Exoplanets and Planetary Systems
Example Sheet 3

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1 Orbits and dynamics

Starting from Kepler's second law and using the ellipse equations we have established in the lecture, demonstrate that

(a)

$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3, \tag{1}$$

(i.e. Kepler's Third Law in it's modern form with evaluated constant).

Hint: On your way of deriving Kepler's third law, you will likely come across the following expression:

$$h = \frac{2\pi}{P} a^2 \sqrt{1 - e^2}. \tag{2}$$

2 Key numbers for Exoplanet Science

Working with estimations and scaling behavior is crucial for physicists and astronomers. Fill out the following lists and remember the scaling ratios between the values, as they will prove to be crucial when working with exoplanets (round to full numbers!):

a) Distances:

$$1\ AU = \_\_\_\_\_\_\_\_\_\_\_\ km \tag{3}$$

$$1\ AU = \_\_\_\_\_\_\_\_\_\_\_\ R_\odot \tag{4}$$

$$1\ R_\odot = \_\_\_\_\_\_\_\_\_\_\_\ R_J \tag{5}$$

$$1\ R_J = \_\_\_\_\_\_\_\_\_\_\_\ R_\oplus \tag{6}$$

b) Masses:

$$1\ M_\odot = \text{\_\_\_\_\_\_\_\_\_\_\_\_}\ kg \tag{7}$$
$$1\ M_\odot = \text{\_\_\_\_\_\_\_\_\_\_\_\_}\ M_J \tag{8}$$
$$1\ M_J = \text{\_\_\_\_\_\_\_\_\_\_\_\_}\ M_\oplus \tag{9}$$

c) Densities:

$$\rho_\odot = \text{\_\_\_\_\_\_\_\_\_\_\_\_}\ g/cm^3 \tag{10}$$
$$\rho_J = \text{\_\_\_\_\_\_\_\_\_\_\_\_}\ g/cm^3 \tag{11}$$
$$\rho_\oplus = \text{\_\_\_\_\_\_\_\_\_\_\_\_}\ g/cm^3 \tag{12}$$

In this exercise we are going to practice the use of these numbers to get familiar with these scales and train our skills to perform quick approximations.

3 Orbits and dynamics

(a) Give the equation for Kepler's third law and solve it for G.

(b) Use only the numbers given above and your knowledge of the system Sun - Earth to calculate G. Express the result in the following way (yr=year):

$$G = \text{\_\_\_\_\_\_\_\_\_\_\_\_}\ \frac{\mathrm{AU}^3}{\mathrm{M}_\odot \cdot \mathrm{yr}^2} \tag{13}$$

(c) Assume now that we detect a Hot Jupiter on a 2 day orbit around a star similar to our sun. What is it's orbital distance a (in AU)?

(d) Assume a Hot Neptune was orbiting at the same orbital distance instead of the Hot Jupiter. What would it's period P be?

Assume our Earth-twin and Hot Jupiter from (b) and (c) are now orbiting an M5-star ($M = 0.21\ \mathrm{M}_\odot$) instead, but the signals are still detected with the same orbital periods $P = 1$ yr and $P = 2$ d, respectively (d=days).

(e) Calculate at which orbital distances you would now expect to find them.

4 Radial velocity

In the lecture we established the equation

$$k = 28.4\ \left(\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot \left(\frac{M_2}{\mathrm{M_J}}\right) \sin i \cdot \left(\frac{P}{\mathrm{yr}}\right)^{-1/3} \cdot \left(\frac{M_1}{\mathrm{M}_\odot}\right)^{-2/3} \cdot \frac{1}{\sqrt{1-e^2}}. \tag{14}$$

for the amplitude k of an RV signal.

4.1

(a) Express the mass, period and eccentricity of Earth, Neptune and Jupiter in units of AU, yr, and M_J.

(b) Assume $\sin i = 1$ and calculate what RV effects Earth, Neptune and Jupiter cause on the Sun.

