Neutron stars and pulsars

Relativistic Astrophysics and Cosmology: Lecture 12

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Pre-lecture question:

What's the fastest something can spin?

Last time

- Jets
- Special relativistic beaming
- Gamma ray bursts

This lecture

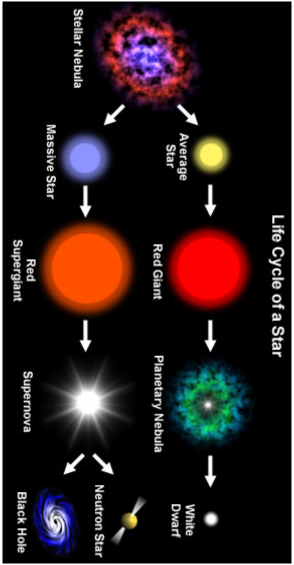
- Neutron stars
- Pulsars
- Elliptical orbits

Next lecture

- Binary systems & Gravitational waves

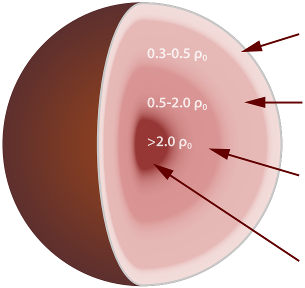
Neutron star formation

- As discussed in Lecture 7, neutron stars are formed in the collapse of large stars $> 8M_\odot$.
- As stars deplete their nuclear fuel, the core is "poisoned" by Iron ash, triggering a runaway process where the star begins to collapse and causes the core to exceed the Chandrasekhar limit $1.4M_\odot$ (the limit for white dwarf formation).
- Inverse beta decay allows the core to convert to pure neutrons, emitting a flood of neutrinos, causing the remainder of the star to bounce and go supernova (Type II, Ib or Ic).
- The core retains most of the angular momentum .
- Since $R_{NS} \ll R_{\rm star}$, expect neutron stars to be spinning very rapidly.



Scales of neutron stars

- Researchers spend careers modelling neutron star equations of state $P(\rho)$ (see Lecture 4 for how one would then use this with the Oppenheimer-Volkov equation).
- One can get a lot of the way using scaling arguments however.



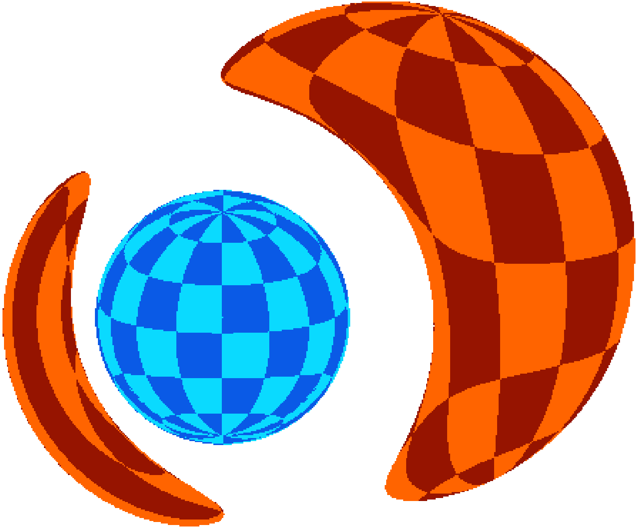
- We know neutron stars are larger than the Chandrasekhar limit, and bounded by the Buchdahl limit $(1.4 < M/M_\odot < 5)$.
- A similar degeneracy argument as found in Lecture 7, but using just neutrons gives:

$$\boxed{M \sim \frac{\hbar^6}{G^3 m_n^8} R^{-3},} \qquad \boxed{R \sim 3\,\mathrm{km} \left(\frac{M}{M_\odot}\right)^{-1/3}.}$$

- Approx diameter from Girton to Homerton.
- In practice this approximates the core, and neutron stars are usually measured $R \sim 10\,\mathrm{km}$.

