Fall 2015 Notes – Atiyah and McDonald, Munkres, Lucier

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Contents

1 Commutative Algebra: Atiyah and McDonald

1.1 Rings and ring homomorphisms

A *ring* A is a set with two binary operations (addition and multiplication) such that

(1) A is an abelian group with respect to addition (so that A has a zero element, denoted by 0, and every $x \in A$ has an (additive) inverse, $-x$).
(2) Multiplication is associative ($(xy)z = x(yz)$) and distributive over addition ($(x(x+z) = xy+xz$, $(y+z)x = yx+zx$). We shall consider only rigs which are *commutative*:
(3) $xy = yx$ for all $x, y \in A$, and have an *identity element* (denoted by 1):
(4) $\exists 1 \in A$ such that $x1 = 1x = x$ for all $x \in A$. The identity element is then unique.

A *ring homomorphism* is a mapping f of a ring A into a ring B such that

(i) $f(x+y) = f(x) + f(y)$ (so that f is a homomorphism of abelian groups, and therefore also $f(x-y) = f(x) - f(y)$, $f(-x) = -f(x)$, $f(0) = 0$),
(ii) $f(xy) = f(x)f(y)$,
(iii) $f(1) = 1$.

2 Topology: Munkres