Instructor: Tatsunari Watanabe **Name:** ____________________.
TA: Carlos Salinas

MA 26500-215 Quiz 11

July 29, 2016

1. (6 points) Find the least squares solution $\bar{\mathbf{x}}$ of the system $A\bar{\mathbf{x}} = \bar{\mathbf{b}}$ where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ -1 & 0 \end{bmatrix}, \qquad \bar{\mathbf{b}} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$$

2. (4 points) Suppose that A and B are conjugate matrices. Show that if λ is an eigenvalue of A then it is an eigenvalue of B.

Solution: Suppose that λ is an eigenvalue of A and that A is conjugate to B. Then, λ is an eigenvalue of A means that there exists a vector (the associated eigenvector) $\mathbf{x}$ such that $A\mathbf{x} = \lambda\mathbf{x}$; while A is conjugate to B means that there exists an invertible matrix P such that $A = PBP^{-1}$. Thus,

$$PBP^{-1}\mathbf{x} = A\mathbf{x} = \lambda\mathbf{x}$$

so

$$\begin{aligned} PBP^{-1}\mathbf{x} &= \lambda\mathbf{x} \\ BP^{-1}\mathbf{x} &= P^{-1}\lambda\mathbf{x} \\ &= \lambda P^{-1}\mathbf{x} \end{aligned}$$

now let $\mathbf{y} = P^{-1}\mathbf{x}$ and we have

$$B\mathbf{y} = \lambda\mathbf{y}.$$

So λ is an eigenvalue of B with associated eigenvector $\mathbf{y} = P^{-1}\mathbf{x}$.

3. (8 points) Suppose that P is an idempotent matrix, i.e., $P^2 = I$. Show that the only possible eigenvalues for P are $\lambda = 0$ and $\lambda = 1$.