# MA 26500-215    Quiz 2

July 29, 2016

1. (10 points)  Given the system of linear equations

$$x_1 - 2x_2 + x_3 - x_4 = 3$$
$$x_1 + x_2 + x_3 - x_4 = 1 \qquad (\star)$$
$$x_1 + x_3 - x_4 = 2$$

find its matrix representation and the reduced row-echelon form of that matrix.

A. $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

E. Not listed.

> **Solution:** Finding the augmented matrix representation of the system $(\star)$ is easy enough
>
> $$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 3 \\ 1 & 1 & 1 & -1 & 1 \\ 1 & 0 & 1 & -1 & 2 \end{array}\right].$$
>
> Performing the appropriate row operations on the matrix above gets you to the matrix in **D**.

2. (10 points)  Given the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & -1 \\ -1 & -4 & 0 \end{bmatrix}$$

and the vector $\mathbf{b} = \begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix}$, find the vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$ by finding $A^{-1}$:

A. $A^{-1} = \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & 1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

B. $A^{-1} = \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & -1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

C. $A^{-1} = \begin{bmatrix} 2 & 3 & -2 \\ 3 & 5 & -4 \\ 1 & 1 & -1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

D. $A^{-1} = \begin{bmatrix} 2 & 3 & -2 \\ 3 & 5 & -4 \\ 1 & 1 & -1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$

E. Does not exist; the matrix $A$ is singular.

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**Solution**: As you learned from Kolman and Hill, you can find the inverse of an $n \times n$ large matrix by augmenting it with the identity matrix $I$ on the right and performing Gaußian elimination on the left. So write

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & 5 & -1 & 0 & 1 & 0 \\ -1 & -4 & 0 & 0 & 0 & 1 \end{array}\right]$$

and after doing Gaußian elimination you get

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & 4 & 3 \\ 0 & 1 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & -3 & 2 & 1 \end{array}\right].$$

Then, the inverse of $A$ is

$$A^{-1} = \begin{bmatrix} -4 & 4 & 3 \\ 1 & -1 & -1 \\ -3 & 2 & 1 \end{bmatrix}.$$

Than only leaves **A** as the possible answer choice.