Notification-Gated Reliability Unit Commitment: Least-Distortionary Out-of-Market Interventions

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***Abstract*—System operators routinely commit generation resources outside of the energy market to ensure reliability, but existing reliability unit commitment (RUC) formulations optimize over the full planning horizon without distinguishing decisions that must be made immediately from those that can be deferred to subsequent market processes. This scope mismatch leads to systematic over-commitment, displacing market-scheduled resources and distorting price signals. We propose a notification-gated RUC formulation that uses physical generator startup lead times to identify the minimal set of non-deferrable commitment decisions at each operational stage. The formulation is solved in two phases: a deterministic mixed-integer program that minimizes reliability intervention costs over the gating window, followed by a second-order cone program that verifies worst-case dispatch feasibility under ellipsoidal uncertainty. When robust verification fails, column-and-constraint generation iterates between the phases to close the gap. The gating framework generalizes across operational horizons — multi-day assessment, day-ahead RUC, and intra-day RUC are all instances with different parameterizations — and recovers standard DARUC as a limiting case. Numerical experiments on the RTS-GMLC test case demonstrate significant reductions in out-of-market commitments while maintaining system reliability under worst-case uncertainty.**

***Index Terms*—Reliability unit commitment, robust optimization, market design, notification time, linear decision rules.**

I. Introduction

RELIABILITY processes are required in power system operations due to a fundamental market failure in the provision of reliability. To address this, system operators routinely commit generation resources outside of the energy market through reliability-driven commitment actions. While effective in preserving system feasibility, empirical evidence and operational experience indicate that the heuristics used to trigger these commitments are often overly conservative, leading to systematic over-commitment and distortion of price signals in both the day-ahead and real-time markets.

Motivated by this gap, this work proposes a robust optimization framework to identify the minimal set of generation commitment decisions that are strictly necessary prior to the next day-ahead market clearing to ensure system reliability. Rather than optimizing all commitment and dispatch decisions under uncertainty, the proposed methodology focuses exclusively on commitment decisions that cannot be deferred without risking infeasibility under adverse realizations of load and renewable generation.

System reliability is enforced with respect to a worst-case forecast model of net demand, capturing uncertainty in both load and renewable production. Commitment decisions are selected so as to guarantee feasibility for all realizations within this uncertainty set, while minimizing the extent of out-of-market intervention. In this sense, the resulting commitments may be interpreted as the least-distortionary reliability actions, providing a principled defense of operator interventions while preserving market signals to the greatest extent possible.

A. Contributions

This paper makes the following contributions.

First, we develop a notification-gated formulation for reliability unit commitment that explicitly models the interaction between physical generator lead times and the sequencing of market and operational decision processes. The proposed framework identifies the subset of commitment decisions that cannot be deferred to subsequent market stages and restricts out-of-market interventions to this non-deferrable decision set.

Second, we introduce a minimal-intervention reliability objective that minimizes only startup actions and associated no-load costs induced by non-deferrable commitments. This objective decouples reliability enforcement from full economic re-optimization, thereby preserving market-based commitment and dispatch outcomes while ensuring worst-case feasibility under uncertainty.

Third, we demonstrate that the proposed formulation generalizes naturally across operational horizons, including multi-day reliability assessment and day-ahead reliability unit commitment, enabling coordinated treatment of long-lead thermal resources and near-term reliability adjustments within a unified optimization framework.

Fourth, we conduct a comprehensive numerical evaluation comparing the proposed approach against standard reliability unit commitment and integrated market-reliability formulations. Results show significant reductions in early commitment interventions and redispatch magnitude without compromising system reliability.

II. Literature Review

A. Reliability Commitment in Practice

Every North American RTO operates a reliability unit commitment process following the day-ahead market clearing. The Southwest Power Pool (SPP) runs a DARUC with a 48-hour look-ahead that may commit additional resources beyond the day-ahead market solution [cite SPP market protocols]. The Midcontinent ISO (MISO) executes a similar reliability assessment commitment (RAC) process to ensure resource adequacy over the next operating day [cite MISO BPM-002]. CAISO's residual unit commitment (RUC) procures capacity to close any gap between day-ahead scheduled supply and

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the operator's demand forecast [cite CAISO BPM]. PJM, NYISO, and ERCOT employ analogous processes under varying nomenclature [cite PJM manual 11; NYISO OATT; ERCOT protocols].

