Forces on plates maintaining their motions

Physics of the Earth as a Planet, Lecture 10

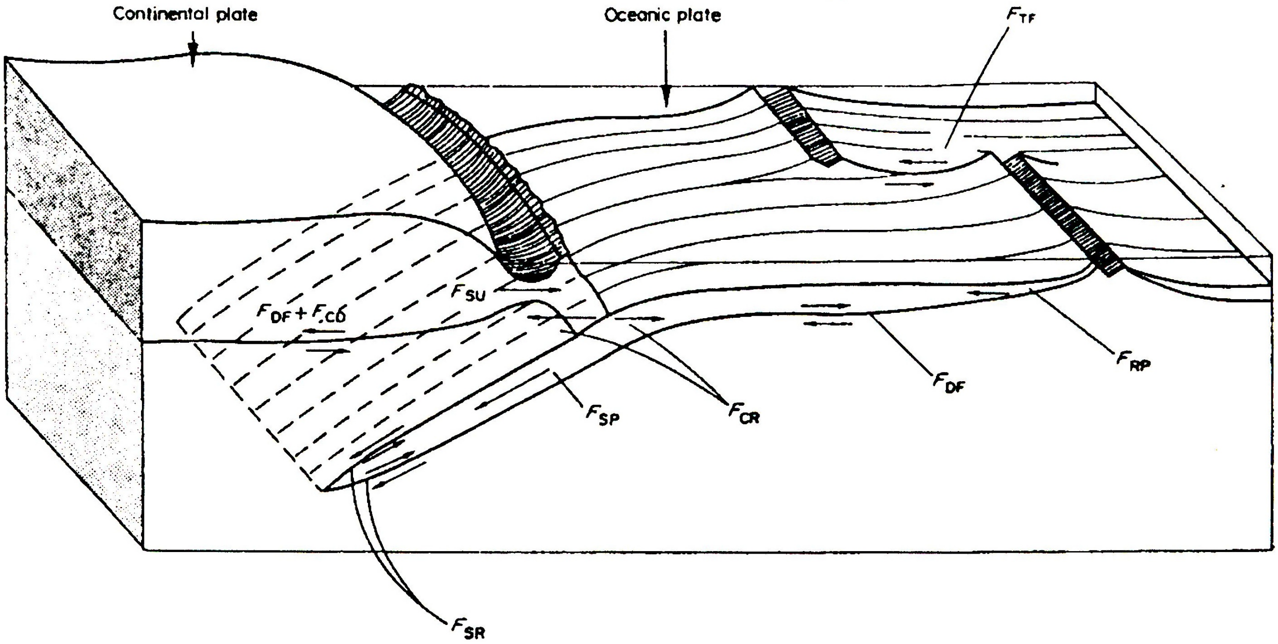

Figure 1: A summary of the different forces acting on plates. The most important driving forces are those of ridge push (RP) and slap pull (SP) which must balance the resistive viscous drag forces (here labelled DF (drag force) and SR (slab resistance)).

The convective circulation that is mapped by using the relationship between the gravity field and residual depth cannot maintain the motions of the larger plates, because there are many rising and sinking regions beneath them, spaced 100 to 1500 km apart. But the plate movements are themselves a form of convection. Beneath island arcs the sinking slab advects cold material into the mantle, and the separation of plates at ridge axes transports hot material upwards. Are the resulting forces sufficient to maintain the motions? This question can be examined in two ways. The first, and most obvious, is to start from the governing equations, include the temperature dependence of the viscosity (without which there will be no rigid plates), and solve the equations on large machines for parameter ranges similar to those of the Earth. An enormous effort has been put into such efforts over the last 30 years, but the results have so far been disappointing. No-one has yet produced numerical solutions of these equations that look anything like the observed plate motions. There are two major problems. The sinking regions in the numerical calculations are plume-like, and are therefore unlike the planar slabs we see beneath island arcs. If the calculations are restricted to two, rather than three, dimensions, so that planar structures are forced to occur, material from both sides of the island arc enters the sinking region, rather than coming from just one side as it does on Earth. It is likely that it is the behaviour of the faults that form the plate boundaries which is responsible for the failure of the numerical solutions to reproduce what we see. The faults on which earthquakes occur are weak, and remain weak even if movement on them ceases. They behave like glass scratched with a diamond, which will always break where it was scratched. If you introduce a weak plane into a two dimensional convection model you can produce asymmetric subduction, but then you have forced the solution to behave in the way that you know it does, and have learnt little. So progress in this area has been slow.

It is, however, easy to show that the thermal structure of ridges and trenches can maintain the observed motions. The model used for this purpose has rigid plates moving on top of mantle with constant viscosity, and all the temperature variation occurs within the plates. The driving forces are then easy to calculate, as is the return flow from trenches to ridges. Since the momentum of the plate motions is unimportant, the driving forces must be exactly balanced by the resistive forces at all times.

The idealised model has two forces driving the plates: ridge push and slab pull (Figure 1). That slabs pull is clear, but it is perhaps less obvious that a compensated ridge exerts a force on the plate, because the pressure at all depths above the base of the plate is greater than it is beneath old plate, falling to zero at the base of the plate. A good exercise in your understanding of isostasy is to show that the mean pressure ΔP exerted by a ridge is

$$\Delta P = \frac{1}{6}\alpha(T_1 - T_0)\rho_{\mathrm{m}} g d\,(1 + \mathcal{O}\,(e/d)) \tag{1}$$

where ρ_{m} is the mantle density at 0°C, α the thermal expansion coefficient, T_0 is the surface temperature, T_1 the temperature at the base of the plate of thickness d, g is the acceleration due to gravity and e the elevation of the ridge above old sea floor. Substitution gives $\Delta P \simeq 20$ MPa.

