**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:

probability that the service manager cannot meet his commitment is Option(B)-(0.2676)

By using Z score value,

xi-*μ/* *σ*

60-55/8=0.625 and python code is 1-stats.norm.cdf (0.625)

=0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:

**A**. Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44 = (44 - 38)/6 = 1 => 84.13 %

=> People above 44 age = 100 - 84.13 = 15.87% ≈ 63    out of 400

Z score for 38 = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is F**ALSE**

**B.** True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As we know that if X ~ N (µ1, σ1^2), and Y ~ N (µ2, σ2^2) are two independent random variables then,

X + Y ~ N (µ1 + µ2, σ1^2 + σ2^2), and

X − Y ~ N (µ1 − µ2, σ1^2 + σ2^2).

Similarly, if Z = aX + bY, where X and Y are as defined above, i.e., Z is linear combination of X and Y then,

Z ~ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question

2X1~ N (2µ,4 σ^2) and

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4µ,6 σ^2)

**Normal distribution. Parameters are same**.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D.

The Probability of getting value between a and b should be 0.99.

So, the Probability of going wrong, or the Probability outside the a and b area is 0.01 (i.e., 1-0.99).

The Probability towards left from a = -0.005 (i.e., 0.01/2).

The Probability towards right from b = +0.005 (i.e., 0.01/2).

So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z= (X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z (-0.005) \*20+100 = -(-2.57) \*20+100 = 151.5

Z (+0.005) \*20+100 = (-2.57) \*20+100 = 48.5.

So, the option ‘D’ is correct.

Ans: D-48.5, 151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

1. Specify a Rupee range (centred on the mean) such that it contains 95% probability for the annual profit of the company.

Mean Profit is Rs 540 million.

Standard Deviation is Rs 225.0 million

Range is Rs (99.00810347848784, 980.9918965215122) in Millions.

**B.**5th percentile of profit (in Million Rupees) is 170.0

**C**. Making 1 loss: - Confidence Interval: 0.0477903522728147.

Making 2 loss: Confidence Interval: 0.040059156863817086.

Probability of Division 1 making a loss in a given year is more than Division 2.(profit2)