MA52300 Fall 2016

Homework Assignment 2

Due Fri, Sep 9, 2016

1. Verify assertion (36) in [E,§3.2.3], that when Γ is not flat near x^0, the noncharacteristic condition is
$$D_pF(p^0, z^0, x^0) \cdot \nu(x^0) \neq 0.$$
(Here $\nu(x^0)$ denotes the normal to the hypersurface Γ at x^0).

2. Show that the solution of the quasilinear PDE
$$u_t + a(u)u_x = 0$$
with initial conditions $u(x, 0) = g(x)$ is given implicitly by
$$u = g(x - a(u)t).$$
Show that the solution develops a shock (becomes singular) for some $t > 0$, unless $a(g(x))$ is a nondecreasing function of x.

3. Show that the function $u(x, t)$ defined for $t \geq 0$ by
$$u(x,t) = \begin{cases} -\frac{2}{3}(t + \sqrt{3x + t^2}) & \text{for } 4x + t^2 > 0 \\ 0 & \text{for } 4x + t^2 < 0 \end{cases}$$
is an (unbounded) entropy solution of the conservation law $u_t + (u^2/2)_x = 0$ (*inviscid Burger's equation*).