LUTE TUNING AND TEMPERAMENT IN THE SIXTEENTH AND SEVENTEENTH CENTURIES

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Introduction

Today's standard of tuning is equal temperament and just about every instrument that anyone purchases is usually built with that temperament in mind. This has been the case for most of the twentieth century, but when performing music from earlier periods in Western music literature, the equal temperament standard does not apply. Generally speaking, the standard for most music from the sixteenth and early seventeenth centuries is meantone temperament, not equal temperament. However, meantone temperament was not the same kind of standard in the way that equal temperament is today. Many different kinds of meantone temperaments existed during this period, and variations depended on the time period such as early or late sixteenth century, the geographical location and even the instrument. As these variations in temperament continued into the seventeenth and eighteenth centuries, newer, non-meantone temperaments were invented and the options became even more numerous and complex with majority of them applied to keyboard instruments because of the fixed nature of their pitches.

Fretted instruments such as lutes and theorbos also had to follow the same meantone standard, but the execution of these temperaments on a lute differed from other instruments. Keyboard instruments, for example, had their own procedures for producing different varieties of meantone temperament. Lutes and other similar fretted instruments could produce these as well, but the process was different and the limitations a particular temperament imposed on a keyboard instrument could be different than those imposed on a fretted instrument. The implications for a lute player at this time were that, depending on the temperament, the player would need to alter his or her technique, and in some cases, the instrument itself, in order to accomodate a particular temperament. This raises the question of how performers at that time dealt with the issue. More importantly, it should give today's performers pause when determining what temperament to use. Just as we cannot apply an all-encompassing modern temperament to older music, neither can we apply all-encompassing historically-informed one either.

The principle difference between today's equal temperament and the temperaments used at the turn of the seventeenth century is that every temperament at that time had semitones of non-uniform or varied size. The variation could either be regular, where the variation between the sizes of semitones within an octave followed a predictable pattern, or the variation could be irregular with semitone size varying greatly from one note to the next. When tuning a keyboard instrument where each string or pipe can be pitched independently of the others, this presents no problem. Bowed instruments without frets may vary their pitches with the placement of the left-hand fingers. Wind instruments have fingerings and the performer's embouchure to provide small adjustments in pitch to match any kind of temperament.

Because fretted instruments place their frets so that one fret intersects all of the strings on the instrument, the inherent problem with attempting to use semitones of varying size is that when you set the size of a semitone for one string of the instrument, you have set the same sized semitone for all the other notes on the adjacent strings. This creates unavoidable problems if the temperament you are trying to emulate requires a small semitone on one string but a larger, differently sized one on the string next to it. The result is that as you progress down the fingerboard and set the size of each of your semitones, whether they are the same size or not, you must choose what sized semitone benefits the majority of the pitches on that fret. If the majority of the pitches on a fret are best suited to a small semitone, then you must have a semitone of the same small size for all of those pitches. Alternatively, if the majority of the pitches require a large semitone, then all the pitches get the same sized, larger semitone. The problem is that there is usually always one pitch on the fret somewhere that does not fit and needs to have a semitone of a different size than all the others.

Lute players have dealt with these problems of varied semitone size for centuries, and there is no perfect solution to the problem, but there are many combinations of different kinds of solutions that produce good results. What follows is an examination of these solutions in detail. Before we get to these, however, we need a a brief explanation of the history of tuning and temperament. Chapter one provides this as well as some context and a working technical knowledge of temperament. The second chapter examines historical fretting systems published during the sixteenth and early seventeenth centuries. Here we will see how lute players at the time dealt with temperament and the shortcomings of their solutions. Chapter three moves beyond historical fretting systems and attempts to a provide unified fretting system for use in performance by combining historical techniques with modern practices. We will see how historical fretting systems must be modified in order to be successful and what other techniques may used to achieve a working system. The last chapter provides a conclusion to our predicament and summarizes the many different ways lute players today may navigate temperaments in modern day performance contexts.

Chapter 1

Tuning and Temperament

Tuning and temperament is one of the most well-documented and discussed issues in music. While I cannot present any new information on the subject, we should have an understanding of it as it is discussed within the context of this paper. What follows is a short summary of the history of western tuning methods and systems of temperament. Because there are so many different kinds of temperaments, I will only concentrate on the types that will be used for discussion and comparison later. These main types are: 1) modern-day equal temperament and other temperaments with equal semitones; 2) temperaments that utilize unequal semitones such as the regular meantone temperaments called quarter-comma and sixth-comma meantone; 3) irregular temperaments, which also use unequal semitones and are also referred to as "well" temperaments; and, 4) the tuning systems attributed to Pythagoras and Ptolemy.

The majority of my information comes from Murray Barbour's 1951 book *Tuning and Temperament: A historical survey* as well as more recent books such as Ross

Duffin's *How Equal Temperament Ruined Harmony (and why you should care)*. As the latter title suggests, discussions of temperament usually revolve around the concept of equal temperament and whether or not its purpose is justified during certain periods of music history. Barbour's book, while considered one of the most thorough compendiums of information, generally portrayed temperaments other than equal temperament as inferior. Authors such as Duffin and others in the historical performance field feel that equal temperament has degraded the effect of musics for which it was not originally intended.

It is not my purpose to extoll the virtues of one temperament over another. The matter is quite subjective, a fact supported by the vast number of temperaments available to a lutenist at this time. While some of these temperaments conformed to the contemporary norms of meantone, others resembled modern-day equal temperament. It was not an unknown temperament at this time and historical fretting sources match it quite closely using systems of equal division and other complex methods of dividing a string. Its use on other instruments was discouraged because of the consequences to harmony; however, its favor with fretted instruments remained. Before we can get to the reasons why, we must first discuss the history of tuning and temperament, and the main problem it poses to us.

1.1 The Greeks' debate

The history of tuning begins with the ancient Greeks and their philosophers who proposed the first solutions to tuning notes within the span of an octave. Three of the most influential figures in this area were Pythagoras, Aristoxenus and Claudius Ptolemy. Pythagoras is generally credited with discovering the concept of tuning, although none of his original writings survive. He established the first mathematical principles that apply to tuning; however, it was Aristoxenus and Ptolemy that began the debate about tuning which is still continues today. One of the fundamental teachings of the Pythagoreans was that the universe could be explained according to simple numbers and ratios. An example of this numerical simplicity is found in the tetractys, a common symbol associated with the Pythagoreans because it represented the basic sequence of numbers: 1, 2, 3, and 4. Pythagoreans believed that everything in the world could be reduced to these simple numbers. The number four, for example,

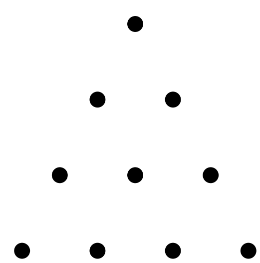

Figure 1.1: The Pythagorean tetractys

could be used to explain the four seasons of the year or the four elements of earth, wind, fire and water while the sum of the numbers (1 + 2 + 3 + 4) gave you 10 and that was the basis for their entire system of arithmetic.[1] For the Pythagoreans, numbers meant everything.

Today, such notions of numeric significance are usually found in numerology and astrology. To us, such associations seem almost nonsensical because we are taught a more scientifically developed understanding of the universe; however, basic math-

1. Catherine Nolan, “Music theory and mathematics,” in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 273.

ematical principles of number underpin even our most complex theories. To regard the Pythagoreans' notions of number and meaning as absurd belies the fact that our notions of meaning are tied to same kinds of numbers, but just in a different way. For them, just as much as us today, everything operated according to numbers and music was no exception.

For the Pythagoreans, the most important intervals were the unison, octave, fifth and fourth. Not only were these the first pitches in the harmonic series, but you could also express them using very simple numeric ratios: 1:1 for the unison, 2:1 for the octave, 3:2 for the fifth and 4:3 for the fourth. They proved these ratios using the monochord, which was a single string divided into different parts. In order to produce the intervals in a Pythagorean system, the monochord was divided into a certain number of parts and then stopped with either a finger or small bridge. For example, a string divided into two parts and stopped at the first yielded a distance with the ratio of 2:1. This distance also produced the musical interval of an octave. For the perfect fifth, it was divided into three parts and stopped at the second, creating the ratio 3:2. While other intervals could be produced, they stopped at the fourth, with its ratio of 4:3 because creating other intervals used numbers greater than four. Not only did these numbers fit perfectly into the Pythagorean notion of the tetractys, they also described how music fit into the physical world. Ratios of pitch could also be expressed as ratios of weight and distance. Because their world was so dependent on these numbers, any other intervals in a scale ultimately had to be derived from these original four, regardless of what the actual pitches were in reality.[2]

In Pythagorean tuning, other intervals were calculated by subtracting or adding these original four intervals in different combinations. In terms of arithmetic, the sum of two ratios meant a product of the two, while subtraction of two ratios meant using division. So a Pythagorean would calculate a wholetone by subtracting the fourth from the fifth.

$$3:2 \div 4:3 = 9:8 \tag{1.1}$$

This produced a wholetone with a ratio of 9:8. A semitone was then calculated by subtracting two of these wholetones from the original fourth.

$$(4:3 \div 9:8) \div 9:8 = 256:243 \tag{1.2}$$

This produced a ratio of 256:243. Of course these ratios were not part of the tetractys, but that did not matter because the resulting semitone was created from intervals that were. In this case, the fourth, already a member of the original four intervals, and the wholetone created from the fourth and fifth.

While the numerical simplicity of using only unisons, octaves, fifths and fourths fit perfectly with the Pythagorean ideal, using them as the basis for a tuning system produced unacceptable results when applied in a musical context. While the Pythagoreans had found a way to tune their scale, the also uncovered the central problem that has plagued us ever since. Using their system or any other temperament created since then, it is not possible to create a twelve-tone scale that is internally consistent. We

2. Nolan, "Music theory and mathematics," 274.

can tune a chromatic scale using only Pythagorean fifths by starting with the interval C to G and continuing through the circle of fifths for all twelve pitches, returning the the original note C. After proceeding through twelve fifths from our starting C, the

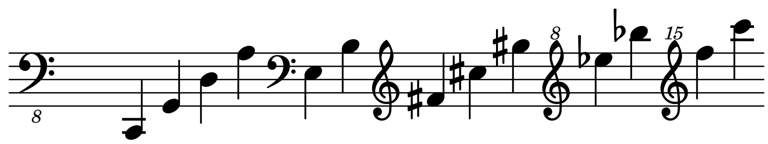

Figure 1.2: The circle of twelve fifths

final C is seven octaves above it. If Pythagorean tuning was internally consistent, the

Figure 1.3: The note C spanning seven octaves

C seven octaves above would be the exact same pitch as the C resulting from twelve fifths. We can test this mathematically, by adding together twelve ratios of 3:2 and comparing them with the sum of seven octaves using the ratio 2:1.[3]

$$\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{531441}{4096} = 129.7463 \qquad (1.3)$$

$$\frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{2}{1} = \frac{128}{1} = 128 \qquad (1.4)$$

The mathematical result is that the sum of the twelve fifths is greater than the sum of the seven octaves. In musical terms, the C resulting from twelve fifths is sharper than what we expect to hear, a C seven octaves above the C from which we started.

3. Ross W. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)* (New York, NY: W.W. Norton & Company, 2007), 25.

Pythagorean tuning presented another problem as well. Not only was it not internally consistent but was that it was not consistent with the rest of the harmonic series either. The first three pitches in the harmonic series matched Pythagorean ratios perfectly. The pure intervals of the octave, fifth and fourth in the harmonic series were exactly in tune when compared to the ratios of their Pythagorean counterparts. However, the fourth pitch in the harmonic series, the major third, when tuned acoustically pure as it naturally occurred in the series, actually vibrated at a ratio of 5:4 and not 81:64, as the Pythagoreans would have calculated by adding together two of their 9:8 wholetones: $9 : 8 \times 9 : 8 = 81 : 64$.

These two tuning discrepancies that result from using only Pythagorean intervals to build a scale of notes are called *commas*. The *ditonic comma* results when building a chromatic scale upon successive fifths, such as the difference between twelve fifths and seven octaves, and the *syntonic comma* results from the difference between the pure harmonic 5:4 major third and the Pythagorean 81:64 major third. The Pythagoreans themselves were well aware of these problems and tried to overcome them, but it was only Aristoxenus and Ptolemy who could propose any real solutions. Aristoxenus proposed a solution by ignoring the Pythagorean ideals of mathematical simplicity and used tuning systems that divided the string according to parts and not ratios. He described tuning by using equal parts of the string and was the first to develop the concept of tuning using equal semitones which would later be the foundation of equal temperament. Ptolemy on the other hand continued with the Pythagorean notion of ratio, except he modified it slightly. In Ptolemy's view, tuning should only use ratios that are superparticular in nature, meaning that the first number of the ratio should always be one unit great than the other. So Pythagorean ratios like 2:1, 3:2, 3:4 and even 9:8 were acceptable, but so too were other ratios such as 5:4, the pure major third and 6:5, the pure minor third.

Ptolemy's solution rectified a lot of the Pythagoreans' mathematical ratios with nature's own internal tuning system but it still had problems when it came to musical execution. In a twelve-tone scale, there are three major thirds. Starting on the note C, we can fit three of them within an octave: C to E, E to G-sharp, and A-flat to C. However, adding these intervals together using Ptolmey's ratios does not get us to a

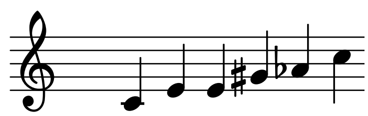

Figure 1.4: Three major thirds within an octave

complete octave either. The sum of three thirds should sound the same as an octave with the ratio 2:1, but the mathematical result it slightly less than 2, which sounds flat.

$$\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times = \frac{125}{64} = 1.953125 \tag{1.5}$$

While the note E could remain fixed in relationship to the major third starting from C and the major third ending on G-sharp, the fact that G-sharp and A-flat are used for the other two major thirds implies that they are different notes. Most modern musicians regard the enharmonic respelling of a note as a kind of music homonym: an alternative word to express the same thing. While the different names of G-sharp and A-flat can indicate different functions, such as the third degree of the E major scale or the first degree of the A-flat major, we today regard them as notes having the same vibrating pitch. However, as Ptolemy's tuning problem shows us, the two notes are not only functionally different, they are musically different are therefore tuned differently.

Ptolemy's solution of tuning with only pure intervals is known as just intonation. While it succeeded in correcting the problems associated with Pythagorean tuning, it made it impossible to tune certain notes in the chromatic scale because they required a different tuning depending on musical context in which they occurred. Instruments of fixed pitches, such as any keyboard instrument or fretted instrument, whose semitones were fixed and immobile, could not accommodate a tuning of only pure intervals, or at least, not in a scale of twelve tones. Instruments of variable pitch, such as fretless bowed instruments, wind instruments or the human voice were exempt from this problem because they were able to adjust any note sharper or flatter depending on the need of the situation. Violinists, for example can place their fingers at any position along the fingerboard and wind players who may vary the pitch of their notes using their embouchure.

