### **Homework 3**

### **Recitation Exercises**

```
In [1]: N support = lambda occurances, records: occurances / records
confidence = lambda supAB, supA: supAB / supA
```

#### 5.2

2. Consider the data set shown in Table 5.20 I.

Table 5.20. Example of market basket transactions

Customer ID	Transaction ID	Items Bought
1	0001	{a, d, e}
1	0024	{a, b, c, e}
2	0012	{a, b, d, e}
2	0031	{a, c, d, e}
3	0015	{b, c e}
3	0022	{b, d, e}
4	0029	{c d}
4	0040	{a, b, c}
5	0033	{a, d, e}
5	0038	{a, b, e}

- a. Compute the support for itemsets  $\{e\}$ ,  $\{b, d\}$ , and  $\{b, d, e\}$  by treating each transaction ID as a market basket.
- b. Use the results in part (a) to compute the confidence for the association rules  $\{b,\ d\} \to \{e\}$  and  $\{e\} \to \{b,\ d\}$ . Is confidence a symmetric measure?
- c. Repeat part (a) by treating each customer ID as a market basket. Each item should be treated as a binary variable (1 if an item appears in at least one transaction bought by the customer, and 0 otherwise).
- d. Use the results in part (c) to compute the confidence for the association rules  $\{b,\ d\} \to \{e\}$  and  $\{e\} \to \{b,\ d\}$ .
- e. Suppose  $s_1$  and  $c_1$  are the support and confidence values of an association rule r when treating each transaction ID as a market basket. Also, let  $s_2$  and  $c_2$  be the support and confidence values of r when treating each customer ID as a market basket. Discuss whether there are any relationships between  $s_1$  and  $s_2$  or  $c_1$  and  $c_2$ .

A:

```
In [2]: | print("Support for {e}: ", support (8, 10))
    print("Support for {b, d}: ", support (2, 10))
    print("Support for {b, d, e}: ", support (2, 10))

Support for {e}: 0.8
    Support for {b, d}: 0.2
    Support for {b, d, e}: 0.2
```

B:

• Confidence is evidently not a symetric measure, which is good, becuase it shows that if item set {b, d} is purchased we can say with confidence item set {e} will probably be purchased but if item set {e} is purchased it will not say much about wheather itemset {b, d} will be purchased

C:

```
In [5]: 
Arr print("Confidence for \{b, d\} -> \{e\} = s(\{b, d, e\}) / s(\{b, d\}) = ", confidence(support(4, 5), support(4, print("Confidence for <math>\{e\} -> \{b, d\} = s(\{b, d, e\}) / s(\{e\}) = ", confidence(support(4, 5), support(5, 5)))

Confidence for \{b, d\} -> \{e\} = s(\{b, d, e\}) / s(\{b, d\}) = 1.0

Confidence for \{e\} -> \{b, d\} = s(\{b, d, e\}) / s(\{e\}) = 0.8
```

E:

• When we use the transactionID as a market basket, we're looking to predict the assocation between two itemsets for a given transaction, but with CustomerID, we're trying to predict the association between two itemsets from a customer's purchase history. It would be dangerous to say that a high confidence or support in one may lead to high confidence or support in another. If a customer has ever bought a common item at anypoint in time say "batteries" and later buys another common item say "eggs" it is very easy to show high support/confidence. However if each customer had only purchased those items in a few transactions relative to their total transactions, the support/confidence would lower when using transactionID as a market basket.

#### 5.6

- 6. Consider the market basket transactions shown in Table 5.21
- a. What is the maximum number of association rules that can be extracted from this data (including rules that have zero support)?
- b. What is the maximum size of frequent itemsets that can be extracted (assuming minsup > 0)?

able	5.21.	Market	basket	transactions.

