Futures pricing in electricity markets

based on Benth et al. (2014)

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Agenda

1. Energy Markets
   - History
   - Spot Markets
   - Economics of Spot Prices
   - Futures Market
2. Electricity Derivative Pricing
   - Classical Theory
   - Electricity futures pricing model
   - Empirical Results
   - Conclusions

Pre-Liberalisation

Liberalisation of the German electricity market started in April 1998

Before liberalisation: system based on calculatory costs, prices according to ‘cost-plus’ rule

- Integrated value-chain: production, grid, distribution
- Electricity production to secure supply within a regional monopole
- Long-term supply contracts
- No liquid market on the whole sale market
- Regulated consumer prices, regulated investments

Post-Liberalisation

System is market based: higher volatility of prices, flexibility

- Unbundling of value-chain
- Power plants are used optimally (no oblication to secure supply)
- New players and products
- Trading in Long- and Short-positions on a liquid whole sale market
- Investments based on market expectations

Markets

Power can be traded at

- Nordpool
- European Energy Exchange
- Energy Exchange Austraia

All exchanges have established spot and futures markets.

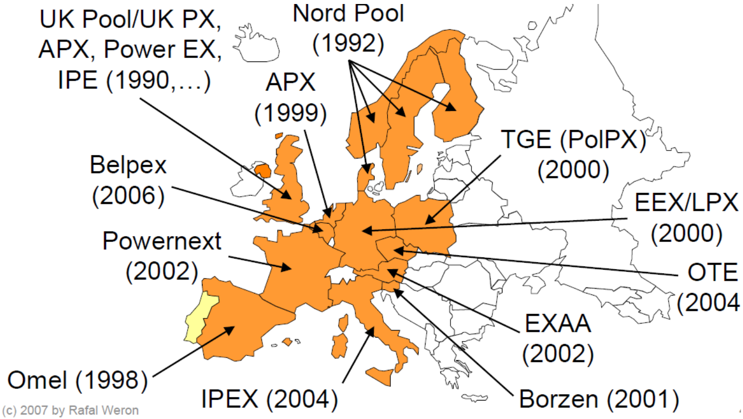
EEX Spot Market

Trading in

- Power
- Natural gas
- CO_2 emission rights
- Power day-ahead auctions (DE, AU, FR, CH)
  - 24 hours of respective next day traded in one-hour intervals or block orders:
  - Baseload 1-24h; Peakload 9-20h; Night 1-6h; Rush hour 17-20h; Business 9-16h
- Continuous power intraday trading (DE, FR), until 75 minutes before delivery (delivery on same or following day in single hours or blocks)

- Participants submit their price offers|bit curves
- EEX system prices are equilibrium prices that clear the market
- EEX day prices are the average of 24-single hours
- Similar structures can be found on other power exchanges (Nord Pool, APX, etc.)

Electricity Markets in Europe



EXAA Spot Market Price Processes

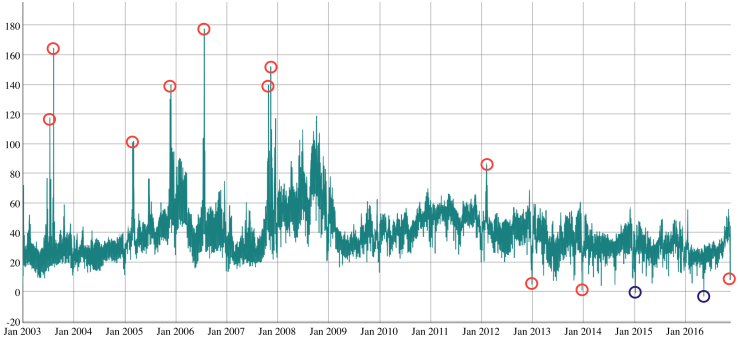


Figure 1: EXAA daily spot prices 2003-2016 EXAA

Why is electricity special?

