MA166: Recitation 8 Prep

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1 Homework Solutions

Section 1.1: Homework 18

Problem 1.1. The masses m_i are located at the points P_i. Find the moments M_x and M_y and the center of mass of the system.

$$m_1 = 2, \qquad m_2 = 1, \qquad m_3 = 7;$$
$$P_1(2, -5), \qquad P_2(-3, 1), \qquad P_3(3, 5).$$

Solution. The definitions for the moment of the system about the y-axis is

$$M_y = \sum_{i=1}^{n} m_i x_i, \tag{1}$$

and for the moment of the system about the x-axis is

$$M_x = \sum_{i=1}^{n} m_i y_i. \tag{2}$$

So all you need to do for this problem is to plug in the values

$$M_y = m_1x_1 + m_2x_2 + m_3x_3 = \boxed{-4,}$$

and

$$M_x = m_1y_1 + m_2y_2 + m_3y_3 = \boxed{-2.}$$

Then the total mass is $M = 10$ so

$$(\bar{x}, \bar{y}) = \boxed{\left(-\frac{2}{5}, -\frac{1}{5}\right).}$$

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Problem 1.2. Sketch the region bounded by the curves, and visually estimate the location of the centroid.

$$y = 4x, \qquad y = 0, \qquad x = 1.$$

Solution. The image you can find yourself. It's at the centroid of the triangle (assuming uniform distribution of mass) and there's a very simple formula for finding the centroid of a triangle, from a purely geometric perspective, it is

$$(\bar{x}, \bar{y}) = \frac{1}{3}(x_1 + x_2 + x_3, y_1 + y_2 + y_3) \tag{3}$$

where (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of the triangle. The vertices are very clearly $(0, 0)$, $(1, 0)$ and $(1, 4)$ hence

$$(\bar{x}, \bar{y}) = \boxed{\left(\frac{2}{3}, \frac{4}{3}\right).}$$

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Problem 1.3. Sketch the region bounded by the curves, and visually estimate the location of the centroid. Find the exact coordinates of the centroid.

$$y = e^x, \qquad y = 0, \qquad x = 5.$$

Find the exact coordinates of the centroid.

Solution. I'll assume you can plot this on your own. Having me do it is asking for too much this late at night :-). Now, recall the definition of the moments about the axes

$$M_y = \int_a^b x(f(x) - g(x))\, dx \tag{4}$$

and

$$M_x = \int \frac{(f(x) - g(x))^2}{2}\, dx \tag{5}$$

and of course the formula for the centroid

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{A}, \frac{M_x}{A}\right). \tag{6}$$

Now the first thing we need to do is to calculate the area

$$A = \int_0^5 e^x\, dx = e^5 - 1.$$

Next, we calculate M_y and M_x like so

$$\begin{aligned} M_x &= \int_0^5 xe^x\, dx & \qquad M_y &= \int_0^5 \frac{e^{2x}}{2}\, dx \\ &= [xe^x - e^x]_0^5 & &= \frac{1}{4}\left[e^{2x}\right]_0^5 \\ &= 4e^5 + 1 & &= \frac{e^{10} - 1}{4}. \end{aligned}$$

So the centroid is

$$\boxed{(\bar{x}, \bar{y}) = \left(\frac{1 + 4e^5}{e^5 - 1}, \frac{e^5 + 1}{4}\right).}$$

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Problem 1.4. Find the centroid of the region bounded by the given curves.

$$y = 6 \sin 5x, \qquad y = 6 \cos 5x, \qquad x = 0, \qquad x = \frac{\pi}{20}.$$

Solution. What a horrible calculation. Spare my poor fingers having to type this out in details :^).

The area is

$$A = 6 \int_0^{\pi/12} \cos 3x - \sin 3x\, dx$$

$$= 2[\sin 3x + \cos 3x]_0^{\pi/12}$$
$$= \boxed{2(\sqrt{2}-1).}$$

Skipping straight to the centroid, we have the following

$$\bar{x} = \frac{1}{2(\sqrt{2}-1)} \int_0^{\pi/12} x\cos 3x - x \sin 3x \, dx \qquad \bar{y} = \frac{1}{4(\sqrt{2}-1)} \int_0^{\pi/12} \cos^2 3x - \sin^2 3x \, dx$$
$$=$$

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Problem 1.5. Find the centroid of the region bounded by the given curves.

$$y = x^3, \qquad x + y = 10, \qquad y = 0.$$

Solution. ☺

Problem 1.6. Calculate the moments M_x, M_y and the center of mass of the lamina with the given density and shape.

Solution. ☺

Problem 1.7.

Solution. ☺

Section 1.2: Homework 19

Problem 1.8.

Solution. ☺

Problem 1.9.

Solution. ☺

Problem 1.10.

Solution. ☺

Problem 1.11.

Solution. ☺

Problem 1.12.

Solution. ☺

Problem 1.13.

Solution. ☺

Problem 1.14.

Solution. ☺

Problem 1.15.

Solution. ☺

Problem 1.16.

Solution. ☺

Problem 1.17.

Solution. ☺

Problem 1.18.

Solution. ☺

Section 1.3: Homework 20

Problem 1.19.

Solution. ☺

Problem 1.20.

Solution. ☺

Problem 1.21.

Solution. ☺

Problem 1.22.

Solution. ☺

Problem 1.23.

Solution. ☺

Problem 1.24.

Solution. ☺

Problem 1.25.

Solution. ☺

Problem 1.26.

Solution. ☺

Problem 1.27.

Solution. ☺

2 Past Exam Problems

Problem 2.1.

Solution. ☺