MA 572: Homework 3

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PROBLEM 3.1 (HATCHER §2.1, EX. 17)

(a) Compute the homology groups $H_n(X, A)$ when X is S^2 or $S^1 \times S^1$ and A is a finite set of points in X.

(b) Compute the groups $H_n(X, A)$ and $H_n(X, B)$ for X a closed orientable surface of genus two with A and B the circles shown. [What are X/A and X/B?]

Proof. (a) Since A is a finite collection of points in S^2, let us enumerate the set A via $\{a_1, ..., a_n\}$ and denote by A_k the subset $\{a_1, ..., a_k\}$ of A, where $k \leq n$. Now, by the generalization of theorem 2.16 to triples, we have the long exact sequence

$$\cdots \longrightarrow H_m(A_n, A_{n-1}) \longrightarrow H_m(S^2, A_{n-1}) \longrightarrow H_m(S^2, A_n) \longrightarrow H_{m-1}(A_n, A_{n-1}) \longrightarrow \cdots . \quad (1)$$

Exactness of (1) tells us that for $m \geq 2$ we have $H(S^2, A_{n-1}) \cong H(S^2, A_n)$ since

$$H_m(A_n, A_{n-1}) = 0 \longrightarrow H_m(S^2, A_{n-1}) \longrightarrow H_m(S^2, A_n) \longrightarrow 0 = H_{m-1}(A_n, A_{n-1})$$

is exact. Evidently, $H_m(A_n, A_{n-1}) = 0$ for $m > 1$.[1]

(b)   ∎

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[1] I will prove this if time permits.

Problem 3.2

Proof. ∎

Problem 3.3

Proof.   ∎

Problem 3.4

Proof. ∎