you must give the `\paper` **command**

question@11 question1@object11

Gautam Buddha University

Mid Semester Examinations

5 Years Integrated B.Tech.+ M.Tech./M.B.A.
(Biotechnology) Semester-II, (March 2012)

Course Name: Mathematics-II — Maximum Marks: 50
Course Code: MA-104 — Time: 2:00 Hours

Q.1. Attempt ALL parts of the following: $(10 \times 2.5 = 25)$

(a) Discuss the continuity of the function f given by $f(x) = |x|$ at $x = 0$.

(b) Find the derivative of $f(x) = \tan^{-1} x$. (Do not use direct formula)

(c) Differentiate $\cos(\log x + e^x)$, $x > 0$ with respect to x.

(d) Find $\dfrac{dy}{dx}$ for $\sin^2 y + \cos xy = k$, where k is some constant.

(e) Differentiate the following w.r.t. x,

$$\left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$$

(f) Find $\dfrac{dy}{dx}$, if $x = a(t + \sin t)$, $y = a(1 - \cos t)$

(g) If $y = 3e^{2x} + 2e^{3x}$, prove that $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0$

(h) Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

(i) If $y = \tan^{-1} x$, prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$

(j) Expand $\log x$ in powers of $(x-1)$ by Taylor's theorem and hence find the value of $\log 1.1$

Q.2. Attempt ALL parts of the following: $(10 \times 2.5 = 25)$

(a) $\displaystyle\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

(b) $\displaystyle\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

(c) $\displaystyle\int \sin^3(2x+1) dx$

(d) $\displaystyle\int \cos 2x \cos 4x \cos 6x \, dx$

P.T.O.

(e) Show that $\displaystyle\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

(f) $\displaystyle\int \frac{3x-1}{(x-1)(x-2)(x-3)}dx$

(g) $\displaystyle\int \frac{x\,\sin^{-1}x}{\sqrt{1-x^2}}dx$

(h) $\displaystyle\int_0^{\pi/4} \sin^3 2x\,\cos 2x\,dx$

(i) $\displaystyle\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx$

(j) Use integration to find the area enclosed by the circle $x^2+y^2=a^2$