MA 523: Homework 3

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CARLOS SALINAS PROBLEM 3.1

Problem 3.1

Consider the initial value problem

$$u_t = \sin u_x; \qquad u(x,0) = \frac{\pi}{4}x.$$

Verify that the assumptions of the Cauchy–Kovalevskaya theorem are satisfied and obtain the Taylor series of the solution about the origin.

Solution. ▶

MA 523: Homework 3

CARLOS SALINAS PROBLEM 3.2

Problem 3.2

Consider the Cauchy problem for u(x,y)

$$u_y = a(x, y, u)u_x + b(x, y, u)$$
$$u(x, 0) = 0$$

Let a and b be analytic functions of their arguments. Assume that $D^{\alpha}a(0,0,0) \geq 0$ and $D^{\alpha}b(0,0,0) \geq 0$ for all α . (Remember by definition, if $\alpha = 0$ then $D^{\alpha}f = f$.)

- (a) Show that $D^{\beta}u(0,0) \geq 0$ for all $|\beta| \leq 2$.
- (b) Prove that $D^{\beta}u(0,0) \geq 0$ for all $\beta = (\beta_1, \beta_2)$. (*Hint:* Argue as in the proof of the Cauchy–Kovalevskaya theorem; i.e., use induction in β_2)

Solution. ▶

MA 523: Homework 3

CARLOS SALINAS PROBLEM 3.3

Problem 3.3

(Kovalevskaya's example) Show that the line $\{t=0\}$ is characteristic for the heat equation $u_t=u_{xx}$. Show there does not exist an analytic solution of the heat equation in $\mathbb{R} \times \mathbb{R}$, with $u=1/(1+x^2)$ on $\{t=0\}$. (*Hint:* Assume there is an analytic solution, compute its coefficients, and show that the resulting power series diverges in any neighborhood of (0,0).)

Solution. ▶

MA 523: Homework 3