- Non-storable
- Homogeneous
- Produced through various methods
- Production should be when there is demand
- High fluctuation in demand
- No short-term elasticity in demand
- Negative prices are possible

Basic economic concepts

- A producer produces only if marginal costs are met
- There is only one price of a homogeneous product
- Only producers with marginal costs below the market price will produce
- Production which only meets marginal costs (MC) does not cover the fixed costs

Economics of Electricity Production

- MC for power plants $\approx$ prices of fuel and CO_2 certificates
- Order of power plant use
  - wind
  - solar
  - water
  - nuclear
  - coal
  - gas
  - oil
- To meet demand power plants are added in order of increasing MC (merit order)
- The marginal power plant fixes the market price **for all plants in use**

Merit order (no trade)

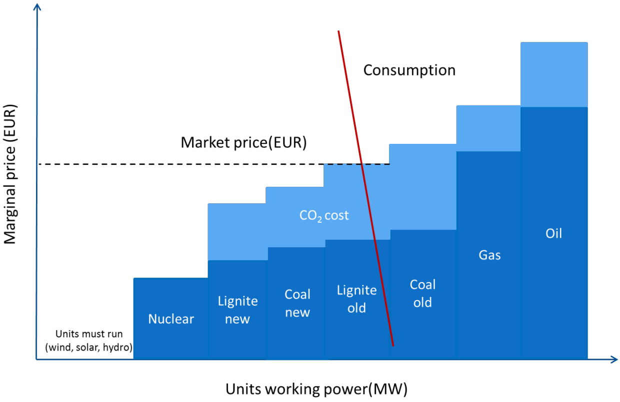


Figure 2: Merit order. Source: Mentor EBS

EEX Futures Market

Traded products

- Futures contracts for power, natural gas, emissions, coal, wind power
- Phelix Futures and Phelix Baseload or Peakload montly power index for the current month, the next nine months, eleven quarters and six years with cash settlement
- Baseload and Peakload FR/DE Power Futures for the current month, next six months, seven quarters and six years with physical settlement, obliging for continuous delivery of 1MW during a month, quarter, a year
- Actively exchange traded: 7 months, 5 quarters, 2-3 years
- OTC transactions

Spot-Forward Relationship: classical theory

Under the no-arbitrage assumption we have the spot-forward relationship

$$F(t,T) = S(t)\exp\{(r-y)(T-t)\}$$

where r is the interest rate at time t for maturity T and y is the convenience yield (on holding inventories).

- In the stochastic model this means

$$F(t, T) = \mathsf{E}_Q\{S(t)|\mathcal{F}_t\}$$

  where $\mathcal{F}_t$ is the accumulated available market information (in most models the information generated by the spot price)
- Q is the risk-neutral probability
  - discounted spot price is a Q-martingale
  - or the expected return under Q is r

We observe backwardation: Futures prices are below spot price

- Producers accept paying a premium for securing future production
- This may be caused by hedging pressure for long term investments
- Convenience yield larger than risk-free rate

Most models give either normal backwardation or contango

- No stochastic change of sign (risk premium)
- True even for jump models

Example: Energy fuels

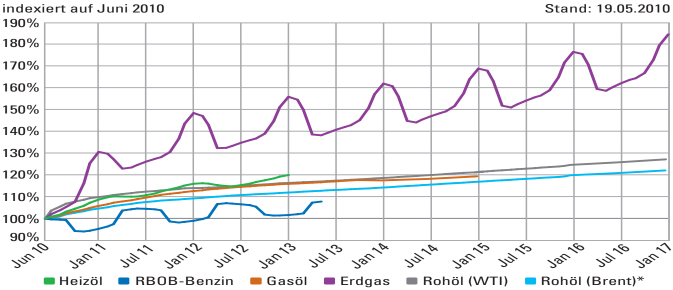


Figure 3: Most energy fuels show contango. Source: Bloomberg L.P.

Example: Metals

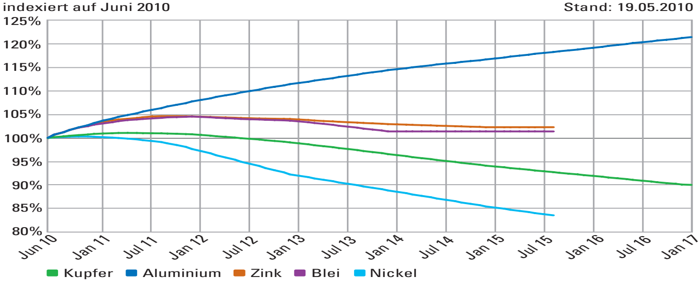


Figure 4: Only aluminium shows untypical contango behaviour. Source: Bloomberg L.P.

