



Introduction to the Fundamentals of Particle Accelerators

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Overview Of This Talk

- My background
- Uses for particle accelerators
- How to accelerate charged particles
- Longitudinal dynamics fundamentals
- Modern particle accelerator types
- Overview of Fermilab accelerator complex
- Beam optics fundamentals

Drawings and diagrams pulled heavily from the Fermilab Operations Department Concepts Rookie Book, a training manual for new Operators for which I was a co-author:

<http://beamdocs.fnal.gov/AD-public/DocDB>ShowDocument?docid=4444>



My Background



- Summer Intern in Fermilab Antiproton Source Target Hall group for 3 summers
- B.S. in Engineering Physics @ University of Illinois
- Main Control Room Operator
- M.S. in Accelerator Physics/Technology from Indiana University
 - via U.S. Particle Accelerator School (“USPAS”)
- Accelerator Division External Beamlines Dept., Engineering Physicist
- Accelerator Division Controls Dept., ACORN project and Robotics



United States Particle Accelerator School

Fermilab runs the United States Particle Accelerator School (“USPAS”) in cooperation with major U.S. universities. Students can attend specific courses of interest and may attain graduate credit if desired. Many HEP physicists use USPAS as an opportunity to learn about how the accelerators work. Each USPAS session is hosted by a different university every 6 months, and lasts for two weeks.

<http://uspas.fnal.gov/index.shtml>



Uses for Particle Accelerators

An incomplete list:

- Experimental High-Energy Particle Physics
- Industrial
 - Rubber galvanizing
 - Food sterilization
 - Waste-water sanitization
 - Semi-conductor manufacture (ion implantation and x-ray lithography)
 - Accelerator-driven sub-critical nuclear reactors
- Material Science
 - Radiation damage
- Imaging
 - High-speed imaging of explosives (proton radiography at LANL)
 - Live biological processes (accelerator-produced x-rays at Argonne APC)
 - Electron radiography for nuclear fissile contraband in shipping containers
 - Proton medical tomography imaging
- Cancer therapy
 - Direct radiation therapy (photons, electrons, protons, neutrons, heavy ions)
 - Medical isotope production for imaging and treatment (molybdenum 99)

Industrial Applications of Accelerators (USPAS):

<http://uspas.fnal.gov/programs/2018/msu/18MSUHistory.shtml>



How to Accelerate Charged Particles

Lorentz Force Equation:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

The force on a charged particle is:

- Proportional to its charge
- Parallel to an electric field applying the force
- Perpendicular to both an external magnetic field applying the force and the direction of motion of the particle

Acceleration means a change in a particle's velocity vector. This can mean either or both of:

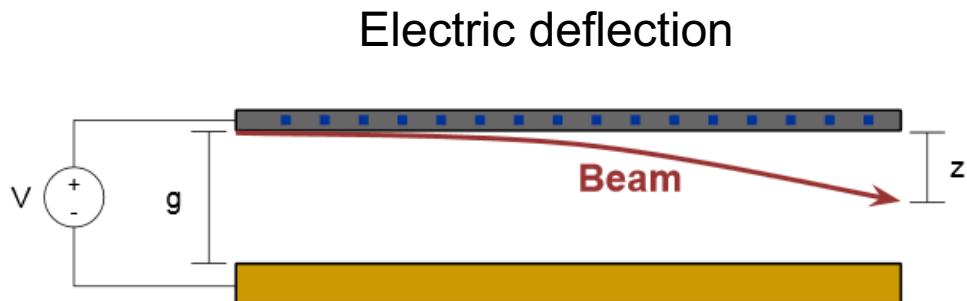
- A change in speed (velocity vector magnitude)
- A change in travel direction (velocity vector direction)

How to Accelerate Charged Particles

Lorentz Force Equation:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

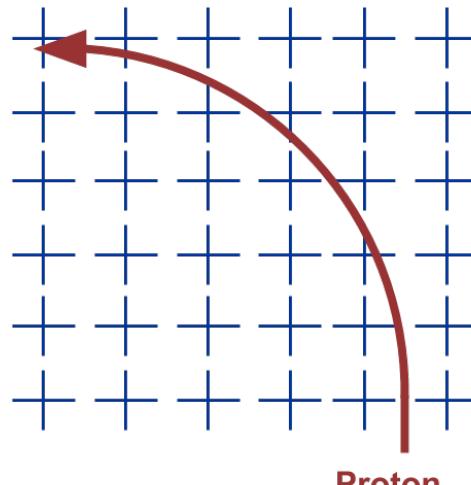
Both an Electric and a Magnetic field can “bend” a particle's trajectory:



$$Z_{deflect} = \frac{qVL^2}{2g\beta^2 E_{particle}}$$

Magnetic deflection

Magnetic field into page



$$R_{bend} = \frac{p}{qB}$$

How to Accelerate Charged Particles

However, trajectory deflection is not usually the type of acceleration we mean when we call something a “particle accelerator”. We typically mean the first definition, i.e. a change in the particle's velocity, and thus its kinetic energy.

Work-Energy Theorem:

$$W = \Delta T$$

Work: $W = \int \vec{F} \cdot \vec{v} dt$ Kinetic energy: $T = \frac{1}{2}m|\vec{v}|^2$

The Work-Energy theorem therefore translates as:

A force parallel to a particle's direction of motion is required to change its kinetic energy.

How to Accelerate Charged Particles

Can either a static electric or magnetic field change a particle's kinetic energy?

Magnetic field:

$$W = \int q(\vec{v} \times \vec{B}) \cdot \vec{v} dt = 0$$

Electric field:

$$W = \int q\vec{E} \cdot \vec{v} dt$$

No. The force from a static magnetic field is always perpendicular to the particle velocity, so cannot do work to change the kinetic energy.

Yes. The component of the electric field that is parallel to the particle velocity can do work to change the kinetic energy.

Caveat: Faraday's law shows that a time-varying magnetic field can induce an electric field, and is thus able to change particle kinetic energy. This is the operating principle of the "betatron" type of accelerator.



Relativistic energy and momentum

A particle's total energy is comprised of its intrinsic "rest energy" and its kinetic energy.
Often rest energy is also referred to as "rest mass".

$$E_{total} = E_{rest} + E_{kinetic}$$

We use the "electron-volt", or "eV" unit of energy for convenience. An electron passing through a potential difference of 1 Volt gains 1 eV of kinetic energy, or 1.602E-19 J. Metric multipliers apply:
 $1E6 \text{ eV} = 1 \text{ MeV}$, etc.

Rest energy is intrinsic to the particle type (proton = 938 MeV, electron = 0.511 MeV).
Another relationship for the total energy is the following:

$$E_{total} = \gamma mc^2$$

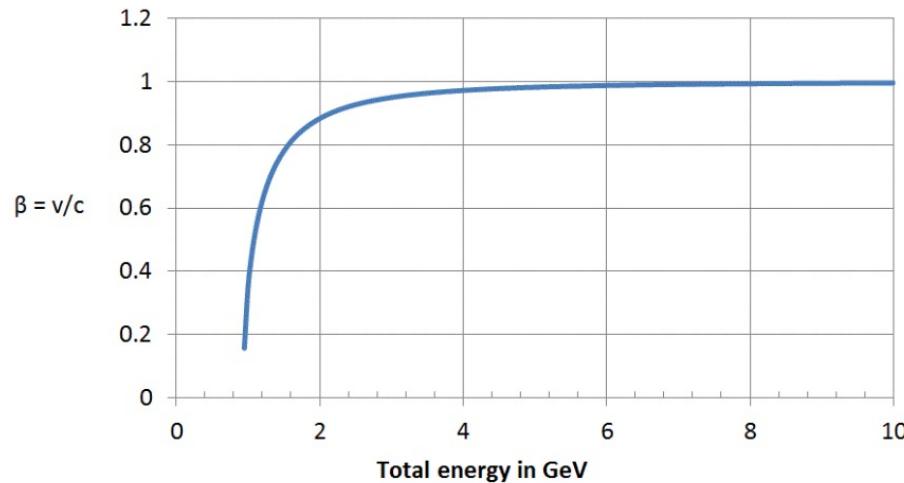
Where the "Lorentz" relativistic factors are functions of the particle velocity compared to the speed of light in vacuum "c":

$$\beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Relativistic energy and momentum

As an accelerator increases a particle's kinetic energy, and thus its total energy, the velocity asymptotically approaches the speed of light.

Proton velocity vs. total energy



For example, the highest-energy beam at Fermilab is the 120 GeV proton beam coming out of the Main Injector. This corresponds to a velocity of 99.997% the speed of light in vacuum.

At low energy, particles with slightly different energy have noticeably different velocity.

At high energy, the velocities are all about the same.

Electrostatic Acceleration

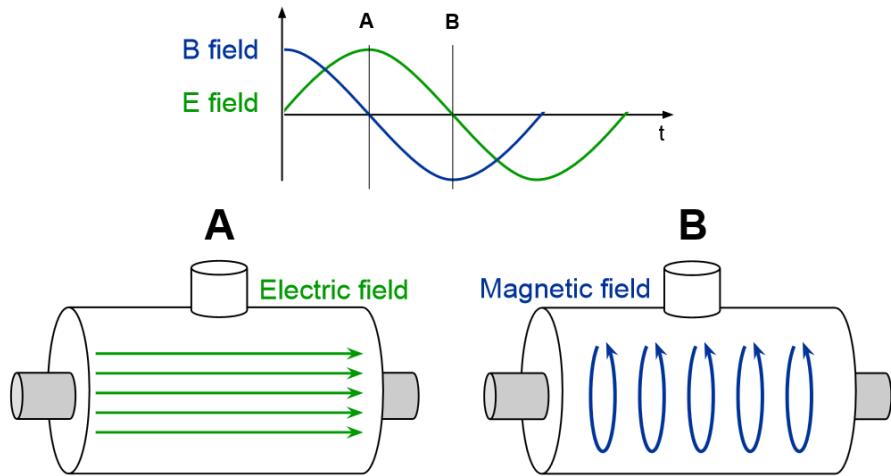
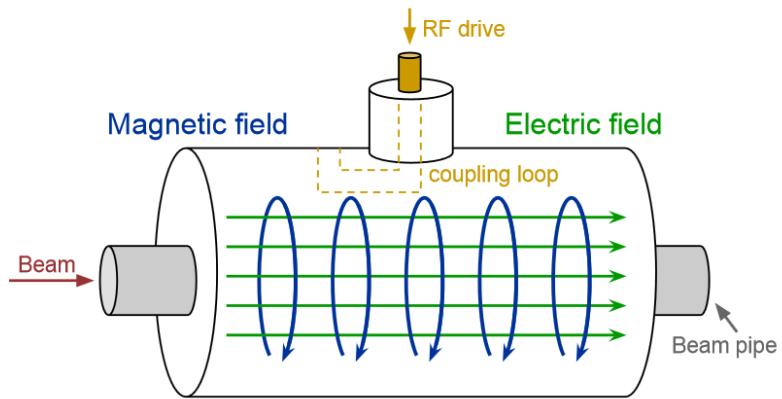
Static electric fields can be used to accelerate charged particles. The start of the Fermilab beam used to use a 35 kV “Cockcroft-Walton” accelerator to accelerate hydrogen ions up to 35 KeV.

Electrostatic accelerator becomes prohibitively difficult at higher beam energies: a 120 GeV electrostatic accelerator would require a potential of 120 billion volts. The dielectric strength of air is about 3E6 volts/meter, so the accelerator's anode and cathode would have to be separated by *40 kilometers* (almost 25 miles) to prevent arcing. This number is “only” ~3 km in vacuum.



Radio-frequency Cavities

Modern high-energy accelerators typically use standing-wave electromagnetic resonant cavities to accelerate particles. The simplest form, shown below and called a “pillbox cavity”, is simply a conducting hollow structure sized to propagate a TEM electromagnetic wave.



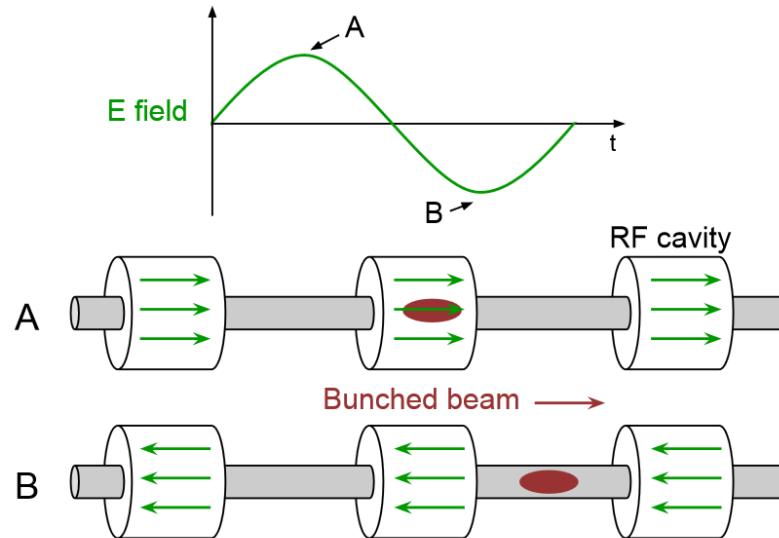
The resulting standing wave provides an electric field of very high magnitude that accelerates particles forward. The electric and magnetic fields oscillate in amplitude (and direction) with a fixed phase relationship to each other. Notice that for the TEM fundamental mode pictured, the E and B fields are 90 degrees out of phase.

Radio-frequency Cavities

For maximal acceleration, particles should only arrive in an RF cavity when the oscillating electric field is strongest and pointing in the same direction as the particle velocity. For this to occur, the RF frequency f_0 times the distance L between cavities must be an integer n multiple of the particle velocity v :

$$f_0 L = nv$$

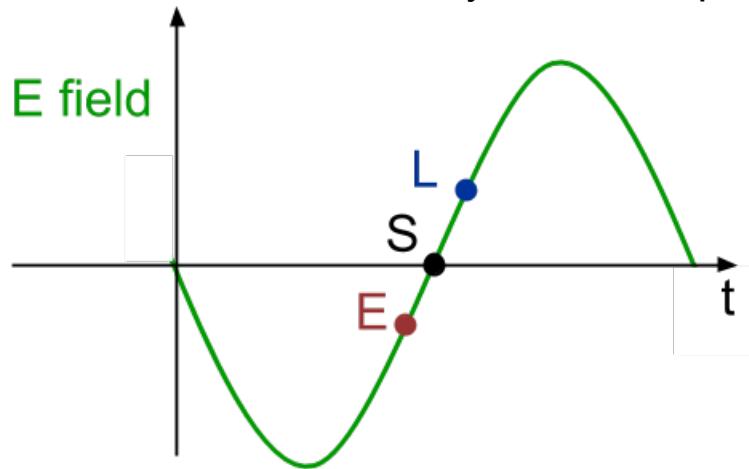
This is visually represented below, and is called the “synchronicity condition”. Particles with velocity outside this relationship will not experience maximal acceleration, and may even be decelerated if they are far enough away from synchronicity.



