The fluid dynamics of mantle convection

Physics of the Earth as a Planet, Lecture 8

The critical Rayleigh number for convection

Lord Rayleigh (1916) Phil. Mag. 32, pp 529-546

The governing equations for convection consist of statements of conservation of mass,

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

conservation of momentum,

$$0 = -\nabla P + \eta \nabla^2 \mathbf{v} - \rho g \hat{\mathbf{z}}, \tag{2}$$

conservation of energy,

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \kappa \nabla^2 T, \tag{3}$$

and an equation of state

$$\rho = \rho_0 (1 - \alpha T). \tag{4}$$

In writing these equations, we have made what is known as the Boussinesq approximation. This states that variations in density may be neglected except in the term involving gravity (this term produces the convective flow). It is the Boussinesq approximation that allows us to write mass conservation (1) as an incompressibility relation despite the fact that the density varies.

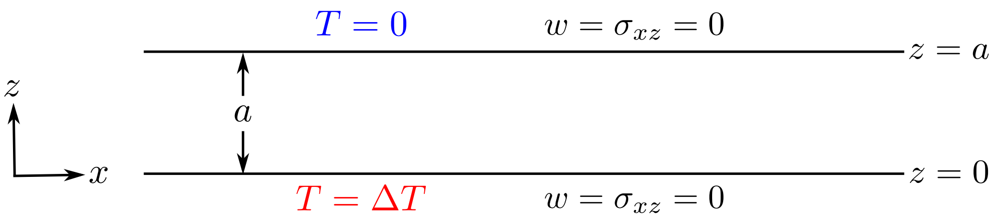

Figure 1: Model geometry.

The simplest problem to solve is that of a layer of thickness a with impermeable and shear stress-free boundaries maintained at constant temperature difference ΔT (Figure 1). When there is no heating from below ($\Delta T = 0$) the pressure in the fluid is hydrostatic i.e. $P = \rho_0 g(a - z)$. It is useful to introduce a perturbed pressure $\mathcal{P}$ which represents the difference from this hydrostatic base state:

$$P = \rho_0 g(a - z) + \mathcal{P}. \tag{5}$$

Using (4), conservation of momentum (2) then becomes

$$0 = -\nabla \mathcal{P} + \eta \nabla^2 \mathbf{v} + \rho_0 g \alpha T \hat{\mathbf{z}}. \tag{6}$$

We will now make the equations dimensionless. Substitution of

$$\begin{aligned} (x, y, z) &= a(x', y', z') \\ T &= \Delta T\, T' \\ t &= \frac{a^2}{\kappa} t' \\ \mathbf{v} &= \frac{\kappa}{a} \mathbf{v}' \\ \mathcal{P} &= \frac{\eta \kappa}{a^2} \mathcal{P}' \end{aligned}$$

and dropping the primes gives the non-dimensional governing equations as

$$\nabla \cdot \mathbf{v} = 0, \tag{7}$$

$$-\nabla \mathcal{P} + \nabla^2 \mathbf{v} = -\mathrm{Ra} T \hat{\mathbf{z}}, \tag{8}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T, \tag{9}$$

where

$$\mathrm{Ra} = \frac{\rho_0 g \alpha \Delta T a^3}{\kappa \eta} \tag{10}$$

is the Rayleigh number, the key dimensionless number governing convection.

As in the previous lecture, we can simplify the momentum equation (8) by taking the curl,

$$\nabla^2 \nabla \times \mathbf{v} = -\mathrm{Ra} \nabla \times \hat{\mathbf{z}} T \tag{11}$$

and introducing a stream function $\psi(x, z, t)$,

$$\mathbf{v} = (u, 0, w) = \left(-\frac{\partial \psi}{\partial z}, 0, \frac{\partial \psi}{\partial x} \right). \tag{12}$$

Equation (7) is then automatically satisfied, and (11) becomes

$$\nabla^4 \psi = -\mathrm{Ra} \frac{\partial T}{\partial x}, \tag{13}$$

and (9) gives

$$\frac{\partial T}{\partial t} + \left(-\frac{\partial \psi}{\partial z}\frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial z} \right) = \nabla^2 T. \tag{14}$$

Equations (13) and (14) are nonlinear and no general analytic techniques are available to solve them. If, however, the convection is weak, as it will be when it can only just occur, then the nonlinear term in (14) will be small, the temperature will differ little from the conductive solution, and the equations can be *linearised*

$$T = 1 - z + \theta, \tag{15}$$

where $\theta \ll 1$. Then the term in brackets on the left of (14) becomes

$$-\frac{\partial \psi}{\partial z}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} + \frac{\partial \theta}{\partial z}\frac{\partial \psi}{\partial x}. \tag{16}$$

Since (13) shows that $\psi \sim \theta$ and both are small, the middle term is much larger than the other two, and (14) becomes

$$\frac{\partial \theta}{\partial t} - \frac{\partial \psi}{\partial x} = \nabla^2 \theta \tag{17}$$

which is a linear equation.

If the normal component of the velocity is to vanish on $z = 0, 1$, then $\psi = 0$ on these planes. Since $T = 1, 0$, (15) shows that $\theta = 0$ on both also. The final pair of boundary conditions arise from the absence of shear stress

$$\begin{aligned} \sigma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = -\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial x^2} \\ &= 0 \text{ on } z = 0, 1 \end{aligned}$$

Since $\psi = 0$ on the boundaries, so is $\partial^2 \psi / \partial x^2$. Therefore the shear stress is zero if $\partial^2 \psi / \partial z^2 = 0$ on the boundaries.

To determine whether the layer is convectively stable we introduce a small disturbance and find the conditions which must be satisfied for the disturbance to grow. Equations (13) and (17) are linear and can be solved in the usual way by separation of variables. Substituting

$$\begin{aligned} \theta &= \Theta(z)\mathrm{e}^{\sigma t+ikx}, \\ \psi &= \Psi(z)\mathrm{e}^{\sigma t+ikx}, \end{aligned}$$

the layer will be stable if $\mathrm{Re}(\sigma) < 0$ and unstable if $\mathrm{Re}(\sigma) > 0$.

Substitution into (13) and (17) gives

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - k^2\right)^2 \Psi = -ik\mathrm{Ra}\Theta, \tag{18}$$

$$\sigma\Theta - ik\Psi = \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - k^2\right)\Theta, \tag{19}$$

or

$$\sigma\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - k^2\right)^2 \Theta - \mathrm{Ra}\, k^2\Theta = \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} - k^2\right)^3 \Theta. \tag{20}$$

The set of solutions to (20) which satisfy all the boundary conditions are of the form

$$\Theta = \Theta_0 \sin n\pi z, \tag{21}$$

where n is an integer. Then

$$\sigma = \frac{k^2\ \mathrm{Ra}}{(n^2\pi^2 + k^2)^2} - (n^2\pi^2 + k^2). \tag{22}$$

σ is real and is a function of the wavenumber k. We now seek the minimum value of Ra for which σ is zero, since this will be the Rayleigh number at which the layer first becomes unstable to convective disturbances. If σ is zero

$$\mathrm{Ra_c} = \frac{(n^2\pi^2 + k^2)^3}{k^2}. \tag{23}$$

$\mathrm{Ra_c}$ is plotted as a function of k in Figure 2. The minimum value of $\mathrm{Ra_c}$ as a function of k is

$$\mathrm{Ra_c} = \frac{27}{4}n^4\pi^4 \tag{24}$$

at $k = n\pi/\sqrt{2}$. Thus the smallest critical Rayleigh number is when $n = 1$, where

$$\mathrm{Ra_c} = \frac{27}{4}\pi^4 = 657.51\ldots \tag{25}$$

and

$$\begin{aligned} k &= \frac{\pi}{\sqrt{2}} = 2.221\ldots \\ \lambda &= 2\sqrt{2} = 2.828\ldots \end{aligned}$$

At the onset of convection the flow is organised into rolls (Figure 3).

When the boundary conditions are different from those used here, for instance when both components of the velocity are zero on both boundaries, (21) no longer satisfies the boundary

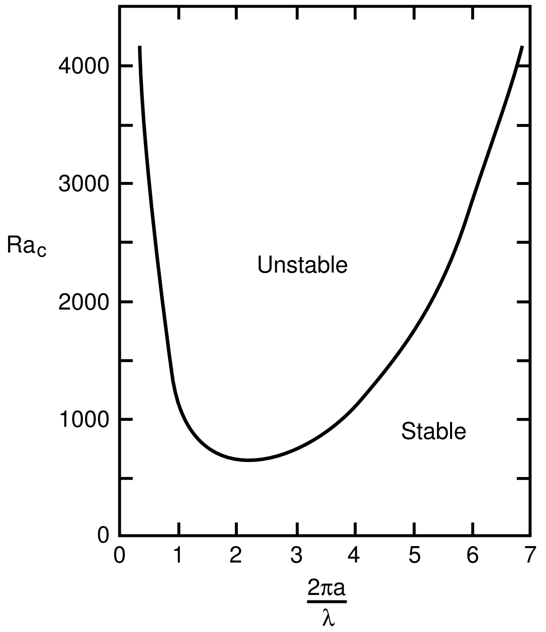

Figure 2: Critical Rayleigh number $\mathrm{Ra_c}$ for the onset of thermal convection as a function of dimensionless wave number k (see (23)). Only the curve for most unstable mode $n = 1$ is shown.