Example: Crop

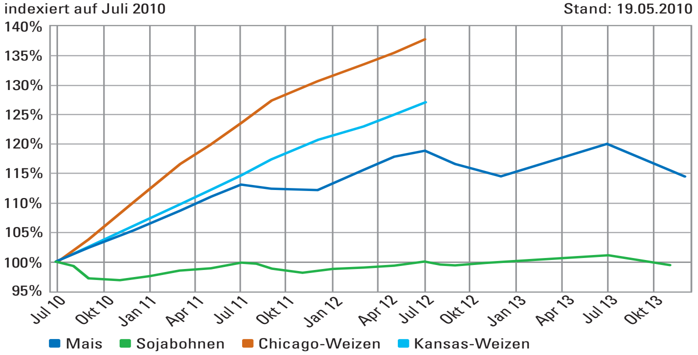


Figure 5: Contango and backwardation for crop. Source: Bloomberg L.P.

Example: Precious Metals

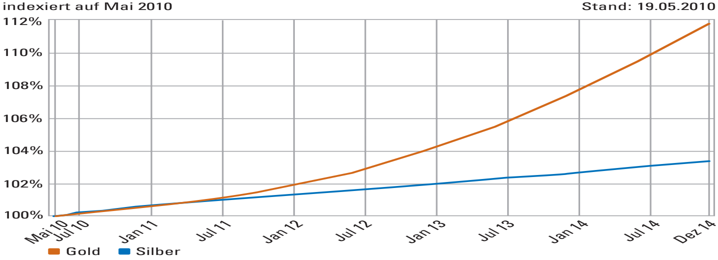


Figure 6: Contango for precious metals. Source: Bloomberg L.P.

The shape of commodities forward curves for different delivery periods indicates the market players' (producers, retailers and speculators) 'attitudes' towards risk bearing in these markets

Contrast to equity: Here, if interest rates and dividends are assumed deterministic, simple no-arbitrage arguments are employed to show that the arbitrage-free forward price will be the cost of borrowing net of collected dividends yielded by the equity.

- Storage of spot is not possible (only indirectly in water reservoirs or expensive large scale accumulators)
- Delivery periods for Futures
- Buy-and-Hold strategy fails
- No foundation for classical spot forward relations

In electricity markets one typically observes that,

- for ‘long’ dated forward contracts, markets are in backwardation (forward below spot)
- for ‘shorter’ maturities the markets are in contango (forward above spot)

Market Risk Premium

The market risk premium or forward bias $\pi(t, T)$ relates forward and expected spot prices

It is defined as the difference, calculated at time t, between the forward $F(t, T)$ at time t with delivery T and expected spot price:

$$\pi(t, T) = F(t, T) - \mathsf{E}^P\{S(T)|\mathcal{F}_t\}$$

Here E^P is the expectation operator, under the historical measure P, with information up until time t and $S(T)$ is the spot price at time T

FEB4

The Bessembinder-Lemmon Model

- One-period model
- Power companies are able to forecast demand in the immediate future with precision
- N_P identical producers; N_R identical retailers that buy power in the wholesale market and sell it to final consumers at fixed unit price
- P_R fixed unit price that consumers pay
- Q_{R_i} an exogenous random variable that denotes the realised demand for retailer i

The cost function

- Each producer i has cost function

$$TC_i = F + \frac{a}{c}(Q_{P_i})^c,$$

  where F are fixed costs, Q_{P_i} is the output of producer i, and $c \leq 2$
- The cost function implied that the marginal production costs increase with output
- If $c > 2$ marginal costs increase at an increasing rate with output
- Moreover, the distribution of power prices will be positively skewed even when the distribution of power demand is symmetric

Clearing prices

- First, assume that forward prices are given
- Obtain optimal behaviour in the spot market
- Work back and find optimal positions in the forward market

The wholesale spot market

- Producers sell to retailers who in turn distribute to power consumers
- P_W denotes the wholesale spot price, $Q^W_{P_i}$ quantity sold by producer i in the wholesale spot market, $Q^F_{P_i}$ quantity that producer i has agreed to deliver (purchase if negative) in the forward market at the fixed forward price P_F.
- The ex-post profit of producer i is given by

$$\pi_{P_i} = P_W Q^W_{P_i} + P_F Q^F_{P_i} - F - \frac{a}{c}(Q_{P_i})^c$$

where each producer's physical production, Q_{P_i} is the sum of its spot and forward sales $Q^W_{P_I} + Q^F_{P_I}$