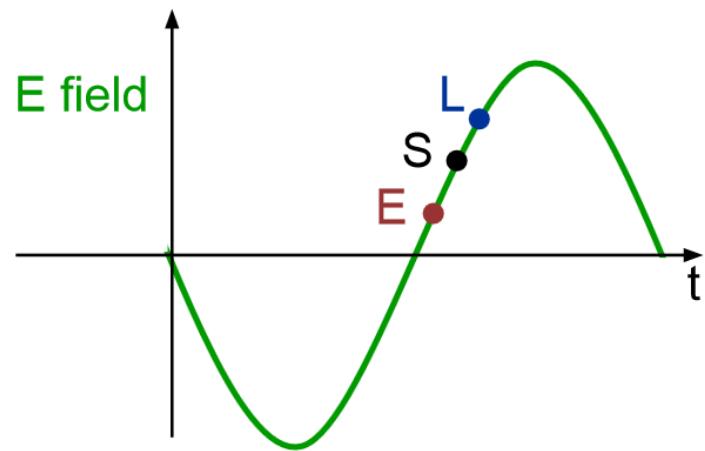
RF Phase Focusing and Bunching

The electric field magnitude in an RF cavity oscillates sinusoidally, which provides for a feature known as “phase focusing” where particles with slightly asynchronous arrival times are corrected by the RF curve.

Consider three particles that arrive in an RF cavity at slightly different times (or equivalently, phases in the E-field oscillation). One particle arrives at the correct time and is considered the synchronous (“S”) reference particle. The other two are slightly early (“E”) and slightly late (“L”). The varying E-field each particle sees encourages them to “bunch” up around the synchronous phase, i.e. “phase focusing”.



No net acceleration of the beam, just bunching

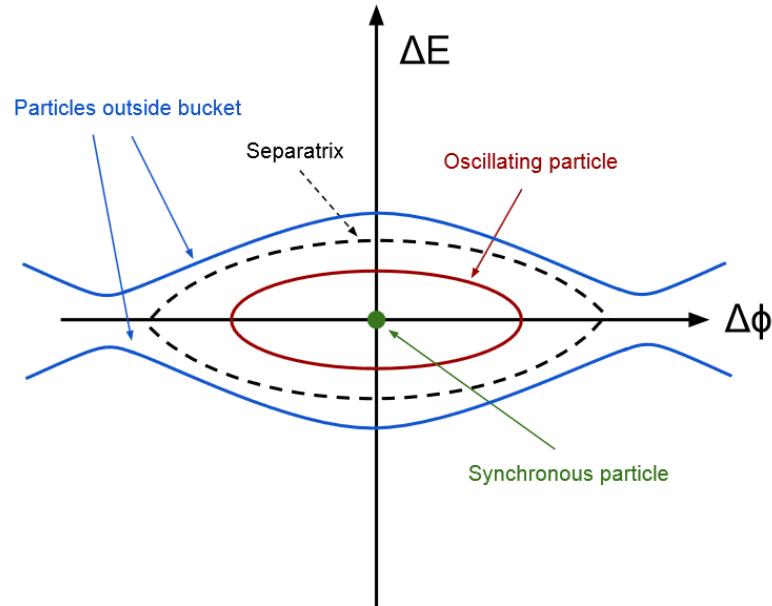
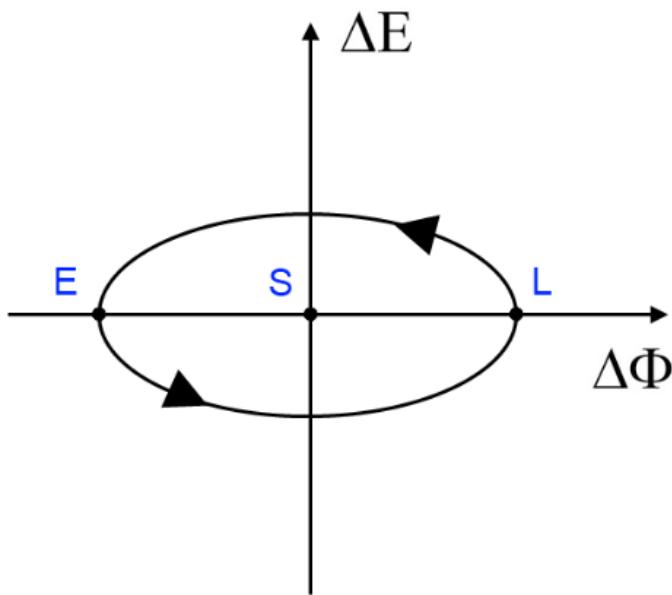


Net acceleration of the beam

Longitudinal Phase Space

Phase focusing causes every particle (except the synchronous particle) to undergo time-energy oscillation about the synchronous energy and phase. This is known as “synchrotron oscillation”, and can be visualized in longitudinal phase space as an approximately elliptical path (i.e. quasi-linear oscillation).

There is a maximum deviation in energy and phase (or time) that a particle can have and still be phase-focused. The line in phase space that separates stable (phase focused) and unstable oscillation is called the “separatrix”. The area inside the separatrix is referred to as the “bucket”. Practically speaking, more available RF voltage means a larger bucket area, and thus more beam that can be accelerated cleanly.

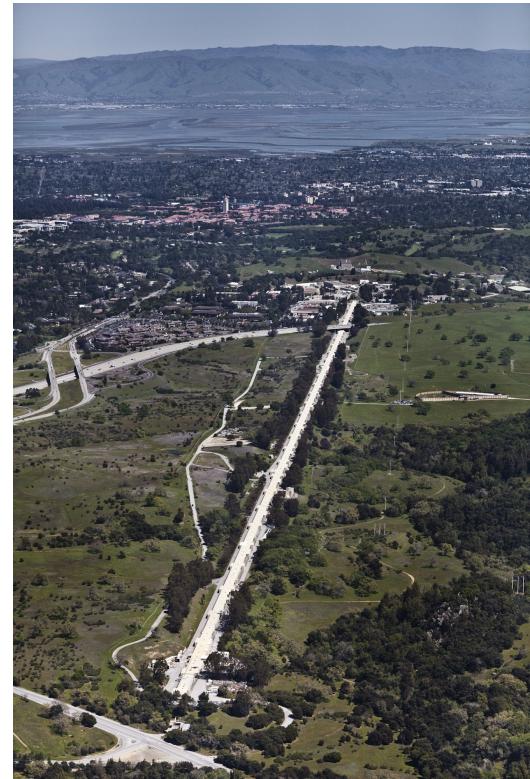


Linear Accelerator

Linear accelerators, or “Linacs”, are primarily composed of RF cavities to maximize the amount of acceleration provided. Each RF cavity only has one chance to accelerate a given particle, so every extra meter of space is typically taken up by an RF cavity. Magnets take up space between RF cavities to provide trajectory corrections and beam focusing. Linacs are usually very straight, so large magnets are not typically needed.



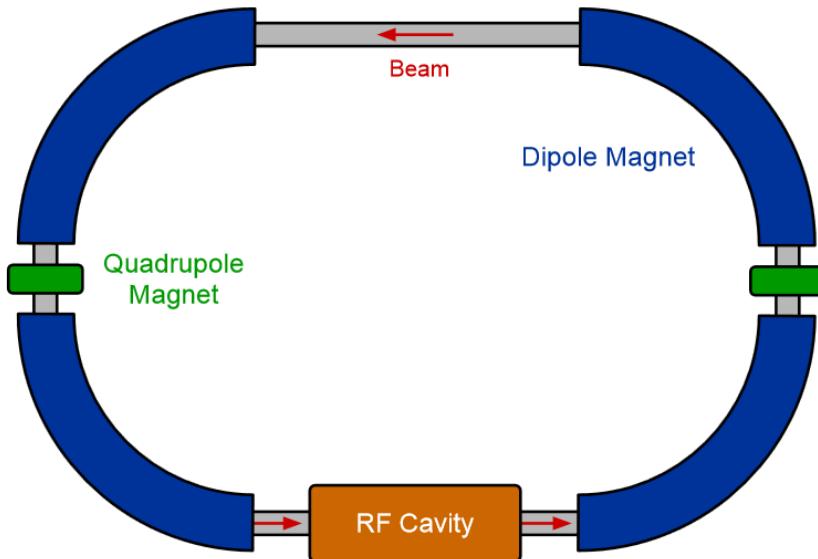
Fermilab Linac. 400 MeV H⁻ ions. Source: fnal.gov



Stanford Linear Accelerator
50 GeV e+/e-, 2 miles long
Source: slac.stanford.edu

Synchrotron

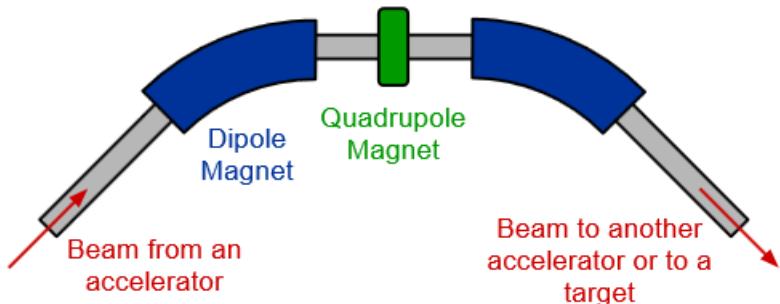
Synchrotrons are modern circular accelerators whose RF cavities repeatedly add kinetic energy to the particles with every pass around the ring. To maintain synchronicity with the particle bunches, the RF frequency must increase as beam kinetic energy increases. The beam becomes more difficult to bend with magnetic fields at higher kinetic energy, so the dipole magnet strength must also increase. Thus the beam kinetic energy, RF frequency, and dipole field must all remain synchronized throughout the acceleration cycle. Synchrotrons are currently the highest-energy particle accelerators, and can either provide beam to fixed-target experiments or serve as colliders themselves.



Main Injector. Source: fnal.gov

Beamlines

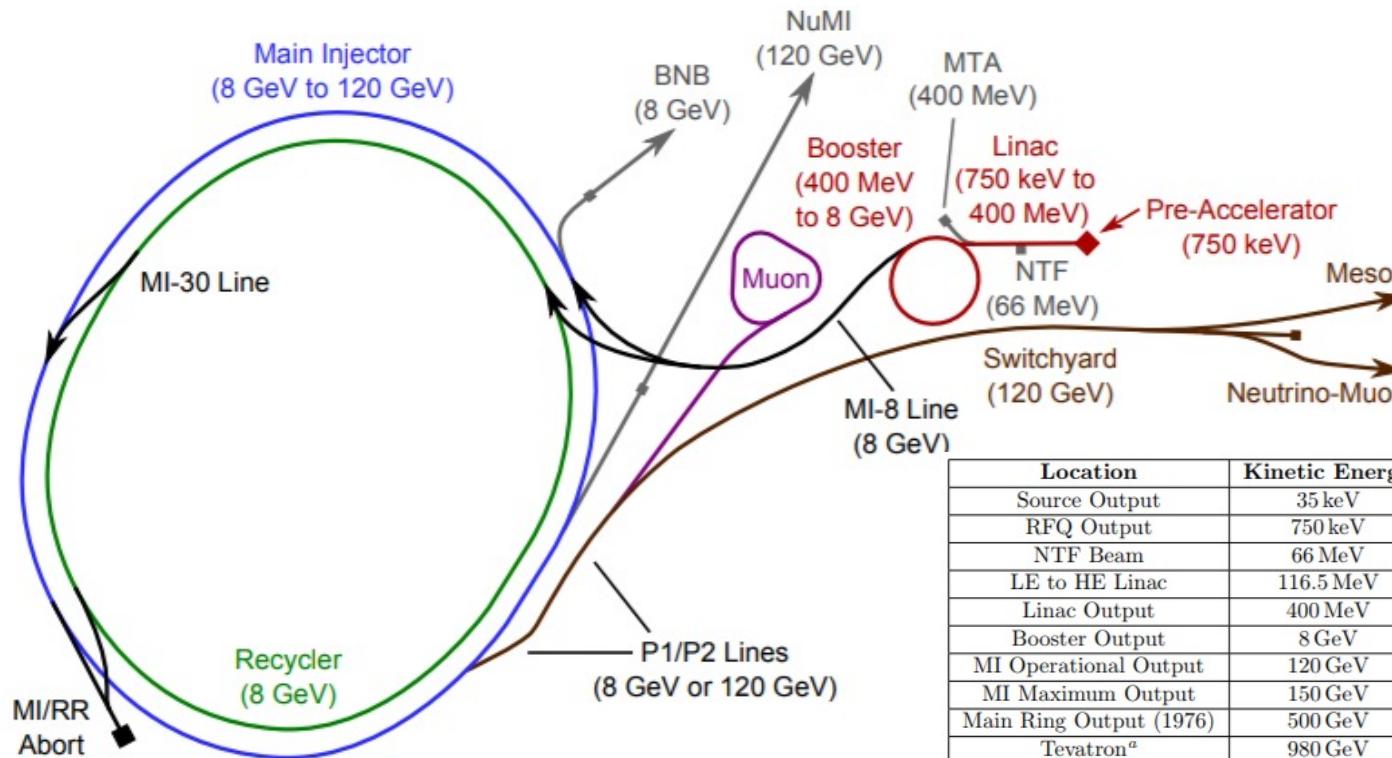
Beamlines transfer beam at a fixed energy between accelerators, or from a final accelerator to an experiment or target. These lines rarely contain RF cavities unless some complicated manipulation is required. Experimental beamlines can also direct beam into a target to create a secondary (or tertiary) beam of new particles, then select and focus a specific subset of those particles to direct to the experiment.



