MA161Lesson Plan MicroTeachingSession

Carlos Salinas

October 8, 2015

1 Indeterminate Forms and L'Hospital's Rule

Limit of the form

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

where both $f(x) \to 0$ and $g(x) \to 0$ as $x \to a$ is called an *indeterminate form of type* $\frac{0}{0}$.

Theorem 1 (L'Hospital's Rule)**.** *Suppose f and g are differentiable and $g'(x) \neq 0$ on an open interval I that contains a (except possibly at a). Suppose that*

$$\lim_{x \to a} f(x) = 0 \qquad \textit{and} \qquad \lim_{x \to a} g(x) = 0$$

or that

$$\lim_{x \to a} f(x) = \pm\infty \qquad \textit{and} \qquad \lim_{x \to a} g(x) = \pm\infty.$$

(In other words, we have an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.) Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).

1.1 Indeterminate Products

Limit of the form

$$\lim_{x \to a} [f(x)g(x)]$$

where $f(x) \to 0$ and $g(x) \to \pm\infty$ as $x \to a$ is called an *indeterminate form of type* $0 \cdot \infty$. We can deal with it by writing the product fg as a quotient:

$$fg = \frac{f}{1/g} \qquad \text{or} \qquad fg = \frac{g}{1/f}.$$

1.2 Indeterminate Differences

Limit of the form

$$\lim_{x \to a} [f(x) - g(x)]$$

where $f(x) \to \infty$ and $g(x) \to \infty$ as $x \to a$ is called an *indeterminate form of type* $\infty - \infty$. Try to convert the difference into a quotient (e.g., by using a common denominator, or rationalization, or factoring out a common factor) so that we have an indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.