Extreme objects

- Images shows ray tracing diagram of neutron star (blue) and a companion star (red).
- Gravitational lensing near the surface heavily bends light.
- Note you can see both poles of NS and $\sim 70\%$ of the surface.
- Note since $R_{\mathrm{NS}} \sim \mathcal{O}(\mu)$ not unusual for photons near the surface to have close to circular orbits (3μ).
- Very strong surface forces mean "mountains" on neutron stars are < 1 mm [arxiv:2105.06493].



Neutron star spin

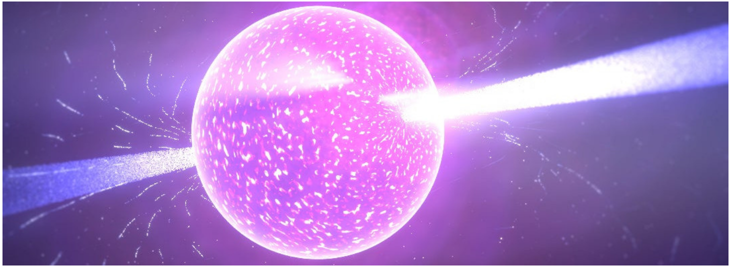
- Consider a situation where a collapsing star converts converts all of its mass and angular momentum into a neutron star.
- We know that angular momentum $L \sim MR^2\Omega$ (since moment of inertia $I \sim MR^2$).
- If $R_{\mathrm{NS}} \sim 10\,\mathrm{km}$, and $R_\odot \sim 700\,000\,\mathrm{km}$, and equatorial solar rotation period is $P_\odot \sim 24\,\mathrm{days}$, then $P_{\mathrm{NS}} = P_\odot \left(\frac{R_{\mathrm{NS}}}{R_\odot}\right)^2$.
- Plugging the numbers tells us that Neutron star rotation periods are in milliseconds.
- Can also turn this around: since the rotation/pulsation of any self-gravitating object cannot spin faster than breakup speed or oscillate faster than the escape velocity:

$$v_{\mathrm{escape}}^2 \sim R^2\Omega^2 \leqslant \frac{GM}{R} \Rightarrow \boxed{\Omega^2 \leqslant G\rho}$$

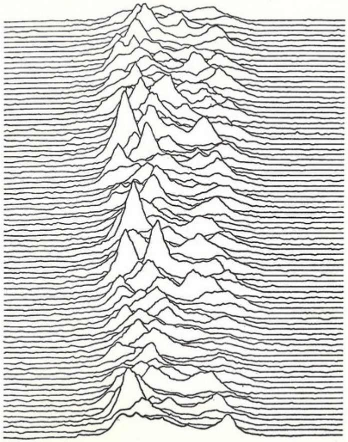
- If an object's rotation period is $P < 1\,\mathrm{s}$, then $\rho \sim \rho_{\mathrm{nuclear}}$.
- i.e. self-gravitating objects with periods faster than a second must be neutron stars.

Pulsars: lone neutron stars

- Pulsars are rapidly spinning neutron stars.
- Strong magnetic fields emanate from their magnetic poles.
- If the orientation of the lighthouse beam is lucky enough to intersect with earth, we observe pulses of radio waves.
- Pulsars have steadily increasing periods between 1.4–9 ms.
- Discovered in Cambridge 1967 by Jocelyn Bell Burnell (supervised by Anthony Hewish).

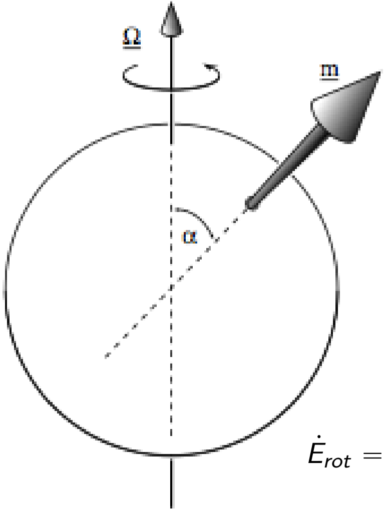


6.7: *Successive pulses from the first pulsar discovered, CP 1919, are here superimposed vertically. The pulses occur every 1.337 seconds. They are caused by a rapidly-spinning neutron star.*



The magnetic dipole model

- Premise: a spinning sphere, which has magnetic dipole moment m, surface magnetic field B_p and angular velocity Ω, emits magnetic dipole radiation of power $\mathcal{P}$.
- The equivalent Larmor's formula involves the square of the acceleration of magnetic moment.



