MA 572: Homework 3

Carlos Salinas

February 9, 2016

PROBLEM 3.1 (HATCHER §2.1, EX. 17)

(a) Compute the homology groups $H_n(X, A)$ when X is S^2 or $S^1 \times S^1$ and A is a finite set of points in X.

(b) Compute the groups $H_n(X, A)$ and $H_n(X, B)$ for X a closed orientable surface of genus two with A and B the circles shown. [What are X/A and X/B?]

Proof. (a) Let A' be the set A minus a point. Now, by the generalization of theorem 2.16 to triples, we have the long exact sequence

$$\cdots \longrightarrow H_m(A, A') \longrightarrow H_m(X, A') \longrightarrow H_m(X, A) \longrightarrow H_{m-1}(A, A') \longrightarrow \cdots. \tag{1}$$

Exactness of (1) tells us that for for all m we have $H_m(X, A') \cong H_m(X, A)$ since $H_m(A, A') = 0$ gives us the sequence

$$H_m(A, A') = 0 \longrightarrow H_m(X, A') \longrightarrow H_m(X, A) \longrightarrow 0 = H_{m-1}(A, A'),$$

which by exactness

(b)   ∎

PROBLEM 3.2 (HATCHER §2.2, EX. 1)

Prove the Brouwer fixed point theorem for maps $f \colon D^n \to D^n$ by applying degree theory to the map $S^n \to S^n$ that sends both the northern and southern hemispheres of S^n to the southern hemisphere via f. [This was Brouwer's original proof.]

Proof.   ∎

Problem 3.3 (Hatcher §2.2, Ex. 6)

Show that every map $S^n \to S^n$ can be homotoped to have a fixed point if $n > 0$.

Proof.　　　　　　　　　　　　　　　　　　　　　　　　　　　■

Problem 3.4

Let $\mathcal{U}$ be an open cover of X. Prove that the inclusion of $C_*^{\mathcal{U}}(C)$ into $C_*(X)$ is a chain homotopy equivalence.

Proof. ∎