MA 166 - SPRING 2007

QUIZZES

These are "cooperative" or "learning" quizzes. Their purpose is to try to make sure that every student either knows, or learns how to solve all the problems correctly.

The quizzes are closed book.

Each student should work on every problem and, if necessary, he or she may ask for help from the TA or other students. Students are allowed and even encouraged to talk to each other, but they should not divide up problems amongst groups.

During the quiz, the TA should play a very active role. He or she should circulate among the students and look at their work. If someone is having difficulty with a problem, the TA should give that person a hint, without actually doing the problem.

Students should be frequently encouraged to cooperate with the "learning spirit" of the quizzes. Those who simply copy solutions from others will not learn how to do the problems, and consequently will do poorly in the tests and final exam. On the other hand, students who are in a position to help others should be encouraged to do so, without doing the problems for them. It should be pointed out that according to the grading policy for the course, if all students of a particular TA get an A on the exams, they will all get an A for the course.

REVIEW EXERCISES FOR THE QUIZ. The students are expected to review the lessons covered by the quiz, do the assigned review exercises and bring their work to class. In the first 10-15 minutes of class, the TA should answer questions on the review exercises and then write the quiz problems on the blackboard.

The students should turn in the review exercises together with the quiz. The TA should glance at the review exercises to determine whether they were substantially done. If not, two points out of ten should be subtracted from the quiz grade.

MA 166 - Quiz 1 (Brief Review of MA 165)
(35 minutes)

1. Complete the following tables:

(a)

x	-1	$-\sqrt{3}/2$	$-\sqrt{2}/2$	$-1/2$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\sin^{-1} x$									

(b)

x	$-\sqrt{3}$	-1	$-\sqrt{3}/3$	0	$\sqrt{3}/3$	1	$\sqrt{3}$
$\tan^{-1} x$							

2. Find the derivatives of the following functions

(a) $f(x) = (\sin^{-1}(3x))(\tan^{-1} x^2)$

(b) $y = \dfrac{\sin(\pi x^2)}{e^{3x} - x^2}$

(c) $g(x) = e^{-2x}\cos(3x)$

(d) $f(x) = (\tan x)\ln(x^2 \sec x)$

3. Evaluate the integrals:

(a) $\displaystyle\int \frac{x^3}{1+x^4}\,dx$

(b) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

(c) $\displaystyle\int_{-1}^{1} \frac{\sin x}{1+x^2}\,dx$ (Hint: $\sin(-x) = -\sin x$)

(d) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2\theta\, d\theta$

(e) $\displaystyle\int \frac{\sec x \tan x}{1+\sec x}\,dx$

MA 166 - Quiz 2 (Lessons 1–2)
(40 minutes)

1. Show that
$$x^2 + y^2 + z^2 = 4x + 2y - 6z$$
is an equation of a sphere. Find the center and the radius of the sphere.

2. Describe in words the region in $\mathbb{R}^3$ represented by
   (a) $x = 10$
   (b) $x^2 + y^2 + z^2 > 1$
   (c) $x = y$

3. Let $\vec{a} = 2\,\vec{\imath} + 2\,\vec{\jmath} + \vec{k}$ and $\vec{b} = \vec{\imath} + 2\,\vec{\jmath} - 3\,\vec{k}$. Find the following:
   (a) $2\,\vec{a} - \vec{b}$
   (b) $|\vec{a}|$
   (c) $|\vec{b}|$
   (d) a unit vector in the direction of $\vec{a}$
   (e) a vector of length 10 whose direction is opposite to that of $\vec{a}$.

4. Consider the three points $P(2, 1, 1)$, $Q(3, 3, -1)$, and $R(1, 2, 0)$.
   (a) Write $\overrightarrow{RP}$ and $\overrightarrow{RQ}$ in the form $a\,\vec{\imath} + b\,\vec{\jmath} + c\,\vec{k}$.
   (b) Plot the points P, Q, and R and sketch the triangle PQR.

MA 166 - Quiz 3 (Lessons 3–5)
(40 minutes)

1. Let $\vec{a} = 2\vec{\imath} + 2\vec{\jmath} + \vec{k}$ and $\vec{b} = \vec{\imath} + 2\vec{\jmath} - 3\vec{k}$. Find the following:
   (a) $\vec{a} \cdot \vec{b}$.
   (b) The cosine of the angle between $\vec{a}$ and $\vec{b}$.
   (c) The scalar projection of $\vec{b}$ onto $\vec{a}$: $\text{comp}_{\vec{a}}\vec{b}$.
   (d) The vector projection of $\vec{b}$ onto $\vec{a}$: $\text{proj}_{\vec{a}}\vec{b}$.
   (e) $\vec{a} \times \vec{b}$.
   (f) A vector orthogonal to both $\vec{a}$ and $\vec{b}$.
   (g) A unit vector $\vec{u}$ orthogonal to both $\vec{a}$ and $\vec{b}$ and having negative $\vec{k}$ component.

2. Consider the three points $P(1, 0, 2)$, $Q(2, 4, -3)$, and $R(1, 2, 1)$.
   (a) Find a vector orthogonal to the plane through these points.
   (b) Find the area of the triangle PQR.

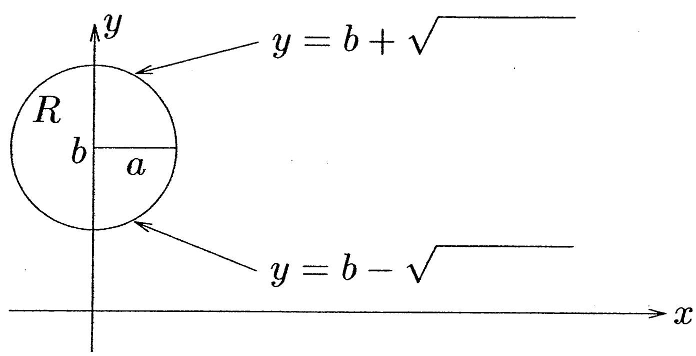
3. Sketch the region enclosed by the curves
   $$y = x + 1 \text{ and } y = 3 - x^2,$$
   and set up a definite integral that gives the area of the region. Do not evaluate the integral.

4. Sketch the region enclosed by the curves
   $$y = \frac{1}{x},\ x = 0,\ y = 1,\ y = 2$$
   and find the area of the region.

