The primordial universe

Relativistic Astrophysics and Cosmology: Lecture 22

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Friday 24$^{\text{th}}$ November 2023

Pre-lecture question:

Where is the edge of the Universe?

Feedback request

https://cambridge.eu.qualtrics.com/jfe/form/SV_bsl6yIsiZgvsPC6

- Feedback encouraged.
- Deadline Sunday 26$^{\text{th}}$ November (after Lecture 22).
- 11 short, multiple choice questions.
- The free-form (optional) box is particularly useful for improving the course, e.g:
  - What would you have like to see more/less of?
  - What worked well/not-so-well?
  - How well does this fit with Part II/III Physics?
- Important for influencing future students, department and lecturer.

Last time

- Observational cosmology: CMB, BAO & SNe

This lecture

- Horizons in cosmology
- The horizon, flatness & monopole problems
- Scalar fields & inflation
- The cosmological constant

Next lecture

- Perturbations – beyond homogeneity

Horizons in general relativity

- We have encountered the event horizon of a black hole (a point of no return).
- In special relativity, uniformly accelerating observers have “apparent horizons”.
- These are obviously related under the equivalence principle (if local gravity is acceleration, the black hole event horizon has many of the hallmarks of an apparent horizon).
- Will discuss three horizons used in cosmology:
  - The particle horizon,
  - The event horizon,
  - The Hubble horizon.
- Note that cosmologists may mean any of the three when they speak of “the horizon”.
- Usually they will be referring to the Hubble horizon.

The Particle Horizon

- We have already encountered the particle horizon briefly in Lecture 19.
- The particle horizon is the furthest a photon can have travelled in the lifetime of the universe until now

$$\boxed{\chi_p = \int_0^{t_0} \frac{c dt}{R}.}$$

- We may interpret this as the "conformal time to the big bang" since $\chi = \eta$ for a photon.
- We may measure it in terms of the comoving horizon χ, or the physical horizon $R_0\chi$.
- By symmetry, this is the furthest we can see.
- For an Einstein-de-Sitter universe, we have $\boxed{\chi_p^{\mathrm{EdS}} = 3ct_0.}$
- This is quite a good mnemonic – if the big bang was roughly 14 by ago, then in comoving distance the edge of the observable universe is roughly 42 bly away.

The Event Horizon

- The event horizon is the dual of the particle horizon.
- It is the furthest a photon will be able to travel by the end of the universe

$$\boxed{\chi_e = \int_{t_0}^{\infty} \frac{cdt}{R}.}$$

- By symmetry, objects further away than this will never be seen by us.
- In many cases (e.g. Einstein-de-Sitter), $\chi_e = \infty$, i.e. eventually we'll be able to see the whole universe given enough time.
- However, universes with a cosmological constant have finite χ_e.

The Hubble Horizon

- We have already encountered the Hubble time H_0^{-1}, as the rough age of the Universe.
- In particular, in a constant velocity universe, $\dot{R} = \text{const}$ and H_0^{-1} is the age.
- Depending on the acceleration $\ddot{R}$, the real age can be larger or smaller than this, if the $R - t$ curve is concave or convex.
- The Hubble horizon, or Hubble radius is the equivalent, but in distance $\boxed{R_h = cH^{-1}}$.
- It defines the region of 'superluminal' expansion – objects further than the Hubble radius have a recession 'velocity' greater than the speed of light.
- It is also useful to define the comoving Hubble radius $\boxed{\chi_h = \frac{R_h}{R} = \frac{c}{RH} = \frac{c}{\dot{R}}}$, the same concept but measured in comoving coordinates.
- Note that in an EdS universe χ_h increases with time, and points on it are not really a "horizon" (i.e. a point of no return).

How does the horizon change with time?

