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MA 26500-215 Quiz 8

July 19, 2016

1. Let $\mathscr{P}_2(\mathbb{R})$ be the set of all polynomials of degree less than or equal to 2 with coefficients in $\mathbb{R}$, i.e., if $p(t) = at^2 + bt + c$ is a polynomial in $\mathscr{P}_2(\mathbb{R})$, then $a, b, c \in \mathbb{R}$.

   (a) (4 points) Show that the set $\mathscr{P}_2(\mathbb{R})$ is closed under addition and multiplication by scalars. What is a *zero* for this set?

   (b) (4 points) The set $\mathscr{P}_2(\mathbb{R})$ is in fact a vector space. Find a basis for $\mathscr{P}_2(\mathbb{R})$.

   (c) (12 points) Define an inner product $\langle -, - \rangle \colon \mathscr{P}_2(\mathbb{R}) \times \mathscr{P}_2(\mathbb{R}) \to \mathbb{R}$ by

   $$\langle p(t), q(t) \rangle \longmapsto \int_0^1 p(t)q(t)\,\mathrm{d}t.$$

   Find a polynomial $q \in \mathscr{P}_2(\mathbb{R})$ such that $\langle p, q \rangle = p(1/2)$ for every $p \in \mathscr{P}_2(\mathbb{R})$. [Hint: You should start by looking at the basis you found in part (b). If you chose a nice basis t^2 should be in your basis. Now for a general $q(t) = at^2 + bt + c \in \mathscr{P}_2(\mathbb{R})$ we have

   $$\langle t^2, p(t) \rangle = \int_0^1 t^2 q(t)\,\mathrm{d}t = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

   Can you come up with enough equations to solve for the unknowns a, b, c?]