Roll No. | | | | | | | | |

Gautam Buddha University

Mid Semester Examinations

Ph.D. (Applied Mathematics) Odd Semester, (November, 2013)

Course Name: Linear Algebra — Maximum Marks: 25

Course Code: MA-601 — Time: 2:00 Hours

Q.1. Find all values of k for which the resulting linear system has (i) no solution (ii) unique solution (iii) infinitely many solutions.

$$\begin{aligned} x + y - z &= 2 \\ x + 2y + z &= 3 \\ x + y + (k^2 - 5)z &= k \end{aligned}$$

Q.2. Suppose $A = (a_{ij})_{m \times n}$. Now regarding A as a linear map from

$$\mathbb{R}^n \to \mathbb{R}^m$$

Show that

(a) S, the set of all solutions of homogeneous system $AX = 0$, $x \in \mathbb{R}^n$ is a subspace of $\mathbb{R}^n$ and hence it is precisely the kernel of A.

(b) $\mathcal{R}(A)$, the range of A is the subspace of $\mathbb{R}^m$, and hence it is precisely the column space of A.

Q.3. Let $S = \{v_1, v_2, v_3, v_4\}$, where

$$v_1 = (1, -2, 0, 3, -4), \quad v_2 = (3, 2, 8, 1, 4), \quad v_3 = (2, 3, 7, 2, 3), \quad v_4 = (-1, 2, 0, 4, -3),$$

and let $V = span(S)$. Then find a basis using row operation, for V and hence the dimension of V.

Q.4. (a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$. Verify that the trace equals the sum of the eigenvalues, and determinant equals their product.

(b) With the same above matrix A, solve the differential equation $\frac{du}{dt} = Au$, $u(0) = \begin{bmatrix} 0 & 6 \end{bmatrix}^T$. What are the two pure exponential solutions.

Q.5. Let V be the vector space consisting of all functions of the form

$$\alpha e^{2x} \cos x + \beta e^{2x} \sin x$$

Consider the following linear transformation $L : V \to V$:

$$L(f) = f' + f$$

(a) Find the matrix representing L with respect to the basis $\{e^{2x} \cos x, e^{2x} \sin x\}$.

(b) Use your answer from part (a) to find one solution to the following differential equation:

$$y' + y = e^{2x} \cos x$$