ARUC and DARUC Formulations
(as implemented in code)

1 Sets and Indices

$i \in \mathcal{I}$	Generators (I total)
$n \in \mathcal{N}$	Buses (N total)
$\ell \in \mathcal{L}$	Lines (L total)
$t \in \mathcal{T}$	Time periods (T total, $t = 0, \ldots, T-1$)
$b \in \mathcal{B}$	Cost blocks (B total)
$k \in \mathcal{K}$	Wind generators / uncertainty dimensions (K total)
$\mathcal{I}_n$	Generators at bus n
$\mathcal{W} \subseteq \mathcal{I}$	Wind generators
$\mathcal{Z} \subseteq \mathcal{I}$	Z-eligible generators (thermal + wind)

2 Parameters

C_i^{NL}	No-load cost
C_i^{SU}	Startup cost
C_i^{SD}	Shutdown cost
$C_{i,b}^{\mathrm{p}}$	Block b marginal cost for generator i
$\overline{P}_{i,b}$	Block b capacity for generator i
$P_i^{\min}$	Minimum output when committed
$P_{i,t}^{\max}$	Maximum output (time-varying for wind = forecast)
R_i^U, R_i^D	Ramp-up / ramp-down limits
$T_i^{\mathrm{up}}, T_i^{\mathrm{dn}}$	Minimum up-time / down-time
$d_{n,t}$	Load at bus n, period t
$D_t = \sum_n d_{n,t}$	System load
$H_{\ell,n}$	PTDF matrix entry
$F_\ell^{\max}$	Line flow limit
M^p	Power balance slack penalty
u_i^0	Initial commitment status
$\mathbf{\Sigma}_t \in \mathbb{R}^{K \times K}$	Covariance matrix (time-varying)
$\mathbf{L}_t$	Cholesky factor: $\mathbf{\Sigma}_t = \mathbf{L}_t \mathbf{L}_t^\top$
ρ_t	Ellipsoid radius (time-varying)
ρ_t^{line}	Line-flow ellipsoid radius ($= \gamma \cdot \rho_t$, where $\gamma =$ `rho_lines_frac`)

3 DAM: Deterministic Day-Ahead UC

Variables. $u_{i,t} \in \{0,1\}$ (commitment), $v_{i,t} \in \{0,1\}$ (startup), $w_{i,t} \in \{0,1\}$ (shutdown), $p_{i,t,b} \geq 0$ (block dispatch), $p_{i,t} \geq 0$ (total dispatch), $s_t^p \geq 0$ (power balance slack).

$$
\begin{aligned}
\min \quad & \sum_{i,t}\Big[C_i^{\mathrm{NL}}u_{i,t}+C_i^{\mathrm{SU}}v_{i,t}+C_i^{\mathrm{SD}}w_{i,t}+\sum_b C_{i,b}^{\mathrm{p}}\,p_{i,t,b}\Big]+\sum_t M^p s_t^p && && \text{(1a)}\\
\text{s.t.} \quad & \sum_i p_{i,t}+s_t^p=D_t && \forall\, t && \text{(1b)}\\
& p_{i,t,b}\le \overline{P}_{i,b}\,u_{i,t} && \forall\, i,t,b && \text{(1c)}\\
& p_{i,t}=\sum_b p_{i,t,b} && \forall\, i,t && \text{(1d)}\\
& P_i^{\min}\,u_{i,t}\le p_{i,t}\le P_{i,t}^{\max}\,u_{i,t} && \forall\, i,t && \text{(1e)}\\
& p_{i,t}-p_{i,t-1}\le R_i^U\,u_{i,t-1}+R_i^U\,v_{i,t} && \forall\, i,\ t\ge 1 && \text{(1f)}\\
& p_{i,t-1}-p_{i,t}\le R_i^D\,u_{i,t}+R_i^D\,w_{i,t} && \forall\, i,\ t\ge 1 && \text{(1g)}\\
& u_{i,t}-u_{i,t-1}=v_{i,t}-w_{i,t} && \forall\, i,t \quad (u_{i,-1}:=u_i^0) && \text{(1h)}\\
& \sum_{\tau=t-T_i^{\mathrm{up}}+1}^{t} v_{i,\tau}\le u_{i,t} && \forall\, i,\ t\ge T_i^{\mathrm{up}}-1 && \text{(1i)}\\
& \sum_{\tau=t-T_i^{\mathrm{dn}}+1}^{t} w_{i,\tau}\le 1-u_{i,t} && \forall\, i,\ t\ge T_i^{\mathrm{dn}}-1 && \text{(1j)}\\
& -F_\ell^{\max}\le \sum_n H_{\ell,n}\Big(\sum_{i\in\mathcal{I}_n} p_{i,t}-d_{n,t}\Big)\le F_\ell^{\max} && \forall\, \ell,t && \text{(1k)}
\end{aligned}
$$

