

## MA 26500-215 Quiz 2

July 19, 2016

1. (10 points) Given the system of linear equations

$$\begin{aligned}x_1 - 2x_2 + x_3 - x_4 &= 3 \\x_1 + x_2 + x_3 - x_4 &= 1 \\x_1 + x_3 - x_4 &= 2\end{aligned}\tag{★}$$

find its matrix representation and the reduced row-echelon form of that matrix.

- A.  $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$   
B.  $\begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$   
C.  $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$   
**D.**  $\begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$   
E. Not listed.

**Solution:** Finding the augmented matrix representation of the system (★) is easy enough

$$\left[ \begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 1 & 1 & 1 & -1 & 1 \\ 1 & 0 & 1 & -1 & 2 \end{array} \right].$$

Performing the appropriate row operations on the matrix above gets you to the matrix in **D**.

2. (10 points) Given the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & -1 \\ -1 & -4 & 0 \end{bmatrix}$$

and the vector  $\mathbf{b} = \begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix}$ , find the vector  $\mathbf{x}$  such that  $A\mathbf{x} = \mathbf{b}$  by finding  $A^{-1}$ :

- A.**  $A^{-1} = \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & 1 \end{bmatrix}$ ,  $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$   
B.  $A^{-1} = \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & -1 \end{bmatrix}$ ,  $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$   
C.  $A^{-1} = \begin{bmatrix} 2 & 3 & -2 \\ 3 & 5 & -4 \\ 1 & 1 & -1 \end{bmatrix}$ ,  $\mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$   
D.  $A^{-1} = \begin{bmatrix} 2 & 3 & -2 \\ 3 & 5 & -4 \\ 1 & 1 & -1 \end{bmatrix}$ ,  $\mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$   
E. Does not exist; the matrix  $A$  is singular.

**Solution:** As you learned from Kolman and Hill, you can find the inverse of an  $n \times n$  large matrix by augmenting it with the identity matrix  $I$  on the right and performing Gaussian elimination on the left. So write

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & 5 & -1 & 0 & 1 & 0 \\ -1 & -4 & 0 & 0 & 0 & 1 \end{array} \right]$$

and after doing Gaussian elimination you get

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & 4 & 3 \\ 0 & 1 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -3 & 2 & 1 \end{array} \right].$$

Then, the inverse of  $A$  is

$$A^{-1} = \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & 1 \end{bmatrix}.$$

Than only leaves **A** as the possible answer choice.