MA 166 - Quiz 4 (Lessons 6–8)
(40 minutes)

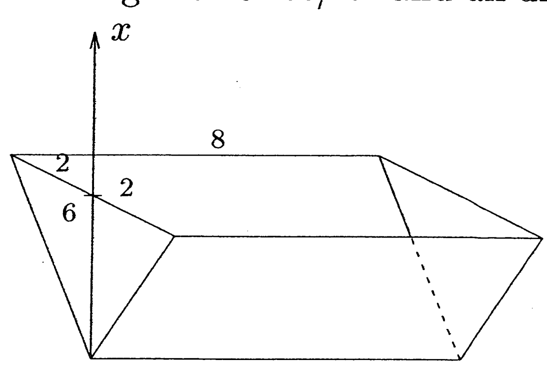
1. Find the volume V of the solid obtained by rotating the region enclosed by the curves $y = x^2$ and $y = x^3$ about the x-axis.

2. Let R be the region inside the circle
$$x^2 + (y-b)^2 = a^2 \qquad (a < b)$$
as shown in the figure below. Derive the following formula for the volume of the solid (torus) obtained by rotating R about the x-axis.
$V = 4\pi b \int_{-a}^{a} \sqrt{a^2 - x^2}\, dx.$



3. The region inside the circle $x^2 + y^2 = 1$ and to the right of the line $x = \dfrac{1}{2}$ is rotated about the y-axis. Use the method of shells to find the volume of the resulting solid.

4. The tank pictured below is full of water. Set up an integral which gives the work required to pump all the water over the top. Do not evaluate the integral. (Water weighs 62.5 lbs/ft^3 and all dimensions are in feet).



MA 166 - Quiz 5 (Lessons 10–12)
(40 minutes)

Evaluate the integrals

1. $\displaystyle\int \sin^4 x \cos^3 x \, dx$

2. $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

3. $\displaystyle\int \tan^3 x \sec x \, dx$

4. $\displaystyle\int \frac{x^2}{\sqrt{4 - x^2}} \, dx$

5. $\displaystyle\int_0^1 \frac{1}{(x^2 + 1)^{3/2}} \, dx$

MA 166 - Quiz 6 (Lessons 13–15)
(40 minutes)

1. $\displaystyle\int \frac{x^2}{x^2-1}\,dx$

2. $\displaystyle\int \frac{x}{x^2+6x+9}\,dx$

3. $\displaystyle\int \frac{1}{x^3+x}\,dx$

4. $\displaystyle\int \frac{e^x}{1-e^{2x}}\,dx$ [Hint: First use a substitution and then partial fractions].

MA 166 - Quiz 7 (lessons 16–18)
(40 minutes)

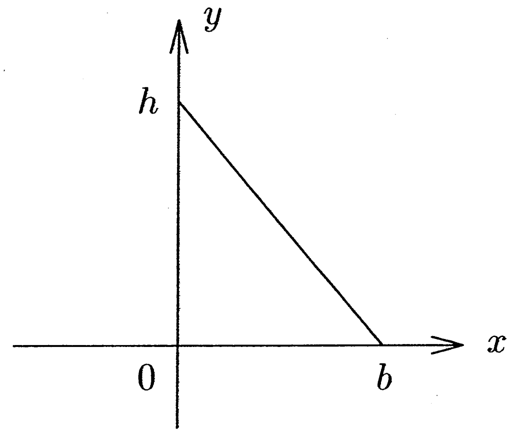
1. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

   (a) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$ (b) $\displaystyle\int_1^5 \frac{1}{(5-x)^2}\,dx$ (c) $\displaystyle\int_0^\infty xe^{-x^2}\,dx$

2. Find the length of the curve

   $$y = \ln(x^2 - 1), \quad 2 \le x \le 5$$

3. Consider the triangular lamina with density $\rho = 1$ shown below.



   Use calculus to find the following:

   (a) The moment M_y of the lamina about the y-axis.

   (b) The moment M_x of the lamina about the x-axis.

   (c) The center of mass $(\bar{x}, \bar{y})$ of the lamina.

MA 166 - Quiz 8 (Lessons 20–22)
(40 minutes)

1. Determine whether the series is convergent or divergent. If it is convergent, find its sum. You must justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{3^{n+1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^{n-1}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{7}{4^{n+2}}$

2. Determine whether the series is convergent or divergent. You must state clearly what test you are using and verify that the conditions of the test are satisfied.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)^{3/2}}$ (d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-3}}$

MA 166 - Quiz 9 (Lessons 23–25)
(40 minutes)

1. Determine whether the series is convergent or divergent. You must state clearly what test you are using and verify that the conditions of the test are satisfied

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^3 - n - 4}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{3n^2+5}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n}}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{3n+1}$

2. Find the smallest number of terms that we need to add in order to find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$ with an error < 0.01.

3. Determine whether the series is absolutely convergent, conditionally convergent, or divergent. You must justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{3^n}$

MA 166 - Quiz 10 (Lessons 26–28)
(35 minutes)

1. Find the radius of convergence and the interval of convergence of the power series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\,5^n}\, x^n$ (b) $\displaystyle\sum_{n=1}^{\infty} n(x-2)^n$

2. Find a power series representation for each of the functions. Also write down the first four nonzero terms and give the interval of convergence. Example:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots, \qquad -1 < x < 1$$

   (a) $\dfrac{1}{1+x}$ (b) $\dfrac{1}{3-x}$

3. Use a power series to approximate the value of the definite integral

$$\int_0^{0.1} \frac{1}{1+x^3}\, dx$$

with an error $< 10^{-7}$. (Leave your answer as a sum of terms).

MA 166 - Quiz 11 (Lessons 30–32)
(40 minutes)

1. Write down the Maclaurin series for each of the functions. Also write down the first four nonzero terms and give the interval of convergence. Example:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, -\infty < x < \infty$$

(a) $\dfrac{1}{1-x}$, (b) e^{-3x} (c) $\dfrac{1}{1+x^2}$, (d) $\sin x$, (e) $\cos x$,

(f) $\sin 2x$.

2. Sketch the curve C with the given parametric equations, and indicate with an arrow the direction in which the curve is traced as t increases. Also, find the Cartesian equation of the curve and the intervals for x and y.

(a) $x = 2\cos t$, $y = 2\sin t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

(b) $x = \sqrt{4-t^2}$, $y = t$, $-2 \le t \le 2$

(c) $x = 3t$, $y = t$, $1 \le t \le 2$

3. Find the length L of the curve

$$x = \frac{1}{3}\cos^3 t, \quad y = \frac{1}{2}\sin^2 t, \quad \text{for } 0 \le t \le \frac{\pi}{2}.$$

4. For the curve in 2(a), find $\dfrac{dy}{dx}$ at the point corresponding to $t = -\dfrac{\pi}{4}$.

MA 166 - Quiz 12 (Lessons 33–35)
(25 minutes)

DO NOT COLLECT. DISCUSS SOLUTIONS DURING THE LAST 10 MINUTES OF CLASS.

