Relativistic Astrophysics and Cosmology — Examples 4 — 2023

The cosmological constant should be taken as zero in the cosmology questions below, except where models with Λ are specifically mentioned.

1. Show that in a de Sitter universe the angular diameter distance is given by

$$d_\theta = \frac{z}{1+z}\sqrt{\frac{3}{\Lambda}}.$$

   What does this result imply for the angular diameter subtended by an object of fixed proper diameter D as a function of redshift for both low and high redshift?

2. Suppose that in a de Sitter universe a photon is emitted from the origin of coordinates ($\chi = 0$), at time $t = t_0$ (now) towards an object which is currently seen at redshift z. Show that that the photon is received at the object at a time $t_{\rm recep}$ given by

$$ct_{\rm recep} = ct_0 - \sqrt{\frac{3}{\Lambda}}\ln(1-z),$$

   provided $z < 1$, and that it is never received if $z \geqslant 1$.

3. What is meant by a particle horizon? If our universe evolves according to the Einstein-de-Sitter model show that the total mass contained within our particle horizon is $6c^3tG^{-1}$, where t is the time since the big bang.

   Use this result to find the mass contained within the particle horizon when t is so small that the horizon lies within the radius of a proton. Comment on your answer.

4. The energy density in radiation from quasar light is $\sim 10^{-16}{\rm Jm}^{-3}$ now. What is the mean mass density in dead black holes, assuming that most quasars occurred at a redshift of 2? What is the mean mass per galaxy of dead black holes if the density of galaxies is $10^{-2}{\rm Mpc}^{-3}$?

5. If galaxies of a certain type now have a space density of $2 \times 10^{-3}\,{\rm Mpc}^{-3}$, and each of them went through a very luminous phase lasting 10^8 years after they formed at $z = 3$, what is the observed areal density on the sky (measured per square degree) of galaxies in that phase? [Assume an Einstein-de-Sitter universe and a reasonable value for H_0.]

6. Galaxies having a local number density N_0 are surrounded by gaseous haloes, each of which has a cross section σ. If the numbers and sizes of these galaxies do not change with time show that a line of sight to a quasar at redshift z will intersect, on average, n haloes where

$$n = \int_0^z (N_0\,\sigma\,c\,/H_0)\,(1+z)\,(1+\Omega_{m0}z)^{-1/2}\,dz.$$

7. Show that for all reasonable values of Ω_{m0} and H_0, the spatial curvature (i.e. the term kc^2/R^2 in the second field equation) is negligible during the whole of the radiation dominated phase of the universe. Use this to calculate how long the universe is radiation dominated if the epoch of equality (i.e. the time at which the energy density in radiation equals that in matter) is given by $z_{eq} = 9600\,\Omega_{m0}h^2$ (where $h = H_0/50\,\mathrm{kms}^{-1}/\mathrm{Mpc}$). Obtain another estimate of the same quantity by assuming the present universe is matter dominated Einstein-de-Sitter, and finding the time corresponding to z_{eq}. [Answers: $10,410\,(\Omega_{m0}h^2)^{-2}$ and $13,880\,h^{-4}$ years respectively.]

8. If radiation and matter were in thermal equilibrium prior to recombination, and if no condensation into galaxies had subsequently occurred, what would be the present temperature of the cosmic gas? If residual gas at this temperature still exists how would you attempt to detect it? (Assume that after recombination the matter cools according to the adiabatic law $TV^{\gamma-1}$ =constant, where $\gamma = 5/3$.)

9. By assuming that the universe is radiation dominated Einstein-de-Sitter before recombination, and matter dominated Einstein-de-Sitter afterwards, show (a) that the proper distance across the particle horizon at recombination is $2ct_{\rm rec}$, and (b) that this distance subtends an angle $\approx \frac{2}{3}(1+z_{\rm rec})^{-1/2}$ radians today. Comment on the implications for the microwave background.

10. For the dynamics of inflation as discussed in the lectures, derive the scalar field equivalent of the (A) equation (the 'acceleration equation') by differentiating the scalar field (B) ('energy') equation w.r.t. time and employing the scalar field equivalent of the continuity equation to eliminate $\ddot{\phi}$.