In [ ]:
```
using LinearAlgebra
using IJulia
using Plots
using Printf
using Noise
using CSV
using DataFrames
```

** Problem 2.5 **

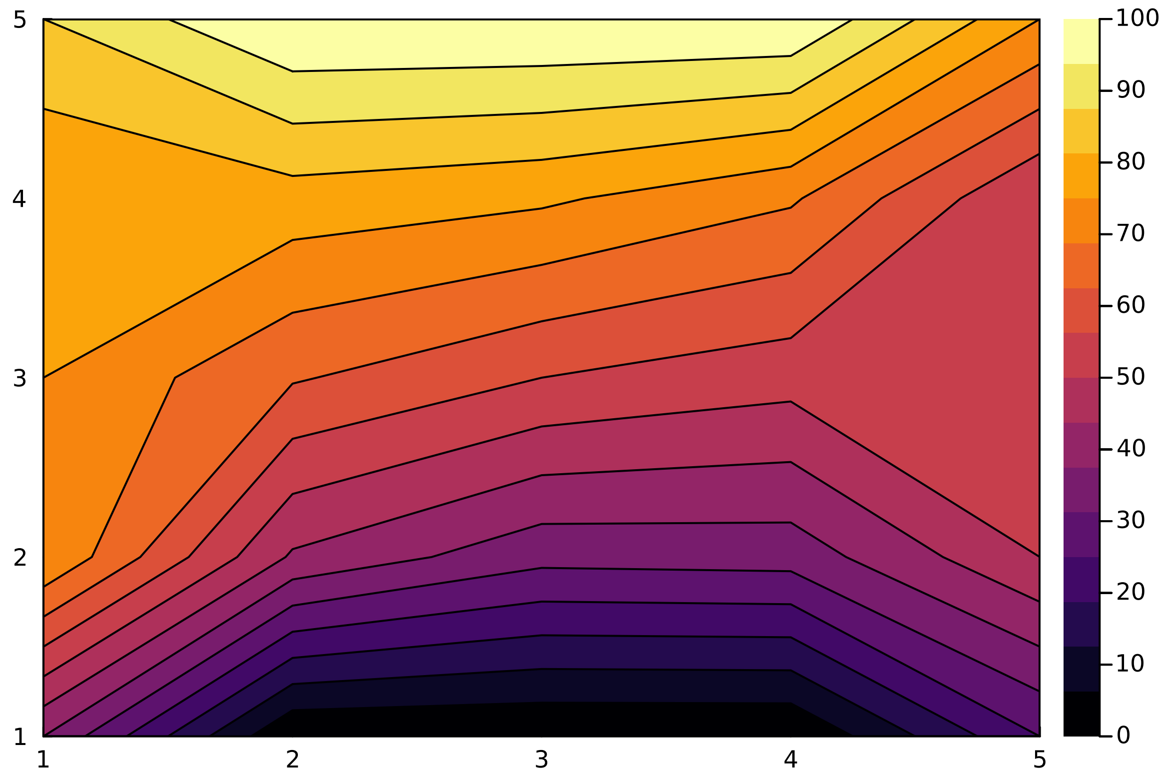
In [ ]:
```
σ = 10* [2 5 3
        5 1 4
        3 4 3]

vals = eigvals(σ)
print("Eigenvalues are ", vals)

## Part 2
vecs = eigvecs(σ)
print("\nEigenvectors are \n")
display(vecs)

## Part 3
τmax = (maximum(vals)-minimum(vals))/2
print("\nτmax is ", τmax)

##Part 4
σᵥ = sqrt(1/2*((σ[1,1]-σ[1,2])^2 + (σ[2,2]-σ[3,3])^2 + (σ[3,3] - σ[1,1])^2 + 6*(σ[1
print("\nσᵥ is " , σᵥ)
```

```
Eigenvalues are [-37.32050807568878, -2.679491924311231, 100.0]
Eigenvectors are
3×3 Matrix{Float64}:
  0.57735   -0.57735   -0.57735
 -0.788675  -0.211325  -0.57735
  0.211325   0.788675  -0.57735
τmax is 68.6602540378444
σᵥ is 125.29964086141668
```

In [ ]:
```
## Part 5
Q = 1/3 * [1 -2 2;
    2 -1 -2;
    2 2 1]

σnew = Q' * σ * Q

vals, vecs = eigen(σnew)
print("The values are the same as before: ", vals , "\nThe vectors are the same as
display(vecs)
τmax = (maximum(vals)-minimum(vals))/2
print("\nτmax is the same as before, ", τmax, "\n")
σᵥ = sqrt(1/2*((σ[1,1]-σ[1,2])^2 + (σ[2,2]-σ[3,3])^2 + (σ[3,3] - σ[1,1])^2 + 6*(σ[1
print("\nσᵥ is the same as before: ", σᵥ)
```

```
The values are the same as before: [-37.32050807568879, -2.6794919243112307, 99.99
999999999994]
The vectors are the same as before:
3×3 Matrix{Float64}:
  0.19245    0.19245    -0.96225
 -0.0188748  0.981125    0.19245
 -0.981125   0.0188748  -0.19245
τmax is the same as before, 68.66025403784437

σᵥ is the same as before: 125.29964086141668
```

