MA 571: Homework # 3 due Monday September 14.

Please read

Section 19,
from page 119 to the top of page 125 in Section 20,
Section 21 (but skip Theorem 21.6).

Please do:

p. 111 # 7(a) (note: you will need to get the definition of $\lim_{x\to a^+}$ from an analysis book)
p. 112 # 13
p. 118 # 2 (product topology only), # 3 (product topology only), # 6, # 7
p. 126 # 3(b), 4(b) (For 4(b), do the sequence **w** only. Prove your answers.)

A) Given: X is a metric space, A is a countable subset of X, and $\bar{A} = X$.
To prove: the topology of X has a countable basis.

B) Given: Y is an ordered set, (a, b) and (c, d) are disjoint open intervals, and there are elements $x \in (a, b)$ and $y \in (c, d)$ with $x < y$.
To prove: every element of (a, b) is less than every element of (c, d).

C) (This problem will be used when we discuss quotient spaces). Let S and T be sets and let $f : S \to T$ be a function. Let $A \subset S$.

(i) Give an example to show that the equation

$$f^{-1}f(A) = A \tag{*}$$

isn't always valid.

(ii) Define an equivalence relation $\sim$ on S by $s \sim s'$ if and only if $f(s) = f(s')$. Using this equivalence relation, describe the subsets A of S for which $(*)$ is true. Prove that your answer is correct.