MA557 Problem Set 2

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Problem 2.1

Let $\mathfrak{a}$ be an R-ideal and M a finite R-module. Show that

$$\sqrt{\operatorname{ann}(M/\mathfrak{a}M)} = \sqrt{\operatorname{ann}(M) + \mathfrak{a}}.$$

Proof. ∎

Problem 2.2

Let R be a local ring and M, N finite R-modules. Show that $M \otimes_R N = 0$ if and only if $M = 0$ or $N = 0$.

Proof. ■

Problem 2.3

Show that $R^n \cong R^m$ if and only if $n = m$.

Proof. ■

Problem 2.4

Prove 2.7.

Proof. ■

Problem 2.5

Prove 2.8.

Proof. ■

Problem 2.6

Prove 2.9.

Proof. ■

Problem 2.7

Prove 2.10.

Proof. ■