4.5: 4, 5, 6, 11, 12, 15, 16
Matlab 6.2: 1, 2, 6
Matlab 6.3: 1

4.5.4) Determine whether $S = \{[3\ 1\ 2], [3\ 8\ -5], [-3\ 6\ -4]\}$ is a linearly independent set in R_3.

$$\text{Form} \begin{vmatrix} 3 & 1 & 2 \\ 3 & 8 & -5 \\ -3 & 6 & -9 \end{vmatrix} \begin{array}{l} \\ -3r_1+r_2 \\ r_1+r_3 \end{array} = \begin{vmatrix} 3 & 1 & 2 \\ 0 & 7 & -7 \\ 0 & 7 & -7 \end{vmatrix} = 0$$

So they are linearly dependent

Each given augmented matrix is derived from Eqn (1) for 4.5.5, 4.5.6. (S = Columns)
Is the set S linearly independent?

4.5.5)

$$\left[\begin{array}{cccc|c} 2 & 1 & 3 & 3 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 2 & 1 & 0 \\ 5 & 1 & 8 & 5 & 0 \end{array}\right] \begin{array}{l} +r_2+r_1 \\ \\ r_2+r_3 \\ 5r_2+r_4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 3 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 2 & 2 & 0 \\ 0 & 1 & 8 & 10 & 0 \end{array}\right] \begin{array}{l} \\ r_1+r_2 \\ \\ r_3+r_4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 3 & 0 \\ 0 & 0 & 3 & 4 & 0 \\ 0 & -1 & 2 & 2 & 0 \\ 0 & 0 & 10 & 12 & 0 \end{array}\right] \begin{array}{l} \\ r_3 \Updownarrow r_2 \\ \\ \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 3 & 3 & 0 \\ 0 & -1 & 3 & 4 & 0 \\ 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 10 & 12 & 0 \end{array}\right] \begin{array}{l} \\ -r_2 \\ \\ -3r_3+r_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 3 & 0 \\ 0 & 1 & -3 & -4 & 0 \\ 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} -r_3+r_1 \\ r_3+r_2 \\ \\ r_3 \Leftrightarrow r_4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 4 & 0 \end{array}\right] \begin{array}{l} \\ \\ \\ -3r_3+r_4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{array}\right] \text{ forces } c_1 = \dots = c_4 = 0$$

So they are linearly independent

4.5.6) $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ is in RREF and since the bottom row is zero, there is a nontrivial solution. Hence they are linearly dependent.

4.5.11 Which of the given vectors in R_3 are linearly dependent? For those which are, express one vector as a linear combination of the rest.

(a) $[1\ 1\ 0],\ [0\ 2\ 3],\ [1\ 2\ 3],\ [3\ 6\ 6]$

(b) $[1\ 1\ 0],\ [3\ 4\ 2]$.

(c) $[1\ 1\ 0],\ [0\ 2\ 3],\ [1\ 2\ 3],\ [0\ 0\ 0]$.

(a) Form from $c_1\vec{v}_1 + \dots c_n\vec{v}_n = \vec{0}$ the augmented matrix

$$\left[\begin{array}{cccc|c}1&0&1&3&0\\1&2&2&6&0\\0&3&3&6&0\end{array}\right]\xrightarrow{-r_1+r_2}\left[\begin{array}{cccc|c}1&0&1&3&0\\0&2&1&3&0\\0&3&3&6&0\end{array}\right]\xrightarrow{-r_3+r_2}\left[\begin{array}{cccc|c}1&0&1&3&0\\0&-1&-2&-3&0\\0&3&3&6&0\end{array}\right]\xrightarrow{-r_2}\left[\begin{array}{cccc|c}1&0&1&3&0\\0&1&2&3&0\\0&3&3&6&0\end{array}\right]\xrightarrow{-3r_2+r_3}$$

$$\left[\begin{array}{cccc|c}1&0&1&3&0\\0&1&2&3&0\\0&0&-3&-3&0\end{array}\right]\xrightarrow{-1/3\,r_3}\left[\begin{array}{cccc|c}1&0&1&3&0\\0&1&2&3&0\\0&0&1&1&0\end{array}\right]\xrightarrow[-2r_3+r_2]{-r_3+r_1}\left[\begin{array}{cccc|c}1&0&0&2&0\\0&1&0&1&0\\0&0&1&1&0\end{array}\right]$$

So $[3\ 6\ 6] = 2[1\ 1\ 0] + 1[0\ 2\ 3] + 1[1\ 2\ 3]$.