The governing equations are:

$$\mathcal{P} = \frac{2}{3c^3}\left(\frac{\mu_0}{4\pi}\right)|\ddot{m}|^2,$$

$$m = \frac{B_p R^3}{2}\left(\frac{4\pi}{\mu_0}\right),$$

$$\ddot{m} = m\sin\alpha\Omega^2,$$

$$\mathcal{P} = -\frac{dE_{rot}}{dt},$$

$$E_{rot} = \frac{1}{2}I\Omega^2,$$

$$\dot{E}_{rot} = I\Omega\dot{\Omega} = -\frac{8\pi}{\mu_o}\frac{B_p^2 R^6 \Omega^4}{3c^3}\sin^2\alpha.$$

The magnetic dipole model – intuition

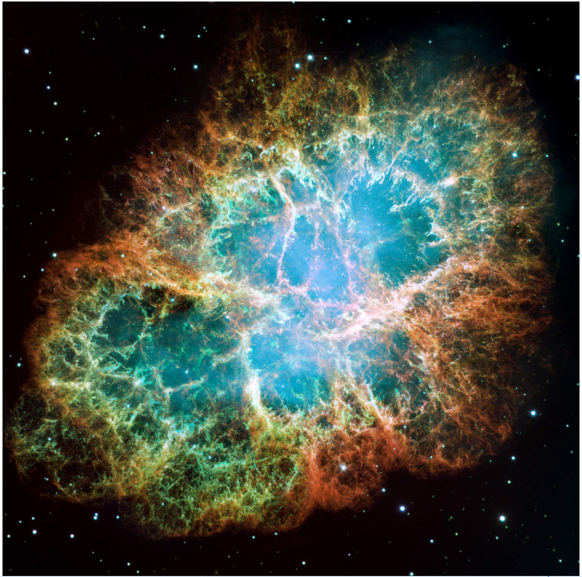
- Imagine a dipole magnet rotating.
- The B field can corotate with NS up until roughly the speed of light cylinder $R_c = c/\Omega$.
- The field there (assuming a dipole all the way) $B_c \sim B_p(R/R_c)^3$.
- The energy density $\varepsilon_c \approx B_c^2 \mu_O^{-1}$.
- This is being 'stripped off' and is flowing away at c as radiation so

$$\mathcal{P} \sim \varepsilon_c R_c^2 c \Rightarrow \frac{B_p^2 R^6 \Omega^4}{\mu_o c^3}.$$

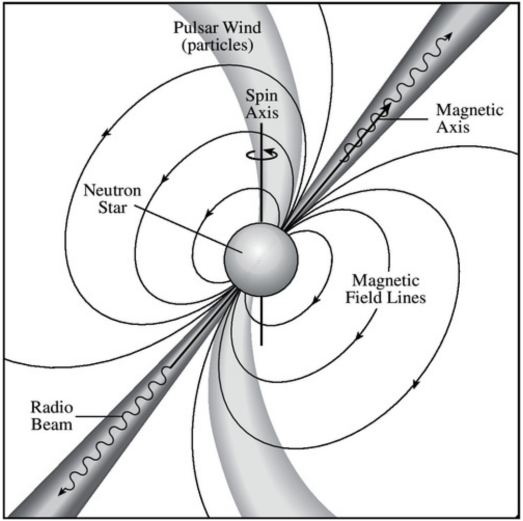
- This is correct to within a geometrical factor $(8\pi/3)$.
- Thus $\dot{\Omega} < 0 \rightarrow$ the pulsar slows down due to the loss rotational energy.
- Characteristic Age $\tau =$ 'spin-down time' $\equiv -\frac{\Omega}{\dot{\Omega}} = \frac{6Ic^3\mu_0}{B_p^2 R^6 \sin\alpha^2 \Omega_0^2 4\pi}$.
- Braking index n: $\dot{\Omega} \propto -\Omega^n$. For a magnetic dipole should be 3 - the measured value is about 2.5 (due to particle outflow-wind?).
- NB this is something measurable, which we compare with theory.