- Retailers buy in the real-time wholesale market the difference between realised retail demand and their forward positions
- $Q^F_{P_i}$ quantity sold (purchased if negative) forward by reatiler j, P_R fixed retail price per unit
- The ex-post profit for each retailer is

$$\pi_{P_j} = P_R Q_{R_j} + P_F Q^F_{R_j} - P_W(Q_{R_j} + Q^F_{R_j}),$$

- The profit maximising quantity for producer i is (FOC wrt $Q^W_{P_i}$)

$$Q^W_{P_i} = \left(\frac{P_W}{a}\right)^{\frac{1}{c-1}} - Q^F_{P_i}$$

- The equilibrium total retail demand is equal to total production and forward contracts are in zero net supply
- Hence we must have that summing over all producers production must equal total demand from retailers

$$N_P \left(\frac{P_W}{a} \right)^{\frac{1}{c-1}} = \sum_{j=1}^{N_R} Q_{R_j}$$

- Therefore the market-clearing wholesale price is

$$P_W = a\left(\frac{Q^D}{N_P}\right)^{c-1},$$

where $Q^D = \sum_{j=1}^{N_R} Q_{R_j}$ is total system demand. We see that when $c > 2$ an increase in demand has a disproportionate effect on power prices

- Each producers sale in the wholesale market is

$$Q_{P_i}^W = \frac{Q^D}{N_P} - Q_{P_i}^F$$

Demand for forward positions

- Producers profit (with no forwards) is

$$\rho_{P_i} = P_W \frac{Q^D}{N_P} - F - \frac{a}{c}\left(\frac{Q^D}{N_p}\right)^c$$

- Retailers profit (with no forwards) is

$$\rho_{R_j} = P_R Q_{R_j} - P_W Q_{R_j}$$

Mean-Variance Analysis for optimal forward position

- Assume that market players

$$\max_{Q^F_{\{P_i,R_j\}}} \mathrm{E}[\pi_{\{P_i,R_j\}}] - \frac{A}{2}\mathrm{Var}[\pi_{\{P_i,R_j\}}]$$

where, for example, producers have the profit function

$$\pi_{P_i} = \rho_{P_i} + P^F Q^F - P_W Q^F$$

FOCs imply

$$Q^F_{P_i,R_j} = \frac{P^F - \mathrm{E}(P_W)}{A\,\mathrm{Var}(P_W)} + \frac{\mathrm{Cov}[\rho_{\{P_i,R_j\}}, P_W]}{\mathrm{Var}(P_W)}$$

- The optimal forward position contains two components
  - The first term reflects the position taken in response to the bias $P^F - \mathrm{E}(P_W)$
  - The second term is the quantity sold or bought forward to minimize the variance of profits
- Forward hedging can reduce risk precisely because the covariance term is generally non-zero

Equilibrium forward price

- Using the market clearing condition one can show that

$$P_F = \mathsf{E}(P_W) - \frac{N_P}{Nca^{\frac{1}{c-1}}} \left\{ cP_R \operatorname{Cov}(P_W^{\frac{1}{c-1}}, P_W) - \operatorname{Cov}(P_W^{\frac{c}{c-1}}, P_W) \right\},$$

  where $N = \frac{N_R + N_P}{A}$ reflects the number of firms in the industry and the degree to which they are concerned with risk

- The forward price will be less than the expected wholesale price, if the first term in brackets, which reflects retail risk, is larger than the second term, which reflects production cost risk

Forward bias

We can approximate $P^{\frac{1}{c-1}}$ and $P^{\frac{c}{c-1}}$ using a Taylor series to see

$$P_F = \mathrm{E}(P_W) - \alpha \,\mathrm{Var}(P_W) + \gamma S(P_W)$$

where

$$\alpha = \frac{N_P(\frac{c}{c-1})}{Nca^{\frac{1}{c-1}}} \left[\{\mathrm{E}(P_W)\}^{\frac{1}{c-1}} - P_R \{\mathrm{E}(P_W)\}^{\frac{c+1}{c-1}} \right]$$

and

$$\gamma = \frac{N_P(\frac{c}{c-1})}{2Nca^{\frac{1}{c-1}}} \left[\frac{1}{c-1} \{\mathrm{E}(P_W)\}^{\frac{c+1}{c-1}} - \frac{c+1}{c-1} P_R \{\mathrm{E}(P_W)\}^{\frac{2}{c-1}} \right]$$

