Introduction

Relativistic Astrophysics and Cosmology: Lecture 1

Sandro Tacchella

Friday 6th October 2023

Housekeeping

- I'm Dr Sandro Tacchella, Assistant Professor in Extragalactic Astrophysics at the Cavendish.
- Research interests: observations and models of galaxy formation and evolution; co-evolution of dark matter and baryon; evolution of the first galaxies and black holes in the early Universe.
- Tools: largest telescope in space (e.g., JWST) and on the ground (e.g., VLT); Bayesian statistics and Machine Learning; development and analysis of numerical and analytical simulation and models.
- Handouts will be available on Moodle & Teaching Information System (TiS).
- Feedback is very welcome (especially typos), either via email, the forms toward the end of term or in-person after lecture finishes.
- Lectures are Fridays, Mondays and Wednesdays at 10:00 am.
- Lectures are being recorded and made available via Moodle.

Housekeeping

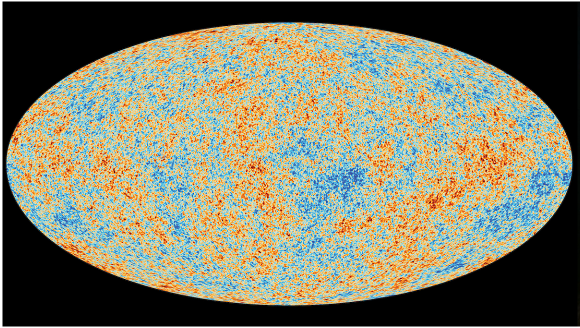
- Supervisions: discuss examples, answer questions, and prepare for the exam.
- Supervisions will be done by David Puskas and Sandro Tacchella.
- Examples Sheet 1 will be available on Monday.
- 1st supervision: Oct 16 - Oct 18
- 2nd supervision: Oct 30 - Nov 1
- 3rd supervision: Nov 13 - Nov 15
- 4th supervision: Nov 27 - Nov 29
- Sign-up sheets with further information will be distributed on Monday.

Big Thanks

- to Dr. Will Handley, who has upgraded and provided most of the teaching material
- to previous lecturers, in particular Prof. Andy Fabian and Prof. Anthony Lasenby

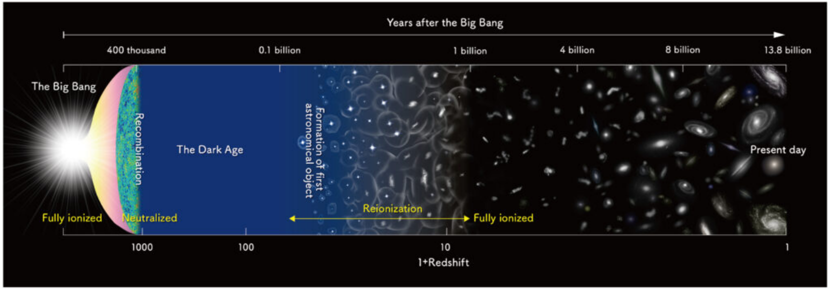
From Big Bang to Black Holes

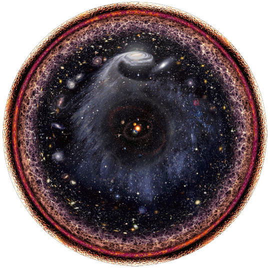
 

- Course covers some of the most extreme physics we know.
- Gives an overview of our Universe from one end of spacetime to the other.
- Touches on some of the most exciting results in 20th and 21st century physics.

Course contents: Cosmology

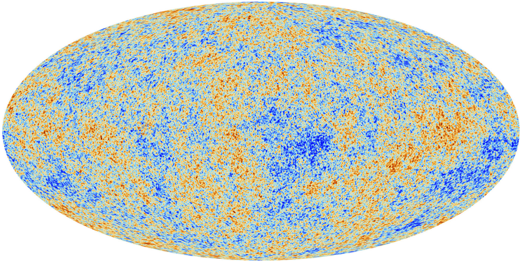
- Precision answers to the oldest questions:
  - How old is the Universe? (13.8 $\pm$ 0.1 Gyr)
  - How did it begin? (Hot big bang)
  - How will it end? (exponential expansion)
  - What's its shape? (flat)
  - How big is it (infinite, but only just)
  - What's in it? (70% dark energy, 25%/5% dark/visible matter)

