Part IB Physics A: Lent 2022

QUANTUM PHYSICS EXAMPLES I

Prof. C. Castelnovo

1. In one of Millikan's experiments, a clean sodium surface was irradiated by light having various wavelenths λ, and the emitted electrons subjected to a retarding voltage V. The electron current was determined from the deflection d of an electrometer accumulating charge for 30 s. From the following tabulated results, estimate the stopping potential for each wavelength, and hence determine Planck'c constant $\hbar$, and the work function W_{Na} of sodium.

$\lambda = 546$ (nm)		434		405		365		313	
V (V)	d (mm)	V	d	V	d	V	d	V	d
0.253	28	0.829	44	0.934	82	1.353	67.5	1.929	52
0.305	14	0.881	20	0.986	55	1.405	36	1.981	29
0.358	7	0.934	10	1.039	36	1.458	19	2.034	12
0.410	3	0.986	4	1.091	24	1.510	11	2.086	5
				1.143	3	1.562	4	2.138	2.5

2. Calculate the de Broglie wavelength of:

(a) an 80 kg person walking at 6 km h^{-1};
(b) a photon of energy 20 eV;
(c) an electron of kinetic energy 20 eV;

Comment in each case on the scale of your result in relation to measurable effects.

3. A beam of electromagnetic radiation passes through a 50% beam splitter that is inclined at an angle of 45° with respect to the axis of the beam. Two detectors are used to measure the power that passes through the beam splitter, and the power that is reflected off of the beam splitter. For high intensity beams, the two detectors each read

50% of the power in the beam, as expected. The intensity of the beam is now reduced until photons pass through the system one at a time. If the detectors are sufficiently fast and sensitive, describe how they behave.

4. In an alternative universe, $\hbar$ has the value 10^{-6} J s instead of 10^{-34} J s. A dart with mass 1 kg is dropped from a height of 1 m, the intention being to hit a point target on the ground below. What limitation is imposed by the uncertainty principle on the accuracy that can be achieved?

(Neglect uncertainties in the vertical position and momentum, which produce only second-order effects.)

5. Use the uncertainty principle to estimate the ground state energy E_0 of a particle of mass m moving in a one-dimensional harmonic potential $\frac{1}{2}\kappa x^2$.

6. Determine the normalising constants for the following wave functions:

(a) $\psi(x) = A_1 \sin(\pi x/a), \quad 0 \leq x \leq a$;
(b) $\psi(x, y, z) = A_2 \sin(\pi x/a) \sin(\pi y/b) \sin(\pi z/c)$, in a rectangular box with sides of length a, b and c;
(c) $\psi(r) = A_3 \exp(-r/a)$, over all space.

7. Show that for a free particle, having a quadratic dispersion relation, of mass m with wavefunction

$$\psi(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-a^2(k-k_0)^2} \mathrm{e}^{i(kx-\omega t)} \mathrm{d}k,$$

the width of the wave packet is $\sim a$ at $t = 0$, and will double in a time $T = 2\sqrt{3}ma^2/\hbar$.

Calculate T for
(a) a proton localised to within 1 nm;
(b) a 1 kg mass localised to within 0.1 mm.

8. A particle is represented by the wavefunction

$$\psi(x) = Ax\mathrm{e}^{-\alpha x^2}.$$

(a) Calculate the normalising constant A.
(b) Calculate Δx and Δp_x.
(c) Show that $\Delta x \Delta p_x = 3\hbar/2$.

9. A beam of particles travelling in the x-direction with energy E is incident on a one-dimensional potential well with vertical sides of depth V and width a. Show that for certain values of E there is no reflected beam, and sketch the transmission coefficient as a function of E.

The scattering of electrons by Kr atoms shows a (first) minimum when the incident electron kinetic energy is increased from 0 to 0.5 eV. Estimate the effective potential U_{Kr} inside the Kr atoms, assuming that their diameter is 0.4 nm.

10. A particle is confined by the potential:

$$\begin{aligned} V(x) &= 0 \qquad && 0 < x < a \\ V(x) &= \infty && \text{elsewhere} \end{aligned}$$

Find the expectation value of x and the uncertainty in x when the particle is in its n^{th} energy state. Hence show that, as $n \to \infty$, the average value approach the value obtained from classical mechanics.

