a president, vice-president and secretary. In how many ways can the three officers be chosen?