MA 598 PG: Homework 1

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Problem 1.1

(a) Every filter $\mathcal{F}$ is contained in an ultrafilter.

(b) A filter $\mathcal{F}$ in X is an ultrafilter if and only if for each $Y \subset X$, either $Y \in \mathcal{F}$ or $X \setminus Y \in \mathcal{F}$.

(c) For any $x \in X$, the principal filter $\mathcal{F}_{\{x\}}$ is an ultrafilter.

(d) If X is finite, every ultrafilter $\mathcal{F}$ in X is principal.

(e) If X is infinite and $\mathcal{F}$ is a nonprincipal ultrafilter, then $\mathcal{F}$ contains $\mathcal{F}_{\mathrm{cf}}$.

Proof. ■

Problem 1.2

There exists an open subset $U \subset C$ such that $1 \in U$ and $U = U^{-1}$.

Proof. ■

Problem 1.3

Prove that $\bigcap_{i \leq j} E_{i,j} \neq \emptyset$.

Proof. ■