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MA 26500-215 Quiz 1

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1. (10 points) Using the method of elimination, determine if the following system of equations has a solution, no solution or many solutions:

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= -4 \\ 2x_1 + x_2 - 3x_3 &= 4. \end{aligned} \tag{$\star$}$$

Solution: By looking at the number of free variables vs. equations, we can already tell that the system will either have no solutions or many solutions (such a system is said to be *underdetermined*).

Take the first equation and subtract it from the second equation to get

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= -4 \\ x_1 - x_2 &= 8. \end{aligned}$$

Then $x_1 = 8 + x_2$. Now, substitute this into the first equation

$$(8 + x_2) + 2x_2 - 3x_3 = -4$$

so $3x_2 = -4 - 8 + 3x_3$ or $x_2 = -4 + x_3$. Thus

$$\begin{aligned} x_1 &= 8 + x_2 \qquad x_2 = -4 + x_3. \\ &= 8 - 4 + x_3 \\ &= 4 + x_3 \end{aligned} \tag{$\blacklozenge$}$$

Therefore, the system has **many solutions**; you give me a value of x_3 and, using the equations in ($\blacklozenge$), I can find values for x_1 and x_2 that solve the system ($\star$).

2. (5 points) Given the matrices

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

find

$$2A + 3B^{\intercal}.$$

Solution: The calculation is straightforward

$$
\begin{aligned}
2A + 3B^{\intercal} &= 2\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{\intercal} \\
&= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix} \\
&= \begin{bmatrix} 2+3 & -2+0 \\ -2+3 & 2+3 \end{bmatrix} \\
&= \begin{bmatrix} 5 & -2 \\ 1 & 5 \end{bmatrix}.
\end{aligned}
$$