What about Aristoxenus? While he did invent an alternative solution of equal semitones, he rejected both the notions of Pythagorean numeric purity and the Ptolemaic notion of harmonic purity. His method of tuning divided an octave into equal parts, making intervals as groups of these individual parts instead of using ratios at all. His idea of equal parts formed the basis for equal temperament. While this allowed for a tuning of fixed pitches in a chromatic scale, none of these intervals were tuned in a way that precisely matched the tuning in the harmonic series. This was an unacceptable solution for most theorists and musicians during the history of western classical music, and it took many years for equal temperament to gain acceptance.

Because the notion of equal semitones did not sit well with early western music theorists and tuning harmonically pure intervals was impossible in a fixed pitch system, they gravitated towards the Pythagoreans' system of tuning. For one reason, it was simple and operated using the idea of small number of simple ratios. For another reason, it matched the musical tastes of the medieval period such as monophonic chants and polyphonic forms based on fifths, unisons and octaves. Pythagorean ratios persist even to this very day, but as music changed in the early Renaissance period, composers sought to incorporate more consonant thirds into their music, which the strict Pythagorean system of ratios did not easily accommodate. Thus, the old Greek debate between Pythagoras, Ptolemy and Artistoxenus resurfaced again.

1.2 Temperament

A temperament is method of tuning a scale that alters an existing system, usually Pythagorean tuning. For most of the Middle Ages, tuning was described according to the same ratios that Pythagoras had determined hundreds of years earlier. However, as musical tastes changed during the Renaissance, there was an increasing preference for the sounds of thirds instead of unisons and fifths. This represented the problem of rectifying thirds within the Pythagoran tuning scheme that had plagued the Greeks so many years before. The first attempts at a solution appeared at the end of the fifteenth century, and each one took the same approach towards solving the problem. The idea was to change the size of the fifths within the existing Pythagorean system.

One of the first writers to publish a system that broke with the Pythagorean tradition of tuning was Bartolomeus Ramis de Pareja. In his *Musica Practica* of 1482, he created a chromatic scale using two different groups of fifths. Each group was tuned in pure fifths, just like Pythagorean tuning, except the one group was slightly sharper than the other.[4] Although he did not call it a temperament, it technically was not Pythagorean tuning either. Barbour credits Franchinus Gafurius's *Practica musica* of 1496 with the first mention of the idea of temperament. In it, Gafurius says that organists tune their fifths slightly flat, but does not go into any specific details.[5] Similarly, Arnolt Schlick's 1511 treatise *Spiegel der Orgelmacher und Organisten* only gives us a general idea when he refers to tuning the instrument's fifths flat, "as much as the ear will permit."[6] So while the idea was present as early as 1482, it took another several years for it to develop into a system.

Tempering is a process of compromise where the acoustical purity of one interval is changed slightly in order to accommodate the acoustical impurity of another interval. In this case, it is the process of changing the size of the fifth in order to accommodate the major third which is not pure within the standard Pythagorean tuning. Usually this means shrinking the size of the fifth slightly so that is narrower, or flatter than the pure ratio of 3:2, although in some cases, a fifth may be tuned wider or sharper than pure. The problem with Pythagorean thirds were that they were much sharper than pure, differing by the syntonic comma which was the difference between the

4. J. Murray Barbour, *Tuning and Temperament: A Historical Survey* (Mineola, New York: Dover Publications, Inc., 2004), 88.

5. Ibid., 25.

6. Rudolf Rasch, "Tuning and temperament," in *The Cambridge History of Western Music Theory*, ed. Thomas Christensen (Cambridge: Cambridge University Press, 2002), 202.

Pythagorean third at 81:64 and the pure harmonic third at 5:4. Shrinking the size of the fifth resulted in thirds that were flatter than the Pythagorean ratio of 81:54, closer to the harmonically pure ratio of 5:4, and thus more appropriate sounding for music that favored more thirds. Depending on the amount in which you flattened the fifth, you could come very close to a pure 5:4 third, enough so that the listener would not notice the difference. This was at the expense of the fifth, which was now flatter than pure but it was an acceptable compromise because a flat fifth sounded less disconcerting than the sharp third of Pythagorean tuning. The strategy was to shrink several fifths by the same or differing amounts. The result was that instead of one set of intervals being completely pure in nature and another being completely unusable, all the fifths were slightly out-of-tune in differing amounts, while some of the thirds came very close to pure. Which thirds approached pure and which fifths were made flat all depended on the kind of temperament, and there were quite a lot of them.

Murray Barbour classifies temperaments into four basic types: regular meantone, irregular temperaments, equal division, and equal temperament. Whereas a tuning can be defined as a method of obtaining intervals according to Pythagoras, as in Pythagorean tuning, or according to Ptolemy, as in just intonation, temperaments alter the ratios of the intervals so that they are often somewhere in between the two systems. In regular meantone temperaments, most or all of the fifths of the Pythagorean tuning system are tuned flat by the same amount. Depending on the amount in which the fifths are flattened, this can result in thirds that are very close to to pure; however, the fifths and fourths are always not pure. The amount that each fifth was changed from its pure ratio often depended on the musical context. A meantone temperament that tuned fifths very flat resulted in a temperament that had thirds quite close to pure, but because of the mutated nature of the fifths, was only playable in a limited number of keys. Conversely, a meantone temperament that tuned its fifths less flat made more keys playable, but resulted in sharper, less pure thirds.

1.2.1 Regular Meantone and Irregular Temperaments

The first attempts at a systematic application of tempering fifths resulted in the class of temperaments known as meantone temperaments. No one is entirely sure who invented the first complete meantone temperament, but the general consensus is that it is a shared prize between Pietro Aron and Gioseffo Zarlino. Zarlino gets the credit for an exact system that narrows each fifth by $\frac{2}{7}$ of a syntonic comma. His method, however, was far from practical. Although he provides a monochord diagram, his process of tempering the fifths by that much actually creates thirds that are smaller than pure, or too flat.

Pietro Aron is credited with the first practical meantone temperament. While not as theoretically exact as Zarlino's, Aron's instructions can be used to create a workable meantone temperament that narrows fifths enough to create pure thirds, while leaving the remainder of the intervals playable. His instructions are not mathematical in nature, unlike Zarlino's, and requires the person to tune according to the ear and not rely on measurements. Aron's idea was to start with pure thirds and build tempered fifths around them. He begins with a pure third between C and E and then proceeds to tune four fifths that are all equally flat with additional pure thirds between A and C♯, and D and F♯. While Aron says nothing about the division of the comma, Barbour takes these instructions and mathematically proves that if the tuner is able to create Aron's pure thirds by ear and match the size of the other fifths so that they are all tempered by the same amount, each of those fifths will be flatter than pure by $\frac{1}{4}$ of a syntonic comma.[7] Because this is the measurable amount that each fifth is reduced in size, Aron's temperament is referred to as "quarter-comma meantone temperament." While Aron never referred to it this way, nor did he use any math to calculate lengths of a string, theorists and musicians took his ideas and later produced exact calculations that reproduced Aron's temperament.

Regular meantone temperaments contained eleven fifths that were all narrowed by the same amount. This was always measured in terms of a fraction of the syntonic comma. Narrowing four consecutive fifths in the circle of fifths by $\frac{1}{4}$ of a syntonic comma was enough to make eight thirds in a 12-tone scale pure while leaving the remaining four practically unplayable.[8] This was an effective solution for sixteenth-century music because it resulted in more serviceable keys than Pythagorean tuning. The trick to using it was avoiding the keys in which the unplayable thirds resided. This meant that meantone did not solve all of the problems associated with Pythagorean tuning. Just as Pythagorean tuning has a fifth that too wide to be used, meantone temperaments also had an unusable fifth. The so-called "wolf" fifth existed in every

7. Barbour, *Tuning and Temperament: A Historical Survey*, 27.
8. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)*, 33.

temperament, but its size and location depending on the kind of temperament.

As the sixteenth century continued, musicians attempted to correct the problems with quarter-comma meantone, such as its wolf fifth and the other thirds that were not playable. These tuning systems narrowed the fifth in smaller amounts, such as $\frac{1}{5}$ or $\frac{1}{6}$ of a syntonic comma. The result was that the thirds were now sharper than pure but much less so than in Pythagorean tuning. While this created more playable thirds in the scale, the wolf fifth remained. Sixth-comma meantone was popular with fretted instruments, as we will see in a later chapter, but even sixth-comma still contained a wolf fifth and unequal thirds, just as quarter-comma did.

The next major change in temperaments occurred in the seventeenth century when theorists began using systems that narrowed fifths by varying amounts instead of equal amounts, as was the case with regular meantone temperaments. So called irregular temperaments, or well temperaments, became very popular and widespread in the seventeenth and eighteenth centuries. The advantage of well temperaments was that musicians had finer control over which intervals were problematic. For example, in quarter-comma meantone, the fifth between E-flat and A-flat will always be too wide to be used. Technically, this is because the A-flat is really a G-sharp.[9] Similarly, the thirds B to D-sharp, D-flat to F and F-sharp to A-sharp would also be unplayable. With irregular temperaments, fifths were narrowed by differing amounts so the wolf fifth was eliminated and other thirds that would normally be problematic in standard quarter-comma meantone could be moved to other intervals and hidden more effectively. These irregular temperaments did the same thing that a regular

9. Ibid., 35.

meantone temperament did, except that one fifth might be lowered one-quarter of a comma, another might be lowered only by a sixth or less. The result was that the composer had more control over which keys had better-sounding thirds and which did not.

The result of irregular temperaments created an explosion in the number and variety of temperaments that were available to musicians in the seventeenth and eighteenth centuries. Theorists such as Valloti, Young, Werckmeister and others all developed their own systems of temperament that favored different intervals in ways that regular meantone temperaments could not. J. S. Bach also used his own temperaments as well in the performance of his keyboard works. His compositions from *The Well-Tempered Clavier* were expressly written for a keyboard instrument that was tempered using an irregular meantone temperament.

1.2.2 Equal division

All of the methods described thus far, are practical ways to create temperaments. For example, Aaron's method requires no complex mathematical calculations, can be done using the ear alone and produces a good quarter-comma meantone temperament. Later methods of tuning other meantone temperaments and well temperaments used the same techniques of narrowing or widening intervals by certain amounts, either by ear or by some process of calculation. The problem with some of these methods is that it they can be imprecise. The fifth is essentially narrowed by an indeterminate amount to match a pure third or intervals are determined by counting the number of beats that occur as the notes essentially clash against one another. Most importantly, the resulting temperaments contain semitones that vary in size depending on the placement of the tempered intervals. If we are to compare different temperaments, and their specific pitches, we need a more exact way of calculating a given pitch in a given temperament.

Recalling Aristoxenus in our survey of Greek tuning methods, his approach consisted of dividing a string or octave into equal parts and creating intervals by grouping tones into sets of these equal parts. Using this same method, we can create several different regular meantone temperaments by dividing our octave into multiple parts and grouping each wholetone and semitone into a groups of parts. The advantage to using this method as opposed to other methods like Aaron's, is that we can compare semitone sizes more accurately across different kinds of temperaments and we may also quantify the difference in the sizes of our semitones. Barbour refers to this process of calculating temperaments as equal division. Equal division was used in the sixteenth century to calculate string lengths for both quarter and sixth comma meantone temperaments, and its principles remained in use well into the seventeenth and eighteenth centuries.

The first theorist to use equal division was Vicentino in 1555, who divided his octave into 31 parts using a harpsichord that contained six ranks of keys. Vicentino was attempting to create a temperament like quarter-comma meantone, but instead used the principles of equal division instead of Aron's tempered fifths. In order to create an octave of 12 tones out of a total of 31 parts, he grouped each wholetone into five parts and divided the semitone into two different sizes, a smaller semitone with two parts and a larger semitone with three. The size of the semitone depended on where it occurred within the scale. Musicians during the sixteenth and seventeenth centuries were familiar with these differences in semitone size and referred to the larger semitone as the *major semitone* and the smaller semitone as the *minor semitone*. The difference in size between the two semitones was referred to as a comma. We should not confuse this with the diatonic or syntonic comma, which are differences of pitch as well but have no correlation in systems of equal division.

Vicentino's 31-part system solved the same kinds of problems that meantone temperament did, namely that of creating pure thirds at the expense of pure fifths. However, it had the added advantage of clearly indicating that semitones differed in size by one comma. Other approaches to meantone temperament, such as Aaron's, were less specific and would tune certain semitones to a midway point between the wholetone it was dividing. This was usually reserved for the wolf fifth or the chromatic notes which were less likely to be used. As musicians developed systems of equal division, they utilized keyboards that could realize the distinction between semitones more accurately and used split keys that enabled one to play either the diatonic or chromatic semitones. The use of enharmonic instruments that had more than twelve notes per octave dated back to the late fifteenth century, and Patrizio Barbieri's recent book documents them extensively.[10] Vicentino's multi-manual keyboard was one example of such an instrument, but was too impractical to use. More practical examples of keyboards with split keys were found later in the seventeenth century, especially in

10. Patrizio Barbieri, *Enharmonic: instruments and music 1470-1900*, Tastata. Studi e documenti, vol. 2, Revised and translated studies (Latina: Il Levante Libreria Editrice, 2008).

Italy. These instruments had two different keys for some or all of the accidentals and composers such as Handel and Werkmeister continued to use them into the eighteenth century.[11]

Examining the 31-part system in more detail, let us take the C major scale as an example containing five wholetones: C, D, F, G, A; and two semitones: E and B. In our system, we reach 31 parts by assigning 5 parts to each wholetone and 3 parts to each semitone in the scale. Figure 1.5 shows the arrangement of each pitch in the scale according to its number of parts. Five wholetones at 5 parts apiece makes

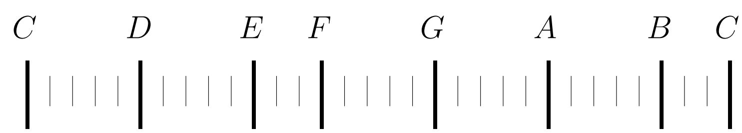

Figure 1.5: The C major scale in a 31-part system

25 parts, and adding the remaining semitones at 3 parts each gives us a total of 31: $25 + 3 + 3 = 31$. Note that semitones E and B have 3 parts as opposed to 2. As we divide our scale further into chromatic pitches, each wholetone is divided into 2 and 3 parts. Pitches that are diatonic within a scale, such as E and B, will always be the larger semitone. This is why these pitches have 3 parts as opposed to 2 in our example scale. The smaller semitone is reserved for chromatic notes that lie outside the scale. For this reason, the larger semitone is sometimes referred to as the *diatonic semitone* while the smaller of the two is called the *chromatic semitone*. Figure 1.6 shows the rest of our C major scale divided into its chromatic and diatonic semitones.

