Modern cosmological data

Relativistic Astrophysics and Cosmology: Lecture 21

Sandro Tacchella

Wednesday 22nd November 2023

Pre-lecture question:

How do we measure the parameters of our Universe?

Last time

- The constituents and timeline of the universe

This lecture

- Supernovae
- The cosmic microwave background
- Baryon Acoustic Oscillations

Next lecture

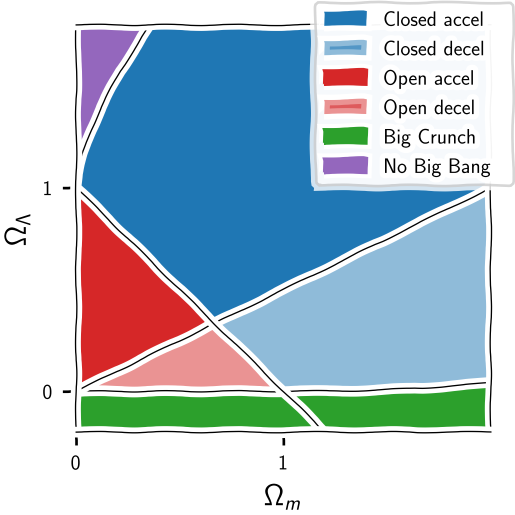
- The primordial universe & inflation

Cosmological data

- In this lecture we aim to cover some of the primary tools we use to constrain our cosmological models:
  - Cosmic Microwave Background Anisotropies,
  - Supernovae cosmology,
  - Baryon Acoustic Oscillations.
- This is necessarily abbreviated, and in particular we won't have time to cover
  - 21cm cosmology,
  - Weak lensing,
  - Strong lensing,
  - Large scale structure,
  - Standard sirens (gravitational waves),
  - Cosmic chronometers.

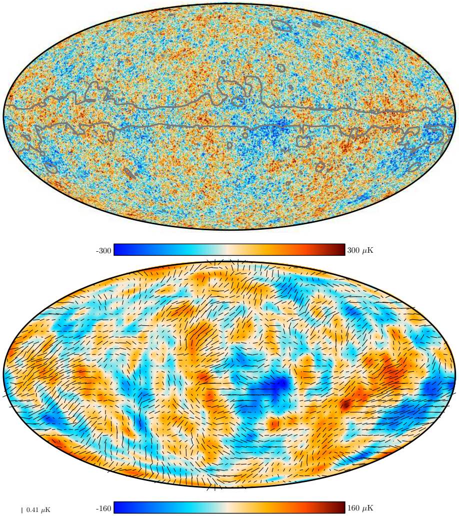
What are the parameters of our Universe?

- Assume that $\Omega_r \approx 0$, so universe completely defined by place in $(\Omega_m, \Omega_\Lambda)$ plane.
- Flat Λ dividing line at $\Omega_\Lambda + \Omega_m = 1$
- Acceleration line at $\Omega_\Lambda - \frac{1}{2}\Omega_m > 0$
- EdS at $\Lambda = 0$, dS at (0,1).
- If Λ is large enough, for closed universes we get a "bounce" and no big bang.
  $\Omega_\Lambda \geqslant 1 - \Omega_m + \frac{3}{2}\left(\Omega_m^2 X^{-1} + X\right)$
  where complex $X = \sqrt[3]{\Omega_m^2 - \Omega_m^3 + \sqrt{\Omega_m^4 - 2\Omega_m^5}}$
- If $\Omega_m < 1$ then negative Λ gives a big crunch. If $\Omega_m \geqslant 1$, then closed universes can sometimes recollapse, even with $\Lambda > 0$
  $\Omega_\Lambda \leqslant 1 - \Omega_m - \frac{3}{2}\left(\frac{1-\sqrt{3}i}{2}\Omega_m^2 X^{-1} + \frac{1+\sqrt{3}i}{2}X\right)$
- The question is: where are we?



