Micro-teaching Session

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Contents

1 Script

This is my script for the *Micro-teaching recitation presentation* on Monday, October 3, 2016. I have attached a sample 15-minute quiz at the end the document.

1.1 L'Hôpital's rule

Today we go over some of your WebAssign problems to show you how to use l'Hôpital's rule to evaluate the limits of quotients f/g and products fg.

The problems we will be discussing in today's recitations are problems 2, 3, 4, 7, 8, 9, and 10. But first, a vote. (Draw a table on the chalkboard

Problem	Votes	Problem	Votes
2		3	
4		7	
8		9	
10			

)

Raise your hand if you want to see a detailed solution to problem 2 [pause], problem 3, etc.

1.2 Exercises

Problem (WebAssign, # 2). Find the limit. Use l'Hôpital's rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to 0}\frac{\sin 2x}{\sin 3x}.$$

Solution. First, let's look at the limit of the numerator and the limit of the denominator, individually. For the numerator, we have

$$\lim_{x\to 0}\sin 2x = 0$$

and, similarly, for the denominator

$$\lim_{x\to 0}\sin 3x = 0.$$

As you may remember for class, this is a limit of the type $0/0$ and a prime candidate for l'Hôpital's rule.

Remember that l'Hôpital's rule says that the limit of a quotient f/g is the limit of the quotient of their derivatives f'/g', i.e.,

$$\lim_{x\to 0}\frac{\sin 2x}{\sin 3x} = \lim_{x\to 0}\frac{2\cos 2x}{3\cos 3x}.$$

Now, the limit of the cos in the numerator and denominator, as $x\to 0$, is 1, so

$$\lim_{x\to 0}\frac{\sin 2x}{\sin 3x} = \frac{2}{3}\left[\frac{\lim_{x\to 0}\cos 2x}{\lim_{x\to 0}\cos 3x}\right] = \frac{3}{2}.$$

easy, right?

Let's have a look at the next problem. ■

Problem (WebAssign, # 3). Find the limit. Use l'Hôpital's rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to 0} \frac{e^{7x} - 1 - 7x}{x^2}.$$

Solution. For this problem we have

$$\lim_{x\to 0} \frac{e^{7x} - 1 - 7x}{x^2}.$$

Note that as $x \to 0$, the denominator goes to 0. Thus, by l'Hôpital's rule

$$\lim_{x\to 0} \frac{e^{7x} - 1 - 7x}{x^2} = \lim_{x\to 0} \frac{7e^{7x} - 7}{x},$$

but here the denominator still goes to 0, so we use l'Hôpital's rule again

$$\begin{aligned} &= \lim_{x\to 0} \frac{49e^{7x}}{1} \\ &= 49. \end{aligned}$$

■

Problem (WebAssign, # 4). Find the limit. Use l'Hôpital's rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to\infty} \frac{\big(\ln(x)\big)^2}{5x}.$$

Solution. Both the numerator and denominator go to ∞. This is a limit of the type ∞/∞. Here, we have

$$\begin{aligned} \lim_{x\to\infty} \frac{\big(\ln(x)\big)^2}{5x} &= \lim_{x\to\infty} \frac{2(\ln x)(1/x)}{5} \\ &= \lim_{x\to\infty} \frac{2\ln x}{5x}, \end{aligned}$$

using l'Hôpital's rule again, we have

$$\begin{aligned} &= \lim_{x\to\infty} \frac{2(1/x)}{5} \\ &= \lim_{x\to\infty} \frac{2}{5x} \\ &= 0. \end{aligned}$$

■

PROBLEM (WebAssign, # 7). Find the limit. Use l'Hôpital's rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to\infty} x\tan(5/x).$$

SOLUTION. Here we do something you may not be completely familiar with, we do what is called a *change of variables*. Setting $u := 1/x$ we see that as $x \to \infty$, $u \to 0$ so we can turn the problem

$$\lim_{x\to\infty} x\tan(5/x)$$

into the equivalent problem

$$\lim_{u\to 0} \frac{\tan(5u)}{u}.$$

You see this, right?