1. Sketch the curve with the polar equation:

   (a) $r = \cos\theta$ (b) $r = 1 + \sin\theta$

   (c) $r = \cos(3\theta)$; $-\dfrac{\pi}{6} \le \theta \le \dfrac{\pi}{6}$

2. Write the following complex numbers in the form $a + bi$:

   (a) $\dfrac{2+3i}{1-5i}$ (b) $e^{1+i\frac{3\pi}{4}}$

MA 166 - Quiz 1 - solutions

1. (a)

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\sin^{-1}x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

x	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\tan^{-1}x$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$

2. (a) $f'(x) = \left(\sin^{-1}(3x)\right)\frac{1}{1+x^4}2x + \frac{1}{\sqrt{1-9x^2}}\cdot 3\left(\tan^{-1}x^2\right)$

(b) $\dfrac{dy}{dx} = \dfrac{(e^{3x}-x^2)\cos(\pi x^2)(2\pi x) - \sin(\pi x^2)\left(3e^{3x}-2x\right)}{(e^{3x}-x^2)^2}$

(c) $g'(x) = e^{-2x}\left(-\sin(3x)\right)3 - 2e^{-2x}\cos(3x)$

(d) $f'(x) = (\tan x)\dfrac{1}{x^2\sec x}\left(x^2\sec x\tan x + 2x\sec x\right)$

$+ (\sec^2 x)\ln\left(x^2\sec x\right)$

~~3.~~ ~~(a)~~ $\cancel{\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+C}$

~~(b)~~ $\cancel{\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C}$

3. (a) $\displaystyle\int \frac{x}{1+x^4}dx = \frac{1}{2}\int \frac{1}{1+u^2}du = \frac{1}{2}\tan^{-1}u + C$

$u = x^2,\ du = 2x\,dx$ $\qquad = \frac{1}{2}\tan^{-1}(x^2) + C$

(b) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int e^u du = 2e^u + C = 2e^{\sqrt{x}} + C$

$u = \sqrt{x}\quad du = \frac{1}{2\sqrt{x}}dx$

MA166 Quiz 1 solutions (continued)

3 (a) $\displaystyle\int \frac{x^3}{1+x^4}\,dx = \frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln|u| + C = \frac{1}{4}\ln|1+x^4| + C$

$u = 1+x^4$, $du = 4x^3\,dx$

(b) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C$

$u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}}\,dx$

(c) $f(x) = \dfrac{\sin x}{1+x^2}$, $f(-x) = -\dfrac{\sin x}{1+x^2} = -f(x)$

$\therefore$ f is odd and hence $\int_{-a}^{a} f(x)\,dx = 0$

$$\int_{-1}^{1} \frac{\sin x}{1+x^2}\,dx = 0$$

(d) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2\theta\,d\theta = \tan\theta\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \tan\frac{\pi}{3} - \tan\frac{\pi}{6} = \sqrt{3} - \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$

(e) $\displaystyle\int \frac{\sec x \tan x}{1+\sec x}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln|1+\sec x| + C$

$u = 1+\sec x$, $du = \sec x \tan x\,dx$

MA 166 Quiz 2 solutions

1. $x^2 - 4x + 4 + y^2 - 2y + 1 + z^2 + 6z + 9 = 4 + 1 + 9$

$(x-2)^2 + (y-1)^2 + (z+3)^2 = 14$

This is an equation of the sphere with center $(2,1,-3)$ and radius $\sqrt{14}$

2. (a) Plane perpendicular to the x-axis at $x = 10$.

(b) Region exterior to the sphere centered at the origin and radius 1.

(c) Plane containing the z-axis and intersecting the (x,y)-plane along the line $y = x$

3. (a) $2\vec{a} - \vec{b} = 2(2\vec{i} + 2\vec{j} + \vec{k}) - (\vec{i} + 2\vec{j} - 3\vec{k}) = 3\vec{i} + 2\vec{j} + 5\vec{k}$

(b) $|\vec{a}| = \sqrt{4+4+1} = \sqrt{9} = 3$

(c) $|\vec{b}| = \sqrt{1+4+9} = \sqrt{14}$

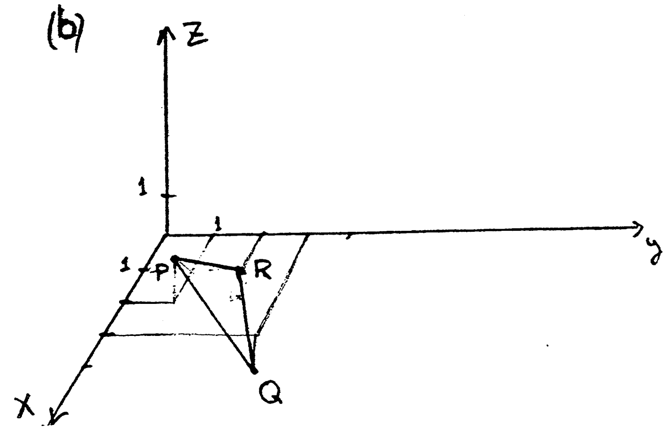
(d) $\dfrac{\vec{a}}{|\vec{a}|} = \frac{1}{3}(2\vec{i} + 2\vec{j} + \vec{k}) = \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{1}{3}\vec{k}$

(e) $-(10)\dfrac{\vec{a}}{|\vec{a}|} = -\frac{20}{3}\vec{i} - \frac{20}{3}\vec{j} - \frac{10}{3}\vec{k}.$

4. (a) $\overrightarrow{RP} = (2-1)\vec{i} + (1-2)\vec{j} + (1-0)\vec{k} = \vec{i} - \vec{j} + \vec{k}$

$\overrightarrow{RQ} = (3-1)\vec{i} + (3-2)\vec{j} + (-1-0)\vec{k} = 2\vec{i} + \vec{j} - \vec{k}$

(b)



MA 166 - Quiz 3 - solutions

(a) (b) - see bottom of next page

1\. (c) $\text{comp}_{\vec{a}} \vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \dfrac{2\cdot 1 + 2\cdot 2 + 1(-3)}{\sqrt{2^2+2^2+1}} = 1$