In [ ]:
```
A = zeros(Float64, 9,9)
A[1,:] = [4, -1, 0, -1, 0, 0, 0, 0, 0]
A[2,:] = [-1, 4, -1, 0, -1, 0, 0, 0, 0]
A[3,:] = [0, -1, 4, 0, 0, -1, 0, 0, 0]
A[4,:] = [-1, 0, 0, 4, -1, 0, -1, 0, 0]
A[5,:] = [0, -1, 0, -1, 4, -1, 0, -1, 0]
A[6,:] = [0, 0, -1, 0, -1, 4, 0, 0, -1]
A[7,:] = [0, 0, 0, -1, 0, 0, 4, -1, 0]
A[8,:] = [0, 0, 0, 0, -1, 0, -1, 4, -1]
A[9,:] = [0, 0, 0, 0 ,0, -1, 0, -1, 4]

b = [75; 0; 50; 75; 0; 50; 175; 100; 150]



x = A\b

newx = reshape(x,3,3)

boarder = zeros(Float64, 5,5)
boarder[1,:] = [87.5 100 100 100 75]
boarder[2,:] = [ 75 x[7:9]' 50]
boarder[3,:] = [75 x[4:6]' 50]
boarder[4,:] = [75 x[1:3]' 50]
boarder[5,:] = [75/2 0 0 0 25]

con = contour(reverse(boarder, dims = 1), fill=true)

xForLater = x
display(con)
```

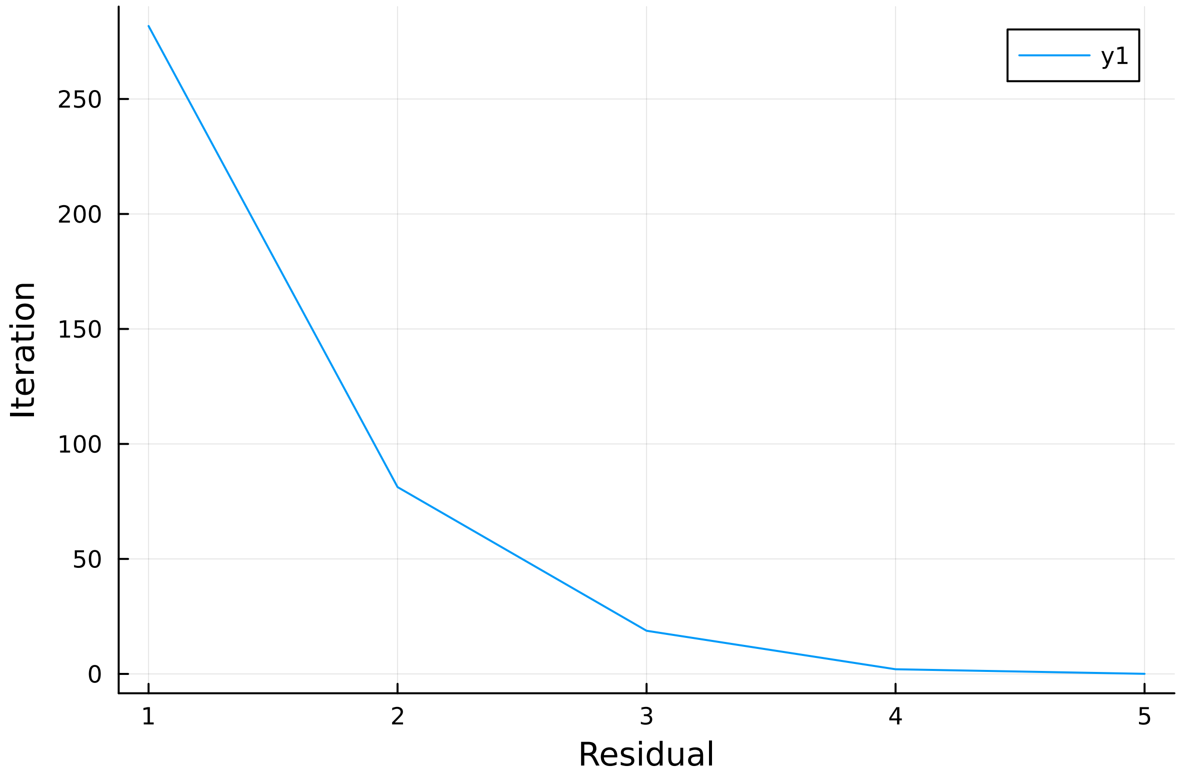


In [ ]:
```
##PART 2
eigStuff = eigen(A)
eigVals = eigStuff.values

K =  maximum(eigVals)/minimum(eigVals)
print("my condition is:")
print(K)

print("\nthe built in condition function is:")
print(cond(A))
```

```
my condition is:5.828427124746191
the built in condition function is:5.8284271247461925
```