- Consider the rate of change of χ_p with time

$$\chi_p = \int_0^{t_0} \frac{c\,dt}{R(t)}, \quad \Rightarrow \quad \frac{d\chi_p}{dt_0} = \frac{c}{R(t_0)} > 0, \quad \text{i.e. the horizon is expanding.}$$

- Parts of the universe which were not in view before, slowly come into view in time.
- Does this mean that a galaxy suddenly appears in the sky?
- Recall:

$$\chi^{\mathrm{EdS}} = \int_0^z \frac{cdz}{R_0 H(z)} = \int_0^z \frac{cdz}{R_0 H_0 (1+z)^{3/2}} = \frac{3ct_0}{R_0}\left(1 - \frac{1}{\sqrt{1+z}}\right).$$

- Comparing with $\chi_p = \frac{2c}{R_0 H_0}$ for EdS tells us immediately that objects, when they appear over the horizon, do so at infinite redshift, and thus do not suddenly burst into appearance.
- The velocity of the event horizon is similarly $\frac{d\chi_e}{dt} = -c/R < 0$, i.e. approaching us (unless it is infinite), but similarly, this does not mean that galaxies that we see now will disappear, since the event horizon is a future surface and not something we see.

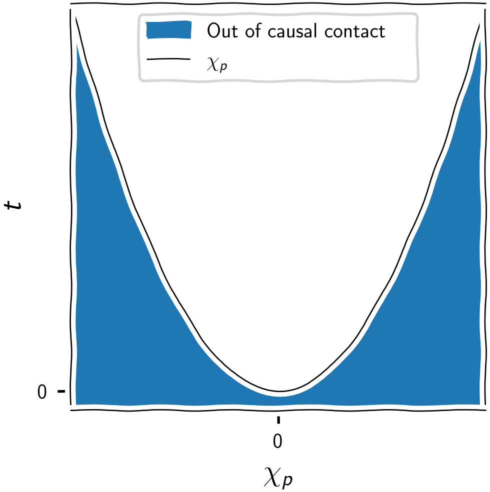
Particle horizons in general w-cosmologies

- Consider a cosmological model with dominant equation of state w

$$\frac{R}{R_0} = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} \quad H = \tfrac{2}{3(1+w)t} \quad \chi_p = \int_0^{t_0} \frac{c\,dt}{R(t)} = \frac{3ct_0}{R_0}\frac{1+w}{1+3w} = \frac{2cH_0^{-1}}{R_0(1+3w)} = \frac{2\chi_h}{1+3w}.$$

- This tends to a *finite* quantity if $w > -\frac{1}{3}$.
- A particle horizon exists if $\chi_p < \infty$, and the above shows that one exists in every Friedmann model, at least in the early stages ($w = \frac{1}{3}$ from Lecture 22).

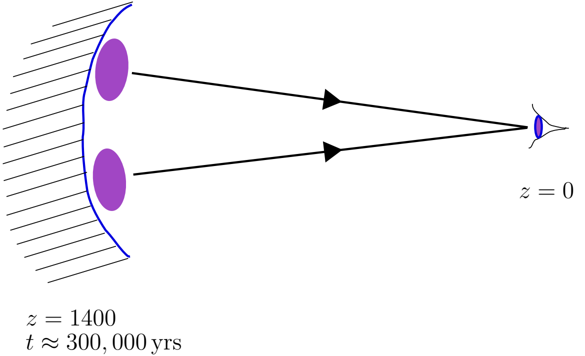
- Points with $\chi > \chi_p$ (which must exist since the EdS universe is spatially infinite), are such that we cannot observe their radiation at the present epoch, since the light from them hasn't had time to reach us!
- We are not, and never have been, in causal contact with regions of the universe with $\chi > \chi_p$.
- This is unexpected: whole universe comes from an initial singularity, but we predict that there are still causally disconnected regions today.
- Brought about by fact that spacetime curvature $\ddot{R}/R$, is very large in the early universe (although purely spatial curvature is negligible).



The Horizon problem

- The observation of the near-perfect uniformity of the CMB presents a real problem
- You will go through this more carefully on Examples Sheet 4, but

- the universe was flat and radiation dominated, so $R \propto t^{1/2}$, *before* recombination, and
- the universe was flat and matter dominated (i.e. EdS), so $R \propto t^{2/3}$, *after* recombination.
- We therefore find that $\chi_p = 2ct_{\rm rec}$
- The angle subtended by this distance, is $\Delta\theta = \frac{2}{3}(1 + z_{\rm rec})^{-1/2}$ radians today.
- Evaluating this for $z_{\rm rec} \approx 1400$, yields an angle of about 1°.



