Constituents of the universe

Relativistic Astrophysics and Cosmology: Lecture 20

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Monday 20$^{\text{th}}$ November 2023

Pre-lecture question:

Why should we believe in dark matter?

Last time

- Measuring the universe

This lecture

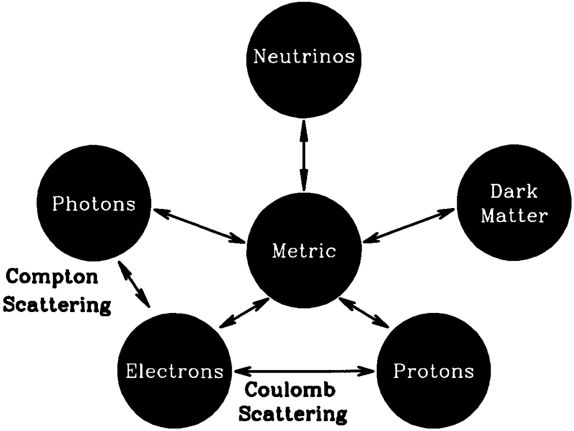
- The make-up of our universe
- Thermal history of the early universe
- Dark matter in the late universe

Next lecture

- Modern cosmological data

The contents of the Universe

- Diagram on the left shows at the broadest level the components of our universe, and how they interact.
- At early times (before recombination) photons, electrons and protons exist as a tightly coupled plasma.
- At later times (after recombination), photons decouple from the atomic electrons and protons.
- At later times still (after reionisation), photons interact once again with ionised atoms.



The composition of the Universe today

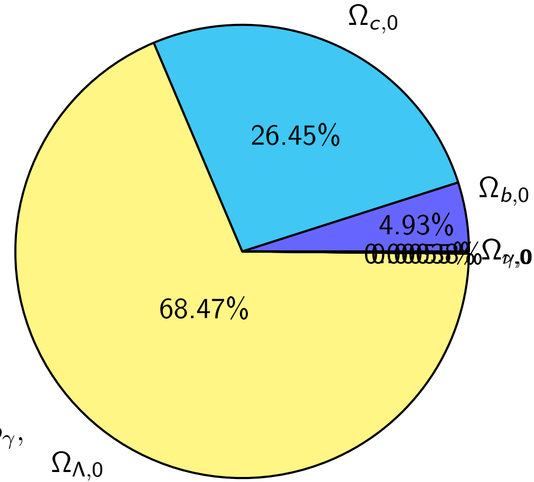
- Planck parameters [arxiv:1807.06209]

4.93%	[Ω_b] Baryonic matter,
26.45%	[Ω_c] (Cold) dark matter,
68.47%	[Ω_Λ] Dark energy,
0.0055%	[Ω_r] Radiation,
0.00038%	[Ω_ν] Neutrinos.

- Planck measures the first three, whilst COBE is still(!) our best measurement of CMB temperature $T_0 = 2.7255\,\mathrm{K}$.
- Radiation and neutrinos computed with:

$$\rho_\gamma = \frac{a}{c^2} T_0^4 = \frac{\pi^2 k_B^4}{15\hbar^3 c^3} T_0^4, \quad \rho_\nu = 3.046 \cdot \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \rho_\gamma,$$

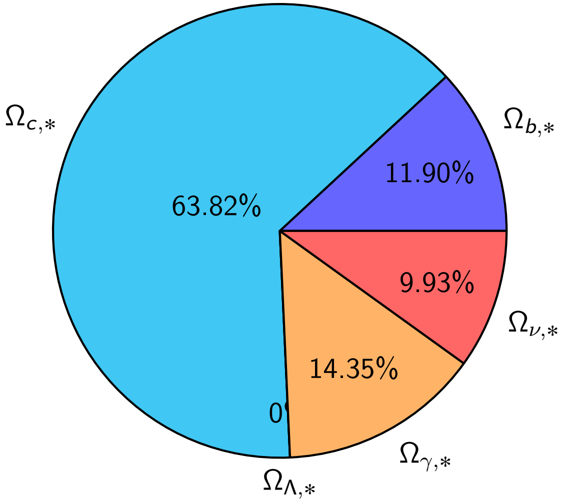
$$\Omega = \frac{3H_0^2 \rho}{8\pi G}, \; H_0 = 67.36\,\mathrm{km s^{-1} Mpc^{-1}}.$$



The composition of the Universe over time

- Note that you now have all the tools to compute how this changes with time.
- For example, evaluating the same plot at the redshift of the CMB $z_* = 1089.91$ yields a very different universe
- $\Omega_m \sim (1+z)^3$ for baryonic and dark matter
- $\Omega_r \sim (1+z)^4$ for photons and neutrinos.

