Advanced Statistical Mechanics (Part III Minor Topic)

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Prerequisites

There are no formal prerequisites, as all students are expected to have attended the Part II Thermal & Statistical Physics course (or equivalent, for the MASt cohort). It would be helpful to have attended the Part II Soft Condensed Matter option; for those who have not, the lectures cover the essential material.

Learning Outcomes and Assessment

This is a theoretical course that focuses on the microscopic processes that drive fluctuations, examining the fundamentals and modern techniques of statistical mechanics, with applications in non-equilibrium thermal physics, diffusion and viscoelasticity, genetics and evolution, and stock markets. However, the in-depth topics are all "physical", covering the mean first-passage time, non-Markov noise and anomalous diffusion, and the crossover between quantum and statistical uncertainty.

Synopsis

<u>Introduction</u>**:** Equilibrium statistical mechanics revised via partition functions / free energy, and via rates / detailed balance.

<u>Stochastic processes</u>**:** Markov process, Poisson and Wiener processes, Stochastic differential equations; conversion from Langevin to Fokker-Planck and Smoluchowski equations; Example 1 – Black-Scholes equation for the price of stocks. Detailed balance; Correlation functions; Fluctuation-dissipation theorem.

<u>Applications of Langevin dynamics</u>**:** Diffusion in shear flow. Example 2 – viscosity of a simple fluid. Ornstein-Uhlenbeck theory; Diffusion in external potentials; Periodic and random potentials; Kramers escape theory and mean first-passage time. Example 3 – "hitting a small target".

<u>Generalised Langevin dynamics</u>**:** Covariant formulation of multi-variable stochastic processes; Fokker-Planck and Smoluchowski equations; Non-Markov stochastic process, viscoelasticity and memory functions; Generalised (viscoelastic) Langevin equation and anomalous diffusion; Dissipation as coupling to an ensemble of oscillators; Mori equation; Liouville equation for dynamical variable and for distribution function (Heisenberg and Schrodinger formalisms). Example 4 – Kubo formula.

<u>Quantum crossover</u>**:** Quantum and stochastic probabilities; statistics of the density matrix; Dissipation as coupling to an ensemble of oscillators. Example 5 – Marcus equation for electron transfer. Example 6 – rate of escape over and tunnelling under the potential barrier

KEY BOOKS

Nonequilibrium Statistical Mechanics, Zwanzig R. (OUP 2001)
Introduction to Stochastic Processes, 2nd Edition, Lawler G. F. (Chapman & Hall 2006)
Brownian Motion: Fluctuations, Dynamics, and Applications, Mazo R. (OUP 2002)
Fokker-Planck Equation, Risken H. (Springer 1996)