Gravity, isostasy and creep

Physics of the Earth as a Planet, Lecture 4

(Fowler, The Solid Earth, CUP, Chapter 5, p. 160-169 1st, p. 193-210 2nd edition)

History

In the 19th century Colonel Everest measured the distance between two points in India on the same longitude in two ways: using theodolites (triangulation) and astronomically (Figure 1). Clearly

$$a(\theta_1 + \theta_2) \text{ should} = d \tag{1}$$

where a is the radius of the Earth. He found a difference between the two values of 162 m, with a probable error of about ± 5 m based on other surveys from this period, over a distance d of 600 km.

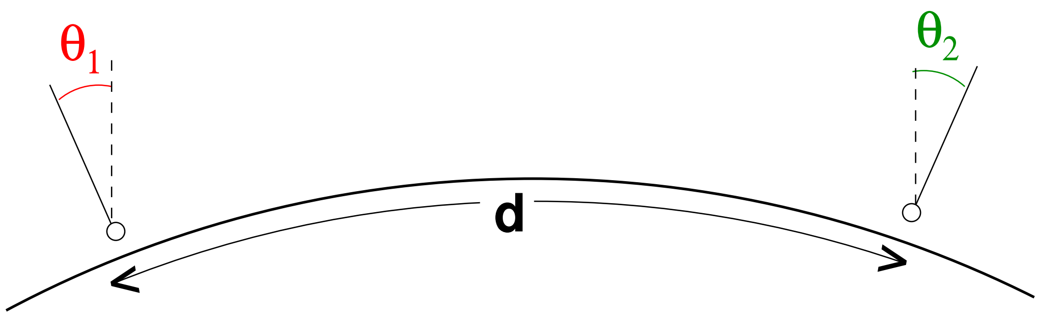


Figure 1: Measuring a distance astronomically.

He believed that the explanation was that the plumb line needed for the astronomical measurements was deflected by the Himalayas (Figure 2). Archdeacon Pratt of Calcutta calculated the deflection, and found it should be 3× that observed.

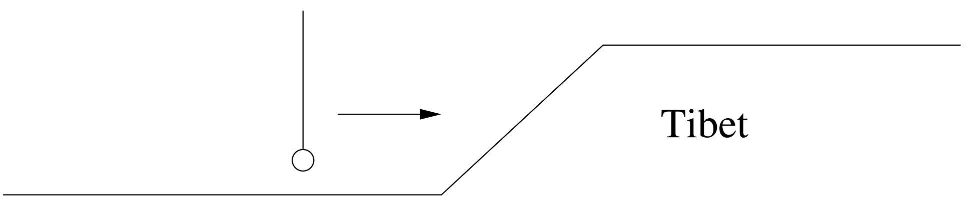
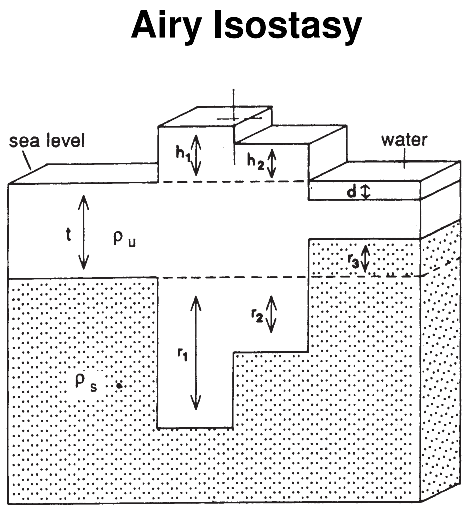


Figure 2: Deflection of the plumb line due to the Himalayas.

Isostasy

Airy suggested that mountains had light roots, and later Pratt proposed that the density of the crust varied laterally. We now know that both mechanisms occur and this is called isostatic compensation.



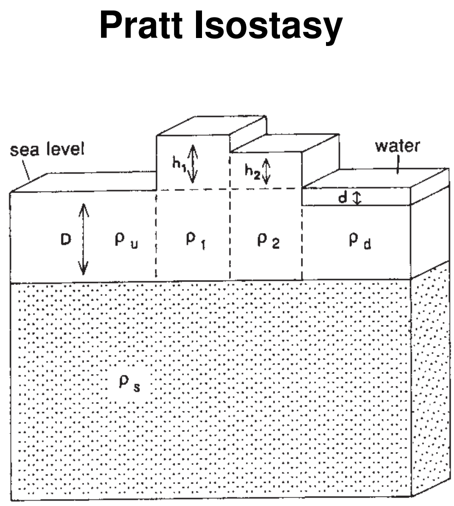


Figure 3: Airy and Pratt isostasy models (after Fowler, The Solid Earth).