11.90%	$[\Omega_b]$ Baryonic matter
63.82%	$[\Omega_c]$ (Cold) dark matter
$\approx$0%	$[\Omega_\Lambda]$ Dark energy
14.35%	$[\Omega_r]$ Radiation
9.93%	$[\Omega_\nu]$ Neutrinos



Obtaining the photon to baryon ratio

- We can calculate the total number of photons in blackbody radiation at temperature T_0, and the number density of protons today

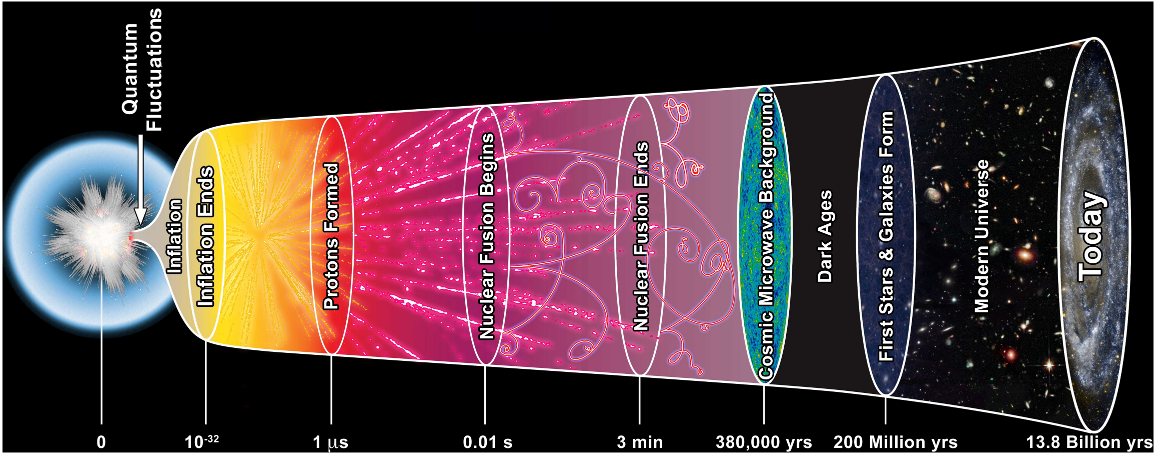
$$n_\gamma = \frac{8\pi}{c^3}\int_0^\infty \frac{\nu^2\,d\nu}{e^{h\nu/kT}-1} = 4.2\times10^8\,\mathrm{m}^{-3} \qquad n_p = \frac{\rho_{0\text{matter}}}{m_p} = \frac{3H_0^2\Omega_{b,0}}{8\pi G m_p},$$

where we have used the contribution to $\Omega_{m,0}$ from ordinary (as against dark) matter.
- Since both n_γ and n_p scale as R^{-3} their ratio will be the same at all times

$$\frac{n_\gamma}{n_p} = \frac{4.2\times10^8\,\mathrm{m}^{-3}}{0.25\,\mathrm{m}^{-3}} = \left.\frac{\text{no. of photons}}{\text{no. of baryons}}\right|_{\text{today}} = 1.67\times10^9.$$

- Thought that it is associated with whatever it is that gave rise to matter/antimatter asymmetry — e.g. if equal amounts of matter and anti-matter existed originally, but there is a slight asymmetry in their annihilation, by about 1 part in 10^9, then this could explain observed photon/baryon ratio, and also current absence of anti-matter.
- But all of this still pretty unclear.

