Examples Sheet 1: Plate tectonics, heat flow, and flexure

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1 A section of the boundary between two plates consists of two rifts offset by a transform fault. The junctions between the rifts and the transform fault are at A (0°N, 60°E) and C (45°N, 0°E), and the transform fault also passes through the point B (30°N, 30°E). If P is the pole of relative rotation between the two plates and O is the centre of the Earth, show first that

$$\mathbf{OA} \cdot \mathbf{OP} = \mathbf{OB} \cdot \mathbf{OP} = \mathbf{OC} \cdot \mathbf{OP}$$

and hence find P.

Magnetic anomalies show that the spreading rate at the ridge is 100 mm/yr. Find the vector of relative instantaneous rotation in radians/y. (Radius of the Earth is 6371 km).

2 Show that the unit vectors $\mathbf{a}_E$ and $\mathbf{a}_N$, pointing E and N at the point λ latitude, ϕ longitude are

$$\mathbf{a}_E = (-\sin\phi, \cos\phi, 0)$$

$$\mathbf{a}_N = (-\sin\lambda\cos\phi, -\sin\lambda\sin\phi, \cos\lambda)$$

Three plates India, Africa and Antarctica meet at the triple junction of three ridges at (-20°N, 70°E). The angular velocity vectors between the pairs of plates are

$$\text{India-Antarctica } (0°\text{N}, 45°\text{E}), 6\times10^{-7}\ °/\text{yr}$$

$$\text{India-Africa } (15°\text{N}, 45°\text{E}), 6\times10^{-7}\ °/\text{yr}$$

where the angular velocity vector is positive in each case if India is taken to be moving. Find the angular velocity of Africa with respect to Antarctica. Use the angular velocities and the expressions for $\mathbf{a}_E$ and $\mathbf{a}_N$ to plot the velocity triangle for the triple junction. Assume that the ridges spread symmetrically and at right angles to their strike, and hence determine the stability of the triple junction and its motion with respect to each plate. Approximately how fast is the length of each ridge changing? Sketch the resulting magnetic anomalies. (The radius of the Earth is 6371 km).

3 Determine the conditions which must be satisfied if the following triple junctions are to be stable:

(a) Ridge-Ridge-Transform
(b) Transform-Transform-Ridge
(c) Ridge-Trench-Transform (2 cases)

4 The pole of rotation for plate A relative to plate B is at 45°N, 0°E, with a present angular velocity of 2×10^{-8} rad/yr. A convergent plate boundary runs north-south through the point C (0°N, 45°E). Calculate the relative motion between the plates *normal* to the boundary (hint: use expression for $\mathbf{a}_E$ from question 2). Assuming the plate takes 10^7 years to warm up and descends at an angle of 45° to the vertical, estimate the depth to the deepest earthquakes.

5 Derive the formula relating magnetic inclination to latitude, $\tan I = 2\tan\lambda$.

6 Show that the thermal time constant τ for an infinite plane layer of thickness a is

$$\tau = \frac{a^2}{\pi^2 \kappa}$$

where κ is the thermal diffusivity, and the temperature is constant on both boundaries. Use this expression to discuss a) the depths of the oceans, b) the variation of heat flow with age through the sea floor.

7 Show that the half width δ of the heat flow anomaly produced by an oceanic ridge spreading with a constant half-rate velocity of U is

$$\delta = \frac{4Ua^2}{\pi^2 \kappa} \ln 2$$

when the heat flux, rather than the temperature, is fixed at the base of the plate of thickness d and thermal diffusivity κ. Estimate the value of a required to account for the observed variation of heat flow and depth with age, and explain why the model with constant temperature rather than heat flux, is probably a better approximation for the geophysical problem.

8 An old plate of thickness a within which the temperature varies linearly with depth is being thrust into an isothermal mantle beneath an island arc with velocity U at an angle ϕ to the horizontal. Obtain the steady state temperature structure, and approximate expressions for vertical advective heat transport and the buoyancy force in the dip direction.

9 Using the bending equation

$$D\frac{\mathrm{d}^4 w}{\mathrm{d}x^4} + g(\rho_m - \rho_w)w = L(x), \qquad D = \frac{ET_e^3}{12(1-\sigma^2)},$$

determine the deflection $w(x)$ of an elastic plate subject to a line load at $x = 0$. Assume the plate remains unbroken at the loading point ($\mathrm{d}w/\mathrm{d}x = 0$ at $x = 0$). Use your result to determine a) the half-width of the depression (the distance from the line load to the first point where $w = 0$), and b) the position of the maximum amplitude of the forebulge. Estimate the elastic thickness beneath the Hawaiian Islands, using the profile shown in Figure 1, and $\rho_m = 3 \times 10^3$ kg m^{-3}, $\rho_w = 10^3$ kg m^{-3}, $E = 70$ GPa, $\sigma = 0.25$.

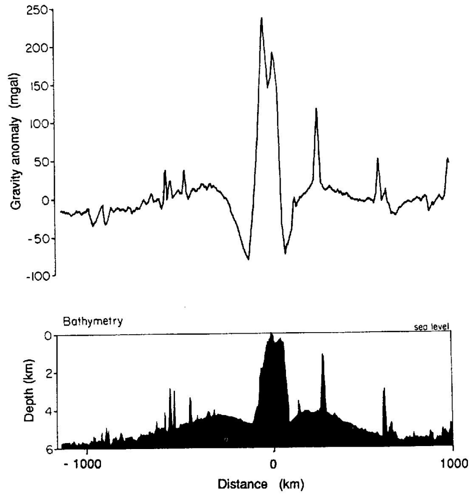

Figure 1: Gravity and bathymetry measurements around Hawaii.