A space X is said to be weakly locally connected at$x$ if for every neighborhood U of x is a connectedd subspace of X contained in U that contains a neighborhood of x.

Show that if X is weakly lacally connected at each of i connects points , then X is lacally connected.

p.162.#6

Let X be locally path connected.
Show that every connected open set in X is path connected.

Show that A_n uncountable product of$\mathbf{R}$(real) with itself is not metrizable.
p.133 9/11 note
Hint; Show that there is a set $A \subset X := \prod_{\alpha \in J} \mathbf{R} \ \ (= \mathbf{R}^J)$
and a point $a \in X$ s.t. $a \in \bar{A}$ but there is no sequence in A converging to a.

Let Y be an ordered set in the order topology.
Let $f, g : x \to Y$ be continuous.
(a) Show that the set $\{x | f(x) \leq g(x)\}$ ia closed in X.
(b) Let $h : X \to Y$ be the function

$$h(x) = \min\{f(x), g(x)\}$$

Show that h is continuous. p.111 #8

Let $A \subset X$; let $f : A \to Y$ be continuous; let Y be hausdorff.

Show that if f may be extended to a continuous function $g : \bar{A} \to Y$, then g is uniquely determined by f.

p.112 #13

Show that if Y is compact, then the projection $\pi_1 : X \times Y \to X$ is a closed map.

p.171 # 7.

Show that every compact subspace of a metric space is bounded in that metric and is closed.

Find a metric space in which not every closed bounded subspace is compact.

p.171, ex4

Show that every order topology is Hausdorff.
p.101 ex10

Let $f : X \to Y$; let Y be compact Hausdorff. Then f is continuous if and only if the graph of f,

$$G_f = \{x \times f(x) | x \in X\},$$

is closed in $X \times Y$.

p.171, ex8

Let X be ta compact Hausdorff space. Let $\mathscr{A}$ be a collection of closed connected subsets of X that is simply ordered by proper inclusion. Then

$$Y = \bigcap_{A \in \mathscr{A}} A$$

is connected.

Let Y be an ordered set in the order topology.
Let $f, g : X \to Y$ be the function

$$h(x) = min\{f(x), g(x)\}$$

Show that h is continuous.
p.112 ex8

Let $p : X \to Y$ be a closed continuous surjective map such that $p^{-1}(\{y\})$ is compact. (Such a map is called a perfect map.)

Show that if Y is compact, then X is compact.

p.172 ex12

Let X be a metric space with metric d ;
let $A \subset X$ be nonempty.
(1) Show that if A is compact, $d(x, A) = d(x, a)$ for some $a \in A$.
(2) Assume that A is compact; let U be an open set containing A.
Show that some ϵ-neighborhood of A is contained in U.
ϵ-neighborhood of A ,$U(A, \epsilon) = \{x | d(x, A) < \epsilon\}$.
p.177 ex2

Let X be a compact Hausdorff space; let $\{A_n\}$ be a countable collection of closed sets of X.

Show that if each set A_n has empty interior in X , then the union $\cup A_n$ has empty interior in X.

p178 ex5

Is $\mathbb{Q}$ locally compact?

p182 example, Mimura p180 ex6

Let $\{X_\alpha\}$ be an indexed family of nonempty spaces.

Show that if $\prod X_\alpha$ is locally compact, then each X_α is locally compact and X_α is compact for all but finitely many value of α.

p.186 ex2 , Mimura p180 ex11

If$f : X_1 \to X_2$ is a homeomorphism of locally compact Hausdorff spaces, show f extends to a homeomorphism of their one-point compactification.

p186. ex5

Show that if X is a Haudorff space that is locally compact at the point x , then for each neighborhood U of x , there is a neighborhood V of x such that $\bar{V}$ compact and $\bar{V} \subset U$.

p186. ex 10

Let X be a compact Hausdorff space, let Y be a topological space , and let $p : X \to Y$ be a closed surjective continuous map.

Prove that Y is Hausdorff.