A brief history of time



- As the Universe expands it cools
- It transitions between different epochs
- Important ones we will cover are nucleosynthesis, recombination & reionisation

Temperature history of early universe

- The early universe is $t < 380\,000\,\mathrm{y}$, largely independent of cosmological model.
- Already said that we can ignore spatial curvature during early stages of the universe.
- This means the velocity equation is simply $\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3}$.
- If black body radiation is the main source of energy density, we know $\rho = aT^4/c^2$.
- The latter means $\frac{\dot{R}}{R} = -\frac{\dot{T}}{T}$ and thus

$$\left(\frac{\dot{T}}{T}\right)^2 = \frac{8\pi G a T^4}{3c^2} \quad \Rightarrow \quad T = \left(\frac{3c^2}{32\pi G a}\right)^{1/4} t^{-1/2} \quad \Rightarrow \quad t = 2.3\,\mathrm{s}\left(\frac{10^{10}\mathrm{K}}{T}\right)^2.$$

- There are no adjustable constants in this expression!
- Note this adds yet more monotonic parameters which we can (and do) use to parameterise cosmic epoch:

$$\boxed{T \sim (1+z) \sim \frac{1}{a} \sim \frac{1}{R} \sim t^{-1/2} \sim \rho^{1/4} \sim E}$$

- NB: this fundamentally comes from $\rho \propto R^{-4}$ from cosmology and $\rho \propto T^4$ from blackbody

- We can also relate as equivalent energy $E = k_B T$ via $1\,\mathrm{K} = 8.6 \times 10^{-14}\,\mathrm{GeV}$

$$E = (1.3\,\mathrm{MeV})\,(t/1\,\mathrm{s})^{-1/2}.$$

- Interesting application of this expression is to the *very* early universe, where we can ask what time corresponds to the grand unification theory scale of about $10^{14}\mathrm{GeV}$

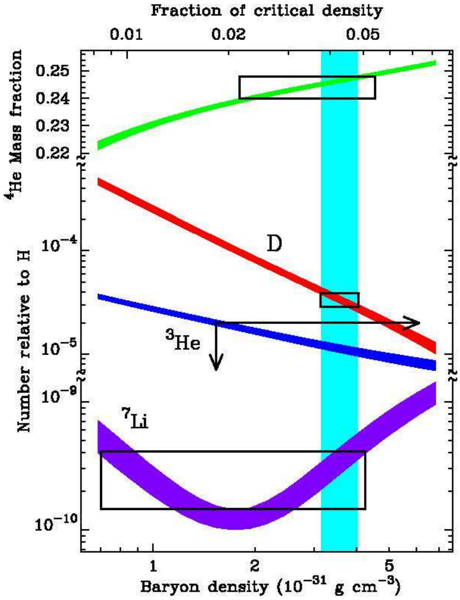
$$t_{\mathrm{GUT}} = 2 \times 10^{-34}\,\mathrm{s}.$$

  confirming the sorts of times we will see are typical for the primordial inflationary Universe
- Around $t = 1$ second we see that the energy corresponds roughly to the energy needed to create electron-positron pairs.
- Thus before this time the universe is flooded with these pairs, but after 1 second they annihilate, leaving only further radiation as their remnant.

Nucleosynthesis

- After these early transitions, the scene is then set for synthesis of light nuclei.
- In the epoch of nucleosynthesis (about 1 second to several minutes) the entire universe has the conditions of a young star – a plasmatic mixture of protons, neutrons, electrons and photons.
- As in stellar nucleosynthesis, over these first three minutes the Universe itself starts synthesising Hydrogen into heavier elements such as Helium, Lithium and Deuterium.
- This primordial nucleosynthesis sets the initial abundances for the first population of stars.
- From the present day (somewhat processed) abundance of these, we can work out the baryon density during primordial nucleosynthesis.
- The abundances of these light elements are sensitive indicators of conditions at that time, and in particular of the overall baryon density Ω_b.

- We look at details of this in the Appendix (non-examinable), and here just note.
- How we work out temperatures in the early universe, when it is radiation dominated — this will give us a guide to why nuclei production begins when it does.
- What are the results of nucleosynthesis constraints for the baryon production.
- Results of cosmic nucleosynthesis in the first three minutes of the Universe.
- The boxes indicate the range of current estimates of the abundances of Helium, Lithium and Deuterium in primordial gas, and the broad line common to the boxes shows the range of allowed baryon density in the Universe.