Transaction ID	Items Bought
1	{Milk, Beer, Diapers}
2	{Bread, Butter, Milk}
3	{Milk, Diapers, Cookies}
4	{Bread, Butter, Cookies}
5	{Beer, Cookies, Diapers}
6	{Milk, Diapers, Bread, Butter}
7	{Bread, Butter, Diapers}
8	{Beer, Diapers}
9	{Milk, Diapers, Bread, Butter}
10	{Beer, Cookies}

- c. Write an expression for the maximum number of size-3 itemsets that can be derived from this data set
- d. Find an itemset (of size 2 or larger) that has the largest support.
- e. Find a pair of items, a and b, such that the rules  $\{a\} o \{b\}$  and  $\{b\} o \{a\}$  have the same confidence.

A:

Max association rules: 602

B:

With minsup > 0, then we just need to find any rule with the most item sets. That would be: 4

C:

Max number of itemsets with size three from a data set of 6 unique items:  $\binom{6}{3} = 20$ 

D:

```
In [7]: N = 10
## for {bread, butter}
suppotCount = 5
print("Max support is {bread, butter} with support:", support(suppotCount, N))
```

Max support is {bread, butter} with support: 0.5

E:

```
We need to find confidences where s\{A, B\} / s\{A\} == s\{A, B\} / s(B)
```

```
S{A} == S{B}
```

{Beer} has 4 occurances

{Cookies} has 4 occurances

{Beer, Coookies} has 2 occurances

{Beer, Cookies} holds this property

In [8]: 
$$\mbox{\begin{tabular}{ll} print("Confidence of {A} -> {B} and {B} -> {A} =", confidence(support(2, 10), support(4, 10))) } \\ & \mbox{\confidence of {A} -> {B} and {B} -> {A} = 0.5} \\ \end{tabular}$$

### 5.8

8. Consider the following set of frequent 3-itemsets:

$$\{1,\,2,\,3\},\,\{1,\,2,\,4\},\,\{1,\,2,\,5\},\,\{1,\,3,\,4\},\,\{1,\,3,\,5\},\,\{2,\,3,\,4\},\,\{2,\,3,\,5\},\,\{3,\,4,\,5\}.$$

Assume that there are only five items in the data set.

- a. List all candidate 4-itemsets obtained by a candidate generation procedure using the  $F_{k-1} imes F_1$  merging strategy.
- b. List all candidate 4-itemsets obtained by the candidate generation procedure in Apriori.
- c. List all candidate 4-itemsets that survive the candidate pruning step of the *Apriori* algorithm.

A:

Frequent 2-Itemsets	Frequent 3-Itemsets	4-Itemset Candidate Generation
{1, 2}	{1, 2, 3}	{1, 2, 3, 4}
{1, 3}	{1, 2, 4}	{1, 2, 3, 5}
{2, 3}	{1, 2, 5}	{1, 2, 4, 5}
{1, 4}	{1, 3, 4}	{1, 3, 4, 5}
{2, 4}	{1, 3, 5}	{2, 3, 4, 5}
{1, 5}	{2, 3, 4}	
{2, 5}	{3, 4, 5}	
{3, 5}		
{3, 4}		
{4, 5}		

B:

Frequent 3-Itemsets	4-Itemset Candidate Generation
{1, 2, 3}	{1, 2, 3, 4}
{1, 2, 4}	{1, 2, 3, 5}
{1, 2, 5}	{1, 2, 4, 5}
{1, 3, 4}	{1, 3, 4, 5}
{1, 3, 5}	{2, 3, 4, 5}
{2, 3, 4}	
{3, 4, 5}	

C:

• The pruning process of apriori is to remove all candidates are not made up of entriely of frequent subsets. In the case of this problem, each 4-ltmeset candidate has 3 subsets, so it needs to be made up of 3 frequent subsets, of this candidates, the only one that makes it past this pruning process is:

 $\{1, 2, 3, 4\}$ 

5.9

9. The *Apriori* algorithm uses a generate-and-count strategy for deriving frequent itemsets. Candidate itemsets of size k+1 are created by joining a pair of frequent itemsets of size k (this is known as the candidate generation step). A candidate is discarded if any one of its subsets is found to be infrequent during the candidate pruning step. Suppose the *Apriori* algorithm is applied to the data set shown in **Table 5.22**  $\square$  with minsup=30%, i.e., any itemset occurring in less than 3 transactions is considered to be infrequent.

