Proof. Let $x_m \to x_0$.

$$\begin{aligned}\lim_{m\to\infty} g(x_m) &= \lim_{m\to\infty} \sup_{n\geq 1} f_n(x_m) \\ &= \sup_{n\geq 1} \lim_{m\to\infty} f_n(x_m) \\ &= \sup_{n\geq 1} f_n(x) \\ &= g(x).\end{aligned}$$

I dont know why we can swtich the order of lim and sup. □