Implementation note. Solar and hydro generators have $u_{i,t}=1$ fixed (all costs zero, commitment is degenerate).

4 ARUC: Adaptive Robust UC with Linear Decision Rules

Decision rule. Dispatch is affine in the uncertainty realization $\mathbf{r}$:

$$p_{i,t}(\mathbf{r})=p_{i,t}^0+\mathbf{Z}_{i,t}^\top\mathbf{r}, \tag{2}$$

where $p_{i,t}^0$ is the nominal dispatch and $\mathbf{Z}_{i,t}\in\mathbb{R}^K$ encodes the linear response to $\mathbf{r}$.

Uncertainty set. Time-varying ellipsoid:

$$\mathcal{U}_t=\big\{\mathbf{r}\in\mathbb{R}^K:\mathbf{r}^\top\mathbf{\Sigma}_t^{-1}\,\mathbf{r}\le\rho_t^2\big\}. \tag{3}$$

Variables. Same commitment variables (u,v,w) as DAM. Nominal dispatch: $p_{i,t}^0\ge 0$, $p_{i,t,b}^0\ge 0$. LDR coefficients: $Z_{i,t,k}\in\mathbb{R}$. Auxiliary SOC variables: $z_{i,t}^{\mathrm{gen}}\ge 0$, $\mathbf{y}_{i,t}^{\mathrm{gen}}\in\mathbb{R}^K$.

$$\min \quad \sum_{i,t}\Big[C_i^{\mathrm{NL}}u_{i,t}+C_i^{\mathrm{SU}}v_{i,t}+C_i^{\mathrm{SD}}w_{i,t}+\sum_b C_{i,b}^{\mathrm{p}}\,p_{i,t,b}^0\Big]+\sum_t M^p s_t^p \tag{4a}$$

subject to commitment constraints (**??**)–(**??**) and:

Nominal power balance and zero-net-response.

$$\sum_i p_{i,t}^0 + s_t^p = D_t \qquad \forall\, t \tag{4b}$$

$$\sum_i Z_{i,t,k} = 0 \qquad \forall\, t, k \tag{4c}$$

Constraint (**??**) ensures power balance holds for all realizations: since $\sum_i p_{i,t}(\mathbf{r}) = \sum_i p_{i,t}^0 + (\sum_i \mathbf{Z}_{i,t})^\top \mathbf{r} = D_t$ for all $\mathbf{r}$.

Nominal ramp constraints.

$$p_{i,t}^0 - p_{i,t-1}^0 \le R_i^U\, u_{i,t-1} + R_i^U\, v_{i,t} \qquad \forall\, i,\ t \ge 1 \tag{4d}$$

$$p_{i,t-1}^0 - p_{i,t}^0 \le R_i^D\, u_{i,t} + R_i^D\, w_{i,t} \qquad \forall\, i,\ t \ge 1 \tag{4e}$$

Robust generator constraints (thermal, $i \notin \mathcal{W}$, $i \in \mathcal{Z}$). Define the auxiliary SOC variables:

$$\mathbf{y}_{i,t}^{\text{gen}} = \mathbf{L}_t\, \mathbf{Z}_{i,t} \tag{4f}$$

$$z_{i,t}^{\text{gen}} \ge \left\|\mathbf{y}_{i,t}^{\text{gen}}\right\|_2 \tag{4g}$$

