MA 519 Fall 2016 (Aaron N. K. Yip)
Homework 5
Due: Thursday, Nov. 17, in class

Ross, 9th edition, Chapter 5:

Problems: #16, 20, 23, 26, 30;

Theoretical Exercises: #11, 31;

Self-Test: #20

Additional Problem:

1. Let $f(x) = \dfrac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{x^2}{2\sigma_1^2}}$ and $g(x) = \dfrac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{x^2}{2\sigma_2^2}}$. Show that

$$\int_{-\infty}^{\infty} f(y)g(x-y)\,dy = \frac{1}{\sqrt{2\pi(\sigma_1^2 + \sigma_2^2)}} e^{-\frac{x^2}{2(\sigma_1^2 + \sigma_2^2)}}.$$

   (Hint: there are many methods to do this problem. One, outlined in class, is to first combine the exponentials and then complete the square for the quadratic function in the exponential.)

2. Let $X_1, \ldots X_n$ be independent random variables with a common mean μ and variance σ^2. Let $M_n = \dfrac{X_1 + \cdots + X_n}{n}$ be the sample mean and $\Sigma_n = \dfrac{\sum_{i=1}^{n}(X_i - M_n)^2}{n}$ be the sample variance. Note that both M_n and Σ_n are random variables.

   Compute EM_n and $E\Sigma_n$. (Express your answers in terms of μ, σ^2 and n.)