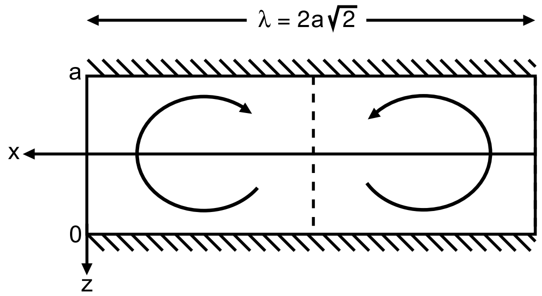

Figure 3: Sketch of two-dimensional counter-rotating cells at the onset of convection in a fluid layer heated from below.

conditions. It is then necessary to solve (18) and (19) with the appropriate boundary conditions, either by analytic or numerical means. The minimum value of the Rayleigh number is then different from (25). The critical Rayleigh numbers for the initial convective instability are between 100 and 3000 for most boundary conditions.

These analytic methods can be extended to Rayleigh numbers of about $2 \times \mathrm{Ra_c}$ with great effort, but then they break down, and progress can only be made with numerical or laboratory experiments. In both cases the experiments are carried out at a number of Rayleigh numbers, and, as of course they must, the results from both types of experiments give the same results.

Convection Experiments

Until the late 1980s most numerical experiments were restricted to two dimensions, whereas laboratory experiments were three dimensional. Nowadays convection at high Rayleigh numbers is routinely simulated in 3D using high performance parallel machines. Such simulations are replacing laboratory experiments, because they are easier to carry out and allow the geophysical observables to be calculated easily. However, a lot of the early insights into convection were obtained using laboratory experiments, which we will discuss now.

The commonest method of studying convection in a tank is called the shadowgraph method. A convective layer is heated and cooled by passing hot and cold water between glass sheets below and above the layer of interest (Figure 4). Light with parallel rays is then shone through the convecting liquid (Figure 5). This method is excellent for examining convective planforms, but provides no information about the temperature structure in the cell.

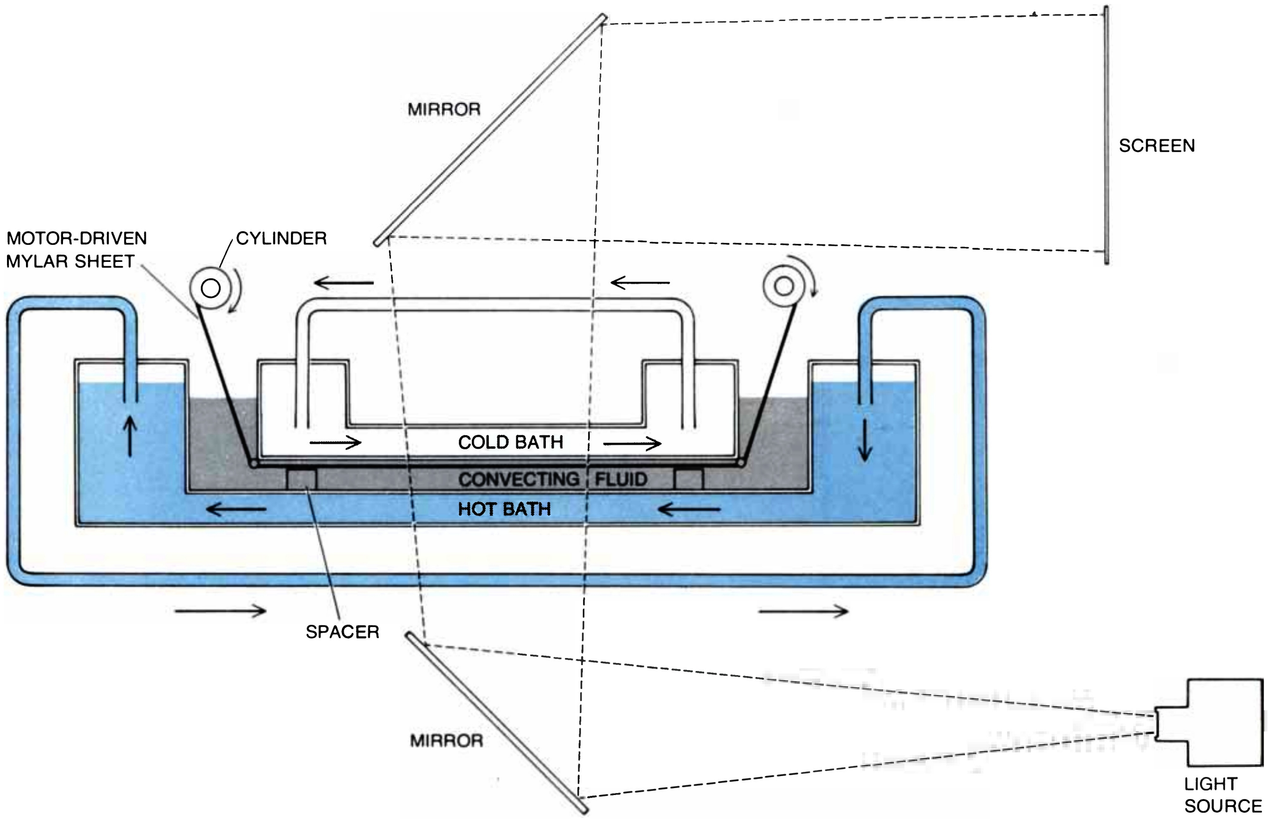

Figure 4: Shadowgraph apparatus for studying convection. The convecting fluid is a viscous silicone oil, which is heated from below and cooled from above. The properties of the oil, in combination with suitable heat flows and depths of fluid, reproduce Rayleigh numbers in the range from about 1,700 to 10^6. The depth of the fluid can be varied from about 1.6 to 7 cm. The convecting region measures about 100 cm on a side. To simulate the effect of a moving plate on the convection patterns a thin sheet of Mylar can be moved across the surface of the oil. Usually only a few hours are needed to reproduce events that would take millions of years on the scale of the Earth. Light shining from below makes the convection patterns visible.

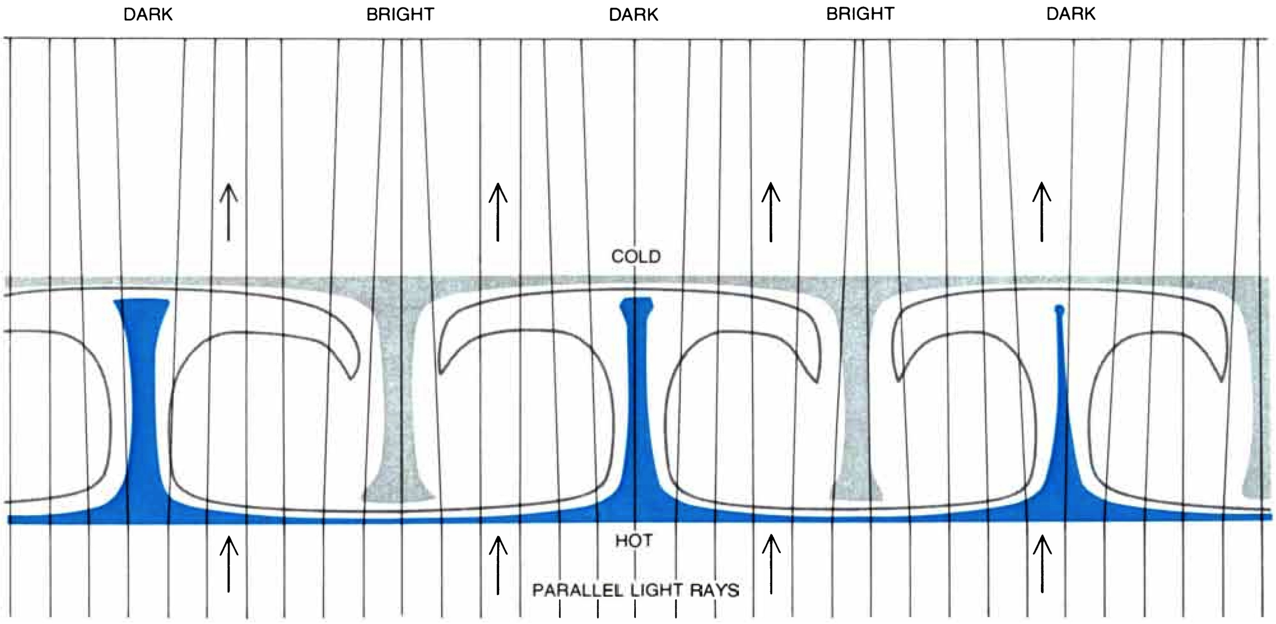

Figure 5: Simple convection cells, consisting of two-dimensional cylindrical rolls, seen here end on, make a shadowgraph like that in Figure 6. Parallel rays of light shine through the fluid from below. The rays are refracted away from hot regions and toward cold ones. Thus hot, rising sheets of fluid are marked by a dark line, which sometimes has a thin bright line on each side. Cold, sinking sheets of fluid are marked by a bright line. Convection cells in general tend to be as wide as they are deep.