NuMI beamline. Source: fnal.gov

Fermilab Accelerators and Beamlines

The Fermilab Accelerator complex consists of a linear accelerator, two synchrotron accelerators, and several experimental beamlines. We deliver beam to a long-baseline neutrino oscillation experimental program (NOvA, MINERvA), a short-baseline neutrino oscillation program (MicroBooNE, SBND, ANNIE), a Nuclear Physics experiment (SpinQuest), the Fermi Test Beam Facility (Meson line), and a rare-process Muon experimental program (G-2 and Mu2e experiments).



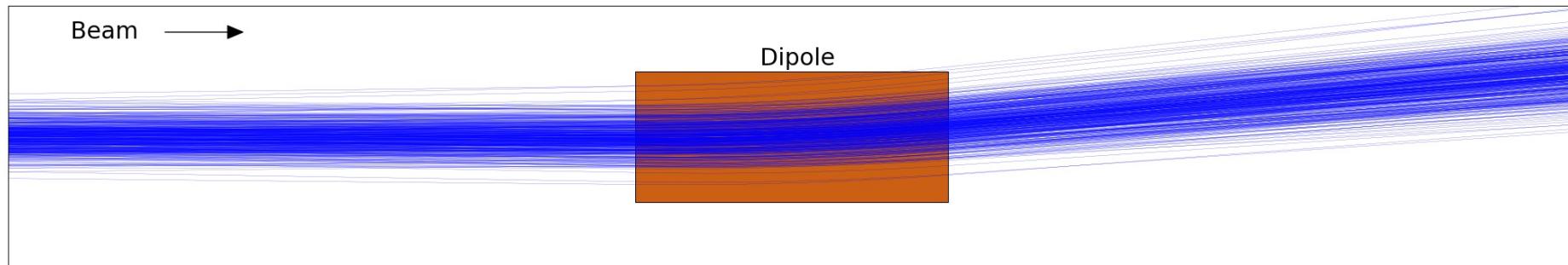
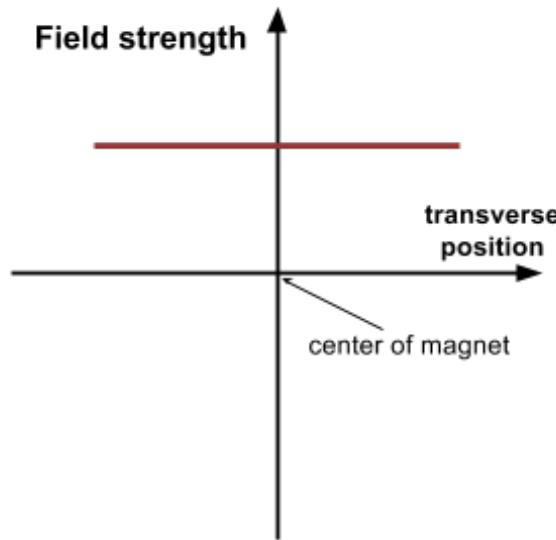
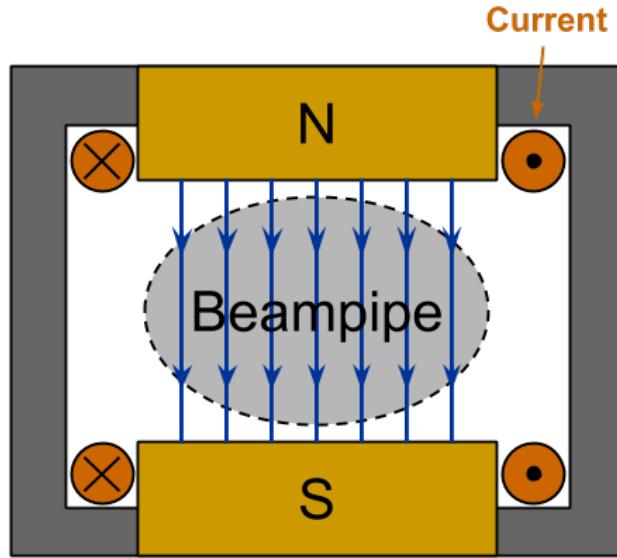
Location	Kinetic Energy	Velocity (m/s)	Velocity (c)
Source Output	35 keV	2.6E6	0.0086
RFQ Output	750 keV	1.2E7	0.04
NTF Beam	66 MeV	1.07E8	0.357
LE to HE Linac	116.5 MeV	1.37E8	0.457
Linac Output	400 MeV	2.14E8	0.713
Booster Output	8 GeV	2.98E8	0.994
MI Operational Output	120 GeV	2.9978E8	0.999969
MI Maximum Output	150 GeV	2.9978E8	0.999981
Main Ring Output (1976)	500 GeV	2.9979E8	0.9999982
Tevatron ^a	980 GeV	2.9979E8	0.99999954
LHC ^b	8 TeV	2.9979E8	0.9999999931

^aNo longer operational

^bThe Large Hadron Collider is located at CERN in Geneva, Switzerland

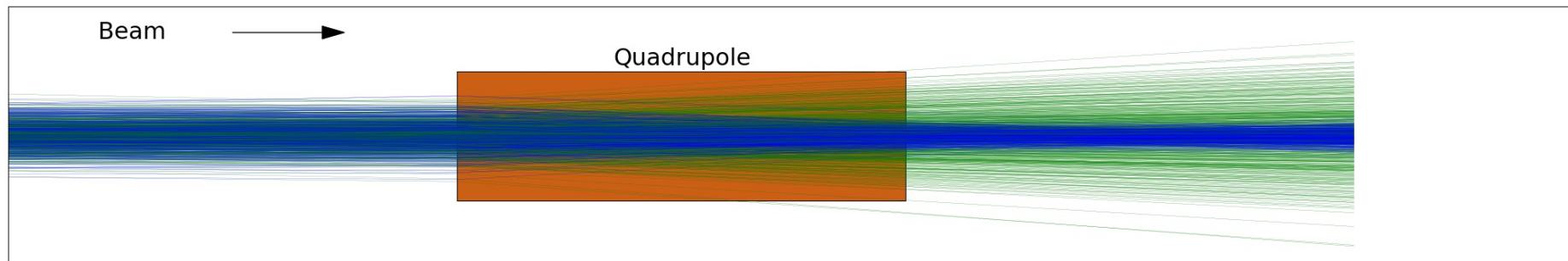
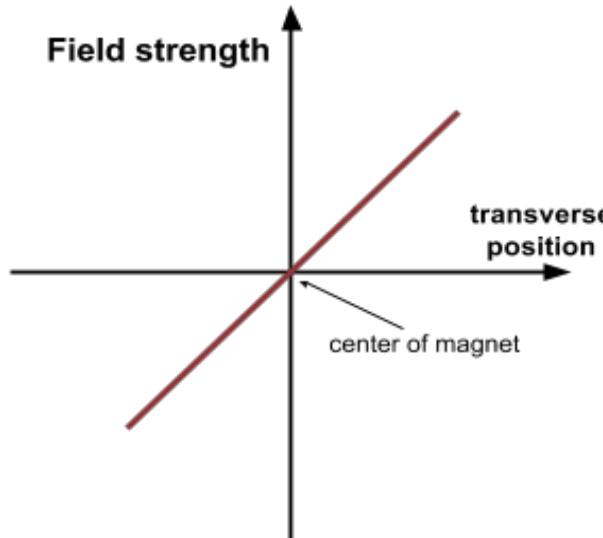
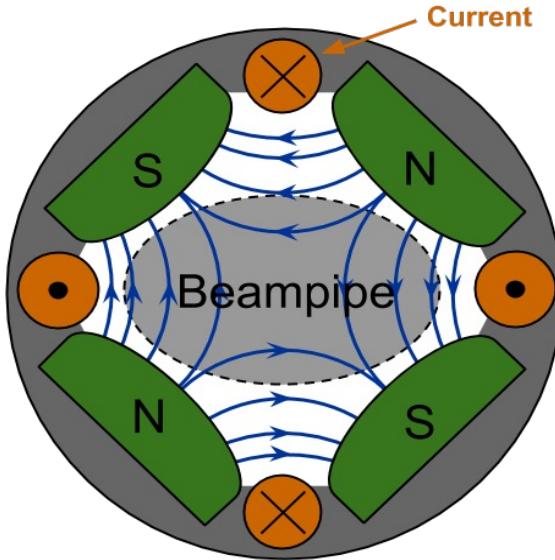
Particle Deflection, Dipole Electromagnet

High energy particle beams are typically directed and focused by magnetic fields. Dipole magnets steer the beam trajectory. However, due to non-zero beam divergence (particle angles), beam size increases over distance.



Particle Transport, Quadrupole Electromagnet

To focus the beam, we typically use quadrupole magnets. These provide a linear restoring force toward the center of the magnet, thus acting like a lens; however, a quadrupole always focuses in one plane and de-focuses in the other**. (Blue is horizontal beam, green is vertical in bottom diagram)



** See Operations Concepts Rookie Book appendix A.1 (pg 83) for derivation of why *linear* focusing in both transverse planes simultaneously is prohibited by Maxwell's equations.

Thin-lens Beam Optics, Single-Particle

Assuming the focal length of the magnetic lens is much longer than the actual magnet (i.e. “thin-lens approximation”), the quadrupole magnet's effect on a single particle's position (x and y) and angle (x' and y') is the following:

$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \quad \begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

The focal length depends on the integrated field strength gradient (B') over the magnet's length (L), as well as the particle's momentum and charge. Higher-momentum particles are “harder” to deflect with magnetic fields; we parametrize this as the “magnetic rigidity”, ($B\rho$).

$$\frac{1}{f} = \frac{B'L}{(B\rho)} \quad (B\rho) = \frac{p}{q}$$

As a side note, the angular deflection from a thin dipole is similarly computed by:

$$\Delta\theta = \frac{BL}{(B\rho)}$$



Thin-lens Beam Optics, Multi-Particle

Now we develop the mathematics for describing the group of particles. Typical beam intensities at Fermilab are \sim trillions of particles, which are too many to keep track of with single-particle theory. Instead, we focus on the statistical distributions of the particles, i.e. first and second moments.

First moments $\langle x \rangle$ and $\langle x' \rangle$ are average of all the particle positions and angles, and propagate the same as the single particle:

$$\begin{pmatrix} \langle x_1 \rangle \\ \langle x'_1 \rangle \end{pmatrix} = M \begin{pmatrix} \langle x_0 \rangle \\ \langle x'_0 \rangle \end{pmatrix}$$

The second moments $\langle x^2 \rangle$, $\langle x'^2 \rangle$, and $\langle xx' \rangle$ are the variances (standard deviation squared) in position and angle, and the average correlation between position and angle. The second moments propagate as follows:

$$\Sigma_x = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix} \quad \Sigma_1 = M\Sigma_0M^T$$

Typically, particle angles are very small and difficult to measure. Usually, we are only able to measure the transverse beam profile at a single point using a profile monitor (like a screen). Thus we can fit a Gaussian curve and compute:

$$\sigma_x = \sqrt{\langle x^2 \rangle}$$



Quadrupole Doublet

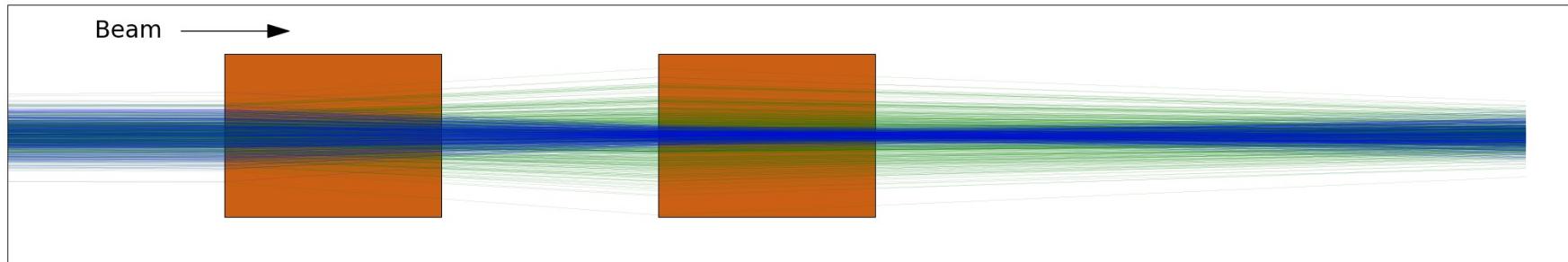
To compute the effect of multiple beamline elements on a single particle vector, simply multiply the transfer matrices together *in the order the beam sees them*; remember, matrix multiplication is not commutative, so the order matters. We can lump all the beamline elements into a single “transfer matrix”, here denoted by “M”.

$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = M \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

For example, let's look at a quadrupole doublet, which is two quadrupole magnets of opposite polarity separated by a small drift (no-magnet space) of length d.

