

# MA 544: Homework 1

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**PROBLEM 1.1 (WHEEDEN & ZYGMUND, CHP. 2, EX. 1)**

Let  $f(x) = x \sin(1/x)$  for  $0 < x \leq 1$  and  $f(0) = 0$ . Show that  $f$  is bounded and continuous on  $[0, 1]$ , but that  $V[f; 0, 1] = +\infty$ .

*Proof.* It is straightforward to see that  $f$  is bounded and continuous on  $[0, 1]$ . To see that  $f$  is continuous we will appeal to an  $\varepsilon$ - $\delta$  argument. Let  $\varepsilon > 0$ , then for  $\delta > 0$  for any  $x \in [0, 1]$ , any  $y \in (\delta - x, x + \delta)$  we have

$$\begin{aligned} |f(x) - f(y)| &\leq |x \sin(1/x) - y \sin(1/y)| \\ &= |(x - y)(\sin(1/x) - \sin(1/y))| \\ &= |x - y| |\sin(1/x) - \sin(1/y)| \\ &\leq \delta \cdot |\sin(1/x) - \sin(1/y)| \end{aligned}$$

*program@epstopdf*

now recall the Taylor expansion of  $\sin$  about 0,  $\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$ , then

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**PROBLEM 1.2 (WHEEDEN & ZYGMUND, CHP. 2, EX. 2)**

Prove theorem (2.1).

*Proof.* Recall the statement of theorem (2.1):

**Theorem** (Wheeden & Zygmund, 2.1). (a) *If  $f$  is of bounded variation on  $[a, b]$ , then  $f$  is bounded on  $[a, b]$ .*

(b) *Let  $f$  and  $g$  be of bounded variation on  $[a, b]$ . Then  $cf$  (for any real constant  $c$ ),  $f + g$ , and  $fg$  are of bounded variation on  $[a, b]$ . Moreover,  $f/g$  is of bounded variation on  $[a, b]$  if there exists an  $\varepsilon > 0$  such that  $|g(x)| \geq \varepsilon$  for  $x \in [a, b]$ .*

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**PROBLEM 1.3 (WHEEDEN & ZYGMUND, CHP. 2, EX. 3)**

If  $[a', b']$  is a subinterval of  $[a, b]$  show that  $P[a', b'] \leq P[a, b]$  and  $N[a', b'] \leq N[a, b]$ .

*Proof.*

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