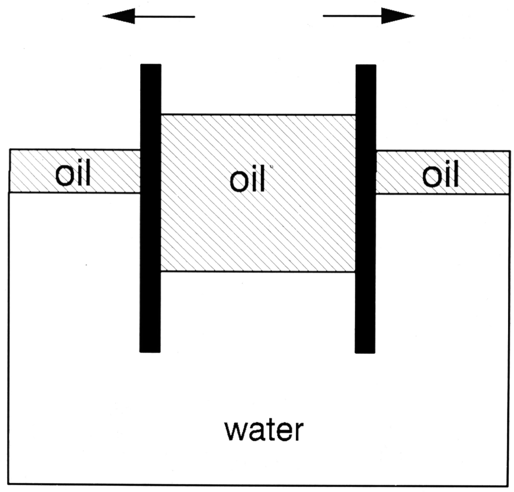

Figure 2: An analogue model of the ridge push force. Oil is floated on water. The two vertical baffles are pushed apart due the pressure difference between the thick and thin layers of oil.

It is important to realise that slab pull is also a convective force, a pull due to the negative buoyancy of the sinking slab. If the slab does not break the stress it exerts on the plate is about an order of magnitude larger than ridge push ($\sigma_{\mathrm{SP}} \sim 200$ MPa). However, this is unlikely to be an accurate estimate of the stress, because there is some, as yet not very good, evidence that the normal faults which form when the subducting plate bends as is moves beneath the overriding plate cut right through the plate. It is therefore likely that slab pull cannot exceed the frictional stress that can be transmitted from one side of a fault to the other, or about 10-20 MPa.

The importance of slab pull is illustrated in Figure 5, which suggests that the velocities of plates are independent of the area of the plates, independent of the ridge length but strongly dependent on the length of the sinking slab attached. Though the number of plates is too small for it to be possible to carry out proper statistics, these observations strongly suggest that slab pull is the major driving force of plate tectonics.

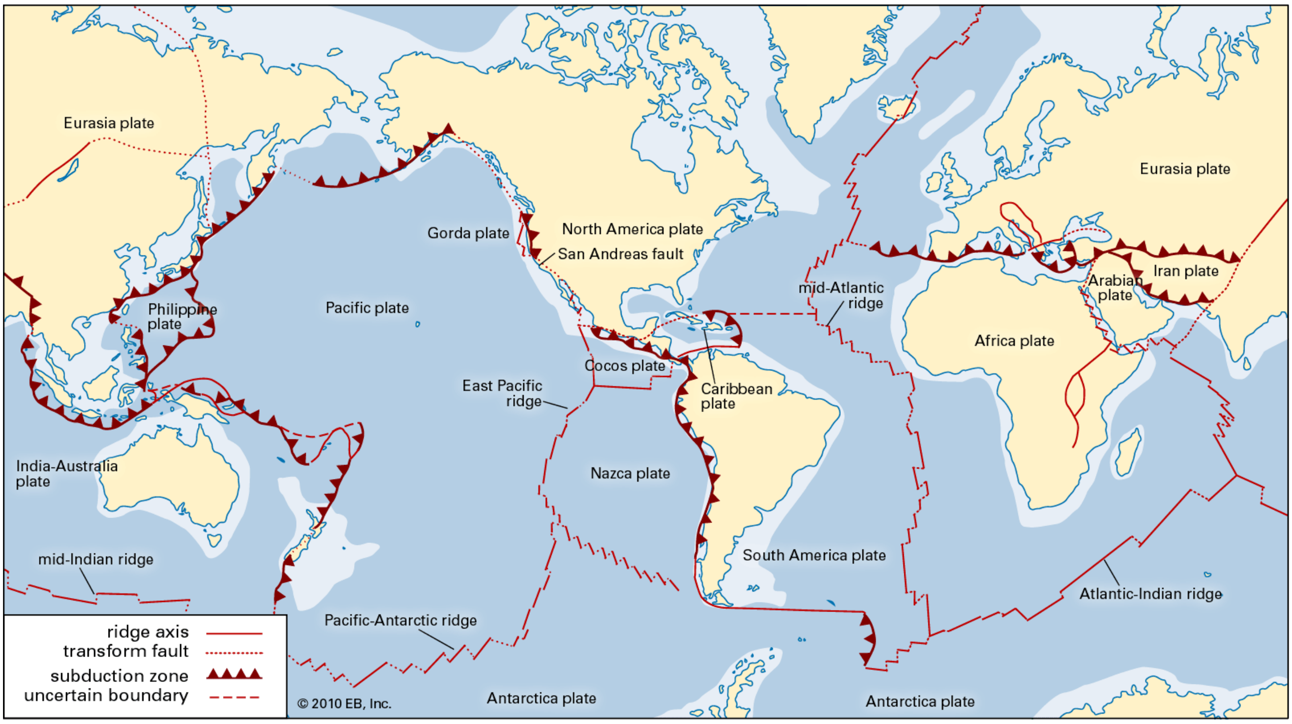

Figure 3: Plate boundaries.