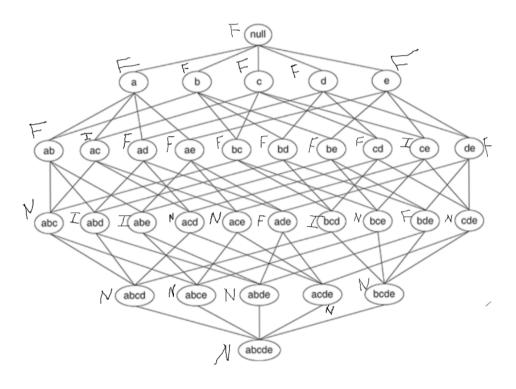
Table 5.22. Example of market basket transactions.

Transaction ID	Items Bought
1	{a, b, d, e}
2	{b, c d}
3	{a, b, d, e}
4	$\{a, c, d, e\}$
5	{b, c, d, e}
6	{b, d, e}
7	{c, d}
8	{a, b, c}
9	{a, d, e}
10	{b, d}

- a. Draw an itemset lattice representing the data set given in Table 5.22 🖳. Label each node in the lattice with the following letter(s):
  - N: If the itemset is not considered to be a candidate itemset by the Apriori algorithm. There are two reasons for an itemset not to be considered as a candidate itemset: (1) it is not generated at all during the candidate generation step, or (2) it is generated during the candidate generation step but is subsequently removed during the candidate pruning step because one of its subsets is found to be infrequent.
  - F: If the candidate itemset is found to be frequent by the *Apriori* algorithm.
  - I: If the candidate itemset is found to be infrequent after support counting.
- b. What is the percentage of frequent itemsets (with respect to all itemsets in the lattice)?
- c. What is the pruning ratio of the *Apriori* algorithm on this data set? (Pruning ratio is defined as the percentage of itemsets not considered to be a candidate because (1) they are not generated during candidate generation or (2) they are pruned during the candidate pruning step.)
- d. What is the false alarm rate (i.e., percentage of candidate itemsets that are found to be infrequent after performing support counting)?

#### A:

#### file:///home/aakef/Downloads/5-9-lattice.png



sketchpad.pro

### B:

 $\frac{16}{32} = 50\%$  of itemsets are frequent

### C:

The number of "N"s that are produced is 11, so  $\frac{11}{32} = 34\%$  of itemsets that were pruned

### D:

The number of "I"s that are produced is 5, so  $\frac{5}{32} = 16\%$  of itemsets that were labled as infrequent by the apriori algorithm after support counting