(d) $\text{proj}_{\vec{a}} \vec{b} = \left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\dfrac{\vec{a}}{|\vec{a}|} = 1\ \dfrac{2\vec{i}+2\vec{j}+\vec{k}}{\sqrt{2^2+2^2+1}} = \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{1}{3}\vec{k}$

(e)
$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 1 \\ 1 & 2 & -3 \end{vmatrix} = \vec{i}\big(2(-3) - 2\cdot 1\big) - \vec{j}\big(2(-3) - 1\cdot 1\big) + \vec{k}\big(2\cdot 2 - 2\cdot 1\big)$$
$$= -8\vec{i} + 7\vec{j} + 2\vec{k}$$

(f) $\vec{a}\times\vec{b} = -8\vec{i} + 7\vec{j} + 2\vec{k}$

(g) $|\vec{a}\times\vec{b}| = \sqrt{64+49+4} = \sqrt{117}$

$\vec{u} = -\dfrac{1}{\sqrt{117}}\left(-8\vec{i} + 7\vec{j} + 2\vec{k}\right) = \dfrac{8}{\sqrt{117}}\vec{i} - \dfrac{7}{\sqrt{117}}\vec{j} - \dfrac{2}{\sqrt{117}}\vec{k}$

2\. (a) $\vec{PQ} = \vec{i} + 4\vec{j} - 5\vec{k}$ $\quad \vec{PR} = 2j - \vec{k}$

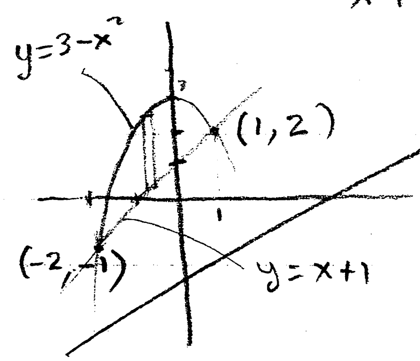
$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & -5 \\ 0 & 2 & -1 \end{vmatrix} = 6\vec{i} + \vec{j} + 2\vec{k}$$

(b) Area of $PQR = \frac{1}{2}|\vec{PQ}\times\vec{PR}| = \frac{1}{2}\sqrt{41}$.

3\. Solve the equations simultaneously to find the points of intersection of the curves.

$x+1 = 3-x^2 \rightarrow x^2+x-2=0 \rightarrow (x+2)(x-1)=0 \rightarrow x=-2,1$

pts of int: $(-2,-1)$, $(1,2)$



$\Delta A = \left[(3-x^2) - (x+1)\right]\Delta x$

$A = \int_{-2}^{1}\left[(3-x^2)-(x+1)\right]dx = \int_{-2}^{1}(2-x^2-x)\,dx$

$= \left(2x - \frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_{-2}^{1} = \left(2-\frac{1}{3}-\frac{1}{2}\right) - \left(-4+\frac{8}{3}-2\right)$

$= 8 - \frac{9}{3} - \frac{1}{2} = \frac{9}{2}$

MA166 - Quiz 3 - solutions (continued)

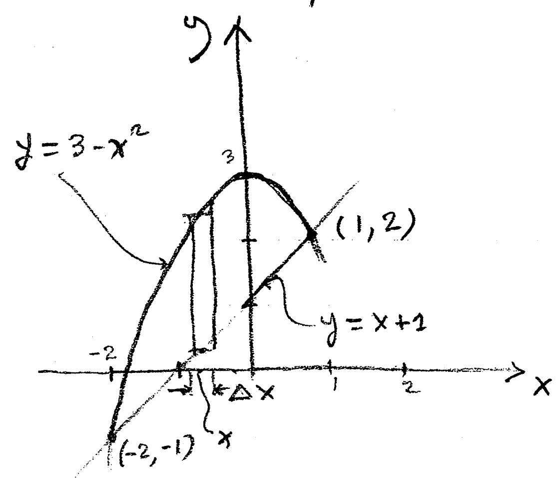
3. We first find the points of intersection of the curves by solving their equations simultaneously.

$y = x+1$, $y = 3-x^2$ $\Rightarrow$ $x+1 = 3-x^2$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0 \longrightarrow x = -2, 1$$

Points of intersection: $(-2,-1)$, $(1,2)$. Sketch the region.



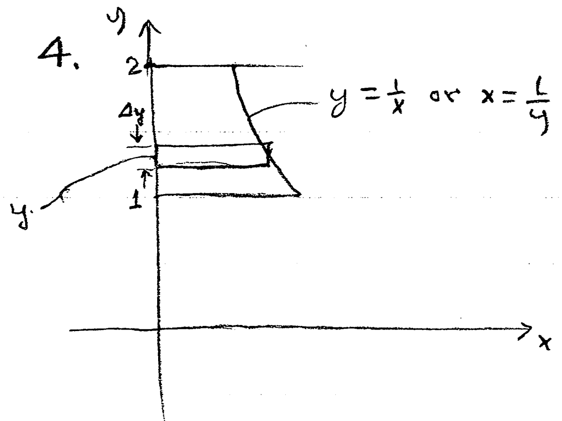
Area of typical rectangle:

$$\Delta A = \left[(3-x^2)-(x+1)\right]\Delta x$$

$$A = \int_{-2}^{1} \left[(3-x^2) - (x+1)\right] dx$$

$$= \int_{-2}^{1} (2 - x^2 - x)\, dx.$$

4.



Area of typical rectangle

$$\Delta A = \left[\frac{1}{y} - 0\right]\Delta y$$

$$A = \int_1^2 \left(\frac{1}{y} - 0\right) dy$$

$$= \ln y \Big|_1^2 = \ln 2.$$

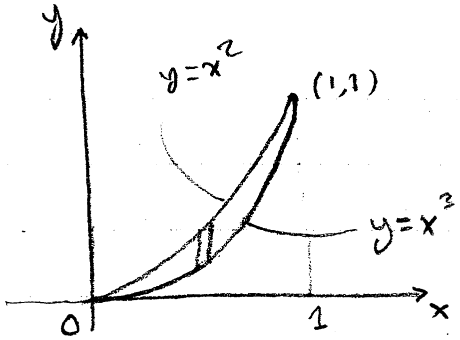
1 (a) $\bar{a} \cdot \bar{b} = 2\cdot 1 + 2\cdot 2 + 1(-3) = 3$

