MA571: Qual Problems

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1 MA571 (Midterm 2015)

Problem 1.1. Prove that a function to a product space is continuous if and only if its components are.

Proof. ■

Problem 1.2. Prove that a subspace is closed if and only if it contains all of its limit points.

Proof. ■

Problem 1.3. Prove that the projection maps for a product are open maps.

Proof. ■

Problem 1.4. Prove that $\partial A = \emptyset$ if and only if A is open and closed.

Proof. ■

Problem 1.5. Prove that a metric space satisfies the 1st countability axiom.

Proof. ■

Problem 1.6. Prove that $\mathbf{R}^\omega$ is not metrizable in the box topology.

Proof. ■

Problem 1.7. Show that the diagonal map is not continuous in the box topology, but it is in the product topology.

Proof. ■

Problem 1.8. Prove the sequence lemma.

Proof. ■

Problem 1.9. Give an example of a surjective map of spaces that is not a quotient map.

Proof. ■

Problem 1.10. Prove that if f_n is a sequence of functions $X \to \mathbf{R}$ considered as elements of $X^{\mathbf{R}}$ with the product topology, then $f_n \to f$ if and only if for each $x \in X$ the sequence $f_n(x)$ converges to the point $f_n(x)$.

Proof. ■

Problem 1.11. Prove that if f_n is a sequence of functions $X \to \mathbf{R}$ considered as elements of $X^{\mathbf{R}}$ with the topology induced by the uniform metric $\bar{\rho}$, then $f_n \to f$ if and only if the sequence of functions f_n converges uniformly to the point f. (Recall that $f_n \colon X \to Y$, with Y a metric space, uniformly converges to f if for any $\varepsilon > 0$ there exists an integer N such that for all $n > N$ and $x \in D$, $d_y(f_n(x), f(x)) < \varepsilon$.)

Proof. ■

Problem 1.12. Give an example of a surjective map of spaces that is not a quotient map.

Proof. ■

Problem 1.13.

Proof. ■

Problem 1.14.

Proof. ■

Problem 1.15.

Proof. ■

Problem 1.16.

Proof. ■

Problem 1.17.

Proof. ■

Problem 1.18.

Proof. ■

Problem 1.19.

Proof. ■

Problem 1.20.

Proof. ■

Problem 1.21.

Proof. ■

Problem 1.22.

Proof. ■

2 MA571 (Final 2015)

Problem 2.1.

Proof. ■

Problem 2.2.

Proof. ■

Problem 2.3.

Proof. ■

Problem 2.4.

Proof. ■

Problem 2.5.

Proof. ■

Problem 2.6.

Proof. ■

Problem 2.7.

Proof. ■

Problem 2.8.

Proof. ■

Problem 2.9.

Proof. ■

Problem 2.10.

Proof. ■

Problem 2.11.

Proof. ■

Problem 2.12.

Proof. ■

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Problem 3.1.

Proof. ■

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