Graphical Abstract

Parallel vehicle scheduling with compatibility constraints and makespan objectives: An application to automated guided vehicle scheduling

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Highlights

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- Research highlight 1
- Research highlight 2

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Abstract

Keywords: Scheduling problem, Minimum makespan, Reentrant blocking job shop, Eligibility constraints, Automated guided vehicle,

1. Introduction

Many features displayed in practice are always not addressed in single "classical" problem. Take a kind of vehicle scheduling problems as an example, the limited capacity of some vehicles forces them to visit specific customers or locations over once, which is not applicable to traditional vehicle routing model pairing the vehicles and arcs among customers, and considering the time products transfered not negligible is not common in scheduling problem.

Two more tricky requirements is that one vehicle can only handle products of one type, and a customer or location can be served by no more than one vehicle at a time.

In this paper, a scheduling problem of multiple vehicles and product types is investigated. The problem has the characteristics of both vehicle problems with pickups and deliveries, and unrelated and uniform parallel machine scheduling problems.

The paper is organized as follows. The vehicle scheduling problem is described in section.

2. Literature review

2.1. Vehicle routing problem to minimize makespan

Vehicle Routing Problems with Pickups and Deliveries

The Vehicle Routing Problem (VRP) is a classical optimization problem, which aims to select the optimal route for vehicles based on one or more depots, under specific conditions (i.e. time windows, capacity constraints).

Various exact or heuristic algorithms are developed to solve different types of VRPs.

2.2. Unrelated and uniform parallel machine scheduling

Machine scheduling problems.

In many cases, jobs/tasks must be assigned to machines/agents.

As an example, consider a factory producing one product for n customers.

Serial production: Only one job can be processed at one time. Each job has its due date. How to schedule the n jobes to minimizie the total number of delay jobs?

In this example, scheduling is nothing but sequencing.

Splitting jobs is not helpful. There are $n!$ ways to formulate this problem (so that a solution may be obtained by solving the model)?

The problems of scheduling jobs/tasks to machines/agents are machine scheduling problems.

Machine scheduling problems may be categorized in multiple ways. By production mode, single machine serial production, multiple parallel machines, flow shop problems, job shop problems. By job splitting, non-preemptive and preemptive problems. By performance measurement, makespan (the time that all jobs are complete), (weighted) total completion time, (weighted) number of delayed jobs, (weighted) total lateness, (weighted) total tardiness, and more.

Subproblems of scheduling problem: 1. Loading involces assigning tasks to resources. if each task can be processed by only one specific resource, no difficulty is presented. 2. Sequencing is concerned with determining the order in which jobs are processed, also within each resource. 3. Scheduling pertains to establishing both the timing and use of resources within an organization.

As another famous optimization problem, Machine Scheduling (MS) is widely applied in manufacturing, logistics.

$R_r||C_{\max}$

2.3. Summary and contributions of the paper

3. Problem formulation

In this section, we first formally define the parallel vehicle scheduling problem with compatibility constraints and makespan objectives, and then propose MILP models.

3.1. Problem definition

To be easier to describe this problem, the notations used in this paper are listed in Table 1.

3.2. Consumption and preprocessing

Gernal assumption in static scheduling problems: A machine can process only one job at a time. A task, once tasken up, is fully completed before the next task is taken. A task of a job is done on only one machine. There is no job splitting. There is no job pre-emption. A task cannot be stopped midway for another task.

To break: No assemblies or divergences. Each task has only one precedent and one successor. There is only a machine of each type (otherwise grouping and balancing)

Johnson's rule, Hodgson's Algorithm

Priority rules.

Total slack:

$Slack = durdate - actualdate - processingtimeofremainingoperations$ Jobs are scheduled in increasing slack, first those with lower slack. If there is negative slack the job will not be on time by th due date.

Critical ratio (CR)

$Criticalratio = \frac{duedate - actualdate}{processingtimeofremainingoperatyions}$ Again, jobs are scheduled in increasing CR, first those with lower CR.