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1 + \frac{d}{f} & d \\ -\frac{d}{f^2} & 1 - \frac{d}{f} \end{pmatrix}$$

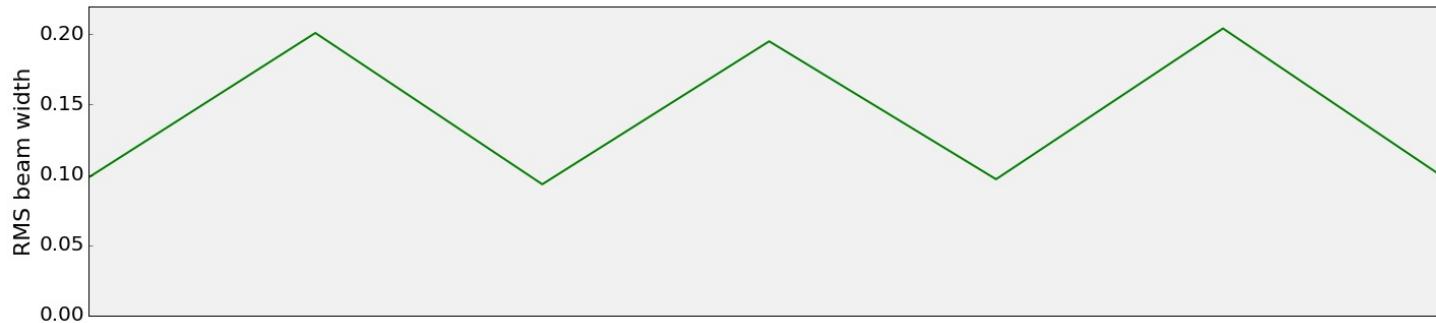
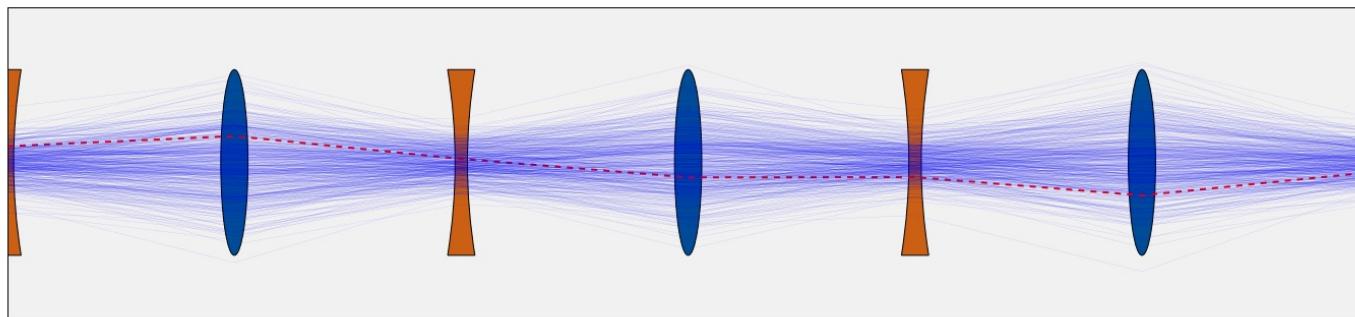
We now have net focusing in both planes with equivalent focal length: $f^* = \frac{f^2}{d}$



Beam Transport, Strong Focusing

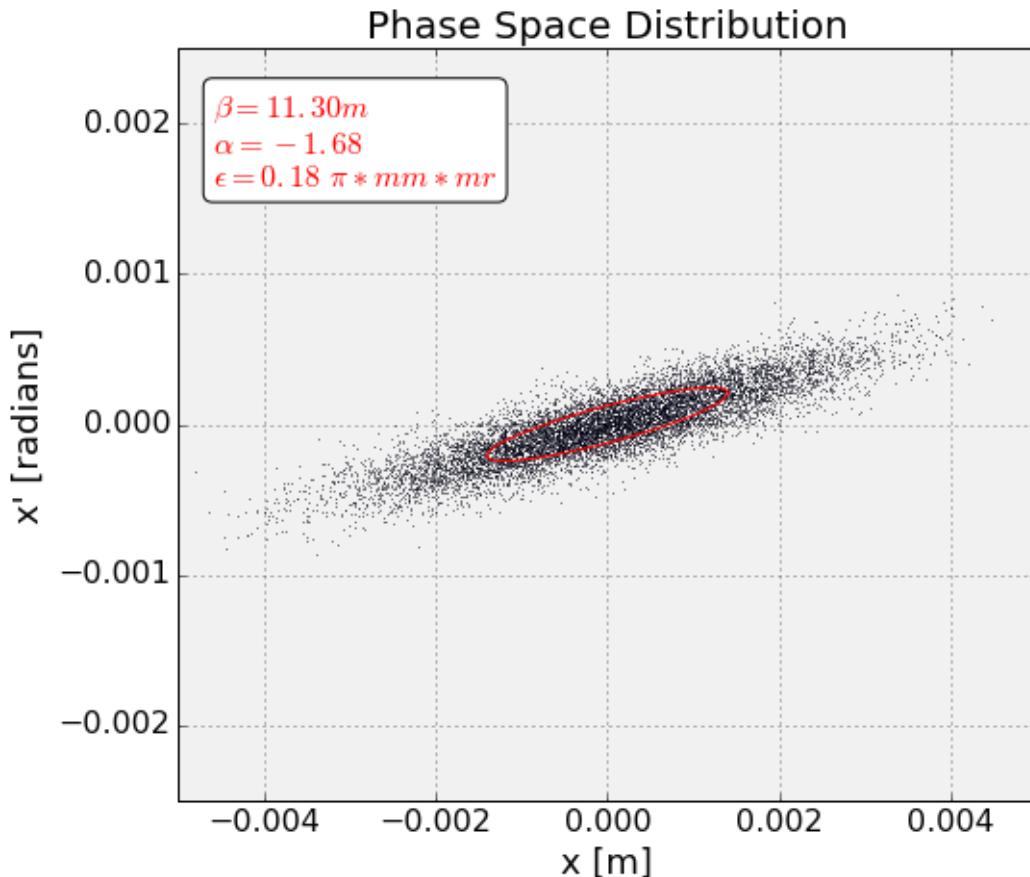
For long-distance beam transport, or to design a stable circular accelerator, it is advantageous to use “Strong Focusing”, or “Alternating Gradient Focusing”. This is a periodic arrangement of quadrupoles that alternate polarity, like stringing doublets together indefinitely. This technique allows for stable transport of beam over arbitrarily-long distances without net increase in the beam size in either plane. Transverse particle oscillations due to strong focusing are known as “betatron oscillations”.

- Christofilos, N. C. (1950). "Focusing System for Ions and Electrons". US Patent No. 2,736,799.
- Courant, E. D.; Snyder, H. S. (Jan 1958). "Theory of the alternating-gradient synchrotron" (PDF). Annals of Physics. 3 (1): 1–48. Bibcode:2000AnPhy.281..360C. doi:10.1006/aphy.2000.6012.



Phase Space and Courant-Snyder Parametrization

A different way to think about oscillating systems is to use phase space, i.e. plot each particle's position on one axis and angle on the other*. Passing through FODO beamline corresponds to rotation in phase space, and the area of the effective ellipse is invariant. (Note that “s” is the beam path variable.)



Equation of ellipse:

$$\epsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

$$\text{where } \alpha = -\frac{1}{2} \frac{\partial \beta}{\partial s} \quad \text{and} \quad \gamma = \frac{1 + \alpha^2}{\beta}$$

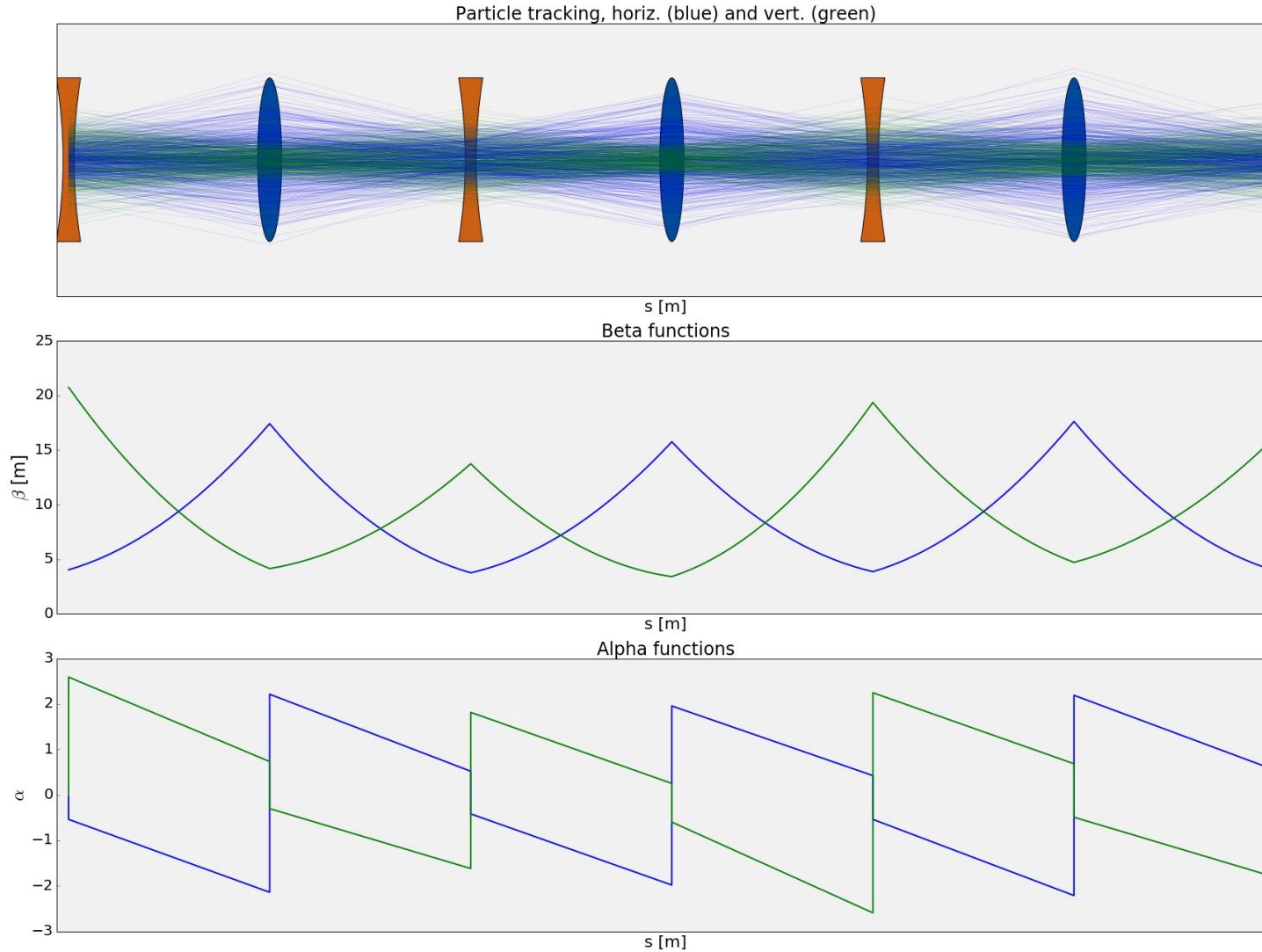
These Courant-Snyder parameters are related to the beam second moments:

$$\epsilon = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

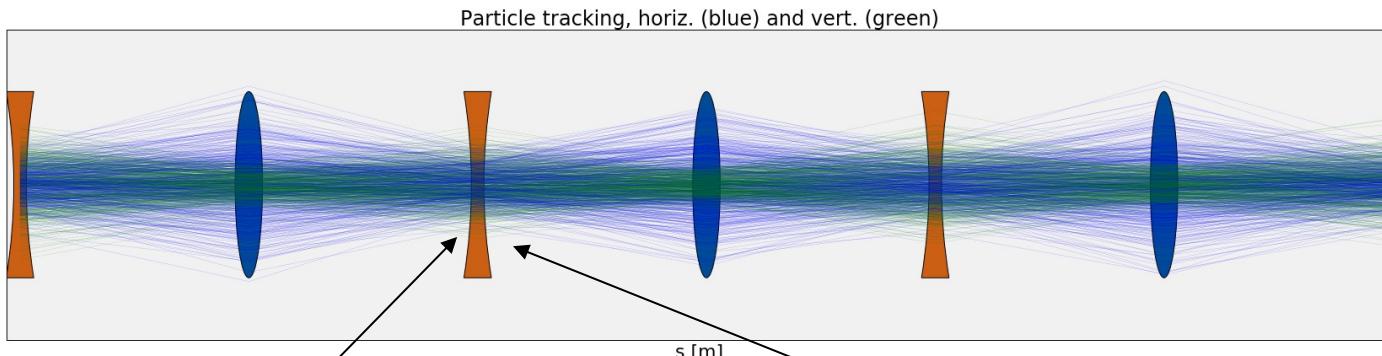
$$\alpha = \frac{-\langle xx' \rangle}{\epsilon} \qquad \beta = \frac{\langle x^2 \rangle}{\epsilon}$$

*Technically, to satisfy Hamilton's equations of motion, you should use the position and momentum for each plane as the phase space pairs. This requires a Relativistic correction: $p_x = m_0 c (\beta \gamma) x'$

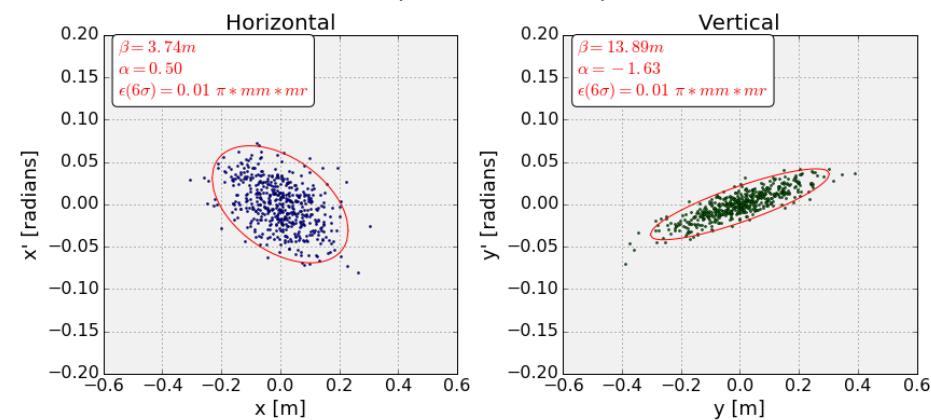
Periodic Optics Example, Courant-Snyder params



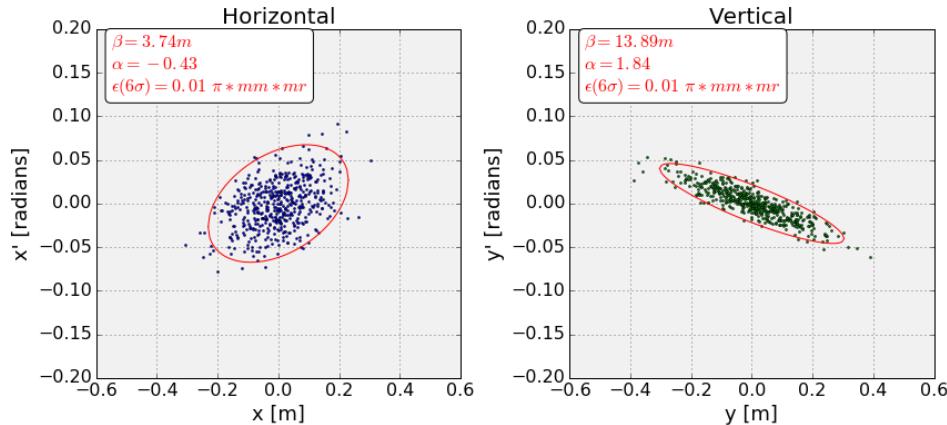
Periodic Optics Example, Ellipse Evolution



