$$\begin{array}{ccc}
\pi_1(S^1, x_0) & \xrightarrow{\ h_* \ } & \pi_1(S^1, x_1) \\
\Big\downarrow{\hat{\beta}} & & \Big\downarrow{\widehat{h \circ \beta}} \\
\pi_1(S^1, y_0) & \xrightarrow{\ h_* \ } & \pi_1(S^1, y_1).
\end{array}$$