MA 519: Homework 3

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September 15, 2016

Problem 3.1 (Handout 3, # 3)

n sticks are broken into one short and one long part. The $2n$ parts are then randomly paired up to form n new sticks. Find the probability that

(a) the parts are joined in their original order, i.e., the new sticks are the same as the old sticks;
(b) each long part is paired up with a short part.

Solution. For part (a): let Ω denote the sample space and let A denote the event that two parts of the broken sticks are paired together in their original order. First, we count the number of sample points in Ω. There are $2n$ ways to chose the first half of a stick and $2n-1$ ways to chose the second. Therefore,

$$\#\Omega = 2n(2n-1).$$

Next we count the number of sample points in A. Once we make a choice from among the $2n$ stick parts, there are ■

Problem 3.2 (Handout 3, # 5)

In a town, there are 3 plumbers. On a certain day, 4 residents need a plumber and they each call one plumber at random.

Solution. ■

Problem 3.3 (Handout 4, # 7)

(Polygraphs). Polygraphs are routinely administered to job applicants for sensitive government positions. Suppose someone actually lying fails the polygraph 90% of the time. But someone telling the truth also fails the polygraph 15% of the time. If a polygraph indicates that an applicant is lying, what is the probability that he is in fact telling the truth? Assume a general prior probability p that the person is telling the truth.

Solution. ■

Problem 3.4 (Handout 4, # 8)

In a bolt factory machines A, B, C manufacture, respectively, 25, 35, and 40 per cent of the total. Of their output 5, 4, and 2 per cent are defective bolts. A bolt is drawn at random from the produce and is found defective. What are the probabilities that it was manufactured by machines A, B, C?

Solution. ∎

Problem 3.5 (Handout 4, # 9)

Suppose that 5 men out of 100 and 25 women out of 10 000 are colorblind. A colorblind person is chosen at random. What is the probability of his being male? (Assume males and females to be in equal numbers.)

Solution. ∎

Problem 3.6 (Handout 4, # 10)

Bridge. In a bridge party West has no ace. What probability should he attribute to the event of his partner having

(a) no ace,
(b) two or more aces?

Verify the result by a direct argument.

Solution. ■

Problem 3.7 (Handout 4, # 12)

A true-false question will be posed to a couple on a game show. The husband and the wife each has a probability p of picking the correct answer. Should they decide to let one of the answer the question, or decide that they will give the common answer if they agree and toss a coin to pick the answer if they disagree?

Solution. ■

Problem 3.8 (Handout 4, # 13)

An urn containing 5 balls has been filled up by taking 5 balls at random from a second urn which originally had 5 black and 5 white balls. A ball is chosen at random from the first urn and is fonud to be black. What is the probability of drawing a white ball if a second ball is chosen from among the remaining 4 balls in the first urn?

Solution. ■

Problem 3.9 (Handout 4, # 15)

Events A, B, C have probabilities p_1, p_2, p_3. Given that exactly two of the three events occured, the probability that C occured is greater than $1/2$ if and only if ... (write down the necessary and sufficient condition).

Solution. ■

Problem 3.10 (Handout 5, # 1)

There are five coins on a desk: 2 are double-headed, 2 are double-tailed, and 1 is a normal coin.

One of the coins is selected at random and tossed. It shows heads.

What is the probability that the other side of this coin is a tail?

Solution. ■

Problem 3.11 (Handout 5, # 2)

(Genetic testing). There is a 50-50 chance that the Queen carries the gene for hemophilia. If she does, then each Prince has a 50-50 chance of carrying it. Three Princesses were recently tested and found to be non-carriers. Find the following probabilities:

(a) that the Queen is a carrier;
(b) that the fourth Princess is a carrier.

Solution. $\blacksquare$

Problem 3.12 (Handout 5, # 4)

(Is Johnny in Jail). Johnny and you are roommates. You are a terrific student and spend Friday evenings drowned in books. Johnny always goes out on Friday evenings. 40% of the times, he goes out with his girlfriend, and 60% of the times he goes to a bar. If he goes out with his girlfriend, 30% of the times he is just too lazy to come back and spends the night at hers. If he goes to a bar, 40% of the times he gets mad at the person sitting on his right, beats him up, and goes to jail.

On one Saturday morning, you wake up to see Johnny is missing. Where is Johnny?

Solution. ■