

# MA571 Problem Set 4

Carlos Salinas

September 17, 2015



**Problem 4.1 (Munkres §20, Ex. #4(a))**

*Proof.*

■

**Problem 4.2 (Munkres §20, Ex. #4(b))**

*Proof.*

■

**Problem 4.3 (Munkres §20, Ex. #6)**

*Proof.*

■

**Problem 4.4 (A)**

Prove Theorem Q.2 from the notes on Quotient Spaces.

*Proof.* Recall the statement of the theorem:

**Theorem** (Theorem Q.2). *A function  $f: X/\sim \rightarrow Y$  is continuous if and only if the composite*

$$X \xrightarrow{q} X/\sim \xrightarrow{f} Y$$

*is continuous.*

■

**Problem 4.5 (B)**

Prove Proposition Q.5 from the notes on Quotient Spaces.

*Proof.* Recall the statement of the proposition:

**Proposition** (Proposition Q.5). *A map  $p: X \rightarrow Y$  satisfies Definition Q.4 if and only if it satisfies the definition at the top of page 137 in Munkres.*

■

**Problem 4.6 (C)**

Prove Proposition Q.6 from the notes on Quotient Spaces.

*Proof.* Recall the statement of the proposition:

**Proposition** (Proposition Q.6). *Let  $p: X \rightarrow Y$  be a Munkres quotient map. A function  $f: Y \rightarrow Z$  is continuous if and only if the composite*

$$X \xrightarrow{p} Y \xrightarrow{f} Z$$

*is continuous.*

■



**Problem 4.7 (D)**

(Do not use Problem E to do this problem). Let  $\sim$  be the equivalence relation on the interval  $[-1, 1]$  defined by  $x \sim y$  if and only if  $x = y$  or  $x = -y$  with  $y \in (-1, 1)$  (you do not have to prove that this is an equivalence relation). Prove that  $[-1, 1]/\sim$  is not Hausdorff.

*Proof.*

■

**Problem 4.8 (E)**

Let  $X$  be a topological space with an equivalence relation  $\sim$ . Suppose that the quotient space  $X/\sim$  is Hausdorff.

Prove that the set

$$S = \{x \times y \in X \times X \mid x \sim y\}$$

is a closed subset of  $X \times X$ .

*Proof.*

■

**Problem 4.9 (F)**

For problem F you need the following definition: if  $Y$  is a topological space and  $S$  is a subset of  $Y$ , we write  $Y/S$  for the quotient space  $Y/\sim$ , where  $\sim$  is defined by  $x \sim y$  if and only if  $x = y$  or  $\{x, y\} \subset S$ . (Intuitively,  $Y/S$  is obtained from  $Y$  by collapsing  $S$  to a point.)

Let  $X$  be a topological space. Let  $U$  be an open set in  $X$ , and let  $A$  be a subset of  $U$ . Give  $U$  the subspace topology. Let  $\iota: U/A \rightarrow X/A$  be the map which takes  $[x]$  to  $[x]$  (you do not have to prove that this is well-defined).

- (i) Prove that  $\iota$  is continuous.
- (ii) Prove that  $\iota$  is an open map.

*Proof.*

■

**Problem 4.10 (G)**

Let  $X$  be a topological space satisfying the first countability axiom (see the bottom of page 130 and the top of page 131). Let  $A \subset X$  and let  $x \in \overline{A}$ . Prove that there is a sequence in  $A$  which converges to  $x$  (see the top of page 131 for a hint).

*Proof.*

■