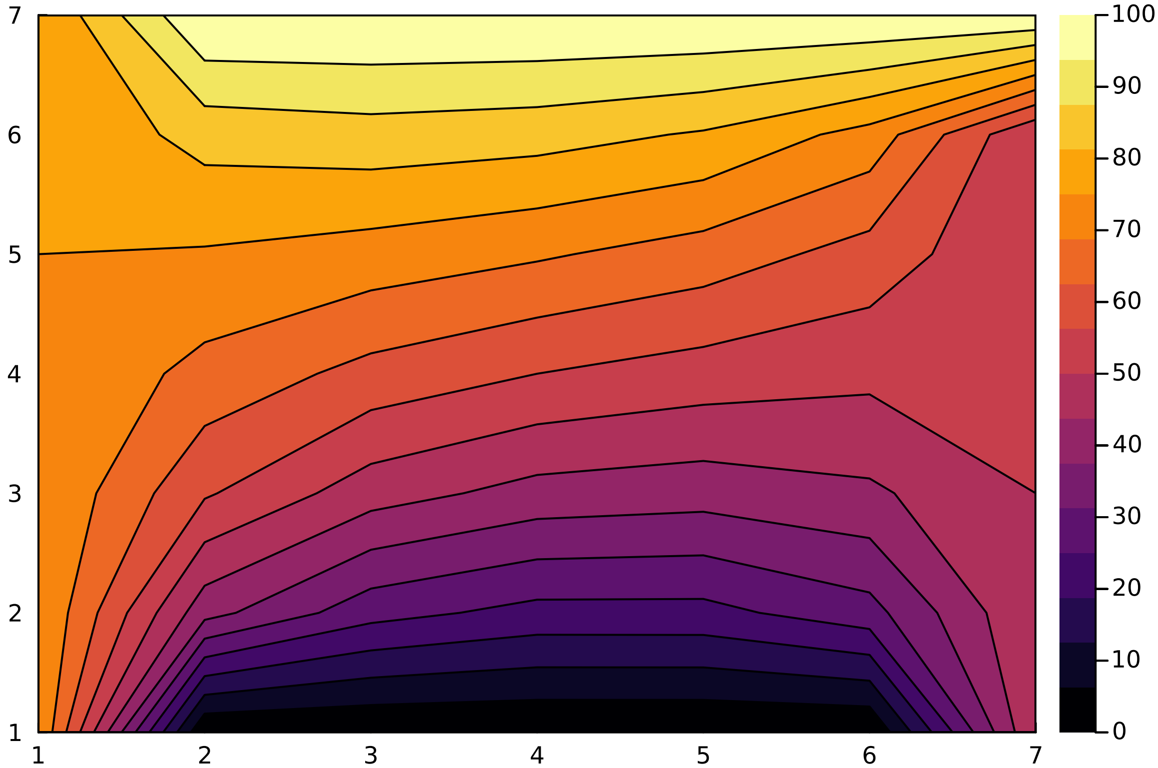
In [ ]:
```
##PART 3

function myBiCSTAB(A, xInitialGuess, b, xForLater, tolerance = 1e-6)

    x = xInitialGuess
    r = b-A*x
    r0hat = r
    if(dot(r0hat,r) == 0)
        print("You fucked up")
    end

    rhoPrev = 1
    alpha = 1
    omega = 1


    v = zeros(Float64, 9, 1)
    p = zeros(Float64, 9, 1)
```

