

MA571 Homework 9

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November 1, 2015

PROBLEM 9.1 (MUNKRES §52, EX. 2)

Let α be a path in X from x_0 to x_1 ; let β be a path in X from x_1 to x_2 . Show that if $\gamma = \alpha * \beta$, then $\hat{\gamma} = \hat{\beta} \circ \hat{\alpha}$.

Proof.

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PROBLEM 9.2 (MUNKRES §52, EX. 3)

Let x_0 and x_1 be points of the path-connected space X . Show that $\pi_1(X, x_0)$ is Abelian if and only if for every pair α and β of paths from x_0 to x_1 , we have $\hat{\alpha} = \hat{\beta}$.

Proof.

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PROBLEM 9.3 (MUNKRES §52, EX. 4)

Let $A \subset X$; suppose $r: X \rightarrow A$ is continuous map such that $r(a) = a$ for each $a \in A$. (The map r is called a *retraction* of X onto A .) If $a_0 \in A$, show that

$$r_*: \pi_1(X, x_0) \longrightarrow \pi_1(A, a_0)$$

is surjective.

Proof.

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PROBLEM 9.4 (MUNKRES §53, EX. 6)

Show that if X is path connected, the homomorphism induced by a continuous map is independent of the base point, up to isomorphisms of the groups involved. More precisely, let $h: X \rightarrow Y$ be continuous, with $h(x_0) = y_0$ and $h(x_1) = y_1$. Let α be a path in X from x_0 to x_1 , and let $\beta = h \circ \alpha$. Show that

$$\hat{\beta} \circ (h_{x_0})_* = (h_{x_1})_* \circ \hat{\alpha}.$$

This equation expresses the fact that the following diagram of maps “commutes”

$$\begin{array}{ccc} \pi_1(X, x_0) & \xrightarrow{(h_{x_0})_*} & \pi_1(Y, y_0) \\ \downarrow \hat{\alpha} & & \downarrow \hat{\beta} \\ \pi_1(X, x_1) & \xrightarrow{(h_{x_1})_*} & \pi_1(Y, y_1). \end{array}$$

Proof.

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PROBLEM 9.5 (MUNKRES §55, EX. 1)

Show that if A is a retract of B^2 , then every continuous map $f: A \rightarrow A$ has a fixed point.

Proof.



PROBLEM 9.6 (MUNKRES §55, EX. 2)

Show that if $h: S^1 \rightarrow S^1$ is nullhomotopic, then h has a fixed point and h maps some point x to its antipode $-x$.

Proof.



PROBLEM 9.7 ((A))

Prove that every m -manifold is locally path-connected.

Proof.

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PROBLEM 9.8 ((B))

Prove that every m -manifold is regular.

Proof.



PROBLEM 9.9 ((C))

Prove that there is no 1-1 continuous function $\iota: S^1 \rightarrow \mathbb{R}$. You may assume any fact about trigonometric functions. (Note: this shows in particular that there is no $\iota: S^1 \rightarrow \mathbb{R}$ with $p \circ \iota$ equal to the identity map, where p is the map in the note on the Fundamental Group of the Circle.)

Proof.

