

MA571 Problem Set 6

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PROBLEM 6.1 (MUNKRES §25, EX. 8)

Let $p: X \rightarrow Y$ be a quotient map. Show that if X is locally connected, then Y is locally connected. [Hint: If C is a component of the open set U of Y , show that $p^{-1}(C)$ is a union of components of $p^{-1}(U)$.]

Proof.

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PROBLEM 6.2 (MUNKRES §25, EX. 10(A,B))

Let X be a space. Let us define $x \sim y$ if there is no separation $X = A \cup B$ of X into disjoint open sets such that $x \in A$ and $y \in B$.

- (a) Show this relation is an equivalence relation. The equivalence classes are called *quasicomponents* of X .
- (b) Show that each component of X lies in a quasicomponent of X , and that the components and quasicomponents of X are the same if X is locally connected.

Proof.

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PROBLEM 6.3 (MUNKRES §26, EX. 4)

Show that every compact subspace of a metric space is bounded in that metric and is closed. Find a metric space in which not every closed bounded subspace is compact.

Proof.

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PROBLEM 6.4 (MUNKRES §26, EX. 5)

Let A and B be disjoint compact subspaces of the Hausdorff space X . Show that there exists disjoint open sets U and V containing A and B , respectively.

Proof.

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PROBLEM 6.5 (MUNKRES §26, EX. 7)

Show that if Y is compact, then the projection $\pi_X: X \times Y \rightarrow X$ is a closed map.

Proof.

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PROBLEM 6.6 (A)

Let X be a compact space and let \sim be an equivalence relation on X . Suppose that the set

$$S = \{ (x, y) \mid x \sim y \}$$

is a closed subset of $X \times X$. Prove that the quotient map $q: X \rightarrow X/\sim$ is a closed map.

Proof.

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PROBLEM 6.7 (B)

Let S^2 be the sphere

$$\{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 + z^2 = 1\}.$$

Let S_+^2 be $S^2 \cap \{z \geq 0\}$ (the upper hemisphere), let S_-^2 be $S^2 \cap \{z \leq 0\}$ (the lower hemisphere), and let E be $S^2 \cap \{z = 0\}$ (the equator). Recall the definition of Y/S from Homework #4. Prove that S^2/S_-^2 is homeomorphic to S_+^2/E . [*Hint:* There are maps in both directions.]

Proof.

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