MA 519: Homework 1

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Problem 1.1 (Handout 1, # 5 [Feller Vol. 1])

A closet contains five pairs of shoes. If four shoes are selected at random, what is the probability that there is at least one complete pair among the four?

Solution. ► First, since the closet contains 5 pairs of shoes, it contains, in total, 10 shoes. Now, let us count the number of points in the sample space: since we are selecting 4 shoes out of 10 and the order does not matter, the order of the sample space Ω is

$$\operatorname{card}\Omega = \binom{10}{4} = \frac{10!}{4!6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{4 \cdot 3 \cdot 2 \cdot 6!} = 10 \cdot 3 \cdot 7 = 210 \tag{1.1}$$

total sample points. Now, which of these points are actually ones we care about? Assuming that no two pairs of shoes are alike, we count the number of favorable sample points: there $\binom{5}{1}$ ways to choose a pair (L, R) and we can arrange them in $2 \cdot 3 + 2 \cdot 2 + 2 \cdot 1 = 12$ ways corresponding to the diagrams

$\mathsf{LR}\star\star$	$\star\mathsf{LR}\star$	$\star\star\mathsf{LR}$
$\mathsf{L}\star\mathsf{R}\star$	$\star\mathsf{L}\star\mathsf{R}$	
$\mathsf{L}\star\star\mathsf{R}$		

where $\star$ here corresponds to any other shoe that is not part of the chosen pair (L, R) (the factor of 2 comes from the permutation of L and R). For the remaining spots, we have

$$\binom{8}{2} = \frac{8!}{2!6!} = \frac{8 \cdot 7 \cdot 6!}{2 \cdot 6!} = 4 \cdot 7 = 24.$$

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PROBLEM 1.2 (HANDOUT 1, # 7 [FELLER VOL. 1])

A gene consists of 10 subunits, each of which is normal or mutant. For a particular cell, there are 3 mutant and 7 normal subunits. Before the cell divides into 2 daughter cells, the gene duplicates. The corresponding gene of cell 1 consists of 10 subunits chosen from the 6 mutant and 14 normal units. Cell 2 gets the rest. What is the probability that one of the cells consists of all normal subunits.

Solution. ►

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Problem 1.3 (Handout 1, # 9 [Feller Vol. 1])

From a sample of size n, r elements are sampled at random. Find the probability that none of the N prespecified elements are included in the sample, if sampling is

(a) with replacement;

(b) without replacement.

Compute it for $r = N = 10$, $n = 100$.

Solution. ▸

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Problem 1.4 (Handout 1, # 11 [Text 1.3])

A telephone number consists of ten digits, of which the first digit is one of $1, 2, \ldots, 9$ and the others can be $0, 1, 2, \ldots, 9$. What is the probability that 0 appears at most once in a telephone number, if all the digits are chosen completely at random?

Solution. ►

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Problem 1.5 (Handout 1, # 12 [Text 1.6])

Events A, B and C are defined in a sample space Ω. Find expressions for the following probabilities in terms of $P(A)$, $P(B)$, $P(C)$, $P(AB)$, $P(AC)$, $P(BC)$ and $P(ABC)$; here AB means $A \cap B$, etc.:

(a) the probability that exactly two of A, B, C occur;

(b) the probability that exactly one of these events occur;

(c) the probability that none of these events occur.

Solution. ► ◄

Problem 1.6 (Handout 1, # 13 [Text 1.8])

Mrs. Jones predicts that if it rains tomorrow it is bound to rain the day after tomorrow. She also thinks that the chance of rain tomorrow is $1/2$ and that the chance of rain the day after tomorrow is $1/3$. Are these subjective probabilities consistent with the axioms and theorems of probability?

Solution. ▸

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PROBLEM 1.7 (HANDOUT 1, # 16)

Consider a particular player, say North, in a Bridge game. Let X be the number of aces in his hand. find the distribution of be the number of aces in his hand. find the distribution of X.

Solution. ►

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Problem 1.8 (Handout 1, # 20)

If 100 balls are distributed completely at random into 100 cells, find the expected value of the number of empty cells.

Replace 100 by n and derive the general expression. Now approximate it as n tends to ∞.

Solution. ► ◄