HW due Oct 23

Let x_0 and x_1 be points of the path-connected space X . Show that $\pi_1(X, x_0)$ is abelian if and only if for every pair α and β of paths from x_0 to x_1 , we have $\hat{\alpha} = \hat{\beta}$

p335 ex3

Show that if $h : S^1 \to S^1$ is nulhomotopic, then h has a fixed point and h maps some point x to its antipode $-x$.

p353 ex2

Show that if Y is locally compact Hausdorff, then composition of maps

$$C(X,Y) \times C(X,Y)$$

is continuous, provided the compact-open topology is used throughout.
p289 ex7

Let $\acute{C}(X,Y)$ denote the set $C(X,Y)$ in some topology $\mathscr{T}$.
Show that if the evaluation map

$$e : X \times \acute{C}(X,Y) \to Y$$

is continuous, then $\mathscr{T}$ contains the compact-open topology.
p289 ex8

Theorem. If $p : A \to B$ is a quotient map and X is locally compact Hausdorff, then $i_X \times p : X \times A \to X \times B$ is a quotient map.

Proof.

(a) Let Y be the quotient space induced by $i_X \times p$; let $q : X \times A \to Y$ be the quotient map. Show there is bijective continuous map $f : Y \to X \times B$ such that $f \circ q = i_X \times p$.

(b) Let $g = f^{-1}$. Let $G : B \to C(X, Y)$ and $Q : A \to C(X, Y)$ be the maps induced by g and q ,respectively. Show that $Q = G \circ p$.

(c) Show that Q is continous ; conclue that G is continuous, so that g is continuous.

p289 ex9

Given spaces X and Y, let $[X, Y]$ denote the set of homotopy classes of maps of X into Y.

(a) Let $I = [0, 1]$. Show that for any X, the set $[X, I]$ had a single element.

(b) Show that if Y is path connected, the set $[I, Y]$ has a single element.

p330 ex2

Let X be a compact space and suppose we are given a nested sequence of sets

$$C_1 \supset C_2 \supset \cdots$$

with all C_i closed. Let U be an open set containing $\cap C_i$.

Prove that there is an i_0 with $C_{i_0} \subset U$.

For midterm2 #5.

Let X be a compact space, and suppose there is a finite family of continuous functions $f_i : X \to \mathbb{R}, i = 1, \cdots, n$, with the following property: give $x \neq y$ in X there is an i such that $f_i(x) \neq f_i(y)$.

Prove that X is homeomorphic to a subspace of $\mathbb{R}^n$.

for midterm2 #6

Prove that the Lubesgue number lemma.

i.e.

Let X be a compact metric space and let $\mathscr{U}$ be a covering of X by open sets.

Prove that there is an $\epsilon > 0$ such that, for each set $S \subset X$ with diameter $< \epsilon$, there is a $U \in \mathscr{U}$ with $S \subset U$.

for midterm2, #7

Let S^1 denote the circle $\{x^2 + y^2 = 1\}$ in $\mathbb{R}^2$.
Define an equivalence relation on S^1 by

$$(x,y) \sim (\acute{x},\acute{y}) \Rightarrow (x,y) = (\acute{x},\acute{y}) \text{ or } (x,y) = (-\acute{x},-\acute{y})$$

(you do not have to prove that this is an equivalence relation).
Prove that the quotient space $S^1/\sim$ is homeomorphic to S^1.
for midterm2, #8

Let X be a locally compact Hausdorff. Explain how to construct the one-point compactification of X, and prove that the space you construct is really compact.

for midterm2 #9

Let X be a locally compact Hausdorff space, let Y be any space, and let the function space$C(X, Y)$ have the compact-open topology.

Prove that the map

$$e : X \times C(X, Y) \to Y$$

defined by the equation $e(x, f) = f(x)$ is continuous.

forr midterm2 #11

Let I be the unit interval, and let Y be a path-connected space.
Prove that any two maps(continuous?) from I to Y are homotopic.
for midterm2 #12

Let X be a topological space and $f : [0, 1] \to X$ any continuous function. Define $\bar{f}$ by $\bar{f} = f(1 - t)$.