Recombination

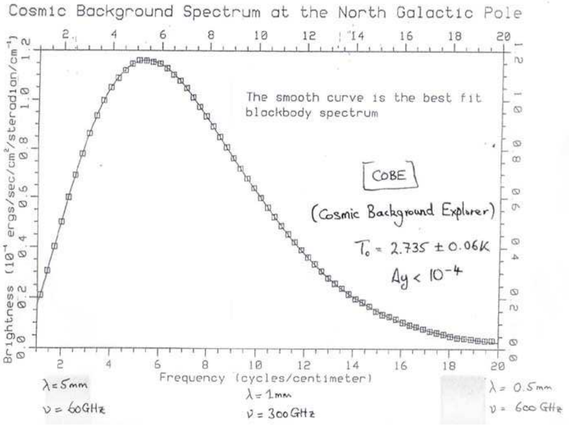
- As the universe cools further, eventually protons and electrons combine to form neutral hydrogen at recombination $z_* \approx 1100$, $T_* \approx 3000\,\mathrm{K}$.
- Ionization potential of hydrogen is $13.6\,\mathrm{eV}$, which corresponds to a temperature of approximately $160\,000\,\mathrm{K}$, so a naive estimate $z_* \approx \frac{160\,000}{2.73} \approx 60\,000$.
- However, recombination cannot be an instantaneous process – e.g. if an electron and proton recombine directly to the ground state get a $13.6\,\mathrm{eV}$ ionizing photon.
- Need to use Saha equation plus consideration of all transitions to get a correct description of the ionization fraction with time.
- Can get approximate answer, however, by requiring that the universe cools to the point where there are no longer sufficient photons of energy $> 13.6\,\mathrm{eV}$.
- Thus can approximate the temperature of recombination by requiring that there be less than one photon of energy $> 13.6\,\mathrm{eV}$ per proton

$$\exp\left(-\frac{13.6\,\mathrm{eV}}{kT_{\mathrm{rec}}}\right) \sim 10^{-9}.$$

- This gives $T_{\mathrm{rec}} \sim 7600\,\mathrm{K}$. This is not too bad an estimate.

Why does the CMB remain blackbody?

- After recombination photons decouple from neutral hydrogen and free stream toward us.
- We measure these photons as the CMB today, so they are a direct sightline to the early universe.
- CMB is blackbody when *emitted* in the early universe, since it is then in thermal equilibrium with the matter.
- How does it maintain its blackbody-form as it propagates towards us, in extreme thermodynamic *dis*equilibrium with the matter?



- Let $n_\nu d\nu$ be the (proper) number density of photons with frequencies in the range $(\nu, \nu + d\nu)$.
- For blackbody radiation since $B_\nu = I_\nu = (c/4\pi)u_\nu$, and $u_\nu = n_\nu h\nu$

$$n_\nu = \frac{8\pi\nu^2}{c^3 \left(e^{h\nu/kT} - 1\right)}.$$

- Consider a *comoving* volume $V(t)$, i.e. one where its proper volume grows as R^3.
- Let t_1 be the epoch immediately after recombination, and t_0 today.
- If photons travel to us completely unhindered from recombination, then the numbers of photons in comoving volume elements should be conserved:

$$n_{\nu_1}(t_1)d\nu_1(t_1)V(t_1) = n_{\nu_0}(t_0)d\nu_0(t_0)V(t_0).$$

- The standard redshift argument gives: $\nu \propto 1/R$, and $d\nu \propto 1/R$:

$$n_{\nu_0}(t_0) = n_{\nu_1}(t_1)\,\frac{d\nu_1(t_1)}{d\nu_0(t_0)}\,\frac{V(t_1)}{V(t_0)} = n_{\nu_1}(t_1)\frac{R_0}{R_1}\left(\frac{R_1}{R_0}\right)^3 = \left(\frac{R_1}{R_0}\right)^2 n_{\nu_1}(t_1).$$

- But $n_{\nu_1}(t_1)$ *does* have a blackbody form, at temperature T_1 say, and thus

$$n_{\nu_0}(t_0) = \left(\frac{R_1}{R_0}\right)^2 \frac{8\pi\nu_1^2}{c^3} \bigg/ \left[\exp(\frac{h\nu_1}{kT_1}) - 1\right].$$

- Finally, $\nu_1 = (R_0/R_1)\nu_0$, so that we can rewrite the last equation as

$$n_{\nu_0}(t_0) = \frac{8\pi\nu_0^2}{c^3} \bigg/ \left[\exp(\frac{h\nu_0}{kT_1 R_1/R_0}) - 1\right].$$

- Radiation still has a Planckian form, but temperature:

$$T_0 = \frac{T_1 R_1}{R_0} = \frac{T_1}{1+z}.$$

- Note this is another proof that $T \propto 1/R$.

