MA 523: Homework 2

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September 1, 2016

CARLOS SALINAS PROBLEM 2.1

Problem 2.1

Verify assertion (36) in [E, §3.2.3], that when Γ is not flat near x^0 the noncharacteristic condition is

$$D_p F(p^0, z^0, x^0) \cdot \nu(x^0) \neq 0.$$

(Here $v(x^0)$ denotes the normal to the hypersurface Γ at x^0).

Solution. ► Integrate with respect to

$$\int_{\mathbb{R}} u_x(t,y) \, dx =$$

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CARLOS SALINAS PROBLEM 2.2

Problem 2.2

Show that the solution of the quasilinear PDE

$$u_t + a(u)u_x = 0$$

with initial conditions u(x, 0) = g(x) is given implicitly by

$$u = g(x - a(u)t).$$

Show that the solution develops a shock (becomes singular) for some t > 0, unless a(g(x)) is a nondecreasing function of x.

Solution. ▶

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CARLOS SALINAS PROBLEM 2.3

Problem 2.3

Show that the function u(x, t) defined by $t \ge 0$ by

$$u(x,t) = \begin{cases} -\frac{2}{3} \left(t + \sqrt{3x + t^2} \right) & \text{for } 4x + t^2 > 0\\ 0 & \text{for } 4x + t^2 < 0 \end{cases}$$

is an (unbounded) entropy solution of the conservation law $u_t + (u^2/2)_x = 0$ (inviscid Burger's equation).

Solution. ►

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