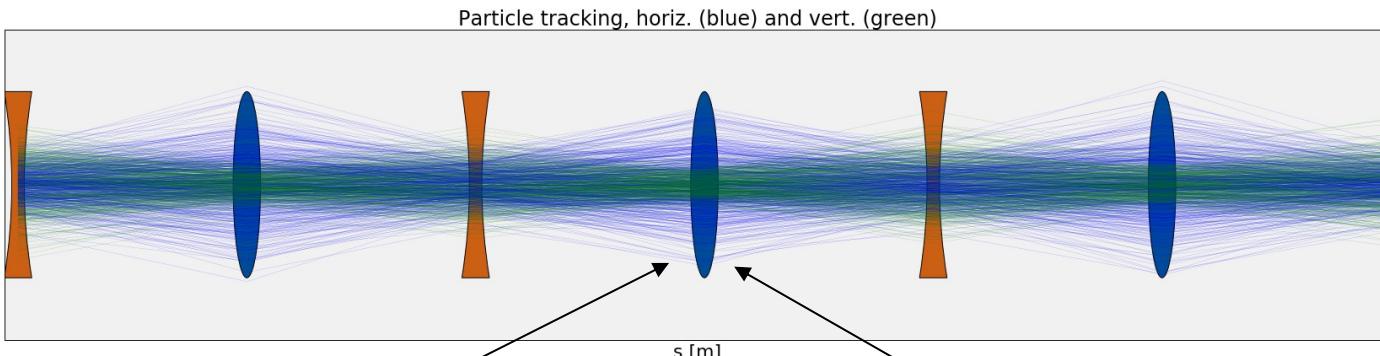
Phase space before D-quad



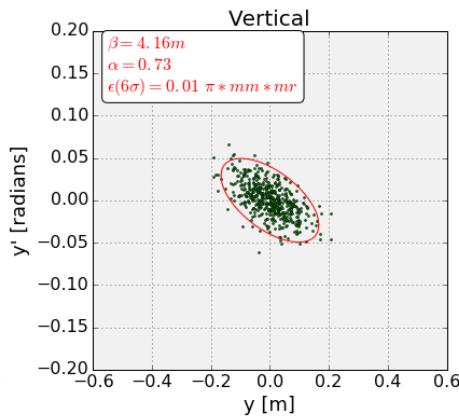
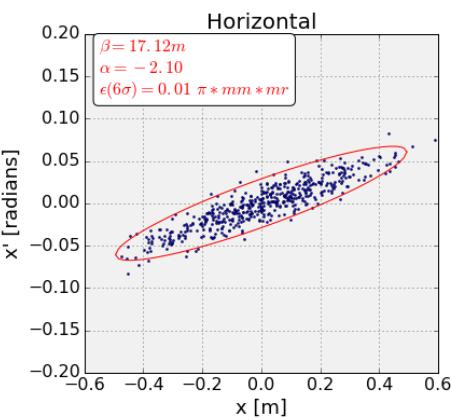
Phase space after D-quad



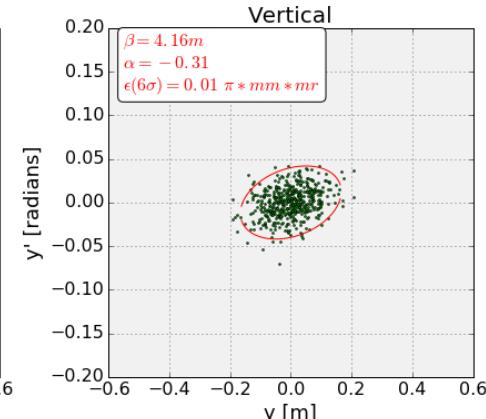
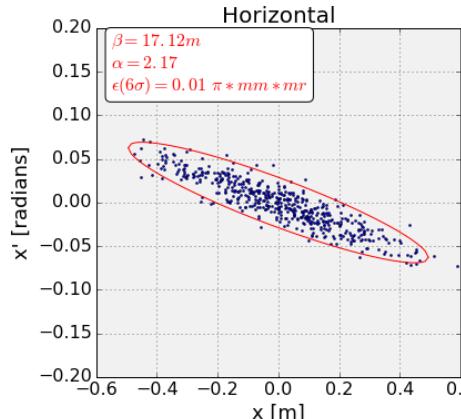
Periodic Optics Example, Ellipse Evolution



Phase space before F-quad



Phase space after F-quad



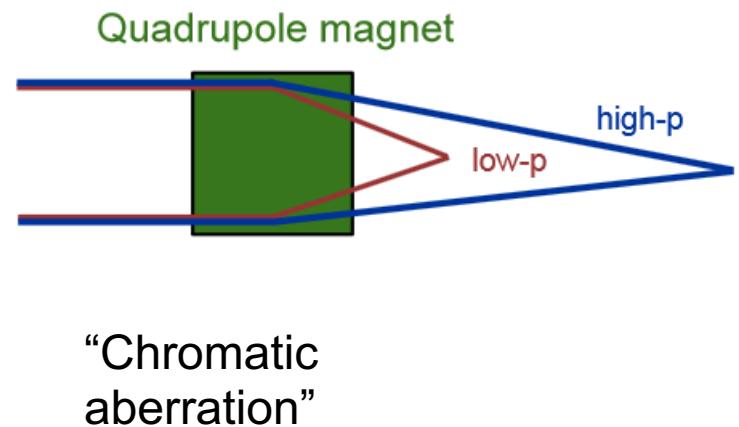
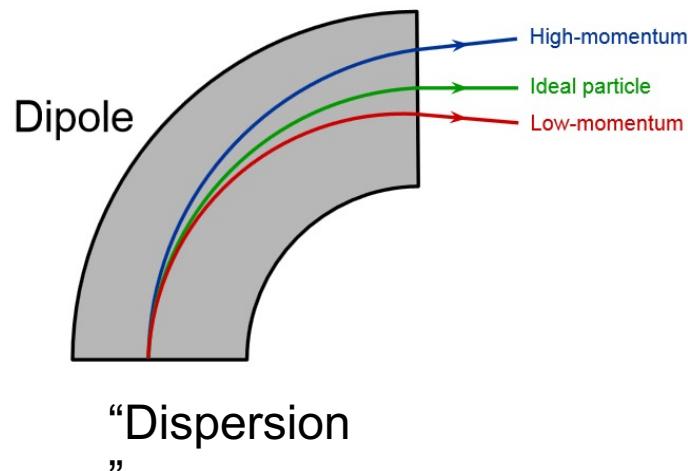
Chromatic Effects

Recall that the bend strength of a dipole and focal length of a quadrupole depends on the particle's momentum. Since a beam will always have some spread in particle momenta, there are “chromatic” effects that occur from this deviation in how the magnetic fields affect the particles.

$$\Delta\theta = \frac{BL}{(B\rho)}$$

$$(B\rho) = \frac{p}{q}$$

$$\frac{1}{f} = \frac{B'L}{(B\rho)}$$



Chromatic Effects

These chromatic effects can be summarized by a “dispersion function” in each transverse plane. These functions, along with the Courant-Snyder ellipse parameters, describe how the beam size and trajectory propagates down a beamline or along a ring as a function of either a particle's momentum or the momentum spread in a beam.

The transverse deviation of a particle is directly proportional to its momentum deviation from the reference momentum. Similarly, this same equation shows how an off-momentum beam's transverse trajectory will deviate from the design trajectory.

Single particle

$$\Delta x = D_x(s) \frac{\Delta p}{p}$$

Multi-particle beam

$$\Delta \langle x \rangle = D_x(s) \langle \frac{\Delta p}{p} \rangle$$

Furthermore, the beam size increases with a spread in the particle momenta.

$$\sigma_x = \sqrt{\epsilon_x \beta_x + [D_x(s) \frac{\sigma_p}{p}]^2}$$



Courant-Snyder in Rings

Around the curvilinear path of the ring ("s"), the change in a particle's angle due to the magnetic field gradient of a quadrupole is:

$$\frac{\Delta x'}{\Delta s} = -\frac{B'(s)}{(B\rho)}x$$

In the limit that Δs goes to zero, this takes the form of a Hill's differential equation:

$$x'' + \frac{B'(s)}{(B\rho)}x = 0$$

The periodic solution to the equation of motion takes the form of:

$$x(s) = \sqrt{\epsilon_x \beta_x(s)} \cos[\psi(s)] \quad \Delta\psi_{1 \rightarrow 2} = \int_{s_1}^{s_2} \frac{1}{\beta(s)} ds$$

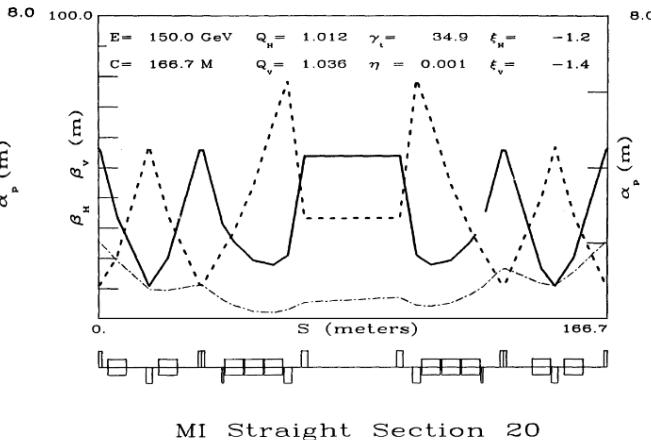
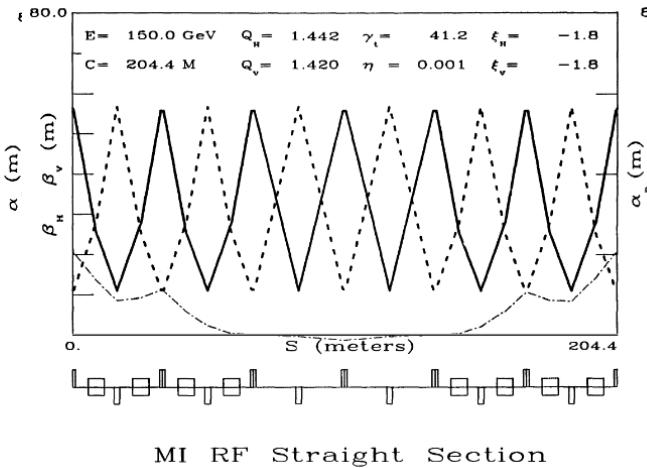
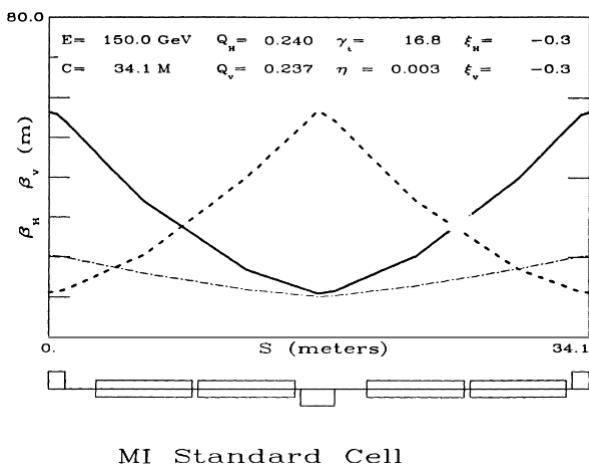
Two of these parameters are familiar elliptical parameters shown a few slides ago. However, a new variable is introduced here known as the "betatron phase". Since the betatron oscillations are periodic and repeatable in a ring, it is useful to talk about the oscillations as described above. This parametrization is known as "Courant-Snyder", or sometimes "Twiss" parameters.

The benefit of describing the ring in terms of the Courant-Snyder parameters is that one does not need to worry about multiplying hundreds of transfer matrices for every element in the ring . In a beamline however, this periodicity is not guaranteed, so it is more common to describe the beam in terms of the second moments and linear transfer matrices.



Courant-Snyder in Rings

Circular accelerators are designed starting with discrete modules that serve specific functions. The more repetition of simple modules, the simpler and more robust the design is. A designer begins with a ring made of “standard cells” that bend and focus the beam around the ring in a stable way. Then special modules are inserted into the pattern to allow for specific needs, such as parallel beam in an extraction region or low dispersion in an RF section.



