COMP 3105 – Assignment 1 Report
– Fall 2025 –

Due: Sunday September 28, 2025 23:59.

Group 51
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Question 1: (7.5%) Linear Regression

(a) (1%) L_2 Regression
Please see A1codes.py for implementation.

(b) (3%) L_∞ Regression

(b.1) (0.25%) For the objective function, we want $\mathbf{c}^T\mathbf{u} = \delta$. What should $\mathbf{c} \in \mathbb{R}^{d+1}$ be?
Recall that $\mathbf{u} = \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix}$
For the objective function to be δ we have:
$\mathbf{c}^T\mathbf{u}$
$= [c_1, c_2, \ldots c_d, c_{d+1}] \cdot \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \\ \delta \end{bmatrix}$
$= c_1w_1 + c_2w_2 + \ldots + w_du_d + c_{d+1}\delta$
Now let $c_1 = c_2 = \ldots = c_d = 0$
And $c_{d+1} = 1$
This gives us:
$0w_1 + 0w_2 + \ldots 0w_d + \delta$
$= \delta.$
Thus $\mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$
Where $c_1 = c_2 = \ldots = c_d = 0$ and $c_{d+1} = 1$.

(b.2) (0.25%) We want $G^{(1)}\mathbf{u} \preceq \mathbf{h}^{(1)} \iff \delta \geq 0$. What should $G^{(1)} \in \mathbb{R}^{1\times(d+1)}$ and $h^{(1)} \in \mathbb{R}$ be?
Here $G\mathbf{u}$ symbolizes the constraints we have for $\delta \geq 0$ and the linear equations formed by $X\mathbf{w} - \mathbf{y} \preceq \delta \cdot \mathbf{1_n}$ and $\mathbf{y} - X\mathbf{w} \preceq \delta \cdot \mathbf{1_n}$. So, the first row of G should symbolize our first constraint $\delta \geq 0$. Since the inequality $G\mathbf{u} \preceq \mathbf{h}$, we can test our first constraint be setting the inner product between G_1 and $\mathbf{u}$ to zero and have h_1 be δ. This gives us:
$G^{(1)} \in \mathbb{R}^{1\times(d+1)} = [0, 0, \ldots, 0]$ and
$\mathbf{h}^{(1)} \in \mathbb{R} = \delta.$
This results in a first linear equation of:
$0w_1 + 0w_2 + \ldots + 0w_d + 0\delta \preceq \delta$
$0 \preceq \delta$
$\delta \geq 0$

(b.3) (0.25%) We want $G^{(2)}\mathbf{u} \preceq \mathbf{h}^{(2)} \iff X\mathbf{w} - \mathbf{y} \preceq \delta \cdot \mathbf{1_n}$. What should $G^{(2)} \in \mathbb{R}^{n\times(d+1)}$ and $h^{(2)} \in \mathbb{R}^n$ be? Recall we want

$$G \cdot \mathbf{u} = \begin{bmatrix} G^{(1)} \\ G^{(2)} \\ G^{(3)} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \preceq \begin{bmatrix} \mathbf{h^{(1)}} \\ \mathbf{h^{(2)}} \\ \mathbf{h^{(3)}} \end{bmatrix} = \mathbf{h}$$

Note that $G^{(2)}$ is in $\mathbb{R}^{n\times(d+1)}$. Our second constraint $X\mathbf{w} - \mathbf{y} \preceq \delta \cdot \mathbf{1_n}$ which gives us:

$$\begin{bmatrix} x_1^T \cdot \mathbf{w} \\ x_2^T \cdot \mathbf{w} \\ \vdots \\ x_n^T \cdot \mathbf{w} \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \preceq \begin{bmatrix} \delta \\ \delta \\ \vdots \\ \delta \end{bmatrix}$$

Which gives us the following linear equations:

$x_1^T\mathbf{w} - y_1 \preceq \delta$

$x_2^T\mathbf{w} - y_2 \preceq \delta$

$\vdots$

$x_n^T\mathbf{w} - y_n \preceq \delta$

Let x_ij denote the i^{th} row and j^{th} column of matrix $X = \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix}$ for $i \in \{1, 2, \dots, n\}$ and $j \in \{1, 2, \dots, d\}$ We can map this into the form $G^{(2)} \cdot \mathbf{u} \preceq \mathbf{h}^{(2)}$ where

$$G^{(2)} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1d} & -1 \\ x_{21} & x_{22} & \dots & x_{2d} & -1 \\ \vdots \\ x_{n1} & x_{n2} & \dots & x_{nd} & -1 \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix}$$

$$\mathbf{h}^{(2)} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$G^{(2)} \cdot \mathbf{u} \preceq \mathbf{h}^{(2)}$

$$\begin{bmatrix} x_{11} & x_{12} & \dots & x_{1d} & -1 \\ x_{21} & x_{22} & \dots & x_{2d} & -1 \\ \vdots \\ x_{n1} & x_{n2} & \dots & x_{nd} & -1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \preceq \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$x_1^T \cdot \mathbf{w} + (-\delta) \preceq y_1$

