MA 519: Homework 2

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Problem 2.1 (Handout 2, # 5)

Four men throw their watches into the sea, and the sea brings each man one watch back at random. What is the probability that at least one man gets his own watch back?

Solution. ► The number of different arrangements, i.e., the sample space Ω, corresponds to the symmetric group on four numbers S_4 and therefore

$$\#\Omega = 4! = 24. \tag{2.1}$$

Let A denote the event that at least one man gets his own watch back. In this case, it is easier to count the sample points in the complement of A, i.e., the event $\Omega \setminus A$ that no man gets his own watch back.

Now we count the sample points in $\Omega \setminus A$: There are 3 ways for the first man to get back a watch other than his own. For the second man, either the first man picked his watch or he did not. In the first case, there are 3 ways for him to get back a watch not his own, whereas in the second only 2 ways. ◄

Problem 2.2 (Handout 2, #7)

Calculate the probability that in Bridge, the hand of at least one player is void in a particular suit.

Solution. ► ◄

Problem 2.3 (Handout 2, # 12)

If n balls are placed at random into n cells, find the probability that exactly 1 cell remains empty.

Solution. ► ◄

Problem 2.4 (Handout 2, # 13)

Spread of rumors. In a town of $n + 1$ inhabitants, a person tells a rumor to a second person, who in turn repeats it to a third person, *etc.* At each step the recipient of the rumor is chosen at random from the n people available. Find the probability that the rumor told r times without:

(a) returning to the originator,
(b) being repeated to any person.

Do the same problem when at each step the rumor told by one person to a gathering of N randomly chosen people. (The first question is the special case $N = 1$).

Solution. ►

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Problem 2.5 (Handout 2, # 14)

A family problem. In a certain family four girls take turns at washing dishes. Out of a total of four breakages, three were caused by the youngest girl, and she was thereafter called clumsy. Was she justified in attributing the frequency of breakages to chance? Discuss the connection with random placement of balls.

Solution. ►

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Problem 2.6 (Handout 2, # 15)

A car is parked among N cars in a row, not at either end. On his return the owner finds exactly r of the N places still occupied. What is the probability that both neighboring places are empty?

Solution. ►

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Problem 2.7 (Handout 2, # 16)

Find the probability that in a random arrangement of 52 bridge card no two aces are adjacent.

Solution. ►

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Problem 2.8 (Handout 2, # 17)

Suppose $P(A) = 3/4$, and $P(B) = 1/3$.

Prove that $P(A \cap B) \geq 1/12$. Can it be equal to $1/12$?

Solution. ►

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Problem 2.9 (Handout 2, # 18)

Suppose you have infinitely many events $A_1, A_2, \ldots$, and each one is sure to occur, i.e., $P(A_i) = 1$ for any i.

Prove that $P\left(\bigcap_{i=1}^{n} A_i\right) = 1$.

Solution. ►

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Problem 2.10 (Handout 2, # 19)

There are n blue, n green, n red, and n white balls in an urn. Four balls are drawn from the urn with replacement. Find the probability that there are balls of at least three different colors among the four drawn.

Solution. ►

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