Then the worst-case constraints become:

$$p_{i,t}^0 + \rho_t\, z_{i,t}^{\text{gen}} \le P_{i,t}^{\max}\, u_{i,t} \tag{4h}$$

$$P_i^{\min}\, u_{i,t} \le p_{i,t}^0 - \rho_t\, z_{i,t}^{\text{gen}} \tag{4i}$$

$$p_{i,t}^0 + \rho_t\, z_{i,t}^{\text{gen}} \le \sum_b p_{i,t,b}^0 \tag{4j}$$

$$p_{i,t,b}^0 \le \overline{P}_{i,b}\, u_{i,t} \qquad \forall\, b \tag{4k}$$

Wind generators ($i \in \mathcal{W}$). Wind generators skip the generic robust $P^{\max}/P^{\min}$ constraints. Instead, dispatch is bounded by the realized wind availability $\bar{P}_{i,t} + r_{k(i)}$:

$$p_{i,t}^0 - \bar{P}_{i,t} + \rho_t \left\|\mathbf{L}_t(\mathbf{Z}_{i,t} - \mathbf{e}_{k(i)})\right\|_2 \le 0 \qquad \forall\, i \in \mathcal{W},\ t \tag{4l}$$

where $\mathbf{e}_{k(i)}$ is the unit vector for the k-th wind generator, and $\bar{P}_{i,t}$ is the wind forecast. Block constraints for wind are nominal only (not robustified).

This is implemented via auxiliary variables $z_{k,t}^{\text{wind}}$, $\mathbf{y}_{k,t}^{\text{wind}}$:

$$\mathbf{y}_{k,t}^{\text{wind}} = \mathbf{L}_t(\mathbf{Z}_{i,t} - \mathbf{e}_k) \tag{4m}$$

$$z_{k,t}^{\text{wind}} \ge \left\|\mathbf{y}_{k,t}^{\text{wind}}\right\|_2 \tag{4n}$$

$$p_{i,t}^0 - \bar{P}_{i,t} + \rho_t\, z_{k,t}^{\text{wind}} \le 0 \tag{4o}$$

Solar/Hydro ($i \notin \mathcal{Z}$). These have $Z_{i,t,k} = 0$ for all k (no uncertainty response). Constraints are nominal only.

Robust line flow constraints. Define the net flow sensitivity to r_k:

$$a_{\ell,t,k} = \sum_n H_{\ell,n} \sum_{i\in\mathcal{I}_n} Z_{i,t,k} \tag{4p}$$

and auxiliary variables:

$$\mathbf{y}_{\ell,t}^{\text{line}} = \mathbf{L}_t\,\mathbf{a}_{\ell,t} \tag{4q}$$

$$z_{\ell,t}^{\text{line}} \geq \left\|\mathbf{y}_{\ell,t}^{\text{line}}\right\|_2 \tag{4r}$$

Then:

$$f_{\ell,t}^{\text{nom}} + \rho_t^{\text{line}}\,z_{\ell,t}^{\text{line}} \leq F_\ell^{\max} \qquad \forall\,\ell,t \tag{4s}$$

$$-f_{\ell,t}^{\text{nom}} + \rho_t^{\text{line}}\,z_{\ell,t}^{\text{line}} \leq F_\ell^{\max} \qquad \forall\,\ell,t \tag{4t}$$

where $f_{\ell,t}^{\text{nom}} = \sum_n H_{\ell,n}(\sum_{i\in\mathcal{I}_n} p_{i,t}^0 - d_{n,t})$ and $\rho_t^{\text{line}} = \gamma\cdot\rho_t$ with $\gamma =$ `rho_lines_frac`.

5 DARUC: Two-Step Day-Ahead Robust UC

Step 1 (Noon). Solve the deterministic DAM (**??**) to obtain $u_{i,t}^{\text{DAM}}$, $v_{i,t}^{\text{DAM}}$, $w_{i,t}^{\text{DAM}}$.