11. A particle of mass m is confined by the potential:

$$\begin{aligned} V(x) &= 0 \qquad && 0 < x < a \\ V(x) &= V_0 && a < x < 2a \\ V(x) &= \infty && \text{elsewhere} \end{aligned}$$

where $V_0 = 2\hbar^2\pi^2/ma^2$. If the particle's energy is $25\hbar^2\pi^2/8ma^2$, what is the probability of finding the particle in the interval $0 < x < a$: (a) quantum mechanically; (b) classically?

12. A one-dimensional rectangular potential well of depth V_0 has width $2a$. Show that there is one and only one bound state for a particle of mass m if

$$\frac{2ma^2V_0}{\hbar^2} < \frac{\pi^2}{4}$$

13. A particle is bound in a one-dimensional potential well:

$$\begin{aligned} V(x) &= \infty \qquad && x < 0 \\ V(x) &= -V < 0 && 0 < x < a \\ V(x) &= 0 && x > a \end{aligned}$$

in the lowest energy state with total energy $-V/4$.

Show that the probability that the particle is outside the attractive part of the well is

$$\frac{9\sqrt{3}}{8\pi + 12\sqrt{3}}$$

14. For a one-dimensional harmonic oscillator oscillating with amplitude a, show that the probability of finding the particle in the interval x to $x + \mathrm{d}x$ is, according to classical mechanics,

$$\begin{aligned} P_{\text{cl}}(x)\,\mathrm{d}x &= \frac{1}{\pi\sqrt{a^2 - x^2}}\,\mathrm{d}x; \qquad && |x| < a \\ &= 0 && |x| > a\ . \end{aligned}$$

With the aid of sketches compare this probability with the quantum mechanical one for the $n = 1$ eigenstate with normalised eigenfunction

$$\psi_1(x) = \frac{\sqrt{2}}{\pi^{1/4}}\frac{x}{x_0^{3/2}}\ \mathrm{e}^{-x^2/2x_0^2}$$

where $x_0 = \sqrt{\hbar/m\omega}$.

(Check the normalization of the classical distribution.)

15. Find, by inspecting the wave functions of a quantum simple harmonic oscillator, the energy eigenvalues of a particle of mass m moving in the potential:

$$\begin{array}{ll} V(x) = \infty & x \leq 0 \\ V(x) = m\omega^2 x^2/2 & x > 0 \ . \end{array}$$

16. Write a few brief notes on the *correspondence principle*, and discuss these with your supervisor.

ANSWERS:

1. The currently accepted values are $\hbar = h/2\pi = 1.05457266(63) \times 10^{-34}$ J s and $W_{\rm Na} = 2 - 3$ eV.

2. (a) 5×10^{-36} m; (b) 6×10^{-8} m; (c) 3×10^{-10} m.

4. $\simeq 1$ mm.

5. $\langle E_0 \rangle \sim \frac{1}{2}\hbar\sqrt{\kappa/m}$.

6. (a) $|A_1| = \sqrt{2/a}$; (b) $|A_2| = \sqrt{8/abc}$; (c) $|A_3| = 1/\sqrt{\pi a^3}$.

7. (a) $\sim 6 \times 10^{-11}$ s;(b) $\sim 3 \times 10^{26}$ s $\approx 10^{19}$ years.

8. (a) $|A| = \sqrt{2^{5/2}\alpha^{3/2}/\pi^{1/2}}$; (b) $\Delta x = \sqrt{3/4\alpha}$; (c) $\Delta p_x = \hbar\sqrt{3\alpha}$.

9. No reflected beam for $E = n^2\pi^2\hbar^2/2ma^2 - V$; $U_{\rm Kr} \sim -1.85$ eV.

10. $\langle x \rangle = a/2$; $\langle x^2 \rangle = (a^2/6)\,(2 - 3/n^2\pi^2)$; $(\Delta x)^2 = (a^2/12)\,(1 - 6/n^2\pi^2)$..

11. (a) 1/2; (b) 3/8.

15. $E_n = \hbar\omega(2n + \frac{3}{2})$, $n = 0, 1, 2, 3, \ldots$.