# Fall 2016 Notes

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# Chapter 1

# Some Category Theory

In an effort to help Vinh prepare for his talk, here are some notes I have compiled on category theory. Here are the books and notes that I used: the *CRing project* notes; *A First Course in Commutative Algebra* by Altman and Kleiman; and *Foundations of Algebraic Geometry* by Ravi Vakil.

## 1.1 Basics

Here are some of the basic ideas (and frankly, the most boring part of category theory).

# Chapter 2

# Probability

Some (mostly discrete) probability theory for MA 51900.

## 2.1 Basics

In this section we will talk about concepts related to discrete probability. Before we begin, we have to define the concepts we will be working with throughout the rest of this section (ye this chapter). First and foremost, to do probability we need a *sample space* $\Omega$ and a probability function $p\colon \mathscr{P}(\Omega) \to [0,1]$ which assigns values between 0 and 1 to subsets of $\Omega$ (usually, one needs to specify a $\sigma$-algebra on $\Omega$, but for the rest of the section, since $\Omega$ is at least countable, the power set $\mathscr{P}(\Omega)$ is a $\sigma$-algebra by default). A *sample point* $\omega$ is a point of $\Omega$ and an event $A \subseteq \mathscr{P}(\Omega)$ is a collection of sample points.

In this section we will talk about concepts related to discrete probability. Before we begin, we introduce the objects we will be working with. First and foremost, to do probability we need a *sample space* $\Omega$ and a *probability* $p\colon \mathscr{M} \to [0,1]$ which assigns values between 0 and 1 to *special* subsets of $\Omega$ which we denote by $\mathscr{M}$ (more formally, this $\mathscr{M}$ is called a *$\sigma$-algebra* and $p$ is called a *probability measure* and there are certain axioms it must satisfy for us to be able to assign consistent values to subsets of $\Omega$ with $p$). An element $\omega \in \Omega$ is called a *sample point* and a (special) collection of $\omega$, $A \in \mathscr{M}$, is called an event. We call the triplet $(\Omega, \mathscr{M}, p)$ a probability space.

With that out of the way, let us get down to the crux of the matter (at least at this point in the class): counting. Since our sample spaces will be finite (at least for now), we need to be able to count sample points in $\Omega$ by way of combinatorics (this is in my opinion, a lot tougher than working with infinite sample spaces for which we must make certain assumptions about the sample points and the probability measure – it is less tedious to solve problems with sane assumptions than it is to count points).