11. Barbour, *Tuning and Temperament: A Historical Survey*, 108.

Chromatic semitones represent the shortest distance between two pitches, such as the

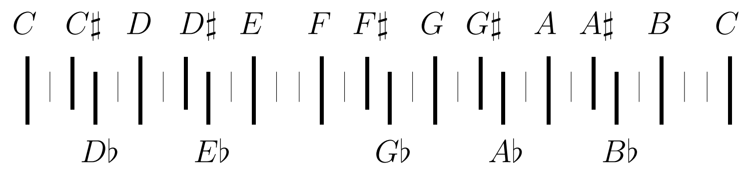

Figure 1.6: The chromatic scale in a 31-part system

distance from C to C♯, but this is also the same distance as G descending to G♭. Likewise, the distance ascending from C to D♭ is the larger, diatonic semitone as well as the descending distance from G to F♯. The term diatonic semitone might be misleading since it implies that D♭ or F♯ are diatonic to a C major scale. For this reason, the minor and major designations are preferable.

Systems of equal division were not limited to just 31 parts. Zarlino and Salinas describe a 19-division octave in the sixteenth century, and later systems included octaves divided into 43 and 55 parts. Each of these systems grouped their wholetones into an odd number of parts and therefore had their semitones divided unequally with a comma's different between them. Praetorius, in his *Syntagma Musicum*, describes a 55-part system with nine parts per wholetone and semitones of four or five parts. He describes this system in reference to the "intermediate" nature of the semitones on fretted instruments:

> Thus the semitones cannot be either major nor minor, but are, perforce, "intermediate" if anything. For I reckon that each fret [...] contains four-and-a-half commas, whereas the major semitone contains five and the minor semitone only four. Since the error is only half a comma either way, the ear hardly notices it with these instruments [...] Major and minor semitones are both produced by the same fret, both sound in tune, [...] especially since by particular applications of the finger to the string,

> over the fret, it is possible to have some control over the pitch of the note produced.[12]

As late as 1723, singer and teacher Pier Francesco Tosi expressed the same idea, though not in reference to fretted instruments, in his treatise *Opinioni de Cantori*:

> A Tone, that gradually passes to another, is divided into nine almost imperceptible Intervals, which are called Comma's, five of which constitute the Semitone Major, and four the Minor.[13]

Praeorius and Tosi are both describing a system of equal division. Tosi was writing in the seventeenth century, but equal division as a tuning method had already been in use for almost a hundred years when it was used to mimic the same features of quarter-comma meantone.

The basic features of each of these systems is outlined in table 1.1, where the corresponding meantone temperament is given. Of the four that are listed, quarter-comma and sixth-comma are the most significant because they were the most frequently used of the meantone systems. Quarter comma was used almost exclusively by keyboard

Number of Divisions	Wholetone	Major semitone	Minor semitone	Meantone equivalent
19	3	2	1	third-comma meantone
31	5	3	2	quarter-comma meantone
43	7	4	3	fifth-comma meantone
55	9	5	4	sixth-comma meantone

Table 1.1: Comparison of systems of equal division

and wind instruments, and was the same kind of temperament that Vicentino was advocating with his system. Sixth-comma turns out to be the same temperament that Praetorius, and later Tosi, were describing although neither refer to it by that name.

12. Michael Praetorius, *Syntagma Musicum II: De Organographia Parts I and II*, Reprinted 2005, Early Music Series 7, Translated and edited by David Z. Crookes (Oxford: Clarendon Press, 1986), 68.

13. Pier. Francesco Tosi, *Observations on the Florid Song*, Translated by Mr. Galliard (London: J. Wilcox, 1743), 20.

I will return to Praetorius's observation about sixth-comma temperament when we examine other specific temperaments for fretted instruments later.

Although they each have different parts, each of the above systems function the same way as Vicentino's original 31-division octave. In a 19-division octave, whole tones consist of three parts, and major and minor semitones are two and one parts respectively. 43-part systems contain whole tones with seven parts each, dividing their semitones between four and three parts, and then a 55-part system has nine parts for its whole tones, divided into five and four. Other systems were possible that created seventh and eighth comma meantone; however, these were not used to any great extent since their tunings began to approximate equal temperament and from a practical standpoint, did not serve as good a purpose.

1.2.3 Equal Temperament

Technically speaking, equal temperament is a type of equal division where the octave is divided into twelve equal parts. In terms of its qualities as a temperament, it compensates for the ditonic comma the same way a meantone temperament does by narrowing the fifth. The difference between equal temperament and meantone temperament is that meantone will narrow its fifths much more than equal temperament. In equal temperament, all twelve fifths of the scale are narrowed by a slight but equal amount. The ditonic comma is therefore dispersed throughout the entire scale so that every pitch is useable, but this ability has its price. The temperament creates thirds that are as sharp as they possibly can be without having too strong an effect on the listener. While they are not as sharp as the thirds of Pythagorean tuning, they lack the pure quality of a quarter-comma meantone temperament and are too sharp to approximate it in the way that other meantone and irregular temperaments do. However, the homogeneous nature of every third being sharp by the same amount tends to veil this lack of harmonic purity from our ears, so that it is not as noticeable.

Even with its overly sharp thirds, theorists in the sixteenth century regarded equal temperament as a technical impossibility. Because everyone based their tunings on the Pythagorean 9:8 wholetone, it was mathematically impossible to divide it into two geometrically equal, whole-number ratios. They could only divide the whole tone unequally into a larger semitone of 17:16 and a smaller one of 18:17.[14] If we recall that adding two geometrical ratios together requires a process of multiplication, then when 18:17 and 17:16 are "geometrically" added together, it produces the 9:8 ratio.

$$\frac{18}{17} \times \frac{17}{16} = \frac{306}{272} = \frac{\frac{306}{34}}{\frac{272}{34}} = \frac{9}{8} \tag{1.6}$$

The difference between the two ratios is similar to the same differences in semitone size found in meantone temperaments and systems of equal division. The smaller 18:17 ratio is the minor semitone and the larger 17:16 ratio is major semitone.

Although equal temperament was impossible according to geometry, musicians knew how to create temperaments that approximated it very closely without having to worry about the math. Despite this ability, most everyone still preferred meantone temperaments because of their pure thirds. Even later in the seventeenth and eigh-

14. Mark Lindley, *Lutes, viols and temperaments* (Cambridge: Cambridge University Press, 1984), 20.

teenth centuries, irregular temperaments were still favored over equal temperament because they made more keys playable and were able to produce some thirds that were much closer to pure than their equally-tempered counterparts.

A tuning method very similar to equal temperament appeared in Giovanni Maria Lanfranco's *Monocordi & Organi* of 1533. His system does not attempt to divide the octave into twelve parts, but instead mimics the quality of the fifth found in systems that do. He describes a system where the fifths are tuned slightly flat and the thirds are made as sharp as the ear can possibly endure.[15] Other writers after him described the same kind of system, sometimes giving him credit and sometimes not. Although Lanfranco's temperament made it possible to use more pitches of the chromatic scale, just as equal temperament does, it still created overly sharp thirds making it inferior to meantone temperament at the time.

One of the first systems to create equal temperament using measurable means was known as the *18:17 rule*. The theorist and lute player Vincenzo Galilei is credited with developing it for practical use.[16] Although technically not true equal temperament, Galilei created twelve semitones of the same size in an octave using a series of Pythagorean minor semitones, or the smaller ratio of the 9:8 wholetone, the 18:17 semitone. His procedure was relatively straightforward. A string is divided into 18 equal parts, and the first part or $\frac{1}{18}$ of the string, is marked as the first semitone. The remaining length or $\frac{17}{18}$ of the string is then redivided into another 18 different parts. The first part of this division is then marked as the second semitone. The process is

15. Barbour, *Tuning and Temperament: A Historical Survey*, 45.
16. Ibid., 57.

then repeated with the remaining string length to mark the third semitone, and so on, each time redividing the remaining amount of string into 18 parts and marking the first part as the next semitone.

Since the Pythagorean minor semitone is slightly smaller than an equally tempered semitone, Galilei's system was not the same as modern equal temperament, but was a way to divide the octave into twelve useable semitones. Since each successive semitone is a bit flat, the fifth is slightly flat as well, very close to an equally-tempered fifth with the octave flat as well and not pure. This issue seemed to go unnoticed on fretted instruments because of the way in which pitches can tend to be sharper at higher frets. Since Galilei was a lutenist, he was using temperaments that he found the best for his instruments. While this worked well for the lute, it does not appear to have transitioned to non-fretted instruments. The same feature of sharp thirds still remained in Galilei's method just as it did with all other temperaments at this time that approximated equal temperament at this time.

Despite the predominance of meantone temperament in the sixteenth century, many shared Galilei's opinions and accepted the fact that lutes and other fretted instruments were tuned with equal semitones. These included such notable musicians as Vicentino, Zarlino, Salinas, Artusi, Praetorius and Mersenne.[17] This did not mean that lutes were tuned in the same kind of equal temperament that we find today. As we will see later, there were different variations in which lutes and fretted instruments approximated equal frets as well as placing their frets unequal to match common meantone temperaments.

17. Lindley, *Lutes, viols and temperaments*, 19.

Equal temperament was not fully accepted in musical practice until at least the eighteenth century, and even as late as the twentieth. Ross Duffin provides ample evidence that musicians were tuning their fifths flatter than is found in today's equal temperament systems well into the nineteenth century, and that true equal temperament really did not become a indisputable fact until 1917 when it was standardized.[18] After that, the advent of modern electronic tuning devices that enabled tuners and musicians to exactly calculate a correctly tempered fifth, solidified equal temperament's place in the modern musical world.

1.3 Describing Temperaments

Throughout the course of this study, we will need to refer to the qualities of the different temperaments explained thus far as well as compare them to other lute-specific temperaments that we will encounter in later chapters. Audio examples can illustrate the differences between temperaments, but sometimes they are so slight that even the most trained ear might have difficulty in distinguishing to the two, or be unable to accurately describe the difference. For example, it might be difficult for any of us to distinguish between a fifth that had been narrowed by one fifth of a comma versus one that was narrowed by a sixth. Furthermore, comparing several different temperaments with audio examples in the context of a written paper can impede the reader, and papers of course do not have speakers built into them.

Expressing temperaments using numerical means is one of the best ways to pin-

18. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)*, 138.

point the exact difference between intervals in a written environment, and we can do this in several different ways. The first, which I have already used, describes intervals using whole-number ratios as the Greeks originally used thousands of years ago. This was also the way that most everyone else used in the sixteenth and seventeenth century as well, including lutenists. For that reason, I will use these whole-number ratios to refer to numerical differences between intervals whenever possible. However, not all intervals are easily defined using these kinds of ratios.

Meantone temperament uses intervals that can be expressed using whole number ratios, but they are not easily derived. We can calculate the ratios of meantone intervals in two ways. The first way involves determining the parts of a comma and subtracting them from the fifth, just as Aaron originally suggested. The only difference is that Aaron was using his ear and achieving the same result, but here we will a more rigorous method of arithmetic to illustrate the same result. Furthermore, we will also need precisely articulate the differences of semitone and interval size in a variety of meantone temperaments.

Because each system of meantone temperament also has a matching system of equal division, we can express its intervals mathematically using the same system of parts or commas that were historically. For example, in Vicentino's 31-part octave, the fifth would comprise three whole tones and a major semitone. Referring to figure 1.5, that makes a total of 18 parts out of the total 31. We can represent this mathematically using the ratio of parts as a logarithmic function: $2^{\frac{18}{31}}$. Applying the same ratios of parts as exponential fractions, we can calculate any interval in any meantone system, such as the quarter-comma major semitone: $2^{\frac{3}{31}}$; or the sixth-comma major semitone: $2^{\frac{5}{55}}$. Incidentally, we can also express intervals in equal temperament the same way. For example, the semitone in equal temperament is: $2^{\frac{1}{12}}$; the whole tone $2^{\frac{2}{12}}$; the fifth: $2^{\frac{5}{12}}$ and so on. The problem with these numeric representations is that are more difficult to comprehend than a simple number, and are not readily comparable to standard Pythagorean ratios.

In order to effectively compare all these different kinds of semitones, we need to evaluate either their ratio or their logarithmic formula to a simple decimal number. In this case, the number could be applied to a string length to determine what length of string would produce a sixth-comma meantone major semitone, a quarter-comma meantone fifth or pure third, and it can also determine the vibrating frequency for those particular pitches as well. Referring to table 1.2, if we compare the Pythagorean ratio 3:2 with its quarter-comma meantone equivalent, expressed using logarithmic functions, we can immediately see that the Pythagorean fifth is slightly larger. We also see that the the quarter-comma third better matches the ratio of the pure third, as opposed to the fifth or sixth-comma third. We know from Aaron's description

Pythagorean fifth:	$3:2 = 1.500$
Quarter-comma meantone fifth:	$2^{\frac{18}{31}} = 1.4955$
Pure third:	$5:4 = 1.2500$
Third-comma meantone third:	$2^{\frac{6}{19}} = 1.2447$
Quarter-comma meantone third:	$2^{\frac{10}{31}} = 1.2506$
Fifth-comma meantone third:	$2^{\frac{14}{43}} = 1.2532$
Sixth-comma meantone third:	$2^{\frac{18}{55}} = 1.2546$

Table 1.2: Comparison of pure and meantone fifths and thirds using decimal equivalents

of tuning meantone temperament that the fifth was tuned slightly flatter than pure, and the chart also bears this out, with the quarter-comma fifth smaller than the pure fifth.

The decimal equivalents that I am choosing to use to express these differences do not have any musical significance. However, the useful feature of these simple decimal numbers is that one can easily rank the different temperaments from smaller to larger, or flatter to sharper. Larger numbers indicate a larger or wider interval that is sharp. Conversely, smaller numbers indicate smaller or narrower intervals that are flat. With the ability to calculate any kind of semitone, whether chromatic or diatonic, in any meantone temperament quarter-comma or otherwise, we can now correctly determine what kind of temperament and semitone lute fretting systems used. That is the matter to which we now turn directly.

Chapter 2

Lute Fretting Systems

In the previous chapter, I set forth several different methods in which we can measure ratios of intervals to determine the quality of their temperament. In this chapter, I will apply these methods to different fretting schemes and determine, from a mathematical standpoint, what kind of tempered interval each fret represents. The overall picture for lute temperaments from the sixteenth through early seventeenth centuries can be outlined as follows: Pythagorean tunings were used exclusively at the beginning of the period, and retained some use for certain intervals later in the period. By the mid-sixteenth century, lute players began to add different types of tempered intervals into the existing Pythagorean scheme. Some of these intervals were meantone, some approximating equal temperament and others were solely original in their quality. By the end of century and onward, fretting systems remained either mixed or used methods of dividing the octave into twelve equal semitones; however, their semitones did not always match those of modern-day equal temperament. The important distinction between these systems and other keyboard temperaments used at the time was that lute temperaments did not fall completely into one kind of tuning or temperament. They were very much to themselves, unlike any other systems at the time.

