Fall 2016 Notes

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Chapter 1

Probability

We will devote this chapter to the material that is covered in MA 51900 (discrete probability) as it was covered in DasGupta's class. We will, for the most part, reference Feller's *An introduction to probability theory and its applications, Volume 1* [5] (especially for the discrete noncalculus portion of the class) and DasGupta's own book *Fundamentals of Probability: A First Course* [3].

1.1 Discrete Probability

The material in this chapter is mostly pulled from Sheldon Ross's *A First Course in Probability Theory* with some examples from [3] and [5]. I find Ross's book to be better structured than the latter two.

Combinatorial Analysis

These are the main results from this section.

Theorem 1.1 (The basic principle of counting)**.** *Suppose that two experiments are to be performed. Then if experiment 1 can result in any one of m possible outcomes and if, for each outcome of experiment 1, there are n possible outcomes of experiment 2, then together there are mn possible outcomes of the two experiments.*

Theorem 1.2 (The generalized principle of counting)**.** *If r experiments that are to be performed are such that the first one may result in any of n_1 possible outcomes; and if, for each of these n_1 possible outcomes, there are n_2 possible outcomes for the second experiment; and if, for each of the possible outcomes of the first two experiments, there are n_3 possible outcomes for the third experiment; etc. ..., then there is a total of $n_1 n_2 \cdots n_r$ possible outcomes of the r experiments.*

Using notation as in [5], the number

$$(n)_r = n(n-1)\cdots(n-r+1)$$

represents the number of different ways that a group of r items could be selected from n items when the order of selection is relevant, and as each group of r items will be counted $r!$ times in this count,

it follows that the number of different groups of r items that could be formed from a set of n items is

$$\frac{(n)_r}{r!} = \frac{n!}{(n-r)!r!}$$

for which we reserve the notation

$$\binom{n}{r}$$

read n choose r. (This is called a binomial coefficient since it appears in the binomial expansion $(a+b)^n$.)

A useful combinatorial identity on binomial coefficients is the following

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

for $1 \leq r \leq n$.

Theorem 1.3 (The binomial theorem)**.**

$$(a+b)^n = \sum_{i=1}^{n} \binom{n}{i} x^i y^{n-i}.$$

Proof. We provide a combinatiorial proof of the theorem. Consider the product

$$(a_1 + b_1) \cdots (a_n + b_n).$$

Its expansion consists of the sum of 2^n terms, each term being the product of n factors. Furthermore, each of the 2^n terms in the sum will contain as a factor either a_i or b_i for each $1 \leq i \leq n$. Now, how many of the 2^n terms in the sum will have k of the a_i and $n-k$ of the b_i as factors? As each term consisting of k of the a_i and $n-k$ of the b_i correspond to a choice of a group of k from the values $a_1, \ldots, a_n$, there are $\binom{n}{k}$ such terms. Thus, letting $a_i = a$, $b_i = b$, $1 \leq i \leq n$, we see that

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}.$$

■

Chapter 2

Introduction to Partial Differential Equations

Here we summarize some important points about PDEs. The material is mostly taken from Evans's *Partial Differential Equations* [4] with occasional detours to Strauss's *Partial Differential Equations: An Introduction* [7].

Chapter 3

Algebraic Geometry

A summary to a course on an introduction to sheaf cohomology. We will mostly reference Donu's notes available here `https://www.math.purdue.edu/~dvb/classroom.html`, but also cite Ravi Vakil's *Fundamentals of Algebraic Geometry* [8] available here `https://math216.wordpress.com/`.

3.1 The statement of de Rham's theorem

These are almost verbatim Arapura's notes on the de Rham Complex and cohomology.

Before doing anything fancy, let's start at the beginning. Let $U \subseteq \mathbf{R}^3$ be an open set. In calculus class, we learn about operations

$$\{\,\text{functions}\,\} \xrightarrow{\nabla} \{\,\text{vector fields}\,\} \xrightarrow{\nabla\times} \{\,\text{vector fields}\,\} \xrightarrow{\nabla\cdot} \{\,\text{functions}\,\}$$

such that $(\nabla\times)(\nabla) = 0$ and $(\nabla\cdot)(\nabla\times) = 0$. This is a prototype for a *complex.* An obvious question: does $\nabla \times v = 0$ imply that v is a gradient? Answer: sometimes yes (e.g. if $U = \mathbf{R}^3$) and sometimes no (e.g. if $U = \mathbf{R}^3$ minus a line).

Chapter 4

Algebraic Topology

From my meetings with Mark. We reference Hatcher's *Algebraic Topology* [6] freely available here `https://www.math.cornell.edu/~hatcher/#ATI`.

4.1 Cohomology

Bibliography

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