### 5.12

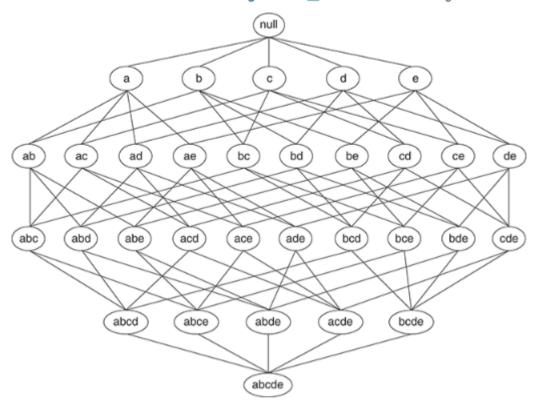


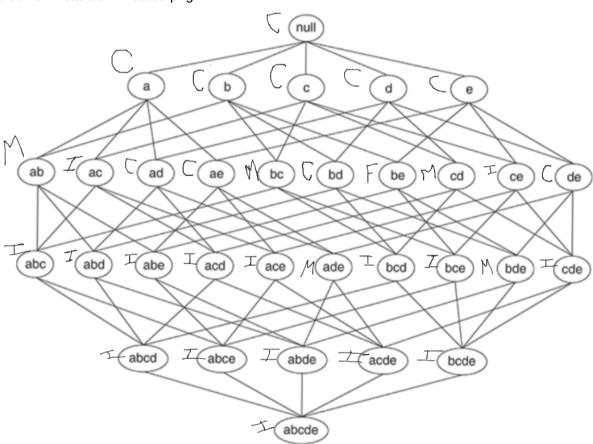
Figure 5.33.

An itemset lattice

- M if the node is a maximal frequent itemset,
- C if it is a closed frequent itemset,
- N if it is frequent but neither maximal nor closed, and
- I if it is infrequent

Assume that the support threshold is equal to 30%.

### file:///home/aakef/Downloads/5-12-lattice.png



sketchpad.pn

13. The original association rule mining formulation uses the support and confidence measures to prune uninteresting rules.

a. Draw a contingency table for each of the following rules using the transactions shown in Table 5.23  $\blacksquare$ .

Table 5.23. Example of market basket transactions.

Transaction ID	Items Bought
1	$\{a,b,d,e\}$
2	{b, c, d}
3	$\{a,b,d,e\}$
4	{a, c, d, e}
5	{b, c, d, e}
6	{b, d, e}
7	{c, a}
8	{a, b, c}
9	{a, d, e}
10	{b, a}

Rules:  $\{b\} 
ightarrow \{c\}, \ \{a\} 
ightarrow \{d\}, \ \{b\} 
ightarrow \{d\}, \ \{e\} 
ightarrow \{c\}, \ \{c\} 
ightarrow \{a\}.$ 

b. Use the contingency tables in part (a) to compute and rank the rules in decreasing order according to the following measures.

- i. Support.
- ii. Confidence.
- iii. Interest  $(X o Y) = rac{P(X,Y)}{P(X)} P(Y).$
- iv.  $\mathrm{IS}(X o Y)=rac{P(X,Y)}{\sqrt{P(X)P(Y)}}$
- v.  $\operatorname{Klosgen}(X o Y) = \sqrt{P(X, \ Y\ )} imes \max(P(Y|X)) P(Y), \ P(X|Y) P(X))$ , where  $P(Y\Big|X) = \frac{P(X, Y)}{P(X)}$ .
- vi. Odds ratio $(X o Y) = rac{P(X,Y)P(\overline{X},\ \overline{Y})}{P(X,\overline{Y})P(\overline{X},Y)}.$

A:

