Hwk 1 p.1

1.1: 1, 2, 4, 6, 10, 14, 16, 22
1.2: 4, 6, 8, 10
ML1.1: 1, 2, 3, 4

Solve the linear system using elimination.

1.1.1) $\begin{cases} x+2y=8 & r_1 \\ 3x-4y=4 & r_2 \end{cases}$

Eliminate y by 2 times first. $r_2 \to 2r_1+r_2$

$\begin{cases} x+2y=8 \\ 3x-4y=4 & r_2 \to 2r_1+r_2 \end{cases}$ $\begin{cases} x+2y=8 \\ 5x+0y=20 \end{cases}$ So $x=4$ and $y=2$.

1.1.2) $\begin{cases} 2x-3y+4z=-12 & r_1 \\ x-2y+z=-5 & r_2 \\ 3x+y+2z=1 & r_3 \end{cases}$

Eliminating x: $r_1 \to r_1-2r_2$, $r_3 \to r_3-3r_2$

$\begin{cases} 2x-3y+4z=-12 & r_1 \to r_1-2r_2 \\ x-2y+z=-5 \\ 3x+y+2z=1 & r_3 \to r_3-3r_2 \end{cases}$ $\begin{cases} y+2z=-2 \\ x-2y+z=-5 \\ 7y-z=16 & r_3 \to r_3-7r_1 \end{cases}$

$\begin{cases} y+2z=-2 \\ x-2y+z=-5 \\ -15z=30 \end{cases}$ So $z=-2$, $y=2$, $x=1$.

1.1.4) $\begin{cases} x+y=5 & r_1 \\ 3x+3y=10 & r_2 \end{cases}$

$\begin{cases} x+y=5 \\ 3x+3y=10 & r_2 \to r_2-3r_1 \end{cases}$ $\begin{cases} x+y=5 \\ 0=-5 \end{cases}$ a contradiction, so the system has no solutions.

1.1.6) $\begin{cases} x+y-2z=5 & r_1 \\ 2x+3y+4z=2 & r_2 \end{cases}$

$\begin{cases} x+y-2z=5 \\ 2x+3y+4z=2 & r_2 \to r_2-2r_1 \end{cases}$ $\begin{cases} x+y-2z=5 & r_1 \to r_1-r_2 \\ y+8z=-8 \end{cases}$ $\begin{cases} x-10z=13 \\ y+8z=-8 \end{cases}$

This has infinitely many solutions $x=10z+13$, $y=-8z-8$, z any real number.

1.1.10 $\begin{cases} x+y=1 & r_1 \\ 2x-y=5 & r_2 \\ 3x+4y=2 & r_3 \end{cases}$

$\begin{cases} x+y=1 & \\ 2x-y=5 & r_2 \to r_2-2r_1 \\ 3x+4y=2 & r_3 \to r_2-3r_1 \end{cases}$ $\begin{cases} x+y=1 \\ -3y=3 \\ y=-1 \end{cases}$ So $y=-1$ and $x=2$

1.1.14 $\begin{cases} 2x+3y-z=6 & r_1 \\ 2x-y+2z=-8 & r_2 \\ 3x-y+z=-7 & r_3 \end{cases}$

$\begin{cases} 2x+3y-z=6 & \\ 2x-y+2z=-8 & r_2 \to r_2-r_1 \\ 3x-y+z=-7 & r_3 \to r_3-r_1 \end{cases}$ $\begin{cases} 2x+3y-z=6 & r_1 \to r_1-2r_3 \\ -4y+3z=-14 & \\ x-4y+2z=-13 & r_3 \to r_3-r_2 \end{cases}$

$\begin{cases} 11y-5z=32 & r_1 \to r_1+3r_2 \\ -4y+3z=-14 & \\ x \quad -z=1 & \end{cases}$ $\begin{cases} -y+4z=-10 & \\ -4y+3z=-14 & r_2 \to r_2-4y \\ x \quad -z=1 & \end{cases}$

$\begin{cases} -y+4z=-10 \\ -13z=26 \\ x \quad -z=1 \end{cases}$ So $z=-2$, $x=-1$, $y=2$.

1.1.16 Given the linear System $\begin{cases} 3x+4y=s \\ 6x+8y=t \end{cases}$

(a) Determine the values of s,t so that the system is consistent.
(b) Determine the values of s,t so that the system is inconsistent.
(c) What relationship between the values of s,t will guarantee the system will be consistent?

$\begin{cases} 3x+4y=s & \\ 6x+8y=t & r_2 \to r_2-2r_1 \end{cases}$ $\begin{cases} 3x+4y=s \\ 0=t-s \end{cases}$

Consistent requires $s=t$ and inconsistent requires $s \neq t$.

