

# Futures pricing in electricity markets

based on Benth et al. (2014)

Awdesch Melzer

Institute of Finance  
Humboldt-Universität zu Berlin  
<https://www.wiwi.hu-berlin.de/en/professuren/bwl/finanz/>



# Agenda

## 1. Energy Markets

- ▶ History
- ▶ Spot Markets
- ▶ Economics of Spot Prices
- ▶ Futures Market

## 2. Electricity Derivative Pricing

- ▶ Classical Theory
- ▶ Spread Options
- ▶ Caps and Floors
- ▶ Swing Options

## Pre-Liberalisation

Liberalisation of the German electricity market started in April 1998

Before liberalisation: system based on calculatory costs, prices according to 'cost-plus' rule

- Integrated value-chain: production, grid, distribution
- Electricity production to secure supply within a regional monopole
- Long-term supply contracts
- No liquid market on the whole sale market
- Regulated consumer prices, regulated investments

## Post-Liberalisation

System is market based: higher volatility of prices, flexibility

- Unbundling of value-chain
- Power plants are used optimally (no obligation to secure supply)
- New players and products
- Trading in Long- and Short-positions on a liquid whole sale market
- Investments based on market expectations

## Markets

Power can be traded at

- Nordpool
- European Energy Exchange
- Energy Exchange Australia

All exchanges have established spot and futures markets.

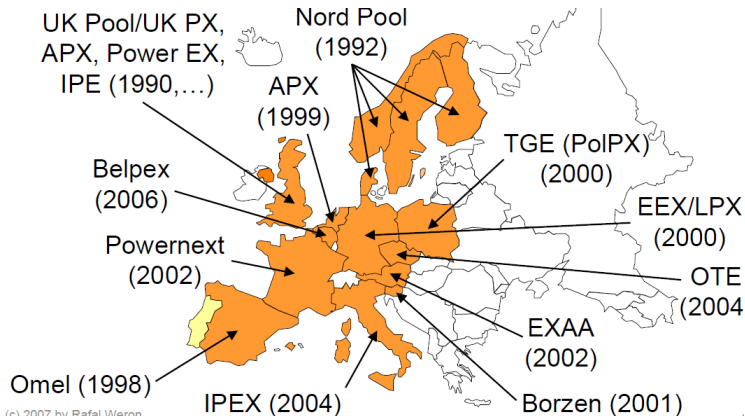
## EEX Spot Market

### Trading in

- Power
- Natural gas
- CO<sub>2</sub> emission rights
- Power day-ahead auctions (DE, AU, FR, CH)
  - ▶ 24 hours of respective next day traded in one-hour intervals or block orders:
  - ▶ Baseload 1-24h; Peakload 9-20h; Night 1-6h; Rush hour 17-20h; Business 9-16h
- Continuous power intraday trading (DE, FR), until 75 minutes before delivery (delivery on same or following day in single hours or blocks)

- Participants submit their price offers|bit curves
- EEX system prices are equilibrium prices that clear the market
- EEX day prices are the average of 24-single hours
- Similar structures can be found on other power exchanges (Nord Pool, APX, etc.)

## Electricity Markets in Europe





## EXAA Spot Market Price Processes

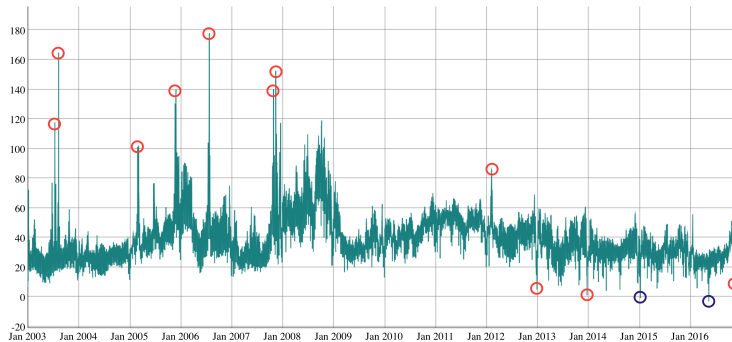



Figure 1: EXAA daily spot prices 2003-2016  EXAA

## Why is electricity special?

- Non-storable
- Homogeneous
- Produced through various methods
- Production should be when there is demand
- High fluctuation in demand
- No short-term elasticity in demand
- Negative prices are possible

