

# Fall 2016 Notes

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## Contents

<b>Contents</b>	<b>1</b>
<b>1 Probability</b>	<b>2</b>
1.1 Discrete Probability . . . . .	2
<b>2 Introduction to Partial Differential Equations</b>	<b>4</b>
<b>3 Algebraic Geometry</b>	<b>5</b>
3.1 The de Rham Complex . . . . .	5
<b>4 Algebraic Topology</b>	<b>6</b>
4.1 The de Rham Complex . . . . .	6
<b>5 Classical Mechanics</b>	<b>7</b>
5.1 Ньютонова Механика . . . . .	7
5.2 Экспериментальные фаткы . . . . .	7
<b>Bibliography</b>	<b>8</b>

# Chapter 1

## Probability

We will devote this chapter to the material that is covered in MA 51900 (discrete probability) as it was covered in DasGupta's class. We will, for the most part, reference Feller's *An introduction to probability theory and its applications, Volume 1* [5] (especially for the discrete noncalculus portion of the class) and DasGupta's own book *Fundamentals of Probability: A First Course* [3].

### 1.1 Discrete Probability

The material in this section is pulled almost entirely from [5] with minor detours to [3]. We will not reference any particular pages in either book (unless we feel particularly lazy).

#### Background

Given a discrete sample space  $\Omega$  with sample points  $\omega_1, \omega_2, \dots$ , we shall assume that with each point  $\omega_j$  there is associated a number, called the probability of  $\omega_j$  and denoted by  $P(\omega_i)$ . It is nonnegative and such that

$$\sum_{i \in \mathbb{N}} P(\omega_i) = 1. \quad (1.1)$$

**Definition 1.1.** The probability  $P(A)$  of an event  $A$  is the sum of the probabilities of all sample points in it.

Since the probability of  $\Omega$  is 1 by (1.1), it follows that for any event  $A$

$$0 \leq P(A) \leq 1. \quad (1.2)$$

Let  $A_1$  and  $A_2$  be arbitrary events. To compute the probability  $P(A_1 \cup A_2)$  that either  $A_1$  or  $A_2$  or both occur, we have to add the probabilities of the sample points contained either in  $A_1$  or in  $A_2$ , but each point is to be counted only once. Therefore, we have

$$P(A_1 \cup A_2) \leq P(A_1) + P(A_2). \quad (1.3)$$

Now, if  $\omega$  is any point contained in both  $A_1$  and  $A_2$  the probability of  $\omega$ ,  $P(\omega)$ , appears on the right-hand side of (1.3) twice but only once in the left-hand side. This analysis leads us to conclude that the probability  $P(A_1 \cap A_2)$  occurs twice on right-hand side of (1.3), and we have the important result

**Theorem 1.2.** For any two events  $A_1$  and  $A_2$  the probability that either  $A_1$  or  $A_2$  or both occur is given by

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2). \quad (1.4)$$

If  $A_1 \cap A_2 = \emptyset$ , that is, if  $A_1$  and  $A_2$  are mutually exclusive, then (1.4) reduces to

$$P(A_1 \cup A_2) = P(A_1) + P(A_2).$$

We may similarly continue to consider the probability of (countably) arbitrarily many events  $A_1, A_2, \dots$ ,

$$P\left(\bigcup_{i \in \mathbb{N}} A_i\right) \leq \sum_{i \in \mathbb{N}} P(A_i). \quad (1.5)$$

This equation is referred to as *Boole's inequality*. In the special case where the events  $A_1, A_2, \dots$  are mutually exclusive, we have

$$P\left(\bigcup_{i \in \mathbb{N}} A_i\right) = \sum_{i \in \mathbb{N}} P(A_i).$$

## Chapter 2

# Introduction to Partial Differential Equations

Here we summarize some important points about PDEs. The material is mostly taken from Evans's *Partial Differential Equations* [4].

## Chapter 3

# Algebraic Geometry

A summary to a course on an introduction to sheaf cohomology. We will mostly reference Donu's notes available here <https://www.math.purdue.edu/~dvb/classroom.html>, but also cite Ravi Vakil's *Fundamentals of Algebraic Geometry* [7] available here <https://math216.wordpress.com/>.

### 3.1 The de Rham Complex

Donu began his first lecture by talking about the de Rham complex and de Rham cohomology so let us also begin by on his lecture. For this section, we cite the first chapter of Bott and Tu's *Differential Forms in Algebraic Topology* [2]. Then, the integral

$$\iiint_{\partial\Omega} F dS$$

## Chapter 4

# Algebraic Topology

From my meetings with Mark. We reference Hatcher's *Algebraic Topology* [6] freely available here <https://www.math.cornell.edu/~hatcher/#ATI>.

### 4.1 The de Rham Complex

## Chapter 5

# Classical Mechanics

This section is devoted to notes and problems from Владимир Арнольд's *Математические методы классической механики* [1].

### 5.1 Ньютонова Механика

Ньютонова механика изучает движение системы материальных точек в  $n$ -мерном евклидовом пространстве. В евклидовом пространстве действует  $n$ -мерная группа движений пространства. Основные понятия и теоремы ньютоновой механики (даже если они и формулируются в терминах декартовых координат) инварианты относительно этой группы.

Ньютонова потенциальная механическая система задается массами точек и потенциальной энергией. Движениям пространства, оставляющим потенциальную энергию неизменной, соответствуют законы сохранения.

Уравнения Ньютона позволяют исследовать до конца ряд важных задач механики, например задачу о движении в центральном поле.

### 5.2 Экспериментальные факты

В этой главе описаны основные экспериментальные факты, лежащие в основе механики: принцип относительности Галилея

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