- Note including Λ changes the results slightly, but not in any major way.

- CMB photons from regions of the sky separated by more than a thumbs width on the sky, must therefore have come, on the standard big bang model, from regions that had never been in causal contact in their past!
- The CMB anisotropy on these scales is about 1 part in 10^5 – it is a real problem for the standard big bang as to how these regions managed to synchronize their temperatures to such amazing accuracy.

Other cosmological problems

- Why is the universe close to flat?
- It turns out that without a cosmological constant, then if the universe starts out not at exactly critical density, then there is a runaway behaviour for Ω_m, moving ever more rapidly away from flatness.
- One needs incredible fine tuning in the past in order to get Ω_m close to one today
- More quantitatively, isolating the curvature parameter from the evolution equations

$$\frac{d\log|\Omega_k|}{d\log R} = (1-\Omega_k)(1+3w)$$

  This is known as the 'flatness problem'.
- Why isn't the universe filled with topological defects, like monopoles, strings, domain walls etc.
- Phase transitions and symmetry breaking in the early universe are expected to lead to these, just as occurs in a liquid crystal — where are they today?
- Why is the microwave background so smooth? (The horizon problem as discussed earlier)

Acceleration $\Leftrightarrow w < -\frac{1}{3}$

- We have already found hints of how this might be explained.
- The particle horizon diverges if $w < -\frac{1}{3}$, or equivalently if $P < -\frac{1}{3}\rho c^2$, substantially modifying the $\chi_p = 2ct$ for radiation dominated EdS.
- The curvature parameter is driven toward flatness if $w < -\frac{1}{3}$.
- Examining the acceleration equation

$$\frac{\ddot{R}}{R} = -\frac{8\pi G}{c^2}(P + \tfrac{1}{3}\rho c^2),$$

- we can see that this requirement on w is really a requirement that the universe is accelerating.
- Accelerating universes cause the comoving Hubble horizon $\chi_h = \frac{c}{\dot{R}}$ to shrink: $\dot{\chi}_h = -\frac{\ddot{R}c}{\dot{R}^2}$, and the universe to expand "superluminally".
- Intuitively, an accelerating universe carries material far outside it's usual causal patch, and flattening the spatial curvature.

What could cause an early period of acceleration?

- The current approach generates acceleration in the early universe using the theory of inflation – an early period of exponential expansion (i.e. hyperinflation).
- Note that bouncing cosmologies would also solve some of the problems, since a bounce is an acceleration.
- These "Ekypyrotic" models are not without their own problems however.
- In particular in their simplest form they require curvature (remember from Lecture 21 that closed Λ cosmologies can have bouncing solutions), so these cannot solve the curvature problem.

Scalar fields in cosmology – note $c = \hbar = 1, \Lambda = 0$

- It turns out it's actually quite easy to get universes to accelerate!
- Consider a scalar field $\phi(x^\mu)$, with Lagrangian, stress-energy tensor and equation of motion

$$\mathcal{L} = \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - V(\phi), \quad T^{\mu\nu} = \nabla^\mu\phi\nabla^\nu\phi - \left(\tfrac{1}{2}\nabla^\alpha\phi\nabla_\alpha\phi - V(\phi)\right)g^{\mu\nu}, \quad \nabla^\mu\nabla_\mu\phi + \frac{d}{d\phi}V(\phi) = 0.$$

- You may encountered this in some form or another as the Klein-Gordon equation, with the free field potential $V(\phi) = \frac{1}{2}m^2\phi^2$ and GR turned off (so $\nabla = \partial$).
- Now fill the universe with a homogeneous scalar field $\phi(x^\mu) = \phi(t)$.
- Homogeneity under the cosmological principle means we can remove all spatial derivatives

$$\boxed{T^{\mu\nu} = \dot\phi^2\delta^\mu_t\delta^\nu_t - \left(\tfrac{1}{2}\dot\phi^2 - V(\phi)\right)g^{\mu\nu}.}$$

- By inspection, this is identical to a perfect fluid comoving with fundamental observers:

