MA 572: Homework 1

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Problem 1.1 (Hatcher §2.1, Ex. 11)

Show that if A is a retract of X then the map $H_n(A) \to H_n(X)$ induced by the inclusion $A \subset X$ is injective.

Proof. ■

Problem 1.2 (Hatcher §2.1, Ex. 12)

Show that chain homotopy of chain maps is an equivalence relation.

Proof. ■

Problem 1.3 (Hatcher §2.1, Ex. 16)

(a) Show that $H_0(X, A) = 0$ iff A meets each path-component of X.

(b) Show that $H_1(X, A) = 0$ iff $H_1(A) \to H_1(X)$ is surjective and each path-component of X contains at most one path-component of A.

Proof. ■

Problem 1.4 (Hatcher §2.1, Ex. 17)

(a) Compute the homology groups $H_n(X, A)$ when X is S^2 or $S^1 \times S^1$ and A is a finite set of points in X.

(b) Compute the groups $H_n(X, A)$ and $H_n(X, B)$ for X a closed orientable surface of genus two with A and B the circles shown. [What are X/A and X/B?]

Proof. ■