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Gautam Buddha University

Mid Semester Examinations

M.Sc. Applied Mathematics First Semester, (September, 2013)

Course Name: Linear Algebra — Maximum Marks: 50

Course Code: MA-401 — Time: 2:00 Hours

Q.1. Attempt ALL parts of the following: $(5 \times 2 = 10)$

(a) Find the dimension of the vector space $\mathbb{C}(\mathbb{R})$ (with usual notations).

(b) Is vector $(3,-1,0,-1)$ spanned by the vectors $(2,-1,3,2)$, $(-1,1,1-3)$, and (1,1,9,-5)?

(c) Find three vectors in $\mathbb{R}^3$ which are linearly dependent, and are such that any two of them are linearly independent.

(d) Is the mapping $T:\mathbb{R}^2 \to \mathbb{R}^2$ linear where $T(x,y) = (\sin x, y)$.

(e) Find the range, rank, null space, and nullity for the zero transformation and the identity transformation on a finite dimensional vector space V.

Q.2. Attempt ALL parts of the following: $(2 \times 5 = 10)$

(a) Let V be the set of pairs (x,y) of real numbers and let $\mathbb{F}$ be the field of real numbers. Given $(x_1,y_1)+(x_2+y_2) = (x_1+x_2,0)$ and $c(x,y) = (cx,0)$. Is V, with these operations, a vector space?

(b) Prove that number of elements in a basis of a vector space V are unique.

Q.3. Attempt ALL parts of the following: $(2 \times 5 = 10)$

(a) Let V be a vector space over the field F. Show that, the intersection of any two subspaces of V is subspace of V.

(b) Show that the vectors $(1,0,-1)$, $(1,2,1)$, and $(0,-3,2)$ form a basis for $\mathbb{R}^3$. Express each of the standard basis vectors as linear combinations of these three vectors.

Q.4. Attempt ALL parts of the following: $(2 \times 5 = 10)$

(a) State and Prove Rank Nullity Theorem.

(b) If T be the unique linear operator on $\mathbb{C}^3$ for which $T(e_1) = (1,0,i)$, $T(e_2) = (0,1,1)$, $T(e_3) = (i,1,0)$. Is T invertible?

Q.5. Attempt ALL parts of the following: $(2 \times 5 = 10)$

If T be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by $T(x,y,z) = (x+y, 2z-x)$. Then

(a) If B is the standard ordered basis for $\mathbb{R}^3$ and B' is the ordered basis for $\mathbb{R}^2$, what is the matrix of T relative to pair B, B' .

(b) If $B = \{(1,0,-1),(1,1,1),(1,0,0)\}$ and $B' = \{(0,1),(1,0)\}$, what is the matrix of T relative to pair B, B'.