MA 544: Homework 1

Carlos Salinas

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Problem 1.1 (Wheeden & Zygmund, Chp. 2, Ex. 1)

Let $f(x) = x\sin(1/x)$ for $0 < x \leq 1$ and $f(0) = 0$. Show that f is bounded and continuous on $[0,1]$, but that $V[f;0,1] = +\infty$.

Proof. ∎

PROBLEM 1.2 (WHEEDEN & ZYGMUND, CHP. 2, EX. 2)

Prove theorem (2.1).

Proof. Recall the statement of theorem (2.1):

Theorem (Wheeden & Zygmund, 2.1). *(a) If f is of bounded variation on $[a, b]$, then f is bounded on $[a, b]$.*

(b) Let f and g be of bounded variation on $[a, b]$. Then cf (for any real constant c), $f + g$, and fg are of bounded variation on $[a, b]$. Moreover, f/g is of bounded variation on $[a, b]$ if there exists an $\varepsilon > 0$ such that $|g(x)| \geq \varepsilon$ for $x \in [a, b]$.

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Problem 1.3 (Wheeden & Zygmund, Chp. 2, Ex. 3)

If $[a', b']$ is a subinterval of $[a, b]$ show that $P[a', b'] \leq P[a, b]$ and $N[a', b'] \leq N[a, b]$.

Proof. ∎