

Ex 3:

$$a): m_{w_1} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad m_{w_2} = \begin{bmatrix} +1 \\ +1 \end{bmatrix},$$

$$\text{mean_diff} = m_{w_1} - m_{w_2} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$S_{w_1} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \quad S_{w_2} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix},$$

$$S_{\text{within}} = S_{w_1} + S_{w_2} = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$S_{\text{within}}^{-1} = \frac{1}{\det(S_{\text{within}})} \text{adj}(S_{\text{within}}) = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$w = S_{\text{within}}^{-1} \times \text{Mean_diff} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ -1 \end{bmatrix}$$

$$b): w^T \cdot w_{\perp} = 0$$

$$\begin{bmatrix} -\frac{1}{2} & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$x + 2y = 0.$$

$$u = \begin{bmatrix} x \\ y \end{bmatrix} \in \{ \mathbb{R}^2, x + 2y = 0 \}.$$