MA 519: Homework 1

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Problem 1.1 (Handout 1, # 5 [Feller Vol. 1])

A closet contains five pairs of shoes. If four shoes are selected at random, what is the probability that there is at least one complete pair among the four?

Solution. ▸ First, since the closet contains 5 pairs of shoes, it contains, in total, 10 shoes. Now, let count the number of points in the sample space: since we are selecting 4 shoes out of 10 and the order does not matter, there are

$$\binom{10}{4} = \frac{10!}{4!6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{4 \cdot 3 \cdot 2 \cdot 6!} = 10 \cdot 3 \cdot 7 = 210 \tag{1.1}$$

total sample points. Now, which of those points are actually ones we care about? ◂

Problem 1.2 (Handout 1, # 7 [Feller Vol. 1])

A gene consists of 10 subunits, each of which is normal or mutant. For a particular cell, there are 3 mutant and 7 normal subunits. Before the cell divides into 2 daughter cells, the gene duplicates. The corresponding gene of cell 1 consists of 10 subunits chosen from the 6 mutant and 14 normal units. Cell 2 gets the rest. What is the probability that one of the cells consists of all normal subunits.

Solution. ▸

◂

Problem 1.3 (Handout 1, # 9 [Feller Vol. 1])

From a sample of size n, r elements are sampled at random. Find the probability that none of the N prespecified elements are included in the sample, if sampling is

(a) with replacement;

(b) without replacement.

Compute it for $r = N = 10$, $n = 100$.

Solution. ▸

◂

Problem 1.4 (Handout 1, # 11 [Text 1.3])

A telephone number consists of ten digits, of which the first digit is one of $1, 2, \dots, 9$ and the others can be $0, 1, 2, \dots, 9$. What is the probability that 0 appears at most once in a telephone number, if all the digits are chosen completely at random?

Solution. ▸

◂

Problem 1.5 (Handout 1, # 12 [Text 1.6])

Events A, B and C are defined in a sample space Ω. Find expressions for the following probabilities in terms of $P(A)$, $P(B)$, $P(C)$, $P(AB)$, $P(AC)$, $P(BC)$ and $P(ABC)$; here AB means $A \cap B$, etc.:

(a) the probability that exactly two of A, B, C occur;

(b) the probability that exactly one of these events occur;

(c) the probability that none of these events occur.

Solution. ►

◄

PROBLEM 1.6 (HANDOUT 1, # 13 [TEXT 1.8])

Mrs. Jones predicts that if it rains tomorrow it is bound to rain the day after tomorrow. She also thinks that the chance of rain tomorrow is $1/2$ and that the chance of rain the day after tomorrow is $1/3$. Are these subjective probabilities consistent with the axioms and theorems of probability?

Solution. ►

◄

Problem 1.7 (Handout 1, # 16)

Consider a particular player, say North, in a Bridge game. Let X be the number of aces in his hand. find the distribution of be the number of aces in his hand. find the distribution of X.

Solution. ▸

◂

Problem 1.8 (Handout 1, # 20)

If 100 balls are distributed completely at random into 100 cells, find the expected value of the number of empty cells.

Replace 100 by n and derive the general expression. Now approximate it as n tends to ∞.

Solution. ► ◄