

MA 26500-215 Quiz 3

July 20, 2016

1. Let A be the following matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & t \\ 1 & 4 & t^2 \end{bmatrix}.$$

- (a) (8 points) Compute the determinant of A using any method you like.
(b) (4 points) For what values of t is A nonsingular?

Solution: For part (a) the method of cofactors is fairly straightforward way to find the determinant in this instance. So expand along the top row

$$\begin{aligned} \det A &= 1 \det \begin{pmatrix} 2 & t \\ 4 & t^2 \end{pmatrix} - 1 \det \begin{pmatrix} 1 & t \\ 1 & t^2 \end{pmatrix} + 1 \det \begin{pmatrix} 1 & 2 \\ 1 & 4 \end{pmatrix} \\ &= (2t^2 - 4t) - (t^2 - t) + (4 - 2) \\ &= t^2 - 3t + 2 \\ &= (t - 2)(t - 1). \end{aligned} \quad (\star)$$

For part (b), using the result of (\star) , we see that t is nonsingular if and only if $t \neq 1, 2$.

2. Let A be the following matrix

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}.$$

- (a) (5 points) Find the adjoint matrix of A , $\text{adj } A$.
(b) (3 points) Find the determinant of A .

Solution: For part (a), remember that the adjoint of a matrix $\text{adj } A$ is the transpose of its cofactor matrix. So we need to find the entries of the cofactor matrix

$$\begin{aligned} A_{11} &= \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18, & A_{12} &= -\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17, & A_{13} &= \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6, \\ A_{21} &= -\begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6, & A_{22} &= \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -8, & A_{23} &= -\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = 2, \\ A_{31} &= \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -10, & A_{32} &= -\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1, & A_{33} &= \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28. \end{aligned}$$

So

$$\text{adj } A = \begin{bmatrix} -18 & 17 & -6 \\ -6 & -8 & 2 \\ -10 & -1 & 28 \end{bmatrix}^T = \begin{bmatrix} -18 & -6 & -10 \\ 17 & -8 & -1 \\ -6 & 2 & 28 \end{bmatrix}.$$

For part (b), recall that $A \operatorname{adj} A = \det A I$. Since the identity matrix I only has 1 across the diagonal, $\det A I$ has $\det A$ across the diagonal. So all we need to do is multiply the first row of A times the first column of $\operatorname{adj} A$ to find the determinant of A

$$\det A = \begin{bmatrix} 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} -18 \\ 17 \\ -6 \end{bmatrix} = -3 \cdot 18 - 2 \cdot 17 - 6 = -94.$$