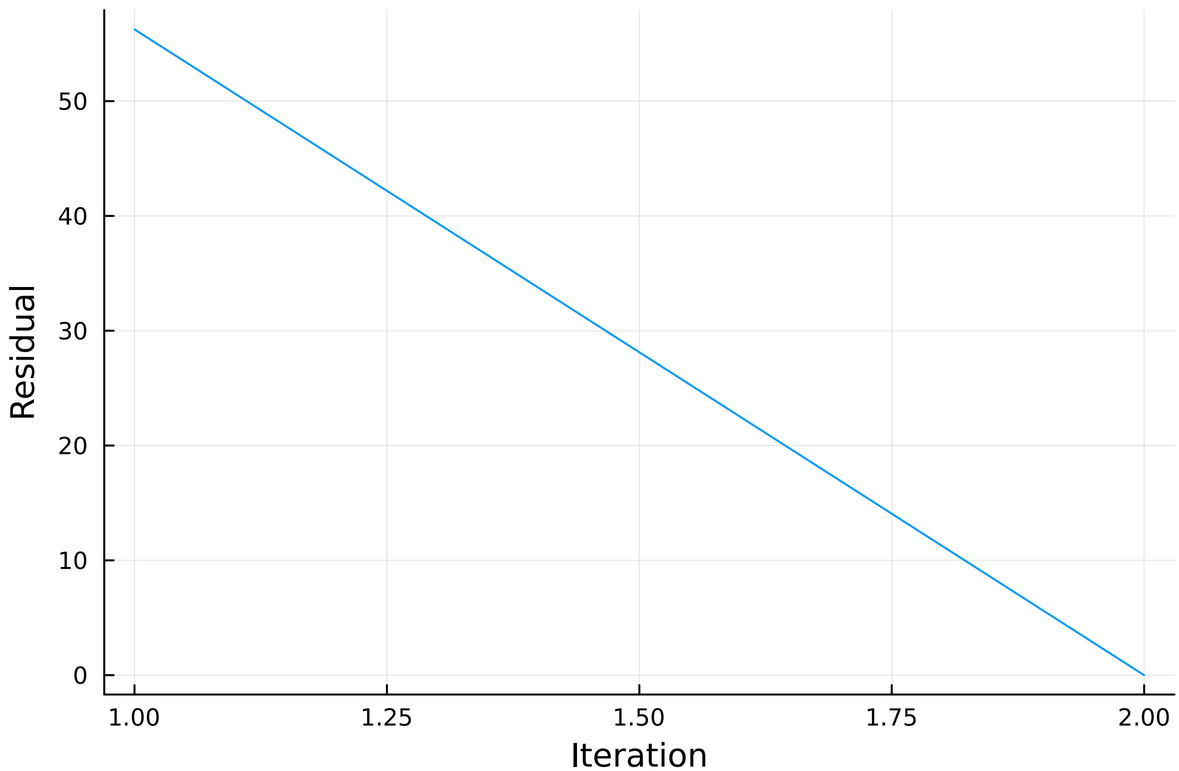
```
    rho = 0.
    beta = 0.
    h = 0.
    s = 0.
    t = 0.

    residual = Vector([])


    while(true)
        push!(residual,norm(r))
        rho = dot(r0hat, r)
        beta = (rho / rhoPrev) * (alpha / omega)
        p = r + beta*(p-omega*v)
        v = A*p
        alpha = rho / dot(r0hat, v)
        h = x + alpha*p
        if(norm(h-xForLater) < 1e-6)
            x = h
            break
        end
        s = r - alpha*v
        t = A*s
        omega = dot(t,s)/dot(t,t)
        x = h + omega*s
        if(norm(x-xForLater) < 1e-6)
            break
        end
        r = s-omega*t
        rhoPrev = rho
    end
    return(x, residual)
end

x = zeros(Float64, 9, 1)

answer = myBiCSTAB(A, x, b, xForLater)
#display(answer[1])
myPlot = plot(answer[2])
xlabel!("Residual")
ylabel!("Iteration")
display(myPlot)
```



In [ ]:
```
function gaussSeidel(nx,ny, tol, maxIter, T0, w; verbose=false)

    T = copy(T0)
    residuals = zeros(nx,ny)

    flag = 0
    iter = 0
    while flag == 0
        iter += 1 # iter = iter + 1

        # update all the open values of T
        # T[i,j] = 1/4*( T[i+1,j] + T[i-1,j] + T[i,j+1] + T[i,j-1]  )

        # Gauss-Seidel (no Tnew)
        for i = 2:nx-1
            for j = 2:ny-1
                Ts = 1/4*( T[i+1,j] + T[i-1,j] + T[i,j+1] + T[i,j-1]  )
                T[i,j] = (1-w)*T[i,j] + w*Ts
            end
        end

        for i = 2:nx-1
            for j = 2:ny-1
                residuals[i,j] = T[i+1,j] + T[i-1,j] + T[i,j+1] + T[i,j-1] - 4*T[i,
            end
        end

        if verbose
            @printf("iter=%3d, |res|=%.3e\n", iter, norm(residuals))
        end
```

```
        if norm(residuals) <= tol
            flag = 1
        elseif iter >= maxIter
            flag = -1
            error("Failed to converge")
        end

    end

    return (T, iter)
end


function createBoarder(topTemp, botTemp, lhsTemp, rhsTemp, gridSize)
    grid = zeros(gridSize, gridSize)
    for i in 1:size(grid, 1)
        grid[1,i] = topTemp
        grid[size(grid,1),i] = botTemp
        grid[i,1] = lhsTemp
        grid[i,size(grid,1)] = rhsTemp
    end
    return grid
end

function getGridCenter(grid)
    centerDistance = Int(floor(size(grid,1)/2)+1)
    return grid[centerDistance,centerDistance]
end

function getBIG()
    converged = false
    i = 5

    centerTemps = [0.0]
    resids = Float64[]
    while !converged
        thisGrid = createBoarder(100,0,75,50,i)
        global tempgrid, numIts =  gaussSeidel(i,i,1e-6,30,thisGrid,1.5)
        if abs( getGridCenter(tempgrid)-centerTemps[Int(length(centerTemps))]) < 1e
            converged = true
        end
        push!(centerTemps, getGridCenter(tempgrid))
        push!(resids,abs(centerTemps[length(centerTemps)] - centerTemps[length(cent
        i += 2
    end
    part2contour = contour(reverse(tempgrid, dims = 1), fill = true)
    part2plot = plot(resids , label = false )
    xlabel!("Iteration")
    ylabel!("Residual")
    return(part2plot, part2contour)
end

part2Plot, part2Contour = getBIG()
display(part2Contour)
display(part2Plot)
```





