MA553: Qual Problems

Carlos Salinas

February 12, 2016

1 Group Theory

1.1 The Sign of a Permutation

Summary of Keith Conrad's blurb by the same name.

Throughout this discussion $n \geq 2$. A cycle in S_n is the (non-unique) product of transpositions: the identity (1), (1 2)(1 2), and a k-cycle with $k \geq 2$ can be written as

$$(i_1\ i_2\ \cdots\ i_k) = (i_1\ i_2)(i_2\ i_3)\cdots(i_{k-1}\ i_k).$$

For example a 3-cycle $(a\ b\ c)$ – which means a, b and c are distict – can be written as

$$(a\ b\ c) = (a\ b)(b\ c).$$

This is not the only way to write $(a\ b\ c)$ using transpositions, e.g., $(a\ b\ c) = (b\ c)(a\ c) = (a\ c)(a\ b)$.