MA166: Exam 2 Prep

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As I promised, here are somewhat detailed solutions to two of the last Exam 2's for MA 166.

1 MA 166 Exam 2, Spring 2015

Problem 1.1. Evaluate the following integral

$$\int_0^\pi \sin^2 x \cos^2 x \, dx$$

Solution. The following trig identities are useful

$$\sin 2\theta = 2 \sin\theta \cos\theta \tag{1}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}. \tag{2}$$

With that in mind we compute the integral

$$\begin{aligned}
\int_0^\pi \sin^2 x \cos^2 x \, dx &= \int_0^\pi (\sin x \cos x)^2 \, dx \\
&= \int_0^\pi \left(\frac{1}{2}\sin 2x\right)^2 \, dx \\
&= \frac{1}{4}\int_0^\pi \sin^2 2x \, dx \\
&= \frac{1}{4}\int_0^\pi \frac{1-\cos 4x}{2} \, dx \\
&= \frac{1}{8}\int_0^\pi 1-\cos 4x \, dx \\
&= \frac{1}{8}\left[x - \frac{1}{4}\sin 4x\right]_0^\pi \\
&= \frac{1}{8}[\pi - 0 - (0-0)] \\
&= \boxed{\frac{\pi}{8}.}
\end{aligned}$$

Answer: B. ☺

Problem 1.2. Evaluate the following integral

$$\int_0^{\pi/4} \sec^4 x \tan x \, dx.$$

Solution. The following identities are useful

$$\sec^2\theta - \tan^2\theta = 1. \tag{3}$$

Substitute $u = \tan x$, $du = \sec^2 x\, dx$

$$\begin{aligned}
\int_0^{\pi/4} \sec^4 x \tan x\, dx &= \int_0^{\pi/4} \sec^2 x \sec^2 x \tan x\, dx \\
&= \int_0^{\pi/4} \sec^2 x(1+\tan^2 x) \tan x\, dx \\
&= \int_0^1 (1+u^2) u\, dx \\
&= \int_0^1 u + u^3\, dx \\
&= \left[\frac{1}{2}u^2 + \frac{1}{4}u^4\right]_0^1 \\
&= \frac{1}{2} + \frac{1}{4} - 0 - 0 \\
&= \boxed{\frac{3}{4}.}
\end{aligned}$$

Answer: A. ☺

Problem 1.3. After the appropriate trigonometric substitution, the integral

$$\int_2^5 \frac{dx}{\sqrt{x^2-4x+13}}$$

becomes

Solution. The general strategy for these types of problems is to complete the square in the denominator and make some sort of trig substitution

$$x^2 - 4x + 13 = \left(x^2 - 4x + 4\right) + 9 = (x-2)^2 + 9.$$

Make the u-substitution $u = (x-2)/3$, $du = dx/3$

$$\begin{aligned}
\int_2^5 \frac{dx}{\sqrt{x^2-4x+13}} &= \int_2^5 \frac{dx}{3\sqrt{(x-2)^2/9+1}} \\
&= \frac{1}{3}\int_2^5 \frac{dx}{\sqrt{\left(\frac{x-2}{3}\right)^2+1}} \\
&= \int_0^1 \frac{du}{\sqrt{u^2+1}}
\end{aligned}$$

follow it up with the trig substitution $\sec\theta = u$, $\sec\theta\tan\theta\, d\theta = du$

$$= \int_{\pi/2}^0$$

☺

Problem 1.4.

Solution. ☺

Problem 1.5.

Solution. ☺

Problem 1.6.

Solution. ☺

Problem 1.7.

Solution. ☺

Problem 1.8.

Solution. ☺

Problem 1.9.

Solution. ☺

Problem 1.10.

Solution. ☺

Problem 1.11.

Solution. ☺

Problem 1.12.

Solution. ☺

2 MA 166 Exam 2, Spring 2014

Problem 2.1.

Solution. ☺

Problem 2.2.

Solution. ☺

Problem 2.3.

Solution. ☺

Problem 2.4.

Solution. ☺

Problem 2.5.

Solution. ☺

Problem 2.6.

Solution. ☺

Problem 2.7.

Solution. ☺

Problem 2.8.

Solution. ☺

Problem 2.9.

Solution. ☺

Problem 2.10.

Solution. ☺

Problem 2.11.

Solution. ☺

Problem 2.12.

Solution. ☺