{b} -> {c}

	$\boldsymbol{B}$	$ar{B}$	
С	3	2	5
$ar{C}$	4	1	5
	7	3	10

 ${a} -> {d}$ 

{b} -> {d}

{e} -> {c}

 $\{c\} -> \{a\}$ 

```
In [9]: ▶ def interest(suppX, suppY, suppXY):
                 return "Interest", (suppXY / suppX) * suppY
             def IS(suppX, suppY, suppXY):
                 return "IS", suppXY / ((suppX * suppY)**0.5)
             def klosgen(suppX, suppY, suppXY):
                 return "Klosgen",((suppXY)**0.5) * (max(confidence(suppXY, suppX) - suppY, confidence(suppXY, suppY)-
             def oddsRatio(suppXY, notXnotY, xNotY, yNotX):
                 return "oddsRatio", (suppXY * notXnotY) / (xNotY*yNotX)
         {b} -> {c}
In [10]: \triangleright suppX = 7/10
             suppY = 5/10
             suppXY = 3/10
             notXnotY = 1/10
             xNotY = 4/10
             yNotX = 2/10
             rules = [interest(suppX, suppY, suppXY), IS(suppX, suppY, suppXY),
                      klosgen(suppX, suppY, suppXY),oddsRatio(suppXY, notXnotY, xNotY, yNotX)]
             rules = sorted(rules, key=lambda tup:tup[1])[::-1]
             print("Sorted rules for {b} -> {c}:\n")
             for i in rules:
                 print(i)
             Sorted rules for {b} -> {c}:
             ('IS', 0.50709255283711)
             ('oddsRatio', 0.374999999999999)
             ('Interest', 0.2142857142857143)
             ('Klosgen', -0.03912303982179756)
         {a} -> {d}
In [11]: \triangleright suppX = 5/10
             suppY = 9/10
             suppXY = 4/10
             notXnotY = 1/10
             xNotY = 1/10
             yNotX = 5/10
             rules = [interest(suppX, suppY, suppXY), IS(suppX, suppY, suppXY),
                      klosgen(suppX, suppY, suppXY),oddsRatio(suppXY, notXnotY, xNotY, yNotX)]
             rules = sorted(rules, key=lambda tup:tup[1])[::-1]
             print("Sorted rules for {a} -> {d}:\n")
             for i in rules:
                 print(i)
             Sorted rules for {a} -> {d}:
             ('oddsRatio', 0.8000000000000002)
             ('Interest', 0.7200000000000001)
```

{b} -> {d}

('IS', 0.5962847939999439)

('Klosgen', -0.03513641844631531)

```
In [12]: \triangleright suppX = 7/10
             suppY = 9/10
             suppXY = 6/10
             notXnotY = 0/10
             xNotY = 1/10
             yNotX = 3/10
             rules = [interest(suppX, suppY, suppXY), IS(suppX, suppY, suppXY),
                      klosgen(suppX, suppY, suppXY),oddsRatio(suppXY, notXnotY, xNotY, yNotX)]
             rules = sorted(rules, key=lambda tup:tup[1])[::-1]
             print("Sorted rules for {b} -> {d}:\n")
             for i in rules:
                 print(i)
             Sorted rules for {b} -> {d}:
             ('Interest', 0.7714285714285715)
             ('IS', 0.7559289460184544)
             ('oddsRatio', 0.0)
             ('Klosgen', -0.02581988897471611)
         {e} -> {c}
In [13]: \triangleright suppX = 6/10
             suppY = 5/10
             suppXY = 2/10
             notXnotY =21/10
             xNotY = 4/10
             yNotX = 4/10
             rules = [interest(suppX, suppY, suppXY), IS(suppX, suppY, suppXY),
                      klosgen(suppX, suppY, suppXY),oddsRatio(suppXY, notXnotY, xNotY, yNotX)]
             rules = sorted(rules, key=lambda tup:tup[1])[::-1]
             print("Sorted rules for {e} -> {c}:\n")
             for i in rules:
                 print(i)
             Sorted rules for {e} -> {c}:
             ('oddsRatio', 2.624999999999999)
             ('IS', 0.36514837167011077)
             ('Interest', 0.166666666666669)
             ('Klosgen', -0.07453559924999296)
         \{c\} -> \{a\}
In [14]: \triangleright suppX = 5/10
             suppY = 5/10
             suppXY = 2/10
             notXnotY = 1/10
             xNotY = 3/10
             yNotX = 3/10
             rules = [interest(suppX, suppY, suppXY), IS(suppX, suppY, suppXY),
                      klosgen(suppX, suppY, suppXY),oddsRatio(suppXY, notXnotY, xNotY, yNotX)]
             rules = sorted(rules, key=lambda tup:tup[1])[::-1]
             print("Sorted rules for {c} -> {a}:\n")
             for i in rules:
                 print(i)
             Sorted rules for {c} -> {a}:
             ('IS', 0.4)
             ('oddsRatio', 0.22222222222222)
             ('Interest', 0.2)
             ('Klosgen', -0.04472135954999578)
```

c. What conclusions can you draw from the results of (a) and (b)?

