2.2.4 Each of the given linear systems is in reduced row echelon form. Solve the system.

(a) $x \quad -2z = 5$
$\quad y + z = 2$

(b) $x = 1$
$y = 2$
$z - w = 4$

(a) Take $z = r$ any real number. Then $x = 5 + 2r$, $y = 2 - r$, $z = r$ is the solution.

(b) Take $w = r$ any real number. Then $x = 1, y = 2, z = 4 + r$ are the solutions.

2.2.6 (i) Find all solutions, if any exist, by the Gaussian elimination method.
(ii) Find all solutions, if any exists, by the Gauss-Jordan reduction method.

(a) $x + y + 2z + 3w = 13$
$x - 2y + z + w = 8$
$3x + y + z - w = 1$

(b) $x + y + z = 1$
$x + y - 2z = 3$
$2x + y + z = 2$

(c) $2x + y + z - 2w = 1$
$3x - 2y + z - 6w = -2$
$x + y - z - w = -1$
$6x + z - 9w = -2$
$5x - y + 2z - 8w = 3$

(a)
$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 1 & -2 & 1 & 1 & 8 \\ 3 & 1 & 1 & -1 & 1 \end{array}\right] \begin{array}{l} -r_1 + r_2 \to r_2 \\ -3r_1 + r_3 \to r_3 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -3 & -1 & -2 & -5 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] \begin{array}{l} -2r_3 + r_2 \to r_2 \\ -r_3 \to r_3 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & 9 & 18 & 71 \\ 0 & 2 & 5 & 10 & 38 \end{array}\right] -2r_2 + r_3 \to r_3 \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & 9 & 18 & 71 \\ 0 & 0 & -13 & -26 & -104 \end{array}\right]$$

$$-\frac{1}{13} r_3 \to r_3 \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & 9 & 18 & 71 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right]$$

(i) This is in row echelon form (Gaussian elimination) so set $w = r$ any real number, then $z = 8 - 2r$, $y = -1$, $x = -2 + r$, $w = r$

(iii) Continuing to RREF (Gauss-Jordan)

$$\begin{array}{l} -2r_3 + r_1 \to r_1 \\ -9r_3 + r_2 \to r_2 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 0 & -1 & -3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right] -r_2 + r_1 \to r_1 \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 2 & 8 \end{array}\right]$$
gives the same solution explicitly as (i).

(b)
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & -2 & 3 \\ 2 & 1 & 1 & 2 \end{array}\right] \begin{array}{l} -r_1 + r_2 \to r_2 \\ -2r_1 + r_3 \to r_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & -1 & -1 & 0 \end{array}\right] r_2 \leftrightarrow r_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -3 & 2 \end{array}\right] \begin{array}{l} -r_2 \to r_2 \\ -\frac{1}{3} r_3 \to r_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2/3 \end{array}\right]$$

(i) Then $z = -2/3$, $y = 2/3$, $x = 1$.

(ii) $\begin{array}{l} -r_2 + r_1 \to r_1 \\ -r_3 + r_2 \to r_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2/3 \\ 0 & 0 & 1 & -2/3 \end{array}\right]$ giving the same.

(c) $\left[\begin{array}{cccc|c} 2 & 1 & 1 & -2 & 1 \\ 3 & -2 & 1 & -6 & -2 \\ 1 & 1 & -1 & -1 & -1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_3} \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 3 & -2 & 1 & -6 & -2 \\ 2 & 1 & 1 & -2 & 1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right] \xrightarrow{\substack{-3r_1+r_2 \to r_2 \\ -2r_1+r_3 \to r_3 \\ -6r_1+r_4 \to r_4 \\ -5r_1+r_5 \to r_5}} \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & -6 & 7 & -3 & 8 \end{array}\right] \xrightarrow{-r_4+r_5 \to r_5} \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right]$

(i) and (ii) no solutions exist.