Despite the ubiquity of these processes, their mathematical formulations are largely proprietary and vary considerably across operators. FERC Order 2222 and subsequent technical conferences have drawn attention to the market impact of reliability commitments, noting that out-of-market actions can suppress energy prices and create uplift charges [cite FERC]. The academic literature has given comparatively little attention to the formulation of the reliability commitment problem itself, typically treating it as a standard unit commitment solved after the market clears. The sequential nature of the decision process — and in particular the question of *which* decisions must be made at each stage — has received almost no formal treatment.

B. Robust and Stochastic Unit Commitment

The robust optimization approach to unit commitment was pioneered by Bertsimas et al. [cite Bertsimas 2012], who introduced a polyhedral uncertainty set for net load and solved the two-stage problem via Benders decomposition. Subsequent work by Jiang et al. [cite Jiang 2012] developed column-and-constraint generation methods that avoid the need for dual-based decomposition and demonstrated improved computational performance. Zhao and Guan [cite Zhao Guan 2013] extended the framework to multi-stage settings and established finite convergence results for the CCG procedure.

Stochastic unit commitment provides an alternative treatment of uncertainty through scenario-based formulations. Papavasiliou et al. [cite Papavasiliou 2011] demonstrated the value of stochastic optimization for wind integration, and Zheng et al. [cite Zheng 2015] developed decomposition methods for large-scale stochastic UC problems. Distributionally robust approaches, which optimize over ambiguity sets of probability distributions, have been explored by Xiong et al. [cite Xiong 2017] and others.

Linear decision rules (LDR) provide a tractable approximation of the recourse policy in robust and adaptive optimization. Ben-Tal et al. [cite Ben-Tal 2004] established the affine adjustable robust counterpart framework, and Lorca and Sun [cite Lorca Sun 2016] applied LDR to unit commitment with ellipsoidal uncertainty sets, obtaining second-order cone reformulations. The present work builds on the LDR-based robust UC framework developed in [paper 2], which introduces learned covariance estimation and conformal prediction for uncertainty set calibration.

A common feature of the existing robust UC literature is that the formulation scope encompasses *all* commitment decisions over the planning horizon. The contribution of this paper is orthogonal to the choice of uncertainty model: rather than proposing a new uncertainty representation, we introduce a *decision scope* restriction that limits the reliability optimization to non-deferrable commitments, regardless of whether the underlying model is deterministic, robust, or stochastic.

C. Market Efficiency and Out-of-Market Actions

Out-of-market reliability actions have well-documented effects on market efficiency. Reliability commitments that deviate from the market solution create make-whole payments (uplift) to committed generators, representing costs that are not reflected in locational marginal prices [cite Hogan 2003; O'Neill 2005]. The convex hull pricing literature [cite Gribik 2007; Schiro 2016] has established that pricing in the presence of non-convexities (including commitment decisions) cannot simultaneously achieve revenue adequacy and cost recovery without some form of side payment.

Reducing the number of out-of-market commitments directly reduces the magnitude of uplift and improves the extent to which energy prices reflect the true marginal cost of supply. Andrianesis et al. [cite Andrianesis 2020] quantify the relationship between commitment interventions and uplift in ISO-NE, finding that reliability commitments account for a substantial fraction of total uplift payments. Ela et al. [cite Ela 2011] analyze the operational and market impacts of wind integration on commitment processes, noting that conservative commitment practices designed to hedge against renewable forecast errors contribute to over-commitment and price distortion.

D. Research Gap

The existing literature treats the reliability commitment problem as a monolithic optimization over the full planning horizon: once the market clears, the reliability process re-optimizes all remaining commitment decisions under its chosen uncertainty model. No existing formulation distinguishes between commitment decisions that must be finalized at the current stage and those that can be deferred to subsequent processes.

This paper addresses this gap by introducing the gating set, which uses physical generator notification times to partition the decision space into deferrable and non-deferrable components. The result is a principled restriction of the reliability optimization scope that preserves market outcomes to the greatest extent permitted by physical constraints.

III. The Reliability Commitment Problem

A. Operational Timeline

North American RTOs share a common sequential structure in their commitment and dispatch processes, though specific timing and nomenclature vary across operators. For notational clarity, we refer to the *Operating Day* (OD) as the day for which the operation plan is being generated. The following sequence is representative of the general pattern; specific timing parameters are given in Section **??**.

The earliest commitment decisions are made through a *multi-day reliability assessment* (MDRA), an advisory process run several days before OD with a look-ahead horizon of four to seven days. The MDRA identifies long-lead resources — typically large thermal units requiring multi-day advance notice — that must be committed to ensure system adequacy.

