

COMP 3105 – Assignment 1 Report

– Fall 2025 –

Due: Sunday September 28, 2025 23:59.

Group 51

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Question 1: (7.5%) Linear Regression

(a) (1%) L_2 Regression

Please see A1codes.py for implementation.

(b) (3%) L_∞ Regression

Here we are going to solve the L_∞ loss regression problem

$$\mathbf{w} = \operatorname{argmin}_{\mathbf{w} \in \mathbb{R}^d} ||X\mathbf{w} - \mathbf{y}||$$

Recall that this optimization can be expressed as a linear programming problem with

the joint parameters $\begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \in \mathbb{R}^{d+1}$ as follows

$$\begin{aligned} & \min_{\mathbf{w}, \delta} \\ & \text{s.t.} \\ & \delta \geq 0 \iff -\delta \leq 0 \\ & X\mathbf{w} - \mathbf{y} \preceq \delta \cdot \mathbf{1}_n \iff X\mathbf{w} - \delta \cdot \mathbf{1}_n \preceq \mathbf{y} \\ & \mathbf{y} - X\mathbf{w} \preceq \delta \cdot \mathbf{1}_n \iff -X\mathbf{w} - \delta \cdot \mathbf{1}_n \preceq -\mathbf{y} \end{aligned}$$

In the following answers, we will convert the optimization to a form that is solvable by the **cxvopt** linear programming (LP) solver, which solves the following form of LP

$$\begin{aligned} & \min_{\mathbf{u}} \mathbf{c}^T \mathbf{u} \\ & \text{s.t. } G\mathbf{u} \preceq \mathbf{h} \end{aligned}$$

Let the unknown variables be $\mathbf{u} = \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \in \mathbb{R}^{d+1}$

For the constraints, since we have three sets of constraints, the matrix G and \mathbf{h} can be decomposed into three parts

$$G \cdot \mathbf{u} = \begin{bmatrix} G^{(1)} \\ G^{(2)} \\ G^{(3)} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \preceq \begin{bmatrix} \mathbf{h}^{(1)} \\ \mathbf{h}^{(2)} \\ \mathbf{h}^{(3)} \end{bmatrix}$$

(b.1) (0.25%) For the objective function, we want $\mathbf{c}^T \mathbf{u} = \delta$. What should $\mathbf{c} \in \mathbb{R}^{d+1}$ be?

Recall that $\mathbf{u} = \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix}$

For the objective function to be δ we have:

$$\mathbf{c}^T \mathbf{u} = [c_1, c_2, \dots, c_d, c_{d+1}] \cdot \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \\ \delta \end{bmatrix} = c_1 w_1 + c_2 w_2 + \dots + w_d u_d + c_{d+1} \delta$$

Now let $c_1 = c_2 = \dots = c_d = 0$

And $c_{d+1} = 1$

This gives us:

$$0w_1 + 0w_2 + \dots + 0w_d + \delta = \delta.$$

$$\text{Thus } \mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Where $c_1 = c_2 = \dots = c_d = 0$ and $c_{d+1} = 1$.

(b.2) (0.25%) We want $G^{(1)} \mathbf{u} \preceq \mathbf{h}^{(1)} \iff \delta \geq 0$. What should $G^{(1)} \in \mathbb{R}^{1 \times (d+1)}$ and $\mathbf{h}^{(1)} \in \mathbb{R}$ be?

Recall the first constraint: $-\delta \leq 0$

We want

$$\underset{1 \times (d+1)}{G^{(1)}} \cdot \mathbf{u} \preceq \underset{1 \times 1}{\mathbf{h}^{(1)}}$$

We will now map our constraint into this form.

$$\begin{bmatrix} G^{(11)}_{1 \times d} & G^{(12)}_{1 \times 1} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \preceq \underset{n \times 1}{\mathbf{h}^{(1)}} \iff -\delta \leq 0$$

$$\underset{1 \times d}{G^{(11)}} \cdot \mathbf{w} + \underset{1 \times 1}{G^{(12)}} \cdot \delta = -\delta \leq 0 = \underset{1 \times 1}{\mathbf{h}^{(1)}}$$

So,

$$\underset{1 \times d}{G^{(11)}} = \mathbf{0}_{1 \times d}$$

$$\underset{1 \times 1}{G^{(12)}} = -1$$

$$\underset{1 \times (d+1)}{G^{(1)}} = [\mathbf{0}_{1 \times d} \quad -1]$$

$$\underset{1 \times 1}{\mathbf{h}^{(1)}} = \mathbf{0}_{1 \times 1}$$

(b.3) (0.25%) We want $G^{(2)} \mathbf{u} \preceq \mathbf{h}^{(2)} \iff X\mathbf{w} - \mathbf{y} \preceq \delta \cdot \mathbf{1}_n$. What should $G^{(2)} \in \mathbb{R}^{n \times (d+1)}$ and $\mathbf{h}^{(2)} \in \mathbb{R}^n$ be?

