

MA544: Qual Problems

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1 MA 544 Spring 2016

1.1 Exam 1 Prep

Problem 1.1. Let $E \subset \mathbb{R}^n$ be a measurable set, $r \in \mathbb{R}$ and define the set $rE = \{r\mathbf{x} \mid \mathbf{x} \in E\}$. Prove that rE is measurable, and that $|rE| = |r|^n|E|$.

Proof.

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Problem 1.2. Let $\{E_k\}$, $k \in \mathbb{N}$ be a collection of measurable sets. Define the set

$$\liminf_{k \rightarrow \infty} E_k = \bigcup_{k=1}^{\infty} \left(\bigcap_{n=k}^{\infty} E_n \right).$$

Show that

$$\left| \liminf_{k \rightarrow \infty} E_k \right| \leq \liminf_{k \rightarrow \infty} |E_k|.$$

Proof.

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Problem 1.3. Let $E \subset \mathbb{R}^n$ be a measurable set, with $|E| = \infty$. Show that for any $C > 0$ there exists a measurable set $F \subset E$ such that $C < |F| < \infty$.

Proof.

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Problem 1.4. Consider the function

$$F(\mathbf{x}) := \begin{cases} |B(x, 0)| & \mathbf{x} > 0 \\ 0 & \mathbf{x} = 0 \end{cases}.$$

Proof.

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Problem 1.5.

Proof.

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Problem 1.6.

Proof.

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Problem 1.7.

Proof.

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Problem 1.8.

Proof.

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Problem 1.9.

Proof.

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