MA 519: Homework 3

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Problem 3.1 (Handout 3, # 3)

n sticks are broken into one short and one long part. The 2n parts are then randomly paired up to form n new sticks. Find the probability that

- (a) the parts are joined in their original order, i.e., the new sticks are the same as the old sticks;
- (b) each long part is paired up with a short part.

Solution. ▶

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Problem 3.2 (Handout 3, # 5)

In a town, there are 3 plumbers. On a certain day, 4 residents need a plumber and they each call one plumber at random.

Solution. ▶

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Problem 3.3 (Handout 4, # 7)

(Polygraphs). Polygraphs are routinely administered to job applicants for sensitive government positions. Suppose someone actually lying fails the polygraph 90% of the time. But someone telling the truth also fails the polygraph 15% of the time. If a polygraph indicates that an applicant is lying, what is the probability that he is in fact telling the truth? Assume a general prior probability p that the person is telling the truth.

Solution. ▶

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Problem 3.4 (Handout 4, # 8)

In a bolt factory machines A, B, C manufacture, respectively, 25, 35, and 40 per cent of the total. Of their output 5, 4, and 2 per cent are defective bolts. A bolt is drawn at random from the produce and is found defective. What are the probabilities that it was manufactured by machines A, B, C?

Solution. ▶

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Problem 3.5 (Handout 4, # 9)

Suppose that 5 men out of 100 and 25 women out of $10\,000$ are colorblind. A colorblind person is chosen at random. What is the probability of his being male? (Assume males and females to be in equal numbers.)

Solution. ▶

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Problem 3.6 (Handout 4, # 10)

Bridge. In a bridge party West has no ace. What probability should he attribute to the event of his partner having

- (a) no ace,
- (b) two or more aces?

Verify the result by a direct argument.

Solution. ▶

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Problem 3.7 (Handout 4, # 12)

A true-false question will be posed to a couple on a game show. The husband and the wife each has a probability p of picking the correct answer. Should they decide to let one of the answer the question, or decide that they will give the common answer if they agree and toss a coin to pick the answer if they disagree?

Solution. ▶

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Problem 3.8 (Handout 4, # 13)

An urn containing 5 balls has been filled up by taking 5 balls at random from a second urn which originally had 5 black and 5 white balls. A ball is chosen at random from the first urn and is found to be black. What is the probability of drawing a white ball if a second ball is chosen from among the remaining 4 balls in the first urn?

Solution. ▶

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Problem 3.9 (Handout 4, # 15)

Events A, B, C have probabilities p_1 , p_2 , p_3 . Given that exactly two of the three events occurred, the probability that C occurred is greater than 1/2 if and only if ... (write down the necessary and sufficient condition).

Solution. ▶

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Problem 3.10 (Handout 5, # 1)

There are five Kkkoins on a desk: 2 are double-headed, 2 are double-tailed, and 1 is a normal kkkoin.

Solution. \blacktriangleright

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Problem 3.11 (Handout 5, # 2)

Solution. \triangleright

Problem 3.12 (Handout 5, # 4)

Solution. \triangleright