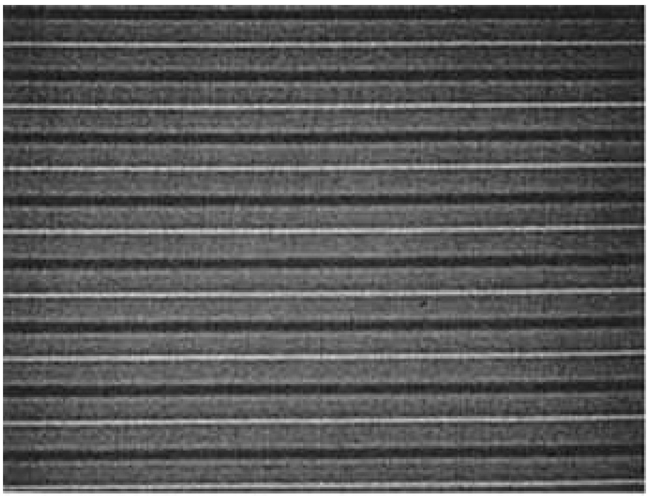

Figure 6: A shadowgraph of convective rolls at $\mathrm{Ra} = 2 \times 10^4$. Bright regions are the downwellings, dark regions are the upwellings

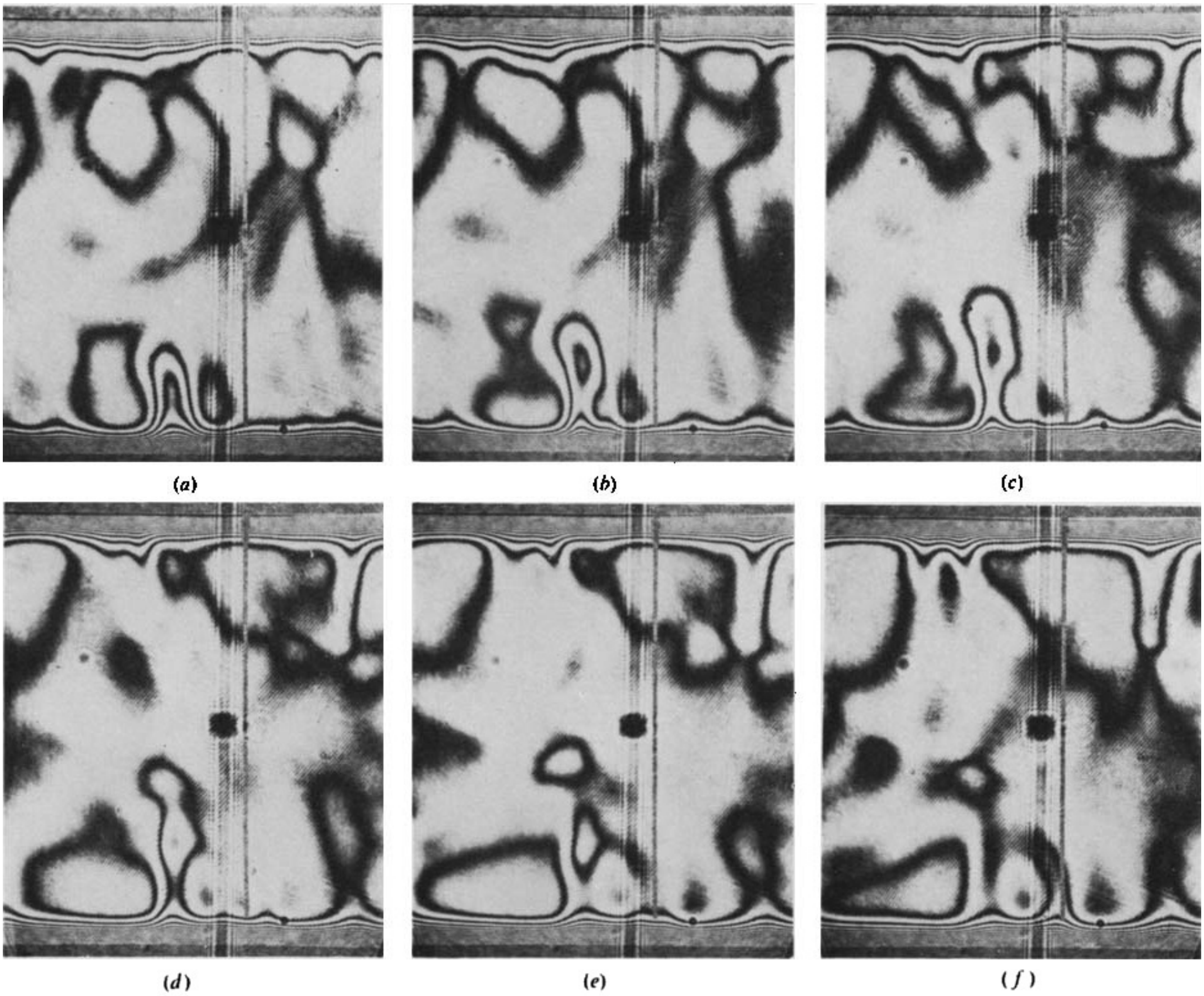

Figure 7: The convection in this layer has been made visible by using a laser. A series of time snapshots are shown. The Rayleigh number is very large, 2×10^6, and the flow is turbulent. The flow is visible because of interference patterns. The travel time of the light depends on the mean density, and hence the mean temperature, of the fluid along a ray path. In this case the illumination is from the side, and the light paths are parallel to the top and bottom of the tank. Hence the interference fringes are controlled by the mean horizontal temperature. The temperature difference between two points depends on the number of dark lines (fringes) between them. Notice the boundary layers at the top and bottom, and the evolution of a hot blob that is produced by instability of the lower boundary layer and rises from near the middle of the lower boundary.

Figure 8: Another method of visualising the flow is to float aluminium particles in the fluid and to leave the camera shutter open for some time. In this picture only the top of the convecting layer has been illuminated with a horizontal slit. The planform and the particle tracks are both clear, but the temperature structure is not visible. This planform is that of bimodal convection, with a Rayleigh number of 2.5×10^4 (see Figure 10).

Instabilities

The next group of pictures (Figures 9, 10 and 11) show shadowgraphs of various types of instabilities that occur on rolls. In these experiments the initial planform is established by placing a mask, made of strips of reflective tape, on top of the tank, and then shining heat lamps onto the top of the tank. Convection then occurs, with the location of the rising limbs of the cells being controlled by the places that are heated. The temperature difference across the tank is then slowly increased to the desired Rayleigh number. In this way a controlled experiment can be carried out, starting with simple initial conditions. Exactly the same sequence is required in the numerical experiments.

The *zigzag instability* occurs when the aspect ratio (distance between rising and sinking regions/depth of the tank) is greater than 1 (Figure 9). It reduces the aspect ratio to about 1.

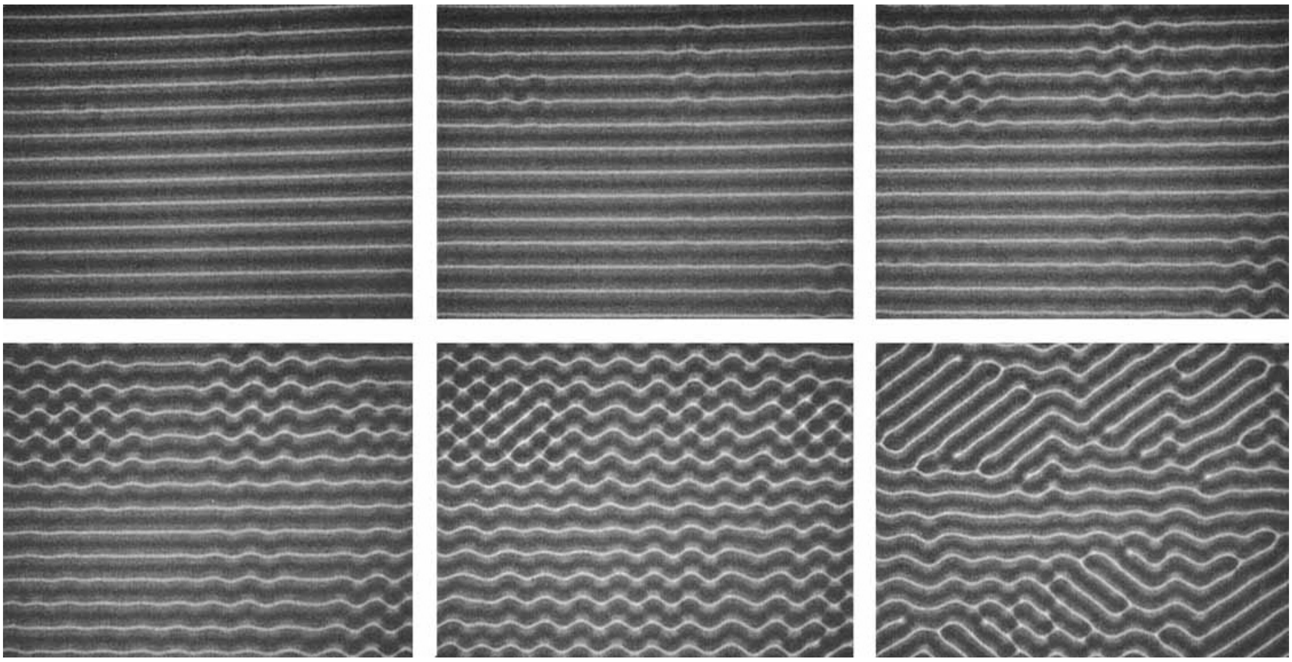

Figure 9: Series of time snapshots showing the development of the zigzag instability at Ra = 3,600.