Step 2 (5 PM). Solve the ARUC (**??**) with additional monotonicity constraints:

$$u_{i,t} \geq u_{i,t}^{\text{DAM}} \qquad \forall\,i,t \tag{5}$$

This means DARUC can only *add* commitments on top of DAM—it cannot decommit any unit that DAM committed.

Deviation tracking. The code defines:

$$u'_{i,t} = u_{i,t} - u_{i,t}^{\text{DAM}} \geq 0 \tag{6}$$

$$v'_{i,t} = v_{i,t} - v_{i,t}^{\text{DAM}} \tag{7}$$

$$w'_{i,t} = w_{i,t} - w_{i,t}^{\text{DAM}} \tag{8}$$

Incremental objective (`incremental_obj=True`). When enabled, the DARUC objective only charges commitment costs for *additionally* committed units, with dispatch costs scaled down to break ties:

$$\min \quad \sum_{\substack{i,t\\u_{i,t}^{\text{DAM}}=0}} \left[C_i^{\text{NL}}u_{i,t} + C_i^{\text{SU}}v_{i,t} + C_i^{\text{SD}}w_{i,t}\right] + \epsilon\sum_{i,t,b} C_{i,b}^{\text{p}}\,p_{i,t,b}^0 + \sum_t M^p s_t^p \tag{9}$$

where $\epsilon =$ `dispatch_cost_scale` (default 0.1). This focuses the optimizer on which additional units to commit for reliability, without re-optimizing dispatch economics.

6 Robust Counterpart Derivation

The robust constraints arise from requiring feasibility for all $\mathbf{r} \in \mathcal{U}_t$. For a generic linear constraint of the form:

$$\mathbf{a}^\top \mathbf{r} \leq c \quad \forall \mathbf{r}^\top \mathbf{\Sigma}_t^{-1} \mathbf{r} \leq \rho_t^2$$

the robust counterpart is:

$$\rho_t \|\mathbf{L}_t \mathbf{a}\|_2 \leq c$$

which is a second-order cone (SOC) constraint. In the Gurobi implementation, this is represented via auxiliary variables $\mathbf{y} = \mathbf{L}_t \mathbf{a}$ and $z \geq \|\mathbf{y}\|_2$, yielding the linear constraint $\rho_t z \leq c$ with an SOC constraint $z^2 \geq \sum_k y_k^2$.

7 Code-to-Formulation Mapping

Code variable	**Math symbol**	**File:line**
`u[i,t]`	$u_{i,t}$	`aruc_model.py:241`
`v[i,t]`	$v_{i,t}$	`aruc_model.py:242`
`w[i,t]`	$w_{i,t}$	`aruc_model.py:243`
`p0[i,t]`	$p^0_{i,t}$	`aruc_model.py:246`
`p0_block[i,t,b]`	$p^0_{i,t,b}$	`aruc_model.py:249`
`Z[i,t,k]`	$Z_{i,t,k}$	`aruc_model.py:271`
`s_p[t]`	s^p_t	`aruc_model.py:252`
`z_gen[i,t]`	$z^{\text{gen}}_{i,t}$	`aruc_model.py:317`
`y_gen[i,t,k]`	$y^{\text{gen}}_{i,t,k}$	`aruc_model.py:318`
`z_wind[k,t]`	$z^{\text{wind}}_{k,t}$	`aruc_model.py:589`
`y_wind[k,t,j]`	$y^{\text{wind}}_{k,t,j}$	`aruc_model.py:591`
`z_line[l,t]`	$z^{\text{line}}_{\ell,t}$	`aruc_model.py:530`
`y_line[l,t,k]`	$y^{\text{line}}_{\ell,t,k}$	`aruc_model.py:531`
`u_prime[i,t]`	$u'_{i,t}$	`aruc_model.py:488`
`sqrt_Sigma`	$\mathbf{L}_t$	`aruc_model.py:204--206`
`rho_lines`	ρ^{line}_t	`aruc_model.py:198`