

# MA 519: Homework 9

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## PROBLEM 9.1 (HANDOUT 13, # 7)

Let  $X$  have a *double exponential* density  $f(x) = \frac{1}{2\sigma}e^{-\frac{|x|}{\sigma}}$ ,  $-\infty < x < \infty$ ,  $\sigma > 0$ .

- (a) Show that all moments exist for this distribution.
- (b) However, show that the MGF exists only for restricted values. Identify them and find a formula.

*SOLUTION.*

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## PROBLEM 9.2 (HANDOUT 13, # 16)

Give an example of each of the following phenomena:

- (i) A continuous random variable taking values in  $[0, 1]$  with equal mean and median.
- (ii) A continuous random variable taking values in  $[0, 1]$  with mean equal to twice the median.
- (iii) A continuous random variable for which the mean does not exist.
- (iv) A continuous random variable for which the mean exists, but the variance does not exist.
- (v) A continuous random variable with a PDF that is not differentiable at zero.
- (vi) a positive continuous random variable for which the mode is zero, but the mean does not exist.
- (vii) A continuous random variable for which all moments exist.
- (viii) A continuous random variable with median equal to zero, and 25<sup>th</sup> and 75<sup>th</sup> percentiles equal to 1.
- (ix) A continuous random variable  $X$  with mean equal to median equal to mode equal to zero, and  $E(\sin X) = 0$ .

SOLUTION. ■

**PROBLEM 9.3 (HANDOUT 13, # 17)**

An exponential random variable with mean 4 is known to be larger than 6. What is the probability that it is larger than 8?

*SOLUTION.*

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## PROBLEM 9.4 (HANDOUT 13, # 18)

(Sum of Gammas). Suppose  $X, Y$  are independent random variables, and  $X \sim G(\alpha, \lambda)$ ,  $Y \sim G(\beta, \lambda)$ . Find the distribution of  $X + Y$  by using moment-generating functions.

SOLUTION. ■

## PROBLEM 9.5 (HANDOUT 13, # 19)

(*Product of Chi Squares*). Suppose  $X_1, X_2, \dots, X_n$  are independent chi square variables, with  $X_i \sim \chi_{m_i}^2$ . Find the mean and variance of  $\prod_{i=1}^n X_i$ .

SOLUTION. ■

## PROBLEM 9.6 (HANDOUT 13, # 20)

Let  $Z \sim N(0, 1)$ . Find

$$P(0.5 < |Z - \tfrac{1}{2}| < 1.5); \quad P\left(\frac{e^Z}{1+e^Z} > \tfrac{3}{4}\right); \quad P(\Phi(Z) < 0.5).$$

SOLUTION. ■



## PROBLEM 9.7 (HANDOUT 13, # 21)

Let  $Z \sim N(0, 1)$ . Find the density of  $\frac{1}{Z}$ . Is the density bounded?

SOLUTION. ■

## PROBLEM 9.8 (HANDOUT 13, # 22)

The 25<sup>th</sup> and the 75<sup>th</sup> percentile of a normally distributed random variable are  $-1$  and  $1$ . What is the probability that the random variable is between  $-2$  and  $2$ ?

*SOLUTION.*

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