(b) $\begin{bmatrix}1&3\\1&4\\0&2\end{bmatrix}\xrightarrow{-r_1+r_2}\begin{bmatrix}1&3\\0&1\\0&2\end{bmatrix}\xrightarrow{-2r_2+r_3}\begin{bmatrix}1&3\\0&1\\0&0\end{bmatrix}\xrightarrow{-3r_2+r_1}\begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix}$ are both linearly independent.

(c) $\begin{bmatrix}1&0&1&0\\1&2&2&0\\0&3&3&0\end{bmatrix}\xrightarrow{\text{by (a)}}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}$ So they are linearly independent and $\begin{bmatrix}0\\0\\0\end{bmatrix}^T = 0\begin{bmatrix}1\\1\\0\end{bmatrix}^T + 0\begin{bmatrix}0\\2\\3\end{bmatrix}^T + 0\begin{bmatrix}1\\2\\3\end{bmatrix}^T$.

4.5.12 Consider the vector space M_{22}. Do as 4.5.11.

(a) $\begin{bmatrix}1&1\\2&1\end{bmatrix},\ \begin{bmatrix}1&0\\0&2\end{bmatrix},\ \begin{bmatrix}0&3\\2&1\end{bmatrix},\ \begin{bmatrix}4&6\\5&6\end{bmatrix}$

(b) $\begin{bmatrix}1&1\\1&1\end{bmatrix},\ \begin{bmatrix}1&0\\0&2\end{bmatrix},\ \begin{bmatrix}0&1\\0&2\end{bmatrix}$

(c) $\begin{bmatrix}1&1\\1&1\end{bmatrix},\ \begin{bmatrix}2&3\\1&2\end{bmatrix},\ \begin{bmatrix}3&1\\2&1\end{bmatrix},\ \begin{bmatrix}2&2\\1&1\end{bmatrix}$

Make these 4x1 columns. Do as in 4.5.11.

(a) $$\begin{bmatrix}1&1&0&4\\1&0&3&6\\2&0&2&8\\1&2&1&6\end{bmatrix}\xrightarrow[\substack{-2r_1+r_3\\-r_1+r_4}]{-r_1+r_2}\begin{bmatrix}1&1&0&4\\0&-1&3&2\\0&-2&2&0\\0&1&1&2\end{bmatrix}\xrightarrow{r_2\updownarrow r_3,\ r_4}\begin{bmatrix}1&1&0&4\\0&1&1&2\\0&-1&3&2\\0&-2&2&0\end{bmatrix}\xrightarrow[\substack{r_2+r_3\\2r_2+r_4}]{-r_2+r_1}\begin{bmatrix}1&0&-1&2\\0&1&1&2\\0&0&4&4\\0&0&4&4\end{bmatrix}\xrightarrow[1/4\,r_4]{1/4\,r_3}\begin{bmatrix}1&0&-1&2\\0&1&1&2\\0&0&1&1\\0&0&1&1\end{bmatrix}\xrightarrow[\substack{-r_3+r_2\\-r_3+r_4}]{r_3+r_1}\begin{bmatrix}1&0&0&3\\0&1&0&1\\0&0&1&1\\0&0&0&0\end{bmatrix}$$

So $3\begin{bmatrix}1&1\\2&1\end{bmatrix} + 1\begin{bmatrix}1&0\\0&2\end{bmatrix} + 1\begin{bmatrix}0&3\\2&1\end{bmatrix} = \begin{bmatrix}4&6\\5&6\end{bmatrix}$.

