Jan 15: Math Review

1.

$$f(x) = \sigma(x) = \frac{1}{1+e^{-x}} = (\underbrace{1+e^{-x}}_{h(x)})^{-1}$$

chain rule: $g'(x) = -x^{-2}$, $g'(h(x)) = -(1+e^{-x})^{-2}$

$h'(x) = 0 + ?$ chain rule again! $e^{-x} = (\underbrace{e^{x}}_{h_1(x)})^{-1}$

$h'(x) = -(e^{x})^{-2} e^{x}$

$h'(x) = -e^{-x}$

$$\therefore f'(x) = \cancel{-}(1+e^{-x})^{-2}(\cancel{-}e^{-x})$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} \mathbin{/\!/}$$

Extra: sigmoid form

$$= \frac{e^{-x}}{(1+e^{-x})(1+e^{-x})} = \underbrace{\left(\frac{1}{1+e^{-x}}\right)}_{\sigma(x)}\left(\frac{e^{-x}}{1+e^{-x}}\right) = \sigma(x)\left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)$$

$$= \sigma(x)\left(\underbrace{\frac{1+e^{-x}}{1+e^{-x}}}_{1} - \underbrace{\frac{1}{1+e^{-x}}}_{\sigma(x)}\right) = \sigma(x)(1-\sigma(x)) \mathbin{/\!/}$$

2. $y = \sigma(x_1), \; x_1 = wx$

$$\frac{dy}{dx} = \frac{dy}{dx_1} \cdot \frac{dx_1}{dx} = \sigma(wx)(1-\sigma(wx)) \cdot w$$

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