MA 598 (Algebraic Geometry): Homework 1

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PROBLEM 1.1 (5-LEMMA)

Given a commutative diagram with exact rows

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

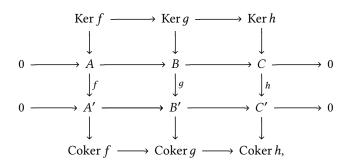
$$\downarrow f \qquad \downarrow g \qquad \downarrow h$$

$$0 \longrightarrow A' \longrightarrow B' \longrightarrow C' \longrightarrow 0,$$

suppose f and h are isomorphisms. Prove that g is an isomorphism.

PROBLEM 1.2 (SNAKE LEMMA)

Given the diagram with exact rows



show that the sequence

$$0 \longrightarrow \operatorname{Ker} f \longrightarrow \operatorname{Ker} g \longrightarrow \operatorname{Ker} h \longrightarrow \operatorname{Coker} f \longrightarrow \operatorname{Coker} f \longrightarrow \operatorname{Coker} h \longrightarrow 0$$

is exact.

PROBLEM 1.3

Given an exact sequence of modules

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

fix any $M \in \mathbf{Mod}_R$.

(a) Show that

$$0 \longrightarrow \operatorname{Hom}(M,A) \longrightarrow \operatorname{Hom}(M,B) \longrightarrow \operatorname{Hom}(M,C)$$

is exact.

(b) Show that

$$0 \longrightarrow \operatorname{Hom}(C, M) \longrightarrow \operatorname{Hom}(B, M) \longrightarrow \operatorname{Hom}(A, M)$$

is exact.

PROBLEM 1.4

PROBLEM 1.5

PROBLEM 1.6