## Basic economic concepts

- A producer produces only if marginal costs are met
- There is only one price of a homogeneous product
- Only producers with marginal costs below the market price will produce
- Production which only meets marginal costs (MC) does not cover the fixed costs

## Economics of Electricity Production

- MC for power plants  $\approx$  prices of fuel and CO<sub>2</sub> certificates
- Order of power plant use
  - ▶ wind
  - ▶ solar
  - ▶ water
  - ▶ nuclear
  - ▶ coal
  - ▶ gas
  - ▶ oil
- To meet demand power plants are added in order of increasing MC (merit order)
- The marginal power plant fixes the market price  
**for all plants in use**

## Merit order (no trade)

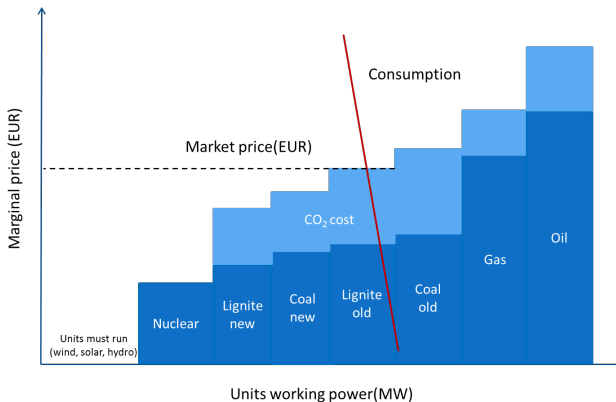


Figure 2: Merit order. Source: Mentor EBS

## EEX Futures Market

### Traded products

- ▣ Futures contracts for power, natural gas, emissions, coal, wind power
- ▣ Phelix Futures and Phelix Baseload or Peakload montly power index for the current month, the next nine months, eleven quarters and six years with cash settlement
- ▣ Baseload and Peakload FR/DE Power Futures for the current month, next six months, seven quarters and six years with physical settlement, obliging for continuous delivery of 1MW during a month, quarter, a year
- ▣ Actively exchange traded: 7 months, 5 quarters, 2-3 years
- ▣ OTC transactions

## EEX Options on Futures

### Traded products

- European-style Phelix Options which lead to opening of the corresponding Phelix Futures position if exercised
- Phelix Futures and Phelix Baseload or Peakload montly power index for the current month, the next nine months, eleven quarters and six years with cash settlement
- Baseload and Peakload FR/DE Power Futures for the current month, next six months, seven quarters and six years with physical settlement, obliging for continuous delivery of 1MW during a month, quarter, a year
- Actively exchange traded: 7 months, 5 quarters, 2-3 years
- OTC transactions

## Spot-Forward Relationship: classical theory

Under the no-arbitrage assumption we have the spot-forward relationship

$$F(t, T) = S(t) \exp\{(r - y)(T - t)\}$$

where  $r$  is the interest rate at time  $t$  for maturity  $T$  and  $y$  is the convenience yield (on holding inventories).



- In the stochastic model this means

$$F(t, T) = E_Q\{S(t)|\mathcal{F}_t\}$$

where  $\mathcal{F}_t$  is the accumulated available market information (in most models the information generated by the spot price)

- $Q$  is the risk-neutral probability
  - ▶ discounted spot price is a  $Q$ -martingale
  - ▶ or the expected return under  $Q$  is  $r$

We observe backwardation: Futures prices are below spot price

- Producers accept paying a premium for securing future production
- This may be caused by hedging pressure for long term investments
- Convenience yield larger than risk-free rate

Most models give either normal backwardation or contango

- No stochastic change of sign (risk premium)
- True even for jump models

## Example: Energy fuels

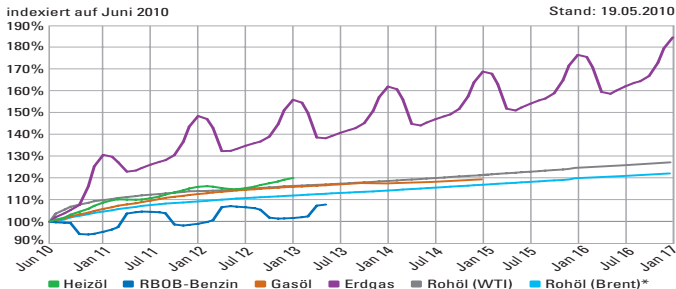


Figure 3: Most energy fuels show contango. Source: Bloomberg L.P.