3.3. Mathematical formulation

Table 1: Notations

sets	definitions
K	set of vehicles
V	set of machines
W	set of product type
D	set of production plan, $D = \{(i,j)\|i,j \in V\backslash\{v_0\}\}$
D_w	set of production plan with particular product type, $w \in W$
R	set of gernal task sequence length, $R = 1, 2, ...$

parameters	definitions
v_0	the depot, $v_0 \in V$
c_k	capacity of vehicles k, $k \in K$
s_i^k	servcing time of vehicle k for machine i, $i \in V, k \in K$
t_{ij}	travel cost from machine i to j, $i, j \in V$
q_{ij}	amount of demand (i, j), $(i, j) \in D$
T_m	maximum makespan
variables	**definitions**
u_w^k	whether vehicle k loading products of w type
x_{ir}^k	whether vehicle k serve machine i as its r-th task
ψ_{ir}^k	reserve of last task of vehicle k in r-th one
ϵ_r^k	waiting time of vehicle k before r-th task
g_{ir}	avaliable time of machine i in r-th task
T_f	gernal finish time
	$w \in W, i, j \in V, k \in K, r \in R$

Unrelated parallel machine scheduling problem with the objective of minimizing the maximal completion time($R||C_{\max}$) and restrict PDP

objective

$$\min T_f \tag{1}$$

subject to

orgin and destination

$$x_{v_0,0}^k, x_{v_0,|R|+1}^k = 1, \forall k \in K \tag{2}$$

$$x_{v_0,r}^k, x_{i,0}^k = 0, \forall i \in V, k \in K, r \in R \tag{3}$$

single occupied

$$\sum_{i \in V, k \in K} x_{ir}^k \leq \sum_{i \in V, k \in K} \psi_{ir}^k = 1, \forall r \in R \tag{4}$$

$$\sum_{i \in V, k \in K} x_{ir}^k \geq \sum_{i \in V, k \in K} x_{i,r+1}^k, \forall r \in R \tag{5}$$

single vehicle sequence

$$\psi_{ir}^k = x_{ir}^k + \psi_{i,r-1}^k \cdot (1 - \sum_{j \in V} x_{jr}^k), \forall i \in V, k \in K, r \in R \tag{6}$$

$$g_{i,r-1} - T_m \cdot \sum_{k \in K} x_{ir}^k \leq g_{ir} \leq g_{i,r-1} + T_m \cdot \sum_{k \in K} x_{ir}^k, \quad \forall i \in V, r \in R \tag{7}$$

$$\epsilon_r^k \leq T_m \cdot \sum_{i \in V} x_{ir}^k, \forall r \in R, k \in K \tag{8}$$

compatibility constraints

$$\sum_{w \in W} u_w^k \leq 1, \forall k \in K \tag{9}$$

$$\psi_{i,r-1}^k \cdot x_{jr}^k \leq u_w^k, \forall (i,j) \in D_w, w \in W, r \in R, k \in K \tag{10}$$

time

$$\sum_{r \in R} (\sum_{i \in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j \in V} t_{ij} \cdot \psi_{i,r-1}^k \cdot x_{jr}^k + \epsilon_r^k) \leq T_f, \forall k \in K \tag{11}$$

fulfill demand

$$\sum_{k \in K, r \in R} c_k \cdot \psi_{i,r-1}^k \cdot x_{jr}^k \geq q_{ij}, \forall (i,j) \in D \tag{12}$$

one by one

$$\begin{aligned}\sum_{r \in \{1,..,\gamma\}} (\sum_{i \in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j \in V} t_{ij} \cdot \psi_{i,r-1}^k \cdot x_{jr}^k + \epsilon_r^k) \geq \\ \sum_{i \in V} g_{i,\gamma} \cdot x_{i,\gamma}^k - T_m \cdot (1 - \sum_{i \in V} x_{ir}^k), \forall k \in K, \gamma \in R\end{aligned} \tag{13}$$

avaliable time

$$\begin{aligned}&\sum_{i \in V} g_{i,\gamma+1} \cdot x_{i,\gamma}^k - T_m \cdot (1 - \sum_{i \in V} x_{ir}^k) \\ &\leq \sum_{r \in \{1,..,\gamma\}} (\sum_{i \in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j \in V} t_{ij} \cdot \psi_{i,r-1}^k \cdot x_{jr}^k + \epsilon_r^k) + \sum_{i \in V} s_i^k \cdot x_{i,r}^k \\ &\leq \sum_{i \in V} g_{i,\gamma+1} \cdot x_{i,\gamma}^k + T_m \cdot (1 - \sum_{i \in V} x_{ir}^k), \forall k \in K, \gamma \in R\end{aligned} \tag{14}$$

$$g_{ir} \leq T_m, \forall i \in V, r \in R \tag{15}$$

domain of definition

$$u_k^w, x_{ir}^k \in \{0,1\}, \forall w \in W, i \in V, r \in R, k \in K \tag{16}$$

$$\epsilon_r^k, g_{ir}, \tau_s^k, T_f \geq 0, \forall i \in V, k \in K, r \in R \tag{17}$$

