
Midterm 1

1. Legibly print your name below.
2. Do not open this test booklet until you are directed to do so.
3. You are not allowed to use a calculator on this exam.
4. You will have **60** minutes to complete the exam. If you finish early you may leave. However, I would recommend you go back and check your work.
5. Use blank spaces for your computations or whatever you need to write down.
6. Circle your answers clearly. Choose **ONLY ONE** answer for each question, unless stated otherwise.

Problem	Points	Problem	Points
1		6	
2		7	
3		8	
4		9	
5		10	

Total

I have adhered to the Purdue Academic Integrity and Honesty Code (Section B.2.a, Code of Student Conduct) in completing this assignment.

Signed: _____ Print Name: _____

1. Consider the system $A\mathbf{x} = \mathbf{b}$, where \mathbf{b} is some 3×1 column vector and

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix}.$$

Determine which of the following statements are true.

- (I) The system is inconsistent.
- (II) The system is consistent.
- (III) The system has infinitely many solutions.
- (IV) The matrix A is row equivalent to the 3×3 identity matrix.
- (V) The system has a unique solution.

A. (I) only

B. (II) and (V)

C. (II) and (III)

D. (II), (IV), and (V)

E. (IV) only

2. Which matrix is a linear combination of $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$?

A. $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 2 \\ 0 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

E. None of the above

3. If $(2A)^T = \begin{bmatrix} 4 & 2 \\ 6 & 8 \end{bmatrix}$, then A equals

A. $\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 6 \\ 8 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

E. None of the above

4. Consider the system

$$\begin{aligned}x + y + z &= 2 \\x + 2y + z &= 3. \\2x + 3y + (a^2 - 5)z &= a\end{aligned}$$

For which value of a does the system have infinitely many solutions?

A. $a = 5$

B. $a = \sqrt{7}$

C. $a = -\sqrt{7}$

D. $a = -5$

E. No such value of a exists.

5. Evaluate the determinant

$$\begin{vmatrix} 2 & 1 & -4 & 6 \\ 4 & 0 & 2 & 0 \\ -3 & 0 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{vmatrix}$$

A. 0

B. -64

C. 64

D. 88

E. 1

6. Which of the following matrices are invertible (nonsingular)?

$$I. \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$II. \begin{bmatrix} 3 & 0 & 8 \\ 5 & 0 & 11 \\ 12 & 0 & 3 \end{bmatrix}$$

$$III. \begin{bmatrix} 0 & 4 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

- A. I only
- B. I and III
- C. III only
- D. All of the Above
- E. None of the Above

7. Which of the following sets of vectors are subspaces of \mathbb{R}^3 ?

I. The set of all vectors of the form $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, where $x + 2y = x + z$.

II. The set of all vectors of the form $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, where $x + y + z \geq 0$.

III. The set consisting of only the zero vector $\mathbf{0}$.

- A. I only
- B. II only
- C. III only
- D. I and III
- E. All of of the Above

8. Consider the linear system

$$\begin{aligned}x - 2y + 3z &= 3 \\4x + 5y - z &= -1. \\6x + y + 5z &= 5\end{aligned}$$

Find the value of r so that $x = 2$, $y = -2$, $z = r$ is a solution to the above system.

A. $r = 0$

B. $r = 1$

C. $r = -1$

D. $r = 2$

E. None of the above

9. Which of the following matrices corresponds to the elementary row operation given by subtracting three times the 1st row from the 2nd row?

A. $\begin{bmatrix} -3 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

E. None of the above

10. Find the $(2,3)$ -th entry of the inverse of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 2 & 4 \end{bmatrix}$.

A. $\frac{-3}{5}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{1}{5}$

E. $\frac{-1}{5}$