Exoplanets and Planetary Systems
Example Sheet 1

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1 Observing a protoplanetary disk

We have learned that planets are formed in the protoplanetary disks around young stars.

(a) What is a typical scale for a protoplanetary disk's radius and temperature?

(b) What does the term *ice line* refer to? At which distance and temperature does the ice line occur? Why does it not refer to a temperature of $273K$?

(c) What is the typical composition of these disks?

(d) Inform yourself about the telescopes ALMA, Herschel and Spitzer. At which wavelengths can they operate? For the detection of which components in the disk are they hence suitable?

2 Disk structure

2.1 Hydrostatic Equilibrium

In vertical direction one can describe the structure using the assumption of hydrostatic equilibrium. Further assuming the star's gravity dominates over the entire disk, we find

$$\frac{1}{\rho}\frac{\partial P}{\partial z} + \left(\frac{GM_*}{a^2}\right)\left(\frac{z}{a}\right) = 0, \tag{1}$$

where G is the gravitational constant, M_* is the mass of the star, and a is the orbital distance.

(a) What expression can be found for P in dependency of ρ assuming an ideal gas? Use it to substitute for the term dependent on P in the above equation.

(b) Use a reasonable expression for ρ in dependency of the disk scale height H to find an expression for the scale height H.

2.2 Angular momentum conservation

In the radial disk direction we have discussed the conservation of angular momentum under the assumption of a steady flow. In a first order approximation the pressure scales as

$$\partial P \sim \exp -i(\omega t \pm 2\pi r/\lambda), \tag{2}$$

such that we can solve the equation and find

$$\omega^2 = k^2 - \frac{4\pi^2 G\Sigma}{\lambda} + \frac{4\pi^2 c^2}{\lambda^2}. \tag{3}$$

(a) Looking at the case $\omega = 0$, solve this equation for λ. You will see that there are only solutions if a certain criterion is fulfilled. Using your solution, elaborate what this criterion is and what it tells about the disk.

3 Planet formation

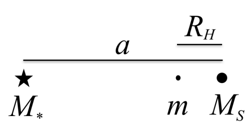
Name the two main theories for planet formation. Which of them is currently believed to be the most plausible scenario? Briefly explain its main steps and current caveats.

4 Growth of planetesimals

4.1 Hill radius

(a) Briefly explain in words what the Hill radius expresses.

(b) A solid body is orbiting a star in a rotating frame (in which the body is resting) in the following configuration:



Assume we put a test mass m into the L_1 Lagrangian point, corresponding to the Hill Radius R_H. Use the balance of forces due to gravitation and rotation in this point, i.e. $\Sigma F = 0$, and find an expression for R_H in dependency of a, M_* and M_S (the solid body's mass).

Hint: Take care to express the influence of the rotating frame, given by Ω, with the correct dependencies. Use that $R_H << a$ at the appropriate steps in your calculation.

4.2 Relative velocity

A planet embryo and a planetesimal orbit a star at distances a and $a + \Delta a$. Show that the relative velocity between these two objects (as an absolute value) can be expressed as

$$v_{rel} \sim \Omega \Delta a. \tag{4}$$

Hint: Start by expressing their velocities using the Keplerian velocity Ω at their specific distance. Use $\Delta a/a << 1$ in your calculation.