Graphical Abstract

Parallel vehicle scheduling with compatibility constraints and makespan objectives: An application to automated guided vehicle scheduling

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Highlights

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- Research highlight 1
- Research highlight 2

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Abstract

Keywords: Scheduling problem, Minimum makespan, Eligibility constraints, Automated guided vehicle, Benders decomposition

1. Introduction

In this paper, a scheduling problem of multiple vehicles and product types is investigated. The problem has the characteristics of both vehicle problems with pickups and deliveries, and unrelated and uniform parallel machine scheduling problems.

2. Literature review

2.1. Vehicle routing problem to minimize makespan

Vehicle Routing Problems with Pickups and Deliveries

The Vehicle Routing Problem (VRP) is a classical optimization problem, which aims to select the optimal route for vehicles based on one or more depots, under specific conditions (i.e. time windows, capacity constraints).

Various exact or heuristic algorithms are developed to solve different types of VRPs.

2.2. Unrelated and uniform parallel machine scheduling

As another famous optimization problem, Machine Scheduling (MS) is widely applied in manufacturing, logistics.

$R_r||C_{\max}$

2.3. Summary and contributions of the paper

3. Problem formulation

In this section, we first formally define the parallel vehicle scheduling problem with compatibility constraints and makespan objectives, and then propose MILP models.

3.1. Problem definition

To be easier to describe this problem, the notations used in this paper are listed in Table 1.

3.2. Consumption and preprocessing

3.3. Mathematical formulation

Table 1: Notations

sets	definitions
K	set of vehicles
V	set of machines
W	set of product type
D	set of production plan, $D = \{(i,j) \mid i,j \in V \backslash \{v_0\}\}$
D_w	set of production plan with particular product type, $w \in W$
R	set of gernal task sequence length, $R = 1, 2, ...$
parameters	**definitions**
v_0	the depot, $v_0 \in V$
c_k	capacity of vehicles k, $k \in K$
s_i^k	servcing time of vehicle k for machine i, $i \in V, k \in K$
t_{ij}	travel cost from machine i to j, $i, j \in V$
q_{ij}	amount of demand (i,j), $(i,j) \in D$
T_m	maximum makespan
variables	**definitions**
u_w^k	whether vehicle k loading products of w type
x_{ir}^k	whether vehicle k serve machine i as its r-th task
ψ_{ir}^k	reserve of last task of vehicle k in r-th one
ϵ_r^k	waiting time of vehicle k before r-th task

g_{ir}	avaliable time of machine i in r-th task
T_f	gernal finish time
	$w \in W, i, j \in V, k \in K, r \in R$

Unrelated parallel machine scheduling problem with the objective of minimizing the maximal completion time($R||C_{\max}$) and restrict PDP

objective

$$\min T_f \tag{1}$$

subject to

orgin and destination

$$x^k_{v_0,0}, x^k_{v_0,|R|+1} = 1, \forall k \in K \tag{2}$$

$$x^k_{v_0,r}, x^k_{i,0} = 0, \forall i \in V, k \in K, r \in R \tag{3}$$

single occupied

$$\sum_{i \in V, k \in K} x^k_{ir} \leq 1, \forall r \in R \tag{4}$$

$$\sum_{i \in V, k \in K} x^k_{ir} \geq \sum_{i \in V, k \in K} x^k_{i,r+1}, \forall r \in R \tag{5}$$

single vehicle sequence

$$\psi^k_{ir} = x^k_{ir} + \psi^k_{i,r-1} \cdot (1 - \sum_{j \in V} x^k_{jr}), \forall i \in V, k \in K, r \in R \tag{6}$$

$$\begin{aligned} g_{i,r-1} - T_m \cdot \sum_{k \in K} x^k_{ir} \leq g_{ir} \leq g_{i,r-1} + T_m \cdot \sum_{k \in K} x^k_{ir}, \\ \forall i \in V, r \in R \end{aligned} \tag{7}$$

$$-T_m \cdot \sum_{i \in V} x^k_{ir} \leq \epsilon^k_r \leq T_m \cdot \sum_{i \in V} x^k_{ir}, \forall r \in R, k \in K \tag{8}$$

compatibility constraints

$$\sum_{w \in W} u^k_w \leq 1, \forall k \in K \tag{9}$$

$$\psi^k_{i,r-1} \cdot x^k_{jr} \leq u^k_w, \forall (i,j) \in D_w, w \in W, r \in R, k \in K \tag{10}$$

time

$$\sum_{r\in R}(\sum_{i\in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j\in V} t_{ij}\cdot \psi_{i,r-1}^k \cdot x_{jr}^k + \epsilon_r^k) \leq T_f, \forall k \in K \tag{11}$$

fulfill demand

$$\sum_{k\in K, r\in R} c_k \cdot \psi_{i,r-1}^k \cdot x_{jr}^k \geq q_{ij}, \forall (i,j) \in D \tag{12}$$

one by one

$$\begin{gathered}\sum_{r\in\{1,..,\gamma\}}(\sum_{i\in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j\in V} t_{ij}\cdot \psi_{i,r-1}^k \cdot x_{jr}^k + \epsilon_r^k) \geq \\ \sum_{i\in V} g_{i,\gamma}\cdot x_{i,\gamma}^k - T_m \cdot (1-\sum_{i\in V} x_{ir}^k), \forall k\in K, \gamma \in R\end{gathered} \tag{13}$$

