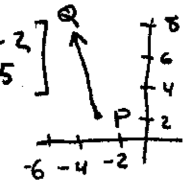
Hwk 9 p.1

4.1: 2, 5, 6, 8, 11, 14, 15, 16, 17, 19

4.1.2 Determine the head of the vector $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ whose tail is $(-3, 2)$. Make a sketch.

$P = (-3, 2)$ is the Tail and $Q = (a, b)$ is the head, Then $\overrightarrow{PQ} = \begin{bmatrix} a-(-3) \\ b-(2) \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$



So $a+3 = -2$ or $a = -5$ and $b - 2 = 5$ so $b = 7$.

4.1.5 For what values of a and b are the vectors $\begin{bmatrix} a-b \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ a+b \end{bmatrix}$ equal?

$\begin{bmatrix} a-b \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ a+b \end{bmatrix}$ then $\begin{matrix} a-b=4 \\ a+b=2 \end{matrix}$. This is $\left[\begin{array}{cc|c} 1 & -1 & 4 \\ 1 & 1 & 2 \end{array}\right] \xrightarrow{r_1+r_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 2 & 0 & 6 \end{array}\right]$

So $a = 3$ $\quad a - b = 4 \quad 3 - b = 4$ and $b = -1$.

4.1.6 For what values of $a, b,$ and c are the vectors $\begin{bmatrix} 2a-b \\ a-2b \\ 6 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 2 \\ a+b-2c \end{bmatrix}$ equal?

This is $\begin{matrix} 2a - b = -2 \\ a - 2b = 2 \\ a + b - 2c = 6 \end{matrix}$. $\left[\begin{array}{ccc|c} 2 & -1 & 0 & -2 \\ 1 & -2 & 0 & 2 \\ 1 & 1 & -2 & 6 \end{array}\right] \begin{matrix} r_1 - r_2 \\ = \\ r_3 - r_2 \end{matrix} \left[\begin{array}{ccc|c} 1 & 1 & 0 & -4 \\ 1 & -2 & 0 & 2 \\ 0 & 3 & -2 & 4 \end{array}\right] \underset{r_2 - r_1}{=} \left[\begin{array}{ccc|c} 1 & 1 & 0 & -4 \\ 0 & -3 & 0 & 6 \\ 0 & 3 & -2 & 4 \end{array}\right] \underset{-\frac{1}{3}r_2}{=} \left[\begin{array}{ccc|c} 1 & \cancel{1} & 0 & -4 \\ 0 & 1 & 0 & -2 \\ 0 & 3 & -2 & 4 \end{array}\right] \begin{matrix} r_1 - r_2 \\ \\ r_3 - 3r_2 \end{matrix}$

$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -2 & 10 \end{array}\right]$ So $a = -2$, $b = -2$, $c = -5$.

4.1.8 Determine the components of each vector $\overrightarrow{PQ}$.

(a) $P(-1, 0)$, $Q(-3, -4)$ $\quad$ (b) $P(1, 1, 2)$, $Q(1, -2, -4)$.

(a) $\overrightarrow{PQ} = \begin{bmatrix} -3-(-1) \\ -4-0 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \end{bmatrix}$ $\qquad$ (b) $\overrightarrow{PQ} = \begin{bmatrix} 1-(1) \\ -2-(1) \\ -4-(2) \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ -6 \end{bmatrix}$

4.1.11 Compute $u+v$, $u-v$, and $3u-2v$ if

(a) $u = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$; (b) $u = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$; (c) $u = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.

(a) $u+v = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$, $u-v = \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$, $3u-2v = 3\begin{bmatrix} 2 \\ 3 \end{bmatrix} - 2\begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix} + \begin{bmatrix} 4 \\ -10 \end{bmatrix} = \begin{bmatrix} 10 \\ -1 \end{bmatrix}$.

(b) $u+v = \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $u-v = \begin{bmatrix} 0 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, $3u-2v = 3\begin{bmatrix} 0 \\ 3 \end{bmatrix} - 2\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \end{bmatrix} + \begin{bmatrix} -6 \\ -4 \end{bmatrix} = \begin{bmatrix} -6 \\ 5 \end{bmatrix}$

(c) $u+v = \begin{bmatrix} 2 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$, $u-v = \begin{bmatrix} 2 \\ 6 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, $3u-2v = 3\begin{bmatrix} 2 \\ 6 \end{bmatrix} - 2\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 18 \end{bmatrix} + \begin{bmatrix} -6 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 14 \end{bmatrix}$.

4.1.14 Let $x=\begin{bmatrix}1\\2\end{bmatrix}$, $y=\begin{bmatrix}-3\\4\end{bmatrix}$, $z=\begin{bmatrix}r\\4\end{bmatrix}$, and $u=\begin{bmatrix}-2\\s\end{bmatrix}$. Find r and s so that (a) $z=2x$, (b) $\frac{3}{2}u=y$, (c) $z+u=x$.