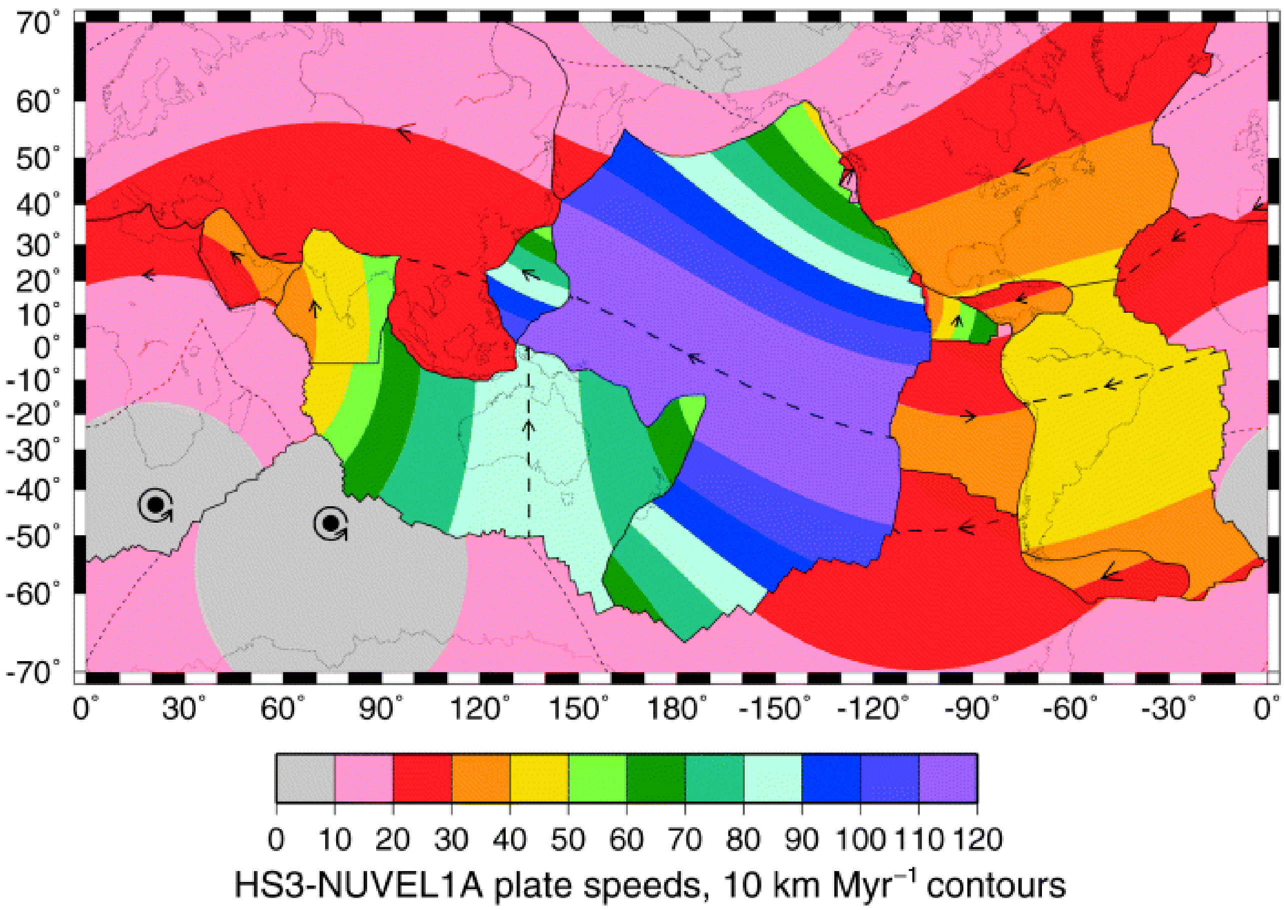

Figure 4: Plate velocities with reference to the hotspots: HS3-NUVEL-1A. Solid black circles, Africa and Antarctica rotation poles, which are each located within their plate; dashed lines, 'equators' for the poles of rotation.