The majority of fretting instructions published during this period provided the player with a practical means of setting frets and did not dwell on theory. Almost all of the sources have very precise rules for determining fret placement with measurement tools such as a straight-edge and compass. These were common tools at the time for making geometrical divisions, and when used for calculating fret placement, could be used to divide a line into any number of equal parts. The remainder of the sources are less precise and instruct the player to place the fret somewhere near another until it sounds agreeable, or to move a fret slightly in one direction or the other without providing any exact measurements.

What follows is a comparison of several fretting systems that appeared in lute and vihuela treatises. For those sources that have instructions on placing frets using a compass, I have determined the exact ratio of the interval using the arithmetic methods I outlined in the previous chapter. The details of the calculations can be found in the appendix at the end of this paper.

2.1 Pythagorean Tunings for Lute

Starting in the 9th century, most if not all fretted instruments were tuned according to Pythagorean ratios. Frets on these instruments yielded pure fifths, fourths, and octaves as well as wholetones with a ratios of 9:8.[1] The result of this, as we saw in the previous chapter, was that major thirds were much wider than pure. What we know of the lute prior to 1500 suggests that it was a four- or five-stringed instrument tuned in fourths. Such a tuning would accommodate a Pythagorean tuning quite well, but after 1500, the lute's repertoire changed and a sixth string was added. The new tuning of the 6-course lute used a minor third between the instrument's third and fourth courses which exposes the major limitation of a Pythagorean system of tuning.

Because theory treatises in the early sixteenth century still advocated Pythagorean systems of tuning because of its mathematical principles, lute players were left to forge their own solutions to the tuning dilemma. One of the mainstays of the lute repertoire throughout the sixteenth century was intabulated polyphonic music. The first books of music printed by Petrucci in the early 1500s included many of these kinds of pieces and composers who played and composed music for the lute would include many of them in their publications. Because this music had many harmonies that relied on thirds, a Pythagorean system of tuning would have presented some obvious shortcomings. It is at this point that we see lute players begin a gradual shift away from Pythagorean tunings and towards meantone temperaments.

During the first half of the sixteenth century, at least three different tuning methods for lute were published. Two of these kept with the existing Pythagorean tradition of the day, while the third departed from that tradition and employed a kind of meantone tuning similar to the tuning that Pietro Aaron and Vicentino were developing at

1. Lindley, *Lutes, viols and temperaments*, 9.

the same time. *Epithoma musice instrumentalis*, published in 1530 by Oronce Finé, was one of the sources that kept with Pythagorean principles. Finé was a professor of mathematics at the University of Paris and published mostly theoretical works about mathematics. It was his personal interest in music that compelled him to write *Epithoma*. Written in Latin, the book includes instructions for tuning a lute, reading tablature and setting frets of a lute. Pierre Attaingnant was a friend of Finé's and it was probably this friendship that resulted in the treatise's publication. An additional set of lute instructions appear in Attaignant's publication *Tres breve et familiere introduction pour entendre et apprendre [...]* in 1529, this time written in French. Some stipulate that there instructions were from Finé and just translated, but this has never been fully proven.[2]

Another example of Pythagorean tuning appeared in a book published about the middle of sixteenth-century. The book was not about music and instead contained various topics concerning France. Appearing in 1556 and carrying the long title of *Discours non plus melancoliques que diverses, de choses mesmement, qui appartiennent a notre FRANCE: & a la fin La maniere de bien &iustement entoucher les Lucs & Guiternes*, it has been attributed to Bonaventure des Periers, but some sources list it as an anonymous author. The subject of tuning lutes and guitars appears in the last chapter of the book, which curiously has nothing to do with any of the other chapters in the book. The final chapter of *Discours [...]* contains instructions for fretting and tuning these instruments as well as a diagram of a mesolab, an ancient

2. Philippe Vendrix, "Finé, Oronce," *Oxford Music Online*:1, http://www.oxfordmusiconline.com/subscriber/article/grove/music/09666.

geometric tool that was first used to solve the issue of dividing a string into equal semitones.

In his assessment of their fretting instructions, Mark Lindley has concluded that Both Finé and the anonymous *Discours [...]* preserved the same characteristics of other Pythagorean tuning methods. These included earlier theorists from the fifteenth century such as Henri Arnaut, Johannes Legreze, Nicola Burzio and Franchino Gafurio, as well as more contemporary theorists like Heinrich Schreiber of Erfurt and Pietro Aron.[3] The features of these fretting schemes resulted in fourths and fifths at ratios of 4:3 and 3:2 respectively, and wholetones with a ratio of 9:8. Semitones depended on which fret the measurements began. In *Discours [...]*, frets 2, 4 and 6 were a series of 9:8 ratios beginning with the nut, while frets 1, 3, 5 and 7 were series of 9:8 ratios beginning with the seventh fret and moving backwards. Finé's method was slightly different, placing frets 6, 8 and 10 at 9:8 ratios starting from the twelfth fret.

The inherent problem with these systems is that the fret placement is based purely on mathematics and maintaining Pythagorean ratios without any deference to practical musicianship. Finé was not a musician, and Lindley suggests that that the author of *Discours [...]* was not either.[4] Furthermore, after attempting to play various pieces from the period, he concludes that "it seems doubtful to me that sensitive players would really have left the pythagorean scheme unaltered."[5] This becomes apparent in later sources from Juan Bermudo and Sylverstro Ganassi who each began with a

3. Lindley, *Lutes, viols and temperaments*, 11.
4. Ibid.
5. Ibid., 13.

Pythagorean tuning scheme and then adjust it to make it more palatable.

Successive instructions on fretting from the mid-sixteenth century onward de-emphasized Pythagorean systems in favor of tempered intervals and methods of equal fretting. While they certainly acknowledged Pythagorean intervals, and writers such as Bermudo and Ganassi offered complete fretting systems using Pythagorean ratios, they were never intended to be a final solution. Most musicians recognized the importance of Pythagorean intervals, such as the fourth, fifth, octave and in some cases the wholetone; however, they also used non-Pythagorean intervals to create a better solution. The primary distinction between sources that advocated Pythagorean tuning versus a tempered kind was that Pythagorean sources tended to focus on more mathematical issues than musical ones.

2.2 Gerle's fretting instructions

The first major break with Pythagorean systems of tuning for lute came from Hans Gerle, a lutenist and composer who published his treatise in 1533. In addition to its specific explanation of fret placement, it is also one of the best sources for lute instruction and practice in the early sixteenth century. It contains detailed explanations of tuning, right-hand and left-hand technique as well as tablature. The most important feature of Gerle's treatment of lute fretting, and the others that I will be analyzing in this chapter, is that Gerle was a lute player and not a theorist. From this viewpoint, matters of theory are discarded in favor of practicality. Gerle probably knew very little of the implications of breaking with the tradition of Pythagorean tuning. His emphasis was on finding a practical way to fret the lute in such a way that it would sound agreeable. The importance of his instructions become more apparent when we see other lutenists such as John Dowland borrowing many of Gerle's methods over 75 years later.

Gerle's instructions are directed towards a player who may or may not have theoretical knowledge of music, but would have had some rudimentary knowledge of arithmetic and geometry. His process involved using a compass and a straight edge, such as a piece of wood or other material that the player cuts to be the same length as the vibrating string length of the instrument. The marks were made on the straight edge using the compass to divide the distances between each mark into different parts. Once all the appropriate marks were made, they could be transferred to the fretboard and the frets placed accordingly.

He uses a very straight-forward, step-by-step approach for setting each fret, beginning with the twelfth.

> Take a straight-edge that is thin or else a flat piece of wood like a ruler, and make it of such a length that at the top it touches the piece of wood that the strings lie on and also touches the bridge that the strings lie on, and when you have made the ruler so that it touches at both ends (don't make it too short; it must touch as I have said), mark the bottom part at the bridge with an a, and the top part with a b, so that you will know which end belongs to the bridge. Then lay the ruler on a table, and take a compass and find the middle of the ruler. Mark it with a point or little dot and put an m there.

By placing the twelfth fret in the middle of the string, he divides the string in half yielding $\frac{1}{2}$, or in terms of a ratio, this would be an octave of 2:1. Compasses were widely used at this time to perform all kinds of geometrical divisions and we can assume most learned individuals at this time would know how to use one in order to execute Gerle's instructions. Euclid, the Greek mathematician, described procedures for dividing lines into equal parts in his famous work *Elements*. This book was first translated into Latin in 1482 and appeared in English translation by 1570. Although Gerle was German, it is reasonable to assume that he could have had access to either a copy in Latin or in German. As far was we know, Gerle was not educated at a university and might have had little or no knowledge of Latin; however, it is still reasonable to assume that with Euclid's work in circulation, he could have come by the knowledge needed to perform the required geometrical calculations.

Gerle continues using pure intervals for the fifth, or seventh fret:

> Then divide from the m to the b [in] three parts; and the first part from the m gives you the seventh and lowest fret. Mark it with a dot and put the number 7 there.

The letter b, marked in the previous step, is the end of the ruler on the side of the nut. Gerle now switches to numbers for frets and indicates the seventh fret with the number 7. The fret is marked at the first of the three parts starting from the twelfth fret or middle of the string. The resulting fret placement is one-third the length of the string, making the vibrating length of the remaining string two-thirds. We can express this semantically as a string divided into three parts with a vibrating length of two parts, or 3:2, which corresponds to the pure fifth. Gerle could have divided the entire string into three parts and place the seventh fret at the first part from the nut, but it seemed more important to base successive frets on the location of existing ones, in this case, the twelfth.

Gerle's instructions continue with the first fret, and similar to his instructions for the seventh, he builds on the calculations of the previous fret to find this distance for this one.

> Then divide elevenfold from the number to the b, and two of the same parts down from the b give you the first fret. Mark this also with a dot and put the number 1 there.

The "number" he is referring to is seven, or the seventh fret that we just marked in the previous step. Here he has us divide the distance from the nut to this fret in eleven parts and mark the second of these parts starting from the nut. This creates a ratio of 33:31, which is not Pythagorean but instead somewhere between the diatonic semitone 16:15 and the ratio 17:16. Mark Lindley has determined that this distance is actually a one-sixth comma semitone, but what kind? In chapter two, we examined systems of equal division that divided the octave into multiple parts greater than twelve. Sixth-comma meantone corresponded to an octave divided into 55 parts and the first fret on the lute would be the first semitone in a 55-division system. If the semitone is was minor or chromatic, it has four parts, and if it is major or chromatic, it has five.

To determine which semitone Gerle was using we can compare our calculations of his fret distance with the known values of different semitones. If we express Gerle's 33:31 ratio as a decimal by dividing 33 by 31, we get $\frac{33}{31} = 1.0645$ rounded off to the nearest fourth decimal place. Comparing that value to the values of other semitones

Equal temperament semitone:	$2^{\frac{1}{12}} = 1.0600$
Sixth comma chromatic semitone:	$2^{\frac{4}{55}} = 1.0517$
Sixth comma diatonic semitone:	$2^{\frac{5}{55}} = 1.0650$

Table 2.1: Equal and sixth-comma semitones

in table 2.1 we can see that Gerle's first fret matches the diatonic semitone in sixth comma meantone temperament.

Gerle has departed from Pythagorean tuning and opted for a tempered semitone at the first fret. This is not to say that he is advocating a sixth comma meantone tuning overall. Gerle still has us using a pure fifth at the seventh fret, which is not the same as a fifth tuned in sixth comma. A sixth-comma fifth is slightly flatter. The fifth is comprised of three whole tones and one diatonic semitone. In a sixth-

Pure fifth:	3:2	$= 1.5000$
Sixth comma fifth:	$2^{\frac{32}{55}}$	$= 1.4967$

Table 2.2: Comparison of pure and sixth-comma fifths

comma system this makes up 32 parts or dieses, and the resulting interval is slightly smaller than the pure Pythagorean fifth, which is what we should expect in any regular meantone temperament (see table 2.2). This will have implications when we examine the internal tuning issues of lutes later.

After having placed the first fret, Gerle continues with the placement of the second, or the whole tone. Here, he returns to the standard Pythagorean 9:8 ratio:

> Then divide again from the number 7 to the b threefold, and the first part down from the b gives you the second fret. Mark it also with a dot and put the number 2 there.

Here we have a three part division from the seventh fret to b, which is the nut. By returning to our earlier calculation of the seventh fret, we can formulate the second fret distance and vibrating length accordingly which results in the Pythagorean wholetone.

So far, Gerle has mixed Pythagorean pure intervals with tempered ones, such as the fifth, the major second and the tempered minor second. As he moves to the perfect fourth at the fifth fret, he writes:

> Then divide from the m to the b in two parts, and the one part gives you the fifth fret. Mark it with a dot and put the number 5 there.

Starting from the twelfth fret, Gerle divides the distance into two parts and takes the first part from the nut as the mark for the fret. The letter m is our octave fret, the exact midpoint of the string. Since he is now dividing that again in half, this produces a Pythagorean interval of the fourth with a ratio of 4:3.

Now Gerle turns his attention to the tritone, or sixth fret:

> Then put the sixth fret in the middle of the fifth and seventh frets. Make it with a dot and put the number 6 there.

If we assume that Gerle is using "middle" to denote the arithmetic mean, as opposed to the geometric mean, we can calculate the middle distance between the between the fifth and seventh frets and get a vibrating ratio of 24:17. However, an alternative explanation is that Gerle is being intentionally vague here as if to tell the player to put the fret somewhere between the two frets, and not necessarily exactly in the middle. If we assume this, then it seems reasonable to also assume that the player could alter the placement until it produced a good result.

Regardless of our interpretation of Gerle's instruction, the tritone is neither Pythagorean ratio nor sixth-comma meantone, but it may be close enough to be considered equally tempered. In a sixth comma meantone with a 55-division octave, the trione contains either 27 or 28 dieses depending on whether or not the semitone above the fourth is chromatic or diatonic. For a lute tuned in G, this would be the difference between a D-sharp and an E-flat. Calculating the decimal equivalents for these two semitones shows us that Gerle's tritone falls somewhere in between the chromatic and diatonic semitones. Although numerically this fret is not identical to the equally-tempered tritone, if we look at a scaled drawing depicting the location the fret in relation to the other types of intervals, Gerle's fret and the location of the equally-tempered fret look nearly identical. We must note that Gerle places the fret in the middle of the other two frets without the use of a compass. Furthermore, while Gerle's sixth fret may come close enough to equal temperament that we could call it an equally tempered interval, he is not using equal temperament in the modern sense. In fact, Gerle was using meantone in its original sense, the way in which Aaron would have tuned.

