

MA166: Recitation 12

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1 Homework

1.1 This Week's Summary

Here's a summary of the material that was (presumably) covered this week. Sections from Stewart

§10.1: Parametric Equations and Polar Coordinates

Suppose that x and y are defined in terms of another third variable t (called the *parameter*) by the equations

$$x = f(t) \quad y = g(t).$$

(called *parametric equations*). Each value of t determines a point (x, y) which we can plot in the coordinate plane \mathbb{R}^2 . As t varies, the point $(x, y) = (f(t), g(t))$ varies and traces out a curve C , which we call the *parametric curve*. The parameter f does not necessarily represent time, and in fact,

1.2 Homework Problems

Solutions to selected problems:

Homework 31

Problem 1 (WebAssign HW 31, # 1). Select the curve generated by the parametric equations. Indicate with an arrow the direction in which the curve is traced as t increases.

$$x = e^{-t} + t, \quad y = e^t - t, \quad -2 \leq t \leq 2.$$

Problem 2 (WebAssign HW 31, # 2). Consider the following equations.

$$x = 1 - t^2, \quad y = t - 3, \quad -2 \leq t \leq 2.$$

- (a) Sketch the curve using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced out as t increases.
- (b) Eliminate the parameter to find a Cartesian equation of the curve for $-5 \leq y \leq -1$.

Problem 3 (WebAssign HW 31, # 3). Consider the parametric equations below.

$$x = \sqrt{t}, \quad y = 11 - t.$$

- (a) Sketch the curve using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced out as t increases.
- (b) Eliminate the parameter to find a Cartesian equation of the curve for $x \geq 0$.

Problem 4 (WebAssign HW 31, # 4). Consider the following.

$$x = \sin \frac{1}{2}\theta, \quad y = \cos \frac{1}{2}\theta, \quad -\pi \leq \theta \leq \pi.$$

- (a) Sketch the curve using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced out as t increases.
- (b) Eliminate the parameter to find a Cartesian equation of the curve for $-5 \leq y \leq -1$.

Problem 5 (WebAssign HW 31, # 5). Consider the following.

$$x = \sin t, \quad y = \csc t \quad 0 < t < \pi/2.$$

- (a) Eliminate the parameter to find a Cartesian equation of the curve.
- (b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

Problem 6 (WebAssign HW 31, # 6). Describe the motion of a particle with position (x, y) as t varies in the given interval.

$$x = 2 + 2 \cos t, \quad y = 1 + 2 \sin t, \quad \pi/2 \leq t \leq 3\pi/2.$$

Problem 7 (WebAssign HW 31, # 7). Describe the motion of a particle with position (x, y) as t varies in the given interval.

$$x = 2 \sin t, \quad y = 1 + \cos t, \quad 0 \leq t \leq 3\pi/2.$$

Problem 8 (WebAssign HW 31, # 8). Describe the motion of a particle with position (x, y) as t varies in the given interval.

$$x = 4 \sin t, \quad y = 9 \cos t, \quad -\pi \leq t \leq 9\pi.$$

Problem 9 (WebAssign HW 31, # 9). Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)–(d) with the parametric curves labeled I–IV.

Problem 10 (WebAssign HW 31, # 10). Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as t increases.

Homework 32

Problem 11 (WebAssign HW 32, # 1). Find dy/dx .

$$x = t \sin t, \quad y = t^2 + 3t.$$

Problem 12 (WebAssign HW 32, # 2). Find dy/dx .

$$x = 7/t, \quad y = \sqrt{t}e^{-t}.$$

Problem 13 (WebAssign HW 32, # 3). Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$$x = t - t^{-1}, \quad y = 9 + t^2, \quad t = 1.$$

Problem 14 (WebAssign HW 32, # 4). Find dy/dx and d^2y/dx^2 .

$$x = e^t, \quad y = te^{-t}.$$

For which values of t is the curve concave upward?

Problem 15 (WebAssign HW 32, # 5). Find dy/dx and d^2y/dx^2 .

$$x = 2 \sin t, \quad y = 3 \cos t, \quad 0 < t < 2\pi.$$

For which values of t is the curve concave upward?

Problem 16 (WebAssign HW 32, # 6). Find the exact length of the curve.

$$x = 3 + t^2, \quad y = 3 + 2t^3, \quad 0 \leq t \leq 2.$$

Problem 17 (WebAssign HW 32, # 7). Find the exact length of the curve

$$x = e^t + e^{-t}, \quad y = 5 - 2t, \quad 0 \leq t \leq 4.$$

Problem 18 (WebAssign HW 32, # 8). Find the distance traveled by a particle with position (x, y) as t varies in the given time interval.

$$x = 3 \sin^2 t, \quad y = 3 \cos^2 t, \quad 0 \leq t \leq 3\pi.$$

What is the length of the curve?

Problem 19 (WebAssign HW 33, # 1). Find two other pairs of polar coordinates of the given polar coordinate, one with $r > 0$ and one with $r < 0$. Then plot the point.

- (a) $(5, \pi/4)$
- (b) $(4, -2\pi/3)$
- (c) $(-4, \pi/6)$

Problem 20 (WebAssign HW 33, # 2). Find the Cartesian coordinates of the given polar coordinates. Then plot the point.

- (a) $(5, \pi)$
- (b) $(6, -2\pi/3)$
- (c) $(-6, 3\pi/4)$

Problem 21 (WebAssign HW 33, # 3). The Cartesian coordinates of a point are given.

