

MA 519: Homework 9

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PROBLEM 9.1 (HANDOUT 13, # 7)

Let X have a *double exponential* density $f(x) = \frac{1}{2\sigma}e^{-\frac{|x|}{\sigma}}$, $-\infty < x < \infty$, $\sigma > 0$.

- (a) Show that all moments exist for this distribution.
- (b) However, show that the MGF exists only for restricted values. Identify them and find a formula.

SOLUTION.

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PROBLEM 9.2 (HANDOUT 13, # 16)

Give an example of each of the following phenomena:

- (a) A continuous random variable taking values in $[0, 1]$ with equal mean and median.
- (b) A continuous random variable taking values in $[0, 1]$ with mean equal to twice the median.
- (c) A continuous random variable for which the mean does not exist.
- (d) A continuous random variable for which the mean exists, but the variance does not exist.
- (e) A continuous random variable with a PDF that is not differentiable at zero.
- (f) a positive continuous random variable for which the mode is zero, but the mean does not exist.
- (g) A continuous random variable for which all moments exist.
- (h) A continuous random variable with median equal to zero, and 25th and 75th percentiles equal to 1.
- (i) A continuous random variable X with mean equal to median equal to mode equal to zero, and $E(\sin X) = 0$.

SOLUTION. ■

PROBLEM 9.3 (HANDOUT 13, # 17)

An exponential random variable with mean 4 is known to be larger than 6. What is the probability that it is larger than 8?

SOLUTION.

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PROBLEM 9.4 (HANDOUT 13, # 18)

(Sum of Gammas). Suppose X, Y are independent random variables, and $X \sim G(\alpha, \lambda)$, $Y \sim G(\beta, \lambda)$. Find the distribution of $X + Y$ by using moment-generating functions.

SOLUTION. ■

PROBLEM 9.5 (HANDOUT 13, # 19)

(*Product of Chi Squares*). Suppose X_1, X_2, \dots, X_n are independent chi square variables, with $X_i \sim \chi_{m_i}^2$. Find the mean and variance of $\prod_{i=1}^n X_i$.

SOLUTION. ■

PROBLEM 9.6 (HANDOUT 13, # 20)

Let $Z \sim N(0, 1)$. Find

$$P(0.5 < |Z - \tfrac{1}{2}| < 1.5); \quad P\left(\frac{e^Z}{1+e^Z} > \tfrac{3}{4}\right); \quad P(\Phi(Z) < 0.5).$$

SOLUTION. ■

PROBLEM 9.7 (HANDOUT 13, # 21)

Let $Z \sim N(0, 1)$. Find the density of $\frac{1}{Z}$. Is the density bounded?

SOLUTION. ■

PROBLEM 9.8 (HANDOUT 13, # 22)

The 25th and the 75th percentile of a normally distributed random variable are -1 and 1 . What is the probability that the random variable is between -2 and 2 ?

SOLUTION.

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