

MA 166: Quiz 9

TA: Carlos Salinas

March 30, 2016

You have **15 minutes** to complete this quiz. You may work in groups, but you are not allowed to use any other resources.

Problem 1. Determine whether the series is convergent or divergent. You must state clearly what test you are using and verify the conditions of the test are satisfied.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n}}.$$

Problem 2. Determine whether the series is absolutely convergent, conditionally convergent, or divergent. You must justify your answer.

$$\sum_{n=1}^{\infty} \frac{(n+1)!}{3^n}.$$

Solutions

Solution to Problem 1. This series is convergent by the alternating series test. Set $a_n := n^{-1/3}$. Note that $1/a_n = n^{1/3} < (n+1)^{1/3} = 1/na_{n+1}$ so $a_{n+1} \leq a_n$ and (ii) $\lim_{n \rightarrow \infty} a_n = 0$. Hence, the series converges. \odot

Solution to Problem 2. Set $a_n := (n+1)!/3^n$ and use the ratio test. By the ratio test, we have

$$\frac{a_{n+1}}{a_n} = \frac{(n+2)!/3^{n+1}}{(n+1)!/3^n} = \frac{(n+2)(n+1)!}{3(n+1)!} = \frac{n+2}{3}$$

which goes to ∞ as $n \rightarrow \infty$ so the ratio test says that the series is divergent. \odot