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## MA 26500-215 Quiz 10

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1. Let  $V$  and  $W$  be subspaces of  $\mathbb{R}^n$  such that  $\dim V \leq \dim W$ . Show that there exists linear map  $L: V \rightarrow W$  with full rank.

**Solution:** Since  $V$  and  $W$  are subspaces of  $\mathbb{R}^n$ ,  $V$  and  $W$  have a basis, say,  $\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$  and  $\{\mathbf{w}_1, \dots, \mathbf{w}_m\}$  where  $n = \dim V$ ,  $m = \dim W$  and  $n \leq m$ . Now, let  $L: V \rightarrow W$  be the map

$$L(\mathbf{v}_i) = \begin{cases} L(\mathbf{w}_i) & \text{if } 1 \leq i \leq n, \\ 0 & \text{if } n < i \leq m. \end{cases}$$

2. Does there exist a linear transformation  $L: V \rightarrow W$  such that

$$\|L(x)\| \leq 1$$

for all  $x \in V$ ? How can you change the statement  $L: V \rightarrow W$  so that it changes the outcome of the last theorem.