MA 571: Homework # 8 due Friday October 23.

Please read Section 44 (but it will not be covered on any homework or exam problem). For a proof that doesn't rely on pictures, see M.H.A. Newman, Elements of the topology of plane sets of points, Section IV.9 (pages 89–92 in the Dover edition).

In Section 46, please read the definition on page 285 (note that $\mathcal{C}(X, Y)$ denotes the set of all continuous functions from X to Y) and then read from the middle of page 286 to the end of the section (but you may skip the proof of Theorem 46.11). Note that in the definition at the top of page 287 the function f should be assumed to be continuous. The top of page 288 is especially important.

Please do:

p. 288 # 6 (Do the "Hausdorff" part only, not the "regular" part.)

p. 289 # 7, 8

For the next problem we need a definition, which **replaces** the definition at the top of page 185.

Definition. If X is a locally compact Hausdorff space then the space Y given by Theorem 29.1 is called the *one-point compactification of X.*

(The difference between this and Munkres's definition is that Munkres says that a compact Hausdorff space does not have a one-point compactification; see the second paragraph on page 185).

A) Let X be a compact Hausdorff space and let W be an open subset of X (so W is locally compact by Corollary 29.3) with $W \neq X$. Prove that the one-point compactification of W is homeomorphic to the quotient space $X/(X - W)$.

B) Let X be a compact Hausdorff space, let Y be a topological space, and let $p : X \to Y$ be a closed surjective continuous map. Prove that Y is Hausdorff. (Hint: one ingredient in the proof is p. 171 # 5.) Note: combining this with HW 4 Problem E and HW 6 Problem A gives a necessary and sufficient condition for a quotient of a compact Hausdorff space to be Hausdorff.

C) Let $S^2 \subset \mathbb{R}^3$ be the subspace

$$\{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$$

Prove that S^2 is a 2-manifold. (The definition of m-manifold, where m is a positive whole number, is given at the top of page 225.)

D) Prove that the union of the x and y axes in $\mathbb{R}^2$ is not a 1-manifold.