The cosmic microwave background anisotropies

- The cosmic microwave background is the start location of photons that have free streamed since the epoch of recombination.
- Images of the CMB are effectively images of the surface of last scattering.
- Plots here show the anisotropies, i.e. what is left over after subtracting off the constant and velocity terms (monopole and dipole), as well as the galaxy.
- Amplitude and polarisation are shown.
- First thing to notice about these anisotropies is that they are noisy i.e. random.
- The second less obvious thing is the randomness is not white noise.



Fourier transforms and power spectra

- We may represent any function by its Fourier decomposition

$$f(x) = \int \tilde{f}(k) e^{ikx} \frac{dk}{2\pi} \qquad \Leftrightarrow \qquad \tilde{f}(k) = \int f(x) e^{-ikx} dx.$$

- For many physical processes particularly where f is inherently random, the power spectrum, given by $\boxed{P(k) \propto |\tilde{f}(k)|^2}$, is where signals express themselves.
- For concrete examples of this, think about
  - an electromagnetic spectrum (e.g. the cones in your eye being excited by specific "wavelengths"),
  - an acoustic spectrum (e.g. different thickness hairs in your cochlear being excited by different frequencies of sound).
- Both ears and eyes act to compute a Fourier power spectrum in a waveband.
- As do astronomical spectrometers.
- Statistical surveys generally compress their measurements of galaxies/anisotropies into a power spectrum to extract science (specifically by computing "two-point functions" of the data).

Fourier transforms on the sky

- If we are working on the sky, then we need to use the Fourier transform appropriate for a sphere, which are the same spherical harmonic functions you first met in quantum mechanics/chemistry:

$$T(\theta,\phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi) \qquad \Leftrightarrow \qquad a_{\ell m} = \int d\Omega\, Y^*_{\ell m}(\theta,\phi)\, T(\theta,\phi).$$

- As with any Fourier transform, the Fourier coefficients $a_{\ell m}$ are in general complex, although reality of T imposes $a^*_{\ell m} = a_{\ell -m}$.
- As with the Fourier transform "small $k \leftrightarrow$ large x" becomes "low $\ell \leftrightarrow$ large angle".
- We know the monopole $a_{00} = T_0 = 2.7255(6)\,\mathrm{K}$.
- The dipole, measured by components $(a_{1,-1}, a_{1,0}, a_{1,+1})$ has magnitude $3.362(1)\,\mathrm{mK}$, and as discussed in Lecture 16 comes from our non-fundamental motion relative to the underlying comoving frame.
- The remaining coefficients (quadrupole, octopole and beyond) are assumed primordial, and are measured to be tens of μK.

The CMB power spectrum

- The power spectrum for a spherical decomposition is therefore $|a_{\ell m}|^2$.
- Current cosmological theories however predict that the $a_{\ell m}$ are inherently random, and for the most simple theories (more on this later) are Gaussian random variables with variance C_ℓ, where C_ℓ is determined by theory.
- We can therefore compute the observed C_ℓ power spectrum by averaging over each multipole ℓ

$$C_\ell = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} |a_{\ell m}|^2.$$

- We can see statistical isotropy at work, since the C_ℓ's are independent of direction index m.
- Important to note the degree of compression: Gb's of timestream data $\rightarrow$ millions of map pixels/$a_{\ell m}$'s, $\rightarrow \sim 2000$ C_ℓ's