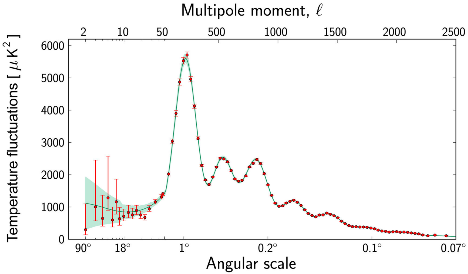




Course contents: Cosmic microwave background

- The Planck satellite 2013 data release heralded the era of precision cosmology.
- Map of the anisotropies is a photograph of the Universe when it was 300,000 years old.
- Winding forward from this, we would see galaxies and galaxy clusters coalescing around regions of higher density.
- Winding backward, we find these fluctuations originate from primordial quantum mechanics.
- The statistical power of this data remains our strongest constraint on models of the Universe.

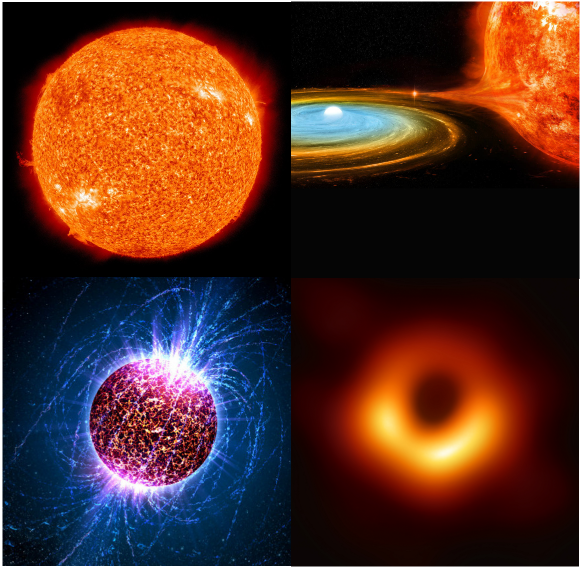




Course contents: Compact objects

- Covering the life cycle and variety of stars:
  - Main sequence stars
  - Red supergiants
  - White dwarfs
  - Accretion disks
  - Neutron Stars/Pulsars
  - Black Holes

  and the interplay between quantum mechanics and gravity that underpins them.

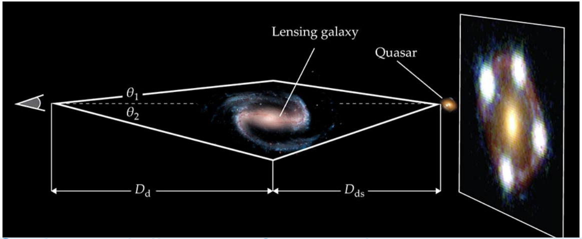


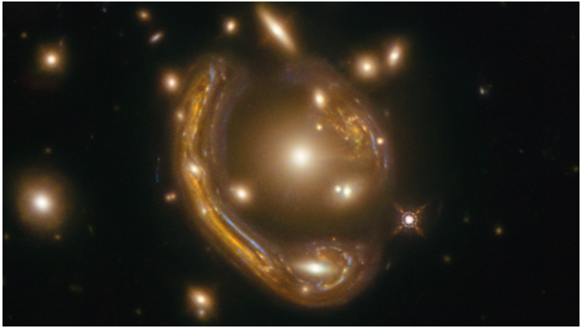
Course contents: Energetic events

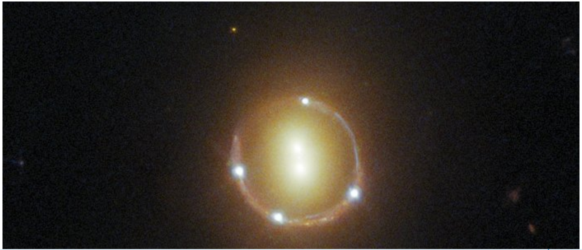
- **Supernovae:** Detonating stars.
- **GRB:** Gamma ray bursts.
- **AGN:** Active galactic nuclei: Supermassive black holes powering galaxies from their centre, manifesting as quasars.
- **Jets:** Ejecta from accretion disks at apparently superluminal velocities.
- **Pulsars:** Neutron star radio emissions.
- **Mergers:** Colliding Black holes and/or neutron stars.