(b) $\bar{a}\cdot\bar{b} = |\bar{a}|\,|\bar{b}| \cos\theta$. $|\bar{a}| = \sqrt{2^2+2^2+1^2} = 3$, $|\bar{b}| = \sqrt{1+4+9} = \sqrt{14}$

$$\cos\theta = \frac{3}{3\cdot\sqrt{14}} = \frac{1}{\sqrt{14}}.$$

MA 166 - Quiz 4 - solutions

1. Points of intersection $x^2 = x^3 \longrightarrow x=0,\ x=1$.



Volume of typical washer

$$\Delta V = \left[\pi (x^2)^2 - \pi (x^3)^2\right]\Delta x$$

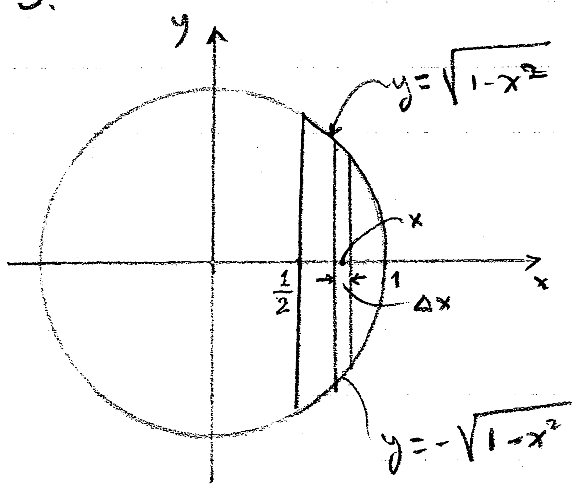
$$V = \int_0^1 \left(\pi x^4 - \pi x^6\right) dx$$

$$= \pi\left(\frac{x^5}{5} - \frac{x^7}{7}\right)\Big|_0^1 = \frac{2\pi}{35}$$

2. $$V = \int_{-a}^{a} \pi\left(b+\sqrt{a^2-x^2}\right)^2 - \pi\left(b-\sqrt{a^2-x^2}\right)^2\Big] dx$$

$$= \pi\int_{-a}^{a} 2b\, 2\sqrt{a^2-x^2}\, dx = 4\pi b \int_{-a}^{a}\sqrt{a^2-x^2}\, dx$$

3.



Volume of typical shell

$$\Delta V = 2\pi x\left[\sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right)\right]\Delta x$$

$$V = \int_{\frac{1}{2}}^{1} 2\pi x\, 2\sqrt{1-x^2}\, dx$$

$$= 4\pi\int_{\frac{1}{2}}^{1} x\sqrt{1-x^2}\, dx =$$

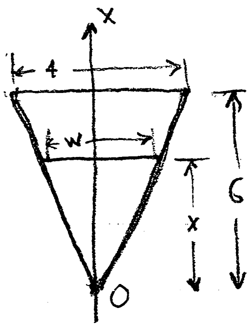
$u = 1-x^2 \quad du = -2x\,dx$

$x = \frac{1}{2} \longrightarrow u = \frac{3}{4}\ ;\ x=1 \rightarrow u=0$

$$= 4\pi\left(-\frac{1}{2}\right)\int_{3/4}^{0}\sqrt{u}\, du = -2\pi\,\frac{u^{3/2}}{\frac{3}{2}}\Big|_{3/4}^{0}$$

$$= \frac{4\pi}{3}\left(\frac{3}{4}\right)^{3/2} = \frac{\pi\sqrt{3}}{2}$$

MA166 - Quiz 4 - solutions (continued)



$$\frac{w}{4} = \frac{x}{6} \rightarrow w = \frac{2}{3}x$$

The weight of a typical layer of water of thickness Δx located at x is

$$62.5\left(\frac{2}{3}x \cdot 8\right)\Delta x$$

and it must be lifted $6-x$ ft.

Thus, the work to pump this typical layer to the top is

$$\Delta W = (6-x)\,62.5\left(\frac{2}{3}x \cdot 8\right)\Delta x$$

and the work to pump all the water over the top is

$$W = \int_0^6 62.5(6-x)\frac{16x}{3}\,dx$$

MA 166 - Quiz 5 - solutions

1. $\int \sin^4 x \cos^3 x \, dx = \int \sin^4 x (1-\sin^2 x)\cos x \, dx$

$u = \sin x \quad du = \cos x \, dx$

$= \int (u^4 - u^6)\, du = \frac{u^5}{5} - \frac{u^7}{7} + C = \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C.$

2. $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2x)\, dx$

$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right)\Big|_0^{\frac{\pi}{2}} = \frac{1}{2}\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right) - \frac{1}{2}\left(0 + \frac{1}{2}\sin 0\right) = \frac{\pi}{4}$

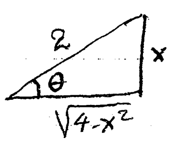
3. $\int \tan^3 x \sec x \, dx = \int \tan^2 x (\sec x \tan x)\, dx$

$= \int (\sec^2 x - 1)\sec x \tan x \, dx = \int (u^2 - 1)\, du$

$u = \sec x \quad du = \sec x \tan x \, dx$

$= \frac{u^3}{3} - u + C = \frac{1}{3}\sec^3 x - \sec x + C$

4 $\int \frac{x^2}{\sqrt{4-x^2}}\, dx$



Set $x = 2\sin\theta \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, \quad \left(\Leftrightarrow \theta = \sin^{-1}\frac{x}{2}\right)$

$dx = 2\cos\theta \, d\theta$

$\sqrt{4-x^2} = \sqrt{4 - 4\sin^2\theta} = 2|\cos\theta| = 2\cos\theta$

(or from triangle)

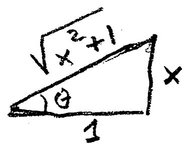
$\therefore \int \frac{x^2}{\sqrt{4-x^2}}\, dx = \int \frac{4\sin^2\theta}{2\cos\theta}\, 2\cos\theta \, d\theta = 4\int \sin^2\theta \, d\theta$

$= 2\int (1 - \cos 2\theta)\, d\theta = 2\theta - \sin 2\theta + C$

$= 2\theta - 2\sin\theta\cos\theta + C = 2\sin^{-1}\frac{x}{2} - 2\,\frac{x}{2}\,\frac{\sqrt{4-x^2}}{2} + C$

MA166 - Quiz 5 - solutions (continued)

5. $\displaystyle\int_0^1 \frac{1}{(x^2+1)^{3/2}}\,dx$



Set $x = \tan\theta \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$dx = \sec^2\theta \, d\theta$