Meantone temperaments that did not use equal division had to approximate some of their semitones by dividing the wholetone into two different halves. For example, Aaron's original instructions did not consider every semitone in the scale, it only covered a set of thirds that were tuned pure leaving some of the other pitches indeterminate. Semitones that were not included in the scheme were divided, splitting a wholetone into two different parts. Since this was probably done by ear and without the aid of exact measurements, the calculation of this fret is an approximation and we should discount the use of arithmetic to prove what kind of semitone Gerle was trying to specify between the fifth and seventh frets.

In order to better visualize these frets that are more like approximations than they are exact calculations, we can can look at a scaled drawing comparing the placement of the different semitones. I will be using many such scaled drawings and each will use the same sample mensur length of 70 centimeters. The purpose of the drawing is not show the exact placement of frets that would normally be approximate, but to demonstrate how one type of inexact fret may be similar to another type of fret that is exact in its placement by showing how close they are to another. The choice of mensur length is based on what best appears on the page. While 70 centimeters is unusual for a lute, a more common mensur length of 60 centimeters is smaller on the page and does not exhibit the differences of semitone size as clearly.

In turning to Gerle's sixth fret, we can look at one of these scaled drawings comparing his fret with the known placements of the chromatic sixth-comma semitone, the diatonic sixth-comma semitone and the equally tempered semitone. These frets are shown in red while Gerle's frets are in black. Gerle's fret favors the chromatic

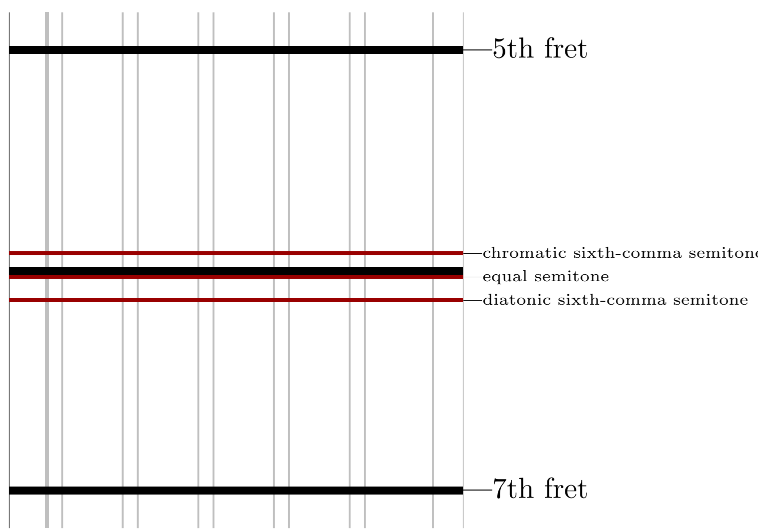

Figure 2.1: Scale drawing of Gerle's sixth fret

side, although given the proximity of their locations, his fret is very near the equally tempered semitone that it could be considered closer. Another point to bear in mind is that as the mensur length decreases, so will the distances between these frets. Conversely, the distances will increase as the mensur increases. This will have more bearing on the theorbo, which has a mensur length of 80 centimeters or more, and is discussed in later chapters where I focus on executing different types of fret placements.

Gerle next instructs us to place the third fret using a new technique whereby he takes the portion of one fret and uses it to compute the distance of another. In this case, he takes a portion of the distance from the nut to the first fret and uses it to find the third.

> Then divide from the number 1 to the b [in] three parts, and when you have the three parts then go with the compass unaltered down from the number 1 again five spans; that gives you the third fret. Mark it with a dot and put the number 3 there.

Assuming that the span is one part of the three parts into which the first fret distance is divided, the total spans from the nut (b) is eight: the five spans from the first fret to the third, plus the original three spans from the nut to the first fret. The resulting 99:83 ratio is closest to a diatonic sixth comma meantone minor third. In looking at

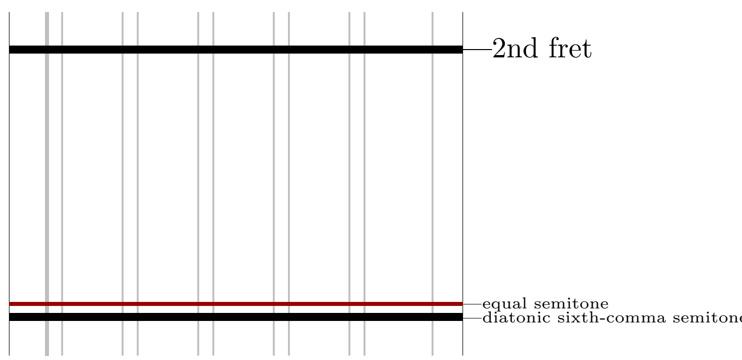

Figure 2.2: Scale drawing of Gerle's third fret

the scale drawing, the black line representing the fret completely occludes the red line of the diatonic sixth-comma semitone, indicating the for all practical purposes the two semitones are the same. Longer mensur lengths might exhibit a slight difference in distance but this different would be minimal at most.

The last fret is the fourth which makes the major third. Similar to the sixth fret, he averages the distance between the third and fifth.

> Then put the fourth fret between the third and the fifth frets. Mark it with a dot and put the number 4 there.

Averaging the distance between the two frets results in the ratio 792:629, which comes closest to the equal tempered major third. Referring to a drawing of the fourth fret, Gerle's major third is flatter than a true equally tempered third, but sharper than a major third in a sixth comma temperament. With our sample mensur length of 70

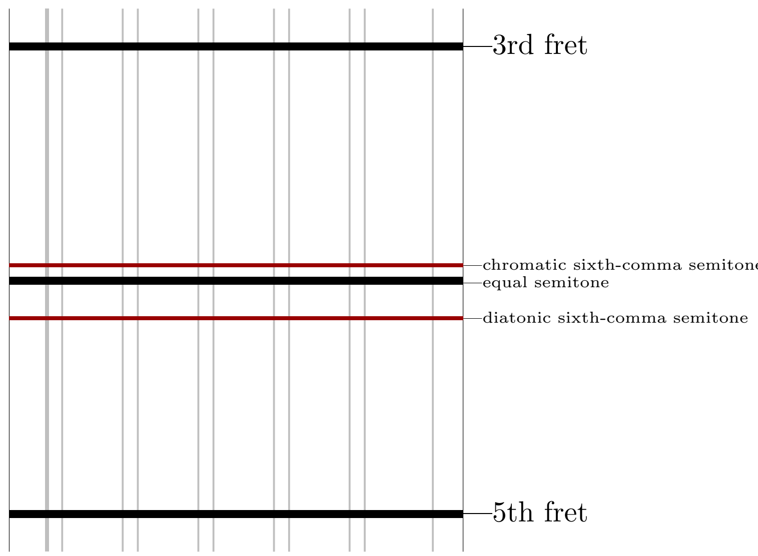

Figure 2.3: Scale drawing of Gerle's fourth fret

centimeters, the difference in actual distance between Gerle's major third and a sixth comma meantone major third is about 2 millimeters. The difference between Gerle's third and a quarter comma meantone third is 4 millimeters. So when speaking of comparisons that account for a few thousandths of a decimal, the numbers themselves are small, but when translated into a real instrument, the differences become large.

Gerle limits his fret placement calculations to the seventh fret only; however, he does provide instructions for placing an eighth but without leaving us any geometrical calculations to do so.

> But if on the lute one wants eight frets, then let him make the eighth fret a little closer to the seventh fret than the sixth is.

Taken at face value, we cannot really determine what Gerle exactly means other than the semitone between seventh and eighth fret is smaller than the semitone between the sixth and seventh. At this point, we are to assume that the player may use his or her ear to adjust the fret accordingly. This same rule would hold true for the rest of the frets as well.

Taken as a whole, Gerle's fretting instructions are a mixed assortment of different intervals. He uses Pythagorean ratios for the basic intervals, but relies on tempered ones for the others. These are either sixth-comma meantone in nature, or completely unique. His approach seems to have taken hold with other players because we see the same kind of heterogeneous technique used for many years afterwards.

2.3 John Dowland's fretting instructions

Gerle's fretting instructions seemed to have survived for quite some time after his death because they appear in an almost identical form an an instruction method by the English lutenist John Dowland. One of the best known lutenists of his day, Dowland was an important composer of songs and solo music for the lute and remains so to this day. During his lifetime, he published five books for lute and voice, as well as music for viol consort and lute accompaniment. His solo works existed mostly in manuscript, with the notable exception of the *Varietie of Lute-Lessons* which was published by his son, Robert Dowland, in 1610 and contained music by John Dowland and some of his contemporaries.

As the title suggests, the *Varietie* was intended as an instruction book, however the music in it demanded a high level of ability. The book begins with two prefaces, one written by John Baptiste Besard and a second by the elder Dowland. Besard's preface, *Necessarie Observations Belonging to the Lute and Lute-playing* appeared several times throughout this period [need dates on Besard's instructions] and was reprinted again here. Dowland's, entitled *Other Necessary Observations belonging to the Lute* offered additional advice on choosing lute strings and a method for setting the frets on the instrument that followed Gerle's very closely. While his instructions postdate Gerle by more that 70 years apart, it is obvious that Dowland must have known of Gerle's book because of the similarity of their instructions. It is also possible that they boy consulted a similar source on geometry and measurement and reached the same conclusions, but it is more likely that Dowland simply knew about Gerle's instructions either through his book directly or from common practice and we repeating these same instructions here.

After giving the usual historical account of Pythagoras discovering harmony by listening to the hammers of blacksmiths, Dowland instructs us to procure a piece of 'whitish' wood that is just as long as the distance from the inward side of the nut to the inward side of the bridge on the lute. From there, his fretting instructions are exactly the same as Gerle's with the exception that Dowland specifies letters instead of numbers since he was using the French style tablature system.

If we account for several printing errors that appear in Dowland's instructions, they read very straightforward.

> Wherefore take a thinne flat ruler of whitish woode, and make it just as long and straight as from the inward side of the Nut to the inward side of the Bridge, then note that wnd which you meane to the Bridge with some small marke, and the other end with the letter A, because you may know which belongeth to the one and to the other. then lay the ruler upon a Table, and take a payre of compasses and seeke out the just middle of the Ruler: that note with a pricke, and set the letter N. upon it, which is a Diapason from the A. as appeareth by the striking of the string open.

After describing the same manner of using a ruler in place of the mensur length of the string, Dowland places the twelfth fret, fret N, at the midpoint of the string creating a pure 2:1 octave.

> Secondly, part the distances from N. to D. in three parts, then the first part gives you the seaventh fret from the Nut, making a Diapente: in that place also set a pricke, and upon it the letter H.

Dowland makes the first of many typographical errors and uses the letter D instead of A. If we allow for that mistake and read the passage 'N. to A.' and not 'N. to D.', it is obvious that Dowland is marking a perfect 3:2 fifth at the letter H. When he sets the first and second frets, he repeats Gerle's measurements almost verbatim, yielding a 33:31 diatonic sixth-comma semitone and a 9:8 Pythagorean wholetone.

> Thirdly, devide the distance from the letter H. to the letter A. in eleaven parts: two of which parts from A. gives the first fret, note that with a pricke, and set the letter B. thereon, which maketh a Semintone. Fourthly, divide the distance from H. to the letter A. in three parts, one of which parts from A. upward sheweth the second fret, note that with a pricke, and set the letter D. upon it, which maketh a whole Tone from A.

When he gets to the fifth fret there is another typo, mistakenly indicating the first fret instead of the fifth, but otherwise the same 4:3 pure fourth that Gerle uses.

> Fifthly, divide the distance from N. to A. into two parts, there the first part sheweth you the first [ie. fifth and not first] fret, sounding a Diastessaron: in that place also set a pricke, and upon it the letter F.

The sixth fret proceeds the same a Gerle's as the average between the fifth and seventh, giving us the same ratio of 24:17.

> The sixt fret with is a G. must be place just in the middle betwixt F. and H. which maketh a Semidiapente.

When Dowland reaches the third fret, he departs from Gerle's geometrical instructions:

> Seventhly, divide the distance from the letter B. to A. in three parts, which being done, measure from the B. upwards foure times and a halfe, and that wil give you the third fret, sounding a Semiditone: mark that also with a prick, & set thereon the letter D.

Whereas Gerle would measure five of his spans from the first fret, Dowland only uses four and a half. This results in a ratio of 198:168 which in our sample diagram appears the same as the sixth comma chromatic semitone. Dowland is intentionally

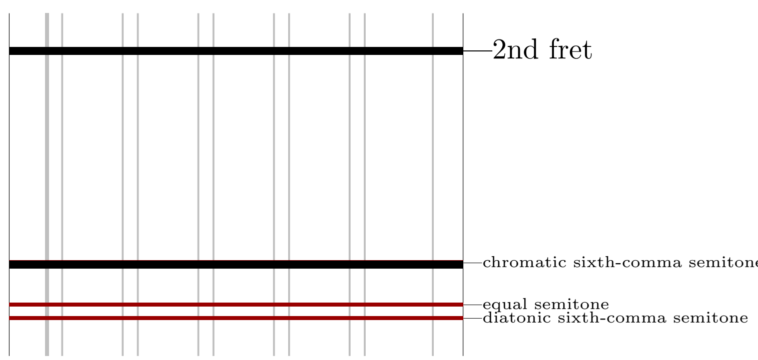

Figure 2.4: Scale drawing of Dowland's third fret

vague here. Instead of using whole numbers like everyone else does when making geometrical divisions with a compass, he uses fractions. If we wanted to be exact as possible, we could find the mean of the original span that is used to find the distance to the third fret and have a true "four and a half spans," but Dowland does not instruct us to do that.

There are several possible explanations for Dowland's odd calculation. First, Dowland could have been leaving the method of calculation to the player. Someone who was able enough to execute Dowland's geometrical calculations would have been able to find the exact number of spans necessary, if he or she was willing to be that exact. Perhaps Dowland simply did not want to go into the extra details. A second explanation is that Dowland knew he preferred a chromatic semitone and used the specification of 4.5 spans as an approximation. The compelling case for this explanation is that the 4.5 span measurement is so close to a chromatic semitone in sixth comma meantone, the same kind of regular meantone temperament that Gerle was using. A third explanation is that Dowland did not know he was expressing the difference between the chromatic and diatonic semitone and arrived at the interval "by ear." Either by experimentation or estimation, he found a way to mathematically represent the fret placement with an approximate measure of 4.5 spans.

As Dowland moves on the fourth fret, he places it using the same arimetic mean that Gerle used in his instructions.

