MA571 Problem Set 1

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Problem 1.1 (Munkres, §17, p. 100, 3)

Show that if A is closed in Y and Y is closed in X, then A is closed in X.

Proof. ∎

Problem 1.2 (Munkres, §17, p. 101, 6(b))

Let A, B and A_α denote subsets of a space X. Prove the following:

(b) $\overline{A \cup B} = \bar{A} \cup \bar{B}$.

Proof. ∎

Problem 1.3 (Munkres, §17, p. 101, 6(c))

Let A, B and A_α denote subsets of a space X. Prove the following:

(b) $\overline{\bigcup A_\alpha} \supset \bigcup \overline{A_\alpha}$.

Proof. ■

Problem 1.4 (Munkres, §17, p. 101, 7)

Criticize the following "proof" that $\overline{\bigcup A_\alpha} \subset \bigcup \bar{A}_\alpha$: if $\{A_\alpha\}$ is a collection of sets in X and if $x \in \overline{\bigcup A_\alpha}$, then every neighborhood U of x intersects $\bigcup A_\alpha$. Thus U must intersect some A_α, so x must belong to the closure of some A_α. Therefore, $x \in \bigcup \bar{A}_\alpha$.

Critique. ∎

Problem 1.5 (Munkres, §17, p. 101, 9)

Let $A \subset X$ and $B \subset Y$. Show that in the space $X \times Y$,

$$\overline{A \times B} = \bar{A} \times \bar{B}.$$

Proof. ∎

Problem 1.6 (Munkres, §17, p. 101, 10)

Show that every order topology is Hausdorff.

Proof. ■

Problem 1.7 (Munkres, §17, p. 101, 13)

Show that X is Hausdorff if and only if the *diagonal* $\Delta = \{ x \times x \mid x \in X \}$ is closed in $X \times X$.

Proof. ■

Problem 1.8 (Munkres, §18, p. 111, 4)

Given $x_0 \in X$ and $y_0 \in Y$, show that the maps $f\colon X \to X \times Y$ and $g\colon Y \to X \times Y$ defined by

$$f(x) = x \times y_0 \quad \text{and} \quad g(y) = x_0 \times y$$

are imbeddings.

Proof. ∎

Problem 1.9 (Munkres, §18, p. 111-112, 8(a,b))

Let Y be an ordered set in the order topology. Let $f, g\colon X \to Y$ be continuous.

(a) Show that the set $\{\, x \mid f(x) \leq g(x) \,\}$ is closed in X.
(b) Let $h\colon X \to Y$ be the function

$$h(x) = \min\{f(x), g(x)\}.$$

Show that h is continuous. [*Hint:* Use the pasting lemma.]

Proof. ■

Problem 1.10

Given: X is a topological space with open sets $U_1, ..., U_n$ such that $\bar{U}_i = X$ for all i. Prove that the closure of $U_1 \cap \cdots \cap U_n$ is X.

Proof. ■