

MA571 Homework 8

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PROBLEM 8.1 (MUNKRES §46, EX. 6)

Show that the compact-open topology, $\mathcal{C}(X, Y)$ is Hausdorff if Y is Hausdorff, and regular if Y is regular. [Hint: If $\overline{\text{line } U} \subset V$, then $S(C, U) \subset S(C, V)$.]

Proof.

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PROBLEM 8.2 (MUNKRES §46, EX. 7)

Show that if Y is locally compact Hausdorff, then composition of maps

$$\mathcal{C}(X, Y) \times \mathcal{C}(Y, Z) \longrightarrow \mathcal{C}(X, Z)$$

is continuous, provided the compact-open topology is used throughout. [*Hint:* If $g \circ f \in S(C, U)$, find V such that $f(C) \subset V$ and $g(\overline{V}) \subset U$.]

Proof.

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PROBLEM 8.3 (MUNKRES §46, EX. 8)

Let $\mathcal{C}'(X, Y)$ denote the set $\mathcal{C}(X, Y)$ in some topology \mathcal{T} . Show that if the evaluation map

$$e: X \times \mathcal{C}'(X, Y) \longrightarrow Y$$

is continuous, then \mathcal{T} contains the compact-open topology. [*Hint:* The induced map $E: \mathcal{C}'(X, Y) \rightarrow \mathcal{C}(X, Y)$ is continuous.]

Proof.

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PROBLEM 8.4 ((A))

Definition 1. Definition. If X is a locally compact Hausdorff space then the space Y given by Theorem 29.1 is called the *one-point compactification* of X .

Let X be a compact Hausdorff space and let W be an open subset of X (so W is locally compact by Corollary 29.3) with $W \neq X$. Prove that the one-point compactification of W is homeomorphic to the quotient space $X/(X - W)$.

Proof.

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PROBLEM 8.5 ((B))

Let X be a compact Hausdorff space, let Y be a topological space, and let $p: X \rightarrow Y$ be a closed surjective continuous map. Prove that Y is Hausdorff. [*Hint*: one ingredient in the proof is p. 171 # 5.]

Note: combining this with HW 4 Problem E and HW 6 Problem A gives a necessary and sufficient condition for a quotient of a compact Hausdorff space to be Hausdorff.

Proof.

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PROBLEM 8.6 ((C))

Let $S^2 \subset \mathbf{R}^3$ be the subspace

$$\{ (x, y, z) \mid x^2 + y^2 + z^2 = 1 \}.$$

Prove that S^2 is a 2-manifold. (The definition of m -manifold, where m is a positive whole number, is given at the top of page 225.)

Proof.

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PROBLEM 8.7 ((D))

Prove that the union of the x and y -axes in \mathbf{R}^2 is not a 1-manifold.

Proof.

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