The Crab Nebula (supernova remnant – Messier 1)

- It contains a pulsar with period $P = 33\,\text{ms}$ and $\dot{P} = 4.3 \times 10^{-13}\,\text{ss}^{-1}$.
- The image of the nebular itself is formed of synchrotron radiation strongly polarised in radio and optical bands.
- Synchrotron radiation for the Crab requires continuous injection of power at $> 10^{31}$ W.
- Modelling estimates $M = 1.4 M_{\odot}$, $R = 12\,\text{km}$, giving $\dot{E} = -6 \times 10^{31}\,\text{W}$.
- The pulsar itself is therefore lighting up this nebula, or equivalently the nebula is acting as a calorimeter for the pulsar.
- $B \sim 5 \times 10^{8}\,\text{T}$ $(10^{13} B_{\oplus}$ or $10^{7} B_{\text{hospital MRI}})$



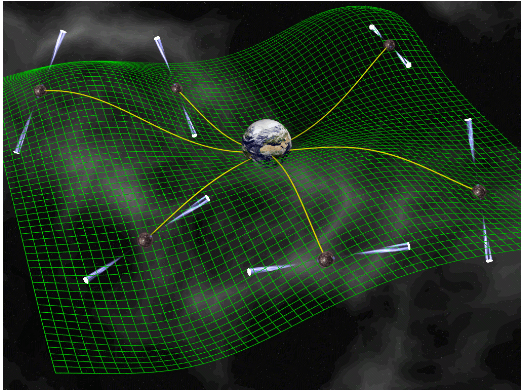
Why are pulsars radio frequency?



- At the magnetic poles have uniform B field.
- Spin induces an electric field $E \sim v \times B \sim \Omega R B$, which gives a voltage of $V \sim \Omega R^2 B$ (e.g. $10^{11} P^{-1}$ V for Crab).
- This rips electric charges via particle creation from the surface, filling the magnetosphere with charged matter.
- Charged particles in strong magnetic fields generate synchrotron radiation.
- Voltage acceleration beams the radiation.
- Voltage decreases as P decreases, and when below $e^{\pm}$ particle creation threshold the radio emission ceases and the pulsar "dies" (although NS lives on).

How are pulsars useful beyond stellar evolution?

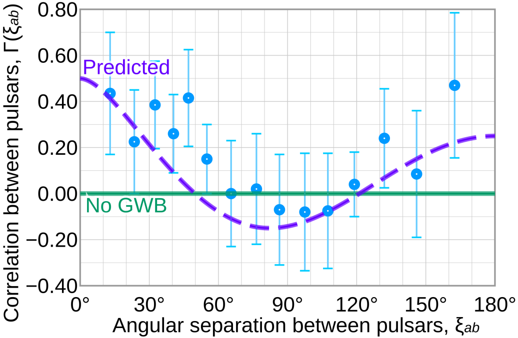
- Pulsars are very accurate clocks, placed throughout spacetime.
- Ideal setup for testing general relativity.
- Bootstrapped together, pulsars could form a new SI standard for defining the second.
- In cosmology, use them as "standard sirens" (c.f. "standard candle" supernovae).
- Pulsar timing arrays are systems of clean millisecond pulsars: statistically averaged become a detector of gravitational waves.



- PTAs provided a benchmark for LIGO since all they needed to do was listen and wait for statistical power to build up over decades to claim discovery. LIGO won.
- They are still useful as they are sensitive to a completely different band of the GW spectrum (e.g. detecting the gravitational wave background).