We balance the weight of different columns (Archimedes' principle of hydrostatic equilibrium) to get the expressions for Airy and Pratt isostasy. Airy compensation is given by

$$(r_1 + h_1)\rho_u = \rho_s r_1 \implies r_1 = \frac{h_1 \rho_u}{\rho_s - \rho_u} \tag{2}$$

where h_1 is the topography and r_1 is the corresponding root. Pratt compensation is given by

$$D\rho_u = \rho_1(h_1 + D) \implies \rho_1 = \rho_u \frac{D}{h_1 + D} \tag{3}$$

which can be used to find the lower density ρ_1.

The reaction of the Indian Survey to Everest's findings was to design an instrument to measure gravity, and did so with pendulums. We now do so by measuring the acceleration with lasers, with spring balances, with vibrating strings (from ICBMs), and by directly measuring the shape of the equipotential (which is close to the shape of the ocean surface).

Gravity

The gravitational acceleration $\mathbf{g}$ satisfies Gauss's law

$$\nabla \cdot \mathbf{g} = -4\pi G\rho \tag{4}$$

where G is the gravitational constant, and ρ is the density. Gravitation is a conservative force, and can be written in terms of a gravitational potential ϕ as

$$\mathbf{g} = -\nabla\phi, \tag{5}$$

where the gravitational potential satisfies Poisson's equation

$$\nabla^2\phi = 4\pi G\rho. \tag{6}$$

In hydrostatic equilibrium the surface of a liquid is an equipotential of ϕ. On Earth, the equipotential surface which corresponds to mean sea level is called the *geoid*. In continental areas the geoid corresponds to level water would be at if imaginary channels were cut through the continents. The geoid varies from that of a reference ellipsoid by ±100 m, which is very small, so the Earth is almost in isostatic equilibrium on a large scale. Geoid highs are found in places where there is subduction. A large geoid low is found over India, which is currently still unexplained as there is no corresponding surface expression (Figure 4).