Course contents: Gravitational Lensing

- Light bends around galaxies and clusters.
- Can be used like any other optical lens as a telescope onto the deep Universe.
- Can "weigh" galaxies and measure the size and shape of the Universe.
- Weak lensing and microlensing can be used as statistical probes of cosmology.

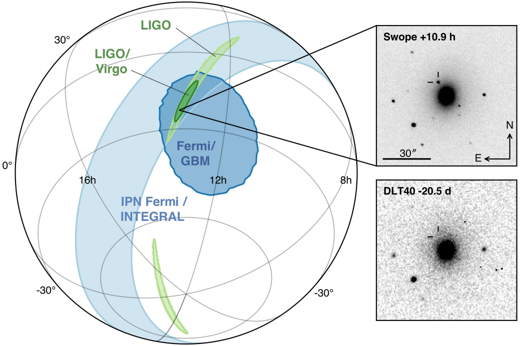


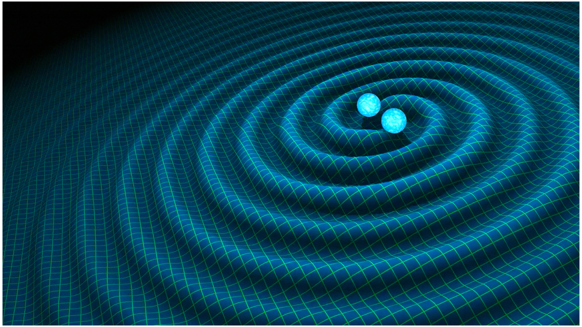


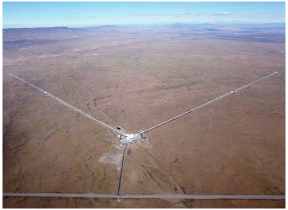


Course contents: Gravitational waves

- In 2015 LIGO made the first direct observation of gravitational waves.
- Confirmed Einstein's 100 year prediction.
- LIGO acts a telescope for the most energetically extreme events in the Universe.





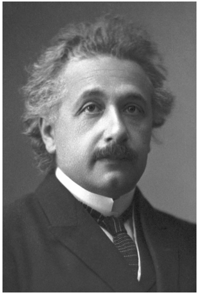


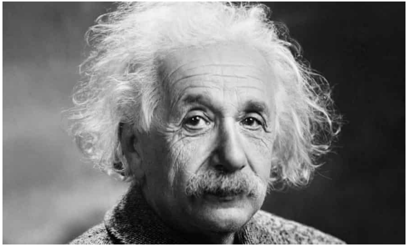
Course contents: Overview

- Lectures 1-3: Basics of General Relativity (Part II)... building intuition
- Lectures 4-6: Beyond Part II, incl. Kerr metric and Hawking black hole thermodynamics
- Lectures 6-12: Compact objects
- Lectures 13-15: Gravitational waves & lensing
- Lectures 16-18: Cosmology Theory
- Lectures 19-21: Measuring our Universe (Observational Cosmology)
- Lectures 22-24: The primordial universe

Basics of General Relativity — Introduction

- We cover all these topics. A common thread, vital to the Physics of all of them, is General Relativity (GR).
- Idea of next few lectures is to review the basics of GR, for which there has already been a course taught in Part II.
- This will not be just revision, however. A major aim is to emphasise the physical basis of GR, and to build up physical intuition by working with some examples of GR in action.
- Also will try to emphasise features and results most useful for other part of course, on the astrophysics of individual objects.
- Then later, will apply to Cosmology!





- So what are the physical underpinnings of GR?
- To lead through to this, we ask a question which was indeed a starting point for Einstein:
- 'Why can't gravity just be treated as a force similar to other forces within special relativity?'
- For example electromagnetism, which even though it was fully described by Maxwell before special relativity (SR), turns out to be entirely compatible with SR.
- To show that we cannot take this route, consider the following:

Einstein's Tower: Incompatibility of Gravity and Special Relativity

- Consider a tower with a device at the bottom which can convert a falling particle into an upward going photon, without loss of energy.
- At the top there is another device, which reconverts the photon into a mass.
- The fact that such devices would be impossible to construct with perfect efficiency does not render the theoretical argument invalid.
- The point is to get a prediction and then compare it with experiments which *can* be done.