The *cross-roll instability* occurs when the aspect ratio is much less than 1, or, as in Figure 10, when the Rayleigh number exceeds about 2×10^4. In the first case the rolls are replaced by new rolls of larger aspect ratio, whereas in the second a new planform results, with two sets of rolls with different wavelength at right angles to each other. This planform is called *bimodal*. The bimodal convection occurs because the cross-roll instability can grow, and is an instability like that at marginal stability. When the Rayleigh number is first increased and then decreased, as it is in this example, the original rolls reappear.

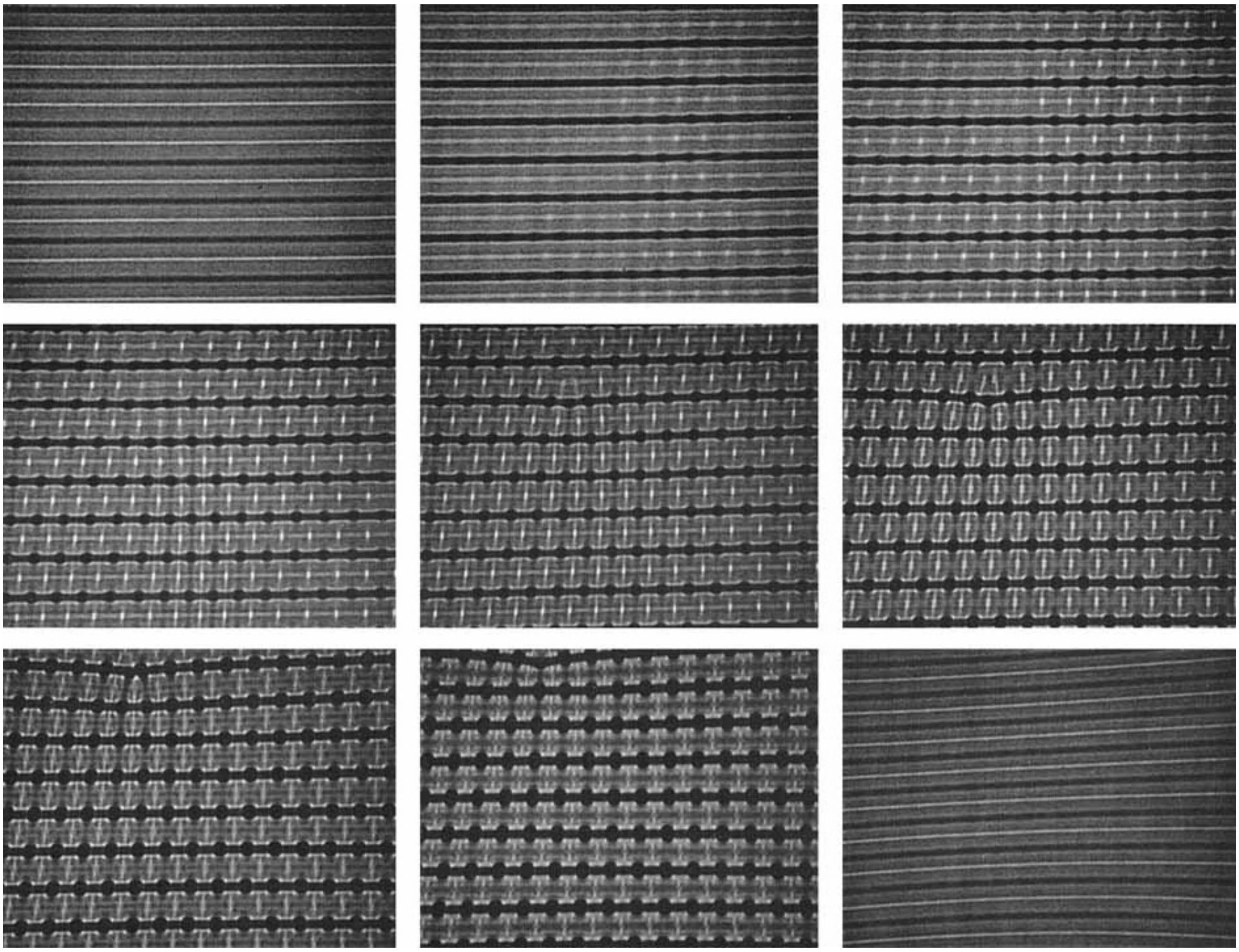

Figure 10: Bimodal convection. Series of time snapshots showing the development of the cross-roll instability. Ra increases slowly in time from 2×10^4 to 6.5×10^4. The last photograph was taken after Ra had been decreased to the original value 2×10^4.

At Rayleigh numbers of more than about 10^5 the bimodal planform breaks down to a *spoke* pattern, so-called because the flow consists of rising and sinking sheets which meet at points like the hub of a wheel (Figure 11). This transition is not reversible. The Rayleigh number of the mantle is at least 10^6, so the type of convection that is expected is spokes (Figure 12).

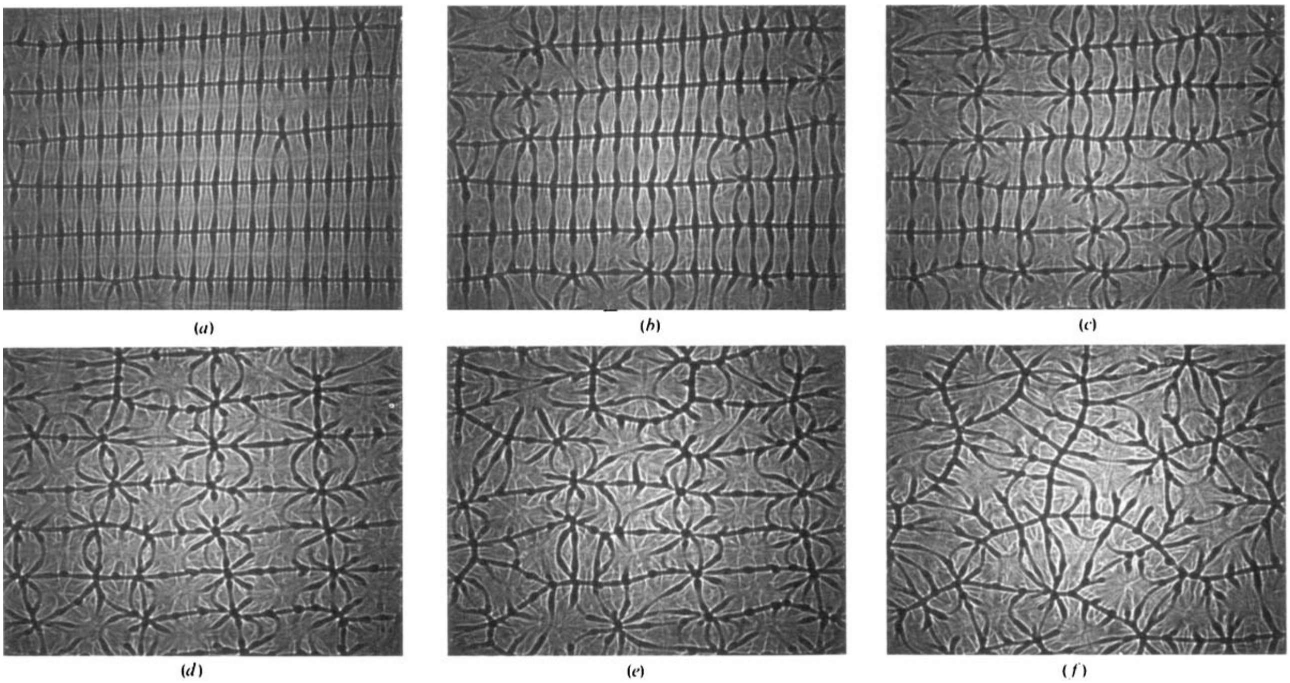

Figure 11: Series of time snapshots showing the transition from bimodal convection to spoke-pattern convection as the Rayleigh number is increased to Ra $= 2.4 \times 10^5$.

