

# MA 523: Homework 8

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## PROBLEM 8.1

Show that the function

$$u(x, t) := \sum_{k=-\infty}^{\infty} (-1)^k \Phi(x - 2k, t)$$

where

$$\Phi(x, t) = \frac{e^{-x^2/4t}}{\sqrt{4\pi t}}$$

is positive for  $|x| < 1$ ,  $t > 0$ .

(*Hint:* Show that  $u$  satisfies  $u_t = u_{xx}$  for  $t > 0$ ,

$$\begin{cases} u = 0 & \text{on } \{|x| = 1\} \times \{t \geq 0\}, \\ u = \delta_0 & \text{on } \{|x| = 1\} \times \{t = 0\}. \end{cases}$$

Then, carefully apply the maximum/minimum principle in a domain  $\{|x| \leq 1\} \times \{\varepsilon \leq t \leq T\}$  for small  $\varepsilon > 0$  and large  $T > 0$  pass to the limit as  $\varepsilon \rightarrow 0^+$  and  $T \rightarrow \infty$ .)

*SOLUTION.*

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## PROBLEM 8.2 (TIKHONOV'S EXAMPLE)

Let

$$g(t) := \begin{cases} e^{-t^2} & t > 0, \\ 0 & t \leq 0. \end{cases}$$

Then  $g \in C^\infty(\mathbb{R})$  and we define

$$u(x, t) := \sum_{k=0}^{\infty} \frac{g^{(k)}(t)}{(2k)!} x^{2k}.$$

Assuming that the series is convergent, show that  $u(x, t)$  solves the heat equation in  $\mathbb{R} \times (0, \infty)$  with the initial condition  $u(x, 0) = 0$ ,  $x \in \mathbb{R}$ . Why doesn't this contradict the uniqueness theorem for the initial value problem.)

*SOLUTION.*

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## PROBLEM 8.3

Evaluate the integral

$$\int_{-\infty}^{\infty} \cos(ax) e^{-x^2} dx, \quad (a > 0).$$

(*Hint:* Use the separation of variables to find the solution of the corresponding initial-value problem for the heat equation.)

*SOLUTION.*

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