

# MA544: Qual Preparation

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Prof. Bañuelos, este es otro problema con el que no he podido avanzar.

**Problem 1.** Let  $f_n: X \rightarrow [0, \infty)$  be a sequence of measurable functions on the measure space  $(X, \mathcal{F}, \mu)$ . Suppose there is a positive constant  $M$  such that the functions  $g_n(x) = f_n(x)\chi_{\{f_n \leq M\}}(x)$  satisfy  $\|g_n\|_1 \leq An^{-4/3}$  and for which  $\mu\{x \in X : f_n(x) > M\} \leq Bn^{-5/4}$ , where  $A$  and  $B$  are positive constants independent of  $n$ . Prove that

$$\sum_{n=1}^{\infty} f_n < \infty$$

almost everywhere.

**Solution.** ▶ Let

$$E = \left\{ x \in X : \sum_{n \in \mathbb{N}} f_n(x) = \infty \right\}.$$

We must show that for every  $\varepsilon > 0$ ,  $\mu(E) < \varepsilon$ , i.e.,  $E$  is a set of measure zero.

Seeking a contradiction, suppose that  $\mu(E) > 0$ . We know that

$$\mu\{f_n > M\} \leq \frac{B}{n^{5/4}}$$

and that

$$\|g_n\|_1 = \int_{\{f_n \leq M\}} f_n(x) \, dx \leq \frac{A}{n^{4/3}}.$$

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