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MA 26500 Quiz 7

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1. For the following problems write **T** for true, **F** for false. You do not need to justify your answers.
   (a) (3 points) For all $m \times n$ matrices A and B, $\text{nullity}(A + B) = \text{nullity}\, A + \text{nullity}\, B$.

   (b) (3 points) For all $n \times n$ matrices A and B, $\text{nullity}(AB) = (\text{nullity}\, A)(\text{nullity}\, B)$.

   (c) (3 points) For all $n \times n$ matrices A and B, where A is an elementary matrix, $\text{nullity}(AB) = \text{nullity}\, B$.

   (d) (3 points) If $\mathbf{x}_p$ is a solution to the system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{y} + \mathbf{x}_p$ is also a solution to $A\mathbf{x} = \mathbf{b}$ for any $\mathbf{y} \in \text{Nullspace}\, A$.

2. (8 points) Prove that if $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are in $\mathbb{R}^3$ and $\mathbf{u}$ is orthogonal to both $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is orthogonal to every vector in $\text{span}\{\mathbf{v}, \mathbf{w}\}$.

   [*Hint*: What does it mean for a vector $\mathbf{x}$ to be in $\text{span}\{\mathbf{v}, \mathbf{w}\}$ and what does it mean for two vectors to be orthogonal?]