# Molecular Gas Dynamics
## Midterm 2 - Part 1

Professor Marco Panesi

## Quasi-1D Steady Adiabatic Nozzle Flow

In this assignment, you will develop a Python code to solve for the steady quasi-one-dimensional adiabatic flow through a converging-diverging nozzle. You will implement the governing equations provided in class and utilize the provided routines to interpolate thermodynamic properties as functions of entropy and enthalpy. The method is discussed in detail in *Molecular Gas Dynamics* by Vincenti and Kruger (pages 183 and following); you are encouraged to consult this reference for guidance.

### Problem Statement

**Objective** Write a Python code to compute the flow properties (velocity, pressure, density, temperature, and Mach number) along the nozzle as a function of the area distribution $A(x)$.

#### Method

- Use the given thermodynamic property tables (in terms of entropy $s$ and enthalpy $h$) to obtain the necessary values for density $\rho$, pressure $p$, temperature $T$, and the speed of sound $a$. The provided routines should be used for interpolation.
- Employ the indirect method, where you assign a value of the enthalpy $h$, compute the corresponding flow velocity $u$, and then determine the corresponding nozzle area $A$ from the continuity equation. From the area distribution $A(x)$, determine the location $x$ along the nozzle.
- Additionally, compare your results with the frozen flow approach, where the specific heat ratio $\gamma$ is assumed constant. Compute the flow properties using both the local thermodynamic equilibrium (LTE) approach and the frozen flow approach, and analyze the differences.

#### Governing Equations

- **Continuity**: $\rho u A = \text{constant}$

- **Energy**: $h + \frac{1}{2}u^2 = h_0$
- **Isentropic Flow**: $s = s_0$

**Reservoir Conditions** Use the following reservoir (stagnation) conditions:

- Total pressure: $p_0 = 5,000,000$ Pa
- Total temperature: $T_0 = 4500$ K

From these, determine the initial enthalpy $h_0$ and entropy $s_0$ for the LTE approach using the provided interpolation routines.

For the frozen flow approach, use the following properties:

- Gas constant:
$$R = \frac{8.3144598}{0.02672963120279829}$$
- Specific heat ratio:
$$\gamma = 1.242431$$

**Task**

1. Using the reservoir conditions, determine $h_0$ and $s_0$ for the LTE approach.
2. For each assigned value of enthalpy $h$ downstream of the reservoir, compute the corresponding velocity $u$ using the energy equation.
3. Calculate the corresponding area $A$ from the continuity equation.
4. Determine the location $x$ in the nozzle using the area distribution function $A(x)$.
5. Compute the flow properties (velocity $u$, pressure $p$, temperature $T$, density $\rho$, and Mach number $M$) along the nozzle using both the LTE approach and the frozen flow approach.
6. Plot the flow properties $u(x)$, $p(x)$, $T(x)$, $\rho(x)$, and $M(x)$ as functions of position along the nozzle for both the LTE and frozen flow approaches.
7. Analyze and explain the differences observed in the flow properties between the LTE and frozen flow models. Discuss your findings in the context of the methods described in Vincenti and Kruger.

**Expected Output**

- Plots of the flow properties $u(x)$, $p(x)$, $T(x)$, $\rho(x)$, and $M(x)$ along the nozzle for both the LTE and frozen flow approaches. Ensure that your plots are clearly labeled and include annotations indicating the throat location (where $A = A^*$) and the subsonic and supersonic regions.
- A comparative analysis explaining the results and discussing the differences between the LTE and frozen flow models.

## Guidance and Code Structure

To assist you in this assignment, the **main structure of the code is provided**. You are expected to write the necessary routines and complete the code as required.

### Functions to Implement

You are expected to write the following key functions:

- `create_reservoir_interpolator(...)`: Creates interpolators for thermodynamic properties for the reservoir.
- `get_reservoir_h_and_s(...)`: Computes enthalpy and entropy from pressure and temperature.
- `compute_hstar_sstar(...)`: Computes $h_*$ and $s_*$ at the throat.
- `compute_rho_star_astar_Fstar(...)`: Computes $a_*$, $\rho_*$, and $f = a_*\rho_*$ at the throat.
- `process_nozzle_indirect_method(...)`: Processes the nozzle flow using the indirect method for the LTE approach.
- `process_nozzle_perfect_gas(...)`: Processes the nozzle flow for the frozen flow approach.

## Additional Instructions

- Use the provided routines and data files to interpolate thermodynamic properties as functions of $h$ and $s$.
- Assume isentropic flow ($s = s_0$) throughout the nozzle for the LTE approach.
- Write modular code with clear functions for each part of the solution.
- Include comments and explanations in your code to clarify the steps.
- Ensure that your plots are clearly labeled, with legends distinguishing between LTE and frozen flow results.
- Read pages 183 and following from *Molecular Gas Dynamics* by Vincenti and Kruger to aid in your analysis and understanding of the flow characteristics.

## Data and Resources

You will be provided with:

- `output.dat`: Thermodynamic property tables for the gas mixture, providing $\rho(h, s)$, $p(h, s)$, $T(h, s)$, and $a(h, s)$.
- `area.dat`: The area distribution function $A(x)$ for the converging-diverging nozzle.
- Example routines for interpolating the thermodynamic properties.
- Reference: *Molecular Gas Dynamics* by Vincenti and Kruger.

## Submission Guidelines

- Submit your Python code along with a LaTeX report summarizing your methodology and findings.
- Include the plots of the flow properties with appropriate labels and annotations.
- Discuss any assumptions made and their potential impact on the results.
- Provide explanations and discussions of the results, focusing on the differences between the LTE and frozen flow approaches.
- Cite any references used, including the textbook by Vincenti and Kruger.