MA 571: Homework # 5 due Monday September 28.

$\boxed{\text{The first midterm will be Monday October 5.}}$

It will consist of three problems selected from the "Problems for the first midterm" posted on the course webpage.

Please read Sections 23, 24 and 25 (but skip Examples 1, 2 and 6 in Section 24).

Please do:

p. 152 # 3, 6, 7, 9 (for # 6, the def of Bd A is at the top of page 102)

p. 157 # 1(ac), 2

p. 162 # 2(b), 4, 6 (# 2(b) is actually easier if you don't use the hint. Also, the meaning of "bounded sequence" is the usual meaning, it's not related to the bounded metric $\bar{d}$)

A) Let X be a topological space. The quotient space $(X \times [0,1])/(X \times 0)$ (see Homework # 4 for the notation) is called the *cone* of X and denoted CX.

Prove that if X is homeomorphic to Y then CX is homeomorphic to CY. (Hint: there are maps in both directions.)

A note on convergence in the box topology on $\mathbb{R}^\omega$.

If you're interested, you might like to prove the following theorem. But this is NOT part of the assigned homework.

Notation: for positive integers i, n, I, N, let us write $(i, n) >> (I, N)$ if $i > I$ and $n > N$.

Theorem: A sequence $\mathbf{x}_i$ in $\mathbb{R}^\omega$ converges to $(0, 0, 0, \ldots)$ in the box topology if and only if two conditions hold:

i) for each n, $\lim_{i \to \infty} (\mathbf{x}_i)_n = 0$, and

ii) there is a pair (I, N) with $(\mathbf{x}_i)_n = 0$ whenever $(i, n) >> (I, N)$.