$x_2^T \cdot \mathbf{w} + (-\delta) \preceq y_2$

$\vdots$

$x_n^T \cdot \mathbf{w} + (-\delta) \preceq y_n$
Which gives us our original constraint:
$x_1^T\mathbf{w} - y_1 \preceq \delta$
$x_2^T\mathbf{w} - y_2 \preceq \delta$
$\vdots$
$x_n^T\mathbf{w} - y_n \preceq \delta$

(b.4) (0.25%) We want $G^{(3)}\mathbf{u} \preceq \mathbf{h}^{(3)} \iff \mathbf{y} - X\mathbf{w} \preceq \delta \cdot \mathbf{1_n}$. What should $G^{(3)} \in \mathbb{R}^{n\times(d+1)}$ and $h^{(3)} \in \mathbb{R}^n$ be? Similar to (b.3) we can model our constraint in terms of G and $\mathbf{u}$.

Again let x_ij denote the i^{th} row and j^{th} column of matrix $X = \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix}$ for $i \in \{1, 2, \ldots, n\}$ and $j \in \{1, 2, \ldots, d\}$.
Our constraint is in the form:
$y_1 - (x_{11}w_1 + x_{12}w_2 + \ldots + x_{1d}w_d) \preceq \delta$
$y_2 - (x_{21}w_1 + x_{22}w_2 + \ldots + x_{2d}w_d) \preceq \delta$
$\vdots$
$y_n - (x_{n1}w_1 + x_{n2}w_2 + \ldots + x_{nd}w_d) \preceq \delta$
Now we map where $G^{(3)} = \begin{bmatrix} -x_{11} & -x_{12} & \ldots & -x_{1d} & -1 \\ -x_{21} & -x_{22} & \ldots & -x_{2d} & -1 \\ \vdots \\ -x_{n1} & -x_{n2} & \ldots & -x_{nd} & -1 \end{bmatrix}$

$\mathbf{u} = \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix}$

$\mathbf{h}^{(3)} = \begin{bmatrix} -y_1 \\ -y_2 \\ \vdots \\ -y_n \end{bmatrix}$

$G^{(3)} \cdot \mathbf{u} \preceq \mathbf{h}^{(3)}$

$$\begin{bmatrix} -x_{11} & -x_{12} & \ldots & -x_{1d} & -1 \\ -x_{21} & -x_{22} & \ldots & -x_{2d} & -1 \\ \vdots \\ -x_{n1} & -x_{n2} & \ldots & -x_{nd} & -1 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \preceq \begin{bmatrix} -y_1 \\ -y_2 \\ \vdots \\ -y_n \end{bmatrix}$$

$-(x_1^T \cdot \mathbf{w}) + (-\delta) \preceq -y_1$
$-(x_2^T \cdot \mathbf{w}) + (-\delta) \preceq -y_2$
$\vdots$
$-(x_n^T \cdot \mathbf{w}) + (-\delta) \preceq -y_n$
Which gives us our original constraint:
$y_1 - (x_1^T \cdot \mathbf{w}) \preceq \delta$

$y_2 - (x_2^T \cdot \mathbf{w}) \preceq \delta$
$\vdots$
$y_n - (x_n^T \cdot \mathbf{w}) \preceq \delta$
$\therefore$ we have the following linear programming optimization solvable by the cxvopt linear programming solver in the form:

$$\min_u \mathbf{c}^T \mathbf{u}$$
$$\text{s.t } G\mathbf{u} \preceq \mathbf{h}$$

where

$$G = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ x_{11} & x_{12} & \dots & x_{1d} & -1 \\ x_{21} & x_{22} & \dots & x_{2d} & -1 \\ \vdots & & & & \\ x_{n1} & x_{n2} & \dots & x_{nd} & -1 \\ -x_{11} & -x_{12} & \dots & -x_{1d} & -1 \\ -x_{21} & -x_{22} & \dots & -x_{2d} & -1 \\ \vdots & & & & \\ -x_{n1} & -x_{n2} & \dots & -x_{nd} & -1 \end{bmatrix}$$

$$\mathbf{u} = \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix}$$

$$h = \begin{bmatrix} \delta \\ y_1 \\ y_2 \\ \vdots \\ y_n \\ -y_1 \\ -y_2 \\ \vdots \\ -y_n \end{bmatrix}$$