The principal scheduling process is the *Day-Ahead Market* (DAMKT), which clears on OD-1 and produces a binding commitment and dispatch schedule for OD. Following the DAMKT, the *Day-Ahead Reliability Unit Commitment* (DARUC) reviews the market solution and commits additional resources as needed to ensure reliability over a horizon covering OD and, in many implementations, OD+1.

During the operating day itself, *Intra-Day RUC* (ID-RUC) processes run on a rolling basis, making near-term commitment adjustments as updated forecasts become available.

The complete sequence — MDRA, DAMKT, DARUC, ID-RUC — forms a nested hierarchy in which each successive process operates over a shorter horizon with more current information, and may revise or supplement the decisions of its predecessor.

B. Unit Commitment Formulation

The following formulation captures the core unit commitment constraints common to both the day-ahead market and reliability processes. Let $\mathcal{I}$ index generating resources, $\mathcal{T}=\{1,\dots,T\}$ index operating periods, $\mathcal{N}$ index buses, and $\mathcal{L}$ index transmission lines.

$$\sum_i p_{i,t} + s^p_t = \sum_n d_{n,t}, \quad \forall t, \tag{1a}$$
$$-\bar{F}^l \le \sum_n S^l_n \Big(\sum_{i\in\mathcal{I}_n} p_{i,t} - d_{n,t}\Big) \le \bar{F}^l, \quad \forall l,t, \tag{1b}$$
$$p_{i,t,b} \le \bar{P}_{i,b}\, u_{i,t}, \ \ \forall i,t,b, \quad p_{i,t} = \textstyle\sum_b p_{i,t,b}, \ \ \forall i,t, \tag{1c}$$
$$p_{j,t} \le \bar{p}^{\text{fcst}}_{j,t}, \quad \forall j \in \mathcal{I}^{\text{ren}},\ t, \tag{1d}$$
$$\underline{P}_i\, u_{i,t} \le p_{i,t} \le \bar{P}_i\, u_{i,t}, \quad \forall i,t, \tag{1e}$$
$$p_{i,t} - p_{i,t-1} \le R^U_i (u_{i,t-1} + v_{i,t}), \quad \forall i,t, \tag{1f}$$
$$p_{i,t-1} - p_{i,t} \le R^D_i (u_{i,t} + w_{i,t}), \quad \forall i,t, \tag{1g}$$
$$u_{i,t} - u_{i,t-1} = v_{i,t} - w_{i,t}, \quad \forall i,t, \tag{1h}$$
$$\sum_{t'=t-\text{MUT}_i+1}^{t} v_{i,t'} \le u_{i,t}, \quad \forall i,\ t \ge \text{MUT}_i, \tag{1i}$$
$$\sum_{t'=t-\text{MDT}_i+1}^{t} w_{i,t'} \le 1 - u_{i,t}, \quad \forall i,\ t \ge \text{MDT}_i, \tag{1j}$$
$$\sum_{t'=1}^{UT_i} u_{i,t'} = UT_i, \quad \sum_{t'=1}^{DT_i} u_{i,t'} = 0, \quad \forall i, \tag{1k}$$
$$\mathbf{p},\, \mathbf{s} \ge 0, \quad \mathbf{u},\, \mathbf{v},\, \mathbf{w} \in \{0,1\}. \tag{1l}$$

We denote this shared constraint set as $\mathcal{X}^{\text{UC}}$.

C. Day-Ahead Market (DAMKT)

The DAMKT minimizes total production cost including energy, no-load, startup, and shutdown costs over all generators and periods:

$$\min_{\mathbf{p},\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{s}} \quad \sum_{i,t}\Big[C^{NL}_i u_{i,t} + C^{SU}_i v_{i,t} + C^{SD}_i w_{i,t} + \sum_b C^p_{i,b}\, p_{i,t,b}\Big] + \sum_t M^p s^p_t \tag{2}$$
$$\text{s.t.} \quad (1).$$

D. Standard Day-Ahead RUC (DARUC)

The standard DARUC uses the same constraint set $\mathcal{X}^{\text{UC}}$ but removes energy costs from the objective, retaining only commitment-related costs:

$$\min_{\mathbf{p},\mathbf{u},\mathbf{v},\mathbf{w},\mathbf{s}} \quad \sum_{i,t}\Big[C^{NL}_i u_{i,t} + C^{SU}_i v_{i,t} + C^{SD}_i w_{i,t}\Big] + \sum_t M^p s^p_t \tag{3}$$
$$\text{s.t.} \quad (1).$$

In practice, the DARUC typically inherits the DAMKT commitment schedule as a lower bound, permitting additional commitments but prohibiting de-commitment of market-cleared resources. The specific implementation varies across operators; some fix the market schedule entirely, while others allow limited modifications. This no-decommitment convention is formalized in Section IV-B.