- (a) $(4, -4)$
 - (i) Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leq \theta < 2\pi$.
 - (ii) Find polar coordinates (r, θ) of the point, where $r < 0$ and $0 \leq \theta < 2\pi$.
- (b) $(-1, \sqrt{3})$

- (i) Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leq \theta < 2\pi$.
- (ii) Find polar coordinates (r, θ) of the point, where $r < 0$ and $0 \leq \theta < 2\pi$.

Problem 22 (WebAssign HW 33, # 4). The Cartesian coordinates of a point are given.

(a) $(2\sqrt{3}, 2)$

- (i) Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leq \theta < 2\pi$.
- (ii) Find polar coordinates (r, θ) of the point, where $r < 0$ and $0 \leq \theta < 2\pi$.

(b) $(1, -3)$

- (i) Find polar coordinates (r, θ) of the point, where $r > 0$ and $0 \leq \theta < 2\pi$.
- (ii) Find polar coordinates (r, θ) of the point, where $r < 0$ and $0 \leq \theta < 2\pi$.

Problem 23 (WebAssign HW 33, # 5). Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions

$$2 < r < 5, \quad 3\pi/2 \leq \theta \leq 5\pi/2.$$

Problem 24 (WebAssign HW 33, # 6). Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions

$$r \geq 5, \quad \pi \leq \theta \leq 2\pi$$

Problem 25 (WebAssign HW 33, # 7). Find a Cartesian equation for the curve and identify it.

$$r^2 \cos 2\theta = 1.$$

Problem 26 (WebAssign HW 33, # 8). Find a Cartesian equation for the curve and identify it.

$$r = 4 \tan \theta \sec \theta.$$

Problem 27 (WebAssign HW 33, # 9). Find a polar equation for the curve represented by the given Cartesian equation.

$$x^2 + y^2 = 10cx.$$

Problem 28 (WebAssign HW 33, # 10). Find a polar equation for the curve represented by the given Cartesian equation.

$$xy = 11.$$

2 Exam Problems

2.1 Exam 3: Spring 2014

Problem 1. Compute

$$\sum_{n=1}^{\infty} \frac{2^{n-1} - (-3)^{n+1}}{4^n}.$$

Problem 2. This series

$$\sum_{n=1}^{\infty} \frac{1}{(n^{2p} + 1)^{1/6}}$$

is convergent if and only if

Problem 3. Test the following series for convergence or divergence.

(a) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right).$

(b) $\sum_{n=1}^{\infty} (-1)^n \arctan n.$

(c) $\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}.$

Problem 4. Determine whether the following series are absolutely convergent, conditionally convergent or divergent.

(a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2n}{3n+5}.$

(b) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^3}.$

(c) $\sum_{n=2}^{\infty} (-1)^{n-1} \frac{\ln n}{n}.$

Problem 5. Test the following series for convergence or divergence.

(a) $\sum_{n=1}^{\infty} \frac{n!}{n^n}.$

(b) $\sum_{n=1}^{\infty} \left(\frac{3n+2}{2n+3}\right)^n.$

(c) $\sum_{n=1}^{\infty} \frac{n}{5^n}.$

Problem 6. Which of the following statements are *always true*?

- (I) If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.
- (II) If $\lim_{n \rightarrow \infty} n^3 |a_n| = 0$, then $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges.
- (III) $\sum_{n=1}^{\infty} (e^n + c)/e^{2n}$ converges for any positive value c .

Problem 7. Given the following series

$$\sum_{n=1}^{\infty} \frac{3}{2^n + n - 1}.$$

Mark, Nancy and David provide the following ingredient of the arguments for convergence or divergence of the series:

- (a) the name of the test to use,
- (b) the conclusion for convergence or divergence

Mark: (a) $b_n = 3/2^n$, comparison test ($0 \leq a_n \leq b_n$); (b) convergent

Nancy: (a) $b_n = 1/n$, limit comparison test ($\lim_{n \rightarrow \infty} a_n/b_n = 3$); (b) divergent

David: (a) ratio test ($\lim_{n \rightarrow \infty} |a_{n+1}|/|a_n| = 1/2$); (b) convergent.

Choose the name(s) of the person(s) with correct arguments.

Problem 8. Consider the Maclaurin series for e^x

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots.$$

By plugging in $x = -1$, one obtains the alternating series

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots.$$

If we compute the sum of the *fewest* terms necessary to guarantee that the error is less than 0.05, using the *estimation theorem for alternating series*, then what is the estimate for e^{-1} ?

Problem 9. Suppose the power series

$$\sum_{n=0}^{\infty} c_n(x-3)^n$$

converges when $x = 1$, but diverges when $x = 7$.

From the above information, which of the following statements can we conclude to be true?

- (I) The radius of convergence is $R \geq 2$.
- (II) The power series converges at $x = 4.5$.
- (III) The power series diverges at $x = 6.5$.

Problem 10. What is the coefficient of x^6 in the power series expansion $2/(1+2x^2)$?

Problem 11. Determine the interval of convergence for the following power series

$$\sum_{n=0}^{\infty} \frac{(-5)^n}{\sqrt{n+2}}(x-3)^n.$$

Problem 12. The power series representation (centered at $a = 0$) for $g(x) = x/(4 - x^2)$ is given by

$$g(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{4^{n+1}}$$

with the interval of convergence $(-2, 2)$.

Find

- (a) the power series representation (centered at $a = 0$), and
- (b) the interval of convergence

for the function

$$f(x) = \ln |4 - x^2|.$$

2.2 Exam 3 Spring 2013