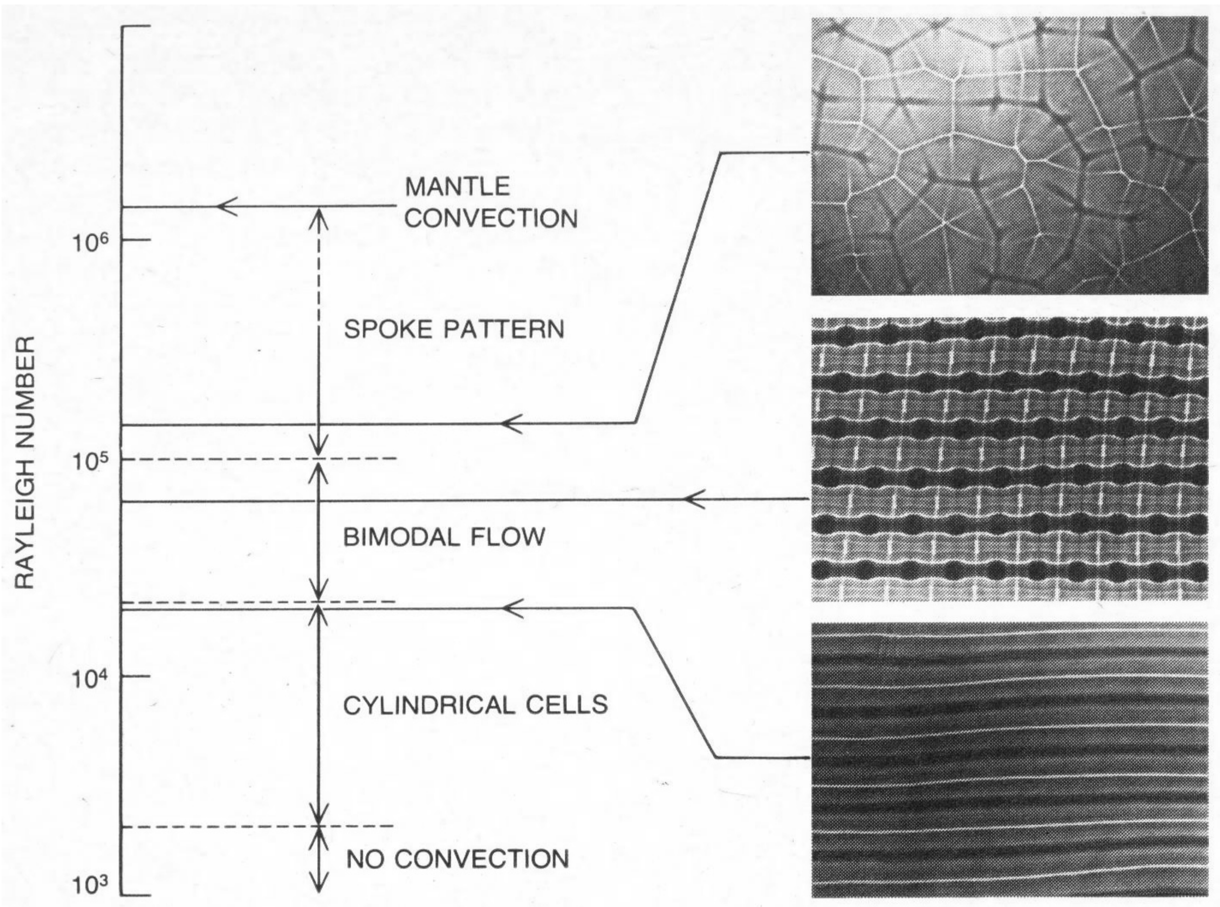

Figure 12: Convection in the mantle is thought to involve Rayleigh numbers lying somewhere between 10^6 and 10^7. Laboratory experiments show that the convection patterns develop in complexity from rolls to bimodal flows to spoke patterns as the Rayleigh number increases.

The behaviour of convection in a layer that is heated from below can be summarised in a diagram that shows the preferred wavenumber ($k = 2\pi/\lambda$) of the flow at different Rayleigh numbers (Figure 13). Each point represents the result of one experiment, in which the Rayleigh number was controlled by fixing the difference in temperature between the top and bottom of the layer, and a mask used to force the initial planform to be rolls. The important feature of this plot is that the aspect ratio of the flow ($= \lambda/2$ or $= k/\pi$) is always between about 1.5 and 0.75. Outside this range instabilities occur that return the aspect ratio to this range.

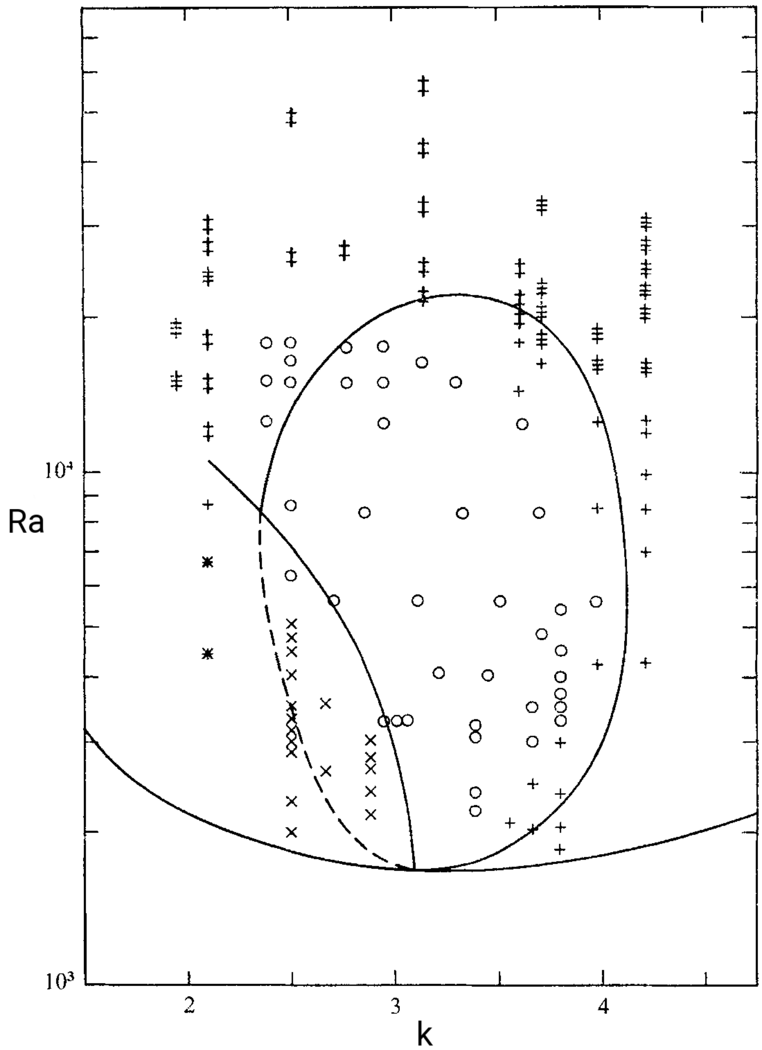

Figure 13: Rayleigh number Ra against preferred wave number k for a series of experiments like those in Figures 9, 10 and 11. $\circ$, stable rolls; $\times$, zigzag instability; $+$, cross-roll instability leading to rolls; $\ddagger$, cross-roll instability leading to bimodal convection; $\ddagger$, cross-roll instability inducing transient rolls. The curves correspond to some theoretical results.

Internal heating

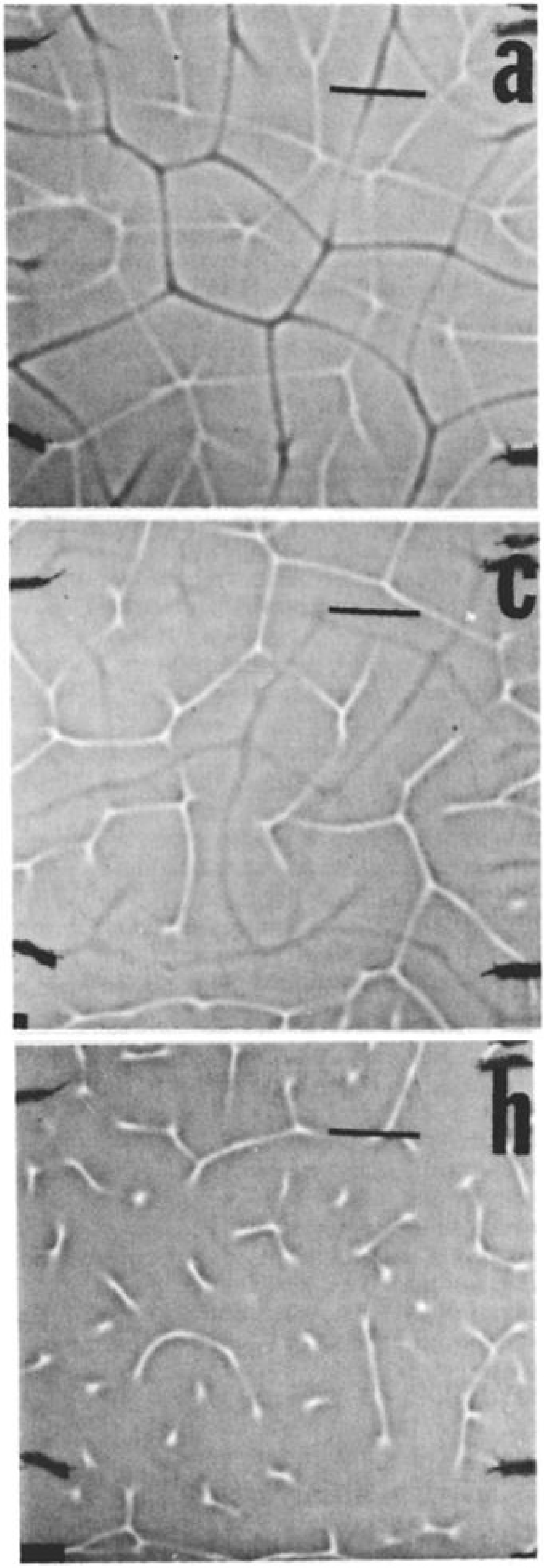

Figure 14: Shadowgraphs showing the effect of internal heating on the planform of convection. The small black bar in each photograph indicates twice the depth of the fluid. a) is a purely basally heated experiment, with Ra $= 1.5 \times 10^5$. Upwellings (dark zones) and downwellings (light zones) are comparable. c) includes some internal heating - notice that the downwellings are now more prominent than the upwellings. h) is dominated by internal heating - the upwellings are now not distinct enough to register in the shadowgraph.