> then set the fourth fret just in the middle, the which wil[l] be a perfect *ditone*:

Since Dowland's third fret was slightly smaller than Gerle's, this will make the placement of his fourth fret different from Gerle's as well, coming to a ratio of 1584:1266 which is very close to a quarter-comma chromatic semitone. Although this could lend support to an argument in support of quarter-comma meantone on fretted instruments, it is the only interval in this scale that appears to be tempered this way. The other intervals are still a mix of Pythagorean and sixth-comma meantone inter-

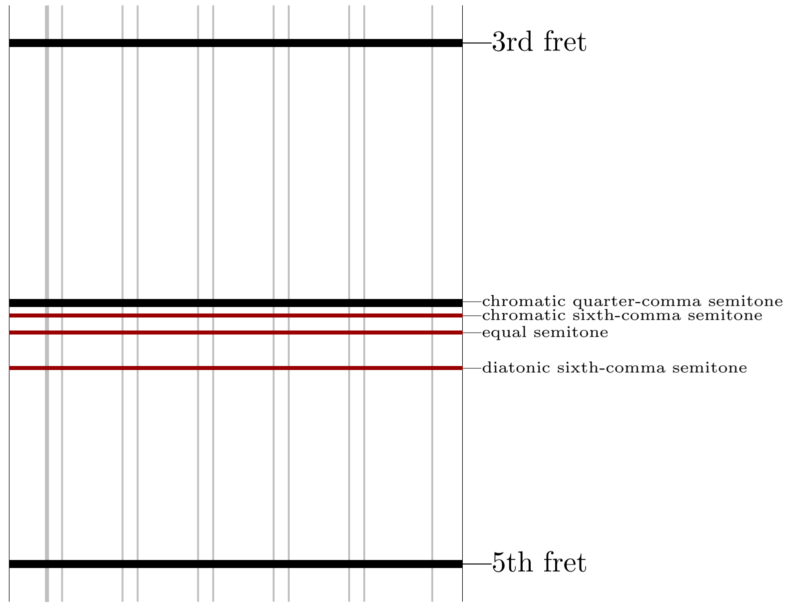

Figure 2.5: Scale drawing of Dowland's fourth fret

vals.

Dowland goes on to give us instructions for placing the eighth, ninth and tenth frets instead of stopping at the seventh as Gerle does; however, it is at this point that his calculations go awry. He places the remaining three frets using the same process:

> then take one third part from B. to the Bridge, and that third part from B. maketh I. which soundeth *Semitonium cum Diapente*, then take a third part from the Bridge to C. [N.B. He means C. to the Bridge and not the other way around] and that third part maketh E. [N.B. Another misprint here, he means the ninth fret, K.] which soundeth *Tonus cum diapente*, or an *Hexachordio maior*. Then take one third part from D. to the Bridge, and that third part from D. maketh L. which soundeth *Ditonus cum Diapente*.

Apart from the errors with the ninth fret, Dowland is repeating the same pattern for each remaining fret. He seems to be calculating these frets placements by dividing the total distance between one fret and the bridge into three parts and then setting the fret one-third the distance from that fret. In looking at the mathematical result of this process, we find the ratios of the higher frets are actually decreasing when they should be increasing. Either we are misinterpreting Dowland's instructions because of further errors in the printed source, or Dowland has simply got his calculations wrong.

99:68 ratio	$\frac{99}{68}$	$= 1.4559$
27:19 ratio	$\frac{27}{19}$	$= 1.4210$
594:426 ratio	$\frac{594}{426}$	$= 1.3944$

Table 2.3: Decimal ratios of Dowland's eighth, ninth and tenth frets

If we return to the calculations that we obtained following Gerle's scheme, the only differences between those and Dowland's are the third and fourth fret. However, in looking at the semantic instructions themselves the real difference lies with the third fret. Gerle required 5 spans from the first to the third fret while Dowland required slightly less, 4.5. The semantic instructions for the fourth fret were the same in both sources: place the fourth fret in the middle, between the other two. The ratio of the fourth fret was only different because the third fret was, having been shorted slightly. If we then entertain the possibility that Dowland was essentially using Gerle's original instructions directly, but with only a slight modification to the third fret, perhaps due to personal preference, we could then see that the eighth, ninth and tenth frets were of Dowland's own doing.

The last three frets of Dowland's scheme had no precedent, if we assume that Dowland was using Gerle as his primary source. Dowland then was on his own to determine in a proper procedure for obtaining the right proportional distance for those last frets. Due to the calculations above, it is clear that one cannot arrive at a useable placement for the last three frets of his scheme. Nor can one use Dowland's own fretting instructions as complete source because there is no mention of the eleventh and twelfth frets which figure prominently in his music, and especially the contents of the book which his instructions preface.

2.4 Ganassi's *Regola Rubertina*

Because Dowland and Gerle hold many similarities in their fretting instructions, we can make a strong case to use them as a single unit for comparison, since all but one of their frets align very closely. An important source that would contrast with their methods is Silvestro Ganassi's instruction method for viola da gamba entitled, *Regola Rubertina*. Although he was a gamba player and his method is intended for that instrument and not a lute, it does contain fretting instructions that are valuable when applied to the lute

Ganassi approches his subject slightly differently than Gerle and Dowland. In the fourth chapter of his treatise, he specifies the locations of each fret using a compass and string division, just as Gerle and Dowland do; however, in the fifth chapter, he adjusts some of these frets to different positions by comparing unisons from one string on the instrument to another. The first, second, third, sixth and eight frets are adjusted flatter than their initial placements, while the fourth fret is adjusted sharper. Some of these changes are markedly different than in other treatieses of the time, namely the fourth fret. Although, the frets that make the wholetone, fourth and fifth, are all kept to their Pythagorean ratios.

For example, Ganassi opens his fourth chapter with the second fret, and uses the same Pythagorean wholetone ratio that everyone else has used at this time:

> Please note that the proportion *sesquioctava* produces pitches expressed by these two number, 9:8. This proportion determines the location of the second fret. If you divide the string, beginning from the nut on the fingerboard and ending at the bridge, where the bow is drawn, into nine parts, the first of the nine parts sets the boundary of the second fret.

He also uses the pure 4:3 ratio of the perfect fourth because he says the open strings that are played against it must be in unison.

> Then, you divide the string into four parts; the first of these four part will set the location fo the fifth fret, which produces the consonance of a fourth, which is created by the proportion of sesquitertia indicated by a ratio of 4:3 [. . .] This produces the consonance of the prefect fourth, because if one then plays the open string, which is at the end of the fourth part of the string length, one achieves the opposite of the 4:3 ratio

Most players would have tuned their open strings using the the fifth fret of the adjacent string. In this case, Ganassi is merely underscoring the fact that the fifth fret should be at a the ratio of a pure fourth because the interval of a fourth between the adjacent strings of the viol should be pure. The seventh fret is kept to a pure ratio as well:

> Then you divide the string length into three parts. The first of the three parts will be the end of the seventh fret, thereby producing the consonance of the perfect fifth, of *diapente*, which is formed with the proportion of *sesquialtera* indicated by the ratio of 3:2.

So for these frets, Ganassi is reiterating what most other theorists believed about tuning the wholetone, fourth and fifth by maintaining their Pythagorean ratios. In chapter five, “Method of Adjusting the Frets,” Ganassi verifies the positions of these three frets by checking the unisons against the various open strings of the instrument. All this is to reify that he wants the all the intervals between open strings to be pure

Pythagorean ratios. It is important to note here that this precludes the use of any meantone temperament since the fifths would need to be tempered flatter than pure. We may also say the same thing about Dowland and Gerle's methods since their fourths and fifths are pure as well.

Once Ganassi has completed setting those frets, he does not revisit them. However, this is not the case for the other frets of the instrument. It is the other intervals that distinguish Ganassi's ideas about fret placement from some his contemporaries. For one, he departs from certain meantone and Pythagorean ratios in favor of his own that are produced by comparing tones from one string and fret, to the tones of a different string or fret.

2.4.1 Ganassi's non-Pythagorean frets

Ganassi's first fret differs in size from both Gerle and Dowland. He begins by having us place the first fret in a Pythagorean ratio, but eventually moves it to a different kind of ratio. In chapter four, he writes:

> After you have positioned the second fret in the manner described above, the first fret should be set half way between the major and minor semitones by their respective proportions. In order not to go into this at length, however, I believe that I have chosen a similar method for finding the first fret which produces a minor semitone, quite easily.[6]

Another translation of the same passage from Mark Lindley's books is slightly different:

6. Silvestro Ganassi, "Ganassi's Regola Rubertina (Conclusion)," Translated by Richard D. Bodig, *Journal of the Viola Da Gamba Society of America* 19 (December 1982): 106.

Dapoi che hauerai touato &	Now when you have found and
terminato il ditto secōdo tasto	located the second fret
cō il modo ditto di sopra	by the method given above,
il tasto primo sera terminato al meza	the first fret is located halfway
tra il scagneleto del manico al	between the nut and the
secondo tasto ma de piu zoe	second fret, but more, i.e.
battēdo di fora la mita della	down the neck by half the
grosseza del tasto . . . & in questo	fret's width. In this regard
ti haueria possuto resonar il	I could have calculated for you the
partimento del semiton maior al minor	division of the major and minor semitones.
. . . il primo tasto elqual fa leffeto	The first fret gives the effect
del semiton minor . . .	of the minor semitone.

Taking both translations into account, the placement of the first fret appears to be halfway between the *scagneleto del manico* or nut and the second fret. When the Pythagorean wholetone (9:8) is divided in half, the resulting ratio is 18:17, or what is the closest to an equal semitone. It is not exactly the same as a true equally

18:17 semitone:	$\frac{18}{17}$	$= 1.0588$
Equal temperament semitone:	$2^{\frac{1}{12}}$	$= 1.0595$

Table 2.4: Comparison of the 18:17 and equally tempered semitones

tempered semitone, but is close enough that many theorists andplayers advocated using a system of semitones solely based on a series of 18:17 divisions.

In the sixth chapter, Ganassi revisits the first fret and adjusts its position so that the unison formed between the first fret on the fourth string and the fifth fret of the fifth string of the instrument is in tune.[7] Because this unison will not be in tune if the first fret is a 18:17 ratio, the fret must be moved towards the nut, rendering it slightly flat. We can determine the decimal equivalent of this new adjusted first fret by referring to a chart of frets and their placements from Richard Bodig's article on

7. Richard D. Bodig, "Silvestro Ganassi's *Regola Rubertina*: Revelations and Questions," *Journal of the Viola Da Gamba Society of America* 14 (December 1977): 67.

Ganassi's *Regola*. He calculates new values for each fret using Ganassi's instructions on where the frets are moved after they are initially placed. The resulting decimal values occur once the player moves the frets to where they are exactly in unison with one another according to Ganassi's instructions.

Using Bodig's calculations of Ganassi's fret adjustments, and comparing them to the fret placements we have see thus far with Gerle and Dowland, Ganassi's first fret is much flatter than either Dowland or Gerle. Curiously, Ganassi's adjusted first fret is almost exactly the same as the chromatic sixth-comma semitone. This differs

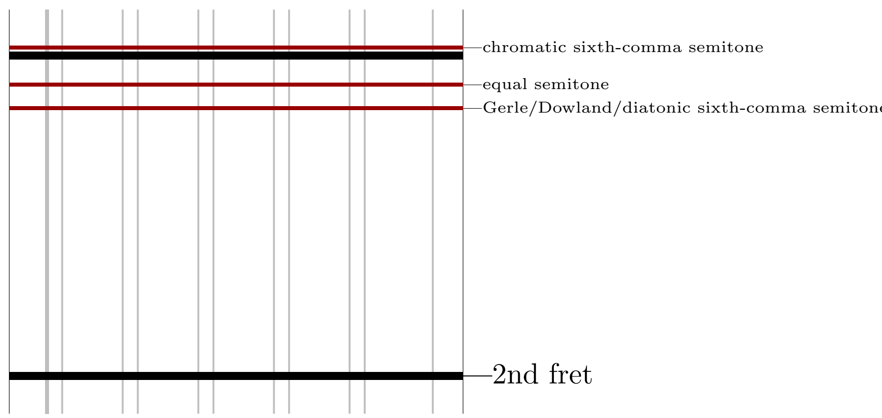

Figure 2.6: Comparison of Ganassi's first fret

from Dowland and Gerle whose first frets come to match the diatonic sixth-comma semitone. For a lute tuned in G, this would be the difference between a G-sharp and an A-flat on the first string.

Ganassi's third fret undergoes a similar transformation. In the fourth chapter, he sets the the fret at as pure minor third. Later in the sixth chapter, he adjusts this and tunes it to the octave formed between the first fret of the third course and the third fret of the sixth course. Returning to Bodig's calculations, this results in lowering Ganassi's third fret from its initial pure minor third to something that is flatter than an equally tempered minor third. The drawing for this fret shows that it

Figure 2.7: Comparison of Ganassi's third fret

is towards the chromatic sixth-comma semitone of Dowland, but not close enough to be considered equal to it, nor any other kind of semitone that we have seen thus far.

Ganassi's initial setting of the fourth fret is in the middle between the third and fifth frets, before the third is adjusted. This gives us a very strange ratio of 48:38. Later, he adjusts the position of the fret so the octave between the second fret of the sixth string and the fourth fret of the fourth string are in tune. As the drawing

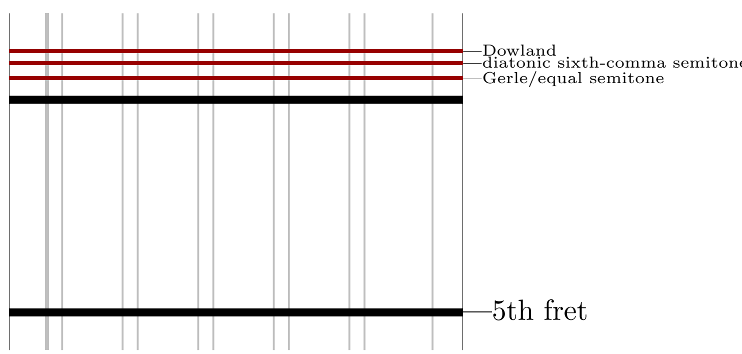

Figure 2.8: Scale drawing of Ganassi's fourth fret

indicates, his fourth fret is almost unplayable since it is yielding a major third that is even sharper than equal temperament. Ganassi provides no further explanation for this, but we must assume that most players would change this fret to something more suitable and dismiss his instructions for this particular fret.

The initial placement for the remaining sixth and eighth frets is similar to the fourth, and like his contemporaries, Ganassi does not provide any information about frets beyond the eighth. His sixth is the same as Gerle's, the arithmetic mean of the distance between the fifth and seventh, or a ratio of 24:17, and the eight fret, which is the most specific we have seen in any source thus far, results in a 8:5 ratio.

> Then the sixth fret is set at the midpoint of the space between the fifth and seventh frets but somewhat less, that is so that the thickness of the fret is within the compass of the distance; that will set its position. The eighth fret will be located so as to have the same spacing as the form the fifth to the sixth frets.