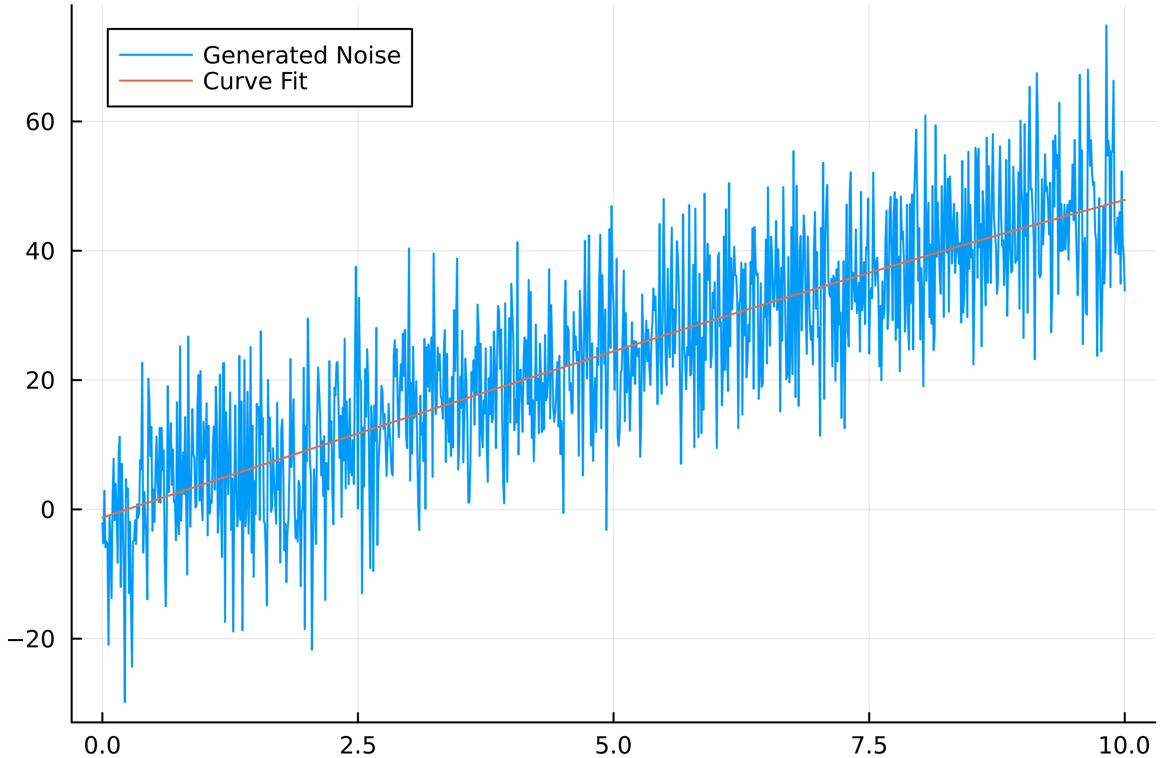
In [ ]:
```
function modelFunction( x, a)
    return a[1]  + a[2]*x + a[3]*x^2
    end

function changeZeroOneArray(onesPlace, arr)
    arr[:] .= 0
    arr[onesPlace] = 1
end
```

