

MA 26500-215 Quiz 1

July 20, 2016

1. (10 points) Using the method of elimination, determine if the following system of equations has a solution, no solution or many solutions:

$$\begin{aligned}x_1 + 2x_2 - 3x_3 &= -4 \\ 2x_1 + x_2 - 3x_3 &= 4.\end{aligned}\tag{★}$$

Solution: By looking at the number of free variables vs. equations, we can already tell that the system will either have no solutions or many solutions (such a system is said to be *underdetermined*).

Take the first equation and subtract it from the second equation to get

$$\begin{aligned}x_1 + 2x_2 - 3x_3 &= -4 \\ x_1 - x_2 &= 8.\end{aligned}$$

Then $x_1 = 8 + x_2$. Now, substitute this into the first equation

$$(8 + x_2) + 2x_2 - 3x_3 = -4$$

so $3x_2 = -4 - 8 + 3x_3$ or $x_2 = -4 + x_3$. Thus

$$\begin{aligned}x_1 &= 8 + x_2 & x_2 &= -4 + x_3. \\ &= 8 - 4 + x_3 & & \\ &= 4 + x_3 & &\end{aligned}\tag{♠}$$

Therefore, the system has **many solutions**; you give me a value of x_3 and, using the equations in (♠), I can find values for x_1 and x_2 that solve the system (★).

2. (5 points) Given the matrices

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

find

$$2A + 3B^T.$$

Solution: The calculation is straightforward

$$\begin{aligned} 2A + 3B^T &= 2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + 3 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^T \\ &= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2+3 & -2+0 \\ -2+3 & 2+3 \end{bmatrix} \\ &= \begin{bmatrix} 5 & -2 \\ 1 & 5 \end{bmatrix}. \end{aligned}$$