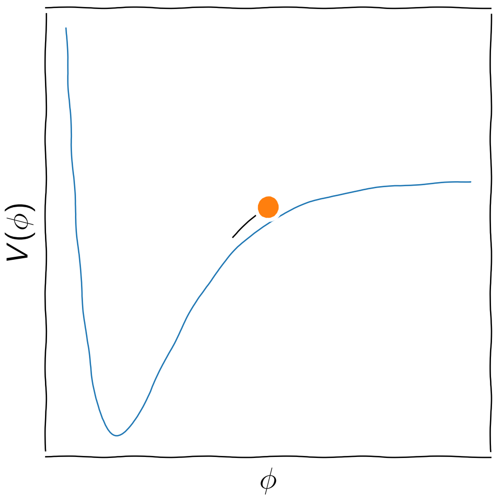
$$\boxed{u^\mu = \delta^\mu_t,} \qquad \boxed{\rho = \tfrac{1}{2}\dot\phi^2 + V(\phi),} \qquad \boxed{P = \tfrac{1}{2}\dot\phi^2 - V(\phi).}$$

- The equation of motion is equally suggestive.
- Noting $\nabla_t = \partial_t + 3H$ we find

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + \frac{d}{d\phi}V(\phi) = 0.}$$

- This[1] is the equation of a particle in a potential $V(\phi)$, with a friction term $\propto H$
- The dynamics of the scalar field in cosmology are therefore relatively easy to conceptualise
- To close the system, we use the velocity equation $H^2 = \frac{8\pi G}{3}\rho - \frac{k}{R^2}$:

$$\boxed{H^2 = \frac{8\pi G}{3}\left(\tfrac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{k}{R^2}}$$



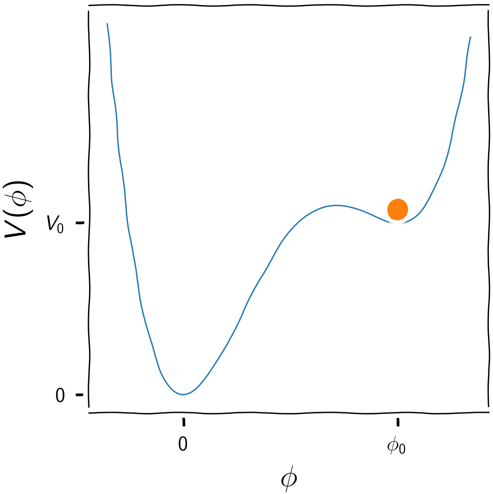
[1] With a bit of thought, this is also the continuity equation $\dot{\rho} = -3H(P+\rho)$ divided by $\dot{\phi}$:

Acceleration from false vacua

- Finally the acceleration equation $\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3P)$ comes out as

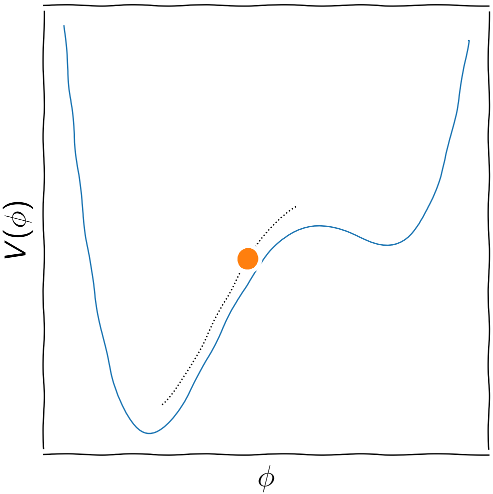
$$\frac{\ddot{R}}{R} = \boxed{\dot{H} + H^2 = -\frac{8\pi G}{3}\left(\dot{\phi}^2 - V(\phi)\right).}$$

- From these equations it is very easy to get acceleration.
- Imagine trapping the field $\dot{\phi} = 0$ in a local minimum at ϕ_0 with potential $V(\phi_0)$.
- In this false vacuum state $\frac{\dot{R}}{R} = H = \frac{8\pi G V_0}{3}$.
- Constant $H \Rightarrow$ de Sitter evolution $\boxed{R \propto e^{Ht}}$.