- Storage of spot is not possible (only indirectly in water reservoirs or expensive large scale accumulators)
- Delivery periods for Futures
- Buy-and-Hold strategy fails
- No foundation for classical spot forward relations

In electricity markets one typically observes that,

- for 'long' dated forward contracts, markets are in backwardation (forward below spot)
- for 'shorter' maturities the markets are in contango (forward above spot)

## Market Risk Premium

The market risk premium or forward bias  $\pi(t, T)$  relates forward and expected spot prices

It is defined as the difference, calculated at time  $t$ , between the forward  $F(t, T)$  at time  $t$  with delivery  $T$  and expected spot price:

$$\pi(t, T) = F(t, T) - E^P\{S(T)|\mathcal{F}_t\}$$

Here  $E^P$  is the expectation operator, under the historical measure  $P$ , with information up until time  $t$  and  $S(T)$  is the spot price at time  $T$

## Market risk premium

- Exposure to the market will differ both between producers and retailers as well as within their own group
- For instance, a large producer will generally be exposed to market uncertainty for a longer period of time, perhaps determined by the remaining life of the assets, whilst retailers will tend to make decisions based on a shorter time scale
- So the need for risk-diversification has a temporal dimension



- These differences in the desire to hedge positions are employed to explain the market risk premium and its sign
- Retailers are less incentivised to contract commodity forwards the further out we look into the market
- In contrast, on the producers' side the need to hedge in the long-term does not fade away as quickly

- We associate situations where  $\pi(t, T) > 0$  with the fact that retailers' desire to cover their positions 'outweighs' those of the producers resulting in a positive market risk premium
- The mirror image is therefore one where the producers' desire to hedge their positions outweighs that of the retailers resulting in a negative market risk premium

## The electricity futures pricing model

Assume that the spot price follows a mean-reverting multi-factor additive process

$$S(t) = \Lambda(t) + Z(t) + Y(t) \quad (1)$$

$\Lambda(t)$  - deterministic seasonal spot price level

$Z(t)$  - low-frequency non-stationary dynamics

$Y(t)$  - short term variations

Benth et al. (2014)

## Stable CARMA( $p, q$ )-Lévy process

$$a(D)\mathbf{Y}_t = b(D)DL(t), \quad D \stackrel{\text{def}}{=} \frac{d}{dt},$$

where the auto-regressive polynomial is given by

$$P(z) = z^p + a_1 z^{p-1} + \dots + a_p$$

and the moving-average polynomial by

$$Q(z) = b_0 + b_1 z^q + \dots + b_{p-1} z^{p-1}.$$

$$Y_t = \mathbf{b}^\top \mathbf{X}_t \quad \text{state equation} \quad (2)$$

$$d\mathbf{X}_t = (\mathbf{A}\mathbf{X}_t)dt + \mathbf{e}_p dL_t \quad \text{observation equation} \quad (3)$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & & & \ddots & \ddots \\ 0 & \dots & \dots & 0 & 1 \\ -\alpha_p & -\alpha_{p-1} & \dots & & -\alpha_1 \end{pmatrix} \quad \mathbf{e}_p = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{p-2} \\ b_{p-1} \end{pmatrix}$$

With Brownian motion  $B_t$  instead of the Lévy process  $L_t$  we have Gaussian CARMA( $p, q$ )

## Characteristic function of $L(t)$

Suppose the Lévy processes are exponentially integrable with  $\kappa > 0$  such that

$$\int_{|z| \geq 1} \exp(\tilde{\kappa} z) l_j(dz) < \infty,$$

for all  $\tilde{\kappa} \leq \kappa$  and  $j = 1, \dots, n$ .