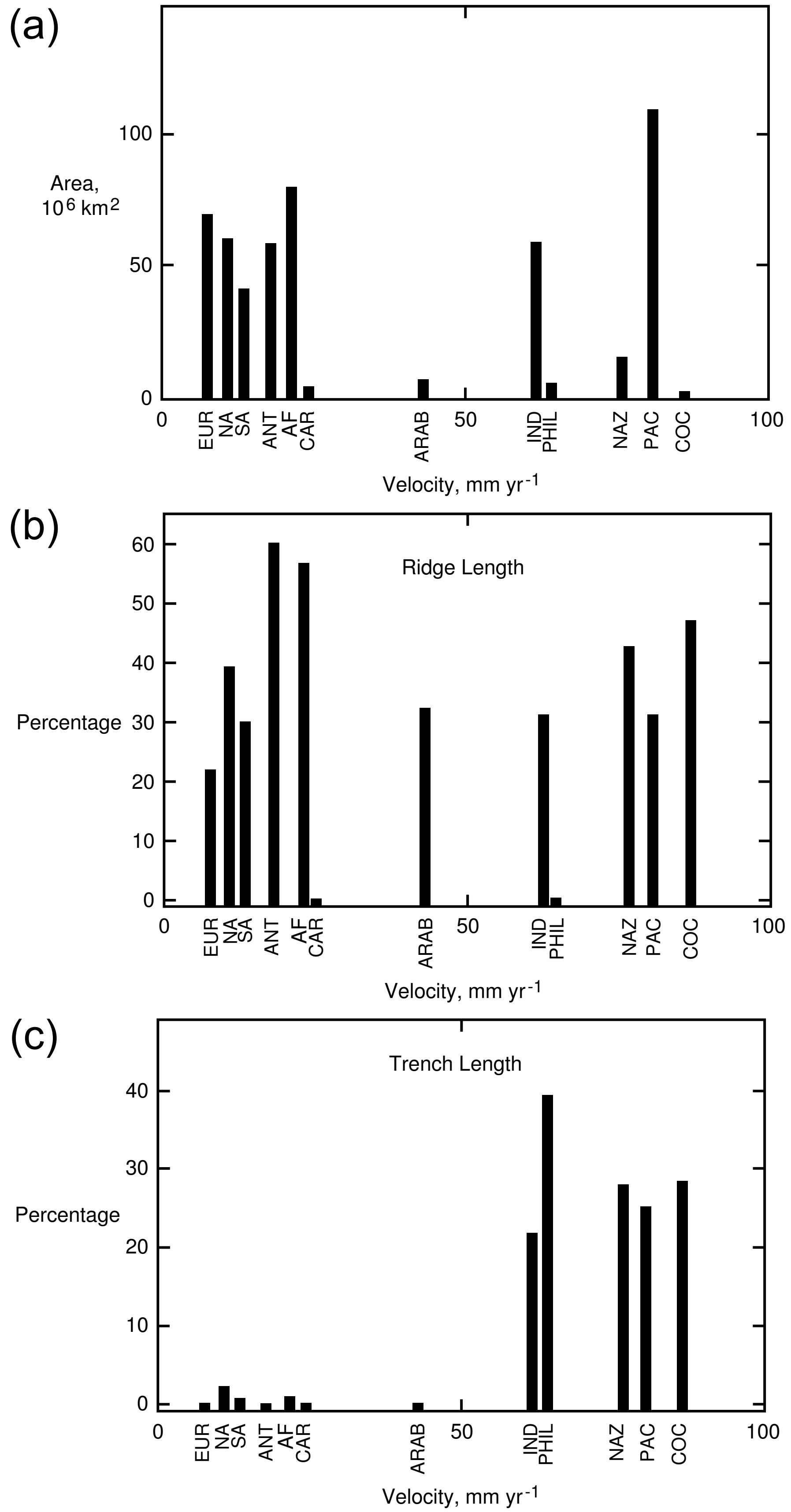

Figure 5: Plate statistics. Plate velocity versus (a) plate area, (b) percentage of perimeter that is ridge and (c) percentage of perimeter that is connected to a downgoing slab. EUR, Eurasia; NA, North America; SA, South America; ANT, Antarctica; AF, Africa; CAR, Caribbean; ARAB, Arabia; IND, India; PHIL, Phillipine; NAZ, Nazca; PAC, Pacific; COC, Cocos.

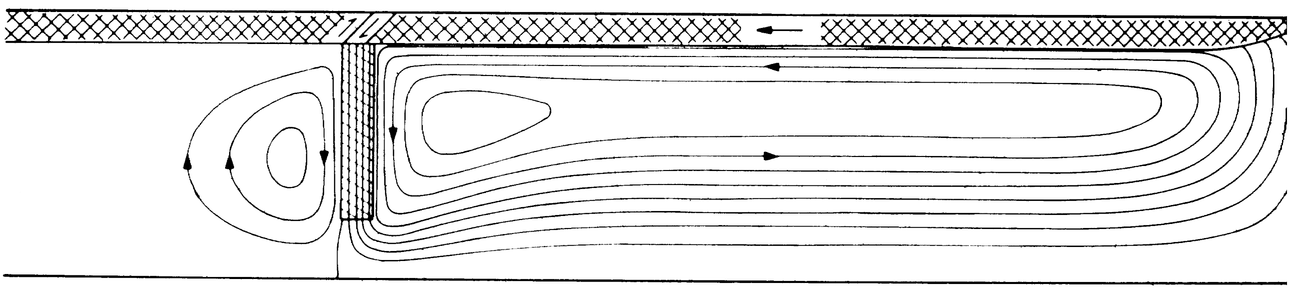

Figure 6: A simple plate model. The plate on the right is assumed to be moving and subducting at a uniform rate. The non-subducting plate on the left is in this case stationary. The resulting flow under the plates is given by the streamlines, which show the return flow under the moving plate and a local circulation, induced by the downgoing slab, under the non-subducting plate.

The magnitude of the resistive forces can be estimated by solving for the viscous flow driven by the plate motion. Figure 6 shows a simple model of flow beneath a plate. Since the driving forces are known, and must equal the resistive forces which are proportional to the viscosity, the force balance can be used to estimate the mantle viscosity. Figure 7 shows such estimates for two plates A and B. A moves at 100 mm a^{-1} and has a length of 10,000 km, whereas B moves at 60 mm a^{-1} and has a length of 6,000 km. These values are similar to those of the Pacific and Indian plates respectively.

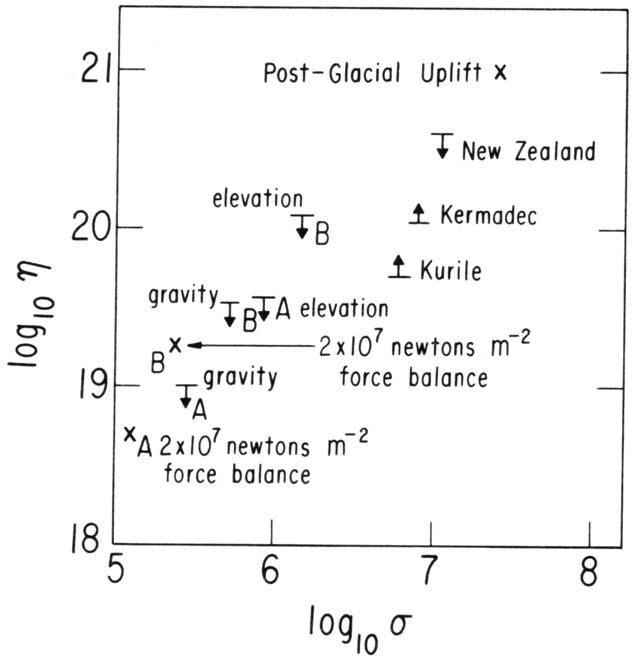

Figure 7: Estimates of viscosity (Pa s) and stress (Pa) for a mantle of constant viscosity. Crosses mark estimated values, horizontal lines with arrows mark bounds; upper bounds if the arrow points downwards, lower if it points upwards. A and B refer to two plate models: estimated values are given based on a force balance (with a maximum driving stress of 20 MPa), bounds are based on gravity and elevation. The bounds marked New Zealand, Kurile, and Kermadec are based on the stress states observed in these slabs from seismology. The estimated mantle viscosity from post-glacial uplift is also shown.