Ganassi as well as other sources at this time such as Dowland, reference the thickness of the gut string used as the fret and allude to the fact that its thickness would have some impact on its placement. For example, a particularly thick fret might offset calculations by as much as a millimeter. For modern lute players, this would be an issue for the lower frets where gut strings of a millimeter or more are often used, but this decreases to thicknesses of less than a millimeter for higher frets. No information exists about the specific thicknesses of the gut strings used for frets on instruments during this time. Ganassi's mention of it here at least indicates that it may have been an issue. His instruction is to simply ensure that no matter how thick the fret is, its entire thickness is within the span of the compass.

The final adjustments to Ganassi's fretting scheme include the sixth and eighth frets, which are moved slightly flatter than where they were originally placed during chapter four. The unison between the sixth fret of the third string and the first fret of the second is used to adjust the sixth fret as needed to make that sound in tune. The eighth fret is then adjusted so that the octave formed between the open fourth string and the eighth fret of the third string is in tune. Using Bodig's measurements, we can see that the final placement of the sixth fret moves it just slightly past the mark for the chromatic sixth-comma semitone, or C♯ on an instrument tuned to a relative pitch of G. It is hard to say if this was intentional or simply a coincidence of

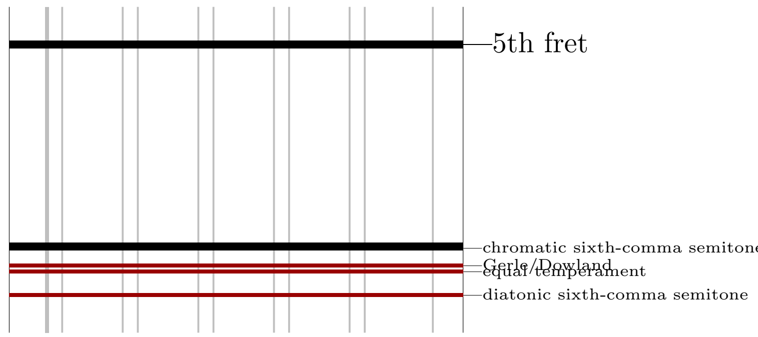

Figure 2.9: Scale drawing of Ganassi's sixth fret

Ganassi's adjustment procedures. This is perhaps more evident in the final placement of the eighth fret which is moved into a position that is not near any one particular temperament, making it unique.

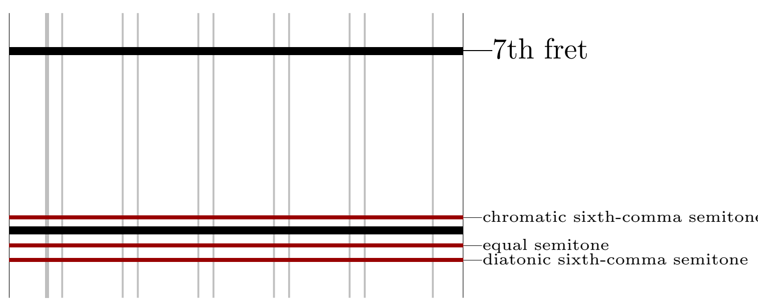

Figure 2.10: Scale drawing of Ganassi's eighth fret

Even though Ganassi spends an entire chapter of this treatise on placing the frets according to geometrical calculations, it is not until the sixth chapter that we see the complete picture of his scheme. Dividing the process such as he does seems to acknowledge the dichotomy between a theoretical system of string division and a practical one that works for music in the realm of performance. Beyond this Platonic differentiation in tuning, we really cannot know what Ganassi's motivations are.

Ganassi holds many similarities with other sources. The second, fifth and seventh frets are the same as Gerle and Dowland. His first fret, however, is noticeably flatter than the Gerle/Dowland model, making it closer to a sixth-comma meantone chromatic semitone instead of a diatonic one which Gerle and Dowland favor. Perhaps the most important feature that Ganassi's fretting scheme shares with others from this period is that combines different types of semitones from different temperaments. This further indicates that lute temperaments were treated differently than temperaments for other instruments.

2.5 Spanish Vihuela Sources

During the sixteenth century, there were a number of publications in Spain of music and instruction on the vihuela. The vihuela shared many similar characteristics with the lute, but the most important to discuss is how its players grappled with the same fretting issues that lutenists did. While the instruments were different, Spanish vihuelists knew of the lute's repertory and technique. For example, in Miguel Fuenllana's *Orphenica lyra*, published in 1554, he discusses the right-handed "foreign style" of playing, which refers to the thumb and index alternation of playing practiced by lutenists at the time.

The schemes for setting frets on the vihuela come from two sources. The first, and earliest of the two Luis Milan's *El Maestro*. Milan was a well-known vihuela composer and poet who seems to have spent most of his active life around Valencia. Although his birth and death dates are not known, it is believed he died some time after 1561 due to a reference in one of the composer's books of poetry, *El Cortesano*.[8] Milan is best described as both a poet and bard who would sing or recite his poetry to his own music. He was active mostly at the court of Valencia where he made his living working for the dukes of Calabria. *El Maestro* is his only musical publication but remains one of the most important vihuela publications we have, not only for its musical content but for its information about performance practice such as tempo and interpretive elements that are absent in other contemporary treatises.

While *El Maestro* contains only a few references to fret placement they are very significant. Milan's own remarks were not as exact as the others we have seen. He gave us no measurements with which to mark frets on a straight edge, but he did give us some clues as to which frets he wanted adjusted to achieve a different quality for certain notes. In Gasser's monograph on performance practices in Milan's music, he translates the following section from *El Maeastro* where Milan discusses changing the position of the fourth fret before the music for Fantasia 14.

> Whenever you play the fourth and third tones in those places through which the fantasia moves, raise the fourth fret of the vihuela a little, so that the note of the fret becomes strong and not weak. [Siempre que tañerais el cuatro y tercero tono por estos tèrminos que est fantasìa anda, alzarèis un poco el cuarto traste de la vihuela para que el punto del dicho traste sea fuerte y no flaco][9]

Additionally, there is another piece of information concerning the same fret before

8. Luis Gásser, *Luis Milán on Sixteenth-Century Performance Practice*, ed. Thomas Binkley, Publications of the Early Music Institute (Bloomington, Indiana: Indiana University Press, 1996), 6.

9. Ibid., 156.

the beginning of the music for *Con pavor recordó el moro.*

> Playing in these pieces on the vihuela, you have to raise the fourth fret a bit toward the pegs. [Tañdo por estas partes en la vihuela habéis de alzar un poco el cuarto traste hacia las clavijas de la vihuela][10]

Both references concern the fourth fret, and both offer an adjustment that raises the fret closer to the pegs, making the interval smaller. This would equate to a narrower major third; however, it also assumes that the fret as been placed in some manner prior to being moved. It is not possible to know by what means Milan would have first placed his fourth fret before moving it, but it seems that for certain pieces, he preferred it to be slightly flatter than its initial placement. This could corroborate Dowland's use of a quarter-comma chromatic semitone for his fourth fret, which is substantially flatter than either an equally-tempered or Pythagorean semitone. However, the problem here is that Dowland achieves a flatter fourth fret by also making his third fret flat as well. Milan mentions no other fret to adjust, so we might assume that all his frets were placed according to standard Pythagorean proportions. Still, both Dowland and Milan do show a preference for flatter fourth frets.

Perhaps the most complete discussion on vihuela fretting appears in Juan Bermudo's *De tañer vihuela*, published in 1555. Bermudo's publication is one of the most extensive on vihuela performance practice, including several chapters on intabulation for the vihuela, and fret placement. Bermudo provides a total of three different fretting schemes. The first of these, which appears in chapter 77, is Pythagorean. The second scheme attempts to correct the semitones in the first scheme by adjusting the first, sixth and eighth frets while leaving the others in their original Pythagorean form.

10. Ibid.

The third and last of Bermudo's schemes is unrelated to either of the previous two and is the closest to modern-day equal temperament.

Dawn Espinosa translated Bermudo's work, with the original Spanish printed alongside the English translation, and prefaced her translation with an informative discussion. In it, she provides a table of Bermudo's fretting schemes and the ratios she has calculated for each fret. For my analysis, I have used these to compare each of Bermudo's fretting schemes with the others that we have seen so far.

Similar to Ganassi, Bermudo gives us a simple Pythagorean-based fretting system but then offers two alternative systems as well. This may help explain the changes in tuning occurring during the time. Prior to the sixteenth century, Pythagorean tuning had been the dominant system, but it had its inadequacies. By starting with a Pythagorean system and then providing alternatives, writers could now acknowledge its importance and at the same time move past its insufficiencies. For example, the second fretting system that Bermudo presents a solution that corrects some of the issues with a Pythagorean system and arrives at the exact same placement for the third fret that Ganassi does. Also similar to many of his contemporaries, his solution

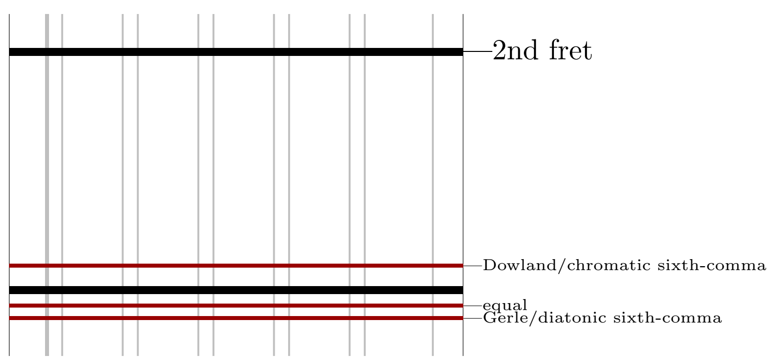

Figure 2.11: Bermudo's third fret

for the first fret clearly advocates the diatonic semitone. However, it is slightly sharper

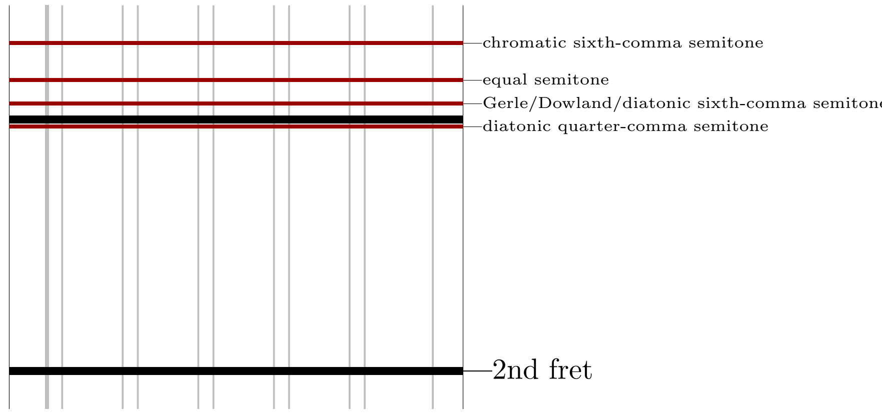

Figure 2.12: Comparison of Bermudo's first fret

than the sixth-comma diatonic semitone of Gerle and Dowland.

Bermudo refers to the "faults" found in fretting systems between notes that are *mi* or *fa*. In terms of sixteenth-century theory, this was the modern-day equivilent of a sharp note versus a flat note. After setting the frets in his first scheme, the Pythagorean one, he discusses the problems found on the eighth fret:

> On the eighth fret there seem to be three faults. This fret should be fa for the seventh, fourth and first strings, but is it [made] mi for all the strings.[11]

Bermudo is highlighting the central problem with fretting instruments: the distance of one semitone on one string is the same for all the other semitones on that string. Depending on the temperament and semitone, a minor semitone might be required for one string while a major semitone would be needed for another.

One of Bermudo's solutions, which we also see with Milan, is to adjust the frets according to the key of the piece. Milan, for example, would adjust the fourth fret when playing the third and fourth tones, while Bermudo describes players who move

11. Dawn Astrid Espinosa, "Juan Bermudo 'On Playing the Vihuela' ('De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555)," *Journal of the Lute Society of America* 38-39 (1995-96): 95.

their frets according to their ears:

> We have seen players who, with their frets set for the sixth, want to play the fourth mode, but are unable to do it without moving the frets as their good ear tells them. What I intend to do here is to give the measurements by which to place the frets, so that those who are not [such good] musicians will be able to place them with ease and exactiude, and thus the vihuela with be more prefect.[12]

Bermudo seems to suggest that it is the good players that are able to move their frets for different modes, while inexperienced or lesser skilled players are unable and should rely on Bermudo's calculations to place the frets for them.

For those players who are unable to correctly place their frets by ear, Bermudo's ultimate solution is a scheme that matches modern equal temperament very closely. His third scheme essentially splits the wholetone into two equal halves, although the composer himself acknowledges that it is not his aim to do so:

> All the [theorists] agree that the whole tone cannot be divided into two equal semitones, but that is what we are presuming to do. The above being the most agreed and true thing among theorists, on the vihuela we find the opposite in practice.[13]

For Bermudo, the issue was a question of theory versus practice. Espinosa summarizes this issue succinctly: "for him, theory has more authority than practice, but he concedes that practice precedes theory."[14]

In table 2.5, the difference between each of Bermudo's fret and modern equal temperament is expressed in both cents and in three different mensur lengths: 600mm, 700mm and 800mm. Millimeters are used for the mensur length because they can express the differences with more accuracy. 700mmm is also the same as 70cm,

12. Espinosa, "Juan Bermudo 'On Playing the Vihuela' ('De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555)," 78.
13. Ibid., xx.
14. Ibid.

which has been the reference mensur length in our other diagrams. Cents, which is a logarithmic measurement, is also used as another point of comparison. Cents are often used with pitches in equal temperament because the system divides the octave into 1200 cents giving each semitone exactly 100 cents, thereby expressing the equal nature of the tuning. Subjectively speaking, the human ear can notice differences of a few cents, but near or less than one cent would be difficult. The negative numbers in

Fret	*In cents*	*At different mensur lengths in mm*		
		600mm	*700mm*	*800mm*
1	0.50	0.16	0.19	0.22
2	0.31	0.09	0.11	0.13
3	**-5.87**	**-1.71**	**-2.00**	**-2.28**
4	0.62	0.17	0.20	0.23
5	-1.96	-0.51	-0.59	-0.68
6	-1.46	-0.36	-0.42	-0.48
7	-1.65	-0.38	-0.44	-0.51
8	**-7.82**	**-1.72**	**-2.00**	**-2.28**
9	-1.33	-0.27	-0.32	-0.37
10	-3.91	-.76	-0.89	-1.02

Table 2.5: Differences bewteen Bermudo's third scheme and true equal temperament

the table show a fret that is flatter than equal temperament, while a positive number indicates a fret sharper than equal temperament.