2.2.8| Solve the linear system, with the given augmented matrix, if it is consistent.

(a) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{array}\right]$ (b) $\left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{array}\right]$

(a) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{array}\right] \xrightarrow{\substack{-r_1+r_2 \to r_2 \\ -r_1+r_3 \to r_3}} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{array}\right] \xrightarrow{2r_2+r_3 \to r_3} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{array}\right] \xrightarrow{-\frac{1}{7}r_3 \to r_3} \left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right]$

$\xrightarrow{\substack{-3r_3+r_1 \to r_1 \\ 3r_3+r_2 \to r_2}} \left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right] \xrightarrow{-2r_2+r_1 \to r_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right]$ is in RREF.

(b) $\left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{array}\right] \xrightarrow{-r_1+r_3 \to r_3} \left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{array}\right] \xrightarrow{r_3+r_2 \to r_2} \left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & -1 & -1 & 2 & -1 \end{array}\right] \xrightarrow{r_2+r_3 \to r_3} \left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & -1 & 1 & 1 \end{array}\right] \xrightarrow{-r_3 \to r_3}$

$\left[\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right] \xrightarrow{-r_2+r_1 \to r_1} \left[\begin{array}{cccc|c} 1 & 0 & 3 & -2 & -2 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right] \xrightarrow{-3r_3+r_1 \to r_1} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right]$ is in RREF.

2.2.10| Find a 2x1 matrix x with entries not all zero such that $Ax = 4x$, where $A = \begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix}$

$Ax = 4x$ is $Ax - 4x = 0$ or $(A - 4I)x = 0$. Set $x = \begin{bmatrix} a \\ b \end{bmatrix}$. Then

$(A - 4I)x = \left(\begin{bmatrix} 4 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right)\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ -2b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ so $b = 0$ and a any real nonzero number.

2.2.12| Find a 3x1 matrix x with entries not all zero such that $Ax = 3x$, where $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$.

$(A-3I)x = \left(\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\right)\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 & 2 & -1 \\ 1 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Augmented matrix is

$\left[\begin{array}{ccc|c} -2 & 2 & -1 & 0 \\ 1 & -3 & 1 & 0 \\ 4 & -4 & 2 & 0 \end{array}\right] \xrightarrow{r_2 \leftrightarrow r_1} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ -2 & 2 & -1 & 0 \\ 4 & -4 & 2 & 0 \end{array}\right] \xrightarrow{+2r_2+r_3 \to r_3} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ -2 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{2r_1+r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{-\frac{1}{4}r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{3r_2+r_1 \to r_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

gives $a + c = 0$, $b = 0$ so any $x = \begin{bmatrix} a \\ 0 \\ -a \end{bmatrix}$ where $a \neq 0$.

2.2.14 | In the following linear system, determine all values of a for which the resulting linear system has (a) no solution; (b) a unique solution; (c) infinitely many solutions:

$$\begin{aligned} x + y \quad - z &= 2 \\ x + 2y \quad + z &= 3 \\ x + y + (a^2-5)z &= a \end{aligned}$$

Put Augmented matrix in REF.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right] \begin{array}{l} \\ -r_1+r_2 \to r_2 \\ -r_1+r_3 \to r_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

Note if $a^2-4 \neq a-2$ we have (a).

If $a^2-4 = a-2$ we have (b). If $a^2-4=0=a-2$ we have (c).

If $a^2-4 = a-2$, then $a^2-a-2=0$ or $(a+1)(a-2)=0$ so $a=-1$ for (b), $a=2$ for (c) and $a \neq -1$ and $a \neq 2$ for (a).