$$N_{\text{pix}} = \sum_{\ell=0}^{N}(2\ell+1) = (N+1)^2.$$

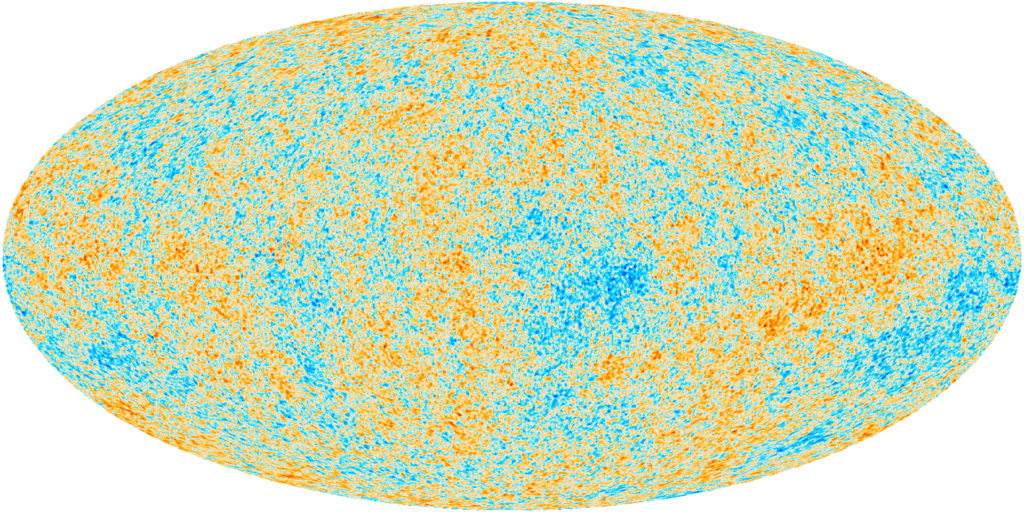
Cosmic variance

- We can also see another critical concept here.
- The theoretical Gaussian coefficients are random, with variance C_ℓ, i.e. $a_{\ell m} \sim 0 \pm \sqrt{C_\ell}$.
- This means that when we estimate the measured spectrum coefficients from the sample average

$$\frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 \sim C_\ell \pm \sqrt{\frac{C_\ell}{2\ell+1}}.$$

- We are left with an intrinsic scatter that gets worse with smaller ℓ (i.e. larger spatial scale).
- This degradation in signal recovery at large scales is called cosmic variance, and stems in essence from the fact that we only have one sky to measure.
- In principle, we could get access to more data by using the fact that other galaxies see different skies, so backscattered CMB measured in a statistical fashion might allow us to get round this – but this is a long way off being feasible at the moment!

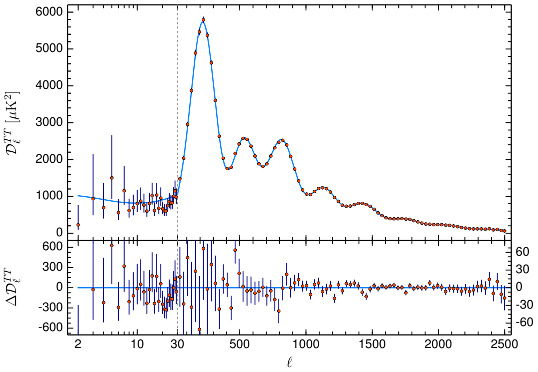
The measured CMB map



- Let's take this theory of noise on the sphere and apply it to our CMB sky.

The measured CMB power spectrum

- This is the power spectrum of the CMB anisotropies, rescaled to $\mathcal{D}_\ell = C_\ell \frac{\ell(\ell+1)}{2\pi}$.
- Note the cosmic variance at low-ℓ, and the upper end of Planck's resolution at high-ℓ.
- The largest peak is the "patchiness" that the eye picks out.
- If the noise were "white" then this power spectrum would be flat $\sim \ell(\ell+1)$.



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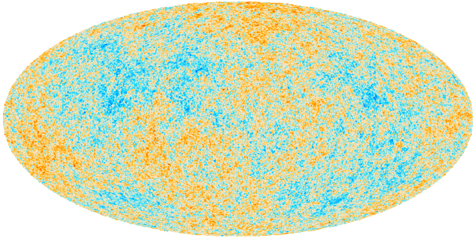
Simulated Planck



