MA166: Recitation 6 Prep

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1 Recitation 6 Prep

Recitation average for Exam 1

Table 1.1 – Section averages for Exam 1.

section	average
294	71.0
151	76.66
112	69.82

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Section 1.1: Homework Solutions

Here are the homework solutions for this week.

Homework 12

Problem 1.1 (WebAssign, HW 12, # 1)**.** Evaluate the integral

$$\int_{3\sqrt{2}}^{6} \frac{1}{t^3\sqrt{t^2-9}}\, dt.$$

Solution. Make the substitution

$$3\sec\theta = t, \tag{1}$$

then $3\sec\theta\tan\theta\, d\theta = dt$ and substituting this and (1) into the integral, making sure to solve for the appropriate values of θ, i.e., the lower bound is $\sec^{-1}\big(\sqrt{2}\big) = \pi/4$ and the upper bound is $\sec^{-1}(2) = \pi/3$

$$\begin{aligned}
\int_{3\sqrt{2}}^{6} \frac{1}{t^3\sqrt{t^2-9}}\, dt &= \int_{\pi/4}^{\pi/3} \frac{3\sec\theta\tan\theta}{3^3\sec^3\theta\sqrt{3^2\sec^2\theta-9}}\, d\theta \\
&= \int_{\pi/4}^{\pi/3} \frac{\tan\theta}{3^2\sec^3\theta\sqrt{3^2\sec^2\theta-3^2}}\, d\theta \\
&= \int_{\pi/4}^{\pi/3} \frac{\tan\theta}{3^2\sec^2\theta\sqrt{3^2(\sec^2\theta-1)}}\, d\theta \\
&= \frac{1}{27}\int_{\pi/4}^{\pi/3} \frac{\tan\theta}{\sec^2\theta\sqrt{\tan^2\theta}}\, d\theta \\
&= \frac{1}{27}\int_{\pi/4}^{\pi/3} \frac{1}{\sec^2\theta}\, d\theta \\
&= \frac{1}{27}\int_{\pi/4}^{\pi/3} \cos^2\theta\, d\theta \\
&= \frac{1}{27}\int_{\pi/4}^{\pi/3} \frac{1+\cos 2\theta}{2}\, d\theta
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{54}\int_{\pi/4}^{\pi/3} 1+\cos 2\theta \; d\theta \\
&= \frac{1}{54}\left(\theta + \frac{\sin 2\theta}{2}\bigg|_{\pi/4}^{\pi/3}\right) \\
&= \boxed{\frac{\pi}{648} + \frac{\sqrt{3}-2}{108}.}
\end{aligned}
$$

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Problem 1.2 (WebAssign, HW 12, # 2)**.** Evaluate the integral. (Use C for the constant of integration.)

$$\int \sqrt{1-25x^2}\; dx.$$

Solution. First, make the substitution $u = 5x$. Then the integral above turns into

$$\frac{1}{5}\int \sqrt{1-u^2}\; du.$$

Now we make the trig substitution $\cos\theta = u$ so $-\sin\theta\; d\theta = du$ and the integral above turns into

$$
\begin{aligned}
\frac{1}{5}\int \sqrt{1-u^2}\; du &= \frac{1}{5}\int \sin\theta(-\sin\theta)\; d\theta \\
&= -\frac{1}{5}\int \sin^2\theta\; d\theta \\
&= -\frac{1}{5}\int \left(\frac{1-\cos 2\theta}{2}\right) d\theta \\
&= -\frac{1}{10}\int 1-\cos 2\theta\; d\theta \\
&= \frac{1}{10}\int \cos 2\theta - 1\; d\theta \\
&= \frac{1}{10}\left(\frac{1}{2}\sin 2\theta - \theta\right) \\
&= \frac{1}{20}\sin 2\theta - \frac{1}{2}\theta + C \\
&= \frac{1}{20}2\sin\theta\cos\theta - \frac{1}{2}\theta + C
\end{aligned}
$$

substituting back u then x, we have

$$
\begin{aligned}
&= \frac{1}{10}\sqrt{1-u^2}u - \frac{1}{2}\cos^{-1}(u) + C \\
&= \frac{1}{10}\sqrt{1-25x^2}(5x) - \frac{1}{2}\cos^{-1}(5x) + C \\
&= \boxed{\frac{\sqrt{1-25x^2}x - \cos^{-1}(5x)}{2} + C.}
\end{aligned}
$$