When does the "early" universe finish?

- We stop being able to use the radiation domination approximation around the epoch of equality, when energy densities are equal $\Omega_r = \Omega_m$.
- Irritatingly for theoreticians this occurs sometime in between nucleosynthesis and recombination:

$$\rho_{\mathrm{rad}} c^2 = a T_0^4 (1+z)^4, \quad \rho_m = \rho_{m0}(1+z)^3 = \Omega_{m0}\frac{3H_0^2}{8\pi G}(1+z)^3 \quad \Rightarrow \quad (1+z_{\mathrm{eq}}) = \frac{3\Omega_{m0} H_0^2 c^2}{8\pi G a T_0^4}.$$

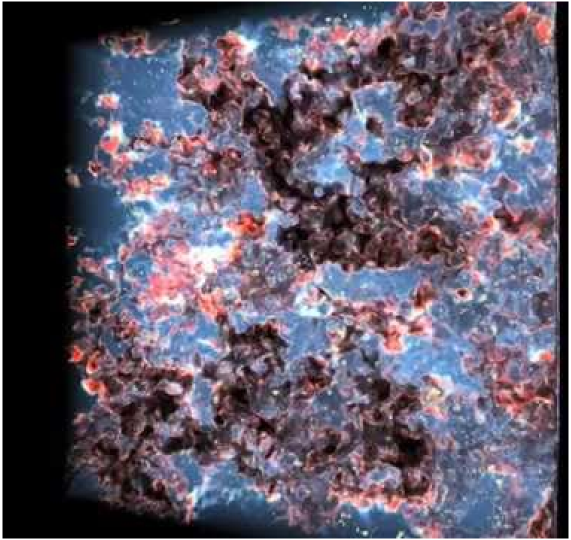
- For current values from earlier $1 + z_{\mathrm{eq}} = 5780$.
- More generally, writing $H_0 = h \times 100\,\mathrm{km s^{-1} Mpc^{-1}}$:

$$1 + z_{\mathrm{eq}} = 4 \times 10^4\, \Omega_{m0}\, h^2,$$

which expresses it in a more cosmology-agnostic form.

Reionisation

- Universe became transparent at recombination, due to material turning into (mainly) hydrogen atoms.
- But universe thought to have reionised by at the latest $z \approx 6$.
- How? — early stars and quasars turned on.
- UV emission led to reionisation of the neutral matter.



The Optical Depth of the Universe

- Can measure in terms of an optical depth to coordinate distance χ:

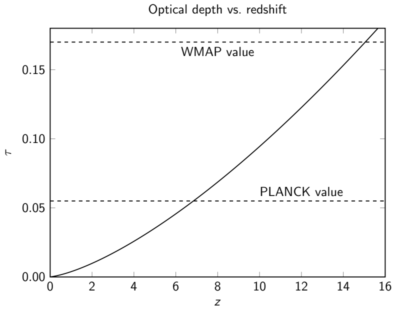
$$\tau(\chi) = \int_0^{\chi} n_e(\chi)\sigma_T R d\chi',$$

  where $n_e(\chi)$ is the function describing the free-electron number density, σ_T is the Thomson scattering cross section ($6.65 \times 10^{-29}\,\mathrm{m}^2$) and $Rd\chi$ is the element of proper distance at coordinate radius χ.
- This follows from counting the number of scatterers with their centres in a tube of constant proper area σ_T stretching between the observer and coordinate radius χ — compare Example sheet 4
- Very important to measure this — when did the early stars, quasars and galaxies turn on?
- How many of them are there?
- τ larger means more early radiating objects.