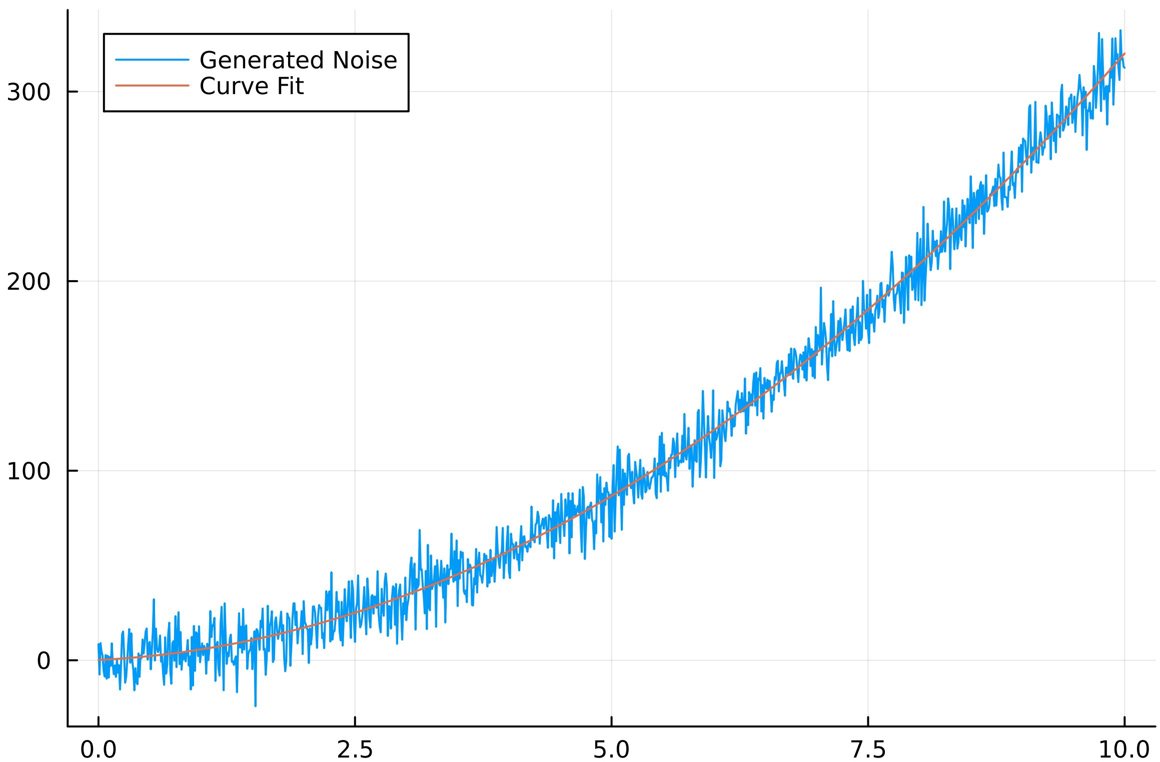
```
function buildX(f, x, n)
    X = zeros(Float64, length(x), n)
    zeroOneArray = zeros(Int, n)
    for i in 1:length(x)
        for j in 1:n
            changeZeroOneArray(j, zeroOneArray)
            X[i, j] = f(x[i], zeroOneArray)
        end
    end
    return X
end

## Part 4

function myCurveFit( X, y)
    a = (transpose(X) * X)^-1 * transpose(X) * y
    return a
end
```

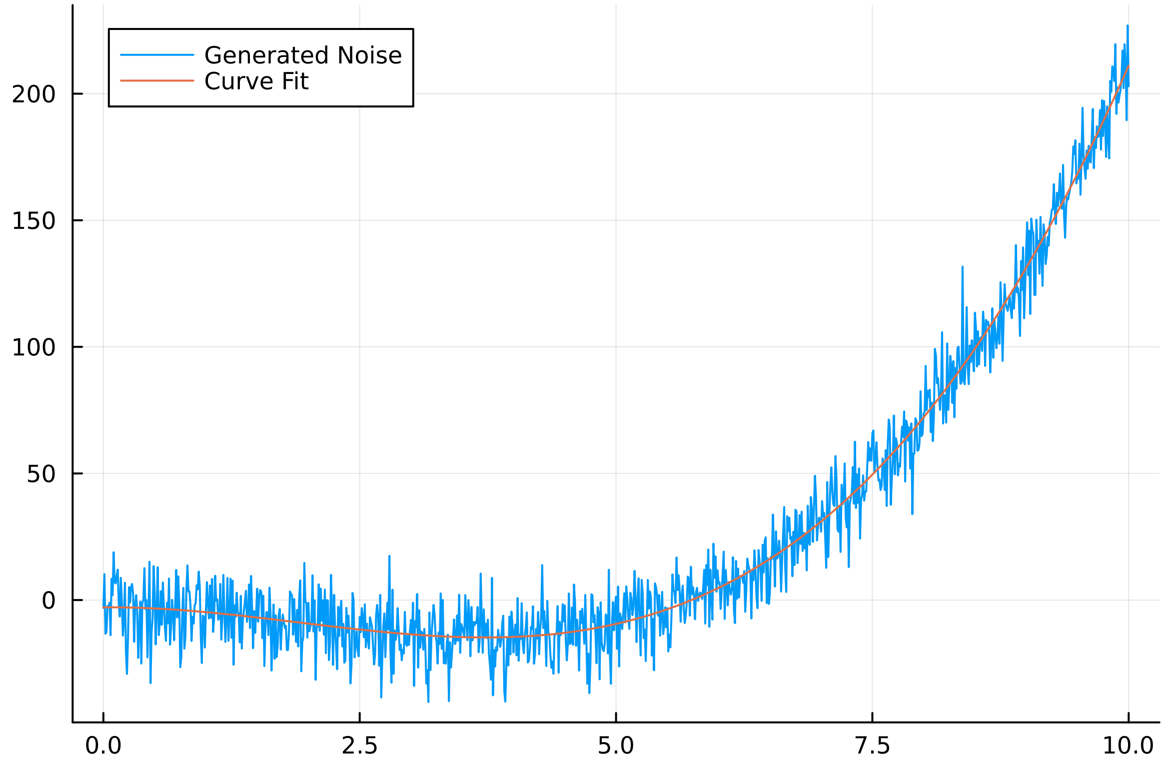
Out[ ]: myCurveFit (generic function with 1 method)

