

$$\begin{array}{ccccc}
H_n(U_i, U_i \setminus \{x_i\}) & \xrightarrow{f_*} & H_n(V, V \setminus \{y\}) \\
\swarrow \cong & \downarrow k_i & \downarrow \cong \\
H_n(S^n, S^n \setminus \{x_i\}) & \xleftarrow{p_i} H_n(S^n, S^n \setminus \{f^{-1}(y)\}) & \xrightarrow{f_*} H_n(S^n, S^n \setminus \{y\}) \\
\swarrow \cong & \uparrow j & \uparrow \cong \\
H_n(S^n) & \xrightarrow{f_*} & H_n(S^n)
\end{array}$$