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MA598: Lie Groups

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CHAPTER 1

Prologue

This summer, we will be making our way through Knapp's *Lie Groups Beyond an Introduction* [2] although, I (the writer of these notes) will occasionally refer to [1] for examples.

1.1 Representation of Finite Groups

Definitions

A *representation* of a finite group G on a finite-dimensional complex vector space V is a homomorphism $\rho\colon G \to \mathrm{GL}(V)$; we say that such a map ρ *gives* V *the structure of a* G*-module.* When there is little ambiguity about the map ρ we will call V itself as a representation of G; in this vein, we supress the symbol ρ and write gv for $\rho(g)(v)$. The dimension of V is sometimes called the *degree* of ρ.

A map φ between two representations V and W of G is a vector space map $\varphi\colon V \to W$ such that

$$\begin{tikzcd} V \arrow[r, "\varphi"] \arrow[d, "g"'] & W \arrow[d, "g"] \\ V \arrow[r, "\varphi"'] & W \end{tikzcd}$$

commutes for every $g \in G$. (We will call this a G*-linear map* when we want to distinguish it from an arbitrary linear map between the vector spaces V and W). We can then define $\operatorname{Ker}\varphi$, $\operatorname{Im}\varphi$, and $\operatorname{Coker}\varphi$, which are also G-modules.

A *subrepresentation* of a representation V is a vector subspace W of V which is invariant under G. A representation V is called *irreducible* if there is no proper nonzero invariant subspace W of V.

Bibliography

[1] B. Hall. *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction.* Graduate Texts in Mathematics. Springer, 2003.

[2] A.W. Knapp. *Lie Groups Beyond an Introduction.* Progress in Mathematics. Birkhäuser Boston, 2002.