## MA 523: Homework 3

Carlos Salinas

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CARLOS SALINAS PROBLEM 3.1

## Problem 3.1

Consider the initial value problem

$$u_t = \sin u_x; \qquad u(x,0) = \frac{\pi}{4}x.$$

Verify that the assumptions of the Cauchy–Kovalevskaya theorem are satisfied and obtain the Taylor series of the solution about the origin.

Solution.

CARLOS SALINAS PROBLEM 3.2

## Problem 3.2

Consider the Cauchy problem for u(x,y)

$$u_y = a(x, y, u)u_x + b(x, y, u)$$
$$u(x, 0) = 0$$

Let a and b be analytic functions of their arguments. Assume that  $D^{\alpha}a(0,0,0) \geq 0$  and  $D^{\alpha}b(0,0,0) \geq 0$  for all  $\alpha$ . (Remember by definition, if  $\alpha = 0$  then  $D^{\alpha}f = f$ .)

- (a) Show that  $D^{\beta}u(0,0) \geq 0$  for all  $|\beta| \leq 2$ .
- (b) Prove that  $D^{\beta}u(0,0) \geq 0$  for all  $\beta = (\beta_1, \beta_2)$ . (*Hint:* Argue as in the proof of the Cauchy–Kovalevskaya theorem; i.e., use induction in  $\beta_2$ )

SOLUTION.

CARLOS SALINAS PROBLEM 3.3

## Problem 3.3

(Kovalevskaya's example) Show that the line  $\{t=0\}$  is characteristic for the heat equation  $u_t=u_{xx}$ . Show there does not exist an analytic solution of the heat equation in  $\mathbf{R} \times \mathbf{R}$ , with  $u=1/(1+x^2)$  on  $\{t=0\}$ . (*Hint:* Assume there is an analytic solution, compute its coefficients, and show that the resulting power series diverges in any neighborhood of (0,0).)

SOLUTION.

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