MA 166: Quiz 7 Solutions

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You have **15 minutes** to complete this quiz. You may work in groups, but you are not allowed to use any other resources.

Problem 1. For **one** of the following integrals, determine whether it is convergent or divergent. If it is convergent, evaluate it

(a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}}$

(b) $\displaystyle\int_1^5 \frac{dx}{(5-x)^2}$

(c) $\displaystyle\int_0^\infty xe^{-x^2}\,dx.$

Problem 2. Find the length of the curve

$$y = \ln\big(x^2-1\big), \qquad 2 \le x \le 5.$$

Solutions

Here are the solutions to the quiz.

Solution. For the following problems, let t be a dummy variable.

(a) Take the limit as $t \to 0$ of the indefinite integral

$$\begin{aligned} I_1 &= \lim_{t\to 0} \int_t^1 \frac{dx}{\sqrt{x}} \\ &= \lim_{t\to 0} \int_t^1 x^{-1/2}\, dx \\ &= \lim_{t\to 0} \left[\frac{1}{2} x^{1/2}\right]_t^1 \\ &= \lim_{t\to 0} \frac{1}{2} - \frac{1}{2} t^{1/2} \\ &= \frac{1}{2}. \end{aligned}$$

So the integral converges and its value is 1/2.

(b) Take the limit as $t \to 5$ of the indefinite integral

$$\begin{aligned} I_2 &= \lim_{t\to 5} \int_t^1 \frac{dx}{(5-x)^2} \\ &= \lim_{t\to 5} \int_1^t (5-x)^{-2}\, dx \end{aligned}$$

make the substitution $u = 5 - x$, $du = -dx$

$$\begin{aligned}
&= \lim_{t\to 5} -\int_{5-t}^{4} u^{-2}\, du \\
&= \lim_{t\to 5} \int_{5-t}^{4} u^{-2}\, du \\
&= \lim_{t\to 5} -\left[-u^{-1}\right]_{5-t}^{4} \\
&= \lim_{t\to 5} \left[u^{-1}\right]_{5-t}^{4} \\
&= \lim_{t\to 5} \frac{1}{4} - \frac{1}{5-t} \\
&= \frac{1}{4} - \lim_{t\to 5} \frac{1}{5-t}
\end{aligned}$$

So the integral converges and its value is 1/2.

(c) Take the limit as $t \to 0$ of the indefinite integral

$$\begin{aligned}
I_3 &= \lim_{t\to 0} \int_t^1 \frac{dx}{\sqrt{x}} \\
&= \lim_{t\to 0} \int_t^1 x^{-1/2}\, dx \\
&= \lim_{t\to 0} \left[\frac{1}{2} x^{1/2}\right]_t^1 \\
&= \lim_{t\to 0} \frac{1}{2} - \frac{1}{2} t^{1/2} \\
&= \frac{1}{2}.
\end{aligned}$$

So the integral converges and its value is 1/2. ☺

Solution. ☺