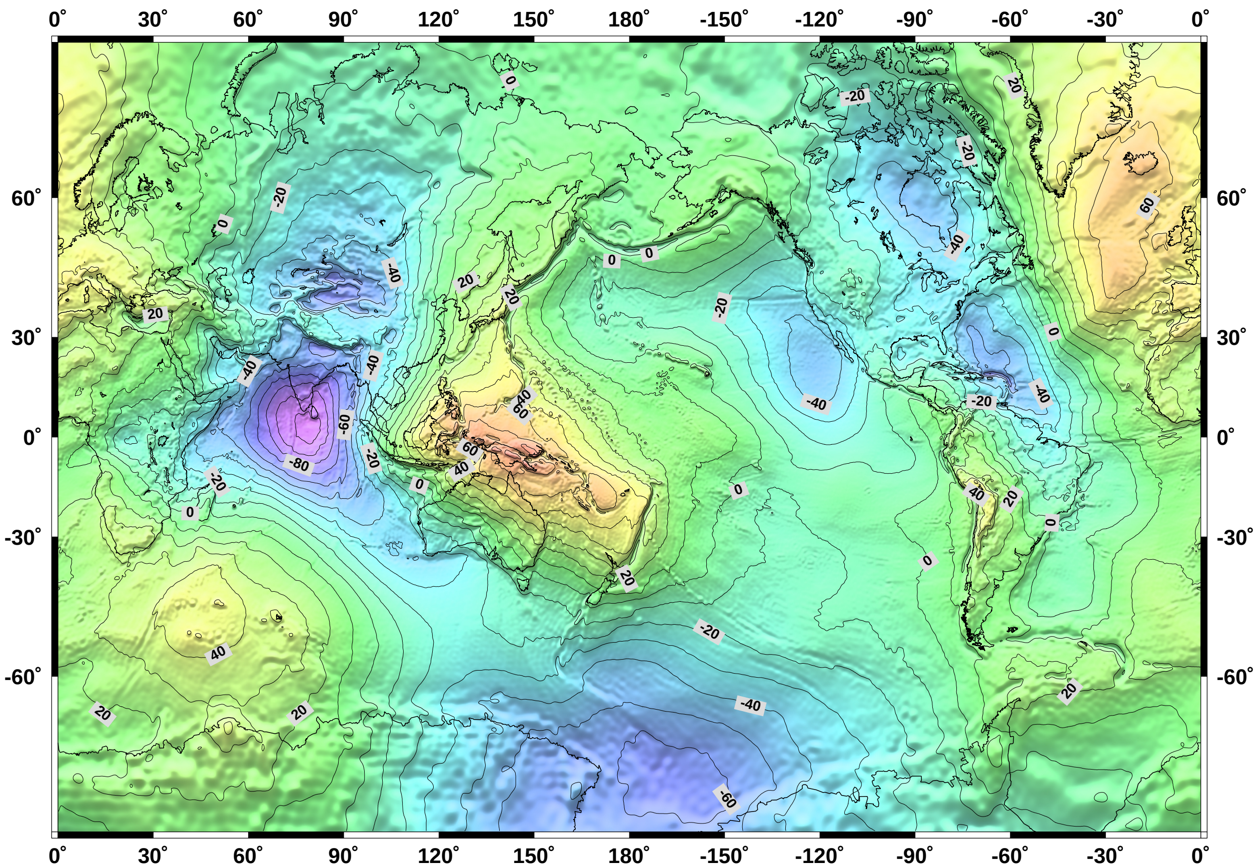


Figure 4: The geoid. Contours are in metres.

Variations in the Earth's gravity field are expressed in two main ways in geophysics. The first is as a geoid anomaly, the deviation from the reference ellipsoid, measured in metres. The second (and more common) is as a gravity anomaly, the deviation in the acceleration due to gravity from that of theoretical gravity, measured in milligals (gals are a unit of acceleration named after Galileo, the first to measure gravity. 1 Gal = 1 cm s^{-2}, 1 milligal = 10^{-5} m s^{-2}). A number of corrections are made to gravity anomalies to allow for easier comparison. The most important of these is the free-air correction, which adjusts the measurement to be that which would have been measured at sea level, assuming no mass exists between the observation point and sea level. Anomalies corrected in this way are called *free-air gravity anomalies*.

Infinite slab approximation

For our purpose it is sufficient to use the infinite slab approximation to calculate gravity anomalies (Figure 5),

$$\Delta g = 2\pi G \Delta\rho t, \tag{7}$$

where Δg is the gravity anomaly, $\Delta\rho$ is the difference in density of the slab from that of its surroundings, and t is the thickness of the slab. See 'Geodynamics' by Turcotte & Schubert, p. 244-246 (3rd ed.), for a derivation of this equation. A very useful approximate version of equation (7) is

$$\Delta g = 42\Delta\rho t \,\text{milligals}, \tag{8}$$

where $\Delta\rho$ is the density contrast in Mg m^{-3} and t is the thickness in km. Most crustal rocks have a density of about 2.7 Mg m^{-3}, and that of sea water is about 1 Mg m^{-3}, so $\Delta\rho = 1.7$ Mg m^{-3}, and the uncompensated gravity anomaly for underwater bathymetry should be about $42 \times 1.7 \approx 70$ milligals/km.

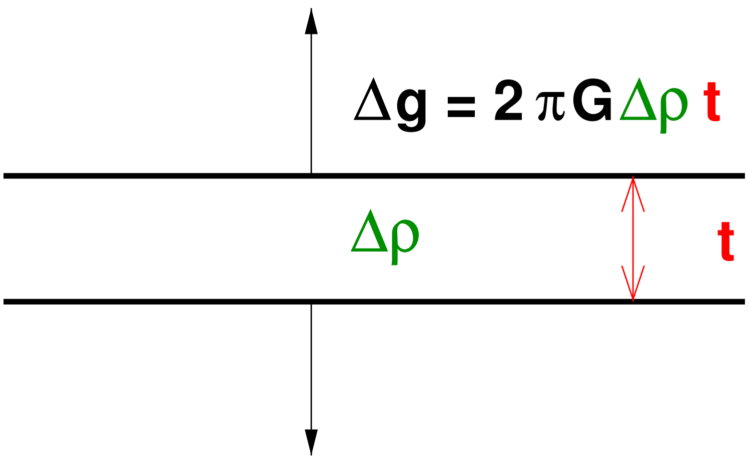


Figure 5: The infinite slab model.

