HwK 5 p.1

2.3: 7, 8, 9, 17, 19, 20

2.3.7 Find the inverse of $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array}\right] \xrightarrow{-2r_1+r_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -2 & -2 & 1 \end{array}\right] \xrightarrow{-\frac{1}{2}r_2 \to r_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & -1/2 \end{array}\right] \xrightarrow{-3r_2+r_1} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 3/2 \\ 0 & 1 & 1 & -1/2 \end{array}\right] \text{ So } A^{-1} = \begin{bmatrix} -2 & 3/2 \\ 1 & -1/2 \end{bmatrix}$$

2.3.8 Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix}$.

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-r_1+r_3 \to r_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow[-2r_3+r_2 \to r_2]{-3r_3+r_1 \to r_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 4 & 0 & -3 \\ 0 & 2 & 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow[1/2 r_2 \to r_2]{-r_2+r_1 \to r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & 1 & 1/2 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

So $A^{-1} = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 1/2 & -1 \\ -1 & 0 & 1 \end{bmatrix}$.

2.3.9 Which of the given matrices are singular? For the nonsingular ones, find the inverse.

(a) $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$

(a) $\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 6 & 0 & 1 \end{array}\right] \xrightarrow{-2r_1+r_2 \to r_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right]$ which, ignoring the augmented part, is singular.

(b) $\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ -2 & 6 & 0 & 1 \end{array}\right] \xrightarrow{2r_1+r_2 \to r_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 12 & 2 & 1 \end{array}\right] \xrightarrow{\frac{1}{12}r_2 \to r_2} \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & \frac{1}{6} & \frac{1}{12} \end{array}\right] \xrightarrow{-3r_2+r_1} \left[\begin{array}{cc|cc} 1 & 0 & 1/2 & -1/4 \\ 0 & 1 & 1/6 & 1/12 \end{array}\right]$

So its inverse is $\begin{bmatrix} 1/2 & -1/4 \\ 1/6 & 1/12 \end{bmatrix}$ and it's nonsingular.

(c) $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-r_1+r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[r_2+r_3 \to r_3]{-r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \xrightarrow[-r_3+r_2 \to r_2]{-3r_3+r_1 \to r_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 4 & -3 & -3 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$

$\xrightarrow{-2r_2+r_1 \to r_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$ So its inverse is $\begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ and it's nonsingular.

(d) $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-r_1+r_2 \to r_2} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_2+r_3 \to r_3} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$ which, ignoring the augmented part, is singular.

2.3.17 Which of the following homogeneous systems have a nontrivial solution?

(a) $\begin{aligned} x+2y+3z&=0\\ 2y+2z&=0\\ x+2y+3z&=0\end{aligned}$ (b) $\begin{aligned} 2x+y-z&=0\\ x-2y-3z&=0\\ -3x-y+2z&=0\end{aligned}$ (c) $\begin{aligned} 3x+y+3z&=0\\ -2x+2y-4z&=0\\ 2x-3y+5z&=0\end{aligned}$

By Thm (2.9), we need to see if the coefficient matrices are singular.

(a) $\begin{bmatrix}1&2&3\\0&2&2\\1&2&3\end{bmatrix} \xrightarrow{r_1+r_3\to r_3} \begin{bmatrix}1&2&3\\0&2&2\\0&0&0\end{bmatrix}$ is singular.

(b) $\begin{bmatrix}2&1&-1\\1&-2&-3\\-3&-1&2\end{bmatrix} \xrightarrow{r_2 \leftrightarrow r_1} \begin{bmatrix}1&-2&-3\\2&1&-1\\-3&-1&2\end{bmatrix} \xrightarrow[3r_1+r_3\to r_3]{-2r_1+r_2\to r_2} \begin{bmatrix}1&-2&-3\\0&5&-5\\0&-7&7\end{bmatrix} \xrightarrow[1/7 r_3\to r_3]{1/5 r_2\to r_2} \begin{bmatrix}1&-2&-3\\0&1&-1\\0&-1&1\end{bmatrix} \xrightarrow{r_2+r_3\to r_3} \begin{bmatrix}1&-2&-3\\0&1&-1\\0&0&0\end{bmatrix}$ is singular.

(c) $\begin{bmatrix}3&1&3\\-2&2&-4\\2&-3&5\end{bmatrix} \xrightarrow[r_2+r_3\to r_3]{r_2+r_1\to r_1} \begin{bmatrix}1&3&-1\\-2&2&-4\\0&-1&1\end{bmatrix} \xrightarrow[-r_3\to r_3]{2r_1+r_2} \begin{bmatrix}1&3&-1\\0&8&-6\\0&1&-1\end{bmatrix} \xrightarrow{-8r_3+r_2\to r_2} \begin{bmatrix}1&3&-1\\0&0&2\\0&1&-1\end{bmatrix} \xrightarrow{r_2 \leftrightarrow r_3} \begin{bmatrix}1&3&-1\\0&1&-1\\0&0&2\end{bmatrix}$

which is nonsingular.

2.3.19 Find all value(s) of a for which the inverse of $A=\begin{bmatrix}1&1&0\\1&0&0\\1&2&a\end{bmatrix}$ exists, what is A^{-1}?

Find A^{-1} to determine conditions on a.

$\left[\begin{array}{ccc|ccc}1&1&0&1&0&0\\1&0&0&0&1&0\\1&2&a&0&0&1\end{array}\right] \xrightarrow[-r_1+r_3\to r_3]{-r_1+r_2\to r_2} \left[\begin{array}{ccc|ccc}1&1&0&1&0&0\\0&-1&0&-1&1&0\\0&1&a&-1&0&1\end{array}\right] \xrightarrow[r_2+r_3\to r_3]{r_2+r_1\to r_1} \left[\begin{array}{ccc|ccc}1&0&0&0&1&0\\0&-1&0&-1&1&0\\0&0&a&-2&1&1\end{array}\right] \xrightarrow{-r_2\to r_2} \left[\begin{array}{ccc|ccc}1&0&0&0&1&0\\0&1&0&1&-1&0\\0&0&a&-2&1&1\end{array}\right] \xrightarrow{\frac{1}{a}r_3\to r_3}$

$\left[\begin{array}{ccc|ccc}1&0&0&0&1&0\\0&1&0&1&-1&0\\0&0&1&-2/a&1/a&1/a\end{array}\right]$ So if $a\neq 0$, $A^{-1}=\begin{bmatrix}0&1&0\\1&-1&0\\-2/a&1/a&1/a\end{bmatrix}$

2.3.20 For what values of a does the homogeneous system $\begin{aligned}(a-1)x+2y&=0\\ 2x+(a-1)y&=0\end{aligned}$ have a nontrivial solution?

Form Augmented matrix and solve.

$\left[\begin{array}{cc|c}a-1&2&0\\2&a-1&0\end{array}\right] \xrightarrow{r_1 \leftrightarrow r_2} \left[\begin{array}{cc|c}2&a-1&0\\a-1&2&0\end{array}\right] \xrightarrow{\frac{1}{2}r_1\to r_1} \left[\begin{array}{cc|c}1&\frac{1}{2}(a-1)&0\\a-1&2&0\end{array}\right] \xrightarrow{-(a-1)r_1+r_2\to r_2} \left[\begin{array}{cc|c}1&\frac{1}{2}(a-1)&0\\0&2-\frac{1}{2}(a-1)^2&0\end{array}\right]$

which has a nontrivial solution provided $2-\frac{1}{2}(a-1)^2=0$

So $(a-1)^2=4$, $a-1=\pm 2$, so $a=1\pm 2=-1,3$