

MA 519: Homework 6

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PROBLEM 6.1 (HANDOUT 8, # 2)

Identify the parameters n and p for each of the following binomial distributions:

- (a) # boys in a family with 5 children;
- (b) # correct answers in a multiple choice test if each question has a 5 alternatives, there are 25 questions, and the student is making guesses at random.

SOLUTION. For part (a), the distribution is binomial with k being the number of children in a given family and p the probability that a child is born, say, male. In this case, we can reasonably assume that $p = 0.5$. Thus, the binomial distribution is given by $\text{Binom}(5, 0.5)$.

For part (b), we use similar reasoning and we have $\text{Binom}(25, 0.2)$ where $k = 25$ is the number of questions and $p = 1/5 = 0.2$ the probability of guessing a question correctly. ■

PROBLEM 6.2 (HANDOUT 8, # 10)

A newsboy purchases papers at 20¢ and sells them for 35¢. He cannot return unsold papers. If the daily demand for papers is modeled as a $\text{Binom}(50, 0.5)$ random variable, what is the optimum number of papers the newsboy should purchase?

SOLUTION. Let $X \sim \text{Binom}(50, 0.5)$ denote the daily demand for papers. Let S denote the number of newspapers the newsboy sells and let B denote the number of newspapers the newsboy buys. Then, we want to optimize the quantity

$$P = 0.35 \cdot S - 0.2 \cdot B$$

■

PROBLEM 6.3 (HANDOUT 8, # 12)

How many independent bridge dealings are required in order for the probability of a preassigned player having four aces at least once to be $1/2$ or better? Solve again for some player instead of a given one.

SOLUTION. ■

PROBLEM 6.4 (HANDOUT 8, # 13)

A book of 500 pages contains 500 misprints. Estimate the chances that a given page contains at least three misprints.

SOLUTION.



PROBLEM 6.5 (HANDOUT 8, # 14)

Colorblindness appears in 1 per cent of the people in a certain population. How large must a random sample (with replacements) be if the probability of its containing a colorblind person is to be 0.95 or more?

SOLUTION. ■

PROBLEM 6.6 (HANDOUT 8, # 15)

Two people toss a true coin n times each. Find the probability that they will score the same number of heads.

SOLUTION. ■

PROBLEM 6.7 (HANDOUT 8, # 16)

Binomial approximation to the hypergeometric distribution. A population of TV elements is divided into red and black elements in the proportion $p : q$ (where $p + q = 1$). A sample of size n is taken without replacement. The probability that it contains exactly k red elements is given by the hypergeometric distribution of II, 6. Show that as $n \rightarrow \infty$ this probability approaches $\text{Binom}(n, p)$.

SOLUTION. ■

PROBLEM 6.8 (HANDOUT 9, # 3)

Suppose X, Y, Z are mutually independent random variables, and $E(X) = 0$, $E(Y) = -1$, $E(Z) = 1$, $E(X^2) = 4$, $E(Y^2) = 3$, $E(Z^2) = 10$. Find the variance and the second moment of $2Z - Y/2 + eX$, where e is the number such that $\ln e = 1$.

SOLUTION. ■

PROBLEM 6.9 (HANDOUT 9, # 14)

(*Variance of Product*). Suppose X, Y are independent random variables. Can it ever be true that $\text{Var}(XY) = \text{Var}(X) \text{Var}(Y)$?

If it can, when?

SOLUTION. ■