MA 544: Homework 6

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Problem 6.1 (Wheeden & Zygmund §4, Ex. 4)

Let f be defined and measurable in $\mathbf{R}^n$. If T is a nonsingular linear transformation of $\mathbf{R}^n$, show that $f(T\mathbf{x})$ is measurable. [If $E_1 = \{\, \mathbf{x} \mid f(\mathbf{x}) > a \,\}$ and $E_2 = \{\, \mathbf{x} \mid f(T\mathbf{x}) > a \,\}$.]

Proof. ∎

Problem 6.2 (Wheeden & Zygmund §4, Ex. 7)

Let f be usc and less that $+\infty$ on a compact set E. Show that f is bounded above on E. Show also that f assumes its maximum on E, i.e., that there exists $\mathbf{x}_0 \in E$ such that $f(\mathbf{x}_0) \geq f(\mathbf{x})$ for all $\mathbf{x} \in E$.

Proof. ∎

Problem 6.3 (Wheeden & Zygmund §4, Ex. 8)

(a) Let f and g be two functions which are usc at $\mathbf{x}_0$. Show that $f+g$ is usc at $\mathbf{x}_0$. If $f-g$ usc at $\mathbf{x}_0$? When is fg usc at $\mathbf{x}_0$?

(b) If $\{f_k\}$ is a sequence of functions are usc at $\mathbf{x}_0$, show that $\inf f_k(\mathbf{x})$ is usc at $\mathbf{x}_0$.

(c) If $\{f_k\}$ is a sequence of functions which are usc at $\mathbf{x}_0$ and which converge uniformly near $\mathbf{x}_0$, show that $\lim f_k$ is usc at $\mathbf{x}_0$.

Proof. ■