

MA 519: Homework 5

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PROBLEM 5.1 (HANDOUT 7, # 6(D, F))

Find the variance of the following random variables

- (d) $X = \#$ of tosses of a fair coin necessary to obtain a head for the first time.
- (f) $X = \#$ matches observed in random sitting of 4 husbands and their wives in opposite sides of a linear table.
This is an example of the *matching problem*.

SOLUTION. Let

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PROBLEM 5.2 (HANDOUT 7, # 8)

(Nonexistence of variance).

- (a) Show that for a suitable positive constant c , the function $p(x) = c/x^3$, $x = 1, \dots$, is a valid probability mass function (PMF).
- (b) Show that in this case, the expectation of the underlying random variable exists, but the variance does not!

SOLUTION. ■

PROBLEM 5.3 (HANDOUT 7, # 9)

In a box, there are 2 black and 4 white balls. These are drawn out one by one at random (without replacement).

- (a) Let X be the draw at which the first black ball comes out. Find the mean and the variance of X .
- (b) Let X be the draw at which the second black ball comes out. Find the mean* the variance of X .

SOLUTION. ■

*What is a mean? How do you pronounce mean? Is it mee-man or muh-man?

PROBLEM 5.4 (HANDOUT 7, # 10)

Suppose X has a *discrete uniform distribution* on the set $\{1, \dots, N\}$.

Find formulas for the mean and the variance of X .

SOLUTION. ■

PROBLEM 5.5 (HANDOUT 7, # 11)

(*Be Original*) Give an example of a random variable with mean 1 and variance 100.

SOLUTION.

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PROBLEM 5.6 (HANDOUT 7, # 13)

(*Be Original*). Suppose a random variable X has the property that its second and fourth moment are both 1.

What can you say about the nature of X ?

SOLUTION. ■

PROBLEM 5.7 (HANDOUT 7, # 14)

(Be Original). One of the following inequalities is true in general for all nonnegative random variables. Identify which one!

$$\begin{aligned} E(X)E(X^4) &\geq E(X^2)E(X^3); \\ E(X)E(X^4) &\leq E(X^2)E(X^2). \end{aligned}$$

SOLUTION. ■

PROBLEM 5.8 (HANDOUT 7, # 15)

Suppose X is the number of heads obtained in 4 tosses of a fair coin.

Find the expected value of the weird function

$$\log\left(2 + \sin\left(\frac{\pi}{4}x\right)\right).$$

SOLUTION.

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PROBLEM 5.9 (HANDOUT 7, # 16)

In a sequence of Bernoulli trials let X be the length of the run (of either successes or failures) started by the first trial.

- (a) Find the distribution of X , $E(X)$, $\text{Var}(X)$.

SOLUTION.

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PROBLEM 5.10 (HANDOUT 7, # 17)

A man with n keys wants to open his door and tries the keys independently and at random. Find the mean and variance of the number of trials

- (a) if unsuccessful keys are not eliminated from further selections;
- (b) if they are.

(Assume that only one key fits the door. The exact distributions are given in II, 7, but are not required for the present problem.)

SOLUTION.

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