Part 5

In [ ]:
```
x = [0:.01:10;]

newYVals(x,a) = a[4] .* x.^3 + x.^2 .* a[3] .+ x .* a[2] .+ a[1]

lin = 5 .* x .- 1
quad = 3 .* x.^2  .+ 2 .* x .+ 1
cube = .5 .* x.^3 - 3 .* x.^2 + 1.9 .* x .- 7

ys = [lin, quad, cube]
i = 2
for y in ys
    yWNoise = add_gauss(y, 10)
    a = myCurveFit(buildX(newYVals, x, 4), yWNoise)

    plotLine = a[4] .* x.^3 + x.^2 .* a[3] .+ x .* a[2] .+ a[1]
    #display(a)
    display(plot(x, [yWNoise, plotLine], label = ["Generated Noise" "Curve Fit"]))
    i += 1
end
```


Generated Noise
Curve Fit
60
40
20
0
−20
0.0
2.5
5.0
7.5
10.0

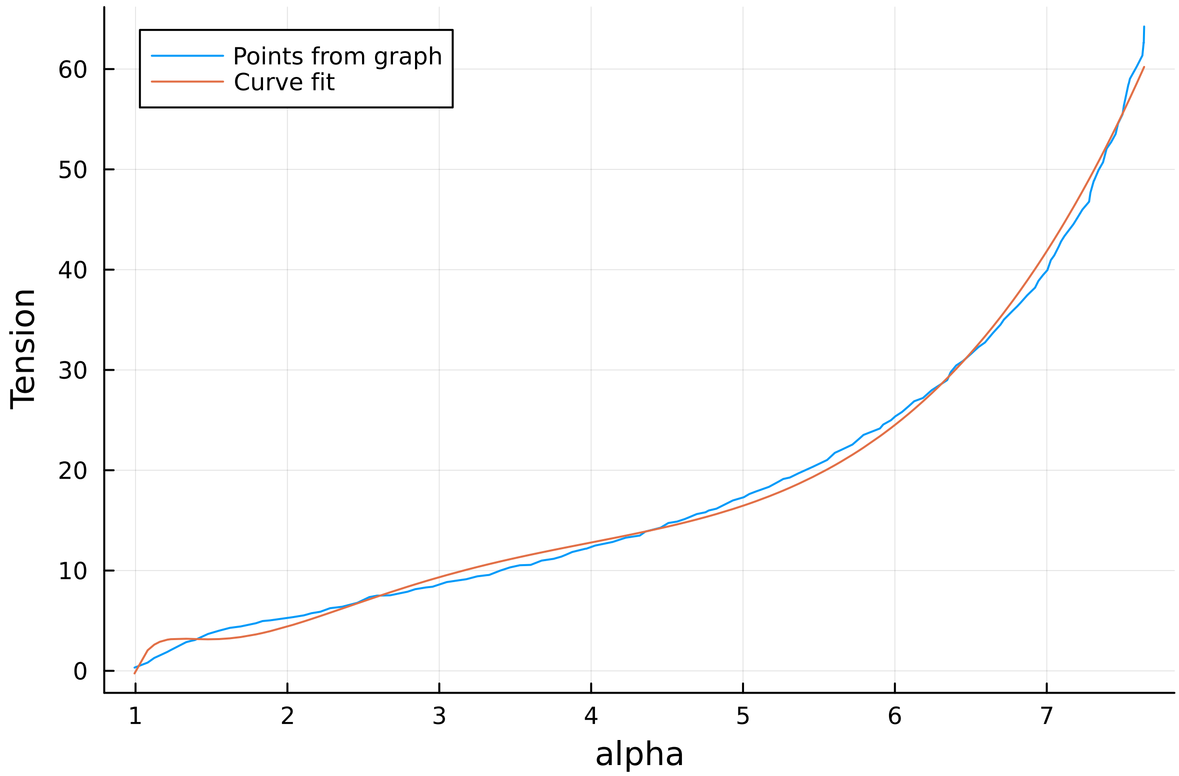

Generated Noise
Curve Fit
300
200
100
0
0.0
2.5
5.0
7.5
10.0



** Part 6 **

In [ ]:
```
digitizerResultsFrame = DataFrame(CSV.File("curvePoints3.csv"))
plot(digitizerResultsFrame[:,1], digitizerResultsFrame[:,2])

function Cnum(i, j)
    if i == 0 && j == 1
        return 1
    elseif i == 1 && j == 0
        return 2
    elseif i == 1 && j == 1
        return 3
    elseif i == 2 && j == 0
        return 4
    elseif i == 0 && j == 2
        return 5
    end
end

function modelFunctionrubber( λ, C)
    result =  2*(λ-λ^-2) * (C[Cnum(1,0)] + C[Cnum(0,1)] * λ^-1 + 2*C[Cnum(2,0)]*(λ^
    return result
end

function evaluateRubber()
    avals = myCurveFit(buildX(modelFunctionrubber, digitizerResultsFrame[:,1], 5),

    resultYs = zeros(Float64, length(digitizerResultsFrame[:,1]))
    for i in 1:length(digitizerResultsFrame[:,1])
        resultYs[i] = modelFunctionrubber(digitizerResultsFrame[i,1], avals)
    end
```

```
    thisPlot = plot(digitizerResultsFrame[:,1], [digitizerResultsFrame[:,2], result
    xlabel!("alpha")
    ylabel!("Tension")
    display(thisPlot)
    return avals

end

avals = evaluateRubber()
print("\nThe values of C are:\n", avals)
```




```
The values of C are:
[24.70905331708491, -18.443361937068403, 27.13641879055649, 0.22515616310374753, -
21.84215041110292]
```