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Problem 1.3 (WebAssign, HW 12, # 3)**.** Evaluate the integral. (Use C for the constant of integration.)

$$\int \sqrt{16+6x-x^2}\,dx.$$

Solution. First we need to complete the square

$$\begin{aligned}(x-3)^2-9 &= x^2-6x+9-9\\ &= x^2-6x.\end{aligned}$$

Then, the integral above turns into

$$\int \sqrt{16+x-x^2}\,dx = \int \sqrt{25+(x-3)^2}\,dx$$

and now we can use the substitution $5u = x-3$ to simplify our integral into

$$\frac{1}{5}\int \sqrt{25-(5u)^2}\,du = \frac{1}{5}\int \sqrt{5^2-5^2u^2}\,du = \int \sqrt{1-u^2}\,du.$$

Now we can use the substitution $\cos\theta = u$ and $-\sin\theta\,d\theta = du$ to get

$$\begin{aligned}\int \sqrt{1-u^2}\,du &= \int \sqrt{1-\cos^2\theta}(-\sin\theta)\,d\theta\\ &= -\int \sqrt{\sin^2\theta}\sin\theta\,d\theta\\ &= -\int \sin^2\theta\,d\theta\end{aligned}$$

which, from our previous problem, we know to be

$$= \frac{1}{2}\sin 2\theta - \theta + C.$$

Substututing back first u then x we get

$$\begin{aligned}\frac{1}{2}\sin 2\theta - \theta + C &= \sin\theta\cos\theta - \theta + C\\ &= \sqrt{1-u^2}u - \cos^{-1}(u) + C\\ &= \sqrt{1-\left(\frac{x-3}{5}\right)^2}\left(\frac{x-3}{5}\right) - \cos^{-1}\left(\frac{x-3}{5}\right) + C\\ &= \boxed{\frac{x-3}{25}\sqrt{16+6x-x^2} - \cos^{-1}\left(\frac{x-3}{5}\right) + C.}\end{aligned}$$

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Problem 1.4 (WebAssign, HW 12, # 4)**.** Evaluate the integral. (Use C for the constant of integration.)

$$\int \frac{1}{\sqrt{t^2-12t+40}}\,dt.$$

Solution. ☺

Problem 1.5 (WebAssign, HW 12, # 5)**.** Evaluate the integral. (Use C for the constant of integration.)

$$\int \sqrt{x^2 + 6x}\, dx.$$

Solution. ☺

Homework 13

Problem 1.6 (WebAssign, HW 13, # 1)**.**

Solution. ☺

Problem 1.7 (WebAssign, HW 13, # 2)**.**

Solution. ☺

Problem 1.8 (WebAssign, HW 13, # 3)**.**

Solution. ☺

Problem 1.9 (WebAssign, HW 13, # 4)**.**

Solution. ☺

Problem 1.10 (WebAssign, HW 13, # 5)**.**

Solution. ☺

Problem 1.11 (WebAssign, HW 13, # 6)**.**

Solution. ☺

Homework 14

Problem 1.12 (WebAssign, HW 14, # 1)**.**

Solution. ☺

Problem 1.13 (WebAssign, HW 14, # 2)**.**

Solution. ☺

Problem 1.14 (WebAssign, HW 14, # 3)**.**

Solution. ☺

Problem 1.15 (WebAssign, HW 14, # 4)**.**

Solution. ☺

Problem 1.16 (WebAssign, HW 14, # 5)**.**

Solution. ☺

Section 1.2: Exam 2 Problems