MA 544: Homework 5

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Problem 5.1 (Wheeden & Zygmund §3, Ex. 14)

Show that the conclusion of part (ii) of Exercise 13 (Problem) is false if $|E|_e = +\infty$.

Proof.   ∎

Problem 5.2 (Wheeden & Zygmund §3, Ex. 16)

Prove (3.34).

Proof.

Lemma. $|P| = v(P)$.

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PROBLEM 5.3 (WHEEDEN & ZYGMUND §3, EX. 18)

Prove that outer measure is *translation invariant*; that is, if $E_{\mathbf{h}} := \{\, \mathbf{x} + \mathbf{h} \mid \frown \in E \,\}$ is the translate of E by $\mathbf{h}$, $\mathbf{h} \in \mathbb{R}^n$, show that $|E_{\mathbf{h}}|_e = |E|_e$. If E is measurable, show that $E_{\mathbf{h}}$ is also measurable. [This fact was used in proving (3.37).]

Proof.   ∎

Problem 5.4 (Wheeden & Zygmund §4, Ex. 1)

Prove corollary (4.2) and theorem (4.8)

Proof.   ∎

PROBLEM 5.5 (WHEEDEN & ZYGMUND §4, EX. 2)

Let f be a simple function, taking its distinct values on disjoint sets $E_1, ..., E_N$. Show that f is measurable if and only if $E_1, ..., E_N$ are measurable.

Proof.   ∎