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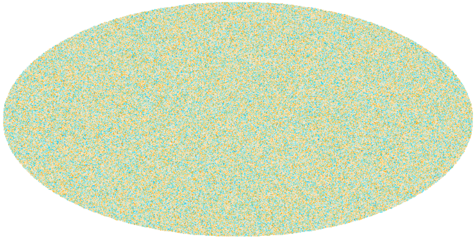
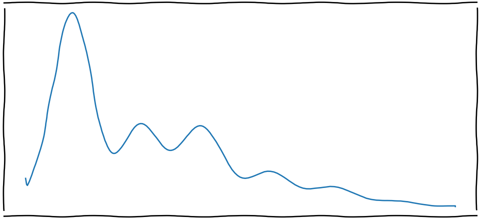
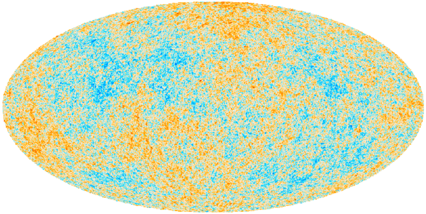
White noise



Why is the CMB power spectrum useful?

- Cosmological parameters Ω_c, Ω_b, Ω_Λ, τ, H_0 influence the location and position of the "acoustic peaks".
- The CMB power spectrum therefore acts like a barcode for our universe's properties.
- A key predicted feature is the decaying series of peaks at the right of this picture, starting at $\ell \sim 200$, or angular scale $\sim 1^\circ$.
- The peaks correspond to plasmatic sound waves at recombination: coupled acoustic oscillations of the coupled photon/matter fluid during recombination.





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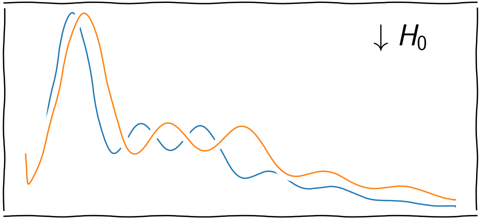

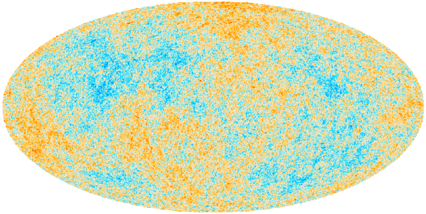

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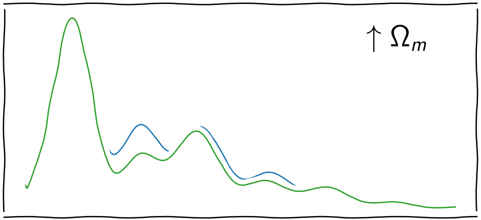

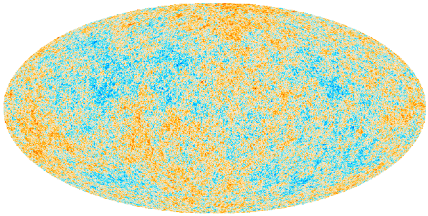

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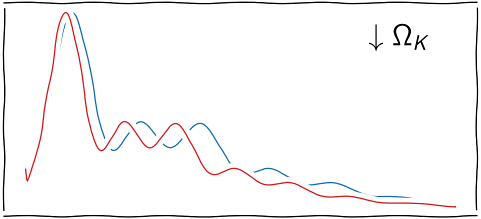