A:

```
print("interest:", interest(0.01, 0.01, 0.09)[1])
In [17]:
          print("correlation coefficient:", ((9*89)-(1*1)) / ((10+10+90+90)**0.5))
          print("Confidence {A} -> {B}:", confidence(9/100, 10/100))
          print("Confidence {B} -> {A}:", confidence(9/100, 10/100))
          interest: 0.09
          correlation coefficient: 56.5685424949238
          B:
In [18]:
        ▶ print("interest:", interest(90/100, 90/100, 89/100)[1])
          print("correlation coefficient:", ((9*89)-(1*1)) / ((10+10+90+90)**0.5))
          print("Confidence {A} -> {B}:", confidence(89/100, 90/100))
          print("Confidence {B} -> {A}}:", confidence(89/100, 90/100))
          interest: 0.89
          correlation coefficient: 56.5685424949238
```

We would ideally want more records in the diagonal, (i.e) AB or Not (A and B) and less in the other boxes. Also corelation is the same

### **Practicum Exercises**

```
In [19]: )
import pandas as pd
import matplotlib.pyplot as plt
import sklearn as sk
import numpy as np
from mlxtend.frequent_patterns import apriori, association_rules
from mlxtend.preprocessing import TransactionEncoder
```

### **Problem 1**

C:

# 2.1 Problem 1

Load the Online Retail dataset (Online Retail.xlsx) from the UCI Machine Learning Repository into Python using a Pandas dataframe. Using the apriori module from the MLxtend library, generate Frequent Itemsets for all transactions for the country of France. What itemset has the largest support? Set the minimum support threshold to 5% and extract frequent itemsets, and use them as input to the association\_rules module. Use each of the confidence and lift metrics to extract the association rules with the highest values, respectively. What are the antecedents and consequents of each rule? Is the rule with the highest confidence the same as the rule with the highest lift? Why or why not?

```
In [20]: path = r"https://archive.ics.uci.edu/ml/machine-learning-databases/00352/Online%20Retail.xlsx"
    df = pd.read_excel(path)
```

### In [22]: ▶ df

#### Out[22]:

	InvoiceNo StockCode		Description	Quantity	InvoiceDate	UnitPrice	CustomerID	Country
0	536365	85123A	WHITE HANGING HEART T-LIGHT HOLDER	6	2010-12-01 08:26:00	2.55	17850.0	United Kingdom
1	536365	71053	WHITE METAL LANTERN	6	2010-12-01 08:26:00	3.39	17850.0	United Kingdom
2	536365	84406B	CREAM CUPID HEARTS COAT HANGER	8	2010-12-01 08:26:00	2.75	17850.0	United Kingdom
3	536365	84029G	KNITTED UNION FLAG HOT WATER BOTTLE	6	2010-12-01 08:26:00	3.39	17850.0	United Kingdom
4	536365	84029E	RED WOOLLY HOTTIE WHITE HEART.	6	2010-12-01 08:26:00	3.39	17850.0	United Kingdom
541904	581587	22613	PACK OF 20 SPACEBOY NAPKINS	12	2011-12-09 12:50:00	0.85	12680.0	France
541905	581587	22899	CHILDREN'S APRON DOLLY GIRL	6	2011-12-09 12:50:00	2.10	12680.0	France
541906	581587	23254	CHILDRENS CUTLERY DOLLY GIRL	4	2011-12-09 12:50:00	4.15	12680.0	France
541907	581587	23255	CHILDRENS CUTLERY CIRCUS PARADE	4	2011-12-09 12:50:00	4.15	12680.0	France
541908	581587	22138	BAKING SET 9 PIECE RETROSPOT	3	2011-12-09 12:50:00	4.95	12680.0	France