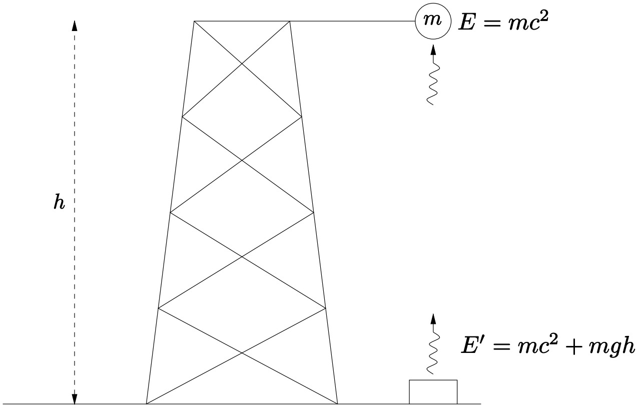


Figure: Tower experiment with mass and photon.

- If the photon arrives at the top with the same energy it had at the bottom, i.e. $E' = E + mgh$, where $E = mc^2$ is the original energy of the particle before dropping, then we would have a perpetual energy machine, providing energy (from nowhere) of $E' - E = mgh > 0$ per cycle.
- Instead, since we're assuming no energy is lost, we must have that the photon energy on arriving at the top $= E$.
- Thus a photon climbing in the Earth's gravitational field satisfies

$$
\begin{aligned}
E_{\text{emission}}\ (= E') &= E_{\text{recep}}\left(1 + \frac{gh}{c^2}\right) \quad (E_{\text{recep}} = E) \quad \text{and} \quad E = 2\pi\hbar c/\lambda, \\
\text{i.e. redshift } z &= \frac{\lambda_{\text{recep}} - \lambda_{\text{emission}}}{\lambda_{\text{emission}}} = 1 + \frac{gh}{c^2} - 1, \\
\text{i.e.} \quad z &= \frac{gh}{c^2} = \frac{\Delta\phi}{c^2}, \qquad (1)
\end{aligned}
$$

where ϕ = Newtonian potential per unit mass.
- This is the *gravitational redshift formula*. We have worked it out using a highly unlikely experimental setup, but its predictions can be checked for any photon moving in a gravitational field.

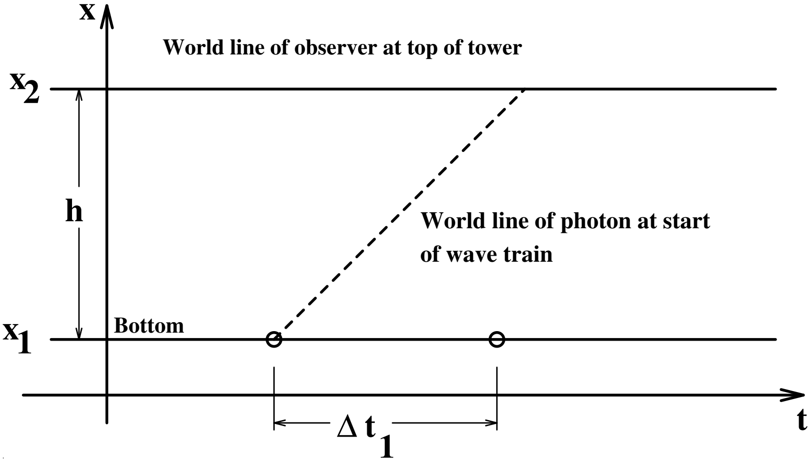
- The most direct check until recently, was carried out by Pound & Rebka (1960) and Pound & Snider (1965) who measured the *difference* in redshift for γ ray photons moving up then down a tower 22.5 m high at Harvard University (The frequency precision was guaranteed by use of the Mossbauer effect).
- The prediction from our formula is

$$z_{\rm up} - z_{\rm down} = \frac{2gh}{c^2} = \frac{2 \times 9.8 \times 22.5}{(3 \times 10^8)^2} = 4.9 \times 10^{-15},$$

  or 4.905×10^{-15} if one uses more precise values. They measured $(4.900 \pm 0.037) \times 10^{-15}$, so the effect was verified to $\sim 1\%$ precision.
- Other applications of the formula are to the spectral shift of lines emitted from the surfaces of white dwarfs, or neutron stars — you'll be asked to estimate these in the first examples sheet.
- Since oscillating systems, like those which produce these lines, can be thought of as *clocks*, formula also has implications for the rate at which time progresses in a gravitational field.