E. Why Standard DARUC Over-Commits

The standard DARUC formulation optimizes commitment decisions $u_{i,t}$ for every generator i across every period t in the planning horizon. However, the vast majority of these decisions will be revisited — and potentially revised — by subsequent processes: the next DAMKT clearing for periods beyond OD, and intra-day RUC for near-term adjustments. The DARUC thus solves an optimization problem whose scope extends well beyond the decisions that are genuinely its to make.

This scope mismatch creates a systematic bias toward early commitment. Because the DARUC minimizes total commitment cost over the full horizon, it may find it optimal to start a unit in an early period to avoid a more expensive startup in a later period — even when the later decision falls within the purview of a subsequent process that will have access to updated forecasts. The DARUC has no mechanism to recognize that such deferred decisions carry no immediate cost: from its perspective, the entire horizon is a single optimization to be solved jointly.

The result is commitment decisions that are feasible but premature. Units are started hours or days before they are operationally required, displacing resources that were optimally scheduled by the market and distorting the price signals that the market was designed to produce.

The fundamental issue is not that the DARUC commits the wrong units in an absolute sense, but that it commits units whose startup decisions could have been deferred without jeopardizing reliability. The formulation lacks a notion of *deferrability*: it treats every commitment decision as equally urgent, regardless of whether the physical lead time of the generator demands an immediate decision or permits deferral to a later stage. Section IV-A introduces the gating set to formalize this distinction.

IV. Notification-Gated Reliability Commitment

The proposed formulation is solved in two phases. Phase 1 determines the minimum-cost set of non-deferrable reliability commitments against the mean forecast. Phase 2 verifies that these commitments are sufficient to guarantee dispatch feasibility under worst-case uncertainty. When Phase 2 identifies a robustness shortfall, the two phases are iterated via column-and-constraint generation until robust feasibility is attained (Section **??**).

A. Gating Sets

The notification (lead) time of a generating unit determines which commitment decisions must be finalized at the current decision stage and which can be deferred to subsequent market or operational processes. Commitment decisions should be made binding only when deferral would violate physical feasibility.

Let decision process k represent the current reliability or market commitment stage (e.g., MDRA, DA-RUC, ID-RUC), and let t^{next} denote the first operating period whose commitment decisions can be modified by the next available process. Let $t \in \mathcal{T}^k = \{1, \dots, T\}$ index the physical operating periods in the look-ahead horizon considered at stage k.

For generating resource i with startup notification time L_i^{SU}, a startup decision affecting operating period t must be issued at the current stage if it cannot be deferred to the next process. This condition defines the *gating set*:

$$\mathcal{T}_i^{\text{gate}} \;:=\; \big\{\, t \in \mathcal{T}^k \;\big|\; t - L_i^{SU} < t^{\text{next}} \big\}. \tag{4}$$

Periods in $\mathcal{T}_i^{\text{gate}}$ correspond to startup decisions that must be initiated before the next commitment process in order to preserve feasibility. Startup decisions outside this set may be deferred and are excluded from the current reliability optimization.

Because $t - L_i^{SU}$ is increasing in t, the gating set is a contiguous block of early periods: if period t is gated, then every earlier period $t' < t$ is also gated. Generators with longer notification times have larger gating sets, reflecting the greater advance notice required to start them.

The gating set restricts *startup* decisions ($v_{i,t}$), not commitment status ($u_{i,t}$). A unit already committed by the market does not require a new startup decision and is therefore not subject to gating.

B. Market-Reliability Decomposition

Let $(\mathbf{u}^m, \mathbf{v}^m, \mathbf{w}^m)$ denote the commitment schedule from the preceding DAMKT solution. The reliability process operates on the *total* commitment variables $(u_{i,t}, v_{i,t}, w_{i,t})$, which are constrained to preserve all market-decided commitments:

$$u_{i,t} \;\geq\; u_{i,t}^m, \qquad \forall\, i, t. \tag{5}$$

The startup and shutdown variables $v_{i,t}$ and $w_{i,t}$ are determined by the logic constraint (1h) applied to the total commitment $u_{i,t}$ and are not independently decomposed.

The *incremental commitment* is defined as

$$u'_{i,t} \;:=\; u_{i,t} - u_{i,t}^m \;\in\; \{0,1\}, \qquad \forall\, i, t, \tag{6}$$

where $u'_{i,t} = 1$ indicates a reliability intervention: a unit brought online beyond the market solution.

