

# MA571 Problem Set 6

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**Problem 6.1 (Munkres §25, Ex. 8)**

Let  $p: X \rightarrow Y$  be a quotient map. Show that if  $X$  is locally connected, then  $Y$  is locally connected.  
[*Hint:* If  $C$  is a component of the open set  $U$  of  $Y$ , show that  $p^{-1}(C)$  is a union of components of  $p^{-1}(U)$ .]

*Proof.*

■

**Problem 6.2 (Munkres §25, Ex. 10(a,b))**

Let  $X$  be a space. Let us define  $x \sim y$  if there is no separation  $X = A \cup B$  of  $X$  into disjoint open sets such that  $x \in A$  and  $y \in B$ .

- (a) Show this relation is an equivalence relation. The equivalence classes are called *quasicomponents* of  $X$ .
- (b) Show that each component of  $X$  lies in a quasicomponent of  $X$ , and that the components and quasicomponents of  $X$  are the same if  $X$  is locally connected.

*Proof.*

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**Problem 6.3 (Munkres §26, Ex. 4)**

Show that every compact subspace of a metric space is bounded in that metric and is closed. Find a metric space in which not every closed bounded subspace is compact.

*Proof.*

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**Problem 6.4 (Munkres §26, Ex. 5)**

Let  $A$  and  $B$  be disjoint compact subspaces of the Hausdorff space  $X$ . Show that there exists disjoint open sets  $U$  and  $V$  containing  $A$  and  $B$ , respectively.

*Proof.*

■

**Problem 6.5 (Munkres §26, Ex. 7)**

Show that if  $Y$  is compact, then the projection  $\pi_X: X \times Y \rightarrow X$  is a closed map.

*Proof.*

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**Problem 6.6 (A)**

Let  $X$  be a compact space and let  $\sim$  be an equivalence relation on  $X$ . Suppose that the set

$$S = \{ (x, y) \mid x \sim y \}$$

is a closed subset of  $X \times X$ . Prove that the quotient map  $q: X \rightarrow X/\sim$  is a closed map.

*Proof.*

■



**Problem 6.7 (B)**

Let  $S^2$  be the sphere

$$\{(x, y, z) \in \mathbf{R}^3 \mid x^2 + y^2 + z^2 = 1\}.$$

Let  $S_+^2$  be  $S^2 \cap \{z \geq 0\}$  (the upper hemisphere), let  $S_-^2$  be  $S^2 \cap \{z \leq 0\}$  (the lower hemisphere), and let  $E$  be  $S^2 \cap \{z = 0\}$  (the equator). Recall the definition of  $Y/S$  from Homework #4. Prove that  $S^2/S_-^2$  is homeomorphic to  $S_+^2/E$ . [*Hint*: There are maps in both directions.]

*Proof.*

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