MA 519: Homework 1

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Problem 1.1 (Handout 1, # 5 [Feller Vol. 1])

A closet contains five pairs of shoes. If four shoes are selected at random, what is the probability that there is at least one complete pair among the four?

Solution. ▸ *We shall assume throughout our homework, unless otherwise stated, that all sample points are equally likely!*

First, since the closet contains 5 pairs of shoes, it contains, in total, 10 shoes. Now, let us consider the case where every shoe we select from the closet is the same kind (either a left shoe or a right shoe); we shall call this event A. In this case, on our first try, we have a $5/10 = 1/2$ chance of grabbing a left shoe (resp. right shoe) from the closet. On our next try, there are 4 left (resp. right) shoes remaining and only 9 shoes in total so our chances of grabbing another left (resp. right) shoe is $4/9$. Proceeding in this fashion, until we select all four shoes, the probability that we picked the left (resp. right) shoe consistently is

$$p(A) = \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7} = \frac{120}{5040} = \frac{1}{42}. \tag{1.1}$$

Now we ask: what is the probability that, in drawing 4 shoes, we have at least 1 pair of shoes. The probability of this event happening   ◂

Problem 1.2 (Handout 1, # 7 [Feller Vol. 1])

A gene consists of 10 subunits, each of which is normal or mutant. For a particular cell, there are 3 mutant and 7 normal subunits. Before the cell divides into 2 daughter cells, the gene duplicates. The corresponding gene of cell 1 consists of 10 subunits chosen from the 6 mutant and 14 normal units. Cell 2 gets the rest. What is the probability that one of the cells consists of all normal subunits.

Solution. ▶

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Problem 1.3 (Handout 1, # 9 [Feller Vol. 1])

From a sample of size n, r elements are sampled at random. Find the probability that none of the N prespecified elements are included in the sample, if sampling is

(a) with replacement;

(b) without replacement.

Compute it for $r = N = 10$, $n = 100$.

Solution. ▶

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Problem 1.4 (Handout 1, # 11 [Text 1.3])

A telephone number consists of ten digits, of which the first digit is one of $1, 2, \ldots, 9$ and the others can be $0, 1, 2, \ldots, 9$. What is the probability that 0 appears at most once in a telephone number, if all the digits are chosen completely at random?

Solution. ▸

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Problem 1.5 (Handout 1, # 12 [Text 1.6])

Events *A, B* and *C* are defined in a sample space Ω. Find expressions for the following probabilities in terms of $P(A)$, $P(B)$, $P(C)$, $P(AB)$, $P(AC)$, $P(BC)$ and $P(ABC)$; here *AB* means $A \cap B$, etc.:

(a) the probability that exactly two of *A, B, C* occur;

(b) the probability that exactly one of these events occur;

(c) the probability that none of these events occur.

Solution. ▶

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Problem 1.6 (Handout 1, # 13 [Text 1.8])

Mrs. Jones predicts that if it rains tomorrow it is bound to rain the day after tomorrow. She also thinks that the chance of rain tomorrow is 1/2 and that the chance of rain the day after tomorrow is 1/3. Are these subjective probabilities consistent with the axioms and theorems of probability?

Solution. ▶

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Problem 1.7 (Handout 1, # 16)

Consider a particular player, say North, in a Bridge game. Let X be the number of aces in his hand. find the distribution of be the number of aces in his hand. find the distribution of X.

Solution. ▸

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PROBLEM 1.8 (HANDOUT 1, # 20)

If 100 balls are distributed completely at random into 100 cells, find the expected value of the number of empty cells.

Replace 100 by n and derive the general expression. Now approximate it as n tends to ∞.

Solution. ▶   ◀