Source: Main Injector Conceptual Design Report



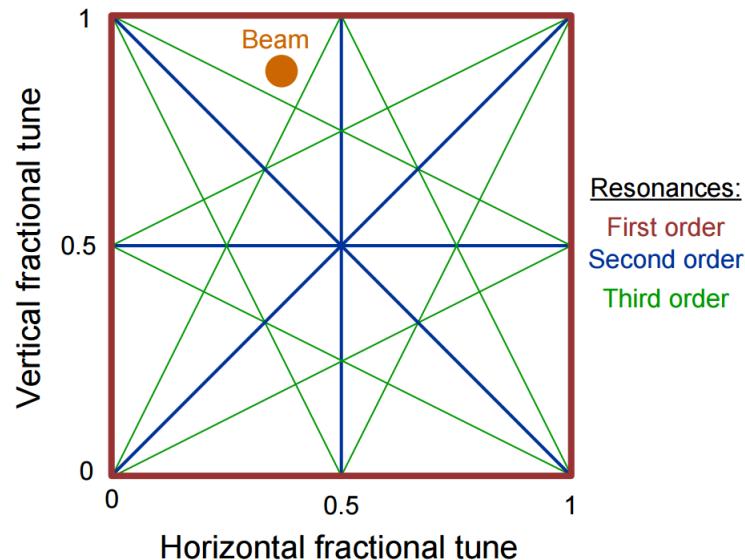
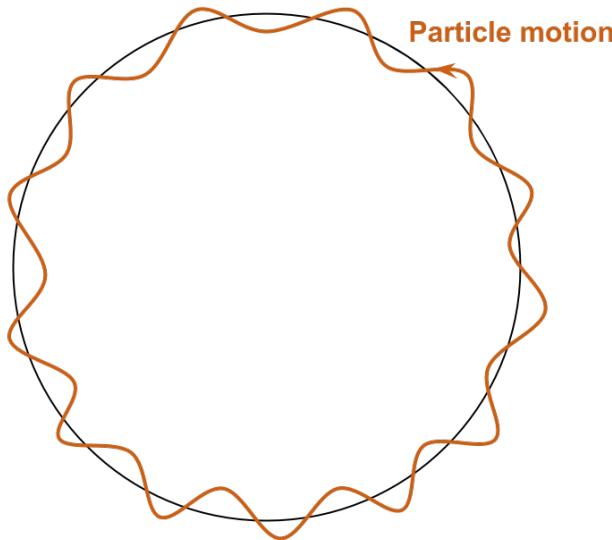
Betatron Oscillations

Alternating focusing/defocusing pattern (“lattice”) in a circular accelerator creates transverse oscillation of particles known as “Betatron motion”. The number of oscillations per revolution is called the “tune” ν , which is computed by integrating the betatron phase around the entire ring. The “chromaticity” ξ describes how the tune shifts with beam momentum offset.

$$\nu = \frac{1}{2\pi} \oint \frac{1}{\beta(s)} ds$$

$$\Delta\nu = \xi \frac{\Delta p}{p}$$

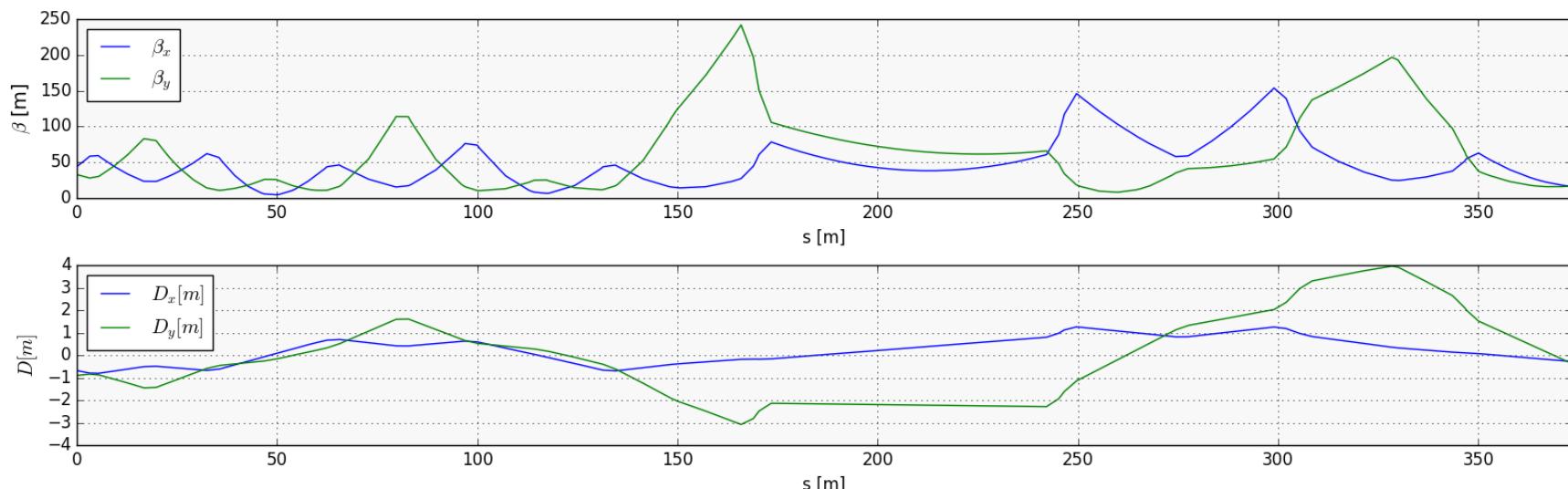
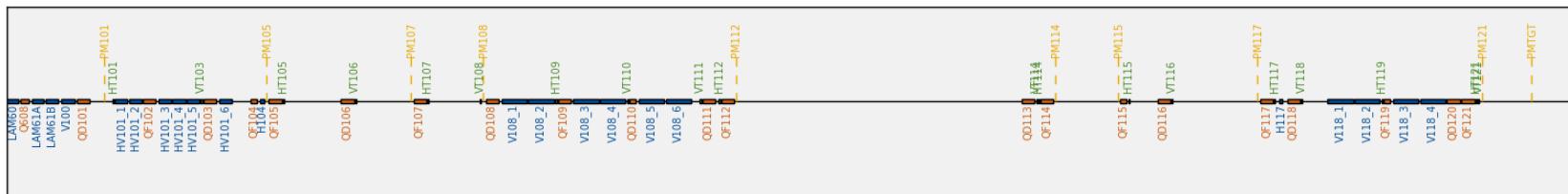
As with any driven oscillation, betatron motion is susceptible to resonance conditions. In particular, fractional-integer tunes allow sensitivity to nonlinear magnet fields in ring; these can be deliberate (sextupoles, octupoles), field imperfections in magnets, or alignment errors.



Courant-Snyder in NuMI Beamline

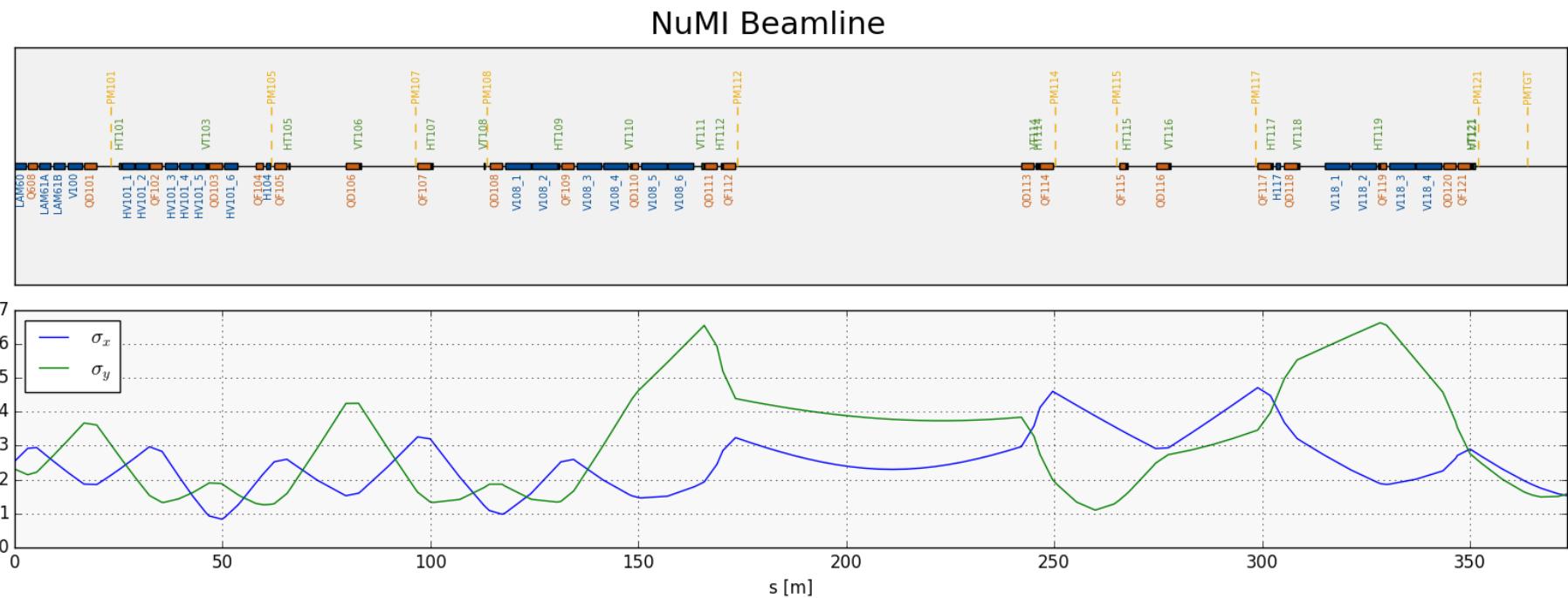
Here is an example of how the ellipse parameters propagate through a real beamline. Notice that there are discrete sections to the optics. The NuMI beamline begins with a very periodic structure before transitioning into a long drift through the carrier pipe where no magnets can fit. After the carrier pipe, the optics transition into a final focus onto the NuMI target. Aggressive vertical dipole magnets push the beam down through the underground water table as quickly as possible, then level the beam back out; the optics must also control the vertical dispersion created by this necessary trajectory.

NuMI Beamline



Beam Size in NuMI Beamline

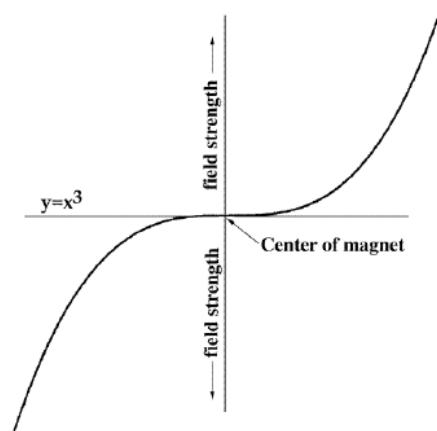
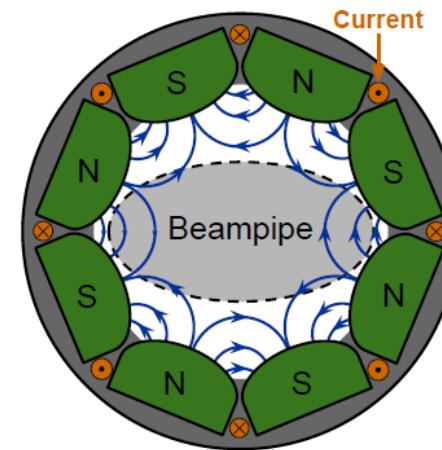
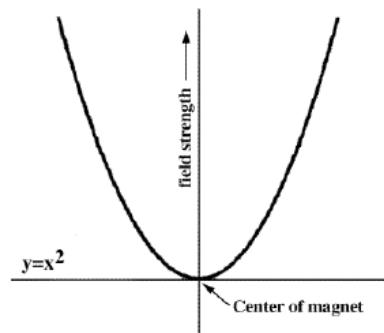
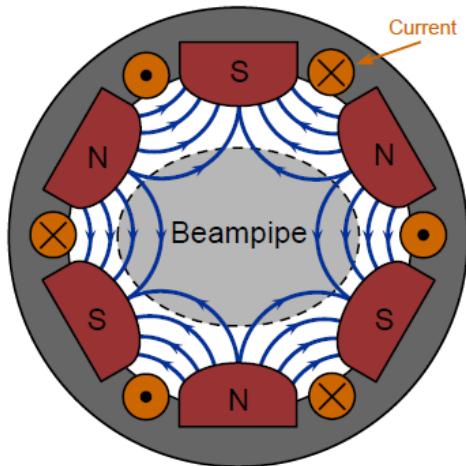
Below is the computed one-sigma beam size; this represents the beam's half-width at 68% containment. This plot helps determine whether certain beam metrics are met, such as spot size on target, and whether the beam ever gets larger than the available apertures. As shown in an earlier slide, the one-sigma beamsize is a combination of the beta function, emittance, momentum spread, and dispersion function.



Non-linear Magnetic Fields

Magnetic fields are not always linear with transverse displacement, as they are with the dipole (linear, slope zero), and quadrupole. Higher-order magnets such as the Sextupole and Octupole have a non-linear field dependence on the transverse particle position.

These magnets are used for more advanced beam manipulation. However, all magnets have small non-zero higher-order terms that cause non-linear effects (manufacturing or alignment errors, etc.) Sometimes these effects are not so small (MI quadrupoles, for example)!

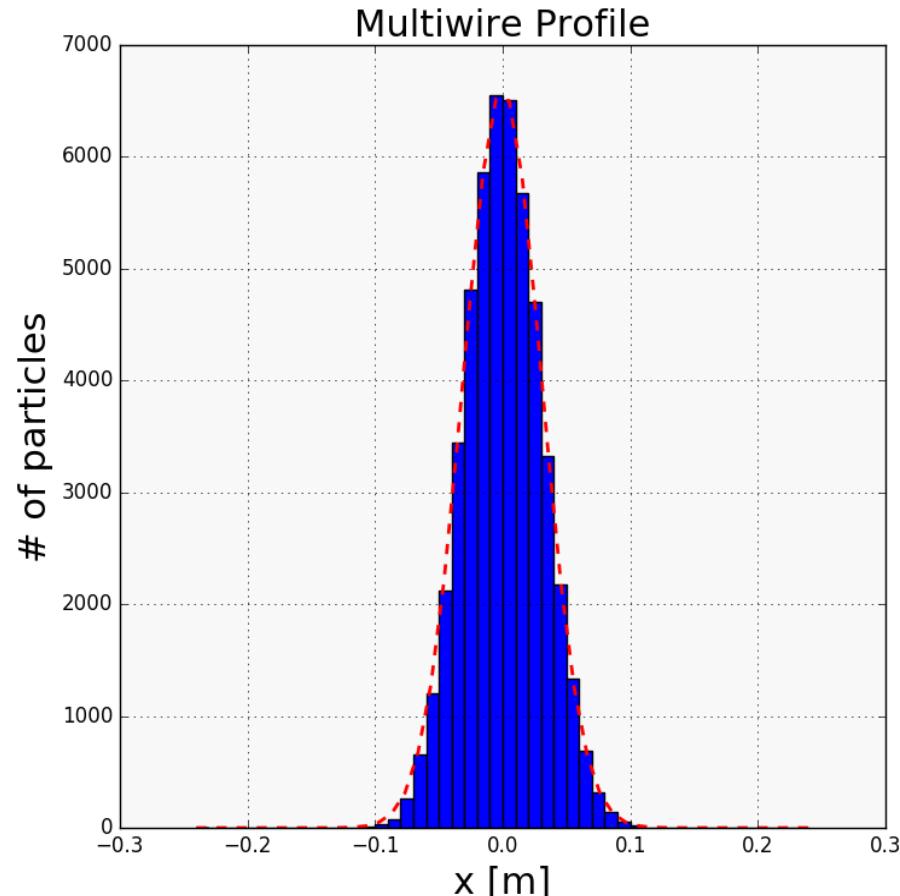
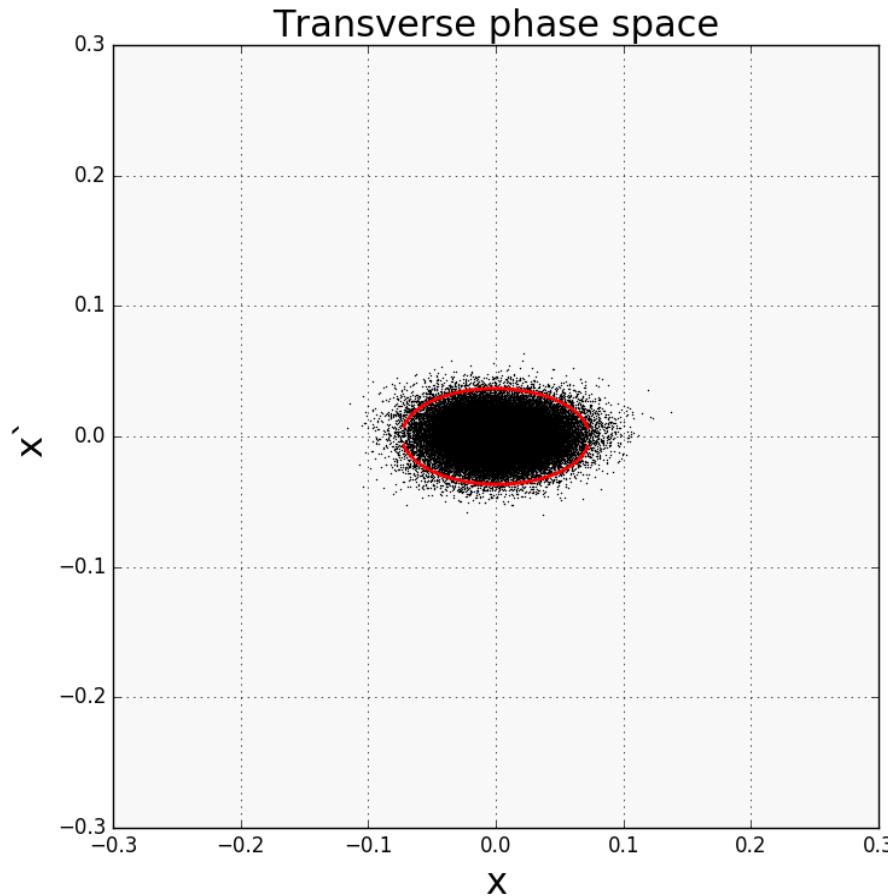