Now, by l'Hôpital's rule

$$\begin{aligned}\lim_{x\to\infty} x\tan(5/x) &= \lim_{u\to 0} \frac{\tan(5u)}{u} \\ &= \lim_{u\to 0} \frac{5\sec^2 u}{1} \\ &= \lim_{u\to 0} \frac{5}{\cos^2 u} \\ &= 5.\end{aligned}$$

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PROBLEM (WebAssign, # 8). Find the limit. Use l'Hôpital's rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to 0}(\csc x - \cot x).$$

SOLUTION. Let's write $\csc(x) - \cot(x)$ under a single quotient

$$\begin{aligned}\csc x - \cot x &= \frac{1}{\sin x} - \frac{1}{\tan x} \\ &= \frac{1}{\sin x} - \frac{\cos x}{\sin x} \\ &= \frac{1-\cos x}{\sin x}.\end{aligned}$$

Now we can start looking at the limit.

By l'Hôpital's rule, we have

$$\begin{aligned}\lim_{x\to 0}(\csc x - \cot x) &= \lim_{x\to 0}\left[\frac{1-\cos x}{\sin x}\right] \\ &= \lim_{x\to 0}\frac{\sin x}{\cos x} \\ &= \lim_{x\to 0}\tan x \\ &= 0.\end{aligned}$$

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Problem (WebAssign, # 9). Find the limit. Use l'Hôpital's rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to 0}(1-8x)^{1/x}.$$

Solution. For this problem, we can again use the *change of variables* $u = (1/x)$ and solve the problem

$$\lim_{u\to\infty}\left(1-\frac{8}{u}\right)^u$$

You may have seen this limit before in yours study of sequences, if you have, you will immediately recognize the limit of this function as e^{-8}.

If you don't, that's alright; we'll provide some details. Suppose for a moment that the limit of this function is L. Then, using log rules, we have

$$\frac{\ln L}{\ln\big((u-8)/u\big)} = u$$
$$\ln L = u\ln\left(\frac{u-8}{u}\right).$$

Now, let's take the limit

$$\lim_{u\to\infty} u\ln\left(\frac{u-8}{u}\right) = \lim_{u\to\infty}\frac{\ln\big((u-8)/u\big)}{1/u},$$

which, by l'Hôpital's rule, becomes

$$= \lim_{u\to\infty}\frac{\big((u-u+8)/u^2\big)\big(u/(u-8)\big)}{-1/u^2}$$
$$= \lim_{u\to\infty}\frac{-8u}{u-8}$$

and again

$$= \lim_{u\to\infty} -8.$$

Thus, the log of the limit is -8, i.e.,

$$\ln L = -8$$

so

$$L = e^{-8}.$$

■

Problem (WebAssign, # 10). Find the limit. Use l'Hôpital's rule if appropriate. If there is a more elementary method, consider using it.

$$\lim_{x\to\infty} x^{8/x}.$$

Solution. We use the same approach. Let $u := 1/x$. Then,

$$L = \lim_{x\to\infty} x^{8/x} = \lim_{u\to 0}\left(\frac{1}{u}\right)^{8u},$$

if it exists.

Thus, taking the natural log of both sides

$$\begin{aligned}\frac{\ln L}{\ln(1/u)} &= 8u\\ \ln L &= 8u\ln(1/u).\end{aligned}$$

Now,

$$\lim_{u\to 0} u\ln(1/u) = \lim_{u\to 0}\frac{\ln(1/u)}{(1/u)}$$

which, by l'Hôpital's rule, becomes

$$\begin{aligned}&= \lim_{u\to 0}\frac{u(-1/u^2)}{-1/u^2}\\ &= \lim_{u\to 0} u\\ &= 0.\end{aligned}$$

Thus,

$$\begin{aligned}\ln L &= 0\\ L &= e^0 = 1.\end{aligned}$$

■

1.3 Sample Quiz

You have **15 minutes** to complete this quiz. You may work in groups, but you are not allowed to use any other resources.

PROBLEM (A). Evaluate the limit

$$\lim_{t\to\infty} \frac{\ln t}{t^2}.$$

PROBLEM (B). Evaluate the limit

$$\lim_{x\to 4} \frac{x^2 - 16}{x - 4}.$$

PROBLEM (C). Evaluate the limit

$$\lim_{x\to 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6}.$$