Exercises on Larson's Problem Solving Through Problems

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1 Heuristics

Typical heuristics that have helped people solve problems in the past

(1) Search for a pattern.
(2) Draw a figure.
(3) Formulate an equivalent problem.
(4) Modify the problem.
(5) Choose effective notation.
(6) Exploit symmetry.
(7) Divide into cases.
(8) Work backward.
(9) Argue by contradiction.
(10) Pursue parity.
(11) Consider extreme cases.
(12) Generalize.

Exercise 1.0.1. Prove that the set of n (different) elements has exactly 2^n (different) subsets.

Proof. For simplicity's sake, let us denote the set of n different elements by $S \coloneqq \{1, ..., n\}$. Now, suppose $T \subset S$. Then, either $T = \emptyset$ or $T \neq \emptyset$. If $T \neq \emptyset$, there is at least one element $i \in T$ and there are n possibilities. Hence, we have $n + 1$ possibilities for $T \subset S$ with at most one element. Now, suppose $T \subset S$ has two elements $i \neq j$, then we have $n(n-1)$ possibilities for T. Continuing in this way, we see that for T_m with $m < n$ the subset consisting of m elements of S, $|T_m| = n(n-1)\cdots(n-m) = n!/(m-n+1)!$. Hence, the total number of possible subsets are

$$\sum_{k=1}^{n} \frac{n!}{(n-k+1)!}$$

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