Exoplanets and Planetary Systems
Example Sheet 2

Prof. Dr. Didier Queloz,
Cavendish Astrophysics, University of Cambridge

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1 Planet formation

1.1 Limitation of the gas accretion rate

The gas accretion phase, essential for the formation of giant gas planets like Jupiter, is limited by two essential factors which are expressed as the Hill and Bondi radii, R_H and R_B. The radius in which gas can be captured is, depending on the scenario, then given as $R_{gc} = \min(R_H, R_B)$.

(a) What are the Hill and Bondi radii and what do they describe?

(b) Calculate their growth factors $1/\tau_H$ and $1/\tau_B$. How do they depend on the mass of the accreting body?

Hint: The growth timescale can be calculated as

$$\tau = M \cdot \left(\frac{dM}{dt}\right)^{-1}, \tag{1}$$

whereby the gas mass accretion rate is given by

$$\frac{dM}{dt} \sim \frac{\Sigma}{H}\Omega R_{gc}^3. \tag{2}$$

1.2 Impuls approximation

Using the impuls approximation for an encounter between a planet and a particle we found a change of angular momentum

$$\Delta J \sim M a^2 \Omega_P \left(\frac{M_P}{M_*}\right)^2 \left(\frac{a}{x}\right)^5. \tag{3}$$

The torque on the planet resulting from the outer parts (at $r \geq a + \Delta r$) of the disk is given by

$$\mathcal{T}_p^{out} \sim -\int_{\Delta r}^{\infty} \frac{\Delta J}{\Delta t} \frac{dM}{M} \tag{4}$$

(a) Calculate the torque from outer disk regions $\mathcal{T}_p^{out}$ and the total torque $\mathcal{T}_p$.

Hint: Use that $\mathcal{T}_p \sim \frac{\Delta r}{a} \cdot \mathcal{T}_p^{out}$*, which can be evaluated from the scaling behavior of the surface density (not to be proven here!).*

2 Planet migration

Similar to the calculation of timescales in exercise 1.1, we can study the timescale for a type I migration where $\Delta r \sim H$ as

$$\tau_I = a \left(\frac{da}{dt} \right)^{-1}. \tag{5}$$

Thereby we can express

$$\frac{da}{dt} = \frac{2\mathcal{T}_p}{M_p a \Omega}, \tag{6}$$

whit $\mathcal{T}_p$ from exercise 1.2.

(a) Calculate the migration timescale τ_I.