$\sqrt{x^2+1} = \sec\theta$

$x = 0 \to \theta = 0 \,, \quad x = 1 \to \theta = \frac{\pi}{4}$

$$\int_0^1 \frac{1}{(x^2+1)^{3/2}}\,dx = \int_0^{\frac{\pi}{4}} \frac{1}{\sec^3\theta} \sec^2\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{\sec\theta}\, d\theta = \int_0^{\pi/4} \cos\theta \, d\theta = \sin\theta \Big|_0^{\pi/4} = \frac{\sqrt{2}}{2}.$$

MA 166 – Quiz 5 – solutions

1. $$\int \frac{x^2}{x^2-1}\,dx = \int\left(1+\frac{1}{x^2-1}\right)dx$$

$$= x + \int \frac{1}{x^2-1}\,dx$$

$$\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} = \frac{A}{x-1}+\frac{B}{x+1}$$

$$1 = Ax + A + Bx - B \rightarrow \left[\begin{array}{l} A+B=0 \\ A-B=1 \end{array}\right.$$

$$\rightarrow A=\tfrac{1}{2},\ B=-\tfrac{1}{2}$$

$$\frac{1}{x^2-1} = \frac{1}{2}\,\frac{1}{x-1} - \frac{1}{2}\,\frac{1}{x+1}$$

$$\int \frac{1}{x^2-1}\,dx = \frac{1}{2}\int\frac{1}{x-1}\,dx - \frac{1}{2}\int\frac{1}{x+1}\,dx = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$$

$$\therefore \int \frac{x^2}{x^2-1}\,dx = x + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C$$

2. $$\frac{x}{x^2+6x+9} = \frac{x}{(x+3)^2} = \frac{A}{x+3}+\frac{B}{(x+3)^2}$$

$$x = Ax + 3A + B \rightarrow A=1,\ B=-3$$

$$\frac{x}{x^2+6x+9} = \frac{1}{x+3} - \frac{3}{(x+3)^2}$$

$$\int \frac{x}{x^2+6x+9}\,dx = \int\frac{1}{x+3}\,dx - 3\int\frac{1}{(x+3)^2}\,dx$$

$$= \ln|x+3| + \frac{3}{x+3} + C$$

MA 166 - Quiz 5 - Solutions (cont)

3. $$\frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$1 = Ax^2 + A + Bx^2 + Cx$$

$$\left.\begin{array}{l} 0 = A+B \\ 0 = C \\ 1 = A \end{array}\right] \rightarrow A=1,\ B=-1,\ C=0$$

$$\frac{1}{x^3+x} = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\int \frac{1}{x^3+x}\,dx = \int \frac{1}{x}\,dx - \int \frac{x}{x^2+1}\,dx$$

$$= \ln|x| - \frac{1}{2}\ln(x^2+1) + C$$

4. $$\int \frac{e^x}{1-e^{2x}}\,dx = \int \frac{1}{1-u^2}\,du$$

set $u=e^x$; $du = e^x dx$

$$\frac{1}{1-u^2} = \frac{A}{1+u} + \frac{B}{1-u}$$

$$1 = A - Au + B + Bu$$

$$\left.\begin{array}{l} 0 = B-A \\ 1 = A+B \end{array}\right] \rightarrow A = \frac{1}{2},\ B = \frac{1}{2}$$

$$\frac{1}{1-u^2} = \frac{1}{2}\frac{1}{1+u} + \frac{1}{2}\frac{1}{1-u}$$

$$\int \frac{1}{1-u^2}\,du = \frac{1}{2}\int \frac{1}{1+u}\,du + \frac{1}{2}\int \frac{1}{1-u}\,du$$

$$= \frac{1}{2}\ln|1+u| - \frac{1}{2}\ln|1-u| + C$$

$$\therefore \int \frac{e^x}{1-e^{2x}}\,dx = \frac{1}{2}\ln\left|\frac{1+e^x}{1-e^x}\right| + C$$

MA166 - Quiz 7 - solutions

1(a) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{t\to 0^+}\int_t^1 \frac{1}{\sqrt{x}}\,dx = \lim_{t\to 0^+}\left[2\sqrt{x}\right]_t^1 = \lim_{t\to 0^+}(2-2\sqrt{t}) = 2.$

(b) $\displaystyle\int_1^5 \frac{1}{(5-x)^2}\,dx = \lim_{t\to 5^-}\int_1^t \frac{1}{(5-x)^2}\,dx = \lim_{t\to 5^-}\left[\frac{1}{5-x}\right]_1^t = \lim_{t\to 5^-}\left(\frac{1}{5-t}-\frac{1}{4}\right)=\infty$

$\therefore$ the integral diverges.

(c) $\displaystyle\int_0^\infty x e^{-x^2}\,dx = \lim_{t\to\infty}\int_0^t x e^{-x^2}\,dx = \lim_{t\to\infty}\left[-\frac{1}{2}e^{-x^2}\right]_0^t =$

$\displaystyle= \lim_{t\to\infty}\left(-\frac{1}{2}e^{-t^2}+\frac{1}{2}\right) = \frac{1}{2}$

2. $\displaystyle L = \int_2^5 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_2^5 \sqrt{1+\left(\frac{2x}{x^2-1}\right)^2}\,dx$

$\displaystyle= \int_2^5 \sqrt{\frac{x^4-2x^2+1+4x^2}{(x^2-1)^2}}\,dx = \int_2^5 \sqrt{\left(\frac{x^2+1}{x^2-1}\right)^2}\,dx$

$\displaystyle= \int_2^5 \frac{x^2+1}{x^2-1}\,dx = \int_2^5 \left(1+\frac{2}{x^2-1}\right)dx$

$\displaystyle\frac{2}{x^2-1} = \frac{2}{(x-1)(x+1)} = \frac{A}{x-1}+\frac{B}{x+1}$ Partial fractions

$2 = Ax + A + Bx - B$

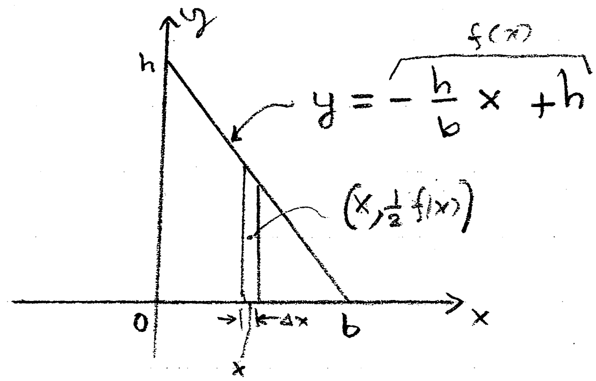
$\left.\begin{array}{l} A+B=0 \\ A-B=2 \end{array}\right] \to A=1,\ B=-1$

$\displaystyle\therefore\ L = \int_2^5 1 + \frac{1}{x-1} - \frac{1}{x+1})\,dx = \left[x + \ln\frac{x-1}{x+1}\right]_2^5$

$\displaystyle= 5 + \ln\frac{4}{6} - 2 - \ln\frac{1}{3} = 3 + \ln\frac{\frac{2}{3}}{\frac{1}{3}} = 3 + \ln 2$

MA166 - Quiz 7 - solutions (continued)

3.