The viscosity of the upper mantle can be estimated from a number of observations. There must be a pressure difference between trenches and ridges that moves the mantle material back to the ridges. Since the horizontal scale of the plates is so large, such a pressure difference must be expressed by a difference in elevation, with the trenches being elevated above the ridges. There must also be a corresponding gravity anomaly. Neither are seen, and the upper bounds in Figure 7 marked 'gravity' and 'elevation' are calculated by assuming that such gravity anomalies are less than 10 milligals and the elevation difference is less than a kilometre. The figure shows that the mantle viscosity estimated from the force balance is about two orders of magnitude smaller than that from postglacial uplift. The viscosity is plotted as a function of stress to discover if the estimates decrease as the stress increases, as they would if dislocation creep was involved. They do not.

The most likely reason that the viscosity estimates are so variable is that plate movement is lubricated by a thin low viscosity layer beneath the plates, which allows them to move without dragging the whole of the upper mantle with them.

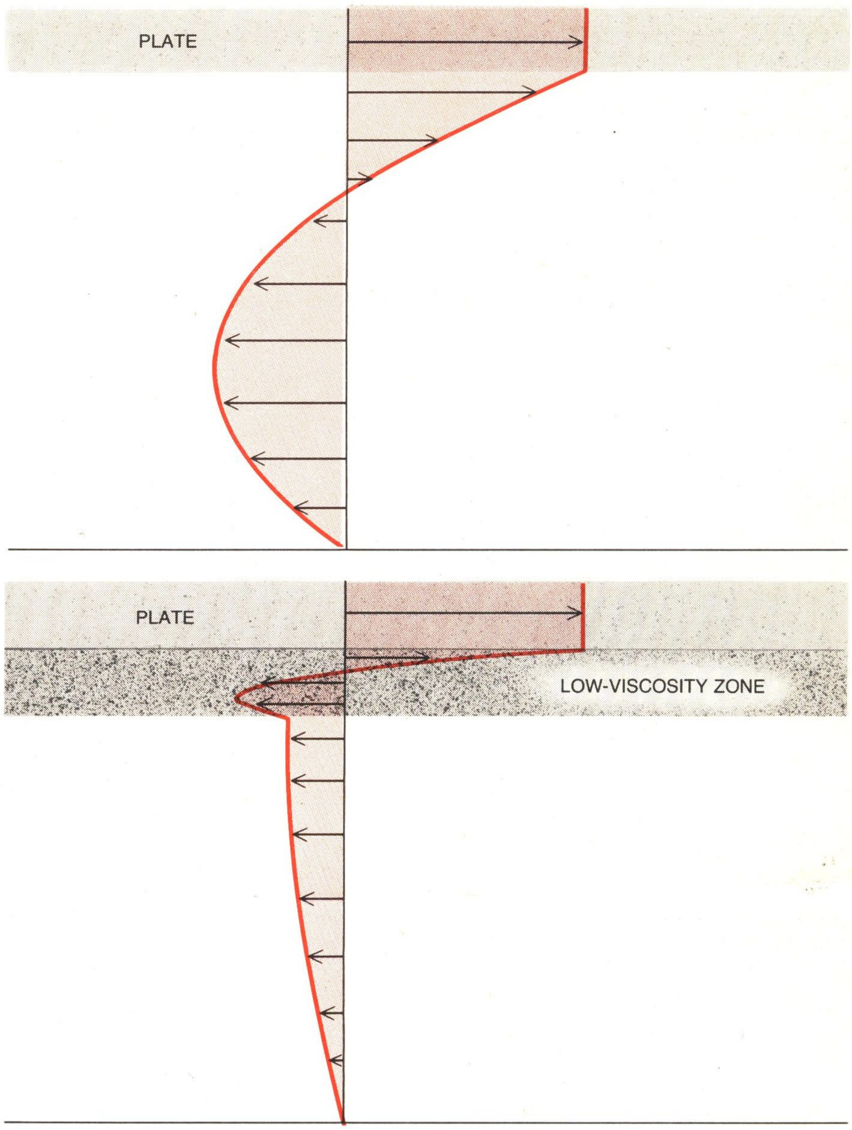

Figure 8: Flow under plates may follow either of two general schemes. In one model (top) the viscosity of the material under the plates is uniform. Thus a considerable layer of fluid is swept along by the plate motion. In order for mass to be conserved there must be a strong reverse flow deeper in the mantle. If, however, there is a thin layer of low-viscosity material under the plates (bottom), surface motions are decoupled from mantle and basically only the mass of the plates themselves need be carried by the return flow. Geophysical observations favour the decoupled model.

The presence of such a low viscosity layer can explain why the plate velocities are not dependent on their areas. Since the homologous temperature is highest just below the plate, this is where the viscosity is expected to be lowest.

