

# MA553: Qual Problems

Carlos Salinas

January 21, 2016

## 1 Ulrich's MA 553 Exercises for Spring '16

### 1.1 Homework 1

**Exercise 1.1.** Let  $G$  be a group,  $a \in G$  an element of finite order  $m$ , and  $n$  a positive integer. Prove that

$$|a^n| = \frac{m}{\gcd(m, n)}.$$

*Proof.* ■

**Exercise 1.2.** Let  $G$  be a group, and let  $a, b$  be elements of finite order  $m, n$  respectively. Show that if  $ba = ab$  and  $\langle a \rangle \cap \langle b \rangle = \{e\}$ , then  $|ab| = \text{lcm}(m, n)$ .

*Proof.* ■

**Exercise 1.3.** Let  $G$  be a group and  $H, K$  normal subgroups with  $H \cap K = \{e\}$ . Show that

- (a)  $hk = kh$  for every  $h \in H, k \in K$ .
- (b)  $HK$  is a subgroup of  $G$  with  $HK \cong H \times K$ .

*Proof.* ■

**Exercise 1.4.** Show that  $A_4$  has no subgroup of order 6 (although  $6 \mid 12 = |A_4|$ ).

*Proof.* ■

### 1.2 Homework 2

**Exercise 1.5.**

*Proof.* ■