MA 571: Homework # 14 due Friday December 11.

Please do:

p. 454 # 6

p. 457 # 1, 2, 3(abc) (See the last paragraph on p. 453 for a description of $P^2 \# T$. For problem 2, "directly" means using the labelling scheme for the Klein bottle at the top of page 454.)

A) Define P^n to be the space $S^n/\sim$, where $z \sim z'$ if and only if $z = z'$ or $z = -z'$. Use the Seifert-van Kampen Theorem to calculate $\pi_1 P^n$. (Hint: induction starting from the case $n = 2$ that was done in class.)

B) A topological space X is called *homogeneous* if for every pair of points $x, y \in X$ there is a homeomorphism $\phi : X \to X$ with $\phi(x) = y$. **Prove** that every connected 2-manifold is homogeneous. (Hint: use the optional problem from the previous assignment.)

Optional problem

i) Let $X \subset \mathbb{R}^3$ be the cylinder

$$\{(x, y, z) \mid x^2 + y^2 = \tfrac{1}{\sqrt{2}} \text{ and } |z| \le \tfrac{1}{\sqrt{2}}\}$$

and let $f : X \to \mathbb{R}^3$ be the map

$$f(x, y, z) = (2^{1/4} x\sqrt{1 - z^2}, 2^{1/4} y\sqrt{1 - z^2}, z).$$

Prove that f is a homeomorphism from X to the subspace

$$Y = S^2 \cap \{(x, y, z) \mid |z| \le \tfrac{1}{\sqrt{2}}\}.$$

ii) Prove that the Möbius band is homeomorphic to P^2 with an open disk removed (think of P^2 as $S^2/\sim$ and use part (i)).