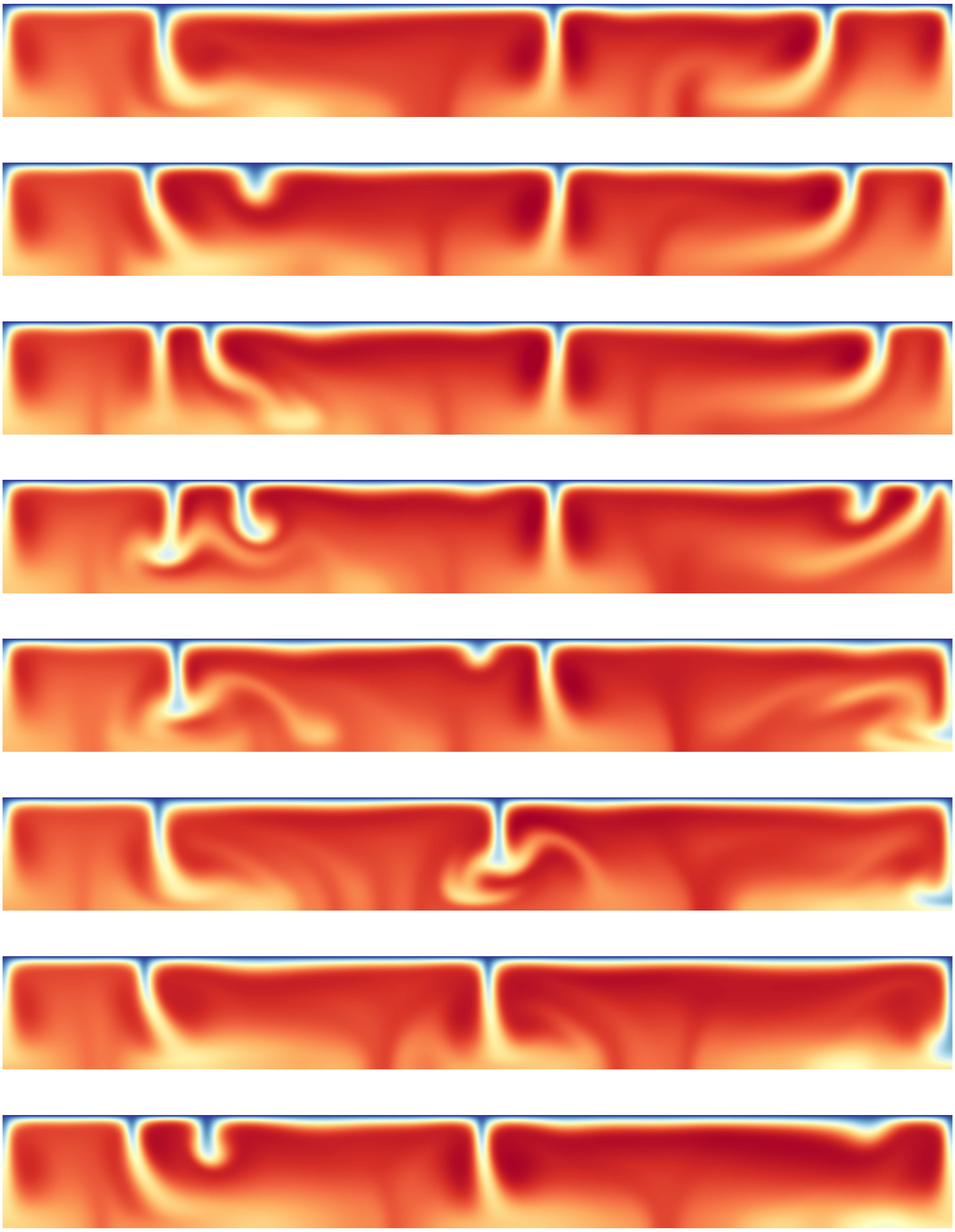

Figure 15: This plot shows a series of time snapshots of dimensionless temperature for a two dimensional numerical experiment where the fluid is heated entirely from within. Such plots are easily produced, as are those of the flow lines. As the shadowgraph of this type of convection shows (Figure 14h), the true planform of this type of convection consists of sinking plumes, whereas that of the two dimensional calculation is rolls. But the numerical experiment has captured the basic physics of the flow, which is time dependent and dominated by the behaviour of the sinking plumes. The time dependence occurs because the plumes slowly move together, and then suddenly join. New plumes then arise because of the instability of the cold boundary layer. Experiments like this one, but in three dimensions not two, have now largely replaced tank experiments. They have a number of important advantages. It is easy to study the temperature and flow without interfering with the circulation. The geophysical observables can be calculated directly, and the true rheological properties of the fluids can be used.

Temperature dependent viscosity

A larger number of stable planforms are allowed in fluids where the viscosity varies with temperature than those whose viscosity is constant. A useful fluid to use for such experiments is corn syrup, whose viscosity changes by about factor of 10^6 between $-10°C$ and $50°C$ (Figure 16).

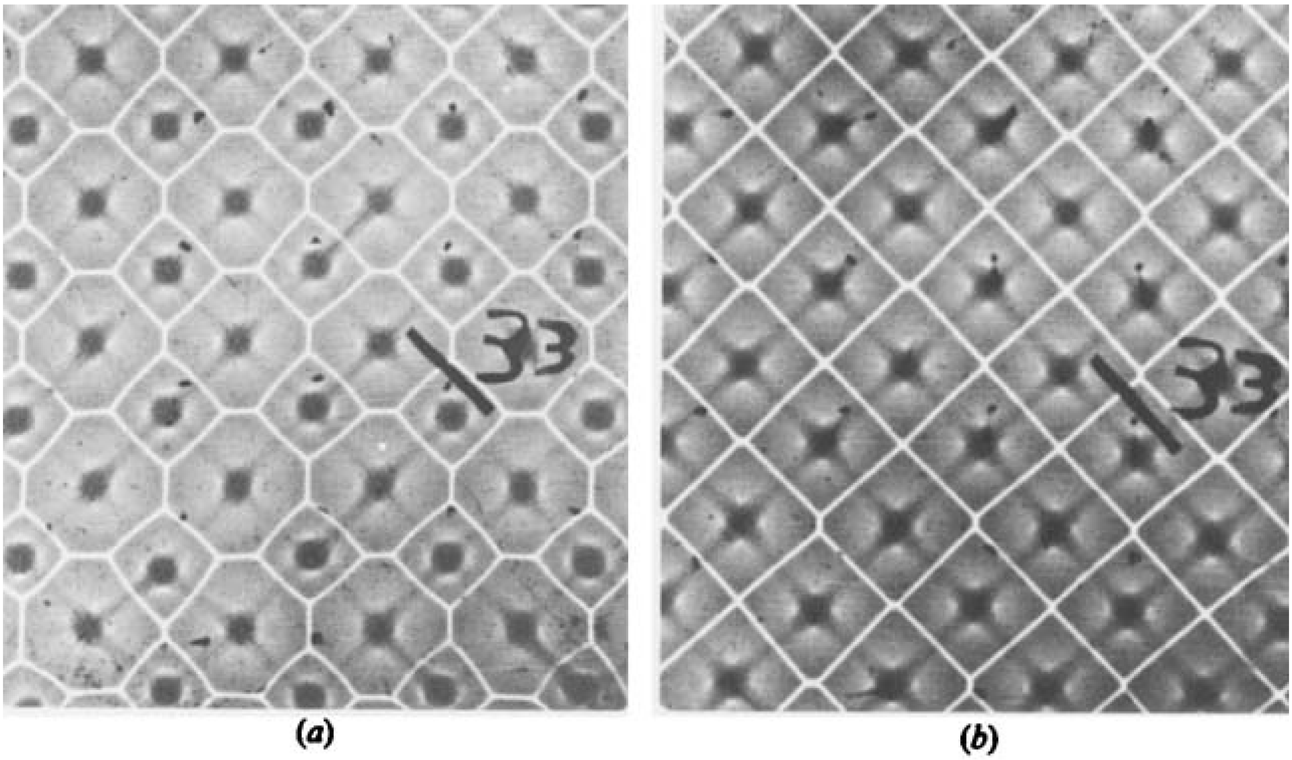

Figure 16: The mosaic instability observed in squares with k = 2.62 induced at a Rayleigh number of 14,800 and a viscosity variation of 50. The left picture shows the initial circulation, driven by heating at the centres of the octagons. The little square cells arose of their own accord, and grew at the expense of the octagons when the mask was removed (right picture). The length of the short black bar is twice the depth of the layer.