A reliability startup at period t may *absorb* a market startup at a later period: if the market planned to start unit i at period $t{+}2$, but the reliability process starts it at t, the unit is already online at $t{+}2$ and the market startup is superseded. This interaction is handled automatically by the logic constraint (1h) operating on total u.

C. Phase 1: Deterministic Gated LD-RUC

Phase 1 identifies the minimum-cost reliability commitments that ensure feasibility under the mean renewable forecast $\boldsymbol{\mu}$. The formulation uses the full constraint set $\mathcal{X}^{\text{UC}}$ over the entire planning horizon; only the *objective* is restricted to the gating window.

$$\min_{\substack{\mathbf{u},\mathbf{v},\mathbf{w},\\ \mathbf{p},\mathbf{s}}} \quad \sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{T}_i^{\text{gate}}} \Big[C_i^{SU}\, v_{i,t} \;+\; C_i^{NL}\, u'_{i,t} \Big] \;+\; M \sum_t s_t^p \tag{7a}$$

$$\text{s.t.} \quad (1) \quad \text{with } \bar{p}_{j,t}^{\text{fcst}} = \mu_{j,t}, \tag{7b}$$

$$u_{i,t} \geq u_{i,t}^m, \quad \forall\, i, t, \tag{7c}$$

$$u'_{i,t} = u_{i,t} - u_{i,t}^m, \quad \forall\, i, t. \tag{7d}$$

The objective (**??**) penalizes startup costs $C_i^{SU} v_{i,t}$ and incremental no-load costs $C_i^{NL} u'_{i,t}$ only within the gating window $\mathcal{T}_i^{\text{gate}}$. Startup terms corresponding to the DAMKT solution ($v_{i,t} = v_{i,t}^m$) contribute a constant under the no-decommitment constraint (**??**) and do not influence the optimization. Commitment variables outside the gating window are free but carry no objective cost; the optimizer adds non-gated commitments only when forced by minimum-up-time propagation or system feasibility.

Windowed no-load variant. A gated startup at period t commits unit i for at least MUT_i periods, some of which may fall beyond the gating window. A refined objective attributes no-load cost over the irrevocable commitment window rather than period-by-period:

$$\sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{T}_i^{\text{gate}}} \left[C_i^{SU}\, v_{i,t} \;+ \sum_{\tau=t}^{\min\{t+\text{MUT}_i-1,\,T\}} C_i^{NL}\, v'_{i,t} \right] + M \sum_t s_t^p, \tag{8}$$

where $v'_{i,t} := v_{i,t}\,(1 - v_{i,t}^m)$ isolates reliability startups from market startups.

Output. Phase 1 produces a candidate commitment schedule $\hat{\mathbf{u}} = \mathbf{u}^m + \hat{\mathbf{u}}'$, along with the associated startup and shutdown indicators $(\hat{\mathbf{v}}, \hat{\mathbf{w}})$.

D. Phase 2: Robust Feasibility Verification

Phase 2 takes the commitment decisions from Phase 1 as fixed and checks whether a feasible dispatch policy exists for all uncertainty realizations. Because all binary variables are fixed, Phase 2 is a continuous optimization problem.

1) Uncertainty model: Let $\mathbf{r} \in \mathbb{R}^K$ represent deviations of renewable generation from the mean forecast, where K is the number of uncertain renewable sources. The uncertainty set is an ellipsoid parameterized by a positive definite covariance matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{K\times K}$ and a scalar budget $\rho > 0$:

$$\mathcal{U} \;:=\; \big\{\, \mathbf{r} \in \mathbb{R}^K \;\big|\; \mathbf{r}^\top \boldsymbol{\Sigma}^{-1} \mathbf{r} \leq \rho^2 \big\}. \tag{9}$$

The construction of $\mathbf{\Sigma}$ and calibration of ρ via conformal prediction are detailed in [paper 2]; here we treat $(\mathbf{\Sigma}, \rho)$ as given inputs.

2) Linear decision rules: Dispatch is modeled as an affine function of the uncertainty realization:

$$p_{i,t}(\mathbf{r}) \;=\; p_{i,t}^0 \;+\; \mathbf{z}_{i,t}^\top \mathbf{r}, \qquad \forall\, i, t, \tag{10}$$

where $p_{i,t}^0$ is the nominal dispatch and $\mathbf{z}_{i,t} \in \mathbb{R}^K$ are the linear decision rule (LDR) coefficients governing the response of generator i at period t to the uncertainty realization.

