

# MA557 Problem Set 3

Carlos Salinas

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**PROBLEM 3.1**

Let  $R$  be a domain and  $\Gamma$  the set of all principal ideals in  $R$ . Show that  $R$  is a unique factorization domain if and only if  $\Gamma$  satisfies the ascending chain condition and every irreducible element of  $R$  is prime.

*Proof.*



**PROBLEM 3.2**

Let  $M$  be an Artinian  $R$ -module. Show that every injective  $R$ -linear map  $\varphi: M \rightarrow M$  is an isomorphism.

*Proof.*

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**PROBLEM 3.3**

Let  $M$  be a finitely generated Artinian module. Show that  $M$  is Noetherian.

*Proof.*



**PROBLEM 3.4**

Let  $R$  be a ring that is Artinian or Noetherian, and  $x \in R$ . Show that for some  $n > 0$ , the image of  $x$  in  $R/(0 : x)^n$  is a nonzero-divisor on that ring.

*Proof.*

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**PROBLEM 3.5**

Let  $R$  be an Artinian ring. Show that  $R \cong R_1 \times \cdots \times R_n$  with  $R_i$  Artinian local rings.

*Proof.*

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**PROBLEM 3.6**

Let  $R$  be an Artinian ring all of whose maximal ideals are principal. Show that every ideal in  $R$  is principal.

*Proof.*





**PROBLEM 3.7**

Prove 2.12.

*Proof.* Recall the statement of Theorem 2.12:

**Theorem.** *Let  $R$  be a ring,  $M$ ,  $M'$  and  $M''$  be  $R$ -modules. Then*

(a) *The following are equivalent:*

- (1)  $0 \rightarrow M' \xrightarrow{\varphi} M \xrightarrow{\psi} M''$  is exact
- (2)  $0 \rightarrow \text{hom}(N, M') \xrightarrow{\text{hom}(N, \varphi)} \text{hom}(N, M) \xrightarrow{\text{hom}(N, \psi)} \text{hom}(N, M'')$  is exact for all modules  $N$ .

(b) *The following are equivalent:*

- (1)  $M' \xrightarrow{\varphi} M \xrightarrow{\psi} M'' \rightarrow 0$  is exact.
- (2)  $0 \rightarrow \text{hom}(M'', N) \xrightarrow{\text{hom}(\psi, N)} \text{hom}(M, N) \xrightarrow{\text{hom}(\varphi, N)} \text{hom}(M', N)$  is exact for all modules  $N$ .
- (3)  $M' \otimes N \xrightarrow{\varphi \otimes N} M \otimes N \xrightarrow{\psi \otimes N} M'' \otimes N \rightarrow 0$  is exact for all modules  $N$ .

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