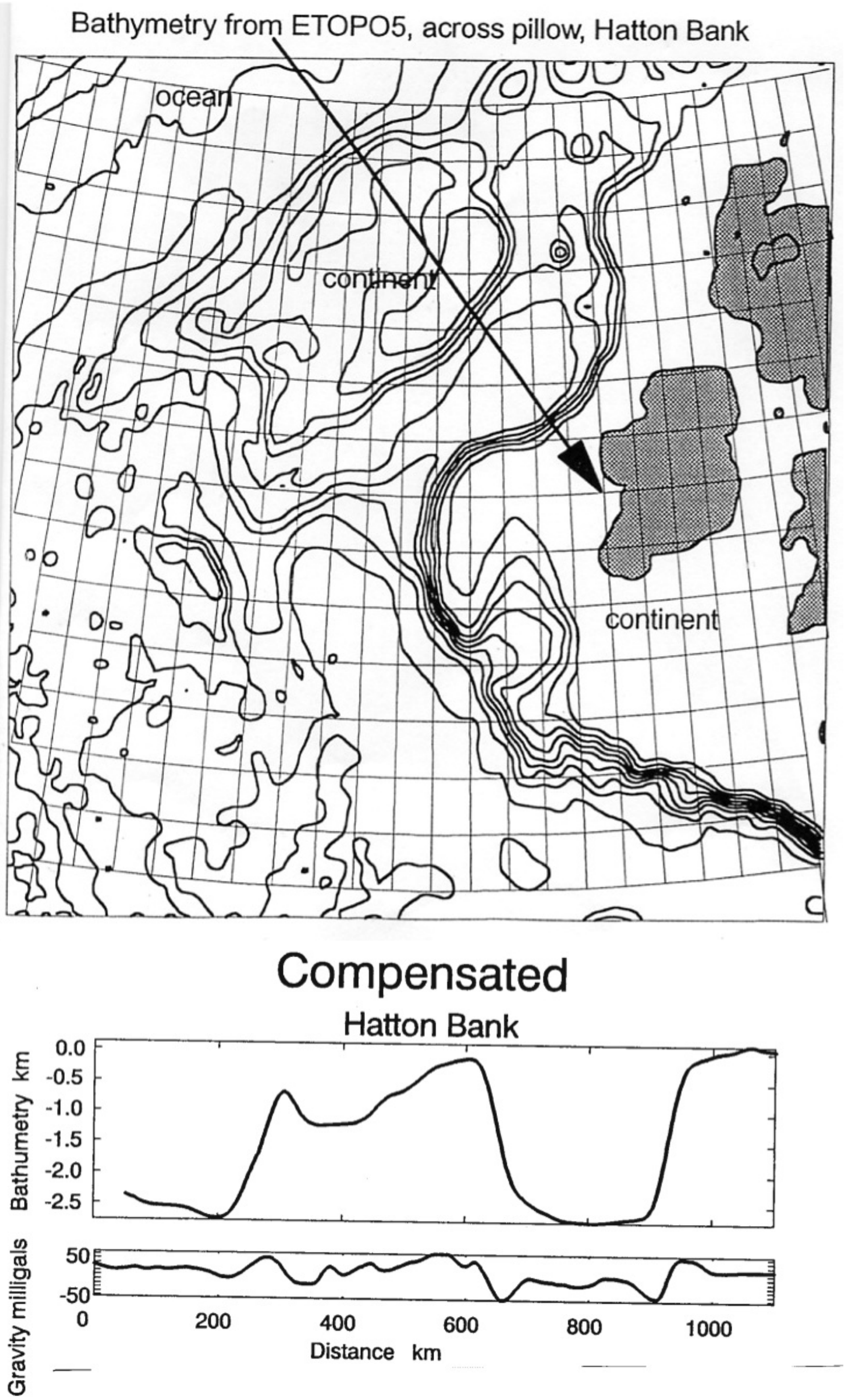


Figure 6: The gravity across Hatton-Rockall, where $t \approx 2$ km, and the gravity anomaly should be about 140 milligals. The observed anomaly is only about 20 milligals, so the topography is compensated.