1.1.22| Is there a value of r so that $x=1, y=2, z=r$ is a solution to the following linear system? If there is, find it.

$$\begin{cases} 2x+3y-z=11 \\ x-y+2z=-7 \\ 4x+y-2z=12 \end{cases}$$

If $x=1, y=2, z=r$ is a solution, then this is

$$\begin{cases} 2+6-r=11 \\ 1-2+2r=-7 \\ 4+2-2r=12 \end{cases} \text{ gives } \begin{cases} 8-r=11 \\ -1+2r=-7 \\ 6-2r=12 \end{cases} \text{ or } \begin{cases} r=-3 \\ r=-3 \\ r=-3. \end{cases}$$

Since $r=-3$ solves all three, it is a solution.

1.2.4| If $\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix}$, find a, b, c, and d.

This is $a+b=4$, $c+d=6$, $c-d=10$, $a-b=2$ giving two systems

(1) $\begin{cases} a+b=4 \\ a-b=2 \end{cases}$ and (2) $\begin{cases} c+d=6 \\ c-d=10 \end{cases}$. For (1), adding gives $2a=6$ so $a=3$, $b=1$;

and for (2) adding gives $2c=16$ so $c=8$, $d=-2$.

Let $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$, $C=\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$, $D=\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$, $E=\begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$, $F=\begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix}$, $O=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

1.2.6| If possible, compute the indicated linear combination:

(a) $C+E$ and $E+C$ (b) $A+B$ (c) $D-F$ (d) $-3C+5O$ (e) $2C-3E$ (f) $2B+F$

(a) $C+E = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -5 & 8 \\ 4 & 2 & 9 \\ 5 & 3 & 4 \end{bmatrix} = E+C$

(b) $A+B$ is not possible for the matrices are not the same size.

(c) $D-F = \begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} -4 & 5 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -7 \\ 0 & 1 \end{bmatrix}$

(d) $-3C+5O = -3\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} + 5\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -9 & 3 & -9 \\ -12 & -3 & -15 \\ -6 & -3 & -9 \end{bmatrix}$

(e) $2C-3E = 2\begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} - 3\begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 6 \\ 8 & 2 & 10 \\ 4 & 2 & 6 \end{bmatrix} - \begin{bmatrix} 6 & 12 & -15 \\ 0 & -3 & -12 \\ -9 & -6 & -3 \end{bmatrix} = \begin{bmatrix} 0 & -14 & 21 \\ 8 & 5 & 22 \\ 13 & 8 & 9 \end{bmatrix}$

(f) $2B+F$ is not possible for the matrices are not the same size.

1.2.8 If possible, compute the following

(a) A^T and $(A^T)^T$ (b) $(C+E)^T$ and C^T+E^T (c) $(2D+3F)^T$ (d) $D-D^T$

(e) $2A^T+B$ (f) $(3D-2F)^T$

(a) $A^T = \begin{bmatrix}1&2&3\\2&1&4\end{bmatrix}^T = \begin{bmatrix}1&2\\2&1\\3&4\end{bmatrix}$ and $(A^T)^T = \begin{bmatrix}1&2\\2&1\\3&4\end{bmatrix}^T = \begin{bmatrix}1&2&3\\2&1&4\end{bmatrix} = A$

(b) $(C+E)^T = \begin{bmatrix}5&-5&8\\4&2&9\\5&3&4\end{bmatrix}^T = \begin{bmatrix}5&4&5\\-5&2&3\\8&9&4\end{bmatrix}$

$C^T+E^T = \begin{bmatrix}3&-1&3\\4&1&5\\2&1&3\end{bmatrix}^T + \begin{bmatrix}2&-4&5\\0&1&4\\3&2&1\end{bmatrix}^T = \begin{bmatrix}3&4&2\\-1&1&1\\3&5&3\end{bmatrix} + \begin{bmatrix}2&0&3\\-4&1&2\\5&4&1\end{bmatrix} = \begin{bmatrix}5&4&5\\-5&2&3\\8&9&4\end{bmatrix}$

(c) $(2D+3F)^T = \left(2\begin{bmatrix}3&-2\\2&4\end{bmatrix} + 3\begin{bmatrix}-4&5\\2&3\end{bmatrix}\right)^T = \left(\begin{bmatrix}6&-4\\4&8\end{bmatrix} + \begin{bmatrix}-12&15\\6&9\end{bmatrix}\right)^T = \begin{bmatrix}-6&11\\10&17\end{bmatrix}^T = \begin{bmatrix}-6&10\\11&17\end{bmatrix}$