(b) $$\begin{bmatrix}1&1&0\\1&0&1\\1&0&0\\1&2&2\end{bmatrix}\xrightarrow[\substack{-r_1+r_3\\-r_1+r_4}]{-r_1+r_2}\begin{bmatrix}1&1&0\\0&-1&1\\0&-1&0\\0&1&2\end{bmatrix}\xrightarrow{-r_3}\begin{bmatrix}1&1&0\\0&-1&1\\0&1&0\\0&1&2\end{bmatrix}\xrightarrow[\substack{r_3+r_2\\-r_3+r_4}]{-r_3+r_1}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\\0&0&2\end{bmatrix}\xrightarrow{r_2\updownarrow r_3}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&2\end{bmatrix}\xrightarrow{-2r_3+r_4}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}$$ These are linearly independent.

(c) $$\begin{bmatrix}1&2&3&2\\1&3&1&2\\1&1&2&1\\1&2&1&1\end{bmatrix}\xrightarrow[\substack{-r_1+r_3\\-r_1+r_4}]{-r_1+r_2}\begin{bmatrix}1&2&3&2\\0&1&-2&0\\0&-1&-1&-1\\0&0&-2&-1\end{bmatrix}\xrightarrow[r_2+r_3]{-2r_2+r_1}\begin{bmatrix}1&0&7&2\\0&1&-2&0\\0&0&-3&0\\0&0&-2&-1\end{bmatrix}\xrightarrow[-r_4]{-1/3\,r_3}\begin{bmatrix}1&0&7&2\\0&1&-2&0\\0&0&1&0\\0&0&+2&+1\end{bmatrix}\xrightarrow[\substack{2r_3+r_4\\-2r_3+r_4}]{-7r_3+r_1}\begin{bmatrix}1&0&0&2\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\xrightarrow{-2r_4+r_1}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

Thus this is a linearly independent set.

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4.5.15 Consider the vector space R^3. Do as 4.5.11.

(a) $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\-1\end{bmatrix}$ (b) $\begin{bmatrix}1\\1\\-1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\-2\end{bmatrix}$ (c) $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\2\\1\end{bmatrix}$

And more...

(a) $\begin{bmatrix}1&0&1\\0&1&2\\0&1&-1\end{bmatrix} \xrightarrow{-r_2+r_3} \begin{bmatrix}1&0&1\\0&1&2\\0&0&-3\end{bmatrix} \xrightarrow{-\frac{1}{3}r_3} \begin{bmatrix}1&0&1\\0&1&2\\0&0&1\end{bmatrix} \xrightarrow[-2r_3+r_1]{-r_3+r_1} \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ and thus are linearly independent.

(b) $\begin{bmatrix}1&0&1&1\\1&1&1&2\\-1&1&1&-2\end{bmatrix} \xrightarrow[r_1+r_3]{-r_1+r_2} \begin{bmatrix}1&0&1&1\\0&1&0&1\\0&1&2&-1\end{bmatrix} \xrightarrow{-r_2+r_3} \begin{bmatrix}1&0&1&1\\0&1&0&1\\0&0&2&-2\end{bmatrix} \xrightarrow{\frac{1}{2}r_3} \begin{bmatrix}1&0&1&1\\0&1&0&1\\0&0&1&-1\end{bmatrix} \xrightarrow{-r_3+r_1} \begin{bmatrix}1&0&0&2\\0&1&0&1\\0&0&1&-1\end{bmatrix}$

So $2\begin{bmatrix}1\\1\\-1\end{bmatrix} + 1\begin{bmatrix}0\\1\\1\end{bmatrix} - 1\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\2\\-2\end{bmatrix}$

(c) $\begin{bmatrix}1&2&-1\\0&1&2\\0&1&1\end{bmatrix} \xrightarrow{-r_3+r_2} \begin{bmatrix}1&2&-1\\0&0&1\\0&1&1\end{bmatrix} \xrightarrow{-r_2+r_3} \begin{bmatrix}1&2&-1\\0&0&1\\0&1&0\end{bmatrix} \xrightarrow{-2r_3+r_1} \begin{bmatrix}1&0&-1\\0&0&1\\0&1&0\end{bmatrix} \xrightarrow{r_2+r_1} \begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$ so linearly independent.