(a) $\begin{bmatrix}r\\4\end{bmatrix}=2\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}2\\4\end{bmatrix}$ so $r=2$. (b) $\frac{3}{2}\begin{bmatrix}-2\\s\end{bmatrix}=\begin{bmatrix}-3\\4\end{bmatrix}=\begin{bmatrix}-3\\\frac{3}{2}s\end{bmatrix}$ so $4=\frac{3}{2}s$ and $s=\frac{8}{3}$

(c) $z+u=\begin{bmatrix}r\\4\end{bmatrix}+\begin{bmatrix}-2\\s\end{bmatrix}=\begin{bmatrix}r-2\\s+4\end{bmatrix}=x=\begin{bmatrix}1\\2\end{bmatrix}$ so $r=3$, $s=-2$.

4.1.15 Let $x=\begin{bmatrix}1\\-2\\3\end{bmatrix}$, $y=\begin{bmatrix}-3\\1\\3\end{bmatrix}$, $z=\begin{bmatrix}r\\-1\\s\end{bmatrix}$, and $u=\begin{bmatrix}3\\t\\2\end{bmatrix}$. Find $r,s,$ and t so that (a) $z=\frac{1}{2}x$, (b) $z+u=x$, (c) $z-x=y$.

(a) $z=\begin{bmatrix}r\\-1\\s\end{bmatrix}=\frac{1}{2}x=\frac{1}{2}\begin{bmatrix}1\\-2\\3\end{bmatrix}=\begin{bmatrix}1/2\\-1\\3/2\end{bmatrix}$ so $r=1/2$, $s=3/2$.

(b) $z+u=\begin{bmatrix}r\\-1\\s\end{bmatrix}+\begin{bmatrix}3\\t\\2\end{bmatrix}=\begin{bmatrix}r+3\\t-1\\s+2\end{bmatrix}=x=\begin{bmatrix}1\\-2\\3\end{bmatrix}$ so $r=-2$, $t=-2$, $s=1$.

(c) $z-u=\begin{bmatrix}r\\-1\\s\end{bmatrix}-\begin{bmatrix}3\\t\\2\end{bmatrix}=\begin{bmatrix}r-3\\-1-t\\s-2\end{bmatrix}=y=\begin{bmatrix}-3\\1\\3\end{bmatrix}$ so $r=0$, $t=-2$, $s=5$.

4.1.16 If possible, find scalars c_1 and c_2 so that $c_1\begin{bmatrix}1\\-2\end{bmatrix}+c_2\begin{bmatrix}3\\-4\end{bmatrix}=\begin{bmatrix}-5\\6\end{bmatrix}$.

This is $\begin{bmatrix}1&3\\-2&-4\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix}=\begin{bmatrix}-5\\6\end{bmatrix}$ so $\left[\begin{array}{cc|c}1&3&-5\\-2&-4&6\end{array}\right]\xrightarrow{2r_1+r_2}\left[\begin{array}{cc|c}1&3&-5\\0&2&-4\end{array}\right]\xrightarrow{\frac{1}{2}r_2}\left[\begin{array}{cc|c}1&3&-5\\0&1&-2\end{array}\right]$

so $c_2=-2$, $c_1=-5-3c_2=-5-3(-2)=1$

4.1.17 If possible, find scalars c_1, c_2, and c_3 so that $c_1\begin{bmatrix}1\\2\\-3\end{bmatrix}+c_2\begin{bmatrix}-1\\1\\1\end{bmatrix}+c_3\begin{bmatrix}-1\\4\\-1\end{bmatrix}=\begin{bmatrix}2\\-2\\3\end{bmatrix}$.

This is $\left[\begin{array}{ccc|c}1&-1&-1&2\\2&1&4&-2\\-3&1&-1&3\end{array}\right]\xrightarrow[3r_1+r_3]{-2r_1+r_2}\left[\begin{array}{ccc|c}1&-1&-1&2\\0&3&6&-6\\0&-2&-4&9\end{array}\right]\xrightarrow[-\frac{1}{2}r_3]{\frac{1}{3}r_2}\left[\begin{array}{ccc|c}1&-1&-1&2\\0&1&2&-2\\0&1&2&-9/2\end{array}\right]\xrightarrow{-r_2+r_3}\left[\begin{array}{ccc|c}1&0&1&0\\0&1&2&-2\\0&0&0&-13/2\end{array}\right]$

which is inconsistent, so no such c_1, c_2, c_3 exist.

4.1.19 If possible, find scalars c_1, c_2, and c_3, not all zero, so that $c_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+c_2\begin{bmatrix}1\\3\\-2\end{bmatrix}+c_3\begin{bmatrix}3\\7\\-4\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

This is $\left[\begin{array}{ccc|c}1&1&3&0\\2&3&7&0\\-1&-2&-4&0\end{array}\right]\xrightarrow[r_1+r_3]{-2r_1+r_2}\left[\begin{array}{ccc|c}1&1&3&0\\0&1&1&0\\0&-1&-1&0\end{array}\right]\xrightarrow[r_2+r_3]{-r_2+r_1}\left[\begin{array}{ccc|c}1&0&2&0\\0&1&1&0\\0&0&0&0\end{array}\right]$ So for $c_3=r$ any real number,

$c_1=-2r$, $c_2=-r$, $c_3=r$.