

# MA 598 (Algebraic Geometry): Homework 1

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**PROBLEM 1.1 (5-LEMMA)**

Given a commutative diagram with exact rows

$$\begin{array}{ccccccc}
 0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C \longrightarrow 0 \\
 & & \downarrow f & & \downarrow g & & \downarrow h \\
 0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' \longrightarrow 0,
 \end{array}$$

suppose  $f$  and  $h$  are isomorphisms. Prove that  $g$  is an isomorphism.

**Solution.** ►

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**PROBLEM 1.2 (SNAKE LEMMA)**

Given the diagram with exact rows

$$\begin{array}{ccccccc}
 & & \text{Ker } f & \longrightarrow & \text{Ker } g & \longrightarrow & \text{Ker } h \\
 & & \downarrow & & \downarrow & & \downarrow \\
 0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C \longrightarrow 0 \\
 & & \downarrow f & & \downarrow g & & \downarrow h \\
 0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \downarrow \\
 & & \text{Coker } f & \longrightarrow & \text{Coker } g & \longrightarrow & \text{Coker } h,
 \end{array}$$

show that the sequence

$$0 \longrightarrow \text{Ker } f \longrightarrow \text{Ker } g \longrightarrow \text{Ker } h \longrightarrow \text{Coker } f \longrightarrow \text{Coker } g \longrightarrow \text{Coker } h \longrightarrow 0$$

is exact.

**Solution.** ►

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**PROBLEM 1.3**

Given an exact sequence of modules

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

fix any  $M \in \mathbf{Mod}_R$ .

(a) Show that

$$0 \longrightarrow \mathrm{Hom}(M, A) \longrightarrow \mathrm{Hom}(M, B) \longrightarrow \mathrm{Hom}(M, C)$$

is exact.

(b) Show that

$$0 \longrightarrow \mathrm{Hom}(C, M) \longrightarrow \mathrm{Hom}(B, M) \longrightarrow \mathrm{Hom}(A, M)$$

is exact.

**Solution.** ►

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**PROBLEM 1.4****Solution.** ▶

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**PROBLEM 1.5****Solution.** ►

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**PROBLEM 1.6****Solution.** ▶

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