2.2.16 | Repeat exercise 14 for the linear system

$$\begin{aligned} x + y \quad + z &= 2 \\ x + 2y \quad + z &= 3 \\ x + y + (a^2-5)z &= a \end{aligned}$$

Put Augmented matrix in REF.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right] \begin{array}{l} -r_1+r_2 \to r_2 \\ -r_1+r_3 \to r_3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-6 & a-2 \end{array}\right]$$

So solving $a^2-6 = a-2$

Then $a^2-a-4=0$ gives $a = \frac{1 \pm \sqrt{1^2-4\cdot1\cdot4}}{2} = \frac{1}{2} \pm \frac{\sqrt{15}}{2}i$ showing that (b) happens if $a = \frac{1}{2} \pm \frac{\sqrt{15}}{2}i$, (c) never happens, and (a) happens otherwise.

2.2.20 | Let $f: \mathbb{R}^3 \to \mathbb{R}^3$ be the matrix transformation defined by $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

Find x, y, z so that $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$.

$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$ in Augmented matrix form is

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 4 \\ 2 & -1 & 3 & 5 \\ 2 & 2 & 0 & -1 \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_3} \left[\begin{array}{ccc|c} 2 & 2 & 0 & -1 \\ 2 & -1 & 3 & 5 \\ 4 & 1 & 3 & 4 \end{array}\right] \begin{array}{l} \\ -r_1+r_2 \to r_2 \\ -2r_1+r_3 \to r_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 0 & -1 \\ 0 & -3 & 3 & 6 \\ 0 & -3 & 3 & 6 \end{array}\right] \xrightarrow{-r_2+r_3 \to r_3} \left[\begin{array}{ccc|c} 2 & 2 & 0 & -1 \\ 0 & -3 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{-1/3 r_2 \to r_2} \left[\begin{array}{ccc|c} 2 & 2 & 0 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{-2r_2+r_1 \to r_1} \left[\begin{array}{ccc|c} 2 & 0 & 2 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

So for any real number r, $x = \frac{3-2r}{2}$, $y = r-2$, $z = r$.

2.2.22 Let $f:\mathbb{R}^3 \to \mathbb{R}^3$ be the matrix transformation defined by $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Find an equation relating a, b, and c so that we can always compute values of x, y, and z for which $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

Put in Augmented matrix

$$\left[\begin{array}{ccc|c} 4 & 1 & 3 & a \\ 2 & -1 & 3 & b \\ 2 & 2 & 0 & c \end{array}\right] \xrightarrow{r_1 \leftrightarrow r_3} \left[\begin{array}{ccc|c} 2 & 2 & 0 & c \\ 2 & -1 & 3 & b \\ 4 & 1 & 3 & a \end{array}\right] \xrightarrow[-2r_1+r_3 \to r_3]{-r_1+r_2 \to r_2} \left[\begin{array}{ccc|c} 2 & 2 & 0 & c \\ 0 & -3 & 3 & b-c \\ 0 & -3 & 3 & a-2c \end{array}\right] \xrightarrow{-r_2+r_3 \to r_3}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 0 & c \\ 0 & -3 & 3 & b-c \\ 0 & 0 & 0 & a-b-c \end{array}\right] \xrightarrow{-\frac{1}{3}r_2 \to r_2} \left[\begin{array}{ccc|c} 2 & 2 & 0 & c \\ 0 & 1 & -1 & -\frac{1}{3}(b-c) \\ 0 & 0 & 0 & a-b-c \end{array}\right] \xrightarrow{-2r_2+r_1 \to r_1} \left[\begin{array}{ccc|c} 2 & 0 & 2 & c+\frac{2}{3}(b-c) \\ 0 & 1 & -1 & -\frac{1}{3}(b-c) \\ 0 & 0 & 0 & a-b-c \end{array}\right] \xrightarrow{\frac{1}{2}r_1 \to r_1}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{1}{2}c - \frac{1}{3}c + \frac{1}{3}b \\ 0 & 1 & -1 & -\frac{1}{3}(b-c) \\ 0 & 0 & 0 & a-b-c \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & \frac{1}{3}b + \frac{1}{6}c \\ 0 & 1 & -1 & -\frac{1}{3}b + \frac{1}{3}c \\ 0 & 0 & 0 & a-b-c \end{array}\right]$$

So $a-b-c=0$ and for any real number r,

$x = \frac{1}{3}b + \frac{1}{6}c - r$, $y = r - \frac{1}{3}b + \frac{1}{3}c$, $z = r$.