We see that $\alpha < 0$, since $\mathrm{E}(P_W) < P_R$

Forward bias

- It must be the case that $\mathrm{E}(P_W) < P_R$
- If the distribution of spot prices is not skewed, $P_F < \mathrm{E}(P_W)$
  - The downward bias in the forward price in the zero-skewness case reflects retailer's net hedging demand, who want to sell in the forward market
  - The profits of power retailers are positively exposed, on average, because more retail power is sold when P_W is high
  - To reduce risk, retailers want to sell forwards
- Typically we have skewness ($c > 2$),
  - So $\gamma > 0$ and the forward price increases with increasing skewness
  - This reflects the fact that the industry wants to hedge against price spikes

Market risk premium

- Exposure to the market will differ both between producers and retailers as well as within their own group
- For instance, a large producer will generally be exposed to market uncertainty for a longer period of time, perhaps determined by the remaining life of the assets, whilst retailers will tend to make decisions based on a shorter time scale
- So the need for risk-diversification has a temporal dimension

- These differences in the desire to hedge positions are employed to explain the market risk premium and its sign
- Retailers are less incentivised to contract commodity forwards the further out we look into the market
- In contrast, on the producers' side the need to hedge in the long-term does not fade away as quickly

- We associate situations where $\pi(t, T) > 0$ with the fact that retailers' desire to cover their positions 'outweighs' those of the producers resulting in a positive market risk premium
- The mirror image is therefore one where the producers' desire to hedge their positions outweighs that of the retailers resulting in a negative market risk premium

The electricity futures pricing model

Assume that the spot price follows a mean-reverting multi-factor additive process

$$S(t) = \Lambda(t) + Z(t) + Y(t) \tag{1}$$

- $\Lambda(t)$ deterministic seasonal spot price level
- $Z(t)$ low-frequency non-stationary dynamics
- $Y(t)$ Lévy-driven short term variations with dependency structure

Benth et al. (2014) ▸ Details ▸ FEB4

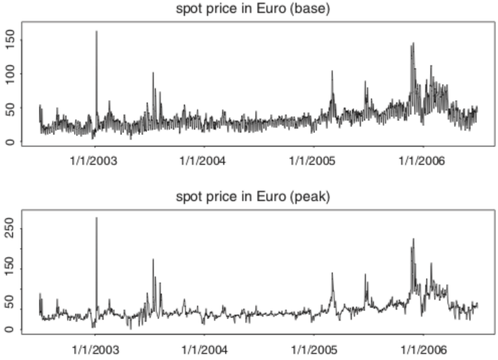


Figure 7: Daily spot prices July 1, 2002 to June 30, 2006 base load (top), peak load (bottom).

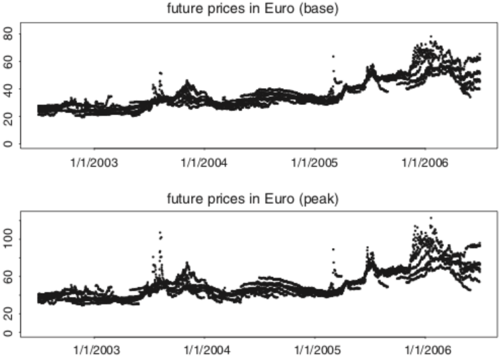


Figure 8: Daily futures prices July 1, 2002 to June 30, 2006 base load (top), peak load (bottom).

Futures price dynamics

The futures price $F(t, T_1, T_2)$ is defined as

$$F(t, T_1, T_2) = \mathsf{E}_Q \left[\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} S(\tau) \mathrm{d}\tau \middle| \mathcal{F}_t \right],$$

where Q is the risk-neutral probability measure, with $S(\tau) \in L^1(Q)$.

The futures price is the *expected value of the average spot price* within the delivery period.

