Solve the following problems by hand, then implement in `numpy` to check your work and familiarize yourself with the library.

1. Given $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, find:

   (a) $4\mathbf{u} - 3\mathbf{v}$

   (b) $\mathbf{u} \cdot \mathbf{v}$

   (c) The angle θ between $\mathbf{u}$ and $\mathbf{v}$, knowing that $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$

   (d) Bonus: plot $\mathbf{u}$ and $\mathbf{v}$ using `matplotlib`. Does your angle θ make sense?

2. Consider a matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ and a vector $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

   (a) Compute $A\mathbf{b}$.

   (b) Find A^{-1}. For 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

   (c) Solve the equation $A\mathbf{x} = \mathbf{b}$ an unknown vector $\mathbf{x}$.