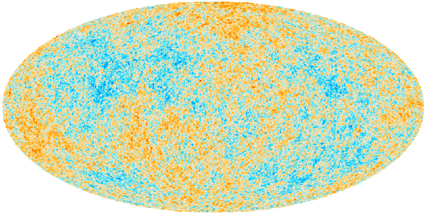

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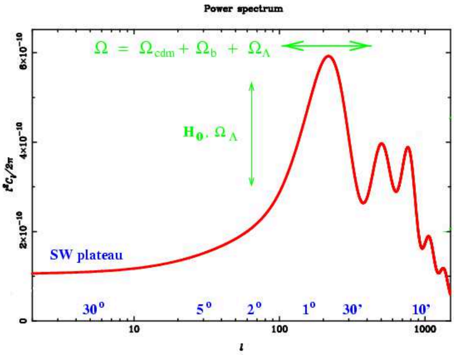
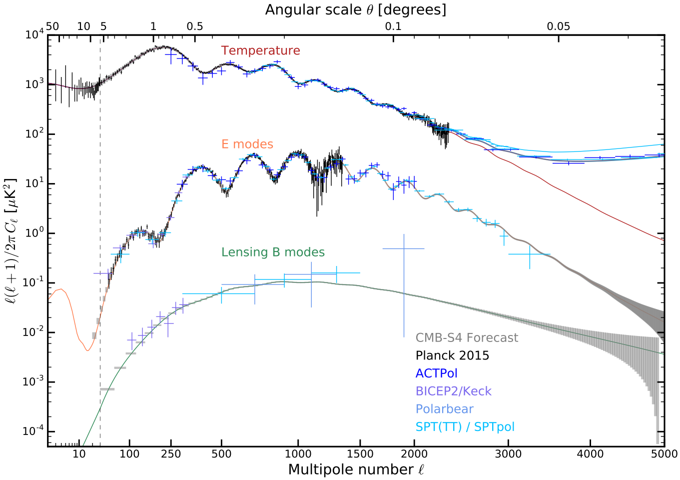


- Different quantities of dark/normal matter/curvature/dark energy in this period change the sound speed and thus the relative importance of different scales.
- Smaller/larger Ω pushes the peaks to the right/left.
- Thus by comparing what we find in experiment with the predictions (such as in the plot, which is for total Ω equal to 1), we can find the value of Ω, and thus whether the universe is open, closed or flat.
- The flatter region at the left, corresponding to where the COBE measurements were made, is called the Sachs-Wolfe plateau, and helps fix the overall normalisation.
- This is also the region where the reprocessing effects on the primordial spectrum which turn it into the CMB power spectrum during recombination are minimal and where we get a direct glimpse of wholly primordial processes.

- Previously shown the TT power spectrum.
- Can also compute the covariance between different components

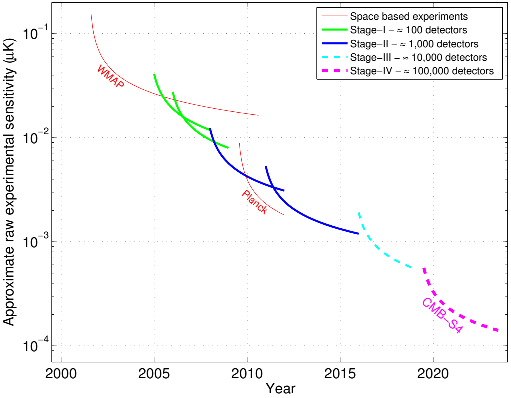
$$C_\ell^{XY} = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} a^{X*}_{\ell m} a^{Y}_{\ell m}.$$

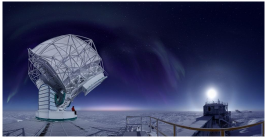
- Here $X, Y \in \{T, E, B, \phi\}$ for 'temperature', 'E mode polarisation' and 'B mode polarisation' and lensing potential.



The future of CMB observation

- For next decade advances will be ground-based
  - ACT (Atacama Cosmology Telescope) end-of-life,
  - SPT (South Pole Telescope) online,
  - SO (Simons Observatory) next few years.
  - CMBS4 (CMB stage 4) 5-10 years.
- These will revolutionise our understanding of high-ℓ power spectra, polarisation and lensing.
- But we need space to get full-sky and complete what Planck started in polarisation.
- Europe (ESA) and US (NASA) have not committed to future CMB space missions.
- Japan (JAXA) and India (ISRO) may yet:
  - LiteBird is a space mission targeted at B-modes,
  - CORE (Cosmic ORigins Explorer) was a template for the next Planck.