Add Y, Z, Π

Table 2: Instrumental variables

variables	definitions
y_{ijr}^k	$\psi_{i,r-1}^k \cdot x_{jr}^k$
z_{ir}^k	$g_{ir} \cdot x_{ir}^k$
π_{ir}^k	$g_{i,r+1} \cdot x_{ir}^k$
	$\forall i, j \in V, r \in R, k \in K$

convert nonlinear to linear

$$\begin{cases} y_{ijr}^k \leq \psi_{i,r-1}^k \\ y_{ijr}^k \leq x_{j,r}^k \\ y_{ijr}^k \geq \psi_{i,r-1}^k + x_{j,r}^k - 1 \end{cases}, \forall i, j \in V, k \in K, r \in R \tag{18}$$

$$\begin{cases} z_{ir}^k \leq T_m \cdot x_{ir}^k \\ z_{ir}^k \leq g_{ir} \\ z_{ir}^k \geq T_m \cdot (x_{ir}^k - 1) + g_{ir} \end{cases}, \forall i \in V, r \in R, k \in K \tag{19}$$

$$\begin{cases} \pi_{ir}^k \leq T_m \cdot x_{ir}^k \\ \pi_{ir}^k \leq g_{i,r+1} \\ \pi_{ir}^k \geq T_m \cdot (x_{ir}^k - 1) + g_{i,r+1} \end{cases}, \forall i \in V, r \in R, k \in K \tag{20}$$

single vehicle sequence

$$\psi_{ir}^k = x_{ir}^k + \psi_{i,r-1}^k - \sum_{j \in V} y_{ijr}^k, \forall i \in V, k \in K, r \in R \tag{21}$$

compatibility constraints

$$y_{ijr}^k \leq u_w^k, \forall (i,j) \in D_w, r \in R, k \in K \tag{22}$$

time

$$\sum_{r \in R} (\sum_{i \in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j \in V} t_{ij} \cdot y_{ijr}^k + \epsilon_r^k) \leq T_f, \forall k \in K \tag{23}$$

fulfill demand

$$\sum_{k \in K, r \in R} c_k \cdot y_{ijr}^k \geq q_{ij}, \forall (i,j) \in D \tag{24}$$

one by one

$$\begin{aligned} \sum_{r \in \{1,..,\gamma\}} (\sum_{i \in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j \in V} t_{ij} \cdot y_{ijr}^k + \epsilon_r^k) \geq \\ \sum_{i \in V} z_{ir}^k + T_m \cdot (1 - \sum_{i \in V} x_{ir}^k), \forall k \in K, \gamma \in R \end{aligned} \tag{25}$$

avaliable time

$$\begin{aligned} &\sum_{i \in V} \pi_{ir}^k - T_m \cdot (1 - \sum_{i \in V} x_{ir}^k) \\ &\leq \sum_{r \in \{1,..,\gamma\}} (\sum_{i \in V} s_i^k \cdot x_{i,r-1}^k + \sum_{i,j \in V} t_{ij} \cdot y_{ijr}^k + \epsilon_r^k) + \sum_{i \in V} s_i^k \cdot x_{i,r}^k \\ &\leq \sum_{i \in V} \pi_{ir}^k + T_m \cdot (1 - \sum_{i \in V} x_{ir}^k), \forall k \in K, \gamma \in R \end{aligned} \tag{26}$$

domain of definition

$$y_{ijr}^k \in \{0,1\}, \forall i,j \in V, r \in R, k \in K \tag{27}$$

$$z_{ir}^k, \pi_{ir}^k \geq 0, \forall i \in V, r \in R, k \in K \tag{28}$$

4. XXXX Algorithm

Heuristic Benders algorithm
master-problem (MP): U, X, Ψ, Y

$$\min 0 + \overline{T_f} \tag{29}$$

s.t. (2), (3), (4), (5), (18), (21), (9), (22), (24) and cuts from sub-problem.
There is no objection in master-problem, just find the feasible solution.
sub-problem (SP): E, G, T_f, Z, Π

$$\min T_f \tag{30}$$

s.t. (7), (8), (19), (20), (23), (24), (25), (26)
try something

$$\max_k\{A_k+B_k+C_k\} \leq \min\{\max_k\{A_k+B_k\}+\max_k\{C_k\}, \max_k\{A_k\}+\max_k\{B_k+C_k\}\} \tag{31}$$

5. Computational analysis

6. Conclusion and future extensions

References