

MA571 Homework 11

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PROBLEM 11.1 (MUNKRES §53, EX. 7(ABCD))

Let G be a topological group with operation \cdot and identity element x_0 . Let $\Omega(G, x_0)$ denote the set of all loops in G based at x_0 . If $f, g \in \Omega(G, x_0)$, let us define a loop $f \otimes g$ by the rule

$$(f \otimes g)(s) = f(s) \cdot g(s).$$

- (a) Show that this operation makes the set $\Omega(G, x_0)$ into a group.
- (b) Show that this operation induces a group operation \otimes on $\pi_1(G, x_0)$.
- (c) Show that the two group operations $*$ and \otimes on $\pi_1(G, x_0)$ are the same. [*Hint:* Compute $(f * e_{x_0}) \otimes (e_{x_0} * g)$.]
- (d) Show that $\pi_1(G, x_0)$ is Abelian.

Proof.

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PROBLEM 11.2 ((A))

Prove Proposition F from the note on the Fundamental Group of the Circle.

Proof.



PROBLEM 11.3 ((B))

Prove Lemma G from the note on the Fundamental Group of the Circle. (Hint: one way to do this is to use the fact, which you don't have to prove, that if \sim is the equivalence relation on $[a, a + 1]$ which identifies a and $a + 1$ then the restriction of p induces a homeomorphism $[a, a + 1]/\sim \rightarrow S^1$.)

Proof.

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PROBLEM 11.4 ((C))

Show that for every point $x \in S^n$ the space $S^n - x$ is homeomorphic to \mathbf{R}^n . You may use the fact, shown in Step 1 of the proof of Theorem 59.3, that S^n with the *north pole* removed is homeomorphic to \mathbf{R}^n . (Hint: linear algebra.)

Proof.



PROBLEM 11.5 ((D))

Show that every loop in S^n which is not onto is path-homotopic to a constant path. (Hint: use Problem C).

Proof.



PROBLEM 11.6 ((E))

Let X be a topological space and let $A \subset X$ be a deformation retract. In the space X/A , the set A is a point (because it is an equivalence class). Show that this point is a deformation retract of X/A . (Hint: use p.289 # 9.)

Proof.

