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## MA 26500-215 Quiz 3

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1. Let *A* be the following matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{bmatrix}.$$

- (a) (8 points) Compute the determinant of A using any method you like.
- (b) (4 points) For what values of *t* is *A* nonsingular?

**Solution**: For part (a) the method of cofactors is fairly straightforward way to find the determinant in this instance. So expand along the top row

$$\det A = 1 \det \begin{pmatrix} 2 & t \\ 4 & t^2 \end{pmatrix} - 1 \det \begin{pmatrix} 1 & t \\ 1 & t^2 \end{pmatrix} 1 \det \begin{pmatrix} 1 & 2 \\ 1 & 4 \end{pmatrix}$$

$$= (2t^2 - 4t) - (t^2 - t) + (4 - 2)$$

$$= t^2 - 3t + 2$$

$$= (t - 2)(t - 1).$$

$$(\star)$$

For part (b), using the result of  $(\star)$ , we see that t is nonsingular if and only if  $t \neq 1, 2$ .

2. Let *A* be the following matrix

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}.$$

- (a) (5 points) Find the adjoint matrix of *A*, adj *A*.
- (b) (3 points) Find the determinant of *A*.

**Solution**: For part (a), remember that the adjoint of a matrix adj *A* is the transpose of its cofactor matrix. So we need to find the entries of the cofactor matrix

$$A_{11} = \begin{bmatrix} 6 & 2 \\ 0 & -3 \end{bmatrix} = -18, \qquad A_{12} = -\begin{bmatrix} 5 & 2 \\ 1 & -3 \end{bmatrix} = 17, \qquad A_{13} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix} = -6,$$

$$A_{21} = -\begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} = -6, \qquad A_{22} = \begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix} = -8, \qquad A_{23} = -\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}, = 2$$

$$A_{31} = \begin{bmatrix} -2 & 1 \\ 6 & 2 \end{bmatrix} = -10, \qquad A_{32} = -\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = -1, \qquad A_{33} = \begin{bmatrix} 3 & -2 \\ 5 & 6 \end{bmatrix} = 28.$$

So

$$\operatorname{adj} A = \begin{bmatrix} -18 & 17 & -6 \\ -6 & -8 & 2 \\ -10 & -1 & 28 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} -18 & -6 & -10 \\ 17 & -8 & -1 \\ -6 & 2 & 28 \end{bmatrix}.$$

For part (b), recall that A adj A = det AI. Since the identity matrix I only has 1 across the diagonal, det AI has det A across the diagonal. So all we need to do is multiply the first row of A times the first column of adj A to find the determinant of A

$$\det A = \begin{bmatrix} 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} -18 \\ 17 \\ -6 \end{bmatrix} = -3 \cdot 18 - 2 \cdot 17 - 6 = -94.$$