The effect of the temperature dependence of the viscosity on the flow is mostly easily seen in numerical experiments (Figure 17). At high Rayleigh numbers the only stable planform of the convection is spokes whatever the temperature dependence of the viscosity.

Online resources

There's an excellent interactive 2D thermal convection simulator available online at:
http://ian-r-rose.github.io/interactive_earth/

There are also a wide variety of movies of Rayleigh-Bénard convection on YouTube.

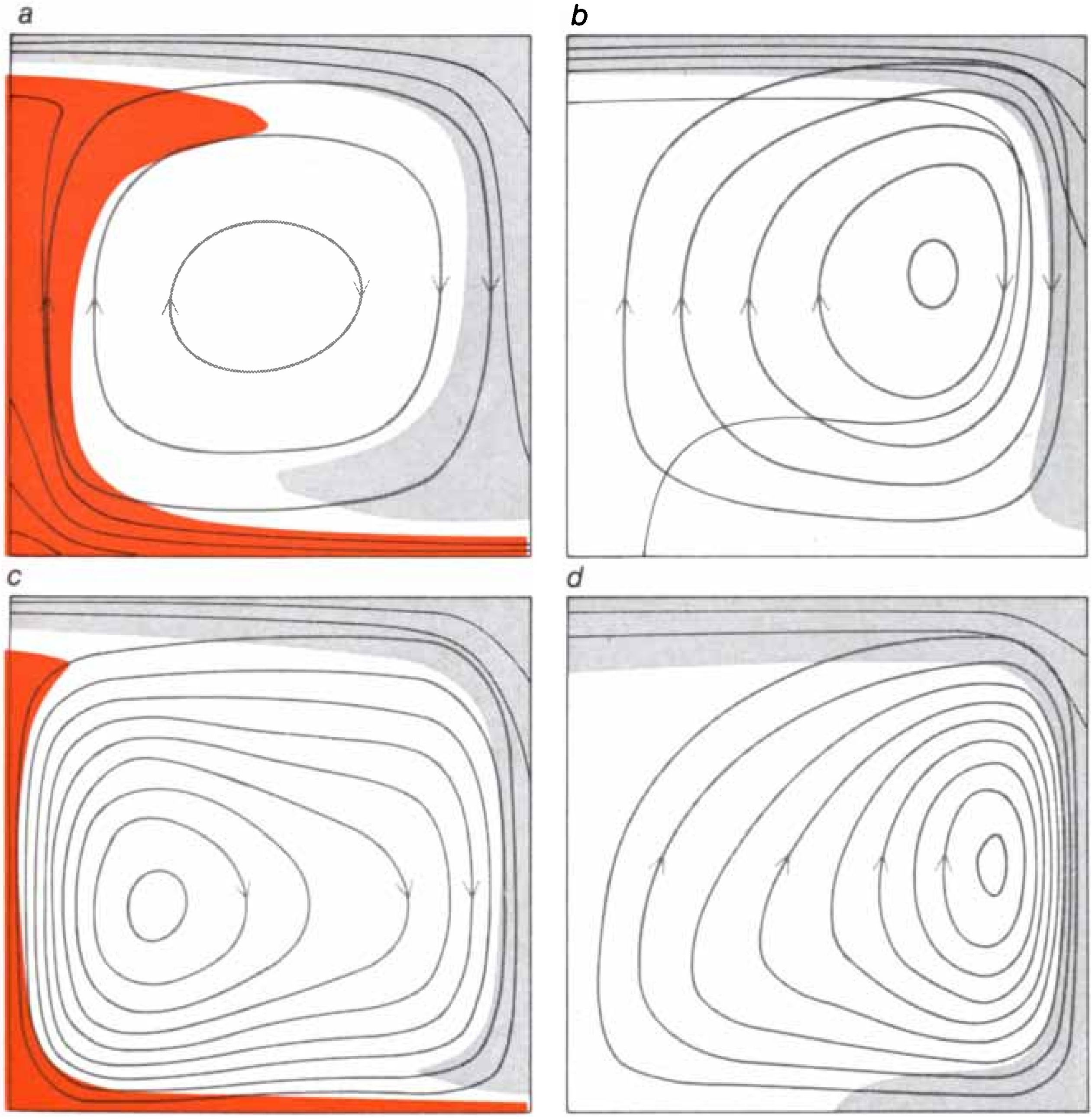

Figure 17: Numerical simulations of convection cells that show how the flow is affected by variations in fluid viscosity and by the mode of heating. When the viscosity is constant and the fluid is heated entirely from below (a), the rising and sinking sheets of fluid are symmetrical. When the heat is supplied uniformly throughout the interior of a fluid of constant viscosity (b), convection consists of thin sinking sheets of cold fluid with hot fluid upwelling everywhere else. When the viscosity decreases markedly with temperature, heating from below (c) produces a convection cell in which the hot rising sheet is significantly thinner than the cold sinking sheet. When the heat is supplied from within a variable-viscosity fluid (d), the pattern is little changed from that seen when the viscosity is constant.

Effect of shear

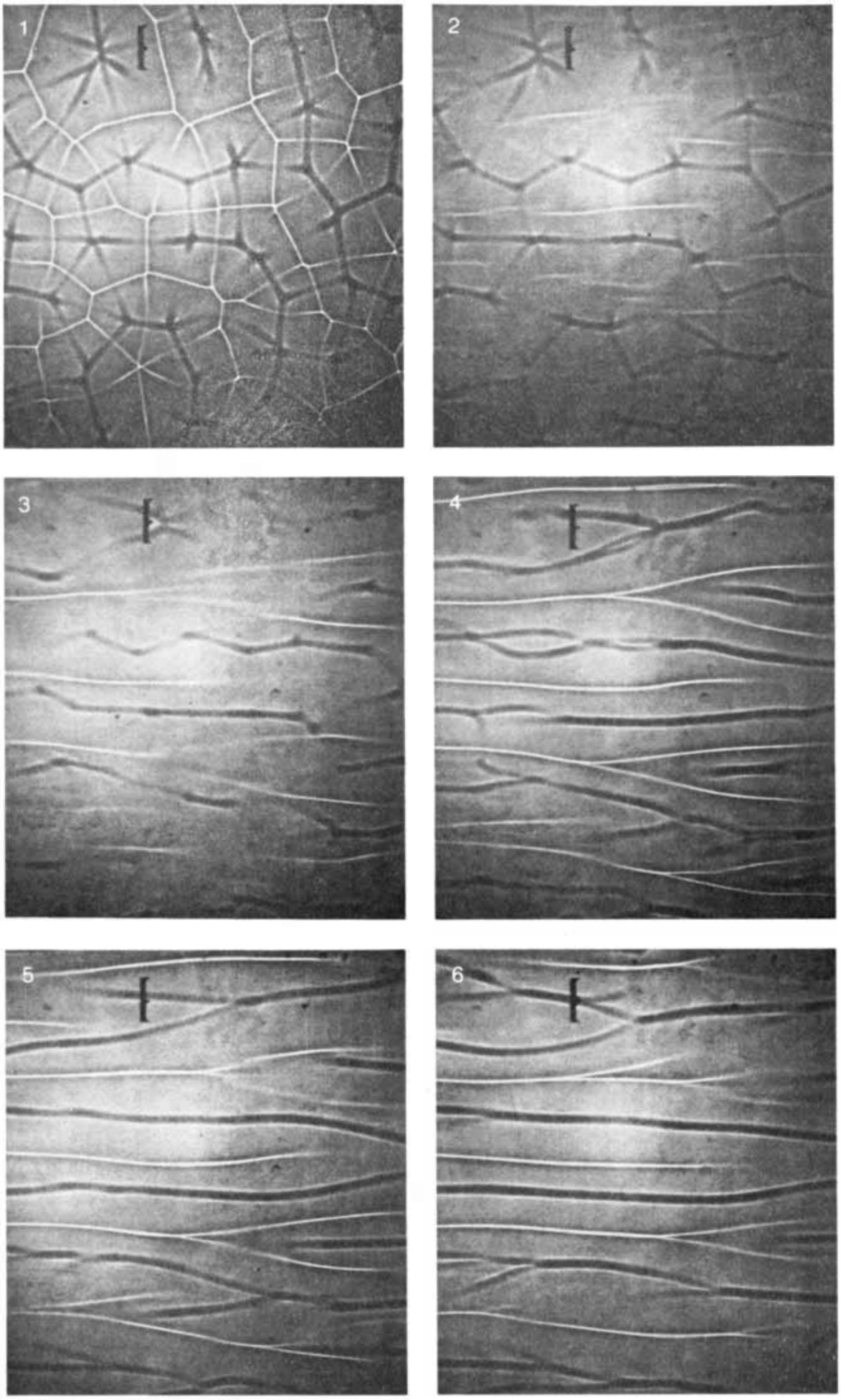

Figure 18: The effect of shear on the spoke pattern has been studied in the apparatus depicted in Figure 4. After a convective pattern characteristic of a Rayleigh number of 1.4×10^5 has become stabilised (first picture) the sheet of Mylar is set in motion to simulate a plate moving across the mantle. Successive pictures, made at equal intervals, show the rearrangement in pattern resulting from the motion of the Mylar plate as it travels from left to right. The shear converts the spoke pattern into rolls whose axes are parallel to the direction of plate motion. The small black bar at upper left in each photograph indicates twice the depth of the fluid.