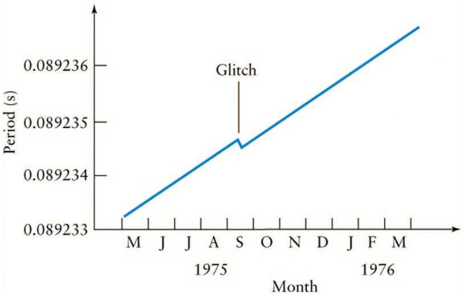
Pulsar timing array (PTA): gravitational wave background

- In June 2023, NANOGrav announced that they found evidence for a gravitational wave background (distinctive Hellings-Downs curve), using 15-year data on 68 pulsars.
- The sources of the gravitational-wave background can not be identified without further observations and analyses, although binaries of supermassive black holes are leading candidates.



- Gravitational wave background is a random background of gravitational waves permeating the Universe. The signal may be intrinsically random, like from stochastic processes in the early Universe, or may be produced by an incoherent superposition of a large number of weak independent unresolved gravitational-wave sources, like supermassive black-hole binaries.

Glitches & Starquakes: Earthquakes on neutron stars



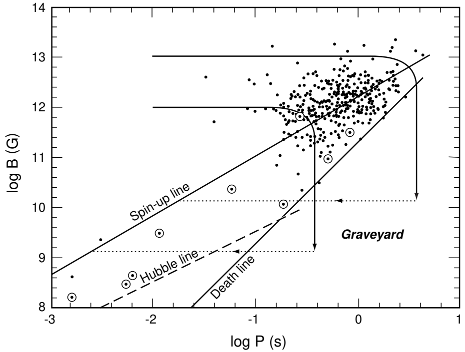
- Glitches are sudden changes in period superimposed on the steady spindown.
- Due either to crustquakes: release of stored elastic energy as the crust relaxes during spindown – the centrifugal force at the equator varies as P^{-2}...
- ...or to the way that the superfluid interior interacts with the crust as it readjusts (pinned superfluid vortices).
- The period changes because the moment of inertia is altered by the rearrangement of crust in the quake, whereas the total angular momentum of course remains unchanged.
- The slow recovery of the spin of the star after a glitch is because the sudden behaviour happens only in the crust and requires days to be transmitted to the superfluid interior i.e. it shows the superfluid nature of the core.

Magnetars

- Magnetars are young neutron stars with surface magnetic fields in the range of 100 times that of the Crab pulsar and above.
- Decay of the magnetic field powers gamma-ray and X-ray emission.
- Their spin has usually slowed to periods of a few seconds.
- The field is so strong that its evolution leads to crustquakes of 32 on the Richter scale resulting in bursts of gamma-rays.
- Such bursts can be lead to the brightest extra-Solar EM fluxes on Earth, even from across the Galaxy.
- Mass extinction if one were to occur within 10ly of earth.
- Rapidly spinning very young magnetars could be responsible for enigmatic Fast Radio Bursts.

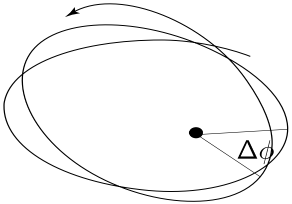
Pulsar evolution

- Plot 403 observed pulsars in $P\ \dot{P}$ plane ($B \propto \sqrt{P\dot{P}}$).
- The arrowed lines indicate evolution with the field decaying exponentially on a 5 My timescale:
- death line a voltage no longer high enough to accelerate particles to make pairs on the magnetic field, then detectable pulsar activity does not occur.
- spin-up line minimum period to which spin-up can occur in Eddington-limited accretion.
- Hubble line pulsar spin-down, $\tau = \frac{P}{\dot{P}} \sim 10^{10}$yr.
- horizontal dashed lines spin-up of a dead pulsar due to accretion.
- black widow pulsars pulsar luminosity destroys companion.



Example: lone pulsar in globular cluster → born with a strong field and spins down to a period of several seconds→ captures a normal star → mass transfer → the pulsar spins up to ∼ ms.