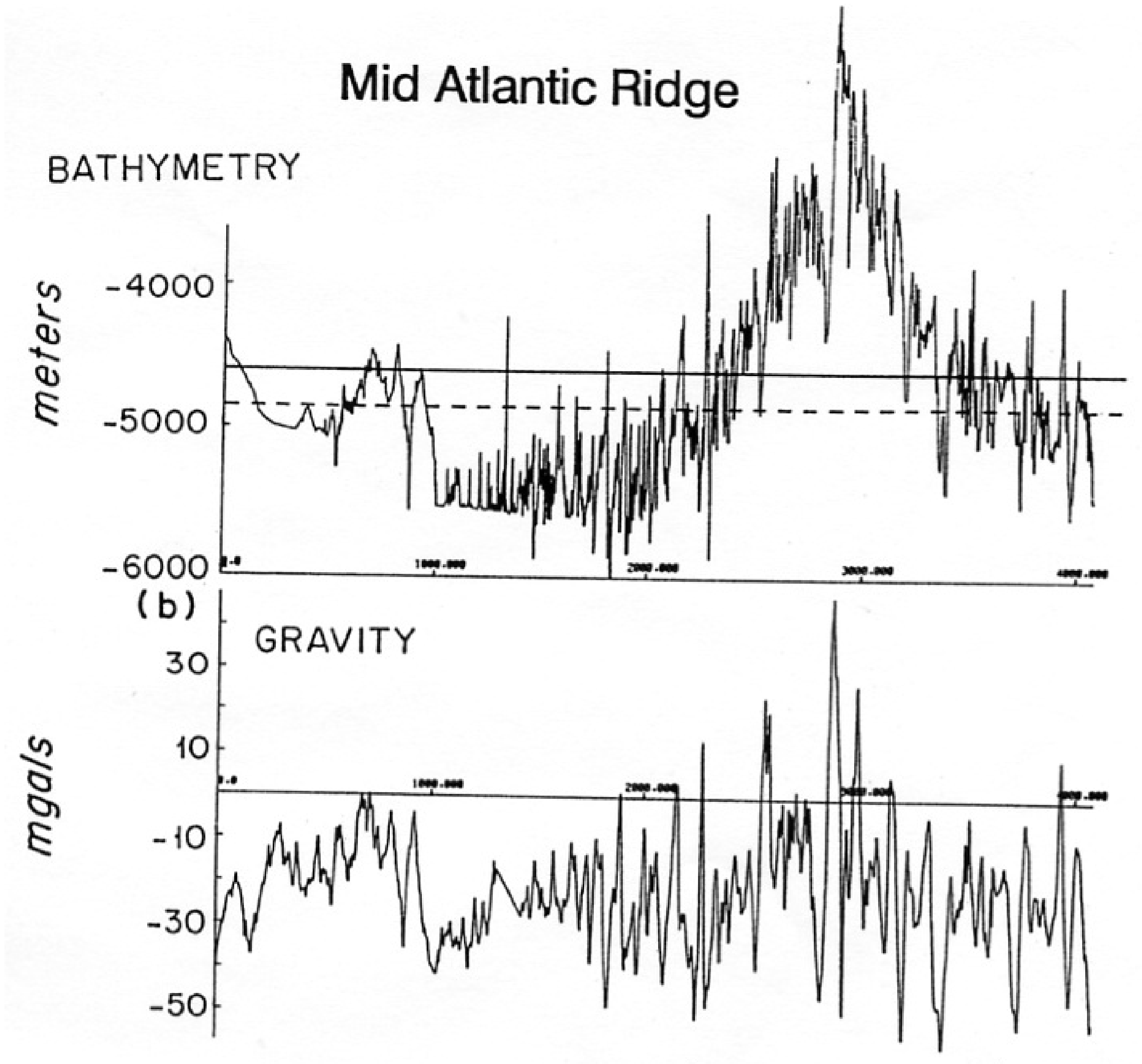


Figure 7: The Mid-Atlantic Ridge, with $t \approx 3$ km. The calculated gravity anomaly is about 200 milligals, and that observed only about 20 milligals. The ridge is therefore compensated.

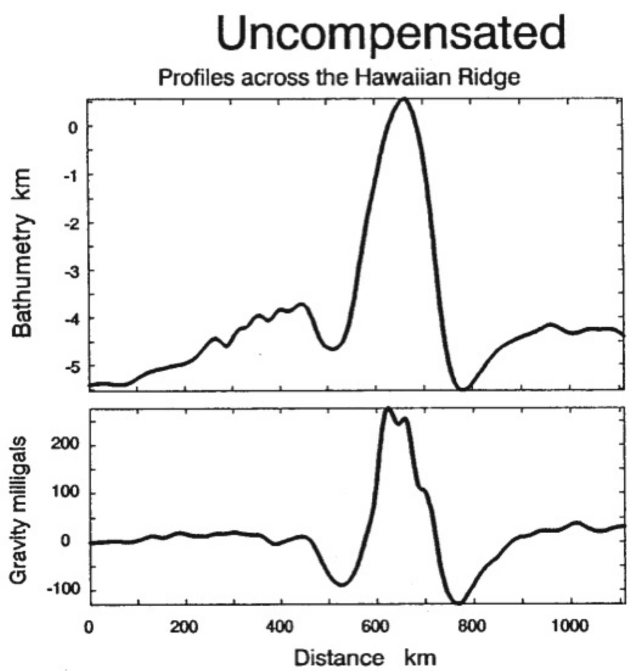


Figure 8: Not all topography is compensated. The Hawaiian Ridge, with $t \approx 4$ km, and both the observed and calculated anomalies are about 300 milligals. So the ridge is not compensated, and must be supported by elastic forces in the plate.

Creep

How does compensation occur? Airy and Pratt thought that the mantle was molten, but this is wrong because it propagates shear waves. We need to understand the long term behaviour of stressed solids, and how they creep. It is useful to define the *homologous temperature*

$$\theta = \frac{T}{T_m} \tag{9}$$

where T is the temperature of the solid, T_m is the melting temperature, both in K, and the *homologous stress* σ/μ, where σ is the stress and μ the shear modulus. Creep, or the long term deformation of solids, is controlled by θ. Only for temperatures larger than a certain homologous temperature θ can creep can occur (Figure 10). The creep mechanism depends on the stress applied.

