Exercises in Basic Mathematics

Carlos Salinas

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Contents

CHAPTER 1

Basic Mathematics Exercises

CHAPTER 2

Algebra Exercises

CHAPTER 3

Algebraic Geometry Exercises

3.1 Elementary Algebraic Geometry

Examples 1. Consider the equation

$$x^2 + y^2 = 1.$$

Over $\mathbf{R}$, a good picture of the solution is a circle.

Over $\mathbf{C}$, it is a 2-sphere without two points. This can be seen as follows. By stereographic projection from the North pole onto the equatorial plane, the complex plane $\mathbf{C}$ is in bijection with the sphere S^2 with the North pole N removed. The equation

$$x^2 + y^2 = 1$$

can be written as

$$(x + iy)(x - iy) = 1,$$

an by letting $w \coloneqq x + iy$ and $z \coloneqq x - iy$, we see that it is equivalent to

$$wz = 1.$$

Clearly, every $w \neq 0$ determines a unique z, and thus, the solution set is indeed $S^2 \setminus \{N, S\}$.

CHAPTER 4

Differential Geometry Exercises

4.1 The Matrix Exponential; Some Matrix Lie Groups

The Exponential Map