Magnets can be characterized by Taylor Series expansion to describe “accidental” non-linear higher-order effects on the beam due to imperfections.

$$H(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots$$

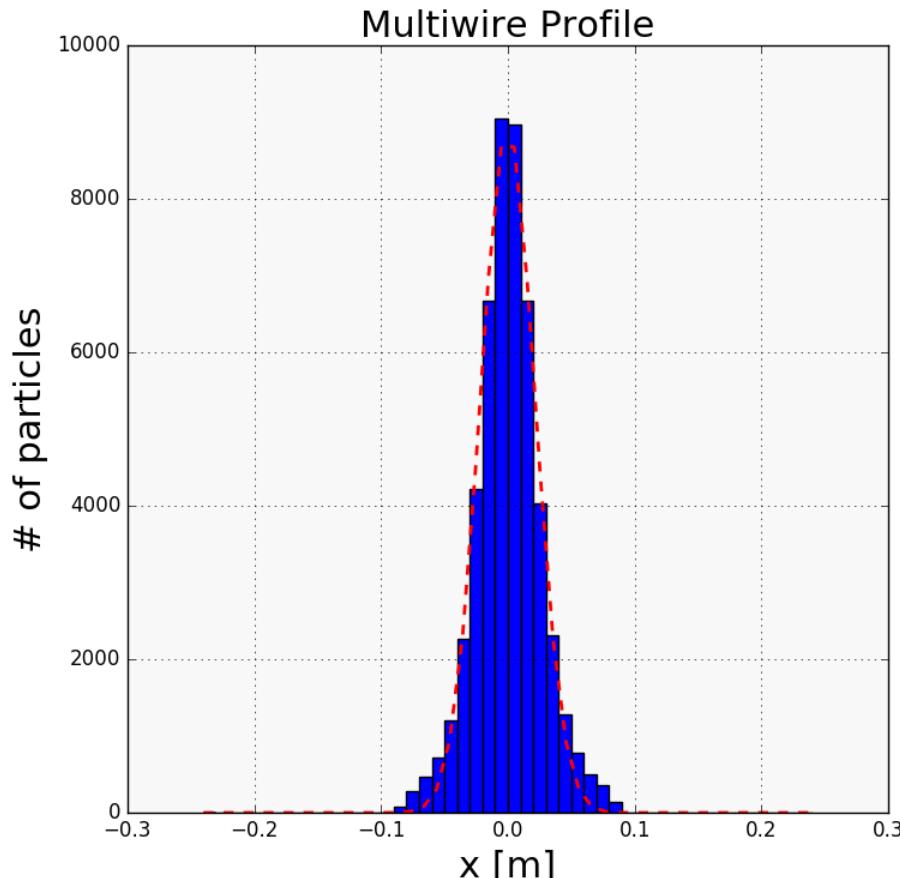
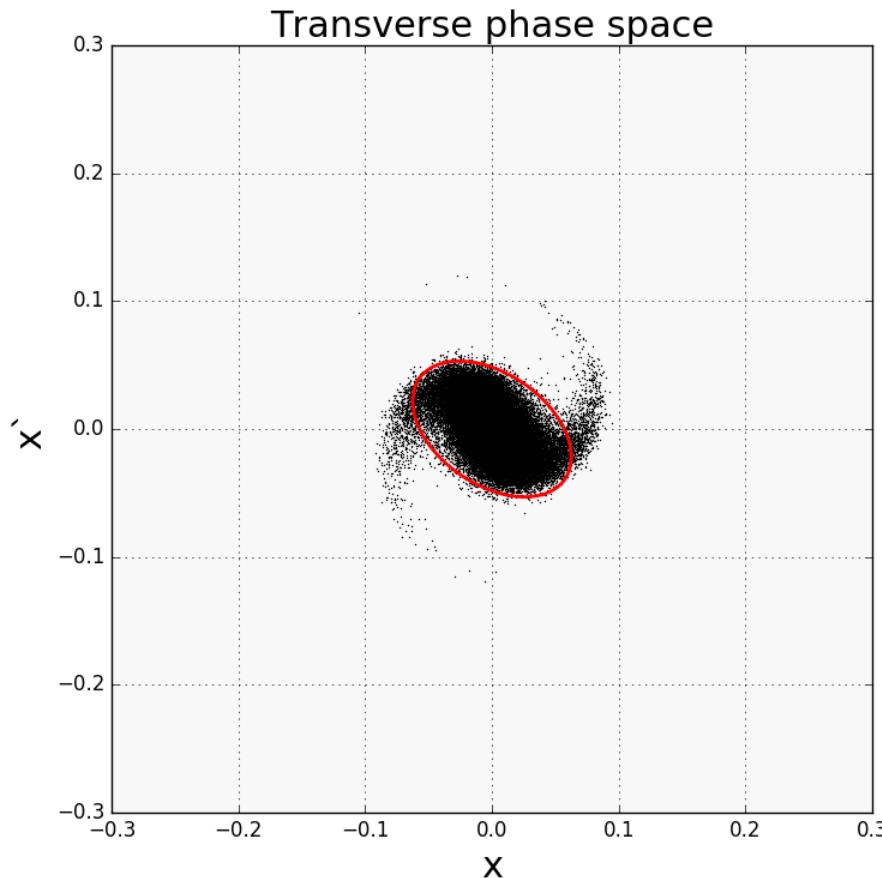
Effect of Non-Linear Fields on Beam

Non-linear fields interact differently with particles on the outside of the ellipse than those toward the core. Thus the fringe particles will rotate at a different rate compared to the core, causing “filamentation”. The beam is no longer elliptical, and requires higher moments to describe. Profiles no longer fit Gaussian distributions as well.