This implies that the spot price process  $S(t)$  has exponential moments up to order  $\kappa$ . The characteristic function of  $L(t)$  with log exponential moments

$$\log E[\exp\{izL_j(t)\}] = t\phi_L(z)$$

is defined as

$$\phi_L(z) = \begin{cases} -\gamma^\alpha |z|^\alpha \left\{ 1 - i\beta(\operatorname{sign} z) \tan\left(\frac{\pi\alpha}{2}\right) \right\} + i\mu z & \text{for } \alpha \neq 1 \\ -\gamma |z| \left\{ 1 + i\beta \frac{2}{\pi} (\operatorname{sign} z) \log |z| \right\} + i\mu z & \text{for } \alpha = 1 \end{cases}$$

## Futures price dynamics

The futures price  $f(t, \tau)$  is defined as

$$f(t, \tau) = \mathbb{E}_Q[S(\tau) | \mathcal{F}_t], \quad 0 \leq t \leq \tau < \infty,$$

where  $Q$  is the risk-neutral probability measure, with  $S(\tau) \in L^1(Q)$ .

## The Bessembinder-Lemon Model

- One-period model
- Power companies are able to forecast demand in the immediate future with precision
- $N_P$  identical producers;  $N_R$  identical retailers that buy power in the wholesale market and sell it to final consumers at fixed unit price
- $P_R$  fixed unit price that consumers pay
- $Q_{R_i}$  an exogenous random variable that denotes the realised demand for retailer  $i$



## The cost function

- Each producer  $i$  has cost function

$$TC_i = F + \frac{a}{c}(Q_{P_i})^c,$$

where  $F$  are fixed costs,  $Q_{P_i}$  is the output of producer  $i$ , and  $c \leq 2$

- The cost function implied that the marginal production costs increase with output
- If  $c > 2$  marginal costs increase at an increasing rate with output
- Moreover, the distribution of power prices will be positively skewed even when the distribution of power demand is symmetric

## Clearing prices

- First, assume that forward prices are given
- Obtain optimal behaviour in the spot market
- Work back and find optimal positions in the forward market

## The wholesale spot market

- Producers sell to retailers who in turn distribute to power consumers
- $P_W$  denotes the wholesale spot price,  $Q_{P_i}^W$  quantity sold by producer  $i$  in the wholesale spot market,  $Q_{P_i}^F$  quantity that producer  $i$  has agreed to deliver (purchase if negative) in the forward market at the fixed forward price  $P_F$ .
- The ex-post profit of producer  $i$  is given by

$$\pi_{P_i} = P_W Q_{P_i}^W + P_F Q_{P_i}^F - F - \frac{a}{c} (Q_{P_i})^c$$

where each producer's physical production,  $Q_{P_i}$  is the sum of its spot and forward sales  $Q_{P_i}^W + Q_{P_i}^F$

- Retailers buy in the real-time wholesale market the difference between realised retail demand and their forward positions
- $Q_{P_i}^F$  quantity sold (purchased if negative) forward by retailer  $j$ ,  $P_R$  fixed retail price per unit
- The ex-post profit for each retailer is

$$\pi_{P_j} = P_R Q_{R_j} + P_F Q_{R_j}^F - P_W (Q_{R_j} + Q_{R_j}^F),$$

- The profit maximising quantity for producer  $i$  is (FOC wrt  $Q_{P_i}^W$ )

$$Q_{P_i}^W = \left( \frac{P_W}{a} \right)^{\frac{1}{c-1}} - Q_{P_i}^F$$

- The equilibrium total retail demand is equal to total production and forward contracts are in zero net supply
- Hence we must have that summing over all producers production must equal total demand from retailers

$$N_P \left( \frac{P_W}{a} \right)^{\frac{1}{c-1}} = \sum_{j=1}^{N_R} Q_{R_j}$$

- Therefore the market-clearing wholesale price is

$$P_W = a \left( \frac{Q^D}{N_P} \right)^{c-1},$$

where  $Q^D = \sum_{j=1}^{N_R} Q_{R_j}$  is total system demand. We see that when  $c > 2$  an increase in demand has a disproportionate effect on power prices

- Each producers sale in the wholesale market is

$$Q_{P_i}^W = \frac{Q^D}{N_P} - Q_{P_i}^F$$

## Demand for forward positions

- Producers profit (with no forwards) is

$$\rho_{P_i} = P_W \frac{Q^D}{N_P} - F - \frac{a}{c} \left( \frac{Q^D}{N_P} \right)^c$$