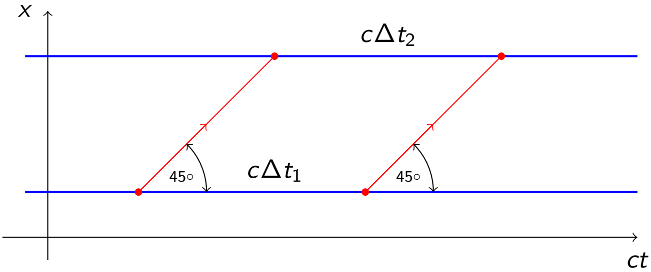
Incompatibility of Gravity with SR

- This is demonstrated most clearly in a slightly different version of the tower experiment discussed by Schild in the 1960's.
- Consider two observers, one at the bottom of the tower, one at the top.
- Suppose there exists a common Lorentz frame (i.e. ordinary SR frame) tied to the Earth in which they are both at rest.
- Can draw a spacetime diagram for them:



- Let the bottom observer emit an electromagnetic wavetrain of frequency ν_1, containing N crests. The time required for this is $\Delta t_1 = N/\nu_1$.
- The question is, how long will the observer at the top take to receive this wavetrain? They must count N crests (this is the invariant), but at a frequency ν_2 where $\nu_2 < \nu_1$ due to the gravitational redshift. Thus the crests will take a longer time

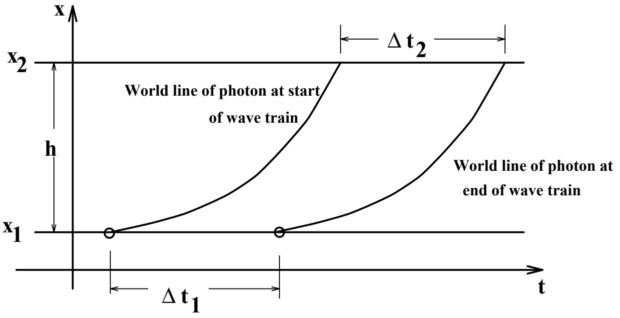
$$\Delta t_2 = \frac{N}{\nu_2} > \frac{N}{\nu_1} = \Delta t_1,$$



- But on the spacetime diagram, ordinary Lorentz geometry with light rays says that the world lines for the photons emitted at the beginning and end of the wavetrain must both climb up at 45°.
- That is, special relativity insists $\Delta t_1 \equiv \Delta t_2$.
- Thus we have a contradiction.

- Now an initial response to this might be to argue that the analysis is oversimplifed. For suppose the gravitational field modifies the way photons travel (We know experimentally that indeed gravity bends light).
- Thus we must allow the photon path corresponding to the start of the wavetrain to be deformed in some way due to this, as shown in the next diagram.

- What we can say with certainty is that nothing about the situation is changing with time.
- Thus the photon paths must be congruent, and so SR *still* predicts $\Delta t_1 = \Delta t_2$, in contradiction to the argument from gravitational redshift.



Same as previously but with gravity allowed to modify the speed at which the photons travel.

Then how do we combine gravity and special relativity?

Einstein's critical insight centres around the observation:

It is remarkable that heavier objects don't fall faster than lighter ones.

- Fundamentally there are two places where mass enters in dynamics:

  Inertial mass: m_I, which enters in Newton's second law $F = m_\mathrm{I} a$, representing an objects resistance to change in motion/momentum.

  Gravitational mass: m_G, which enters in Newton's law of gravity $F = \frac{GM_\mathrm{G} m_\mathrm{G}}{r^2}$, representing an objects "charge" in analogy with electromagnetism.

- The fact that we observe $m_\mathrm{I} = m_\mathrm{G}$ means that equating the two equations above

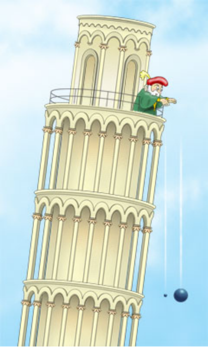
Gravity

$$F_\mathrm{grav} = m_\mathrm{I} a = \frac{GM_\mathrm{G} m_\mathrm{G}}{r^2},$$
$$\Rightarrow a = \frac{GM_\mathrm{G}}{r^2} = g,$$

