

MA 571: Homework # 6 due Wednesday October 7.

Please read Section 26.

Please do:

p. 163 # 8, 10(abc) (For 10(c), do only A and B ; these will be counted as two separate problems. Prove your answers.)

p. 171 # 4, 5, 7

A) Let X be a compact space and let \sim be an equivalence relation on X . Suppose that the set

$$S = \{(x, y) \mid x \sim y\}$$

is a closed subset of $X \times X$. Prove that the quotient map $q : X \rightarrow X/\sim$ is a closed map.

B) Let S^2 be the sphere

$$\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}.$$

Let S_+^2 be $S^2 \cap \{z \geq 0\}$ (the upper hemisphere), let S_-^2 be $S^2 \cap \{z \leq 0\}$ (the lower hemisphere), and let E be $S^2 \cap \{z = 0\}$ (the equator). Recall the definition of Y/S from Homework # 4. Prove that S^2/S_-^2 is homeomorphic to S_+^2/E . (Hint: there are maps in both directions.)