541909 rows × 8 columns

```
In [23]: | isFrance = df["Country"] == "France"
    france_df = df[isFrance][["InvoiceNo", "Description"]]

    france_df[["InvoiceNo", "Description"]]
    cols = france_df["Description"].unique()
    rows = france_df["InvoiceNo"].unique()
    txn_df = pd.DataFrame(columns=cols)
    txn_df.insert(column="InvoiceNo", loc=0, value=rows)
    txn_df = txn_df.replace(np.nan, False)
    for idx, row in france_df.iterrows():
        invoice, item= row["InvoiceNo"], row["Description"]
        txn_df.loc[txn_df['InvoiceNo'] == invoice, item] = True

txn_df = txn_df.drop("InvoiceNo", axis=1)
```

### Out[25]:

	antecedents	consequents	antecedent support	consequent support	support	confidence	lift	leverage	conviction
0	(JUMBO BAG WOODLAND ANIMALS)	(POSTAGE)	0.065076	0.67462	0.065076	1.0	1.482315	0.021174	inf

In [26]: N rules\_lift = association\_rules(freq\_sets, metric="lift", min\_threshold=11)
rules\_lift

### Out[26]:

	antecedents	consequents	antecedent support	consequent support	support	confidence	lift	leverage	conviction	
C	(CHILDRENS CUTLERY SPACEBOY)	(CHILDRENS CUTLERY DOLLY GIRL )	0.058568	0.062907	0.05423	0.925926	14.719029	0.050546	12.650759	
1	(CHILDRENS CUTLERY DOLLY GIRL)	(CHILDRENS CUTLERY SPACEBOY)	0.062907	0.058568	0.05423	0.862069	14.719029	0.050546	6.825380	

- The rule with the highest confidence, does not == the rule with the highest lift
- Confidence simply measures, how often B occurs when A occurs, to predict the liklihood that if A occurs B will occur
- Lift takes into account how often B occurs on it's own. For example if B were to always occur it would have a support of 1, and the lift would be == to the confidence, but the less B occurs the more the lift will increase compared to the confidence, meaning that there is a higher likelihood that A actually causes B to occur, and that B is not just a frequent item on it's own.

## 2.2 Problem 2

Load the Extended Bakery dataset (**75000-out2-binary.csv**) into **Python** using a Pandas dataframe. Calculate the binary correlation coefficient  $\Phi$  for the Chocolate Coffee and Chocolate Cake items. Are these two items symmetric binary variables? Provide supporting calculations. Would the association rules  $\{Chocolate\ Coffee\} \implies \{Chocolate\ Cake\}\ have the same value for <math>\Phi$  as  $\{Chocolate\ Cake\} \implies \{Chocolate\ Coffee\}$ ?

```
▶ local path = r"75000-out2-binary.csv"
In [27]:
              bakery_df = pd.read_csv(local_path)
In [29]:
             item1 name = 'Chocolate Coffee'
              item2_name = 'Chocolate Cake'
              cake_n_coffee = bakery_df[[item1_name,
                                 item2 name]]
              cond 1 = cake n coffee[item1 name] == 1
              cond_2 = cake_n_coffee[item2_name] == 1
              stats = cake_n_coffee.groupby([cond_1,
                                       cond 2]).count()
              stats
   Out[29]:
                                          Chocolate Coffee Chocolate Cake
              Chocolate Coffee Chocolate Cake
                       False
                                    False
                                                   65802
                                                                65802
                                     True
                                                   2962
                                                                 2962
                        True
                                     False
                                                   2933
                                                                 2933
```

```
In [30]: Print(cake_n_coffee["Chocolate Coffee"].corr(cake_n_coffee["Chocolate Cake"]))
print(cake_n_coffee["Chocolate Cake"].corr(cake_n_coffee["Chocolate Coffee"]))

0.48556649252788
```

3303

3303

0.48556649252788003

Yes the items are symetric binary variables

True

Yes the association rules would have the same correlation values going each way becuase we take the product of txns where both items occur and neither occur and subtract the product of the amount of occurances of each item. we then divide that entire difference by the root of the sum of the total amount of occurances and non-occurances of each item. The order of the ruling does not affect this calucation which makes sense becuase correlation simply measures the behavior of both feeatures and compares them.