- If changeover from neutral to ionized matter was approximately instantaneous, and took place at redshift z_r (for $z_\mathrm{reionisation}$), then can write

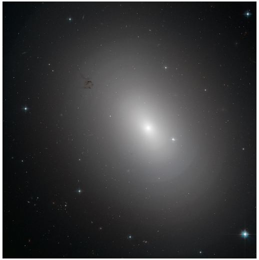
$$\tau(z_\mathrm{r}) = \int_0^{z_\mathrm{r}} n_e(z)\sigma_T R(z)\frac{d\chi}{dz}dz = \int_0^{z_\mathrm{r}} n_e(0)(1+z)^3\sigma_T R(z)\frac{c}{R_0 H(z)}dz,$$
$$= \int_0^{z_\mathrm{r}} \frac{n_e(0)(1+z)^2\sigma_T c}{H(z)}dz.$$

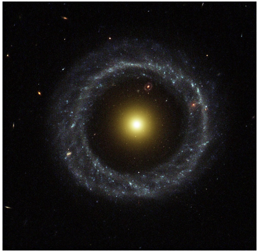
- Estimate $n_e(0)$, the current day density of free electrons, from the baryon density Ω_b (see later) plus assumption about what fraction is hydrogen, how much helium etc., and then just need to specify a cosmology to calculate $H(z)$.
- From the CMB $\tau = 0.054$ [arxiv:1807.06209].
- This value matters for the power of SKA and JWST.

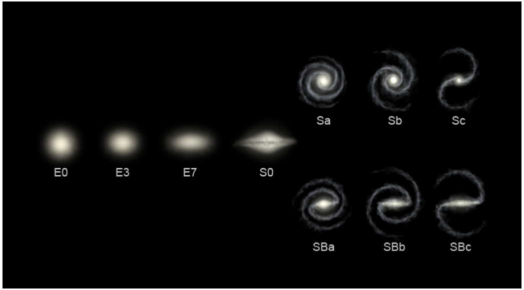


Galaxies

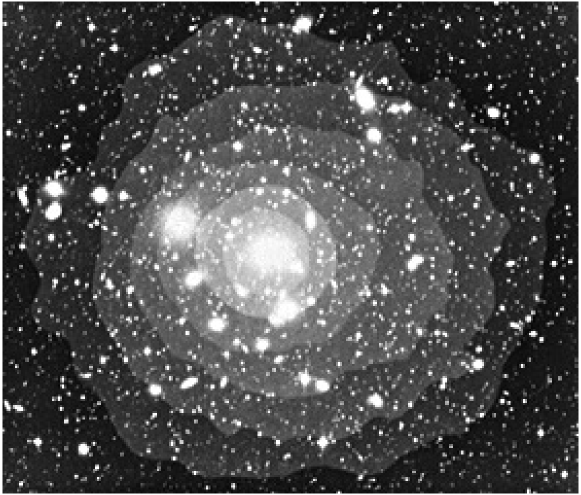
- By reionisation, the late-time universe has begun, and large scale structure begins to form.
- Galaxies are the building blocks of the late-time Universe.
- Gravitationally bound & isolated structures decoupled from the Hubble flow.
- Typically $1-100$kpc in size, separated by $\sim$ Mpc, 10^8-10^{14} stars, (Milky way: 10^{11}).
- Anatomically composed of stars, interstellar medium, active galactic nuclei & dark matter.
- There are approximately 2 trillion galaxies in the observable Universe.
- Come in a wide variety.







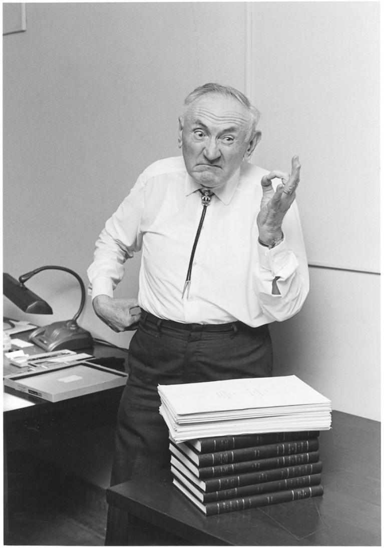
Galaxy clusters



- About one half of all galaxies are in groups of $2 - 10$ large members.
- About 1% are in rich clusters with 100s of members.
- The typical radius of a group or cluster is $1 - 2$Mpc.
- The escape velocity from the core of a massive cluster can be almost 10^4kms^{-1}.

Dark matter

- Hypothesised by Kelvin (1884) & Zwicky (1933).
- It is easy (and healthy) to maintain scepticism around the fact that we have to invent an invisible component of the universe to make the equations add up.
- Dark matter explains a host of findings in a variety of astrophysical and cosmological settings.
- Any replacement of it will have to explain why the universe is trying so hard to look like there is a huge invisible material component.
- One can construct *post hoc* theoretical justifications e.g. If electromagnetic interaction is an exotic phenomenon, then we would expect the majority of the universe to be invisible, incapable of chemistry and boring/non-observing.
- It might have been better to label it "transparent matter".