3) Robust counterpart: Each operational constraint must hold for every $\mathbf{r} \in \mathcal{U}$. For a generic upper-bound constraint $\mathbf{z}_{i,t}^\top \mathbf{r} \le b_{i,t}$, the worst-case over $\mathcal{U}$ yields the second-order cone (SOC) condition

$$\rho \left\| \mathbf{\Sigma}^{1/2} \mathbf{z}_{i,t} \right\|_2 \;\le\; b_{i,t}. \tag{11}$$

Applying this to each constraint in $\mathcal{X}^{\mathrm{UC}}$ with commitment fixed at $\hat{\mathbf{u}}$ gives the Phase 2 formulation:

$$\min_{\mathbf{p}^0,\,\mathbf{Z},\,\mathbf{s}} \quad \sum_t s_t \tag{12a}$$

$$\text{s.t.} \quad \sum_i p_{i,t}^0 + s_t = \sum_n d_{n,t}, \quad \forall\, t, \tag{12b}$$

$$\sum_i z_{i,t,k} = 0, \quad \forall\, k, t, \tag{12c}$$

$$p_{i,t}^0 + \rho \|\mathbf{\Sigma}^{1/2} \mathbf{z}_{i,t}\|_2 \le \bar{P}_i\, \hat{u}_{i,t}, \quad \forall\, i, t, \tag{12d}$$

$$\underline{P}_i\, \hat{u}_{i,t} \le p_{i,t}^0 - \rho \|\mathbf{\Sigma}^{1/2} \mathbf{z}_{i,t}\|_2, \quad \forall\, i, t, \tag{12e}$$

$$(p_{i,t}^0 - p_{i,t-1}^0) + \rho \|\mathbf{\Sigma}^{1/2} (\mathbf{z}_{i,t} - \mathbf{z}_{i,t-1})\|_2 \le R_i^U (\hat{u}_{i,t-1} + \hat{v}_{i,t}), \quad \forall\, i, t, \tag{12f}$$

$$(p_{i,t-1}^0 - p_{i,t}^0) + \rho \|\mathbf{\Sigma}^{1/2} (\mathbf{z}_{i,t-1} - \mathbf{z}_{i,t})\|_2 \le R_i^D (\hat{u}_{i,t} + \hat{w}_{i,t}), \quad \forall\, i, t, \tag{12g}$$

$$\Big| \sum_n S_n^l \Big(\sum_{i \in \mathcal{I}_n} p_{i,t}^0 - d_{n,t} \Big) \Big| + \rho \Big\| \mathbf{\Sigma}^{1/2} \sum_{i \in \mathcal{I}_n} S_n^l\, \mathbf{z}_{i,t} \Big\|_2 \le \bar{F}^l, \quad \forall\, l, t, \tag{12h}$$

$$p_{j,t}^0 + \rho \|\mathbf{\Sigma}^{1/2} (\mathbf{z}_{j,t} - \mathbf{e}_{j,t})\|_2 \le \mu_{j,t}, \quad \forall\, j \in \mathcal{I}^{\mathrm{ren}},\; t, \tag{12i}$$

$$\mathbf{p}^0 \ge 0, \; \mathbf{s} \ge 0. \tag{12j}$$

Here $\mathbf{e}_{j,t} \in \mathbb{R}^K$ is the unit vector selecting the uncertainty dimension corresponding to renewable source j at period t, so that the available renewable output under realization $\mathbf{r}$ is $\mu_{j,t} + \mathbf{e}_{j,t}^\top \mathbf{r}$. Constraint (**??**) ensures that the aggregate dispatch response to any realization sums to zero: when renewable output deviates, the remaining fleet absorbs the imbalance.

Formulation (**??**) is a second-order cone program (SOCP) with no integer variables. If the optimal slack satisfies $\sum_t s_t^* = 0$, the Phase 1 commitments are *robust-sufficient*: a feasible dispatch policy exists for every $\mathbf{r} \in \mathcal{U}$. If $\sum_t s_t^* > 0$, the commitments are insufficient and the *robustness gap* $\sum_t s_t^*$ quantifies the worst-case shortfall in megawatt-hours.

E. Augmentation via Column-and-Constraint Generation

When Phase 2 reports a positive robustness gap, the Phase 1 commitments are insufficient under worst-case uncertainty. We close this gap by iterating between the two phases using column-and-constraint generation (CCG).

Phase 2 not only evaluates the gap but also identifies a *worst-case scenario*: the realization $\mathbf{r}^* \in \mathcal{U}$ at which the dispatch constraints are most stressed. This scenario is added to Phase 1 as an additional set of deterministic constraints, requiring that the commitment schedule be feasible under both the mean forecast and the worst-case realization $\mathbf{r}^*$. Phase 1 is then re-solved, and the updated commitments are passed back to Phase 2 for verification.

