

Homework 1

Due Thursday, September 3

1. Consider the transmission line described in Jackson's problem 1.7. Suppose the conductors carry charges $\pm Q$.
 - (a) Find the electric field at any point of the plane passing through the axes of the conductors. (Please neglect the end effects, i.e., assume the length of the line to be effectively infinite.)
 - (b) Using your result from (a), verify the formula given in the problem for the capacitance. Why is the formula listed as approximate?
2. An uncharged conducting sphere of radius a is placed in the electric field produced by some large distant conductors (basically, the plates of a large parallel capacitor). Without the sphere, the field was uniform (i.e., independent of the location) and equal to $\mathbf{E}_0 = E_0 \hat{\mathbf{z}}$. We will be looking for the field $\mathbf{E}(x, y, z)$ in the presence of the sphere.

Look for a solution to the Laplace equation outside the sphere in the form

$$\Phi(r, \theta) = -E_0 z + \sum_{l=1}^{\infty} \frac{c_l P_l(\cos \theta)}{r^{l+1}},$$

where r, θ, ϕ are spherical coordinates (there is no dependence on ϕ), $z = r \cos \theta$, P_l are the Legendre polynomials, and c_l are the expansion coefficients to be found.

- (a) Use the boundary condition at the surface of the sphere to find all c_l .
- (b) Find the Cartesian components of the electric field \mathbf{E} .
- (c) Find the change in the electrostatic energy caused by the presence of the sphere.