Evidence for dark matter: cluster dynamics

- Estimates of the mass of stars in a cluster from the product of the total visible luminosity and the mass-to-light ratio of a typical galaxy yield values of $M_\star \sim 10^{13} - 10^{14} M_\odot$.
- This conflicts with dynamical mass estimates obtained from the Virial Theorem;

$$M \approx v^2 R/G,$$

where v is assumed to be the dispersion in the radial velocities of the member galaxies,

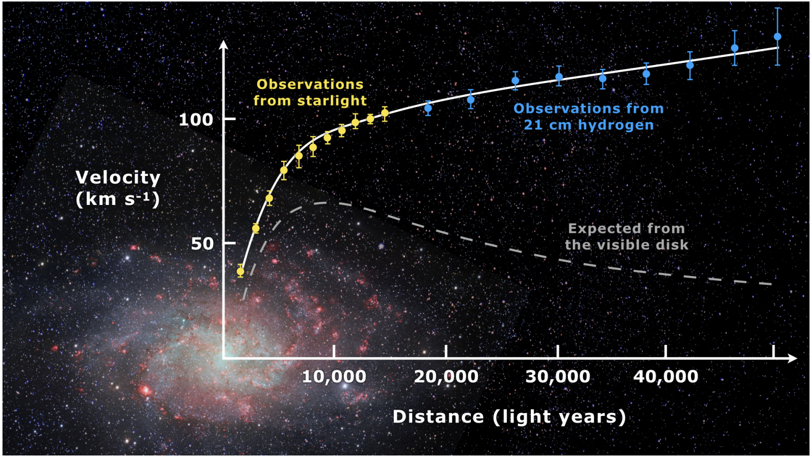
$$M_{dyn} \sim 10^{14} - 10^{15} M_\odot \approx 10 - 50 M_\star.$$

$$\text{i.e.} \quad M_{cluster} \gg \Sigma M_{galaxies}.$$

- This can be explained dark matter (DM) in between the galaxies in a cluster.

Evidence for dark matter: galactic rotation curves

- We can measure rotational velocity using redshift as a function of radius.
- These do not obey Kepler's law.
- Suggest a different distribution of invisible matter.



Dark energy

- Unlike dark matter, this is a much more recent invention.
- It really is just our name for the observation that the universe's expansion is speeding up.
- This can be explained by re-introducing Einstein's cosmological constant.
- There are two cosmological constant problems:
  1. Why is it there at all?
  2. Given that it is there, why is its value so small?
- Given it as an energy associated with empty space which is "created" as the universe expands, it would be great if we could identify it with the vacuum energy of e.g. the Higgs' field.
- However any attempt to do this using our current understanding of QFT gives values that are factors of 10^{120} out!
- Clearly much more work to do on this, and in general cosmologists would likely be more willing to drop dark energy than dark matter.

Summary

- Monotonic parameters for our universe

$$\boxed{T \sim (1+z) \sim \frac{1}{a} \sim \frac{1}{R} \sim t^{-1/2} \sim E^{1/4}}$$

- The photon to baryon ratio 10^9 and matter antimatter asymmetry
- Epochs of nucleosynthesis, recombination and reionisation.
- Dark matter & Dark energy

Next time

Modern cosmological data

Appendix: The baryon density from nucleosynthesis — non-examinable

- Nucleosynthesis starts when the universe has cooled sufficiently for stable protons and neutrons to exist but there is still enough energy for nuclear reactions. This is during the period about 1 second to several minutes.
- Following processes convert protons into neutrons (and vice versa)

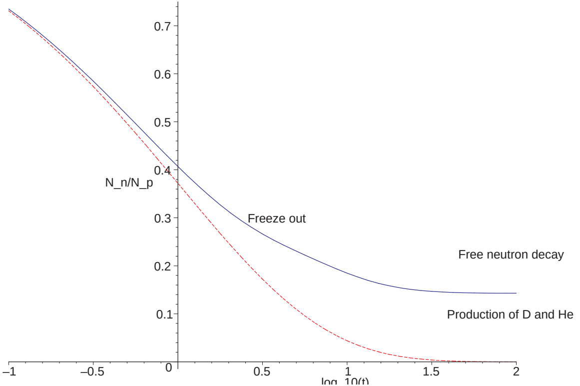
$$\begin{aligned} n &\leftrightarrow p + e^- + \overline{\nu}_e, \\ p + \overline{\nu}_e &\leftrightarrow n + e^+, \\ p + e^- &\leftrightarrow n + \nu_e. \end{aligned}$$

