

# MA 519: Homework 4

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## PROBLEM 4.1 (HANDOUT 5, # 2)

In an urn, there are 12 balls. 4 of these are white. Three players:  $A$ ,  $B$ , and  $C$ , take turns drawing a ball from the urn, in the alphabetical order. The first player to draw a white ball is the winner. Find the respective winning probabilities: assume that at each trial, the ball drawn in the trial before is put back into the urn (i.e., selection *with replacement*).

SOLUTION. ■

## PROBLEM 4.2 (HANDOUT 5, # 8)

Consider  $n$  families with 4 children each. How large must  $n$  be to have a 90% probability that at least 3 of the  $n$  families are all girl families?

SOLUTION. ■

## PROBLEM 4.3 (HANDOUT 5, # 10)

(*Yahtzee*). In Yahtzee, five fair dice are rolled. Find the probability of getting a Full House, which is three rolls of one number and two rolls of another, in Yahtzee.

*SOLUTION.*

■

**PROBLEM 4.4 (HANDOUT 5, # 12)**

The probability that a coin will show all heads or all tails when tossed four times is 0.25. What is the probability that it will show two heads and two tails?

*SOLUTION.*



## PROBLEM 4.5 (HANDOUT 5, # 13)

Let the events  $A_1, A_2, \dots, A_n$  be independent and  $P(A_k) = p_k$ . Find the probability  $p$  that none of the events occurs.

*SOLUTION.*

■

## PROBLEM 4.6 (HANDOUT 6, # 5)

Suppose a fair die is rolled twice and suppose  $X$  is the absolute value of the difference of the two rolls. Find the PMF and the CDF of  $X$  and plot the CDF. Find a median of  $X$ ; is the median unique?

*SOLUTION.*





## PROBLEM 4.7 (HANDOUT 6, # 7)

Find a discrete random variable  $X$  such that  $E(X) = E(X^3) = 0$ ;  $E(X^2) = E(X^4) = 1$ .

*SOLUTION.* Set  $\Omega = \{0, 1\}$  and define a random variable  $X: \Omega \rightarrow \mathbb{R}$  by  $X(0) = -1$ ,  $X(1) = 1$  as well as a probability  $P(0) = P(1) = 1/2$ . Then

$$E[X] = -1(1/2) + 1(1/2) = 0 = (-1)^3(1/2) + 1^3(1/2) = E[X^3],$$

whereas

$$E[X^2] = (-1)^2(1/2) + 1^2(1/2) = 1 = (-1)^4(1/2) + 1^4(1/2) = E[X^4],$$

as desired. ■

## PROBLEM 4.8 (HANDOUT 6, # 9)

(Runs). Suppose a fair die is rolled  $n$  times. By using the indicator variable method, find the expected number of times that a six is followed by at least two other sixes. Now compute the value when  $n = 100$ .

SOLUTION. Let  $\Omega$  denote the sample space and  $A$  denote the event that in a sequence of  $n$  rolls, at least three consecutive sixes come up. Define a random variable  $X: \Omega \rightarrow \mathbb{R}$  by  $X(x) = \chi_A(x)$ , that is,  $X(x) = 1$  if  $x \in A$  and  $X(x) = 0$  otherwise. ■

## PROBLEM 4.9 (HANDOUT 6, # 10)

(*Birthdays*). For a group of  $n$  people find the expected number of days of the year which are birthdays of exactly  $k$  people. (Assume 365 days and that all arrangements are equally probable.)

*SOLUTION.* Let  $\Omega$  denote the sample space and let  $A_k$  denote the event that exactly  $k$  people have the same birthday. We can write  $A_k$  as the intersection of events  $A_{n,k}$  ■

## PROBLEM 4.10 (HANDOUT 6, # 11)

(*Continuation*). Find the expected number of multiple birthdays. How large should  $n$  be to make this expectation exceed 1?

SOLUTION. ■

## PROBLEM 4.11 (HANDOUT 6, # 12)

(*The blood-testing problem*). A large number,  $N$ , of people are subject to a blood test. This can be administered in two ways, (i) Each person can be tested separately. In this case  $N$  tests are required, (ii) The blood samples of  $k$  people can be pooled and analyzed together. If the test is negative, this one test suffices for the  $k$  people. If the test is positive, each of the  $k$  persons must be tested separately, and in all  $k + 1$  tests are required for the  $k$  people. Assume the probability  $p$  that the test is positive is the same for all people and that people are stochastically independent.

- (b) What is the expected value of the number,  $X$ , of tests necessary under plan (ii)?
- (c) Find an equation for the value of  $k$  which will minimize the expected number of tests under the second plan. (Do not try numerical solutions.)

SOLUTION. ■

## PROBLEM 4.12 (HANDOUT 6, # 13)

(*Sample structure*). A population consists of  $r$  (classes whose sizes are in the proportion  $p_1 : p_2 : \cdots : p_r$ ). A random sample of size  $n$  is taken with replacement. Find the expected number of classes not represented in the sample.

SOLUTION. ■