Fall 2016 Notes

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Contents

Chapter 1

Probability

A brief summary of the probability we learned this semester in MA 51900 (discrete probability – yeah, I know). Our references will be (mostly) Feller's *An introduction to probability theory and its applications, Volume 1* [5] and DasGupta's *Fundamentals of Probability: A First Course*[3].

1.1 The Basics

In this section we will talk about concepts related to discrete probability. Before we begin, we introduce the axioms we will be working under. First and foremost, to do probability we need a *sample space* Ω and a *probability* $p\colon \mathscr{M} \to [0,1]$ which assigns values between 0 and 1 to *special* subsets of Ω which we denote by $\mathscr{M}$ (more formally, this $\mathscr{M}$ is called a *σ-algebra* by analysts or a *algebra of events* by probabilists and p is called a *probability measure* and there are certain axioms it must satisfy for us to be able to assign consistent values to subsets of Ω with p). An element $\omega \in \Omega$ is called a *sample point* and a (special) collection of ω, $A \in \mathscr{M}$, is called an event. We call the triplet $(\Omega, \mathscr{M}, p)$ a probability space.

The algebra of events comes $\mathscr{M}$ with a natural multiplication and addition given, naturally, by union and intersection of events (*i.e.* $A + B \coloneqq A \cup B$ and $AB \coloneqq A \cap B$) and an additive as well as multiplicative identity $\emptyset$ and Ω, *etc.* If $AB = \emptyset$ we say that the events A and B are *mutually exclusive*.

Remark 1.1. We won't always use the notation $A + B$ and AB to mean $A \cup B$ and $A \cap B$, respectively (since I prefer the set-theoretic notation over the algebraic one), but Prof. DasGupta makes has a preference for the latter and Feller uses a mix of the two. Now, you may ask "Why introduce this notation at all if you are going to disregard it?" The reason is that I will be using examples from Feller and DasGupta's book and sometimes I will be too rushed to bother translating the notation and though I don't expect anybody but myself to read this, it may very well happen that I pass these notes on to somebody else.

In this section, we shall assume that our sample space Ω is discrete,*i.e.* $\#\Omega < \infty$ or at the very least $\aleph^0$. We additionally require that for each point ω in the space Ω its probability $p(\omega)$ in non-negative and

$$\sum_{\omega \in \Omega} p(\omega) = 1. \tag{1.1}$$

There are of course a whole number of beautiful relationships that p satisfies (those that any sane measure would satisfy like countable additivity, subadditivity, *etc.*), but we shall not talk about them here, instead let us get down to the crux of the matter (at least at this point in the class): counting. Since our sample spaces will be finite (at least for now), we need to be able to count sample points in Ω by way of combinatorics (this is in my opinion, a lot tougher than working with infinite sample spaces for which we must make certain assumptions about the sample points and the probability measure – it is less tedious to solve problems with sane assumptions than it is to count points).

Basic Combinatorics

It is often reasonable to assume that the probability of any particular sample point $\omega \in \Omega$ is just as likely as that of any other sample point. We say that in such a sample space each sample point is *equally likely* to happen. This means that the probability of $\omega \in \Omega$ happening is precisely

$$p(\omega) = \frac{1}{\#\Omega}.$$

Thus, to compute the probability of an event A, we need clever ways of counting points.

The following material is taken, mostly, from Feller's book.

Chapter 2

Introduction to Partial Differential Equations

Here we summarize some important points about PDEs. The material is mostly taken from Evans's *Partial Differential Equations* [4].

Chapter 3

Algebraic Geometry

A summary to a course on an introduction to sheaf cohomology. We will mostly reference Donu's notes available here `https://www.math.purdue.edu/~dvb/classroom.html`, but also cite Ravi Vakil's *Fundamentals of Algebraic Geometry* [7] available here `https://math216.wordpress.com/`.

3.1 The de Rham Complex

Donu began his first lecture by talking about the de Rham complex and de Rham cohomology so let us also begin by on his lecture. For this section, we cite the first chapter of Bott and Tu's *Differential Forms in Algebraic Topology* [2].

Chapter 4

Algebraic Topology

From my meetings with Mark. We reference Hatcher's *Algebraic Topology* [6] freely available here `https://www.math.cornell.edu/~hatcher/#ATI`.

4.1 The de Rham Complex

Bibliography

[1] V.I. Arnold, K. Vogtmann, and A. Weinstein. *Mathematical Methods of Classical Mechanics*. Graduate Texts in Mathematics. Springer New York, 2013.

[2] R. Bott and L.W. Tu. *Differential Forms in Algebraic Topology*. Graduate Texts in Mathematics. Springer New York, 2013.

[3] A. DasGupta. *Fundamentals of Probability: A First Course*. Springer Texts in Statistics. Springer New York, 2010.

[4] L.C. Evans. *Partial Differential Equations*. Graduate studies in mathematics. American Mathematical Society, 2010.

[5] W. Feller. *An introduction to probability theory and its applications*. Number v. 1 in Wiley mathematical statistics series. Wiley, 1950.

[6] A. Hatcher. *Algebraic Topology*. Cambridge University Press, 2002.

[7] R. Vakil. Math 216: Foundations of algebraic geometry, 2016.