MA 571: Homework # 12 due Monday November 23.

Please do:

p. 366 # 7(c), 9(abc)

p. 375 # 2 The definition of P^2 is on page 372; you may use the fact thar P^2 is Hausdorff (proved at the bottom of page 372)

For the next two problems we will allow paths to be defined on $[a, b]$; the concept of path homotopy generalizes to this context in the obvious way.

Definition. Let M be an m-manifold.

i) A *linear* path in $\mathbb{R}^m$ is a path $f : [a, b] \to \mathbb{R}$ with $f(s) = \frac{1}{b-a}[(b - s)z_1 + (s - a)z_2]$ for two points z_1 and z_2.

ii) A *quasi-linear* path in M is a path $g : [a, b] \to M$ for which there is an open set U containing $g([a, b])$ and a homeomorphism h from U to an open set in $\mathbb{R}^m$ such that $h \circ g$ is linear.

iii) A *piecewise quasi-linear* path in M is a path $g : [a, b] \to M$ for which there is a finite partition of $[a, b]$ into subintervals such that the restriction of g to each subinterval of the partition is quasi-linear.

A) (i) Let M be an m-manifold, let U be an open set in M which is homeomorphic to an open *ball* in $\mathbb{R}^m$, and let g be a path in U. Prove that g is path-homotopic to a quasi-linear path. (Hint: straight-line homotopy).

(ii) Prove: every path in an m-manifold is path-homotopic to a piecewise quasi-linear path. (Hint: Theorem 51.3, Lebesgue Lemma and part (i))

B) Prove: a piecewise quasi-linear path in an m-manifold with $m > 1$ cannot be onto. (Hint: use Problem A from HW 2; you may *assume*, without proving it, that the image of a linear path does not contain an open set of $\mathbb{R}^m$ if $m > 1$.)

C) i) S^m is an m-manifold for all m (you don't have to prove this, it follows easily from the solution of HW8 # 3). Prove that S^m is simply connected for $m \geq 2$. Do not use Section 59. (Hint: use Problems A and B from this assignment and Problem C from HW 11.)

ii) Prove that R^n is not homeomorphic to R^2 for $n \neq 2$. (Hint: you may use Theorem A from the note on the Fundamental Group of the Circle).