(a) $\Delta M_y = x\, f(x)\, \Delta x$

$$M_y = \int_0^b x\left(-\frac{h}{b}x + h\right)dx = \left[-\frac{h}{b}\frac{x^3}{3} + h\frac{x^2}{2}\right]_0^b = -\frac{hb^3}{b3} + \frac{hb^2}{2} = \frac{hb^2}{6}$$

(b) $\Delta M_x = \frac{1}{2} f(x) \left(f(x)\,\Delta x\right)$

$$M_x = \int_0^b \frac{1}{2}\left(-\frac{h}{b}x + h\right)^2 dx$$

$$= \frac{1}{2}\left(\frac{h^2}{b^2}\frac{x^3}{3} - 2\frac{h^2}{b}\frac{x^2}{2} + h^2 x\right)\Big|_0^b$$

$$= \frac{1}{2}\left(\frac{1}{3}h^2 b - h^2 b + h^2 b\right) = \frac{h^2 b}{6}$$

(c) $A = \frac{hb}{2}$ $\qquad M_y = \bar{x} A \qquad \bar{x} = \dfrac{\frac{hb^2}{6}}{\frac{hb}{2}} = \dfrac{b}{3}$

$M_x = \bar{y} A \qquad \bar{y} = \dfrac{\frac{h^2 b}{6}}{\frac{hb}{2}} = \dfrac{h}{3}$

$$(\bar{x}, \bar{y}) = \left(\frac{b}{3}, \frac{h}{3}\right)$$

MA 166 - Quiz 8 - solutions

1(a) $a_n = \frac{n}{n+1}$ $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$

$\therefore$ Series diverges by the Test for Divergence

(b) $\sum_{n=1}^{\infty} \frac{5^n}{3^{n+1}} = \frac{5}{3^2} + \frac{5^2}{3^3} + \frac{5^3}{3^4} + \cdots = \frac{5}{3^2}\left[1 + \frac{5}{3} + \left(\frac{5}{3}\right)^2 + \cdots\right]$

Geometric series with $r = \frac{5}{3} > 1$ $\therefore$ series diverges

(c) $\sum_{n=0}^{\infty} \left(-\frac{1}{3}\right)^n = 1 + \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)^2 + \cdots = \frac{1}{1-\left(-\frac{1}{3}\right)} = \frac{3}{4}$

Geometric series with $r = -\frac{1}{3}$ and $|r| = \frac{1}{3} < 1$.

(d) $\sum_{n=1}^{\infty} \frac{7}{4^{n+2}} = \frac{7}{4^3} + \frac{7}{4^4} + \frac{7}{4^5} + \cdots = \frac{7}{4^3}\left[1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \cdots\right]$

$$= \frac{7}{4^3} \cdot \frac{1}{1-\frac{1}{4}} = \frac{7}{48}$$

Geometric series with $r = \frac{1}{4}$, $|r| = \frac{1}{4} < 1$.

2 (a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$ is convergent because it is a p-series with $p = 1.1 > 1$

(b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$. Integral test. Let $f(x) = \frac{1}{x \ln x}$, $x \geq 2$

f is continuous, positive and decreasing and

$$f(n) = \frac{1}{n(\ln n)} = a_n$$

$$\int_2^{\infty} \frac{1}{x \ln x}\,dx = \lim_{t\to\infty} \int_2^t \frac{1}{x \ln x}\,dx = \lim_{t\to\infty} \Big[\ln(\ln x)\Big]_2^t$$

$$= \lim_{t\to\infty} \Big[\ln(\ln t) - \ln(\ln 2)\Big] = \infty$$

$\therefore$ the improper integral and hence the series is divergent

MA 166 – Quiz 8 – solutions (continued)

(c) $\sum_{n=1}^{\infty} \frac{1}{(n+1)^{3/2}}$. Compare with the series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ which is convergent, because it is a p-series with $p=\frac{3}{2}>1$.

$$\frac{1}{(n+1)^{3/2}} \leq \frac{1}{n^{3/2}} \text{ for all } n \geq 1.$$

By the comparison test, $\sum_{n=1}^{\infty} \frac{1}{(n+1)^{3/2}}$ is convergent.

(d) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-3}}$. Compare with the series

$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2}} = \sum_{n=2}^{\infty} \frac{1}{n}$ which is divergent, because it is a p-series with $p=1$ (or because it is the harmonic series).

$$\frac{1}{n} = \frac{1}{\sqrt{n^2}} \leq \frac{1}{\sqrt{n^2-3}} \quad \text{for all } n \geq 2$$

By the comparison test, $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-3}}$ is divergent.

MA166 - Quiz 9 - solutions

1 (a) Compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$ and use Limit Comparison Test.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n-1}{n^3-n-4}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^3-n^2}{n^3-n-4} = 1 > 0$$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series, $p=2>1$) $\therefore \sum_{n=1}^{\infty} \frac{n-1}{n^3-n-4}$ converges.

(b) Compare with $\sum_{n=1}^{\infty} \frac{1}{3n}$ and use Limit Comparison Test,

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+1}{3n^2+5}}{\frac{1}{3n}} = \lim_{n\to\infty} \frac{3n^2+3n}{3n^2+5} = 1 > 0.$$

$\sum_{n=1}^{\infty} \frac{1}{3n} = \frac{1}{3} \sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series) $\therefore \sum_{n=1}^{\infty} \frac{n+1}{3n^2+5}$ diverges.

(c) Alternating series with $b_n = \frac{1}{\sqrt[3]{n}}$,

(i) $b_n \leq b_{n+1}$ because $\frac{1}{\sqrt[3]{n+1}} < \frac{1}{\sqrt[3]{n}}$

(ii) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\sqrt[3]{n}} = 0$

$\therefore$ series converges by the Alternating Series Test.

(d) Alternating series with $b_n = \frac{n}{3n+1}$.

but $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{3n+1} = \frac{1}{3} \neq 0$

and Alt. Ser. Test does not apply.