Low temperature behaviour is controlled by the movement of dislocations, also called dislocation glide. The critical stress is the stress required to move the dislocations, and gives rise to a yield stress. At low temperatures crystals can only deform by dislocation movement, and they only do so when the yield stress is exceeded.

Faults are somewhat like crystal dislocations, and move in earthquakes which involve stresses of ~ 10 MPa, or less. Rocks are harder than metals because many glide planes are not allowed, because of bonding. A typical Young's modulus is $\mu \sim 70$ GPa, and so the homologous stress is

$$\log_{10}(\sigma/\mu) \approx -4 \tag{10}$$

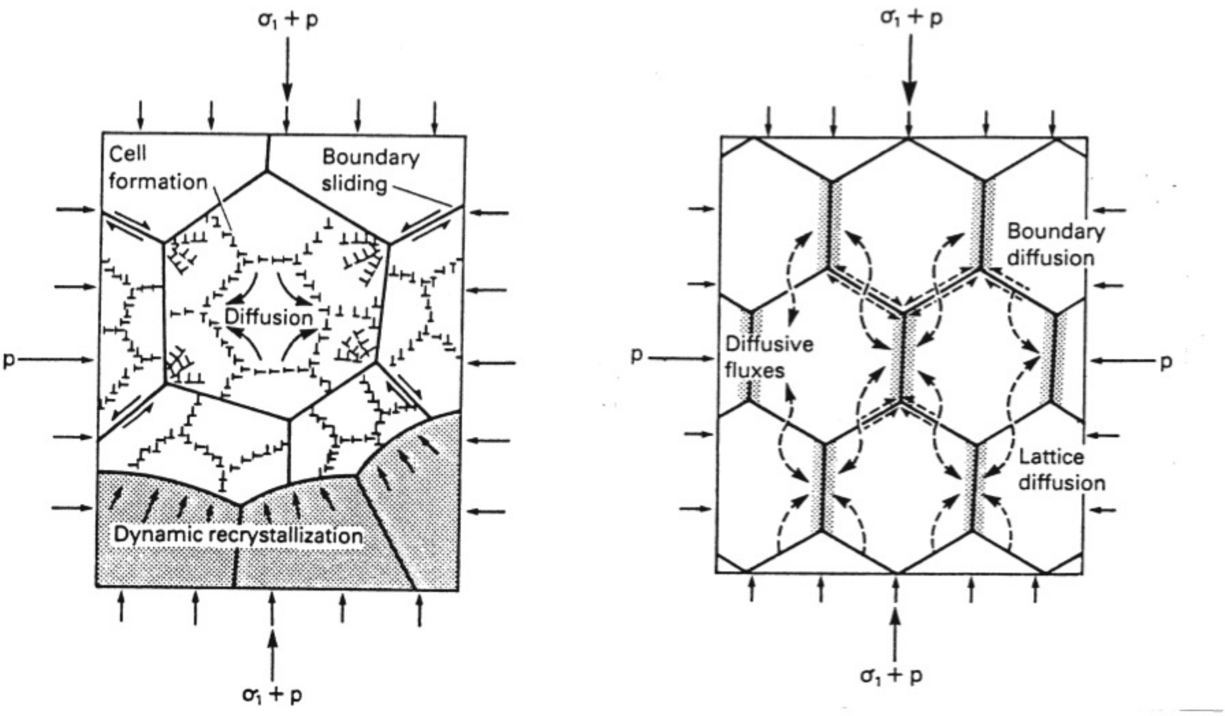


Figure 9: Illustration showing the different creep mechanisms, dislocation (power-law) creep (left) and diffusion creep (right).

High temperature behaviour is controlled by diffusion, which operates in two ways. At moderate stresses, if crystal dislocations can move but get stuck, then diffusion allows atoms to diffuse away. This is called *dislocation creep*; the stress-strain relationship follows a power-law (Figure 9, left panel). At lower stresses, creep still occurs, but by diffusion alone without significant movement of dislocations. This is called *diffusion creep* and has a Newtonian (linear) relationship between stress and strain (Figure 9, right panel). There are two types of diffusion creep, Coble creep and Nabarro-Herring creep. Coble creep dominates at low temperatures and involves diffusion along grain boundaries. High temperature behaviour is dominated by Nabarro-Herring creep, which involves diffusion within grains.