Return

The methodology

Econometrics	Fin. Mathematics
Spot price ▸ Model	$CARMA(p, q)$ ▸ CARMA
$\downarrow$	$\downarrow$
deseasonalise ▸ Λ_t	Futures pricing ▸ F_t
$\downarrow$	$\downarrow$
detrend	Market price of risk ▸ θ
$\downarrow$	
TSA: ARMA(p, q)	

Benth et al. (2007)

EEX price data

Spot prices:

- July 1, 2002 to June 30, 2006
- peak load prices: 5 days a week ($T_p = 1045$)
- base load prices: 7 days a week ($T_p = 1461$)

Futures:

- July 1, 2002 to June 30, 2006
- peak load futures: average for peak hours between 8am & 8pm
- base load futures: average for all hours per day
- Delivery period: 1 Month contracts

Stable CARMA(2,1) model

	CARMA parameters			Stable parameters			
	b_1	a_1	a_2	α_L	β_L	γ_L	$\mathsf{E}[L(1)]$
Base load	0.286	1.485	0.091	1.652	0.391	6.407	0.057
Peak load	0.613	2.334	0.226	1.321	0.065	6.520	−0.045

Table 1: CARMA(2,1) Coefficient estimates of the stable CARMA process

Market Price of Risk

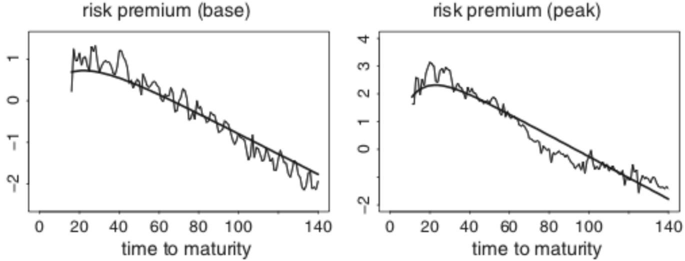


Figure 9: MPR: estimate (bold), empirical (thin) for base (left) and peak load (right). Time to maturity: $u = \frac{1}{2}(T_1 + T_2) - t$

Conclusions

- Two factor model feat. spikes and non-stationarity
- MPR shows typical change of sign depending on time to maturity
- At short end: positive MPR due to skewness in spot prices (Bessembinder & Lemmon (2002))
- At long end (>2 months) negative MPR: Generators want to hedge uncertainty about future prices

Open questions:

- Goodness of fit of the stable CARMA model?
- Negative price spikes are not covered
- Effects drawn by renewables not considered
- Does new data give similar results?

For Further Reading

- OE Barndorff-Nielsen, FE Benth and AED Veraart (2013)
  Modelling energy spot prices by volatility modulated Lévy-driven Volterra processes
  Bernoulli **19**(3), 803-845
- FE Benth, C Klüppelberg, G Müller and L Vos (2014)
  Futures pricing in electricity markets based on stable CARMA spot models
  Energy Economics 44, 392-406
- H Bessembinder and ML Lemmon (2002)
  Equilibrium pricing and potimal hedging in electricity forward markets
  Journal of Finance **57**(3), 1347-1382

Seasonality function Λ: Truncated Fourier Series

$$\Lambda_{p,t} = c_1 + c_2 \cdot t + c_3 \cos\left(\frac{2\pi t}{261}\right) + c_4 \sin\left(\frac{2\pi t}{261}\right)$$

$$\Lambda_{b,t} = c_1 + c_2 \cdot t + c_3 \cos\left(\frac{2\pi t}{365}\right) + c_4 \sin\left(\frac{2\pi t}{365}\right) + c_5 \cos\left(\frac{2\pi t}{7}\right) + c_6 \sin\left(\frac{2\pi t}{7}\right)$$

Estimation via robust MLE, indices p and b stand for peak load spot prices and base load spot prices

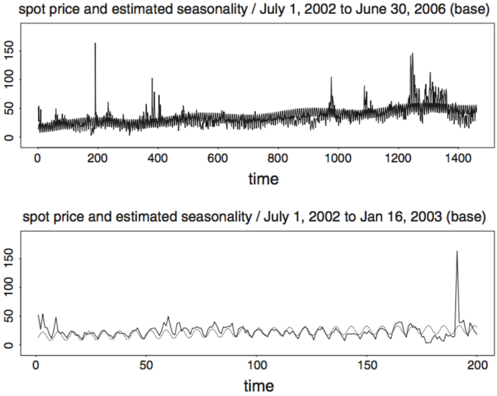


Figure 10: Daily spot prices July 1, 2002 to June 30, 2006 base load with corresponding seasonal component.