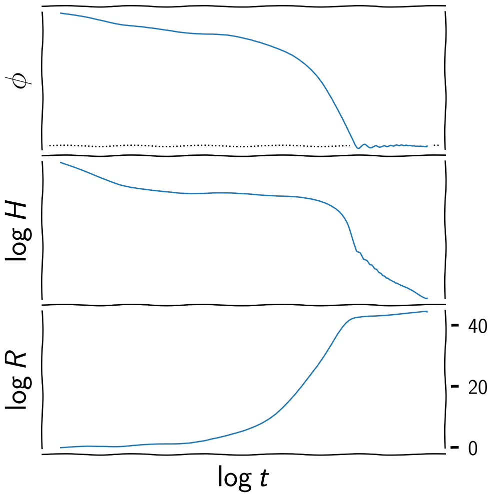
Acceleration from slow roll

- In fact, we don't even need to trap the field!
- Imagine perturbing ϕ away from minimum.
- The field will start to roll downhill, but will be slowed by friction until $\boxed{\dot{\phi}^2 \ll V(\phi)}$, known as the slow-roll approximation.
- We can say than that $\frac{\dot{R}}{R} = H \approx \text{constant}$, and so the universe is an approximate de Sitter state $\boxed{R \sim e^{Ht}}$.
- For most potentials, the slow roll solution is an attractor, which means that whatever the initial conditions, the evolution quickly snaps into a slow-roll state.



Numerical solutions

- These equations are relatively straightforward to solve numerically (try it!).
- The above were produced with the potential $V(\phi) = \frac{1}{2}m^2\phi^2$ with $8\pi G = c = \hbar = 1$.
- For initial conditions these were started at with a relatively high starting velocity to show entry to slow roll.
- Slow roll can be seen as a plateau in ϕ and H, with corresponding exponential rise in R.
- At the end ϕ oscillates at the bottom of the potential $\phi = 0$.
- Care must be taken with the $\log t$ axis (need to integrate backward to set correct $t = 0$ or use kinetic ICs [arxiv:1401.2253]).



Is this sensible?

- We have a mechanism therefore which can generate large amounts of exponential expansion
  - drives the Universe toward flatness,
  - dilutes monopoles and topological defects,
  - adds plenty of conformal time to resolve the horizon problem.
- However, the mechanism requires invoking an unknown scalar field – termed the inflaton.
- It would be great if it were e.g. the Higgs field, but the Higgs boson is too heavy to be the inflaton without modifications to GR.
- Moreover, the energy scale of inflation is thought to be $> 10^{12}$TeV, i.e. over a trillion times higher than the LHC.
- It is somewhat an understatement that the physics is speculative at these energy scales!
- Current thinking justifies these theories by an "effective field theory" approach, which broadly argues that whatever the true physics is, degrees of freedom can be integrated out to yield something that looks like an inflaton for the purposes of dynamics.

Anatomy of the period of inflation

- We therefore believe that between 10^{-34} s and 10^{-32} s of the universe, the universe inflated from virus to galactic scales.
- At the end of this, our current causal patch was about the size of a football (UK unit).
- After this rather speculative inflaton physics, the inflaton field presumably decays into normal matter/radiation in a period called "reheating".
- This is also around the GUT energy scale, so it may be that a grand unified theory will predict an adequate high-energy scalar field.
- All of the causal patch after inflation would be highly uniform, as it is a magnification of a microscopic portion of a presumably stochastic proto-universe.
- As we will come onto in Lecture 24, the real reason we believe inflation is such a good theory of the Universe is not it's postdictions of horizon, flatness and monopole problems, but that it predicts exactly how non-uniform the universe is afterwards.
- Inflation provides a generic explanatory mechanism for setting the initial conditions of the Universe.

