MA 519: Homework 9

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Problem 9.1 (Handout 13, # 7)

Let X have a *double exponential* density $f(x) = \frac{1}{2\sigma}e^{-\frac{|x|}{\sigma}}$, $-\infty < x < \infty$, $\sigma > 0$.

(a) Show that all moments exist for this distribution.
(b) However, show that the MGF exists only for restricted values. Identify them and find a formula.

Solution. ∎

Problem 9.2 (Handout 13, # 16)

Give an example of each of the following phenomena:

(i) a continuous random variable taking values in $[0,1]$ with equal mean and median;
(ii) a continuous random variable taking values in $[0,1]$ with mean equal to twice the median;
(iii) a continuous random variable for which the mean does not exist;
(iv) a continuous random variable for which the mean exists, but the variance does not exist;
(v) a continuous random variable with a PDF that is not differentiable at zero;
(vi) a positive continuous random variable for which the mode is zero, but the mean does not exist;
(vii) a continuous random variable for which all moments exist;
(viii) a continuous random variable with median equal to zero, and 25^{th} and 75^{th} percentiles equal to 1;
(ix) a continuous random variable X with mean equal to median equal to mode equal to zero, and $E(\sin X) = 0$.

Solution. ■

Problem 9.3 (Handout 13, # 17)

An exponential random variable with mean 4 is known to be larger than 6. What is the probability that it is larger than 8?

Solution. ■

Problem 9.4 (Handout 13, # 18)

(Sum of Gammas). Suppose X, Y are independent random variables, and $X \sim G(\alpha, \lambda)$, $Y \sim G(\beta, \lambda)$. Find the distribution of $X + Y$ by using moment-generating functions.

Solution. ■

Problem 9.5 (Handout 13, # 19)

(Product of Chi Squares). Suppose $X_1, X_2, \cdots, X_n$ are independent chi square variables, with $X_i \sim \chi^2_{m_i}$. Find the mean and variance of $\prod_{i=1}^n X_i$.

Solution. ■

Problem 9.6 (Handout 13, # 20)

Let $Z \sim N(0,1)$. Find

$$P\big(0.5 < |Z - \tfrac{1}{2}| < 1.5\big); \quad P\Big(\tfrac{e^Z}{1+e^Z} > \tfrac{3}{4}\Big); \quad P(\Phi(Z) < 0.5).$$

Solution. ■

Problem 9.7 (Handout 13, # 21)

Let $Z \sim N(0,1)$. Find the density of $\frac{1}{Z}$. Is the density bounded?

Solution. ∎

Problem 9.8 (Handout 13, # 22)

The 25^{th} and the 75^{th} percentile of a normally distributed random variable are -1 and 1. What is the probability that the random variable is between -2 and 2?

Solution. ∎