Recall our second constraint is

$$X\mathbf{w} - \mathbf{y} \preceq \delta \cdot \mathbf{1}_n \iff X\mathbf{w} - \delta \cdot \mathbf{1}_n \preceq \mathbf{y}$$

We want

$$\underset{n \times (d+1)}{G^{(2)}} \cdot \mathbf{u} \preceq \underset{n \times 1}{\mathbf{h}^{(2)}}$$

We will now map our constraint into this form.

$$\begin{aligned} \begin{bmatrix} G_{n \times d}^{(21)} & G_{n \times 1}^{(22)} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \preceq \mathbf{h}_{n \times 1}^{(2)} &\iff X\mathbf{w} - \delta \cdot \mathbf{1}_n \preceq \mathbf{y} \\ G_{n \times d}^{(21)} \cdot \mathbf{w} + G_{n \times 1}^{(22)} \cdot \delta = X\mathbf{w} - \delta \cdot \mathbf{1}_n \preceq \mathbf{y} = \mathbf{h}_{n \times 1}^{(1)} \end{aligned}$$

So,

$$G_{n \times d}^{(21)} = X$$

$$G_{n \times 1}^{(22)} = -\mathbf{1}_{n \times 1}$$

$$G_{n \times (d+1)}^{(2)} = \begin{bmatrix} X & -\mathbf{1}_{n \times 1} \end{bmatrix}$$

$$\mathbf{h}_{n \times 1}^{(2)} = \mathbf{y}$$

- (b.4) (0.25%) We want $G^{(3)}\mathbf{u} \preceq \mathbf{h}^{(3)} \iff \mathbf{y} - X\mathbf{w} \preceq \delta \cdot \mathbf{1}_n$. What should $G^{(3)} \in \mathbb{R}^{n \times (d+1)}$ and $\mathbf{h}^{(3)} \in \mathbb{R}^n$ be?

Recall our third constraint is

$$\mathbf{y} - X\mathbf{w} \preceq \delta \cdot \mathbf{1}_n \iff -X\mathbf{w} - \delta \cdot \mathbf{1}_n \preceq -\mathbf{y}$$

We want

$$G_{n \times (d+1)}^{(3)} \cdot \mathbf{u} \preceq \mathbf{h}_{n \times 1}^{(3)}$$

We will now map our constraint into this form.

$$\begin{aligned} \begin{bmatrix} G_{n \times d}^{(31)} & G_{n \times 1}^{(32)} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \preceq \mathbf{h}_{n \times 1}^{(3)} &\iff -X\mathbf{w} - \delta \cdot \mathbf{1}_n \preceq -\mathbf{y} \\ G_{n \times d}^{(31)} \cdot \mathbf{w} + G_{n \times 1}^{(32)} \cdot \delta = -X\mathbf{w} - \delta \cdot \mathbf{1}_n \preceq -\mathbf{y} = \mathbf{h}_{n \times 1}^{(3)} \end{aligned}$$

So,

$$G_{n \times d}^{(31)} = -X$$

$$G_{n \times 1}^{(32)} = -\mathbf{1}_{n \times 1}$$

$$G_{n \times (d+1)}^{(3)} = \begin{bmatrix} -X & -\mathbf{1}_{n \times 1} \end{bmatrix}$$

$$\mathbf{h}_{n \times 1}^{(3)} = -\mathbf{y}$$

- (b.5) (2%) Based on your derivations in (b), implement a Python function

$$\mathbf{w} = \text{minimizeLinf}(X, y)$$

that returns a $d \times 1$ vector of weights/parameters \mathbf{w} corresponding to the solution of minimum L_∞ loss.

$$\mathbf{w} = \operatorname{argmin}_{\mathbf{w} \in \mathbb{R}^d} \|X\mathbf{w} - \mathbf{y}\|$$

Based on our derivations in (b), we result in

$$\mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \in \mathbb{R}^{d+1}$$

$$\mathbf{u} = \begin{bmatrix} \mathbf{w} \\ \delta \end{bmatrix} \in \mathbb{R}^{d+1}$$

$$G = \begin{bmatrix} \mathbf{0}_{1 \times d} & -1 \\ X & -\mathbf{1}_n \\ -X & -\mathbf{1}_n \end{bmatrix} \in \mathbb{R}^{(2n+1) \times (d+1)}$$

$$\mathbf{h} = \begin{bmatrix} 0 \\ \mathbf{y} \\ -\mathbf{y} \end{bmatrix} \in \mathbb{R}^{2n+1}$$

$$\begin{matrix} & \text{s.t.} \\ G_{(2n+1) \times (d+1)} \cdot \mathbf{u}_{(d+1) \times 1} \preceq \mathbf{h}_{(2n+1) \times 1} \end{matrix}$$

Please see `A1codes.py` for full implementation.

(c) (2%) Synthetic Regression Problem

In this part, you will evaluate your implemented algorithms on a synthetic dataset.

(c.1) (1%) Implement a Python function

```
train loss, test loss = synRegExperiments()
```

that returns a 2 x 2 matrix train loss of average training losses and a 2 x 2 matrix test loss of average test losses (See Table 1 and Table 2 below.) It repeats 100 runs as follows

Please see **A1codes.py** for full details.

(c.2) (1%) Looking at your tables from above, analyze the results and discuss any findings you may have and the possible reason behind them.