Chapter 14: Recursion

Instructor: Mark Edmonds

edmonds_mark@smc.edu

Recursion

- We've also talked about functions and the caller/callee relationship
- So what if we have a function call itself?
  - I.e. the caller is the same function as the callee
- This is known as *recursion* and it one of the most powerful ways to control a program.
- The intuition behind this is that we can solve a big problem by breaking it down into a smaller problem.
  - Recursion

Base Cases

- If a function is going to call itself, how will the function eventually stop calling itself?
- If the function doesn't have a way to stop calling itself, the function will call itself for forever (essentially an infinite loop) until your computer runs out of resources.
- We fix this problem by creating a **base case** that doesn't call the function again

Example: A factorial function

- Definition of a factorial:

$$n! = \prod_{k=1}^{n} k$$

- How can we write this in a recursive manner?
- How can we write the factorial of n as a function of the factorial of $n-1$?

$$n! = n * (n-1)!$$

- Ok, so we can write the factorial of n as a function of the factorial of $n-1$. But what should the base case be?
  - When $n=1$, we stop
- In C++, this code is incredibly simple to write:

```
int factorial(int n){
  // base case
  if(n == 1)
  {
    return 1;
  }
  // otherwise, recurs into factorial(n * 1) (this is called the
     recursive case)
  else
  {
    return n * factorial(n-1);
  }
}
```

- How would you write this function using a for loop?

```
int factorial_loop(int n){
  int fac = 1;
  for (int i = 1; i <=n; i++){
    fac *= i;
  }
  return fac;
}
```

- The parallel to the **base case** is the **recursive case** where the function calls itself.
- The recursive case should make some progress towards the base case, otherwise the program may never terminate

The Fibonacci Sequence

- Fibonacci (introduced the idea in 1202) wondered a simple question has an interesting mathematical formulation: how many rabbits could be born in a year?
- He assumed the following conditions:
  - Begin with one male rabbit and female rabbit that have just been born.
  - Rabbits reach sexual maturity after one month.
  - The gestation period of a rabbit is one month. (How long it takes to give birth - for humans it's 9 months typically)
  - After reaching sexual maturity, female rabbits give birth every month.
  - A female rabbit gives birth to one male rabbit and one female rabbit.
  - Rabbits do not die.

- This is best shown with this diagram:

