

MA 519: Homework 12

Max Jeter, Carlos Salinas

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PROBLEM 12.1 (HANDOUT 15, # 10)

Consider the experiment of picking one word at random from the sentence

All is well in the newell family

Let X be the length of the word selected and Y the number of Ls in it. Find in a tabular form the joint PMF of (X, Y) , their marginal PMFs, means, and variances, and the correlation between X and Y .

SOLUTION.



PROBLEM 12.2 (HANDOUT 15, # 11)

Consider the joint PMF $p(x, y) = cxy$, $1 \leq x \leq 3$, $1 \leq y \leq 3$.

- (a) Find the normalizing constant c .
- (b) Are X and Y independent? Prove your claim.
- (c) Find the expectations of X , Y , and XY .

SOLUTION.

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PROBLEM 12.3 (HANDOUT 15, # 12)

A fair die is rolled twice. Let X be the maximum and Y the minimum of the two rolls. By using the joint PMF of X and Y worked out in the text, find the PMF of $\frac{X}{Y}$, and hence the mean of $\frac{X}{Y}$.

SOLUTION. ■

PROBLEM 12.4 (HANDOUT 15, # 13)

Two random variables have the joint PMF $p(x, x+1) = \frac{1}{n+1}$, $x = 0, \dots, n$. Answer the following question with as little calculation as possible.

- (a) Are X and Y independent?
- (b) What is the variance of $Y - X$?
- (c) What is $\text{Var}(Y \mid X = 1)$?

SOLUTION.

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PROBLEM 12.5 (HANDOUT 15, # 14)

(*Binomial Conditional Distribution*). Suppose X and Y are independent random variables, and $X \sim \text{Bin}(m, p)$, $Y \sim \text{Bin}(n, p)$. Show that the conditional distribution of X given by $X + Y = t$ is a hypergeometric distribution; identify the parameters of this hypergeometric distribution.

SOLUTION. ■

PROBLEM 12.6 (HANDOUT 15, # 15)

Suppose a fair die is rolled twice. Let X and Y be the two rolls. Find the following with as little calculation as possible.

- (a) $E(X + Y | Y = y)$.
- (b) $E(XY | Y = y)$.
- (c) $\text{Var}(X^2Y | Y = y)$.
- (d) $\rho_{X+Y, X-Y}$.

SOLUTION. ■

PROBLEM 12.7 (HANDOUT 15, # 16)

(A Standard Deviation Inequality). Let X and Y be two random variables. Show that $\sigma_{X+Y} \leq \sigma_X + \sigma_Y$.

SOLUTION. Suppose σ_X and σ_Y exist and are finite. Then the second moments of both X and Y exist and are finite. Therefore, by Minkowski's inequality, we have

$$E(|X + Y|^2)^{\frac{1}{2}} \leq E(X^2)^{\frac{1}{2}} + E(Y^2)^{\frac{1}{2}} + 2[E(X^2)E(Y^2)]^{\frac{1}{2}}$$

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PROBLEM 12.8 (HANDOUT 15, # 17)

Seven balls are distributed randomly in seven cells. Let X_k be the number of cells containing exactly k balls. Using the probabilities tabulated in II, 5, write down the joint distribution of X_2, X_3 .

SOLUTION.

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PROBLEM 12.9 (HANDOUT 15, # 18)

Two ideal dice are thrown. Let X be the score on the first die and Y be the larger of two scores.

- (a) Write down the joint distribution of X and Y .
- (b) Find the means, the variances, and the covariance.

SOLUTION.



PROBLEM 12.10 (HANDOUT 15, # 19)

Let X_1 and X_2 be independent and have the common geometric distribution $\{q^k p\}$ (as in problem 4). Show without calculations that the *conditional distribution of X_1 given $X_1 + X_2$ is uniform*, that is,

$$P(X_1 = k \mid X_1 + X_2 = n) = \frac{1}{n+1}, \quad k = 0, \dots, n. \quad (12.1)$$

SOLUTION. ■

PROBLEM 12.11 (HANDOUT 15, # 20)

If two random variables X and Y assume only two values each, and if $\text{Cov}(X, Y) = 0$, then X and Y are independent.

SOLUTION. ■