Formally, at CCG iteration ℓ, Phase 1 is augmented with the scenario set $\mathcal{R}^{(\ell)} = \{\mathbf{r}^{*(1)}, \dots, \mathbf{r}^{*(\ell)}\}$ collected from prior Phase 2 solves:

$$\text{(??) with additional constraints: } \mathcal{X}^{\mathrm{UC}}(\mathbf{r}^{*(m)}) \;\; \forall\, \mathbf{r}^{*(m)} \in \mathcal{R}^{(\ell)}, \tag{13}$$

where $\mathcal{X}^{\mathrm{UC}}(\mathbf{r})$ denotes the deterministic UC constraints evaluated at the renewable forecast $\boldsymbol{\mu} + \mathbf{r}$. Phase 1 remains a mixed-integer linear program at each iteration (no SOC constraints); only the number of scenario copies of $\mathcal{X}^{\mathrm{UC}}$ grows.

The CCG procedure converges in a finite number of iterations to the optimal solution of the monolithic robust gated LD-RUC.

This follows from the standard finite-convergence result for two-stage robust optimization with a compact uncertainty set and finite recourse [cite Zeng & Zhao 2013].

In practice, we expect the number of CCG iterations to be small. When the system has sufficient reserve headroom, Phase 2 may certify robust feasibility on the first pass, requiring no augmentation at all. The robustness gap and iteration count are reported in the case study as measures of how far the deterministic gated solution lies from robust sufficiency.

F. Summary

The complete LD-RUC procedure is:

1) Solve Phase 1 (**??**): deterministic gated LD-RUC (MIP) with mean forecast. Obtain candidate commitments $\hat{\mathbf{u}}$.
2) Solve Phase 2 (**??**): robust feasibility verification (SOCP) with fixed $\hat{\mathbf{u}}$.
3) If $\sum_t s_t^* = 0$: terminate. The commitments are robust-sufficient.
4) If $\sum_t s_t^* > 0$: extract worst-case $\mathbf{r}^*$, augment Phase 1 via (**??**), and return to Step 1.

The separation of commitment (integer, Phase 1) from dispatch verification (continuous, Phase 2) offers both computational and structural advantages. Phase 1 is a standard-sized MIP with the same constraint structure as a conventional DARUC; the gating set modifies only the objective. Phase 2 is a SOCP that scales with the number of generators and periods but involves no combinatorial decisions.

V. Generalization Across Decision Stages

The gating set (4) is parameterized by the decision stage k (through t^{next}) and the generator notification times L_i^{SU}. By varying these parameters, the LD-RUC framework instantiates naturally across the operational planning sequence.

A. Multi-Day Reliability Assessment (MDRA)

The MDRA is an advisory process run several days before the operating day, with a horizon of four to seven days. At this stage, t^{next} corresponds to the first period of OD-1 (the day before the operating day), when the DAMKT will clear. Only generators with very long notification times — those whose startup lead time exceeds the interval between the MDRA and the DAMKT — have non-empty gating sets. In practice, this restricts the MDRA's binding decisions to large coal, nuclear, and combined-cycle units requiring multi-day advance notice.

B. Day-Ahead RUC (DA-RUC)

The DA-RUC runs after the DAMKT clears for the operating day. Here t^{next} is the first period of OD$+1$ (the next operating day), since the next DAMKT will cover that horizon. Generators with notification times shorter than 24 hours have gating sets that cover only the operating day itself; those with longer lead times may also gate periods on OD$+1$. This is the primary application of the LD-RUC framework developed in Section **??**.

C. Intra-Day RUC (ID-RUC)

Intra-day reliability processes run on a rolling basis during the operating day. At each invocation, t^{next} is the earliest period that can still be modified by the next intra-day run (typically the current period plus the process cycle time). The gating sets shrink as the operating day progresses and more decisions become irrevocable, leaving only fast-start resources with non-empty gating windows.

D. Relationship to Standard DARUC

The standard DARUC formulation (3) can be viewed as the limiting case of the gating framework in which $\mathcal{T}_i^{\text{gate}} = \mathcal{T}$ for all i — that is, every commitment decision is treated as non-deferrable. This occurs when $t^{\text{next}} \to \infty$ (no subsequent process exists) or equivalently when $L_i^{SU} \to \infty$ for all generators.

