MA166 Recitation Notes and Exercises

Carlos Salinas

January 20, 2016

1 Notes: Vectors and the Geometry of Spaces

Material found in Stewart §12.

1.1 Three-Dimensional Coordinate Systems

Here are some of the most important concepts, equations, and theorems from this section. I know. I know. These are very boring concepts that you have probably seen all your life and you know how to do. But we must start somewhere and here is a perfect place.

The distance between two points $P_1(x, y, z)$ and $P_2(x, y, z)$ in $\mathbb{R}^3$ is given by the formula

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}. \tag{1}$$

This is also called the *Euclidean norm* and generalizes to all dimensions. Note that equation (1) is equivalent to

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} = |P_2P_1|$$

so that the distance between point does not depend on your point-of-view, i.e, whether you think of the line starting connecting P_1 and P_2 as starting at P_1 and ending at P_2 or vice-a-versa.

We often refer to the point $P_1(x, y, z)$ as the tuple (x_1, y_1, z_2) and $P_2(x, y, z)$ as (x_2, y_2, z_2), $P_3(x, y, z)$ as (x_3, y_3, z_3) and so on.

The equation of a sphere with $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2. \tag{2}$$

In particular, if the center is the origin O, then the equation (2) reduces to

$$x^2 + y^2 + z^2 = r^2.$$

1.2 Vector

A particle moves along a line segment from point A to point B. The corresponding displacement vector $\mathbf{v}$ has initial point A and terminal point B and is written $\mathbf{v} = \overrightarrow{AB}$.

Combining Vectors

If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the sum $\mathbf{u} + \mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.

2 Exercises Week 2