EXAA Spot Market Price Processes

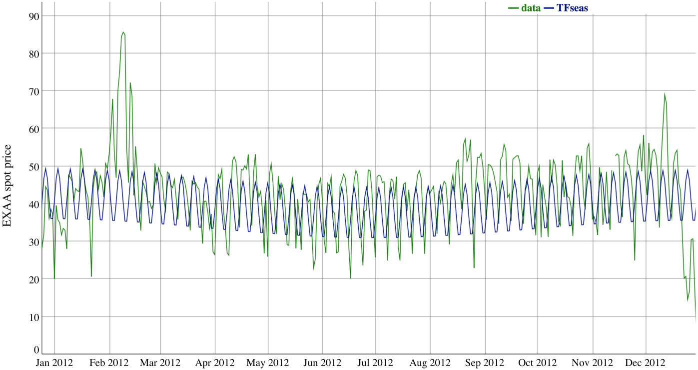


Figure 11: EXAA daily spot prices with seasonality 2003-2016 EXAA

return

Stable CARMA(p, q)-Lévy process

$$a(D)\mathbf{Y_t} = b(D)D\mathbf{L}(t), \quad D \overset{\text{def}}{=} \frac{d}{dt},$$

where the auto-regressive polynomial is given by

$$P(z) = z^p + a_1 z^{p-1} + \ldots + a_p$$

and the moving-average polynomial by

$$Q(z) = b_0 + b_1 z^q + \ldots + b_{p-1} z^{p-1}.$$

$$Y_t = \mathbf{b}^\top \mathbf{X}_t \qquad \text{state equation} \qquad (2)$$

$$d\mathbf{X}_t = (\mathbf{A}\mathbf{X}_t)dt + \mathbf{e}_p d\mathbf{L}_t \qquad \text{observation equation} \qquad (3)$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \dots & \dots & 0 & 1 \\ -\alpha_p & -\alpha_{p-1} & \dots & & -\alpha_1 \end{pmatrix} \quad \mathbf{e}_p = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{p-2} \\ b_{p-1} \end{pmatrix}$$

With Brownian motion B_t instead of the Lévy process L_t we have Gaussian CARMA(p, q)

Characteristic function of $L(t)$

Suppose the Lévy processes are exponentially integrable with $\kappa > 0$ such that

$$\int_{|z|\geq 1} \exp(\tilde{\kappa} z) l_j(\mathrm{d}z) < \infty,$$

for all $\tilde{\kappa} \leq \kappa$ and $j = 1, \ldots, n$.
This implies that the spot price process $S(t)$ has exponential moments up to order κ. The characteristic function of $L(t)$ with log exponential moments

$$\log \mathsf{E}[\exp\{izL_j(t)\}] = t\phi_L(z)$$

is defined as

$$\phi_L(z) = \begin{cases} -\gamma^\alpha |z|^\alpha \left\{1 - i\beta(\operatorname{sign} z)\tan\left(\frac{\pi\alpha}{2}\right)\right\} + i\mu z & \text{for } \alpha \neq 1 \\ -\gamma|z| \left\{1 + i\beta\frac{2}{\pi}(\operatorname{sign} z)\log|z|\right\} + i\mu z & \text{for } \alpha = 1 \end{cases}$$

Stability condition

Assumptions:

(i) The polynomials $a(\cdot)$ and $b(\cdot)$ have no common zeros

(ii) $l_L(\{0\}) = 0$ and $\int_{\mathbb{R}} (q^2 \wedge 1) l_L(dz) < \infty$

(iii) All eigenvalues of A are distinct and have strictly negative real parts: A has full rank, is diagonalisable with eigenvectors U and eigenvalue matrix D: $\exp(At) = U \exp(Dt) U^{-1}$

(iv) To achieve stationarity in the Lévy case: $\sigma_t^2 = \int_{-\infty}^{t} i(t,s) \mathrm{d}U_s$, where U_s is a Lévy subordinator, $i(t,s) \stackrel{!}{=} i^*(t-s) > 0$ and i^* is derministic.

Barndorff-Nielson et al. (2013), Benth et al. (2014)

▸ Model ▸ FEB4