Prove that $f * \bar{f}$ is path-homotopic to the constant path at $f(0)$.

for midterm2 #13.

Let A be a subspace of X; let $j : A \to X$ be the injection map, and let $f : X \to A$ be a continuous map. Suppose there is a homotopy $H : X \times I \to X$ between the map $j \circ f$ and the identity map of X.

(a) Show that if f is a retraction, then j_* is an isomorphism.

(b) show that if H maps $A \times I$ into A, then j_* is an isomorphism.

(c) Gve an example in which j_* is not an isomorphism.

HW due 11/20 ,p366 ex7.

We define the degree of a continuous map $h : S^1 \to S^1$ as follows:

Let b_0 be the point $(0, 1)$ of S^1, choose a path α in S^1 from b_0 to x_0, and define $\gamma(x_0) = \hat{\alpha}(\gamma)$. Then $\gamma(x_0)$ generates $\pi_1(S^1, x_0)$. Show that the element $\gamma(x_0)$ is independent of the choice of the path α.

Now given $h : S^1 \to S^1$, choose $x_0 \in S^1$ and let $h(x_0) = x_1$. Consider the homomorphism

$$h_* : \pi_1(S^1, x_0) \to \pi_1(S^1, x_1).$$

Since both groups are infinite cyclic, we have

$$(*)h_*(\gamma(x_0)) = d \cdot \gamma(x_1)$$

for some integer d, if the group is written additively. The integer d is called the degree of h and is denoted by $\deg h$.

The degree of h is independent of the choice of generator γ; choosing the other generator would merely change the sign of both sides of (*).

(a) Show that d is independent of the choice of x_0.

(b) Show that if $h, k : S^1 \to S^1$are homotopic, they have the same degree.

(c) Show that $\deg(h \circ k) = (\deg h) \cdot (\deg k)$.

HW due 11/20 ,p366 ex9

Prove every path in an m-manifold is path-homotopic to a piecewise quasi-linear path.

HW due Nov 20 C)

Prove a piecewise quasi-linear path in an m-manifold with $m > 1$ cannot be onto.

HW due Nov 20 D)

If $n > 1$, show that the fundamental group of the n-fold torus is not abelian.
HW due Dec 11, p454 ex6

Let X be the quotient space obtained from B^2 by identifying each point x of S^1 with its antipode $-x$.

Show that X is homeomorphic to the projective plane P^2.

p375 ex2 HW due Dec2

Caluculate $H_1(P^2\#T)$. Assuming that the list of compact surfaces given in Theorem 75.5 is a complete list, to which of these surfaces is $P^2\#T$ homeomorphic?

p457 ex1 HW due Dec11

IF K is the Klein bottle, calculate $H_1(K)$ (directly).
p457 ex2 HW due Dec11

A topological space X is called homeogeneous if for everyy pair of points $x, y \in X$ there is a homeomorphism $\phi : X \to X$ with $\phi(x) = y$.

Prove that every connected 2-manifold is homogeneous.

HW due Dec11 A)

Prove that every m-manifold is regular.
HW due Nov4 A)

let X_α be a family of topological spaces. For each α, let A_α be a subset of X_α. Prove that

$$\bar{\prod_\alpha} A_\alpha = \prod_\alpha \bar{A_\alpha}$$

1st mid term

Let X and Y be topological spaces and let $f : X \to Y$ be a function with the property that

$$f(\bar{A}) \subset f\bar{(}A)$$

for all subsets A of X.

Prove that f is continuous.

1st mid term

Let X and Y be topoloogical spaces and let $f : X \to Y$ be a continuous function. Let G_f (called the graph of f) be the subspace $\{(x, f(x))|x \in X\}$ of $X \times Y$.

Prove that if Y is Hausdorff then G_f is closed.

1st mid term

Let X and Y be connected. Prove that $X \times Y$ is connected.
1st mid term