(d) $D-D^T = \begin{bmatrix}3&-2\\2&4\end{bmatrix} - \begin{bmatrix}3&-2\\2&4\end{bmatrix}^T = \begin{bmatrix}3&-2\\2&4\end{bmatrix} - \begin{bmatrix}3&2\\-2&4\end{bmatrix} = \begin{bmatrix}0&-4\\4&0\end{bmatrix}$

(e) $2A^T+B = 2\begin{bmatrix}1&2\\2&1\\3&4\end{bmatrix} + \begin{bmatrix}1&0\\2&1\\3&2\end{bmatrix} = \begin{bmatrix}2&4\\4&2\\6&8\end{bmatrix} + \begin{bmatrix}1&0\\2&1\\3&2\end{bmatrix} = \begin{bmatrix}3&4\\6&3\\9&10\end{bmatrix}$

(f) $(3D-2F)^T = \left(3\begin{bmatrix}3&-2\\2&4\end{bmatrix} - 2\begin{bmatrix}-4&5\\2&3\end{bmatrix}\right)^T = \left(\begin{bmatrix}9&-6\\6&12\end{bmatrix} + \begin{bmatrix}8&-10\\-4&-6\end{bmatrix}\right)^T = \begin{bmatrix}17&-16\\2&6\end{bmatrix}^T = \begin{bmatrix}17&2\\-16&6\end{bmatrix}$

1.2.10 Is the matrix $\begin{bmatrix}3&0\\0&2\end{bmatrix}$ a linear combination of the matrices $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ and $\begin{bmatrix}1&0\\0&0\end{bmatrix}$? Justify your answer.

Yes for $2\begin{bmatrix}1&0\\0&1\end{bmatrix} + \begin{bmatrix}1&0\\0&0\end{bmatrix} = \begin{bmatrix}3&0\\0&2\end{bmatrix}$.

Matlab 1.1 | Enter matrices A, B, and C into Matlab.

$A = \begin{bmatrix} 4 & -3 \\ 2 & 1 \\ 0 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 1 \\ 0 & 1 & 5 \end{bmatrix}$ $C = \begin{bmatrix} 5 \\ 8 \\ 7 \end{bmatrix}$. Exercises 1 and 2 refer to these.

Matlab 1.1.1 | Enter the command that performs the indicated action. Execute it in Matlab.

(a) Display all of A. A

(b) Display only the second row of A. A(2,:)

(c) Display only the (3,2)-entry of A. A(3,2) = 6

(d) Display only column 3 of B. B(:,3)

(e) Display the first two columns of B. B(:,1:2)

(f) Display the last 2 rows of A. A(2:3,:).

Matlab 1.1.2 | Define a new matrix D having the same contents as A by typing the matlab command D = A. On the line, enter the command that performs the indicated action.

(a) Make the (1,1)-entry of D equal to 12. D(1,1) = 12.

(b) Make the (3,2)-entry of D equal to -8. D(3,2) = -8.

(c) Type the command E = [D C]. Describe the contents in terms of D and C.

Makes a 3×3 matrix with first two columns from D and last column C.

(d) Type command F = [D B]. Describe the contents of F in terms of D and B.

Makes a 3×5 matrix with first two columns from D and last three columns from B.

(e) Type the command G = [E; B]. Describe the contents of G in terms of E and B.

Makes a 6×3 matrix with first three rows from E and last three rows from B.

Matlab 1.1.3 | Perform the following in Matlab.

(a) Construct a column c1 with entries 0, -1, 3, 5. c1 = [0; -1; 3; 5]

(b) Construct a column c2 with entries 4, -2, 0, 7. c2 = [4; -2; 0; 7]

(c) Construct a matrix H whose columns are c1 and c2 without retyping entries. H = [c1 c2].

(d) Construct a matrix K whose first two columns are both c1 and whose third column is c2.

K = [c1 c1 c2].

Matlab 1.1.4 | Perform the following in Matlab.

(a) Construct a row r1 with entries 2, -1, 5. r1 = [2 -1 5].

(b) Construct a row r2 with entries 7, 9, -3. r2 = [7 9 -3].

(c) Construct a matrix M whose rows are r1 and r2 without retyping.

M = [r1 ; r2].

(d) Describe the result of the command 3*r1.

Multiples each element of the row matrix by 3.

(e) Describe the command r1 + r2.

Adds component wise the elements in each row matrix.

(f) Describe the command [r1 ; r1 - r2 ; r2]

Creates a matrix with first row r1, second row r1 - r2, and third row r2. It is a 3x3 matrix.