MA 571: Homework # 11 due Wednesday November 18.

Please read pages 359–362 in Section 58 and Step 1 in the proof of Theorem 59.3 on page 369. Also please read from the bottom of page 370 to the top of page 372 (stop just before Theorem 60.3)

Optional reading: Section 56 (note that Step 1 on page 354 doesn't need "covering spaces," just Theorem A from the note on the Fundamental Group of the Circle) and the rest of Section 59 (we'll prove the main theorem of this section two other ways.)

Please do:

p. 335 # 7(abcd) (The definition of topological group is at the bottom of page 145.)

A) Prove Proposition F from the note on the Fundamental Group of the Circle.

B) Prove Lemma G from the note on the Fundamental Group of the Circle. (Hint: one way to do this is to use the fact, which you don't have to prove, that if $\sim$ is the equivalence relation on $[a, a+1]$ which identifies a and $a+1$ then the restriction of p induces a homeomorphism $[a, a+1]/\sim\ \to S^1$.)

C) Show that for every point $x \in S^n$ the space $S^n - \{x\}$ is homeomorphic to $\mathbb{R}^n$. You may use the fact, shown in Step 1 of the proof of Theorem 59.3, that S^n with the *north pole* removed is homeomorphic to $\mathbb{R}^n$. (Hint: linear algebra.)

D) Show that every loop in S^n which is not onto is path-homotopic to a constant path. (Hint: use Problem C).

E) Let X be a topological space and let $A \subset X$ be a deformation retract. In the space X/A, the set A is a point (because it is an equivalence class). Show that this point is a deformation retract of X/A. (Hint: use p. 289 # 9.)