The meaning of some commonly used terms in fluid dynamics

All branches of fluid dynamics make extensive use of *dimensionless numbers*, which are called after someone who was involved in the subject. The most important one for mantle convection is the *Rayleigh number* Ra

$$\mathrm{Ra} = \frac{\rho g \alpha \Delta T a^3}{\kappa \eta}, \tag{26}$$

where g is the acceleration due to gravity, α the thermal expansion coefficient, ΔT the temperature difference between the top and bottom of the layer of depth a, κ the thermal diffusivity, ρ the density and η the viscosity of the fluid. It is a measure of the ratio of the heat carried by the movement of the fluid to that carried by conduction when the convection is self sustaining. It is large if the convection is vigorous. *Convection* is flow maintained by density differences between the bulk of the fluid and those parts which are rising and sinking. Such forces are known as *buoyancy forces*. If they result from density differences resulting from temperature differences the flow is referred to as *thermal convection*. Variations in the concentration of a solute, such as salt in the oceans, produce *solutal convection*. Whether or not it results from buoyancy forces, any transport by the motion of the fluid itself, of for instance energy, momentum, a chemical species, is called *advective* transport, to distinguish it from conductive or diffusive transport. The *dynamic viscosity* η is related to the *kinematic viscosity* ν by

$$\nu = \frac{\eta}{\rho}. \tag{27}$$

ν governs the rate at which momentum diffuses. Compare the equation

$$\frac{DT}{Dt} \equiv \frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla) T = \kappa \nabla^2 T + \frac{H}{C_P} \tag{28}$$

for heat transport, with

$$\frac{D\mathbf{v}}{Dt} \equiv \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \nu \nabla^2 \mathbf{v} - \frac{1}{\rho} \nabla P - g\hat{\mathbf{z}} \tag{29}$$

for momentum transport. The nondiffusive terms on the right of the equations are source terms, of heat and momentum respectively.

If the fluid velocity V is given, as it is in plate tectonic problems, then the ratio of the heat transport by advection to that by diffusion is given by the *Peclet number*

$$\mathrm{Pe} = \frac{Vd}{\kappa} \sim \frac{\mathbf{v} \cdot \nabla T}{\kappa \nabla^2 T}, \tag{30}$$

where d is the thickness of the plate.

The *Prandtl number* Pr measures the ratio of the rate of diffusion of momentum of the fluid by viscosity to that of heat

$$\mathrm{Pr} = \frac{\nu}{\kappa}. \tag{31}$$

Pr for the mantle is $\sim 10^{24}$, but it is difficult to use fluids in the laboratory which have $\mathrm{Pr} > 10^4$ because they are so viscous that they will not pour out of their containers!

The *Reynolds number* Re measures the ratio of the advection of momentum by the fluid motion to that by diffusion

$$\mathrm{Re} = \frac{Va}{\nu} \sim \frac{\mathbf{v} \cdot \nabla \mathbf{v}}{\nu \nabla^2 \mathbf{v}}. \tag{32}$$

Re for the mantle is $\sim 10^{-19}$.

The *Nusselt number* Nu measures the ratio of the heat transported through the convecting layer to that which would be carried under the same conditions in the absence of convection.

The reason why numbers like these are important is because the behaviour of a fluid is controlled by their values and by nothing else. For instance, if you carry out a laboratory experiment at a particular Rayleigh number, all systems with this Rayleigh number will behave in exactly the same way, irrespective of their size. However the time scale over which changes take place obviously depends on the size of the system. Convecting layers of silicone oil with a large viscosity and a thickness of one or two centimetres are easy to use in the laboratory. The whole of geological time for a body the size of the earth then is equivalent to a fraction of an hour.

A useful concept in fluid dynamics is *vorticity* $\boldsymbol{\omega}$, which is defined by

$$\boldsymbol{\omega} = \nabla \times \mathbf{v}. \tag{33}$$

The vorticity of a rigid body rotating with angular velocity Ω is 2Ω. However, the definition of vorticity applies also to a fluid undergoing a general deformation.

A fluid is called *Newtonian* if the stress σ exerted is proportional to the strain rate $\dot{\epsilon}$ to the first power

$$\sigma \propto \dot{\epsilon} \tag{34}$$

Many common fluids are Newtonian. Under some conditions rocks are also, but if the deformation rate or stress is large enough to cause the dislocations to move then

$$\sigma \propto \dot{\epsilon}^{1/n} \tag{35}$$

where n has a value between 3 and 5 at high temperature. The material is then *non-Newtonian*. It is not yet clear whether the mantle motions involve large enough stresses to cause the behaviour to be non-Newtonian.

The geometry of a convecting layer is described by the *planform* and *aspect ratio* of the circulation. The planform (square, hexagonal, rolls) is the geometry when viewed from above. The aspect ratio is the ratio of the distance between rising and sinking regions to the depth of the layer. Some authors call the reciprocal of this ratio the aspect ratio.

A *linear equation* is one such as

$$\frac{dx}{dt} = -\lambda x \tag{36}$$

where x appears only as x, not x^2. In a *nonlinear equation* such as

$$\frac{dx}{dt} = -\lambda x^2 \tag{37}$$

this is not true. Linear equations can easily be solved by standard methods, whereas nonlinear are in general difficult to solve and their solutions sometimes behave in most unexpected ways.

Methods of solving convective equations

The equations governing convection are nonlinear. If the velocity is small, so that the temperature is almost the conductive temperature, an approximate set of equations can be obtained which are linear. This procedure is called *linearisation*. Nearly all the mathematical results which apply to convective systems have been obtained from these linearised equations. The Rayleigh numbers for which they are valid are between about 1000 and 3000. That for the mantle is at least 10^6, so they are little use for our purposes. They are very useful if you want to know whether a layer will convect or not. You introduce a small temperature disturbance and use the equations to find out whether the disturbance grows or decays. If it decays the layer is *stable*, or *stably stratified*. If it grows it can do so in two ways, exponentially or by growing oscillations.

The second is called *overstability*. The layer may be stable to a small disturbance but not to a larger one. This behaviour is known as *finite amplitude instability*. The initiation of a new subduction zone is an example of a finite amplitude instability, since large plates like the Pacific do not subduct of their own accord, even though the lithosphere is denser than the mantle below.

Most fluids undergoing convection can only be studied using the full nonlinear equations, because the Rayleigh number is large. The governing equations can be solved by numerical techniques, and the flow can be studied by properly scaled laboratory experiments. The temperature structure consists of thin *boundary layers*, adjacent to the top and bottom of the layer, across which the temperature varies rapidly, and an *interior flow*. Circular rising regions are referred to as *plumes* if they are produced by buoyancy forces, and as *jets* if they are maintained by the advection of momentum. Heat is transported by hot plumes and sheets which rise from the lower boundary layer and cold plumes and sheets sinking from the top. The thickness of the plumes and sheets is similar to that of the boundary layers. The flow may be *steady*, when it does not change with time, or *unsteady* if it does. The *overturn time* is the time taken for a particle to travel once round the convection cell, and can only be defined if the circulation is steady. If it is unsteady the flow can change slowly or quickly. If it changes rapidly there may be no clear pattern of convective cells and therefore is *disordered*. The terms *laminar* and *turbulent* are often used when discussing convection, but are best avoided, since fluid dynamicists generally use them in connection with the type of mathematical analysis which is used to describe the flow, rather than to describe the flow itself. I follow the definition of turbulence given by Herbert Huppert, a distinguished fluid dynamicist at DAMTP who taught me fluid dynamics: ‘turbulence is that which the speaker does not understand’.

An *adiabatic* process means that no heat is supplied or removed from the material, and should (strictly) be distinguished from an *isentropic* process (meaning constant entropy), which is both adiabatic and reversible. So rubbing your hands together to make them hot is an adiabatic, but not an isentropic, process, whereas the heating of air when you push on your bicycle pump is both adiabatic and isentropic. If you read the convective literature you will find that most people do not distinguish between adiabatic and isentropic processes (though they should!). The movement of fluid in the interior of a convecting layer at high Rayleigh numbers is too rapid for heat conduction to be important. Therefore the temperature only changes because of density changes due to pressure. The temperature gradient which arises under such conditions should be called an *isentropic* temperature gradient, though it is generally called an *adiabatic* temperature gradient. Much of the mantle beneath the plates is probably in this condition.