MA 519: Homework 14

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Problem 14.1 (Handout 16, # 2)

A number N is chosen according to a Poisson distribution with mean 10. Then 5 numbers are chosen from $\{0, 1, \ldots, N\}$. Suppose X is the maximum of these 5 numbers.

What is $P(X > 10)$?

Solution. ■

Problem 14.2 (Handout 18, # 15)

(X, Y) is distributed uniformly inside of the unit circle. Find the density of $X + Y$ and hence the mean of $X + Y$. Was the value of the mean obvious? Why?

Solution. Suppose the random vector $(X, Y) \sim U\big[\{\,(x, y) : x^2 + y^2 < 1\,\}\big]$. ∎

Problem 14.3 (Handout 18, # 16)

Let X be a random number in $[0,1]$. What is the probability that the number 5 is completely missing from the decimal expansion of X?

Solution. Suppose X is picked randomly from the interval $\Omega := [0,1]$. Let A be the set of all real numbers in Ω without a 5 in their decimal expansion. We show that $P(X \in \Omega \setminus A) = 1$ by proving that $P(X \in A) = 0$.

To compute the probability of A we first decompose A as the limit of $\bigcap_{k=1}^{n} A_k$ where

$$A_k := \left\{ x \in \Omega : x = 0.n_1 n_2 \cdots n_k \cdots,\ n_k = 5 \right\}.$$

By properties of the probability measure, $P(A) = \lim_{k\to\infty} P(A_k)$ so we need to find an expression for $P(A_k)$. Let us begin with the case $k = 1$: ■

Problem 14.4 (Handout 18, # 17)

A foot long stick is broken into three pieces. Find the density functions of the length of the longest part, the smallest part, and the medium part. What are the expected values for each part?

Solution. ■