2.2.26 Find an equation relating a, b, and c so that the linear system

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 3y + 3z &= b \\ 5x + 9y - 6z &= c \end{aligned}$$

is consistent for any values of a, b, c that satisfy that equation.

Augmented matrix and REF.

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right] \xrightarrow[-5r_1+r_3 \to r_3]{-2r_1+r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & -1 & 9 & c-5a \end{array}\right] \xrightarrow[-r_2+r_3 \to r_3]{-r_2 \to r_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 1 & -9 & 2a-b \\ 0 & 0 & 0 & 2a-5a-b+c \end{array}\right]$$

So $-3a-b+c=0$.

Matlab 4.2

Matlab 4.2.2 Use rref to find the general solution of the following homogeneous system of linear equations. Record your solution.

$x_1 - x_2 + 2x_3 \qquad + x_5 = 0$

$2x_1 + x_2 + x_3 + x_4 + x_5 = 0$

$x_1 + x_2 \qquad + 2x_4 + 2x_5 = 0$

$S = [1\ {-1}\ 2\ 0\ 1\ 0;\ 2\ 1\ 1\ 1\ 1\ 0;\ 1\ 1\ 0\ 2\ 2\ 0]$

$$\text{rref}(S) = \left[\begin{array}{ccccc|c} 1 & 0 & 1 & 0 & 1/4 & 0 \\ 0 & 1 & -1 & 0 & -3/4 & 0 \\ 0 & 0 & 0 & 1 & 5/4 & 0 \end{array}\right]$$

So $x_1 = -x_3 - \frac{1}{4}x_5$

$x_2 = x_3 + \frac{3}{4}x_5$

$x_4 = -\frac{5}{4}x_5$

Matlab 4.2.3 Let A be the coefficient matrix in Exercise 2. Compute rref(A) and rref(A'). Are they the same?

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 0 & 1/4 \\ 0 & 1 & -1 & 0 & -3/4 \\ 0 & 0 & 0 & 1 & 5/4 \end{bmatrix} \quad \text{rref}(A') = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

They are not the same.

Matlab 4.3

Matlab 4.3.1 Use rref command to find the inverses of each matrix below.

$$A1 = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad A2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\text{rref}([A1\ \text{eye}(\text{size}(A1))]) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 2 \end{bmatrix} \text{ so } A1^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\text{rref}([A2\ \text{eye}(\text{size}(A2))]) = \begin{bmatrix} 1 & 0 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{bmatrix} \text{ so } A2^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 \end{bmatrix}.$$

Matlab 4.3.2 Use invert to determine which of the following matrices are nonsingular. Record if it is singular or nonsingular. Record the inverse if it is nonsingular.

(a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ invert(A) matrix is singular

(b) $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$ $\text{Invert}(B) = \begin{bmatrix} -1.7778 & .8889 & -.1111 \\ 1.5556 & -.7778 & .2222 \\ -.1111 & .2222 & -.1111 \end{bmatrix}$

nonsingular.

(c) $C = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 0 & 6 \\ 7 & 0 & 8 & 9 \\ 0 & 10 & 11 & 12 \end{bmatrix}$ invert(C) = $\begin{bmatrix} .2 & .1067 & .0533 & -.0933 \\ .1818 & .1030 & -.0848 & .0121 \\ .1455 & -.1042 & .0388 & .0230 \\ -.2848 & .0097 & .0352 & .0521 \end{bmatrix}$

nonsingular

(d) $D = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 0 & 6 \\ 7 & 0 & 8 & 9 \\ 1 & 2 & 3 & 0 \end{bmatrix}$ invert(D) matrix is singular.