4.2

We want to develop a new RV instrument to search for exoplanets on perfectly circular orbits - CORVI (Circular Orbit Radial Velocity Instrument, financed by an Italian consortium).

(a) How precise does CORVI have to be to detect a signal from a Hot Jupiter with $P = 5$ d orbiting a sun-like star at inclinations of up to $45°$.

(b) What precise do we need if we want to detect Neptune-mass ($M = 17\ M_\oplus$) or Earth-mass planets at the same orbital distance up to the same inclination?

(c) Since nobody would want to life on these planets, our grant application got rejected. We therefore decided that it would be more interesting to search for Earth-twins in the habitable zone (HZ). Assume the HZ for a G-type star (like our sun) spans roughly from $0.8 - 1.5$ AU. What is the minimal accuracy we would have to reach to find an Earth-twin within the HZ around a sun-like star?

(d) In order to make our life easier, we decided to target smaller stars now, since not only their HZ is closer to the star ($0.25 - 1.3$ AU for an M5-star), but the RV signal is also easier to measure. What accuracy do we have to reach to detect our Earth-twin within the HZ of an M5-star ($M = 0.21\ M_\odot$)?

5 Transit detection

We have learned that the transit duration can be calculated from the stellar properties by

$$T = T_0\sqrt{1-b^2} \tag{15}$$

$$\text{with } T_0 = R_* \cdot M_*^{-1/3} P^{1/3} (4/G\pi)^{1/3} \sim \rho_*^{-1/3} P^{1/3}. \tag{16}$$

Further we have learned that the transit depth can be calculated as

$$\delta = R_p^2/R_*^2. \tag{17}$$

5.1

We detect a signal around a sun-like star with transit duration of 2 h and a period of 10 d.

(a) Use Eq. 13 and calculate T_0.[1]

Hint: Only use the key values we have established in Task 1 and 2! Use e.g. that you know $R_\odot$ in AU.

(b) What does this tell us about the impact parameter? Calculate it and sketch the transit geometry.

(c) Using only the information given so far, can we determine what kind of planet it is?

5.2

Let's assume we measure a transit with the same period around an M5 star with $M = 0.21\ M_\odot$ and $R = 0.32\ R_\odot$.

(a) How does this affect the transit duration? Calculate the new values for T_0 and T.

5.3

Assume the planet detected in both cases (i.e. around the G2-star and the M5-star) was a Hot Jupiter.

(a) Calculate the respective transit depths δ it would have caused.

5.4

We now look at a different system and find a transit with a relative depth of 0.26 % and a period of $P = 60\ d$ around a K0-star ($R = 0.85\ R_\odot$, $M = 0.78\ M_\odot$).

(a) Calculate and express the planet radius in $R_\oplus$. What type of planet is it?

(b) The habitable zone for K-type stars is expected to range roughly from 0.25 to 1.3 AU. Could we live on this planet?

BONUS: What other aspects would we have to consider to really address this question?

[1] *Hint: If you were unable to calculate the value of G simply use a buffer constant C with* $G = C \cdot \frac{AU^3}{M_\odot \cdot y^2}$ *for your calculations.*

6 BONUS: Get a feeling for exoplanets

Play around with the key numbers and scenarios we have established here. Try to compare what happens if you use different planets (Earths, Neptunes, Jupiters) around different stars. Train to get a feeling for what values you need in order to find out more about the systems, and which values are correlated. Invent your own scenarios to get a feeling for what systems are realistic (also considering planet formation and migration) and detectable - and which are not.

7 BONUS: What exoplanets do we know?

Explore the wesbides ”exoplanets.org” and/or ”exoplanetarchive.ipac.caltech.edu” to get an overview of what exoplanets have been found so far. Use the websides' plotting tools (scatter and/or histogram) to gain an overview of the distributions of exoplanets looking at their mass, radii, orbital distance, and other attributes you might find interesting. Discuss the trends you discover. Which may follow from planet formation and migration mechanisms? Which may be caused by the use of different detection methods and their intrinsic bias?