4.5.16 For what values of c are the vectors $[-1\ 0\ -1]$, $[2\ 1\ 2]$, and $[1\ 1\ c]$ in R_3 linearly dependent?

Form
$\begin{bmatrix}-1&2&1\\0&1&1\\-1&2&c\end{bmatrix} \xrightarrow{-r_1} \begin{bmatrix}1&-2&-1\\0&1&1\\-1&2&c\end{bmatrix} \xrightarrow{r_1+r_3} \begin{bmatrix}1&-2&-1\\0&1&1\\0&0&c-1\end{bmatrix} \xrightarrow{2r_2+r_1} \begin{bmatrix}1&0&1\\0&1&1\\0&0&c-1\end{bmatrix}$

So linearly dependent if $c-1=0$ or $c=1$.

Matlab 6.2

Matlab 6.2.1 Determine if $v_1 = [2\ 1\ 0]$, $v_2 = [-1\ \ 1\ 3]$, $v_3 = [0\ -1\ 6]$ spans the vector space of rows with three real entries which has dimension 3.

$\mathrm{rref}([v_1'\ \ v_2'\ \ v_3']) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ gives this is a dim 3 V.S. So it spans a vector space of dimension 3. So yes.

Matlab 6.2.2 Determine if $v_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, $v_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, Spans the vector space of columns with four real entries which has dimension 4.

$\mathrm{rref}([v_1\ \ v_2\ \ v_3\ \ v_4]) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ which gives it spans a vector space of dimension 3. $v_3 = v_1 - v_2$. So No.

Matlab 6.2.6 Let $T = \{v_1, v_2, v_3, v_4\}$ where $v_1 = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$, $v_4 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. Does T span the vector space of all 2×2 matrices with real entries which has dimension 4.

$v_i = \mathrm{reshape}([v_i], 4, 1)$ $\mathrm{rref}([v_1\ \ v_2\ \ v_3\ \ v_4]) = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ so T spans a vector space of dimension 3 and $v_4 = -2v_1 + v_2 + v_3$. So No.

Matlab 6.3

Matlab 6.3.1 Determine if the following sets are linearly independent or linearly dependent.

(a) $S = \{v_1 = [4\ 2\ 1],\ v_2 = [-2\ 3\ 1],\ v_3 = [2\ -11\ -4]\}$

$\mathrm{rref}([v_1'\ \ v_2'\ \ v_3']) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$ Not L.I. $v_3 = -v_1 - 3v_2$.

(b) $S = \{v_1 = [3\ \ 1\ 2],\ v_2 = [-1\ \ 1\ 3],\ v_3 = [7\ 1\ 1]\}$

$\mathrm{rref}([v_1'\ \ v_2'\ \ v_3']) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ Not L.I. $v_3 = 2v_1 - v_2$

(c) $S = \left\{ v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 1 \\ -3 \\ -1 \end{bmatrix}, V_3 = \begin{bmatrix} 1 \\ 2 \\ 6 \\ -5 \end{bmatrix} \right\}$

$\text{rref}([v_1 \ v_2 \ v_3]) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ so they are L.I.

(d) $S = \left\{ v_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \\ 4 \end{bmatrix}, V_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}, V_3 = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

$\text{rref}([v_1 \ v_2 \ v_3]) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ so they are L.I.

(e) $S = \{ P_1(t) = t^2 + 2t + 1, \ P_2(t) = t + 2, \ P_3(t) = 3t^2 + 4t - 1 \}$

$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, V_3 = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}$ $\text{rref}([v_1 \ v_2 \ v_3]) = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

So $P_3 = 3P_1 - 2P_2$ and L.D.

(f) $S = \left\{ V_1 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, V_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, V_3 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \right\}$

$V_i = \text{reshape}(v_i, 4, 1)$. $\text{rref}([V_1 \ V_2 \ V_3]) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

So they are L.I.