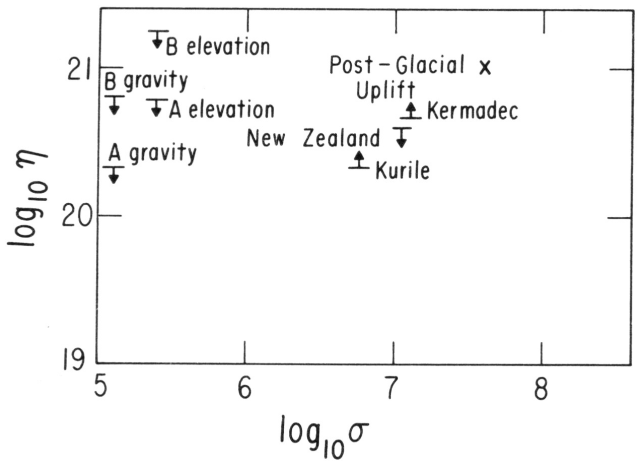

Figure 9: Estimates of viscosity (Pa s) and stress (Pa) for the part of the mantle below a low viscosity layer. Symbols have same meaning as Figure 7.

The same observations as before can be used to estimate the viscosity of the upper mantle below the low viscosity layer, and give estimates similar to that from postglacial rebound (Figure 9). It is harder to estimate the viscosity and thickness of the low viscosity layer, but probable values are $10^{18} - 10^{19}$ Pa s and 100 km. This model shows that the convective driving forces associated with the thermal structure of ridges and trenches are sufficient to maintain the observed plate motions. However, what they clearly cannot do is start the motions or create new plate boundaries. New ridges are seen to start in two ways. Spreading ridges often end on a large transform fault. The corner produces a stress concentration, and the plate breaks, and so increases the length of the ridge. This process happened many times in the central and North Atlantic, as the ridge extended northwards.

The other process that can start a new ridge is the uplift associated with plumes. In East Africa two plumes, beneath Kenya and the Afar, have uplifted the plate by at least a kilometre. Doing so causes the plate to be in extension, as it tries to slide off the top of the plumes. The resulting stresses are $\sim$ 20 MPa, or similar to the other stresses that act on plates. Such stresses are sufficient to produce a new plate boundary in East Africa.

In contrast, starting new trenches is much more difficult, and requires a force to push two parts of an existing plate together at more than 20 mm a^{-1} for about 10 Ma before slab pull exceeds the frictional resistance on the thrust faults. Instabilities of this type, which only are self sustaining when the system is displaced from equilibrium by a finite amplitude disturbance, are known as *finite amplitude instabilities*. Very few examples of trench formation are known, and in all cases they occur on a small part of a plate boundary between two large plates, presumably because the necessary forces can then be supplied by other parts of the plate boundaries.

The Earth's thermal history

	TW	TW
Oceanic Heat Loss (300×10^6 km^2)	32	30–34
Continental Heat Loss (210×10^6 km^2)	14	13–15
Total Surface Heat Loss (510×10^6 km^2)	46	43–49
Radioactive sources (mantle+crust)	20	17–23
Net loss	26	20–32

Table 1: Present day whole Earth energy budget: preferred value and range.

	TW	TW
Continental heat production (crust + lith. mantle)	7	6–8
Total heat flow from convecting mantle	39	35–43
Radioactive heat sources (convecting mantle)	13	9–17
Heat from core	8	5–10
Total input	21	14–27
Net loss (mantle cooling)	18	8–29

Table 2: Present day mantle energy budget: preferred value and range.

All the discussion so far has been concerned with present plate motions. It is obviously of interest to discover whether the tectonics of the Earth has always been controlled by plate motions, and whether there have been any changes in the past. Neither of these questions is easy to answer, because the oldest part of the oceans is only 0.18 Ga old, compared with 3.9 Ga for the oldest continental rocks, or 4.5 Ga for the age of the Earth. So the early history can only be reconstructed from the continental rocks which is hard to do.

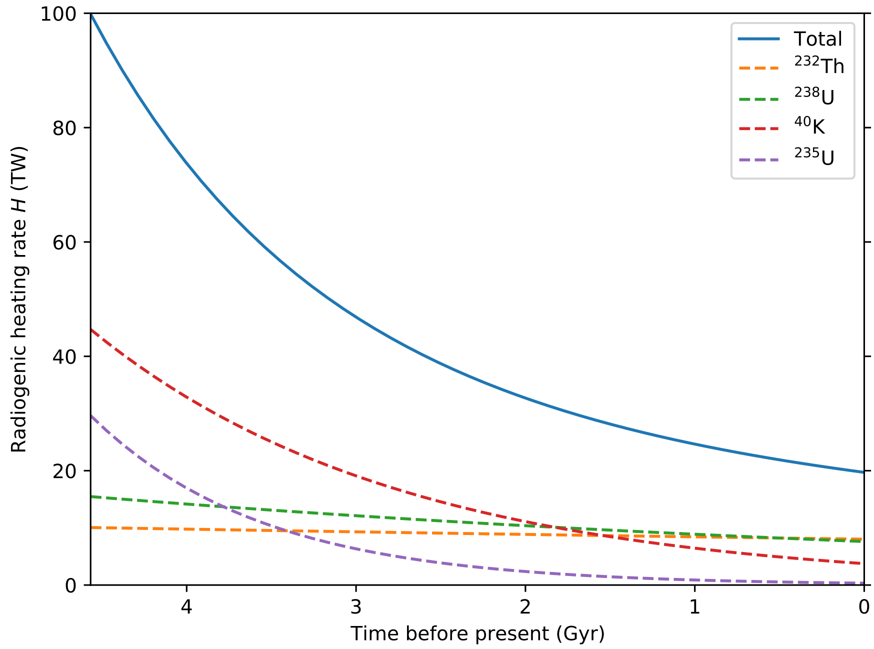

Figure 10: Rate of radiogenic heat production within the Earth over time. The blue curve shows the total heat production; dashed lines show the individual contributions from the four main heat-producing isotopes.