Beyond the CMB: Standard rulers & standard candles

- Recall from Lecture 19 If you knew how bright an object was L, then using measurements on earth F, z we can account for the effects of cosmology on the time-delayed observation to measure how far away it is using the luminosity distance

$$\boxed{d_L = \left(\frac{L}{4\pi F}\right)^{\frac{1}{2}} = R_0 S(\chi)(1+z).}$$

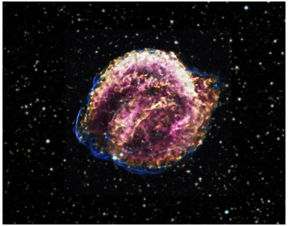
- Similarly, if you knew how large an object was D, then using measurements on earth $\Delta\theta, z$ we can account for the effects of cosmology on the time-delayed observation to measure how far away it is using the angular diameter distance

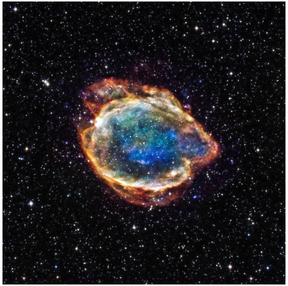
$$\boxed{d_\theta = \frac{D}{\Delta\theta} = \frac{R_0 S(\chi)}{(1+z)}.}$$

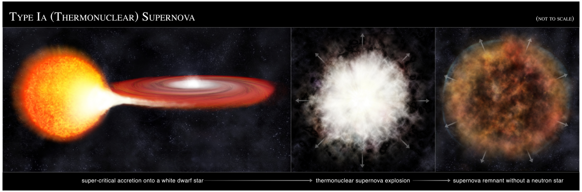
- If you know how far away a set of objects are in the universe from different epochs, you can use these to build up a map of the evolution of the universe e.g. $\chi = \chi_{\Lambda\mathrm{CDM}}(z)$, and provide inferences of the proportions of the constituents.

Standard candles: Type Ia supernovae

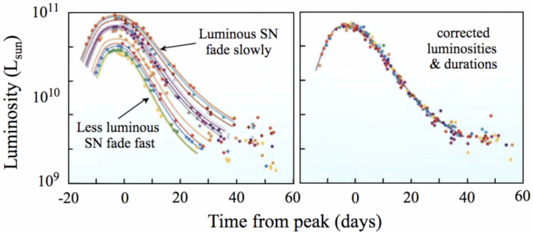
- These supernovae are a subset of Type I's (supernovae lacking Hydrogen).
- As discussed earlier in the course, these supernovae are formed from a binary system.
- Main sequence/red giant star deposits mass onto a white dwarf via Roche lobe overflow.
- The white dwarf is thus gently brought up to above the Chandrasekhar mass, at which point it detonates in a predictable fashion.
- Such supernovae have a characteristic luminosity, and thus form a "Standard candle" L.

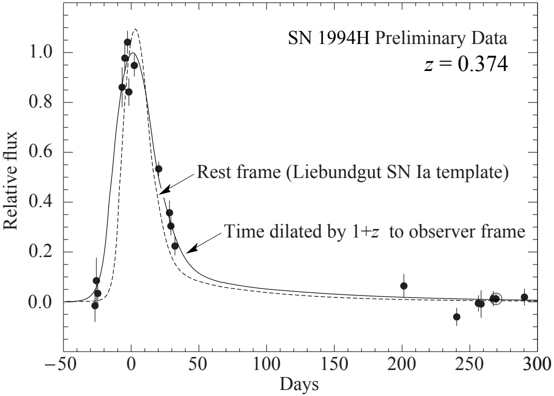




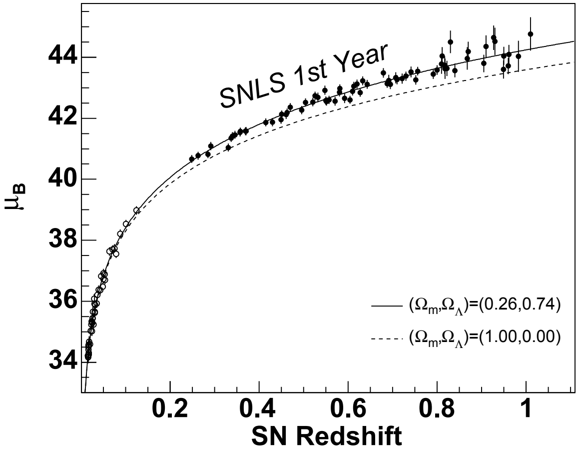


- In practice it's a bit more complicated than this. Rather than a standard luminosity L, they form a standardisable "light curve"
- Using physics we know e.g. time dilation by redshifts (measured by spectroscopy).
- There are still a host of systematic errors to control, and many cosmologists doubt that we're fully on top of them even today.