We know that isostatic compensation occurs by creep in the lithosphere, but at what depths? We can estimate this by using the information in Figure 10, which gives profiles of the temperature T and melting temperature T_m with depth within the Earth (for old oceanic regions), along with the deformation map for olivine (the dominant mantle mineral). Consider adding a layer of material $t = 1$ km thick with a excess density of $\rho = 10^3$ kg m^{-3} to the Earth's surface. This would produce a gravity anomaly of $\Delta g = 40$ milligals, which is fairly typical. The stress exerted by this layer is

$$\sigma \sim \rho g t, \tag{11}$$

so $\sigma \sim 10$ MPa is a typical stress. As a homologous stress this is

$$\log_{10}(\sigma/\mu) \approx -4. \tag{12}$$

For compensation to occur, we want 1 km of vertical motion in a plate of thickness 100 km to occur in geological time. Geological time is $\sim 10^9$ years, and 1 year $\approx \pi \times 10^7$ s. This corresponds to a vertical strain rate of 3×10^{-19} s^{-1}. Figure 10c shows that the homologous temperature must exceed 0.6 before such flow can occur, and Figures 10a and b show that this temperature occurs at a depth of about 60 km. This argument applies to oceans.

Continental material melts at about 650°C, and so is weaker. Flow occurs below a depth of about 30 km.

For future reference, Figure 11 plots the information in the deformation map of Figure 10 in another way: as contours of effective viscosity $\eta = \sigma/\dot{\epsilon}$, a quantity crucial for understanding the dynamics of mantle convection.

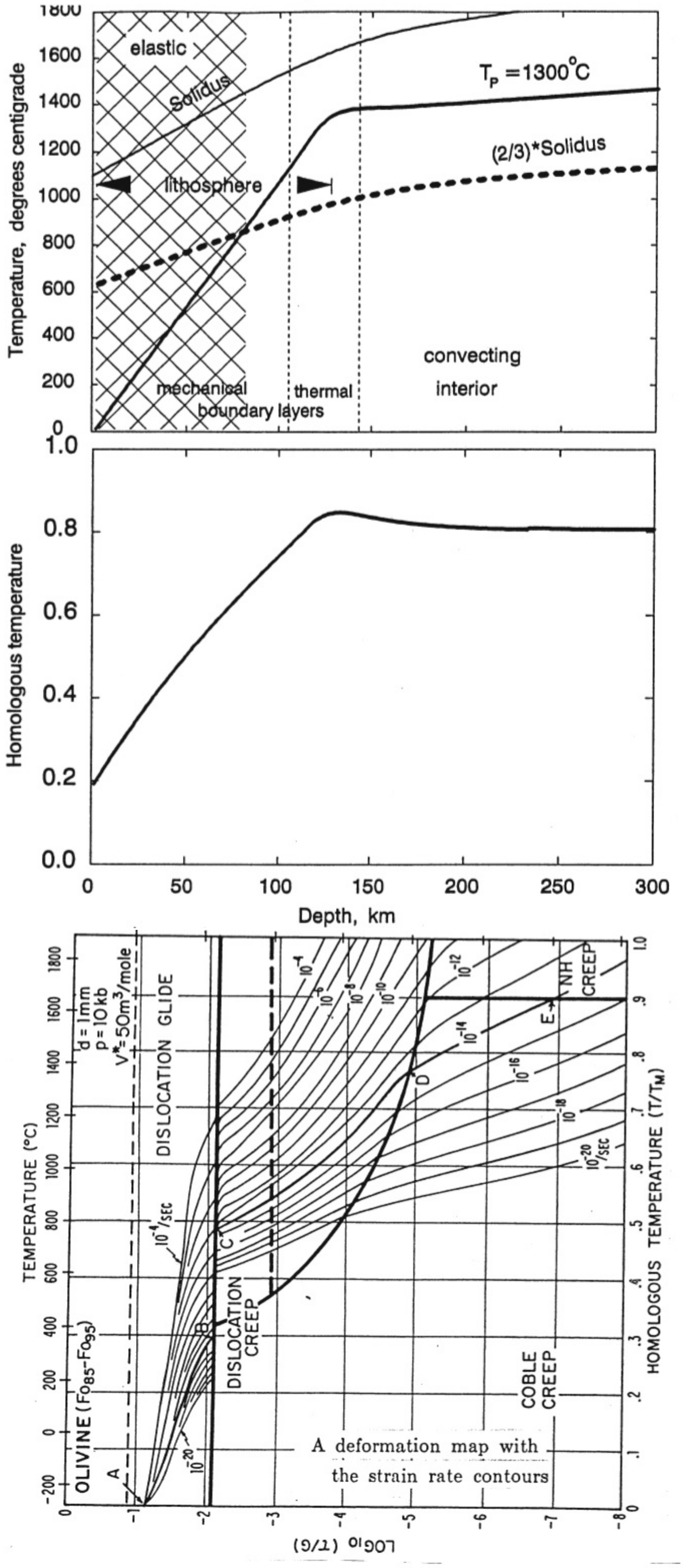


Figure 10: Top panel: Profile of temperature with depth within the Earth (old oceanic regions). Middle panel: Profile of homologous temperature. Bottom panel: Deformation map for olivine (dominant mantle mineral) showing creep mechanisms as a function of homologous temperature and strain rate. G $= \mu =$ 60 GPa.

