MA166: Review Sheet for Exam 1

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1 Review Notes for Exam 1

This is a review sheet for Exam 1 for MA 166. This is by no means the only resource you should use to study for the exam, but I hope it will serve as a good review for some of the techniques you have learned thus far.

1.1 Vectors and the Geometry of Space

Three-Dimensional Coordinate Systems

In this section you first learn about the right-hand rule and right handed coordinate systems. This is really just a mathematical *convention* that we follow because we like the cross product of two "positive" vectors, i.e. vectors in the first quadrant of the xy-plane, to point out of the plane. Keep this in mind as you continue studying the natural sciences.

A cute way to figure out whether you have a right-handed coordinate system is this:

> If you are right-handed, imagine holding a coffee mug with your right hand, your thumb pointing up towards the z-axis, then your fingers wrapped around the handle of the mug will traverse first the x-axis, then end up on the y-axis.

I noticed a lot of students were having trouble with describing equations and inequalities in $\mathbb{R}^2$ and $\mathbb{R}^3$. When you see an equation like "What does the equation $x = 3$ represent in $\mathbb{R}^3$?" your first thought should be, what is a point in the graph of this equation? The equation is telling us, no matter what choice of y and z we make, x will always be 3. Thus, the points $(3, 1, 0)$ and $(3, 0, 1)$ are in the graph of the equation $x = 3$, but $(2, 0, 0)$ is not because we have the constraint that x must equal 3. You can already more or less see what this is going to look like. If you draw the line from $(3, 1, 0)$ to $(3, 0, 1)$ every point on the line will be in the graph of $x = 3$ and if you pick any other point in $x = 3$ and draw the line from $(3, 1, 0)$ to it, the same will be true, so $x = 3$ must be a plane perpendicular to the x-axis which intersects the x-axis at $(3, 0, 0)$.

Now, what does the equation $x = 3$ represent in $\mathbb{R}^2$?

Of course, you should also know the general equation of a sphere centered at (x_0, y_0, z_0) with radius r:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2. \tag{1}$$

When you see a quadratic equation, i.e.,an equation with terms like x^2, y^2, z^2, you should try completing the square and simplifying it. For example, suppose we are asked what the following expression represents

$$x^2 + y^2 + z^2 - 6x - 4y + 6z = 0?$$

First, you gather all your like terms and put them next to each other like this

$$\left(x^2 - 6x\right) + \left(y^2 - 4y\right) + \left(z^2 + 6z\right) = 0.$$

Next, you complete the square, i.e., you add whatever terms you need to add to the parenthesized polynomials to turn it into the square of a linear polynomial (a linear polynomial looks like $ax + b$, or $a'y + b'$, or $a''z + b''$, etc.) so we have

$$(x^2 - 6x + 9)^2 + (y^2 - 4y + 4) + (z^2 + 6z + 9) = 9 + 4 + 9.$$

Don't forget than when you are completing the square, you are adding terms, so you are changing your original equation, you must add the same terms to the right-hand side to balance the equation!

Now you just need to recognize that, because the coefficient in front of x is negative (the same for y) and $(x+a)^2 = x^2 + 2ax + a^2$, then we must be looking at the square of negative -3 (the same is true of the coefficient in front of y) so we have

$$(x-3)^2 + (y-2)^2 + (z+3)^2 = 22.$$

Now we can read off the values: The equation $x^2 + y^2 + z^2 - 6x - 4y + 6z = 0$ represents a sphere of radius $\sqrt{22}$ centered at $(3, 2, -3)$

Now, if you find it a little difficult to remember comp and proj perhaps the following equation will help you see the relationship between the scalar projection and the projection

$$\operatorname{proj}_{\mathbf{v}} \mathbf{w} = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}|^2} \mathbf{v} = \left(\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}|} \right) \frac{\mathbf{v}}{|\mathbf{v}|} = \operatorname{comp}_{\mathbf{v}} \mathbf{w} \frac{\mathbf{v}}{|\mathbf{v}|}. \tag{2}$$

In fact, the scalar projection is just the *signed* magnitude of $\operatorname{proj}_{\mathbf{v}} \mathbf{w}$ since

1.2 Integration