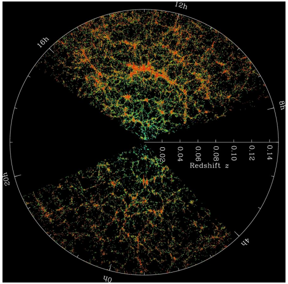
- Next figure shows experimental data from a 2005 compilation of SN Type Ia results on a flux versus redshift plot.
- Here we use magnitude $\boxed{\mu = 5\log_{10}(d_L/10)}$ noting that this is a(n ancient Greek) measure of dimness against redshift z.
- Predicted curves worked out using different values of Ω_{m0} and $\Omega_{\Lambda 0}$
- Clear from this that an Einstein de Sitter universe $((\Omega_{m0}, \Omega_{\Lambda 0}) = (1, 0))$ is going to be strongly ruled out.
- From this type of data, one can create a likelihood of how well a given cosmology parameterised by $(\Omega_{m0}, \Omega_{\Lambda 0})$ fits.

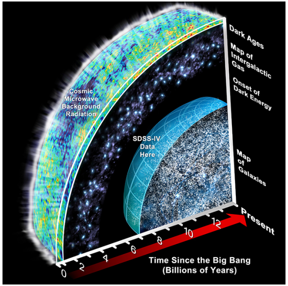


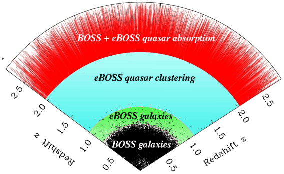
Data from the 'Supernova Legacy Survey'. The curves are essentially of flux (increasing logarithmically downwards*) versus redshift. (From Astier et al., 2005.)*

The Sloan Digital Sky Survey (SDSS)

- The Sloan Digital Sky Survey (SDSS) is a long-running survey of large scale structure.
- Represents the state of the art in three-dimensional mapping of the late-time universe, with spectroscopic measurements for a large portion of the sky.
- Website is an excellent resource for both science and students: sdss.org.
- The BOSS (Baryon Oscillation Spectroscopic Survey) mapped quasars and LRGs (luminous red galaxies) at high redshift in an attempt to measure Baryon Acoustic Oscillations.

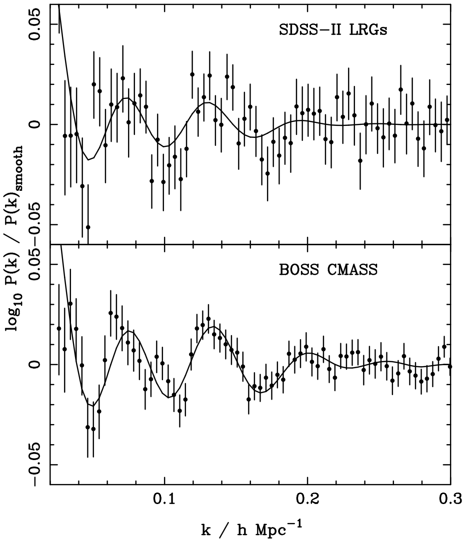






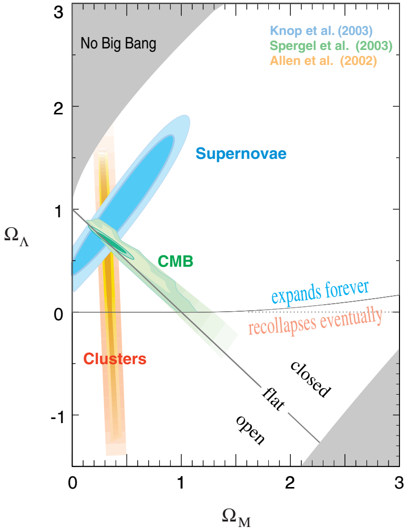
Baryon acoustic oscillations: listening for the size of the Universe

- The same sound waves which we see imprinted in the CMB acoustic peaks can also be seen in large scale structure.
- Pre-recombination, photons are tightly coupled to baryons in the plasma.
- Sound waves in this plasma have high and predictable speed (close to c).
- There is therefore a lengthscale associated with this wavelength, $L \sim ct_* \approx 450\,\mathrm{kly}$ – a standard ruler!
- After recombination, this lengthscale is "frozen out" and visible statistically in the distribution of galaxies.
- In practice this manifests itself in peaks in a power spectrum (this time for 3D spatial location than angular sky position).