The loss of heat from the Earth is controlled by plate motions, and the boundary layer description of high Rayleigh number convection can be used to explore what happens. An important source of heat is radioactive decay, which has decreased with time (Figure 10). The total heat generation rate can be estimated from geochemical arguments, and is 20 ± 3 TW. The dominant form of heat loss from the Earth is plate production on ridges, which accounts for about 70% of the total. Since the rate of heat loss from ridges is easily calculated from the plate model, its value is constrained by bathymetry as well as heat flow measurements. Current estimates give 46 ± 3 TW for the total heat loss from the Earth . Because the heat loss is about twice the heat generation rate, the Earth must be cooling (Table 1). How long it has been doing so depends on the plate production rate in the oceans earlier in its history, about which nothing is yet known.

Thermal history models

A simple way of modelling the Earth's thermal history is through the use of parametrised convection models. These models start from a global statement of conservation of heat for the mantle of the form

$$\rho C_p V \frac{\mathrm{d}T}{\mathrm{d}t} = \rho V h(t) - A q(T), \tag{2}$$

where ρ is the mantle density, C_p is the heat capacity, V is the volume of the mantle, T is the mean temperature, $h(t)$ is the radiogenic heating rate per unit mass (W kg^{-1}), A is the surface area, and $q(T)$ is the surface heat flux (W m^{-2}). For simplicity we neglect here the heat supplied to the base of the mantle by the core. The models rely on a parametrisation of the heat flux $q(T)$ as a function of the mean temperature T. We expect that hotter mantle temperatures lead to greater convective vigour, and thus higher heat losses. The simple boundary layer theory for a constant viscosity mantle predicts that $q(T)$ increases with T as $q(T) \propto T^{4/3}$.

It is helpful to write (2) in a slightly more compact form by introducing an overall mantle heat capacity $C = \rho C_p V$ (with units J K^{-1}), an overall radiogenic heating rate $H(t) = \rho V h(t)$ (in W), and an overall heat loss $Q(T) = A q(T)$ (in W). Equation (2) becomes

$$C \frac{\mathrm{d}T}{\mathrm{d}t} = H(t) - Q(T). \tag{3}$$

In general we expect a nonlinear relation between $Q(T)$ and T, and numerical solutions to equation (3) are often studied. However, to make the analysis here analytically tractable, let us assume that the relationship between $Q(T)$ and T is linear in the form

$$Q(T) = Q(T_0) + (T - T_0)\gamma, \tag{4}$$

for some constant γ, and some reference temperature T_0. This is a leading order Taylor series expansion about the reference temperature T_0, where $\gamma = \frac{\mathrm{d}Q}{\mathrm{d}T}(T_0)$. Note that

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \gamma \frac{\mathrm{d}T}{\mathrm{d}t}, \tag{5}$$

so the heat balance (3) can be written as

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \mu H - \mu Q \tag{6}$$

where

$$\mu = \frac{\gamma}{C}. \tag{7}$$

If H is constant, then (6) has a steady state where the rate of heat loss exactly balances the rate of heat production: $Q = H$. Moreover, any departure from steady state will relax back to steady state, undergoing exponential decay on a convective response time-scale $1/\mu$.

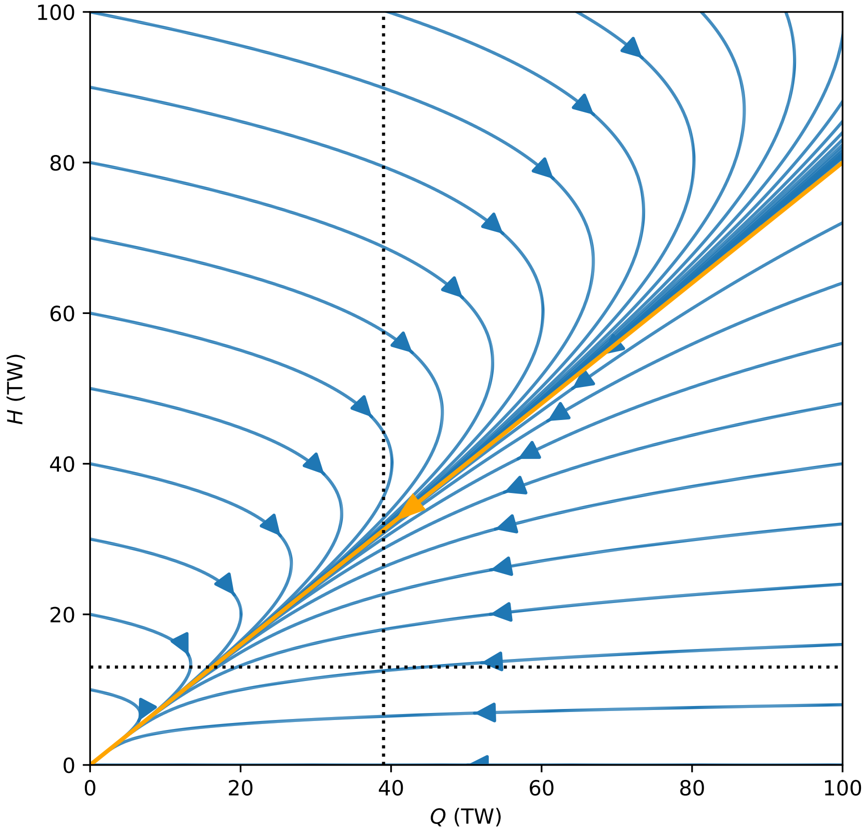

Figure 11: Solution trajectories for the ODEs given in equations (8) and (9) with convective response time $1/\mu = 1.0$ Gyr and radiogenic decay time $1/\lambda = 5.0$ Gyr. All solution trajectories approach the limiting trajectory shown in orange, which has $\mathrm{Ur} \equiv H/Q = 1 - \lambda/\mu = 0.8$. Estimates of the present-day mantle values of $H = 13$ TW and $Q = 39$ TW are shown by the dotted lines.