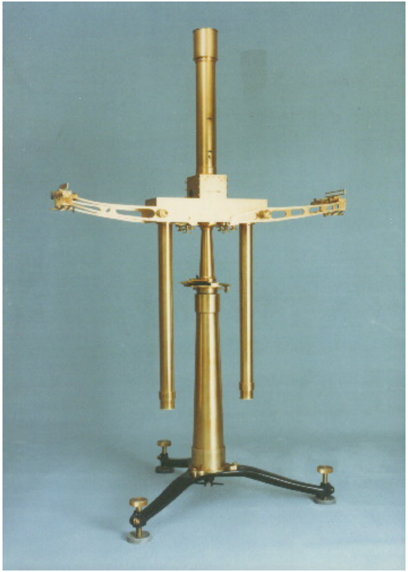
Electromagnetism

$$F_\mathrm{EM} = m_\mathrm{I} a = \frac{Qq}{4\pi\varepsilon_0 r^2},$$
$$\Rightarrow a = \frac{Qq}{4\pi\varepsilon_0 r^2 m_\mathrm{I}} = f\frac{q}{m_\mathrm{I}}.$$

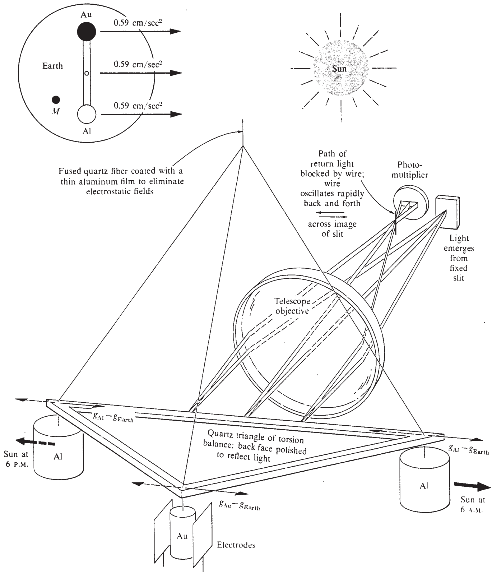
- Historically this has been confirmed to extremely high precision.
- **Galileo:** Objects of different composition were dropped from the Tower of Pisa.
- The fact they had the same time of fall implied the same acceleration, which meant the ratio of gravitational to inertial mass was constant.
- These experiments were not very accurate, but Galileo did some more accurate experiments with inclined planes, to accuracy of maybe 1 part in 100.



- **Eötvös (1922):** looked at the acceleration of wood, copper, platinum, asbestos, water, due to the Earth.
- To do this he used a torsion balance with a different substance on each end in order to measure the difference in acceleration.
- None was found to an accuracy of better than 1 part in 10^9.
- Note that a horizontal-armed balance works, since looking at the combined effects of the Earth's attraction, plus centrifugal force.



- **Roll, Krotkov and Dicke, 1964 and Braginsky and Panov, 1971:**
- These groups, one in the USA and one in Russia, used a modified (three arm) torsion balance to look for 24 hour periodicity induced by the Sun's pull as it sweeps around the Earth.
- The magnitude of this is about $5.9\,\mathrm{mm s^{-2}}$ — compare $9.81\,\mathrm{m s^{-2}}$ for the Earth's pull.
- The substances used (gold versus aluminium for the Roll *et al.* experiment and platinum versus aluminium for Braginsky & Panov) were chosen to have different ratios of neutrons, protons, binding energy to mass energy etc.
- Measured $m_G \approx m_I$ to 1 part in 10^{15}.



Fictitious forces

- Gravity is not alone in this mass cancellation property

$$\begin{aligned} \text{Centrifugal force} \quad F &= -m\omega^2 r \\ \text{Coriolis force} \quad F &= -2m\,\omega \times \vec{v} \\ \text{Euler force} \quad F &= -m\,\dot{\omega} \times \vec{x} \\ \text{G-force} \quad F &= -ma \end{aligned}$$

- These are all fictitious forces, which arise from a choice of non-inertial reference frame.
- They can be removed by changing reference frame (stepping out of car, getting off roundabout).
- Einstein's insight is to build this into his theory of gravity from the start.
- The "correct" inertial reference frame is free-fall
- Dropping into free fall removes the effect of gravity (at least locally)

The Equivalence Principles

The Weak Equivalence Principle (WEP)

