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MA 26500-215 Quiz 3

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1. Let A be the following matrix
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{bmatrix}.$$
   (a) (8 points) Compute the determinant of A using any method you like.
   (b) (4 points) For what values of t is A nonsingular?

Solution: For part (a) the method of cofactors is fairly straightforward way to find the determinant in this instance. So expand along the top row

$$\begin{aligned} \det A &= 1 \det\left(\begin{bmatrix} 2 & t \\ 4 & t^2 \end{bmatrix}\right) - 1 \det\left(\begin{bmatrix} 1 & t \\ 1 & t^2 \end{bmatrix}\right) 1 \det\left(\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}\right) \\ &= (2t^2 - 4t) - (t^2 - t) + (4 - 2) \qquad (\bigstar) \\ &= t^2 - 3t + 2 \\ &= (t-2)(t-1). \end{aligned}$$

For part (b), using the result of ($\bigstar$), we see that t is nonsingular if and only if $t \neq 1, 2$.

2. Let A be the following matrix
$$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}.$$
   (a) (5 points) Find the adjoint matrix of A, $\operatorname{adj} A$.
   (b) (3 points) Find the determinant of A.

Solution: For part (a), remember that the adjoint of a matrix $\operatorname{adj} A$ is the transpose of its cofactor matrix. So we need to find the entries of the cofactor matrix

$$A_{11} = \begin{bmatrix} 6 & 2 \\ 0 & -3 \end{bmatrix} = -18, \qquad A_{12} = -\begin{bmatrix} 5 & 2 \\ 1 & -3 \end{bmatrix} = 17, \qquad A_{13} = \begin{bmatrix} 5 & 6 \\ 1 & 0 \end{bmatrix} = -6,$$
$$A_{21} = -\begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} = -6, \qquad A_{22} = \begin{bmatrix} 3 & 1 \\ 1 & -3 \end{bmatrix} = -8, \qquad A_{23} = -\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}, = 2$$
$$A_{31} = \begin{bmatrix} -2 & 1 \\ 6 & 2 \end{bmatrix} = -10, \qquad A_{32} = -\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = -1, \qquad A_{33} = \begin{bmatrix} 3 & -2 \\ 5 & 6 \end{bmatrix} = 28.$$

So

$$\operatorname{adj} A = \begin{bmatrix} -18 & 17 & -6 \\ -6 & -8 & 2 \\ -10 & -1 & 28 \end{bmatrix}^{\mathsf{T}} = \begin{bmatrix} -18 & -6 & -10 \\ 17 & -8 & -1 \\ -6 & 2 & 28 \end{bmatrix}.$$

For part (b), recall that $A \operatorname{adj} A = \det AI$. Since the identity matrix I only has 1 across the diagonal, $\det AI$ has $\det A$ across the diagonal. So all we need to do is multiply the first row of A times the first column of $\operatorname{adj} A$ to find the determinant of A

$$\det A = \begin{bmatrix} 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} -18 \\ 17 \\ -6 \end{bmatrix} = -3 \cdot 18 - 2 \cdot 17 - 6 = -94.$$