avaliable time

$$\begin{aligned}&\sum_{i\in V} g_{i,\gamma+1}\cdot x_{i,\gamma}^k - T_m \cdot (1-\sum_{i\in V} x_{ir}^k) \\ &\leq \sum_{r\in\{1,..,\gamma\}}(\sum_{i\in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j\in V} t_{ij}\cdot \psi_{i,r-1}^k \cdot x_{jr}^k + \epsilon_r^k) + \sum_{i\in V} s_i^k \cdot x_{i,r}^k \\ &\leq \sum_{i\in V} g_{i,\gamma+1}\cdot x_{i,\gamma}^k + T_m \cdot (1-\sum_{i\in V} x_{ir}^k), \forall k\in K, \gamma\in R\end{aligned} \tag{14}$$

$$g_{ir} \leq T_m, \forall i \in V, r\in R \tag{15}$$

domain of definition

$$u_k^w, x_{ir}^k \in \{0,1\}, \forall w\in W, i\in V, r\in R, k\in K \tag{16}$$

$$\epsilon_r^k, g_{ir}, \tau_s^k, T_f \geq 0, \forall i\in V, k\in K, r\in R \tag{17}$$

Add Y, Z, Π

Table 2: Instrumental variables

variables	definitions

y_{ijr}^k	$\psi_{i,r-1}^k \cdot x_{jr}^k$
z_{ir}^k	$g_{ir} \cdot x_{ir}^k$
π_{ir}^k	$g_{i,r+1} \cdot x_{ir}^k$
	$\forall i, j \in V, r \in R, k \in K$

convert nonlinear to linear

$$\begin{cases} y_{ijr}^k \leq \psi_{i,r-1}^k \\ y_{ijr}^k \leq x_{j,r}^k \\ y_{ijr}^k \geq \psi_{i,r-1}^k + x_{j,r}^k - 1 \end{cases}, \forall i, j \in V, k \in K, r \in R \tag{18}$$

$$\begin{cases} z_{ir}^k \leq T_m \cdot x_{ir}^k \\ z_{ir}^k \leq g_{ir} \\ z_{ir}^k \geq T_m \cdot (x_{ir}^k - 1) + g_{ir} \end{cases}, \forall i \in V, r \in R, k \in K \tag{19}$$

$$\begin{cases} \pi_{ir}^k \leq T_m \cdot x_{ir}^k \\ \pi_{ir}^k \leq g_{i,r+1} \\ \pi_{ir}^k \geq T_m \cdot (x_{ir}^k - 1) + g_{i,r+1} \end{cases}, \forall i \in V, r \in R, k \in K \tag{20}$$

single vehicle sequence

$$\psi_{ir}^k = x_{ir}^k + \psi_{i,r-1}^k - \sum_{j \in V} y_{ijr}^k, \forall i \in V, k \in K, r \in R \tag{21}$$

compatibility constraints

$$y_{ijr}^k \leq u_w^k, \forall (i, j) \in D_w, r \in R, k \in K \tag{22}$$

time

$$\sum_{r \in R} (\sum_{i \in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j \in V} t_{ij} \cdot y_{ijr}^k + \epsilon_r^k) \leq T_f, \forall k \in K \tag{23}$$

fulfill demand

$$\sum_{k \in K, r \in R} c_k \cdot y_{ijr}^k \geq q_{ij}, \forall (i, j) \in D \tag{24}$$

one by one

$$\begin{aligned}\sum_{r\in\{1,..,\gamma\}}(\sum_{i\in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j\in V} t_{ij}\cdot y_{ijr}^k + \epsilon_r^k) \geq \\ \sum_{i\in V} z_{ir}^k + T_m \cdot (1-\sum_{i\in V} x_{ir}^k), \forall k\in K, \gamma \in R\end{aligned} \tag{25}$$

avaliable time

$$\begin{aligned}&\sum_{i\in V}\pi_{ir}^k - T_m\cdot(1-\sum_{i\in V}x_{ir}^k)\\ &\leq \sum_{r\in\{1,..,\gamma\}}(\sum_{i\in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j\in V} t_{ij}\cdot y_{ijr}^k + \epsilon_r^k) + \sum_{i\in V} s_i^k\cdot x_{i,r}^k\\ &\leq \sum_{i\in V}\pi_{ir}^k + T_m\cdot(1-\sum_{i\in V}x_{ir}^k), \forall k\in K, \gamma\in R\end{aligned} \tag{26}$$

domain of definition

$$y_{ijr}^k \in \{0,1\}, \forall i,j\in V, r\in R, k\in K \tag{27}$$

$$z_{ir}^k, \pi_{ir}^k \geq 0, \forall i\in V, r\in R, k\in K \tag{28}$$

4. XXXX Algorithm

Heuristic Benders algorithm
master-problem (MP): U, X, Ψ, Y

$$\min 0 + \overline{T_f} \tag{29}$$

s.t. (2), (3), (4), (5), (18), (21), (9), (22), (24) and cuts from sub-problem. There is no objection in master-problem, just find the feasible solution.
sub-problem (SP): E, G, T_f, Z, Π

$$\min T_f \tag{30}$$

s.t. (7), (8), (19), (20), (23), (24), (25), (26)

5. Computational analysis

6. Conclusion and future extensions

References