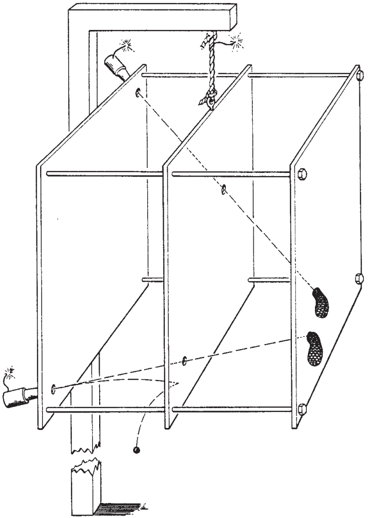
At any point in a gravitational field, then in a frame moving with the free fall acceleration at that point[1], the laws of motion of free test particles have their usual special relativistic form, except for gravity, which disappears locally.

The Strong Equivalence Principle (SEP)

At any point in a gravitational field, then in a frame moving with the free fall acceleration at that point[a], then all the laws of physics (dynamics, electromagnetism, quantum mechanics etc.) have their usual special relativistic form, except for gravity, which disappears locally.

[a] Note that how rapidly this acceleration varies as a function of position determines the region over which the local special relativistic frame is of any use

Weak Equivalence Principle



- One can think of the WEP as saying that instead of thinking of particles as moving under a force — gravity — we instead analyse the situation in a frame *without* gravity, but moving at the free fall acceleration appropriate to that point.
- Consider the experiment shown in the figure (taken from 'Gravitation', by Misner, Thorne & Wheeler).
- It won't work if hung suspended. Ballistics tells us trajectories are parabolas, and the target is a straight line by construction.
- However, if the rope is cut it becomes a local Lorentz frame, and the falling trajectory of the system matches that of the projectile.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- Another, now very familiar example, is the interior of a spacecraft in orbit around the Earth.
- There test particles move on straight lines with uniform velocity — a very good local SR frame — despite the presence of a large gravitating body just nearby.
- Conversely, if we are inside a rocket in some remote region free from gravity we can *simulate* gravity by giving the rocket an acceleration $\boldsymbol{a}$.
- This will be indistinguishable from a uniform gravitational field with $\boldsymbol{g} = -\boldsymbol{a}$.
- Thus the 'equivalence' in the principle of equivalence is between gravity and acceleration.
- Let us take this literally, then the force of gravity at the surface of the Earth is because we are *accelerating upwards, at* $-\boldsymbol{g}$, *relative to free fall*.

Consequences of the Equivalence Principle

- You may recall from your course last year that there is a component of ‘real’ gravity, not reducible just to an equivalent acceleration.
- This is very small near the Earth but quite big near black holes, and manifests itself only over long distances and long times.
- It therefore does not show up in the ‘local’ effects just discussed.
- The motion of a test body in a gravitational field is entirely independent of its nature, mass and composition, and depends only on its position (and instantaneous velocity) in spacetime.
- Note that begins to hint at the (central) idea that gravity has to do with geometry. Nothing about the particle matters, only where it is.
- Gravity lays down ‘tracks’ along which everything must move.
  - Spacetime tells matter how to move
  - Matter tells spacetime how to curve (John Wheeler)

Is this the only view? [non-examinable]

- An interesting counterfactual to consider: What would have happened if Einstein hadn't become interested in modernising Newton's gravity?
- If the task had been left to Feynmann, he would have couched gravity as a Gauge theory, as we do the rest of the standard model.
- Gauge theories of gravity *do* exist [arxiv:0404119].
- In general such theories have (and must have) near-identical predictions to Einstein's differential geometric GR.
- They tend to differ in their topological properties.
- These theories can differ in their large/small scale predictions, and may explain observations usually interpreted as the effects of dark matter/dark energy [arxiv:2003.02690].
- Also interesting in the context of finding a version of gravity compatible with QFT.
- An area of active research in the Cavendish.
- For the rest of the course, we will keep things in the Einsteinian picture.

Summary

- General Relativity is a critical component of astrophysics, driving some of the important and cataclysmic processes in the Universe.
- Einstein's critical insight in unifying special relativity and gravity is that "it *is* odd that heavier objects don't fall faster".
- We should not regard local gravity as a force in the conventional sense, but instead as a manifestation of an improper choice of reference frame.
- The true inertial reference frame (A Lorentz frame) is free fall.

Next time

- Recap of mathematics of general relativity.