Summary

- We have strong evidence to believe that the universe underwent an early accelerated phase.
- This explains the horizon, flatness & monopole problems (post-dictions).
- Can generate acceleration naturally using scalar fields.
- Scalar field stress-energy tensor is a perfect fluid comoving with fundamental observers

$$\boxed{u^\mu = \delta^\mu_t,} \qquad \boxed{\rho = \tfrac{1}{2}\dot{\phi}^2 + V(\phi),} \qquad \boxed{P = \tfrac{1}{2}\dot{\phi}^2 - V(\phi).}$$

- Cosmological evolution equations for a universe filled homogeneously with an inflaton ϕ are

$$\boxed{\ddot{\phi} + 3H\dot{\phi} + \frac{d}{d\phi}V(\phi) = 0,} \quad \boxed{H^2 = \frac{8\pi G}{3}\left(\tfrac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{k}{R^2},} \quad \boxed{\dot{H} + H^2 = -\frac{8\pi G}{3}\left(\dot{\phi}^2 - V(\phi)\right).}$$

- Under slow roll $\boxed{\dot{\phi}^2 \ll V(\phi)}$ attractor solution $\boxed{H \approx \text{constant}}$ and $\boxed{R \sim e^{Ht}}$.

Next time

Perturbations & pre-dictions of inflation.

Appendix: Scalar fields at late times and Λ

- Have seen scalar fields are successful in driving early universe inflation.
- Also the universe is in a state of inflation today (see shortly for quantitative evidence for this).
- So has suggested to many people that maybe scalar fields are responsible for the late-time acceleration of the universe, i.e. provide a form of Dark Energy!
- Indeed, if put $\dot{\phi} = 0$ in above equations would get

$$\rho = \frac{1}{2} m^2 \phi^2, \quad P = -\frac{1}{2} m^2 \phi^2,$$

which would be identical to the fluid density and pressure we found for a cosmological constant on above if

$$\frac{\Lambda}{\kappa} = \frac{1}{2} m^2 \phi^2, \quad \text{i.e.} \quad \Lambda = 4\pi G m^2 \phi^2.$$

- Doesn't in fact work for current model, since only way $\dot{\phi}$ would remain at 0 is if we were at the bottom of the potential — but there $\phi = 0$!
- However, much work has gone into other possible potentials, or indeed theories outside GR (modified gravity) where scalar fields (or scalar field equivalents) may be able to provide behaviour we need.
- Could be a huge breakthrough if we were able explain late time behaviour with a model that doesn't face the problem of empty space having an energy density 10^{122} times below what particle physics suggests for it (which is what Λ explanation faces).

Appendix: Λ as a Lorentz invariant fluid

- If look back to Appendix in Handout 20 (GR derivation of the cosmological field equations), will see that Einstein equations with Λ included are

$$G_{\mu\nu} = -\kappa T_{\mu\nu} - \Lambda g_{\mu\nu},$$

  where

$$T_{\mu\nu} = \left(\rho + \frac{P}{c^2}\right) u_\mu u_\nu - P g_{\mu\nu},$$

  is the Stress-Energy Tensor for the perfect fluid assumed to make up the universe, and we see have added an effective extra SET part, given by a constant multiple times the metric tensor $g_{\mu\nu}$.
- Comparing with the perfect fluid SET expression, this looks just like what one would get for a perfect fluid with energy density $\rho_\Lambda c^2 = \Lambda/\kappa$, and pressure $P_\Lambda = -\Lambda/\kappa$, since then indeed we would have

$$\kappa T^{\Lambda}_{\mu\nu} = \kappa \left(\left(\frac{\Lambda}{\kappa c^2} - \frac{\Lambda}{\kappa c^2} \right) u_\mu u_\nu + \frac{\Lambda}{\kappa} g_{\mu\nu} \right) = \Lambda g_{\mu\nu}.$$

- Notice how this doesn't depend on any assumption we make about being in the IRF (instantaneous rest frame) of any 'fluid'. It happens for any 4-velocity u^μ.
- In fact such a Stress-Energy Tensor is completely Lorentz invariant — any observer, regardless of their state of motion, sees exactly the same thing!
- Thus it's a possible model for a Lorentz invariant vacuum!
- It's this property which effectively makes it the right candidate for producing the curvature of empty space.
- Note also how it ties in with our equation (*) for $\dot{\rho}$, since this is 0 and ρ is constant if $\rho = -P/c^2$.