Andrew Breemer | MENG 451 | HW3

$-X^T(y - Xa) = 0 \Rightarrow a = (X^TX)^{-1}X^Ty$

$-X^Ty + X^TXa = 0$

$X^TXa = X^Ty$

$\boxed{a = (X^TX)^{-1}X^Ty}$

$X^+ = VS^+U^T \qquad S^+ = (S^TS)^{-1}S^T$

$X = USV^T \qquad (X^TX)^{-1}X^T = X^+$

$X^+ = (VS^+\overbrace{U^TU}^{I}SV^T)^{-1}\ VS^+U^T$

$X^+ = (VS^+S\underbrace{V^T)V}_{I}S^+U^T$

$V\underbrace{S^+S}_{I}S^+U^T$

$\boxed{X^+ = VS^+U^T}$

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$$Q = \frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr(\sigma^2_{dev})}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr^2(\sigma - \frac{1}{3} tr(\sigma) I)}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr^2\left(\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} - \frac{1}{3} tr \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} I\right)}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr^2\left(\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} - \begin{bmatrix} \frac{\sigma_1+\sigma_2+\sigma_3}{3} & 0 & 0 \\ 0 & \frac{\sigma_1+\sigma_2+\sigma_3}{3} & 0 \\ 0 & 0 & \frac{\sigma_1+\sigma_2+\sigma_3}{3} \end{bmatrix}\right)}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr^2\left(\begin{bmatrix} \frac{2\sigma_1-\sigma_2-\sigma_3}{3} & 0 & 0 \\ 0 & \frac{-\sigma_1+2\sigma_2-\sigma_3}{3} & 0 \\ 0 & 0 & \frac{-\sigma_1-\sigma_2+2\sigma_3}{3} \end{bmatrix}\right)}$$

$$\sigma_v = \sqrt{\frac{3}{2} tr \begin{bmatrix} \frac{(2\sigma_1-\sigma_2-\sigma_3)^2}{9} & 0 & 0 \\ 0 & \frac{(-\sigma_1+2\sigma_2-\sigma_3)^2}{9} & 0 \\ 0 & 0 & \frac{(-\sigma_1-\sigma_2+2\sigma_3)^2}{9} \end{bmatrix}}$$

$$\sigma_v = \sqrt{\frac{1}{2}((\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2)}$$