

# MA571 Problem Set 1

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**Problem 1.1 (Munkres, §17, p. 100, 3)**

Show that if  $A$  is closed in  $Y$  and  $Y$  is closed in  $X$ , then  $A$  is closed in  $X$ .

*Proof.*

■

**Problem 1.2 (Munkres, §17, p. 101, 6(b))**

Let  $A$ ,  $B$  and  $A_\alpha$  denote subsets of a space  $X$ . Prove the following:

(b)  $\overline{A \cup B} = \bar{A} \cup \bar{B}$ .

*Proof.*

■

**Problem 1.3 (Munkres, §17, p. 101, 6(c))**

Let  $A$ ,  $B$  and  $A_\alpha$  denote subsets of a space  $X$ . Prove the following:

(b)  $\overline{\bigcup A_\alpha} \supset \bigcup \overline{A_\alpha}$ .

*Proof.*

■

**Problem 1.4 (Munkres, §17, p. 101, 7)**

Criticize the following “proof” that  $\overline{\bigcup A_\alpha} \subset \bigcup \bar{A}_\alpha$ : if  $\{A_\alpha\}$  is a collection of sets in  $X$  and if  $x \in \overline{\bigcup A_\alpha}$ , then every neighborhood  $U$  of  $x$  intersects  $\bigcup A_\alpha$ . Thus  $U$  must intersect some  $A_\alpha$ , so  $x$  must belong to the closure of some  $A_\alpha$ . Therefore,  $x \in \bigcup \bar{A}_\alpha$ .

*Critique.*

■

**Problem 1.5 (Munkres, §17, p. 101, 9)**

Let  $A \subset X$  and  $B \subset Y$ . Show that in the space  $X \times Y$ ,

$$\overline{A \times B} = \bar{A} \times \bar{B}.$$

*Proof.*

■

**Problem 1.6 (Munkres, §17, p. 101, 10)**

Show that every order topology is Hausdorff.

*Proof.*

■



**Problem 1.7 (Munkres, §17, p. 101, 13)**

Show that  $X$  is Hausdorff if and only if the *diagonal*  $\Delta = \{x \times x \mid x \in X\}$  is closed in  $X \times X$ .

*Proof.*

■

**Problem 1.8 (Munkres, §18, p. 111, 4)**

Given  $x_0 \in X$  and  $y_0 \in Y$ , show that the maps  $f: X \rightarrow X \times Y$  and  $g: Y \rightarrow X \times Y$  defined by

$$f(x) = x \times y_0 \quad \text{and} \quad g(y) = x_0 \times y$$

are imbeddings.

*Proof.*

■

**Problem 1.9 (Munkres, §18, p. 111-112, 8(a,b))**

Let  $Y$  be an ordered set in the order topology. Let  $f, g: X \rightarrow Y$  be continuous.

- (a) Show that the set  $\{x \mid f(x) \leq g(x)\}$  is closed in  $X$ .
- (b) Let  $h: X \rightarrow Y$  be the function

$$h(x) = \min\{f(x), g(x)\}.$$

Show that  $h$  is continuous. [*Hint:* Use the pasting lemma.]

*Proof.*

■

**Problem 1.10**

Given:  $X$  is a topological space with open sets  $U_1, \dots, U_n$  such that  $\bar{U}_i = X$  for all  $i$ . Prove that the closure of  $U_1 \cap \dots \cap U_n$  is  $X$ .

*Proof.*

■