MA52300 Fall 2016

Homework Assignment 6

Due Mon, Oct 24, 2016

1. For $n = 2$ find Green's function for the quadrant $\{x_1 > 0, x_2 > 0\}$ by repeated reflection.

2. (Precise form of Harnack's inequality) Use Poisson's formula for the ball to prove

$$\frac{r^{n-2}(r - |x|)}{(r + |x|)^{n-1}}\, u(0) \leq u(x) \leq \frac{r^{n-2}(r + |x|)}{(r - |x|)^{n-1}}\, u(0)$$

whenever u is positive and harmonic in $B(0, r) = \{x \in \mathbb{R}^n : |x| < r\}$.

3. Let $P_k(x)$ and $P_m(x)$ be homogeneous harmonic polynomials in $\mathbb{R}^n$ of degrees k and m respectively; i.e.,

$$P_k(\lambda x) = \lambda^k P_k(x), \quad P_m(\lambda x) = \lambda^m P_k(x), \quad \text{for every } x \in \mathbb{R}^n, \lambda > 0$$
$$\Delta P_k = 0, \quad \Delta P_m = 0 \quad \text{in } \mathbb{R}^n.$$

(a) Show that

$$\frac{\partial P_k}{\partial \nu} = k P_k(x), \quad \frac{\partial P_m}{\partial \nu} = m P_m(x) \quad \text{on } \partial B(0, 1),$$

where $B(0, 1) = \{x \in \mathbb{R}^n : |x| < 1\}$ and ν is the outward normal on $\partial B(0, 1)$.

(b) Use (a) and Green's formula to prove that

$$\int_{\partial B(0,1)} P_k(x) P_m(x) d\sigma = 0, \quad \text{if } k \neq m$$