Conversely, the pure market outcome (no reliability intervention) corresponds to $\mathcal{T}_i^{\text{gate}} = \varnothing$ for all i, which occurs when $L_i^{SU} = 0$ (all units can be started instantaneously) or $t^{\text{next}} = 1$ (the next process can modify every period). The gating framework thus spans a continuum between no intervention and full re-optimization, with the physical notification times determining the appropriate point on this spectrum.

VI. Case Study

A. Setup

The proposed LD-RUC formulation is evaluated on the RTS-GMLC test case, a three-area system with 73 generating units (thermal, wind, solar, and hydro) and 120 transmission lines. Wind forecast data are drawn from the Southwest Power Pool (SPP) constellation forecast dataset, scaled to the RTS-GMLC wind fleet capacities. The ellipsoidal uncertainty set parameters $(\boldsymbol{\Sigma}, \rho)$ are constructed using the learned covariance and conformal prediction methodology developed in [paper 2].

Notification times L_i^{SU} are assigned to the generator fleet based on fuel type and capacity:

The DA-RUC decision stage is used as the primary test case, with t^{next} set to the first period of OD$+1$ (period 25 for a 48-hour horizon starting at OD hour 1). All experiments are solved using Gurobi 11 on [hardware description].

B. Standard DARUC vs. LD-RUC

C. Robust Verification Results

D. Comparison with Integrated Robust UC

E. Sensitivity to Notification Times

VII. Conclusion

This paper introduced a notification-gated formulation for reliability unit commitment that restricts out-of-market interventions to the minimal set of commitment decisions that cannot be deferred to subsequent market or operational processes. The gating set, derived from physical generator notification times and the timing of sequential decision stages, provides a principled criterion for distinguishing non-deferrable reliability actions from decisions that can be left to the market.

The proposed two-phase solution approach separates the commitment decision (a deterministic MIP over the gating window) from robust feasibility verification (a continuous SOCP with fixed commitments). This decomposition offers computational advantages and provides diagnostic information: the robustness gap quantifies the worst-case shortfall of the deterministic solution, and the number of CCG iterations measures the additional commitment cost attributable to uncertainty.

The gating framework generalizes across operational horizons. Multi-day reliability assessment, day-ahead RUC, and intra-day RUC are all instances of the same formulation with different parameterizations of the gating set. Standard DARUC is recovered as a limiting case in which all decisions are treated as non-deferrable.

Future work includes analysis of market efficiency impacts (LMP changes and uplift costs resulting from reduced reliability interventions), extension to chance-constrained and distributionally robust uncertainty models, and application to multi-settlement market structures where the interaction between reliability processes and real-time pricing is most consequential.

Appendix A
Nomenclature

Acknowledgment

The authors would like to thank…

Symbol	**Description**
Sets and indices	
$\mathcal{I}$, i	Generating resources
$\mathcal{I}^{\text{ren}}$, j	Renewable generators
$\mathcal{I}_n$	Generators at bus n
$\mathcal{N}$, n	Buses
$\mathcal{L}$, l	Transmission lines
$\mathcal{T}$, t	Operating periods
$\mathcal{T}_i^{\text{gate}}$	Gating set for generator i
$\mathcal{U}$	Uncertainty set
K, k	Uncertainty dimensions
b	Dispatch blocks
Parameters	
$C_i^{NL}, C_i^{SU}, C_i^{SD}$	No-load, startup, shutdown cost
$C_{i,b}^p$	Block energy cost
$\bar{P}_i$, $\underline{P}_i$	Max/min capacity
$\bar{P}_{i,b}$	Block capacity
R_i^U, R_i^D	Ramp-up/down rate
MUT_i, MDT_i	Min up/down time
L_i^{SU}	Startup notification time
$d_{n,t}$	Nodal demand
$\bar{F}^l$	Line flow limit
S_n^l	PTDF entry
$\mu_{j,t}$	Mean renewable forecast
$\mathbf{\Sigma}$	Uncertainty covariance matrix
ρ	Uncertainty budget
M	Slack penalty
$u_{i,t}^m$, $v_{i,t}^m$	DAMKT commitment/startup
t^{next}	First period modifiable by next process
Variables	
$u_{i,t}$	Commitment status (binary)
$v_{i,t}$, $w_{i,t}$	Startup/shutdown (binary)
$u'_{i,t}$	Incremental commitment
$p_{i,t}$	Dispatch (MW)
$p_{i,t}^0$	Nominal dispatch (Phase 2)
$\mathbf{z}_{i,t}$	LDR coefficients (Phase 2)
s_t^p, s_t	Power balance slack