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Name:

MA 26500-215 Quiz 10

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1. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map that sends

$$T(1,0,0) = (3,2,4)$$

$$T(0, 1, 0) = (2, 0, 2)$$

$$T(0, 0, 1) = (4, 2, 3).$$

(a) (4 points) Find the value of T(2, 1, -1).

Solution: Since *T* is a linear map, we know that $T(\mathbf{v} + \mathbf{w}) = T(\mathbf{v}) + T(\mathbf{w})$ and $T(c\mathbf{v}) = cT(\mathbf{v})$ so

$$T(2, 1, -1) = T((2, 0, 0) + (0, 1, 0) + (0, 0, -1))$$

$$= T(2, 0, 0) + T(0, 1, 0) + T(0, 0, -1)$$

$$= 2T(1, 0, 0) + T(0, 1, 0) - T(0, 0, 1)$$

$$= 2(3, 2, 4) + (2, 0, 2) - (4, 2, 3)$$

$$= (6, 4, 8) + (2, 0, 2) + (-4, -2, -3)$$

$$= (6 + 2 - 4, 4 + 0 - 2, 8 + 2 - 3)$$

$$= (4, 2, 7).$$

(b) (6 points) Find the matrix representation of T with respect to the standard basis on \mathbb{R}^3 .

Solution: Using the standard basis on \mathbb{R}^3 which, by the way, is the set $\{(1,0,0),(0,1,0),(0,0,1)\}$, for a general vector

$$\mathbf{x} = x_1(1,0,0) + x_2(0,1,0) + x_3(0,0,1),$$

we have

$$T(\mathbf{x}) = (3x_1 + 2x_2 + 4x_3, 2x_1 + 0x_2 + 2x_3, 4x_1 + 2x_2 + 3x_3).$$

Thus,

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} A_{11}x_1 + A_{12}x_2 + A_{13}x_3 \\ A_{21}x_1 + A_{22}x_2 + A_{23}x_3 \\ A_{31}x_1 + A_{32}x_2 + A_{33}x_3 \end{bmatrix}$$
$$= \begin{bmatrix} 3x_1 + 2x_2 + 4x_3 \\ 2x_1 + 0x_2 + 2x_3 \\ 4x_1 + 2x_2 + 3x_3 \end{bmatrix}.$$

This tells us that the matrix must be

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}.$$

(c) (10 points) Using the matrix representation of *T*, find the characteristic polynomial. **You do not have to simplify it**.

Solution: To find the minimal polynomial of *T*, we find

$$\det(A - \lambda I) = \det \begin{bmatrix} 3 - \lambda & 2 & 4 \\ 2 & 0 - \lambda & 2 \\ 4 & 2 & 3 - \lambda \end{bmatrix}$$
$$= (3 - \lambda) \det \begin{bmatrix} 0 - \lambda & 2 \\ 2 & 3 - \lambda \end{bmatrix} - 2 \det \begin{bmatrix} 2 & 2 \\ 4 & 3 - \lambda \end{bmatrix}$$
$$+ 4 \det \begin{bmatrix} 2 & 0 - \lambda \\ 4 & 2 \end{bmatrix}$$
$$=$$