** Response to part 6 questions ** Based on the collected data points versus the curve fit, this would be a decent model to use if it was okay to be off by 1 or 2. However, this curve fit does not match the low end of the graph very closely and deviates front the whole graph frequently. With more data points this might be improved but the model function wont be perfect.

$$-X^T(y - Xa) = 0 \Rightarrow a = (X^TX)^{-1}X^Ty$$

$$-X^Ty + X^TXa = 0$$

$$X^TXa = X^Ty$$

$$\boxed{a = (X^TX)^{-1}X^Ty}$$

$$X^+ = VS^+U^T \qquad S^+ = (S^TS)^{-1}S^T$$

$$X = USV^T \qquad (X^TX)^{-1}X^T = X^+$$

$$X^+ = (VS^+\underbrace{U^TU}_{I}SV^T)^{-1}\,VS^+U^T$$

$$X^+ = (VS^+S\underbrace{V^T)V}_{I}S^+U^T$$

$$V\underbrace{S^+S}_{I}S^+U^T$$

$$\boxed{X^+ = V\,S^+\,U^T}$$

Andrew Breemer | MENG 451 | HW 3

$$Q = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr(\sigma_{dev}^2)}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr^2(\sigma - \tfrac{1}{3} tr(\sigma) I)}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr^2\left(\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} - \tfrac{1}{3} tr\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} I\right)}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr^2\left(\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} - \begin{bmatrix} \frac{\sigma_1+\sigma_2+\sigma_3}{3} & 0 & 0 \\ 0 & \frac{\sigma_1+\sigma_2+\sigma_3}{3} & 0 \\ 0 & 0 & \frac{\sigma_1+\sigma_2+\sigma_3}{3} \end{bmatrix}\right)}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr^2\left(\begin{bmatrix} \frac{2\sigma_1-\sigma_2-\sigma_3}{3} & 0 & 0 \\ 0 & \frac{-\sigma_1+2\sigma_2-\sigma_3}{3} & 0 \\ 0 & 0 & \frac{-\sigma_1-\sigma_2+2\sigma_3}{3} \end{bmatrix}\right)}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr\begin{bmatrix} \frac{(2\sigma_1-\sigma_2-\sigma_3)^2}{9} & 0 & 0 \\ 0 & \frac{(-\sigma_1+2\sigma_2-\sigma_3)^2}{9} & 0 \\ 0 & 0 & \frac{(-\sigma_1-\sigma_2+2\sigma_3)^2}{9} \end{bmatrix}}$$

$$\sigma_v = \sqrt{\frac{1}{2}\left((\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right)}$$