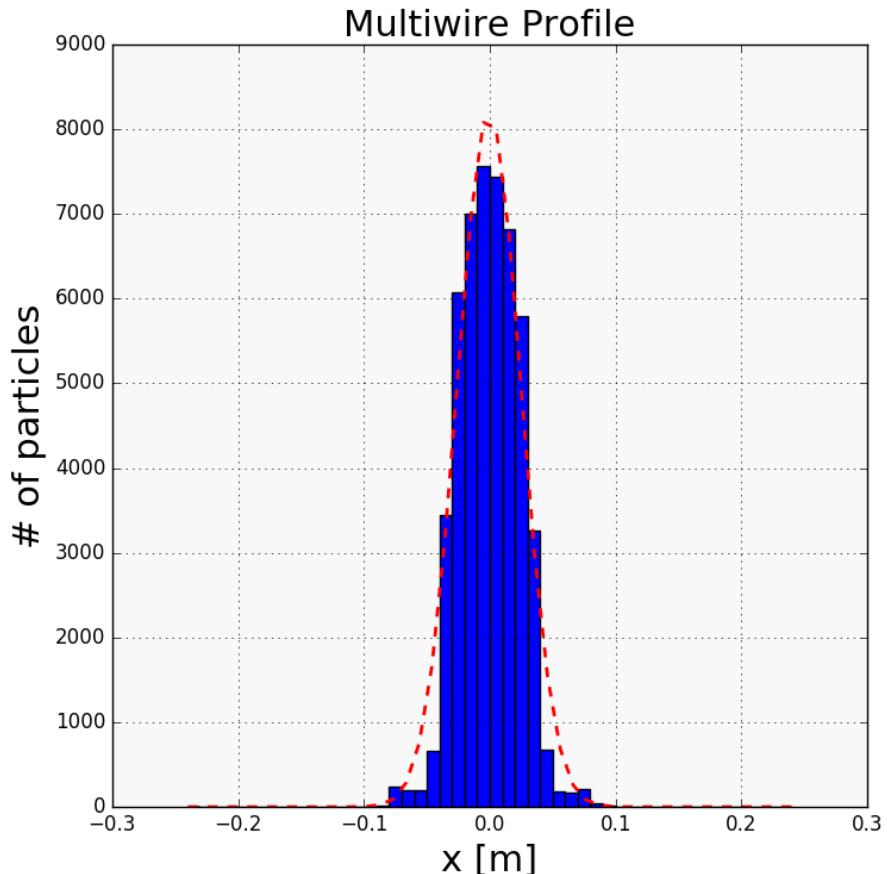
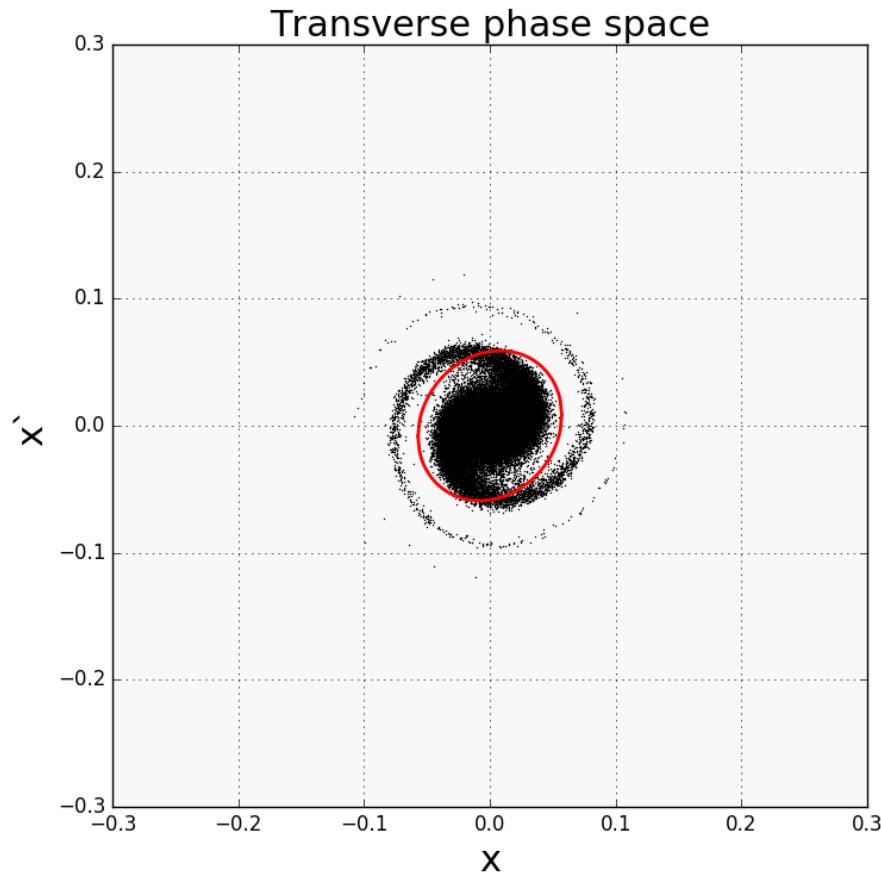
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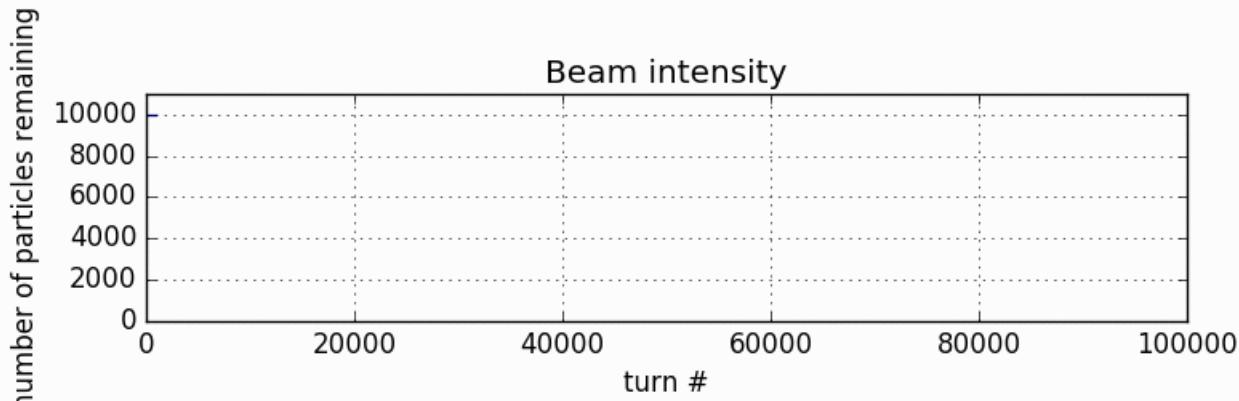
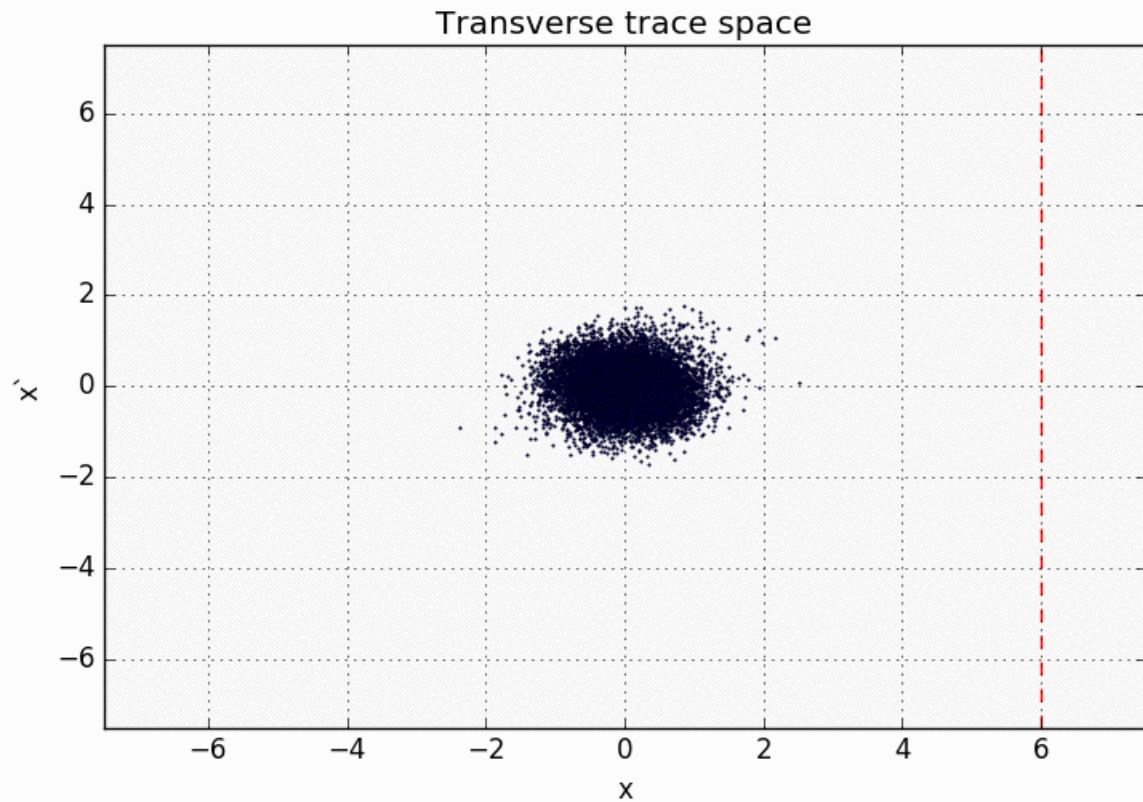
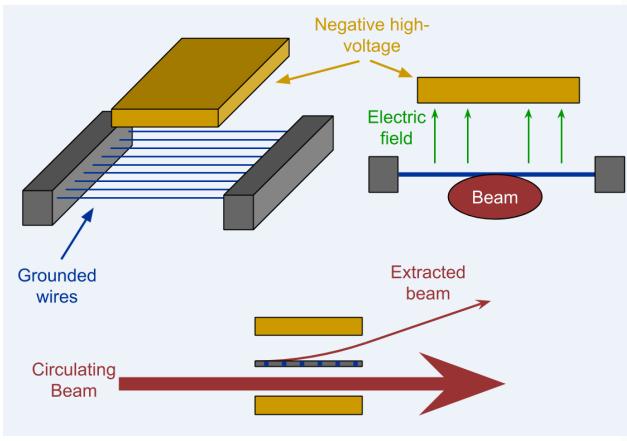
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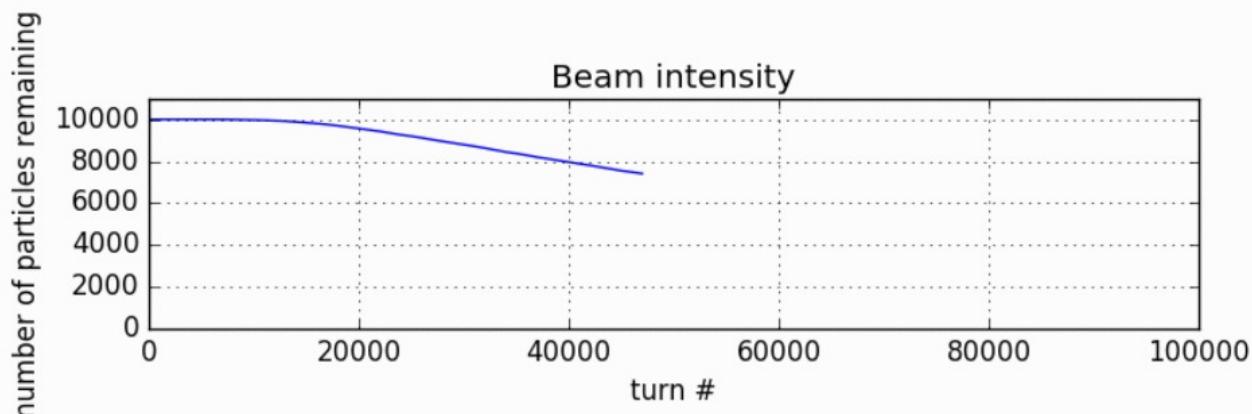
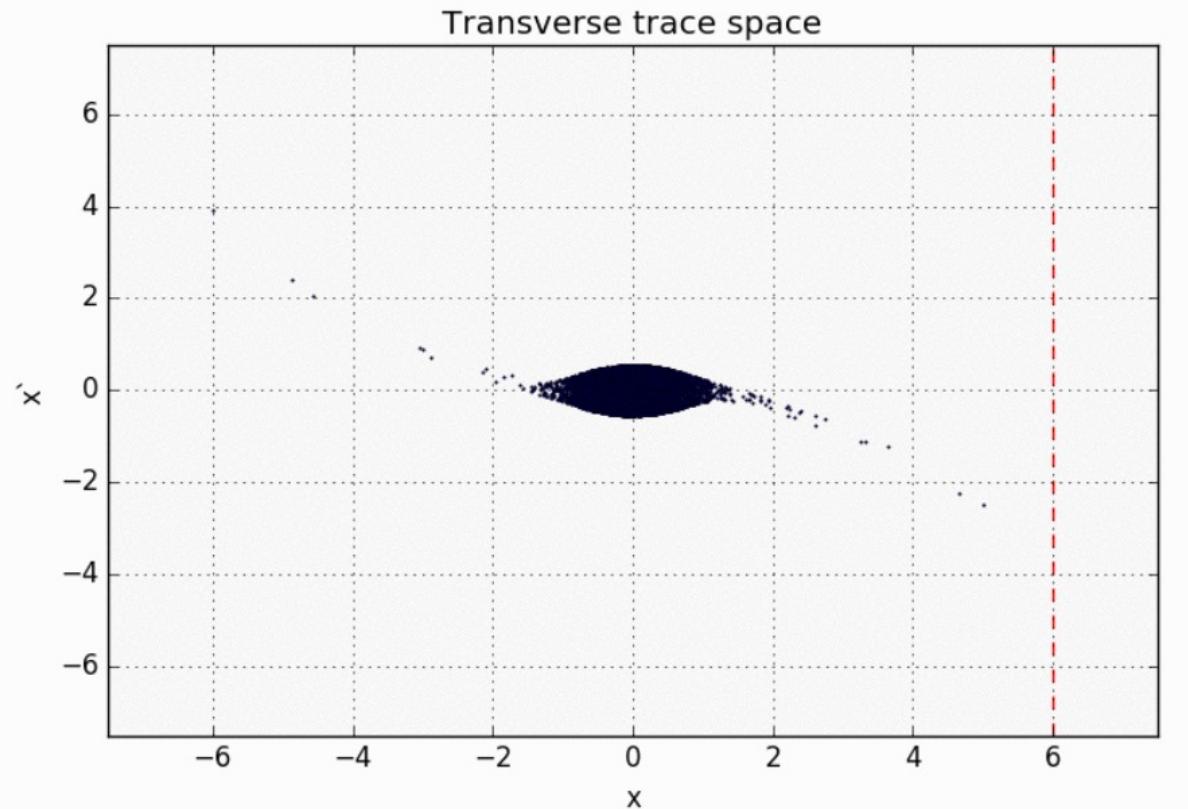
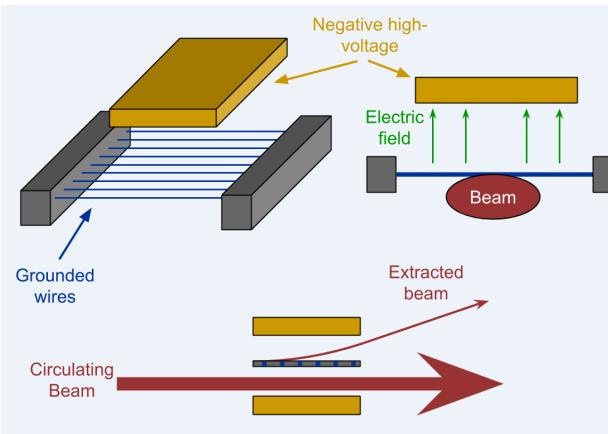
Non-Linear Resonant Extraction

In the Fermilab Switchyard, we take advantage of this nonlinear filamentation, as well as resonance conditions in the betatron oscillations in the Main Injector, to slowly “spill” the beam out to the experiments. The extracted beam is non-elliptical!



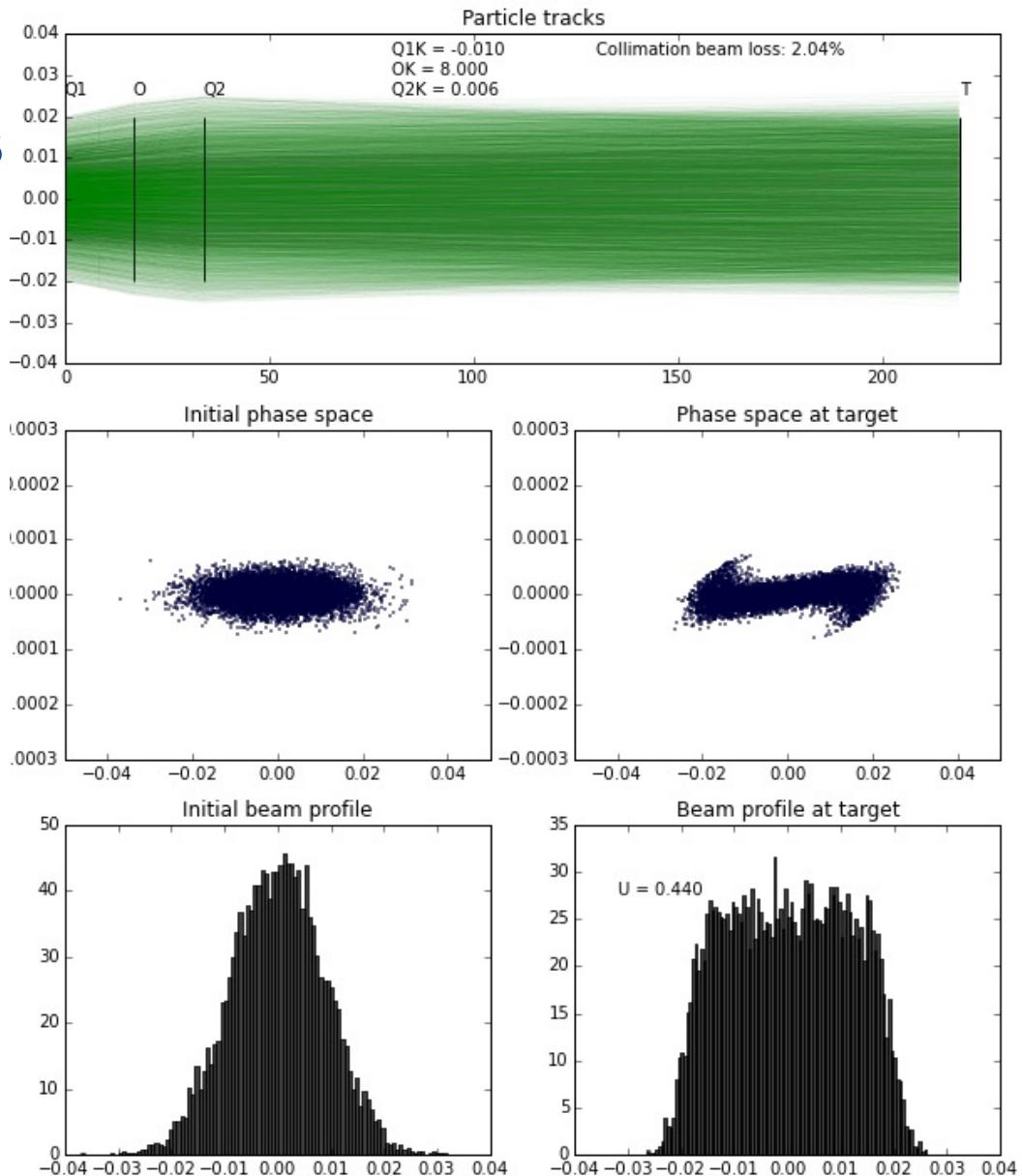
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A Use for Non-Linear Magnets in beamlines

An octupole in a beamline can, if placed appropriately, create a more uniform distribution on an experimental target. For example, if an experiment needs very uniform heating of a cryogenic target, one octupole (or two sextupoles) per plane can achieve the desired profile.



References and Resources

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Thank you!