Orbital Precession



- Primary postdiction for GR: precession of elliptical orbits.
- Result for small changes per orbit is

$$\boxed{\Delta\phi = \frac{6\pi GM}{a(1-e^2)c^2}.}$$

- Mercury's orbit has $a = 5.8 \times 10^{10}$ m, eccentricity $e = 0.2$ and $M_\odot = 2 \times 10^{30}$ kg.
- Therefore predicted precession is

$$\Delta\phi = 5 \times 10^{-7}\,\text{radians} = 0.1'' \quad \text{per orbit.}$$

- Orbital period is 88 days, so expect to accumulate a precession of 43″ per century.
- This is what is observed after correction for the perturbations due to the other planets, which cause a *total* precession of more like 5000″ per century.
- Much more extreme cases, and which start to test 'strong field' regime of GR, now available from pulsars. E.g. the Binary Pulsar (next lecture).

Derivation of orbital precession

- The equation we wish to solve is

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} + \frac{3GM}{c^2}u^2, \qquad (u \equiv \frac{1}{r}). \tag{1}$$

- In the limit that the departure from Newtonian motion is small.
- This would apply to the motion of the planets in our solar system for example.
- The Newtonian solution to this equation is

$$u = \frac{GM}{h^2}(1 + e\cos\phi),$$

- We can use this as a first approximation, and then iterate to get a better one.
- Substituting into the r.h.s. of (1), we obtain the new equation

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} + \frac{3(GM)^3}{c^2h^4}(1 + e\cos\phi)^2.$$

- This will be solved by $u = \frac{GM}{h^2}(1 + e\cos\phi)$ + the particular integral (P.I.) of the equation:

$$\frac{d^2u}{d\phi^2} + u = A(1 + 2e\cos\phi + e^2\cos^2\phi),$$

  where $A = 3(GM)^3/c^2h^4$ is very small. The P.I. can be found to be:

$$A\left(1 + e\phi\sin\phi + e^2\left(\frac{1}{2} - \frac{1}{6}\cos 2\phi\right)\right).$$

- Now, in this expression, the first and third terms are tiny, since A is.
- However, the second term, $Ae\phi\sin\phi$, might be tiny at first, but will gradually grow with time, since the ϕ part (without a cos or sin enclosing it) means it is *cumulative*.
- We must therefore retain it, and our second approximation is

$$u = \frac{GM}{h^2}(1 + e\cos\phi + \delta e\phi\sin\phi),$$

  where

$$\delta = \frac{3(GM)^2}{h^2c^2} \ll 1.$$

- Using

$$\cos(\phi(1-\delta)) = \cos\phi\cos\delta\phi + \sin\phi\sin\delta\phi \approx \cos\phi + \sin\phi\,\delta\phi \quad \text{for} \quad \delta \ll 1,$$

we can therefore write

$$u \approx \frac{GM}{h^2}\left(1 + e\cos[\phi(1-\delta)]\right).$$

- u is therefore periodic, but with period $\frac{2\pi}{1-\delta}$.
- The r values thus repeat on a cycle which is slightly larger than 2π, and we find

$$\Delta\phi = \frac{2\pi}{1-\delta} - 2\pi \approx 2\pi\delta = \frac{6\pi(GM)^2}{h^2c^2}.$$

- But from the geometry of the ellipse, and the Newtonian solution, where we know $l = h^2/GM$ and $l = a(1-e^2)$, we can get the final result:

$$\Delta\phi = \frac{6\pi(GM)^2}{c^2l(GM)} = \frac{6\pi GM}{a(1-e^2)c^2}.$$

Summary

- Neutron stars are the fast spinning cores left over after supernovae.
- When their "lighthouse" radio beams intersect with earth they are observed as pulsars.
- Pulsars move around in the measured P-$\dot{P}$ plane spinning up via accretion and down via dipole radiation.
- Ideal for precision tests of GR as natural clocks placed around spacetime.
- Orbital precession for nearly Newtonian orbits

$$\boxed{\Delta\phi = \frac{6\pi GM}{a(1-e^2)c^2}.}$$

Next time

Binary systems & gravitational waves