Try the Test for Divergence:

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^{n-1} \frac{n}{3n+1}$ does not exist

$\therefore$ series diverges.

2. $$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} = 1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \cdots$$
$$= 1 - \frac{1}{2} + \frac{1}{24} - \frac{1}{720} + \cdots$$

$\frac{1}{24} > \frac{1}{100} = 0.01$, $\frac{1}{720} < \frac{1}{100}$ 3 terms.

MA 166 - Quiz 9 - solutions (continued)

3 (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$. Abs. conv.? $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n}\right| = \sum_{n=1}^{\infty} \frac{1}{n}$ No. (harmonic series)

Convergent? Yes by alt. ser. test.

$\therefore$ series is cond. conv.

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$. Abs. conv.? $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^2+1}\right| = \sum_{n=1}^{\infty} \frac{1}{n^2+1}$ Yes

((Compare with $\sum \frac{1}{n^2}$ which is conv. p-ser. $p = 2 > 1$).

$\therefore$ series is abs. conv.

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$ Abs. conv.? Use ratio test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \frac{2^{n+1}\, n!}{2^n (n+1)!} = \frac{2}{n+1} \xrightarrow[\text{as } n\to\infty]{} 0 < 1$$

$\therefore$ series is abs. conv.

(d) $\sum_{n=1}^{\infty} \frac{(n+1)!}{3^n}$ Abs. conv.? Use ratio test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{(n+2)!}{3^{n+1}}}{\frac{(n+1)!}{3^n}} = \frac{3^n\,(n+2)!}{3^{n+1}\,(n+1)!} = \frac{n+2}{3} \xrightarrow[\text{as } n\to\infty]{} \infty.$$

$\therefore$ series is divergent.

MA166 - Quiz 10 - solutions

1 (a) Let $a_n = \frac{1}{\sqrt{n}\, 5^n} x^n$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{1}{\sqrt{n+1}\,5^{n+1}}x^{n+1}}{\frac{1}{\sqrt{n}\,5^n}x^n}\right| = \lim_{n\to\infty}\frac{1}{5}\sqrt{\frac{n}{n+1}}\,|x| = \frac{|x|}{5}$$

By the ratio test, the series converges for $\frac{|x|}{5} < 1$, or $-5 < x < 5$. Radius of convergence $R = 5$.

When $x = 5$: $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ which diverges (p-series, $p = \frac{1}{2} < 1$)

When $x = -5$: $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ which conv. by the alt. ser. test

Interval of convergence: $[-5, 5)$.

(b) Let $a_n = n(x-2)^n$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)(x-2)^{n+1}}{n(x-2)^n}\right| = \lim_{n\to\infty}\frac{n+1}{n}|x-2| = |x-2|$$

By the ratio test the series converges for $|x-2| < 1$, or $1 < x < 3$. Radius of convergence $R = 1$.

When $x = 1$: $\sum_{n=1}^{\infty}(-1)^n n$ which diverges by the test for divergence ($\lim_{n\to\infty}(-1)^n n$ does not exist)

When $x = 3$: $\sum_{n=1}^{\infty} n$ which div. by the test for div. ($\lim_{n\to\infty} n = \infty$)

Interval of convergence $(1, 3)$.

2. (a) Replace x by $-x$ in Example.

$$\frac{1}{1+x} = \sum_{n=0}^{\infty}(-x)^n = \sum_{n=0}^{\infty}(-1)^n x^n = 1 - x + x^2 - x^3 + \dots, \quad -1 < x < 1.$$

(b)
$$\frac{1}{3-x} = \frac{1}{3}\,\frac{1}{1-\frac{x}{3}} = \frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{x}{3}\right)^n \quad \left(\text{Replace } x \text{ by } \frac{x}{3} \text{ in Ex}\right).$$

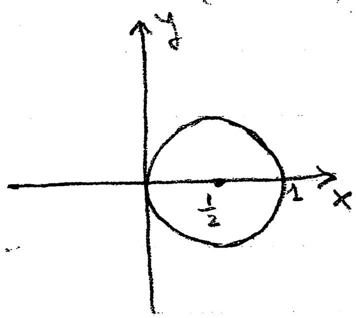
$$= \sum_{n=0}^{\infty}\frac{x^n}{3^{n+1}} = \frac{1}{3} + \frac{x}{3^2} + \frac{x^2}{3^3} + \frac{x^3}{3^4} + \dots, \quad -3 < x < 3.$$

MA 166 - Quiz 12 - solutions

1 (a) $r = \cos\theta \longrightarrow r^2 = r\cos\theta$

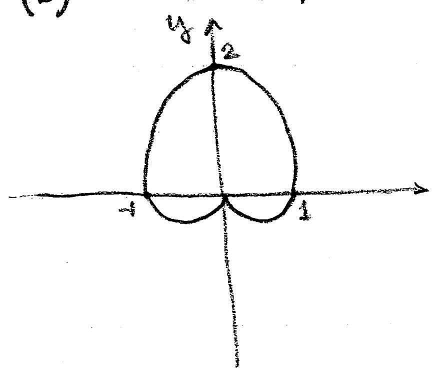
$x^2 + y^2 = x \qquad x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4}$

$(x - \frac{1}{2})^2 + y^2 = \frac{1}{4}$



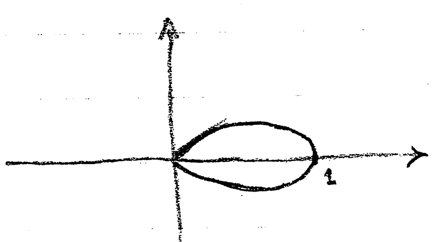
(b) $r = 1 + \sin\theta$ cardioid

θ	r
0	1
$\frac{\pi}{2}$	2
π	1
$\frac{3\pi}{2}$	0
2π	1



(c) $r = \cos(3\theta)$ leaf of a rose

θ	r
$-\frac{\pi}{6}$	0
0	1
$\frac{\pi}{6}$	0



2.(a) $\dfrac{2+3i}{1-5i} = \dfrac{2+3i}{1-5i}\,\dfrac{1+5i}{1+5i} = \dfrac{2+10i+3i-15}{1+25} = \dfrac{-13+13i}{26} = -\frac{1}{2} + \frac{1}{2}i$

(b) $e^{1+i\frac{3\pi}{4}} = e^1\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = e\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$= -e\frac{\sqrt{2}}{2} + e\frac{\sqrt{2}}{2}i$