MA 265 **Name (Print):** _____
Summer 2016
Midterm 1
June 28, 2016
Time Limit: 1:00 hr

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	25	
2	10	
3	10	
4	10	
5	10	
6	25	
Total:	90	

Do not write in the table to the right.

1. The following are **T**rue/**F**alse questions. You do not have to justify your answers. The symbols A, B and C represent $m \times n$ matrices and $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{b} = (b_1, \ldots, b_m)$ be $1 \times n$ and $1 \times m$ vectors, respectively.

   (a) (5 points) ___ If A and B are invertible $n \times n$ matrices then AB is invertible.

   (b) (5 points) ___ Suppose A and B are $n \times n$ matrices such that $AB = I$ then $BA = I$.

   (c) (5 points) ___ Suppose $AB = BA$ then B must equal A^{-1}.

   (d) (5 points) ___ Suppose $AB = AC$ then B must equal C.

   (e) (5 points) ___ Suppose $AA^{\mathrm{T}} = I$ then $\det(A) = \pm 1$.

2. (10 points) Suppose A is a skew-symmetric matrix 3×3 matrix , i.e., $A^{\mathrm{T}} = -A$. Which of the following is **not** true:

   A. $\begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & -4 \\ 1 & 4 & 0 \end{bmatrix}$ is skew-symmetric

   B. $\det(A^{\mathrm{T}}) = \pm 1$

   C. $\det(A) = 0$

   D. All of the above are true

3. (10 points) Suppose A is an idempotent matrix, i.e., $A^2 = A$. Which of the following is true:

   A. A must be the identity matrix I
   B. $(A^{\mathrm{T}})^2 = A$
   C. $\det(A) = 0$
   D. All of the above are true
   E. None of the above are true

4. (10 points) Consider the matrix

$$A := \begin{bmatrix} 0 & 0 & 2 & 0 \\ 2 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}.$$

Which of the following statements is true:

A. The columns are linearly independent

B. The matrix is not invertible

C. The matrix has determinant -2

D. None of the above

5. (10 points) Consider the following system of equations

$$\begin{aligned} x_1 + x_3 &= 5 \\ x_1 - x_2 - x_3 &= 6 \\ x_2 + x_3 &= 7. \end{aligned}$$

The above system of linear equations is:

A. Inconsistent

B. Consistent with a unique solution

C. Consistent with infinitely many solutions

D. None of the above

6. Set

$$A := \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad B := \begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}.$$

(a) (5 points) Compute A^{-1}.

(b) (5 points) Compute ABA^{-1}.

(c) (5 points) Compute $\det(ABA^{-1})$.

(d) (10 points) What is the relationship between $\det(B)$ and $\det(ABA^{-1})$ and why?