- Provided they are fast enough, they will maintain the relative numbers of protons and neutrons in accordance with thermal equilibrium

$$\text{i.e. } \frac{N_n}{N_p} = e^{\frac{-(m_n - m_p)c^2}{kT}} \simeq e^{-\frac{1.5}{T_{10}}},$$

where $(m_n - m_p) = 1.29\,\mathrm{MeV}$ and $T_{10} = T/10^{10}\mathrm{K}$.

- *However*, cross-sections for the (weak) reactions above are all small, and it turns out the timescale for establishing the Boltzmann equilibrium is $t_{weak} \simeq 2.5\,T_{10}^{-5}$ seconds.
- But the expansion timescale (on which the universe is cooling, and the ratio needs to adjust in order to maintain the equilibrium) is $t_{exp} \simeq T_{10}^{-2}$ seconds.
- Therefore N_n/N_p doesn't fall off as steeply as $e^{-1.5/T_{10}}$ after T_{10} falls below unity, but instead "freezes out", declining only on the $\sim 10^3$s timescale of free neutron decay.



Decline of the ratio of neutrons to protons during the period 0.1 to 100 seconds. The red (dashed) curve shows the Boltzmann factor $\exp(-1.5/T_{10})$ *behaviour, while the blue (solid) curve shows the effects of the freeze out.*

- Deuterium can be formed via

$$n + p \leftrightarrow D + \gamma$$

.

- However, because photons outnumber baryons by $\gtrsim 10^8$, the photodissociation process $(\longleftarrow)$ is too rapid to permit a significant D abundance to build up until T has fallen to $\sim 10^9\ ^\circ$K, when there are few photons above the photodissociation energy of 2.2 MeV
- Can work this out using exactly the same approach by which we estimated the temperature at which recombination occurs, and using the same photon/baryon ratio of $\sim 10^9$.
- At this stage, which occurs about 3 minutes after the big bang, the following further reactions can build up ^{4}He:

$$D + D \begin{array}{l} \nearrow \quad {}^3\mathrm{He} + n \rightarrow {}^3H + p \\ \searrow \quad {}^3H + p \end{array}$$

$$^3H + D \rightarrow {}^4\mathrm{He} + n$$

- Almost all D gets processed into He. This means that $N_n/2$ Helium nuclei are formed, requiring N_n of the protons. The resultant ^{4}He abundance (by mass) is therefore

$$\left(\frac{4N_n/2}{4N_n/2 + N_p - N_n}\right)_{t \simeq 100\,\mathrm{s}} \simeq \left(\frac{2N_n}{N_n + N_p}\right)_{t \simeq 100\,\mathrm{s}}.$$

- The N_n/N_p ratio at 100 to 200 s is about 0.15, hence the predicted Helium abundance is about 0.25.
- D and ^{3}He are *intermediate products* on the way to ^{4}He production, so *more* tend to survive when the nucleon density (i.e. Ω_b) is *low*.
- The abundance of D is therefore sensitive to the baryon density in the Universe, and is currently our best indicator of this.
- D is destroyed in stars so measurements of its primordial abundance must be made with gas which has been as undisturbed as possible.
- Best results have been derived from intergalactic clouds which produce Lyman-α absorption by H and D in the spectra of distant quasars.

- Tytler and collaborators have found that the results imply a baryon fraction for the Universe

$$\Omega_b \approx 0.04.$$

- This fits in very well with our earlier estimate from the CMB power spectrum peaks.
- Primordial nucleosynthesis is ineffective above Helium except for trace amounts of a few elements. This is because in two body collisions (all that there is time for) we would have to add either a single proton or another Helium nucleus, to an existing He nucleus, but there are no stable nuclei heavier than Helium with atomic numbers 5 or 8.
- Thus all the heavier elements have to be formed later, in stars. There is thus a nice balance between what the early universe is able to do, and what stars can do later.