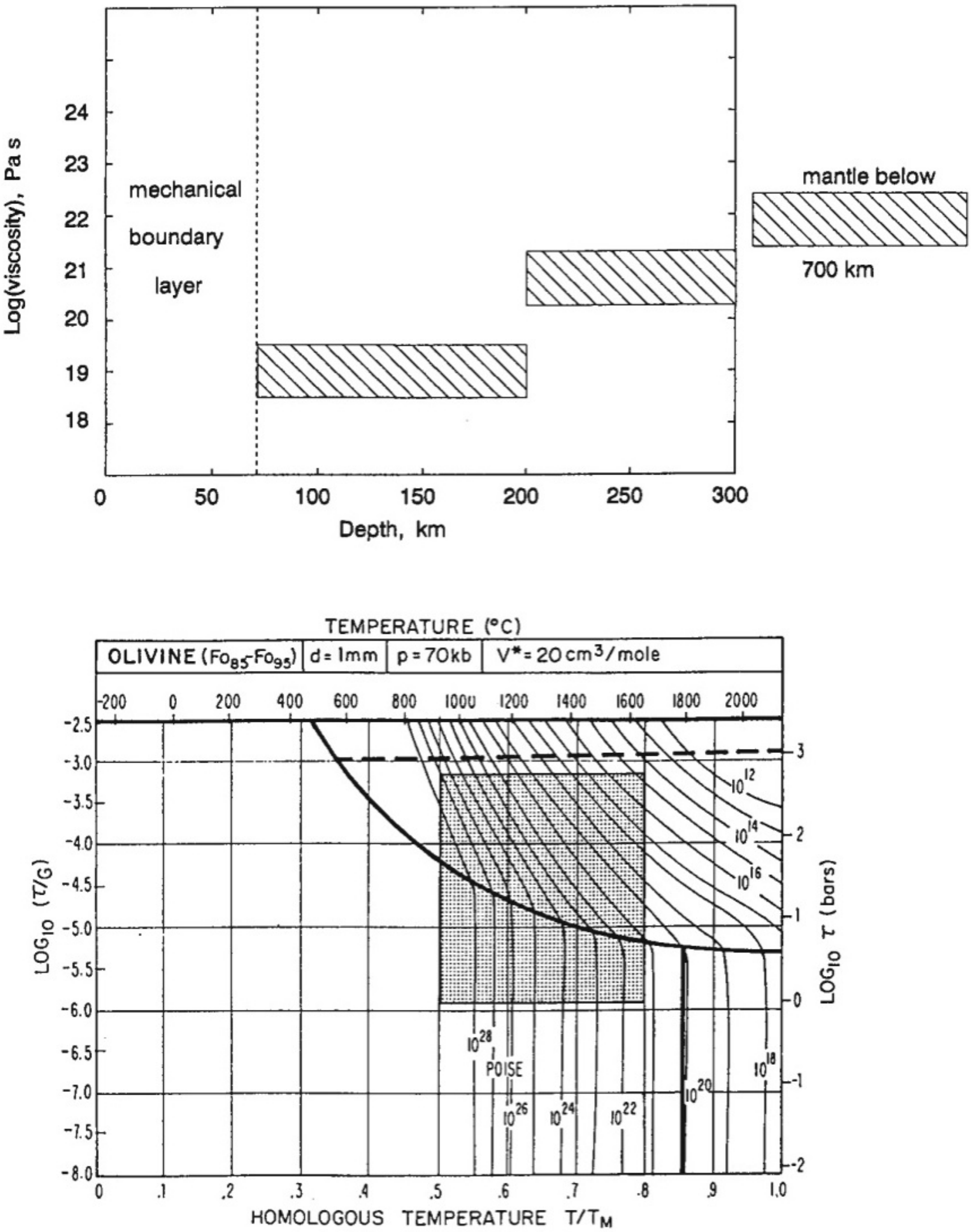


Figure 11: Top panel: Estimated variation of viscosity with depth within the Earth. Bottom panel: An effective viscosity map for olivine under upper mantle conditions. 1 Pa s = 10 poise = 10 g cm^{-1} s^{-1}.