Decaying heating rate

As we have seen in Figure 10, the radiogenic heating rate is not constant in time, but has varied significantly over Earth's history from around 100 TW at the start of the solar system to 20 TW at the present day. A simple way of approximating the decreasing radiogenic heating rate is as an exponential, $H = H_0 \exp(-\lambda t)$ for some initial heating rate H_0 and effective decay constant λ. In detail the radiogenic heating rate depends on a sum of exponential decays for the individual heat-producing isotopes, each with their own decay constants. However, a single exponential demonstrates the main behaviour. With an exponentially decreasing heat production, the thermal history can be described by a set of constant coefficient, coupled first-order ODEs,

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\lambda H, \tag{8}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \mu H - \mu Q. \tag{9}$$

Typically $\mu > \lambda$, i.e. the convective response time is short compared with the time-scale over which the heating rate changes. The ODEs (8) and (9) can be straightforwardly integrated. Example solution trajectories are shown in Figure 11.

Geophysicists typically discuss thermal history models in terms of the *Urey ratio*, defined as the ratio of the radiogenic heating rate to the heat loss,

$$\mathrm{Ur} \equiv \frac{H}{Q}. \tag{10}$$

An important feature of the system described by (8) and (9) is that regardless of initial conditions, all solutions tend to a constant value of the Urey ratio over time, given by

$$\mathrm{Ur} \to 1 - \frac{\lambda}{\mu}. \tag{11}$$

The system is self-regulating. An initially hot Earth will have initially high heat losses and quickly cool down. An initially cold Earth will warm up due to the radiogenic heating before later cooling down as the radiogenic heating rate reduces. If the convective response time is very short compared to the effective radioactive decay time ($\mu \gg \lambda$) then the system comes to a balance where the heat production balances the heat loss and Ur ≈ 1. However, if the convective response time is longer then the heat losses will lag the changes in heating rate and the system tends to a state where Ur < 1.

A typical estimate of the mantle Urey ratio is Ur ~ 0.3, with the mantle losing heat about three times faster than it is produced by radioactive decay. This Urey ratio value is difficult to reconcile with simple convective models which typically have a convective response time $1/\mu \sim 1.0$ Gyr. Since at the present day $1/\lambda \sim 5.0$ Gyr, from (11) we would predict a Urey ratio for the Earth's mantle around Ur ~ 0.8.

There is much debate in the geophysical community as to why the estimates of Urey ratio based on observations are so different from those of the convective models. One might try to appeal to a particular initial condition that yields a present day Ur ratio of 0.8. However, such an initial condition would have to be so hot that much of the mantle would be molten only 1–2 Gyr ago (a feature known as a thermal catastrophe; the rock record rules out such high temperatures).

It may be that heat losses from the Earth are significantly affected by layering in the convective system (e.g. with barriers to material transport between the upper and lower mantle). A layered convective system loses heat less readily than a non-layered system (think double glazing), and this could increase the effective convective response time $1/\mu$, and thus reduce the Urey ratio. However, a layered system may also lead to unrealistically high temperatures in the lower mantle. Alternatively, it may be that the parametrisations for $Q(T)$ commonly used are simply not appropriate to the plate-tectonic style of mantle convection, and there is much work being done to develop alternative parametrisations.

The Age of the Earth

These calculations of the Earth's heat loss are relevant to one of the oldest controversies in geophysics. In the nineteenth century Lord Kelvin estimated the age of the Earth to be about 20 Ma. His argument was simple. He knew the temperature gradient at the surface, from measurements of temperature as a function of depth in mines. He used a model for the Earth where all heat was transported by conduction, and started the calculation with a high initial temperature. He found that he could only produce the observed temperature gradient if the age of the Earth was about 20 Ma. The actual value depends a little on the initial temperature, but all such calculations gave values that were much smaller than the geologists then believed. He carried out a similar calculation for the age of the Sun. He estimated how long the present rate of heat loss could continue before the temperature of the whole body decreased, and obtained an estimate that was similar to his age for the Earth. His simple arguments had enormous influence. Though the geologists thought 20 Ma was far too short a time for the evolution of animals and plants, their own estimates were less soundly based than was that of Kelvin's.

It is often stated that it was Rutherford's discovery of radioactivity that showed that Kelvin's argument was incorrect. This statement is not true: what was wrong with Kelvin's argument was his method of heat transport. The heat capacity of the Earth can sustain the present rate of heat loss for periods of a few Ga without any heat sources. If Kelvin had used the same argument for the Earth as he did for the Sun (as essentially we have done) he would have obtained an age of a few Ga, which is similar to our present beliefs. For the Sun, however, he was entirely wrong, because he was not aware that heat could be produced by thermonuclear reactions.

For a very accessible account of the controversy surrounding Kelvin's age of the Earth, I highly recommend:

England P., Molnar P., Richter F. (2007) *John Perry's neglected critique of Kelvin's age of the Earth: A missed opportunity in geodynamics.* GSA Today 17(1), doi:10.1130/GSAT01701A.1