

# QUALIFYING EXAMINATION

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MATH 519 - Prof. DasGupta

- (20) 1. The number of fish that Anirban catches on any given day has a poisson distribution with mean 20. Due to the legendary softness of his heart, he sets free, on an average, 3 out of 4 fish he catches. Find the mean and the variance of the number of fish Anirban takes home on a given day.
- (20) 2. A fair die is rolled and at the same time a fair coin is tossed. This is done repeatedly. Find the probability that a head occurs (strictly) before a six occurs.
- (20) 3.  $X, Y$  are independent random variables with the common density  $f(x) = \frac{1}{2}e^{-|x|}$ ,  $-\infty < x < \infty$ . Find the density function of  $X + Y^2$ .
- (20) 4. Let  $X_n$  denote the distance between two points chosen independently at random from the unit cube in  $\mathbb{R}^n$ . Evaluate
- $$\lim_{n \rightarrow \infty} \frac{E(X_n)}{\sqrt{n}}.$$
- (20) 5. Let  $X$  be distributed as Uniform  $[0, 1]$ . What is the probability that the digit 5 does not occur in the decimal expansion of  $X$ ?