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MA 26500-215 Quiz 2

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1. (10 points) Given the system of linear equations

$$\begin{aligned} x_1 - 2x_2 + x_3 - x_4 &= 3 \\ x_1 + x_2 + x_3 - x_4 &= 1 \\ x_1 + x_3 - x_4 &= 2 \end{aligned} \qquad (\star)$$

find its matrix representation and the reduced row-echelon form of that matrix.

A. $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

E. Not listed.

Solution: Finding the augmented matrix representation of the system $(\star)$ is easy enough

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 1 & 1 & 1 & -1 & 1 \\ 1 & 0 & 1 & -1 & 2 \end{array}\right].$$

Performing the appropriate row operations on the matrix above gets you to the matrix in D.

2. (10 points) Given the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & -1 \\ -1 & -4 & 0 \end{bmatrix}$$

and the vector $\mathbf{b} = \begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix}$, find the vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$ by finding A^{-1}:

A. $A^{-1} = \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

B. $A^{-1} = \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & -1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

C. $A^{-1} = \begin{bmatrix} 2 & 3 & -2 \\ 3 & 5 & -4 \\ 1 & 1 & -1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

D. $A^{-1} = \begin{bmatrix} 2 & 3 & -2 \\ 3 & 5 & -4 \\ 1 & 1 & -1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

E. Does not exist; the matrix A is singular.

Solution: As you learned from Kolman and Hill, you can find the inverse of an $n \times n$ large matrix by augmenting it with the identity matrix I on the right and performing Gaußian elimination on the left. So write

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & 5 & -1 & 0 & 1 & 0 \\ -1 & -4 & 0 & 0 & 0 & 1 \end{array}\right]$$

and after doing Gaußian elimination you get

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & 4 & 3 \\ 0 & 1 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -3 & 2 & 1 \end{array}\right].$$

Then, the inverse of A is

$$A^{-1} = \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & 1 \end{bmatrix}.$$

Than only leaves A as the possible answer choice.