CS375 Homework 2

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Question 1

a

$$T(n) = 3T\left(\frac{n}{4}\right) + n,\ \text{a} = 3,\ \text{b} = 4,\ f(n) = n$$
$$n^{log_b a} = n^{log_4 3} = O(n^{0.793})$$
$$f(n) = \Omega(n^{log_4 3+\epsilon}), when\ \epsilon \approx 0.2$$
$$\text{Case 3 applies}$$
$$af\left(\frac{n}{b}\right) \leq f(n) \text{ for some } c < 1$$
$$3\left(\frac{n}{4}\right) \leq \left(\frac{3}{4}\right) n \text{ which holds true for } c = 3/4$$

$$\text{By case 3, the solution is } T(n) = \Theta(n)$$

b

$$T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}\lg n,\ \text{a} = 2,\ \text{b} = 4,\ f(n) = \sqrt{n}\lg n$$
$$n^{log_b a} = n^{log_4 2} = O(\sqrt{n})$$
$$\text{Case 2 applies, thus } T(n) = \Theta(\sqrt{n}\lg^2 n)$$

c

$$T(n) = 5T\left(\frac{n}{2}\right) + n^2,\ \text{a} = 5,\ \text{b} = 2,\ f(n) = n^2$$
$$n^{log_b a} = n^{log_2 5-\epsilon} = O(n^2), \text{ when } \epsilon \approx 0.3$$
$$\text{Case 1 applies, thus } T(n) = \Theta(n^{log_2 5})$$

Question 2

$$T(n) = \begin{cases} \Theta(1) & \text{for } n \leq 1 \\ T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right) + n & \text{otherwise} \end{cases}$$

$$\begin{array}{ccccc} & \text{cn} & & \rightarrow \text{cn} & 0 \\ & / \quad \backslash & & & \\ c\left(\frac{n}{4}\right) & & c\left(\frac{3n}{4}\right) & \rightarrow \text{cn} & 1 \\ \vdots & & \vdots & & \\ & & \text{T(1)} & \rightarrow \text{cn} & log_{\frac{4}{3}} n - 1 \end{array}$$

All columns added up: $\Theta(nlgn)$

Question 3

$T(n) = T(n-1) + n\epsilon O\left(n^2\right), T(1) = 1$

$$\begin{aligned} \text{We guess } T(n) &\leq cn^2 : \\ &\leq c(n-1)^2 + n \\ &\leq c(n^2 - 2n + 1) + n \\ &\leq cn^2 - 2cn + c + n \\ &\leq cn^2 \text{ for } c \geq 1 \end{aligned}$$

Question 4

a

Divide the array into $\left(\frac{n}{3}\right)$ sizes.
Conquer each subarray of $\left(\frac{n}{3}\right)$ sizes, comparing thrice on elements which partition the array.
Combine the array once the element has been found.

b

ternarySearch(x, A, left, right)
if $right = left$:
    return x
else if $x < A[left + \left(\frac{right-left}{3}\right)]$:
    return ternarySearch(x, A, left, $left + \left(\frac{right-left}{3}\right)$)
else if $x < A[\left(\frac{right-left}{3}\right) + 2\left(\frac{right-left}{3}\right)]$:
    return ternarySearch(x, A, left, $\left(\frac{right-left}{3}\right), 2\left(\frac{right-left}{3}\right)$)
else if $x < A[2\left(\frac{right-left}{3}\right) + 2\left(\frac{right-left}{3}\right)]$:
    return ternarySearch(x, A, left, $\left(\frac{right-left}{3}\right), 2\left(\frac{right-left}{3}\right)$)

c

$$T(n) = T(n/3) + 1$$

d

$$T(n) = T\left(\frac{n}{3}\right) + 1, \text{ a} = 1, \text{ b} = 3, \ f(n) = 1$$

$$\begin{aligned} n^{log_b a} = n^{log_3 1} &= O(1) \\ \text{Case 2 applies, thus } T(n) &= \Theta(\lg n) \end{aligned}$$

Question 5

a

Divide the array by picking a pivot point, and pushing elements greater than or equal to the pivot to the left, and greater than to the right.
Conquer the larger subarray, repeating the divide step.
Combine the array once the median element has been found.

Question 6 Bonus

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\lg n}$$

$$\begin{array}{ccccc}
 & c\left(\frac{n}{\lg n}\right) & & \rightarrow c\left(\frac{n}{\lg n}\right) & 0 \\
 & / \quad \backslash & & & \\
c\left(\frac{n}{2\lg n}\right) & & c\left(\frac{n}{2\lg n}\right) & \rightarrow c\left(\frac{n}{\lg n}\right) & 1 \\
\vdots & & \vdots & & \\
 & & \mathrm{T(1)} & \rightarrow c\left(\frac{n}{\lg n}\right) & \lg n - 1
\end{array}$$

All columns added up: $\Theta(\left(\frac{n}{\lg n}\right)(\lg n)) = \Theta(n)$