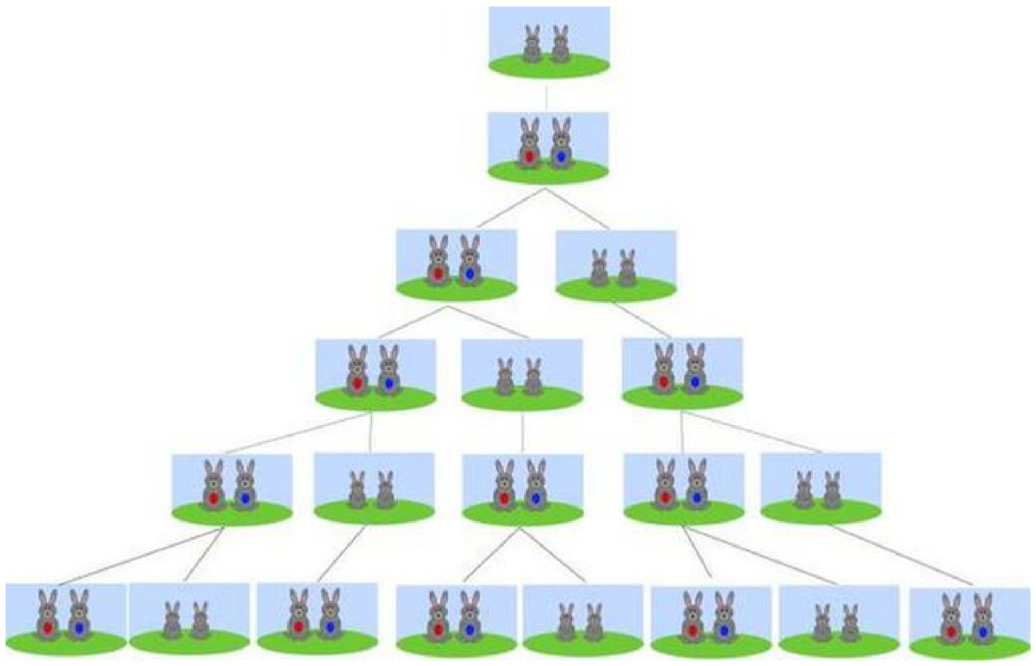

Figure 1: fibonacci_rabbits.jpg

- After one month, the first pair is not yet at sexual maturity and can't mate.
- At two months, the rabbits have mated but not yet given birth, resulting in only one pair of rabbits.
- After three months, the first pair will give birth to another pair, resulting in two pairs.
- At the fourth month mark, the original pair gives birth again, and the second pair mates but does not yet give birth, leaving the total at three pair.
- This continues until a year has passed, in which there will be 233 pairs of rabbits.
- Why Care?
  - Fibonacci's observation extends far beyond breeding rabbits. This pattern shows up in nature everywhere - growth pattern of sunflower seeds, hurricanes, galaxies. Tons of spirals in nature follow this pattern

Formal definition

- $f_n = f_{n-1} + f_{n-2}$
- Initial values at 1 and 2 for f_{n-1} and f_{n-2}, respectively (this is a hint for our base case!
  - $f(0) = 0$
  - $f(1) = 1$
  - $f(n) = f(n-1) + f(n-2)$

- How would you write the recursive version to output

```
int fib(int n){
  if (n == 0)
  {
    return 0;
  }
  else if (n == 1)
  {
    return 1;
  }
  else
  {
    return fib(n-1) + fib(n-2);
  }
}
```

- Think about how this executes in terms of caller/callee
  - The recursive call chases down a ‘rabbit hole’ to get to the base cases, and then starts to return values up to the initial caller, where n is the initial input.
- How would you write this function using a for loop?

```
int fib_loop(int n){
  int first = 0, second = 1, next;
  for (int i = 0 ; i <= n ; i++ )
  {
    if ( i <= 1 )
    {
      next = i;
    }
    else
    {
      next = first + second;
      first = second;
      second = next;
    }
  }
  return next;
}
```

- Possible to write using a loop, but less clear, and farther away from the underlying math.

When to use recursion?

- Often, a problem can be solved using iteration or recursion.
- I would recommend using recursion when writing the iterative solution is overly complex, but sticking to using iteration by default.
- Recursion is actually more resource intensive than iteration, because when a function is called, it enters the *call stack* which requires memory to be allocated. So every recursive call uses memory, something that isn't guaranteed to happen with iterative solutions
  - So iteration can be more efficient
- On the other hand, some problems can be beautifully solved with recursion, but are hard to write or hard to read in an iterative fashion.
  - Fibonacci is the classic example; a recursive solution is much easier to read and think about than the iterative version.

Exercises

1. Write a recursive function that computes the sum of all numbers from 1 to n, where n is given as parameter.

```cpp
#include<iostream>

using namespace std;

int sum_of_range(int);

int main()
{
  int n;
  int sum;

  cout << "Input the last number of the range starting from 1: ";
  cin >> n;

  sum = sum_of_range(n);
  cout << "The sum of numbers from 1 to " << n << " : " << sum << endl;

  return 0;
}

int sum_of_range(int n)
{
  if (n == 1)
  {
    return 1;
  }
  else
  {
    return n + sum_of_range(n - 1);
  }
}
```

2. Write a program in C to count the digits of a given number using recursion

```
#include <iostream>

using namespace std;

int num_digits(int n, int count);

int main()
{
  int n, count = 0;
  cout << "Input  a number: ";
  cin >> n;

  count = num_digits(n, count);

  cout << "The number of digits in the number is : " << count << endl;
  return 0;
}

int num_digits(int n){
  if (n < 10)
  {
    return 1;
  }
  else
  {
    return 1 + num_digits(n/10);
  }
}
```

3. Write a program in C to convert a decimal number to a binary number using recursion.

```cpp
#include <iostream>

using namespace std;

long convert_to_binary(int decimal, long binary, long factor);

int main()
{
  long binary = 0;
  int decimal;

  cout << "Input any decimal number: ";
  cin >> decimal;

  // seed a binary value of 0 and a factor of 1
  binary = convert_to_binary(decimal, 0, 1);
  cout << "The Binary value of decimal number " << decimal << " is: "
      << binary << endl;
  return 0;
}
long convert_to_binary(int decimal, long binary, long factor)
{
  long binary_digit;

  if (decimal == 0)
  {
    return binary;
  }
  else
  {
    binary_digit = decimal % 2;
    binary = binary + binary_digit * factor;
    factor = factor * 10;
    return convert_to_binary(decimal / 2, binary, factor);
  }
}
```