- Retailers profit (with no forwards) is

$$\rho_{R_j} = P_R Q_{R_j} - P_W Q_{R_j}$$

## Mean-Variance Analysis for optimal forward position

- Assume that market players

$$\max_{Q_{\{P_i, R_j\}}^F} E[\pi_{\{P_i, R_j\}}] - \frac{A}{2} \text{Var}[\pi_{\{P_i, R_j\}}]$$

where, for example, producers have the profit function

$$\pi_{P_i} = \rho_{P_i} + P^F Q^F - P_W Q^F$$

FOCs imply

$$Q_{P_i, R_j}^F = \frac{P^F - E(P_W)}{A \text{Var}(P_W)} + \frac{\text{Cov}[\rho_{\{P_i, R_j\}}, P_W]}{\text{Var}(P_W)}$$



- The optimal forward position contains two components
  - ▶ The first term reflects the position taken in response to the bias  $P^F - E(P_W)$
  - ▶ The second term is the quantity sold or bought forward to minimize the variance of profits
- Forward hedging can reduce risk precisely because the covariance term is generally non-zero

## Equilibrium forward price

- Using the market clearing condition one can show that

$$P_F = E(P_W) - \frac{N_P}{N c a^{\frac{1}{c-1}}} \left\{ c P_R \text{Cov}(P_W^{\frac{1}{c-1}}, P_W) - \text{Cov}(P_W^{\frac{c}{c-1}}, P_W) \right\},$$

where  $N = \frac{N_R + N_P}{A}$  reflects the number of firms in the industry and the degree to which they are concerned with risk

- The forward price will be less than the expected wholesale price, if the first term in brackets, which reflects retail risk, is larger than the second term, which reflects production cost risk

## Forward bias

We can approximate  $P^{\frac{1}{c-1}}$  and  $P^{\frac{c}{c-1}}$  using a Taylor series to see

$$P_F = E(P_W) - \alpha \text{Var}(P_W) + \gamma S(P_W)$$

where

$$\alpha = \frac{N_P(\frac{c}{c-1})}{N_C a^{\frac{1}{c-1}}} \left[ \{E(P_W)\}^{\frac{1}{c-1}} - P_R \{E(P_W)\}^{\frac{c+1}{c-1}} \right]$$

and

$$\gamma = \frac{N_P(\frac{c}{c-1})}{2N_C a^{\frac{1}{c-1}}} \left[ \frac{1}{c-1} \{E(P_W)\}^{\frac{c+1}{c-1}} - \frac{c+1}{c-1} P_R \{E(P_W)\}^{\frac{2}{c-1}} \right]$$

We see that  $\alpha < 0$ , since  $E(P_W) < P_R$

## Forward bias

- It must be the case that  $E(P_W) < P_R$
- If the distribution of spot prices is not skewed,  $P_F < E(P_W)$ 
  - ▶ The downward bias in the forward price in the zero-skewness case reflects retailer's net hedging demand, who want to sell in the forward market
  - ▶ The profits of power retailers are positively exposed, on average, because more retail power is sold when  $P_W$  is high
  - ▶ To reduce risk, retailers want to sell forwards
- Typically we have skewness ( $c > 2$ ),
  - ▶ So  $\gamma > 0$  and the forward price increases with increasing skewness
  - ▶ This reflects the fact that the industry wants to hedge against price spikes

## Further Information

BLABLABLA

## For Further Reading



Tobias Oetiker, Hubert Partl, Irene Hyna and Elisabeth Schlegl  
*The Not So Short Introduction to L<sup>A</sup>T<sub>E</sub>X2<sub>ε</sub>*  
available on [www.ctan.org](http://www.ctan.org), 2008



Scott Pakin  
*The Comprehensive L<sup>A</sup>T<sub>E</sub>X Symbol List*  
available on [www.ctan.org](http://www.ctan.org), 2008



Frank Mittelbach and Michel Goossens  
*The L<sup>A</sup>T<sub>E</sub>X Companion – 2nd ed.*  
Addison-Wesley, 2004

## For Further Reading



Mark Trettin and Jürgen Fenn

*An essential guide to  $\LaTeX$ 2e usage*

available on [www.ctan.org](http://www.ctan.org), 2007



Wikipedia Wiki Books

*LaTeX-Wörterbuch: InDeX*

available on [www.wikipedia.de](http://www.wikipedia.de)



Till Tantau

*User Guide to the Beamer Class, Version 3.07*

available on [www.sourceforge.net](http://www.sourceforge.net), 2007