MA571 Problem Set 4

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Problem 4.1 (Munkres §20, Ex. #4(a))

Proof. ∎

Problem 4.2 (Munkres §20, Ex. #4(b))

Proof. ∎

Problem 4.3 (Munkres §20, Ex. #6)

Proof. ∎

Problem 4.4 (A)

Prove Theorem Q.2 from the notes on Quotient Spaces.

Proof. ■

Problem 4.5 (B)

Prove Proposition Q.5 from the notes on Quotient Spaces.

Proof. ■

Problem 4.6 (C)

Prove Proposition Q.5 from the notes on Quotient Spaces.

Proof. ■

Problem 4.7 (D)

(Do not use Problem E to do this problem). Let $\sim$ be the equivalence relation on the interval $[-1, 1]$ defined by $x \sim y$ if and only if $x = y$ or $x = -y$ with $y \in (-1, 1)$ (you do not have to prove that this is an equivalence relation). Prove that $[-1, 1]/\sim$ is not Hausdorff.

Proof. ∎

Problem 4.8 (E)

Let X be a topological space with an equivalence relation $\sim$. Suppose that the quotient space $X/\sim$ is Hausdorff.

Prove that the set

$$S = \{\, x \times y \in X \times X \mid x \sim y \,\}$$

is a closed subset of $X \times X$.

Proof. ∎

Problem 4.9 (F)

For problem F you need the following definition: if Y is a topological space and S is a subset of Y, we write Y/S for the quotient space $Y/\sim$, where $\sim$ is defined by $x \sim y$ if and only if $x = y$ or $\{x, y\} \subset S$. (Intuitively, Y/S is obtained from Y by collapsing S to a point.)

Let X be a topological space. Let U be an open set in X, and let A be a subset of U. Give U the subspace topology. Let $\iota\colon U/A \to X/A$ be the map which takes $[x]$ to $[x]$ (you do not have to prove that this is well-defined).

(i) Prove that ι is continuous.
(ii) Prove that ι is an open map.

Proof. ∎

Problem 4.10 (G)

Let X be a topological space satisfying the first countability axiom (see the bottom of page 130 and the top of page 131). Let $A \subset X$ and let $x \in \overline{A}$. Prove that there is a sequence in A which converges to x (see the top of page 131 for a hint).

Proof. ■