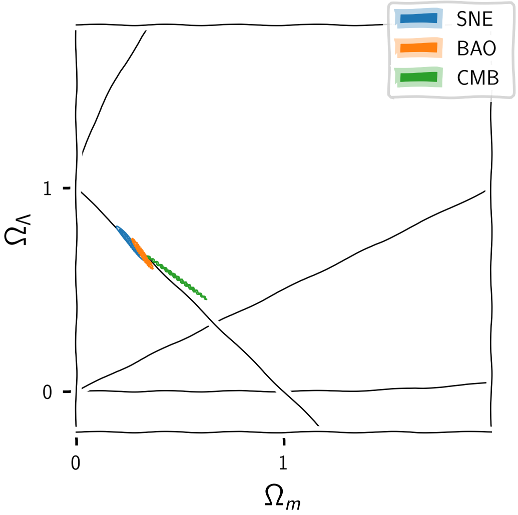
So where are we now?

- This is a famous plot, used since the RAC course was first constructed.
- It shows the same Ω_Λ Ω_m plane discussed on Slide 4 .
- The contours indicate the degree of knowledge provided by CMB (WMAP), Supernovae (SCP) constraints from galaxy clusters (conducted here).
- Note that none of them measure Ω_Λ or Ω_m directly, and this is characteristic in cosmology.
- Cosmologists therefore need quite a sophisticated understanding of inference to disentangle probability distributions in high-dimensional parameter spaces.
- Combined however they give a strong measurement of $\Omega_m \approx 0.3$ and $\Omega_\Lambda \approx 0.7$, i.e. a flat-$\Lambda$ universe of mostly dark energy today.



So where are we now?

- Same plot with today's data.
- Measurements have improved dramatically over the past two decades.
- However, we're now at the stage of precision where the fact that they don't recover consistent answers is important.
- Scientifically speaking this exciting – it means there is some aspect of the Universe we don't understand.
- Remember – big shifts in physics come from apparently "small" problems:
  - mercurial perihelion shift,
  - photoelectric effect,
  - double-slit experiment.



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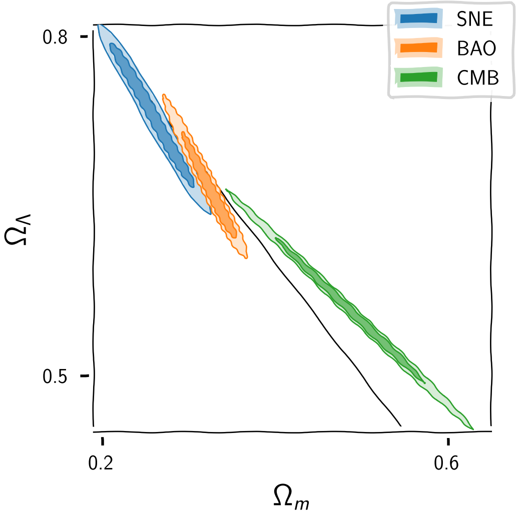

Summary

- The cosmic microwave background anisotropy power spectrum C_ℓ.
- Cosmic variance

$$\boxed{C_\ell = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 \sim C_\ell \pm \sqrt{\frac{C_\ell}{2\ell+1}}.}$$

- Standard candles: Type Ia Supernovae and light curves.
- Standard rulers: Baryon Acoustic Oscillations and the 3D power spectrum $P(k)$.
- Cosmic tensions.

Next time

The primordial universe & inflation – what do we think started everything off?