MA598: Lie Groups

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May 17, 2016

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CHAPTER 1

Prologue

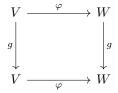
This summer, we will be making our way through Knapp's *Lie Groups Beyond an Introduction* [2] although, I (the writer of these notes) will occasionally refer to [1] for examples.

1.1 Representation of Finite Groups

Definitions

A representation of a finite group G on a finite-dimensional complex vector space V is a homomorphism $\rho \colon G \to \operatorname{GL}(V)$; we say that such a map ρ gives V the structure of a G-module. When there is little ambiguity about the map ρ we will call V itself as a representation of G; in this vein, we supress the symbol ρ and write gv for $\rho(g)(v)$. The dimension of V is sometimes called the degree of ρ .

A map φ between two representations V and W of G is a vector space map $\varphi \colon V \to W$ such that



commutes for every $g \in G$. (We will call this a G-linear map when we want to distinguish it from an arbitrary linear map between the vector spaces V and W). We can then define $\operatorname{Ker} \varphi$, $\operatorname{Im} \varphi$, and $\operatorname{Coker} \varphi$, which are also G-modules.

A subrepresentation of a representation V is a vector subspace W of V which is invariant under G. A representation V is called *irreducible* if there is no proper nonzero invariant subspace W of V.

Bibliography

- [1] B. Hall. Lie Groups, Lie Algebras, and Representations: An Elementary Introduction. Graduate Texts in Mathematics. Springer, 2003.
- $[2]\;$ A.W. Knapp. Lie Groups Beyond an Introduction. Progress in Mathematics. Birkhäuser Boston, 2002.