As the table shows, all but two of this frets are within a few millimeters of an equally-tempered semitone, in a variety of different mensur lengths. The only exceptions here at the third and eighth frets which are flatter than equal temperament. When looking at the differences from the point of cents, these frets are quite flat and most listeners would notice a difference of 5 to 8 cents; however, the differences of fractions of one cent would be virtually undetectable. For what reason Bermudo chose to make these particular frets flatter than the others, by comparison, we cannot know.

Another interesting aspect to Bermudo's scheme is that he counteracts the gradual sharpening of frets as they move to higher positions by making each fret after the fourth gradually flatter than the previous one.

2.6 Sources of Equal Fretting

The last category of fretting systems that we will examine come from the late sixteenth and early seventeenth centuries and advocated dividing the octave into equal semitones. This is not to say that these systems produced the kind of equal temperament that we have today. Creating a temperament with semitones that were exactly equal to one another required a special class of mathematical functions called logarithms, which did not appear until later during the seventeenth century. Despite their ability to correctly divide the octave into equal semitones, equal temperament did not become a true standard until the twentieth century. As Ross Duffin has argued, even during the nineteenth century musicians still tempered their fifths more than that of equal temperament.

Prior to these advances in mathematics, equal semitones where approximated using other methods. Sources during this time used several different systems of measurement that produced temperaments very close modern-day equal temperament, but achieved this through a different means of calculation that did not involve logarithms. The first of these we have already seen with Bermudo's third method of fretting, which approximates equal temperament closely and uses the standard geometrical string divisions common to most lute sources at the time. Additional sources on fretting in equal semitones come from Vicenzo Galilei and Marin Mersenne, as well as numerous other anecdotal sources that Mark Lindley has collected in which contemporary musicians refer to the lute's ability to tune equally.

One of the simplest ways in which a lute player could achieve equal semitones was to divide the octave into a series of Pythagorean minor semitones, at a ratio of 18:17. Called the *18:17 Rule* because if this, both Mersenne and Galilei, as well as numerous other sources, discuss this method. The idea originates with the problem of dividing the Pythagorean 9:8 wholetone into 2 equal parts, which I discussed in the previous chapter. A series of minor semitones produced fairly acceptable results, but as the frets progressed further down the fingerboard, the distances became increasingly smaller, making each fret flatter than the next.

Looking at table /ref18:17rule, we can the 18:17 rule compared with equal temperament when it is executed with a variety of different mensur lengths as well as how it compares in terms of cents difference. The differences between the two systems for

Fret	*In cents*	*At different mensur lengths in mm*		
		600mm	*700mm*	*800mm*
1	-1.05	-0.34	-0.40	-0.46
2	-2.09	-0.65	-0.75	-0.86
3	-3.14	-0.91	-1.07	-1.22
4	-4.18	-1.15	-1.34	-1.54
5	-5.23	-1.36	-1.59	-1.81
6	-6.27	-1.54	-1.80	-2.05
7	-7.32	-1.70	-1.98	-2.26
8	-8.36	-1.83	-2.14	-2.44
9	-9.41	-1.94	-2.27	-2.59
10	-10.45	-2.04	-2.38	-2.72
11	-11.50	-2.12	-2.47	-2.82
12	-12.54	-2.18	-2.55	-2.91

Table 2.6: Comparison of fret placement using the 18:17 system

the first five frets are very slight. Only less than two millimeters between the measurements of these frets at different mensur lengths and at most a five cent difference in pitch. At the higher frets, however, the differences become larger. By the time we reach the octave, the fret is more than twelve cents flatter than the pure octave found in equal temperament. This would be very noticeable to most individuals.

Despite the decrease in distances, using a series of equal minor semitones in the 18:17 rule provided a good usable temperament because its problems were mitigated by two factors: First, most musicians at this time were using some form of temperament that flattened fifths, and at the seventh fret where the fifth occurs, the fret is already flattened by a few millimeters. Second, the tension created from stopping a string at a given fret, especially if the fret itself is quite thick or is doubled as is the case with the viola da gamba, can raise the pith slightly.[15] This effect is compounded by the fact that changes in finger pressure or position have a more dramatic effect on pitch at high positions. Making the frets that flatten gradually as they move to higher positions would help counteract this effect.

Another method of determining equal fret positions used a device called a mesolab, which was an instrument of Greek origin that could determine mean proportionals, or the average distance between two lines. In 1558, Zarlino published a fretting scheme for lute that relied on such a device.[16] There is also a picture of one in the *Discours non plus melancoliques que diverses, de choses mesmement, qui appartiennent a notre FRANCE: & a la fin La maniere de bien &iustement entoucher les Lucs & Guiternes*,

15. Lindley, *Lutes, viols and temperaments*, 21.
16. Ibid., 26.

published in 1556, which was mentioned earlier. However, use of the mesolab did not appear in any practical fretting guides and seems to be relegated to music theory sources where equal temperament was a kind of puzzle to be solved and not really taken seriously in a musical context.

By the seventeenth century, mathematics had advanced enough to employ decimal numbers and logarithms to express musical ratios. Johannes Faulhaber was the first to use logarithms for a fretting scheme in his 1630 publication.[17] In order for it to work, the string had to be divided into 20,000 parts. This idea later found its way into an appendix to a translation of Rene Descartes' *Musicae compendium.* Mark Lindley has concluded that its translator is William Brouncker, who also wrote the appendix. The appendix contained several fretting schemes for lute, including our modern-day one using the $\sqrt[12]{2}$ method, as well as some others of his own devising.

Despite the accuracy of logarithmic calculations in placing frets at equal temperament, the method was largely ignored except in a few treatises. While many authors spoke of the lute's ability to play with equal semitones, the idea of an "equal temperament" was something less certain. From the available evidence, systems approximating equal temperament and utilizing the idea of equal semitones certainly existed during this period, but they still recognized the issue of somehow attempting to reconcile the idea of unequal semitones found in the temperaments of all the other instruments. The central crux of the lute player's struggle was how to navigate these two poles.

17. Ibid., 21.

2.7 Summary

After this lengthy description of the available sources on fretting from the sixteenth century and seventeenth centuries, some of them do agree on the placement of certain frets (see table 2.7). The first, fifth and seventh are almost all uniformly Pythagorean in nature, making the fretted intervals of the perfect fourth and fifth pure. When players to tuned their open strings to one another by using the fifth fret of the adjacent strings as a reference, this would make the interval between the two strings a perfect fourth as well. This is corroborated by every source at this time that contained instructions for tuning the lute's open strings.

<table>
<tr><th>Fret</th><th>Dowland</th><th>Gerle</th><th>Gnasi</th><th>Bermudo II</th></tr>
<tr><td>1</td><td colspan="4">diatonic sixth-comma</td></tr>
<tr><td>2</td><td colspan="4">Pythagorean 9:8 wholetone</td></tr>
<tr><td>3</td><td>chromatic sixth-comma</td><td>diatonic sixth-comma</td><td colspan="2">unique</td></tr>
<tr><td>4</td><td>unique</td><td>unique</td><td>unique</td><td>equal</td></tr>
<tr><td>5</td><td colspan="4">Pythagorean 3:2 fourth</td></tr>
<tr><td>6</td><td colspan="2">chromatic sixth-comma</td><td>unique</td><td>unique</td></tr>
<tr><td>7</td><td colspan="4">Pythagorean 2:3 fifth</td></tr>
<tr><td>8</td><td>n/a</td><td>n/a</td><td colspan="2">unique</td></tr>
</table>

Table 2.7: Comparison of fretting schemes

The use of a Pythagorean wholetone for the second fret seems like a good choice at first glance, since the second fret would be used in most scalar passages as the wholetone above the open strings. However, when used in chords, the second fret often holds the third of the chord. In meantone temperaments, this would mean that the second fret would need to be lower in pitch or moved closer to the nut in order to accommodate either a sixth or quarter comma meantone temperament. Furthermore, the use of pure fourths between open strings, discounts the use of any meantone temperament because of the tempered nature of their fifths.

With the remaining other frets on their instruments, lutenists were experimenting with meantone intervals for these intervals. This included frets one, three, four and six. In the sources examined thus far, there is unanimous agreement that the first fret be a diatonic sixth-comma semitone. This would mean that for instruments tuned in either G or A, their first frets would have to be A♭ or G♭ respectively. However, there is equal disagreement over the remaining chromatic frets. For example, each source has its own technique for placing the fourth fret. Dowland and Gerle seem to favor chromatic frets that closely approximate sixth-comma meantone intervals, while Ganassi and Bermudo arrive at the same placement for their third fret, but neither of them come close to any interval in the meantone scheme of temperament. Additionally, the one glaring error that none of these sources seem to address is that tuning your open strings to perfect fourths would create a very sharp Pythagorean third in the middle between the third and fourth courses. It seems unlikely that anyone would have tolerated that.

Given the available sources we have on fretting, we are unable to resolve all of the discrepancies and inevitably we must make our own choices. We can observe that players embraced a variety of different techniques when addressing the issue and use that as a guiding principle when creating our own. Furthermore, their final solutions were not uniform and no fretting system consisted of entirely Pythagorean intervals or meantone intervals of one type or another. The solutions were always mixed in nature. With that in mind, we can begin the next chapter where I will propose different fretting solutions based on the ones we have seen thus far, and examine them in a musical context, using examples from the period.

Chapter 3

Modern Lute Fretting

If we return to the initial subject at the outset of this paper, the standard in tuning for the sixteenth and early seventeenth centuries was meantone temperament. Many performing ensembles today choose to use quarter-comma meantone temperament because it was a very common temperament at the time for keyboard instruments, and as I explained in chapter one, resulted in harmonious, pure thirds. As we have seen in the previous chapter, none of the major sources on lute fretting employ intervals that are completely meantone in nature, quarter-comma or otherwise. Certain frets are might be meantone, but all of them are not uniformly tuned in one kind of temperament. For any would-be lute players wishing to participate in ensembles today that are employing meantone temperaments, they are going to need practical solutions to make their instruments function in a meantone setting.

Because of the arrangement of semitones on a lute, meantone temperaments must be executed differently than on other instruments. Players in the sixteenth century recognized this problem and proposed various ways of dealing with the problem. Some of them ultimately decided that the lute was simply an equal semitone instrument, or others observed that it was simply different and that music sounding bad on one instrument seemed to sound much better on a lute.[1] However, we are still faced with the reality that lutes performed with other instruments in large and small ensembles, all of which very likely used quarter-comma meantone temperament and other varieties of meantone temperament. So it seems to reason that players then had found ways to deal with the problem at least in some way.

The different historical fretting solutions set forth in the previous chapter fall short of a comprehensive solution to meantone fretting; therefore, we are left to find solutions to problem ourselves. In this chapter, I will address the issue of meantone temperaments in ensembles, specifically quarter-comma meantone temperament because it is the most common temperament associated with music from this period and it is also one of more problematic temperaments to realize on the lute. I will look at quarter-comma meantone in three different contexts: 1) a continuo ensemble with theorbo; 2) a continuo ensemble with lute; and 3) ensembles with lute or theorbo in tablature notation.

The solutions proposed here will build on ideas we have seen from historical sources, but will also incorporante alternative solutions, some of which are modern and contradict the approaches of the history treatises. They include the use of *tastini* or "little frets" made of wood or ivory that only span one string and can be glued on the fret board. These small frets make it possible to have a different kind of semitone on one string, such as a chromatic semitone, while the rest of the strings

1. Lindley, *Lutes, viols and temperaments*, 45.

all have the diatonic semitone. A alternative involves the use of frets placed angles so they are not parallel to the nut or bridge. This results in a fret that could be a diatonic semitone on one end and a chromatic one on the other end. The arguments against such solutions found in historical sources support the notion that the practice was commonly used; however, it was problematic enough that some authors sought to rally against it. Nevertheless, we must consider these solutions if we are able to find a musically acceptable solution to the quarter-comma dilemma.

Every solution to the problem of meantone temperament on the lute is a kind of compromise. Less drastic solutions included alternative fingerings for certain notes and varying the finger pressure of the left hand to alter the pitch. Adjustments such as these could make pitches more agreeable in a tempered setting and enable the instruments to function in given meantone context. There are many different combinations of solutions that can enable a lute to play in a quarter-comma meantone temperament, but as we shall see, each combination has a different effect on the instrument's capabilities.

3.1 The Lute in Ensembles

Musicians who played fretted instruments in ensembles with other instruments, especially keyboard instruments, during the late 16th or early 17th-century would have needed to utilize quarter-comma meantone temperament. This was the standard temperament for keyboard instruments and most, if not all, of bowed string and wind instruments as well. For lutes, the problem is that none of the available historical fretting instructions mention quarter-comma meantone at all. The fretting schemes that were presented in the previous chapter make use of tempered frets, some of which fall near sixth-comma meantone, but none use full quarter-comma meantone fifths or thirds. Either the historical evidence is lacking or there were cases when a lute was tuned to a different temperament than the rest of the ensemble.

Most musicians today are uncomfortable with the idea of deliberately playing in a different temperament than the rest of the instruments in an ensemble, so lute players have found ways in which to make quarter-comma meantone successful on their instruments. While it is possible to realize this temperament on a lute or theorbo, it can create other problems such as limiting the instrument's compass or some of its idiomatic qualities. Common left-hand chord shapes may not be possible in a quarter-comma meantone temperament. However, if the instrument is used in basso continuo, the choice of chord shape is at the player's discretion, so long as it agrees with the harmony. Since the player has more control over the number of voices in the chord and its location on the fretboard, it is easier to overcome the limitations of a meantone fretting system when playing in ensembles. In solo situations, where the exact location of each pitch is dictated by the tablature, a quarter-comma temperament could produce undesirable results and necessitate a different temperament such as sixth-comma.

Before we examine meantone fretting in tablature, we can see how quarter-comma meantone may be realized on the lute. If we recall, the main feature of meantone temperaments are thirds that are much closer to pure than Pythagorean tuning or equal temperament. The two side-effects of this are narrow fifths, or fifths that are flatter than pure, and unequal semitones, namely the diatonic and chromatic semitone. For the keyboard, this means choosing between an F♯ and a G♭, or a D♯ and an E♭. In any meantone temperament, these are two different notes. When tuning a keyboard, the choice between a chromatic or a diatonic semitone can be independently from any other note on the instrument. It is possible to have an F♯, A♭ and C♯ all on the same octave. On the lute, this is not possible because fret placement dictates the semitone size for all notes along that fret. For example, the choice of a chromatic semitone on one course to yield a C♯ might force another course to have G♯ instead of A♭.

In temperaments with unequal semitones, a lutenist can only choose between a chromatic or a diatonic semitone when placing frets on his or her instrument. For example, if we were fretting our instrument in quarter-comma meantone, the first fret would either be a chromatic semitone or a diatonic one. The choice of semitone is up to the player, but the main consideration the player should make is whether or not any pitches on that fret must either be chromatic or diatonic. For a lute in standard G tuning, the second course is tunned to D; therefore, the first fret of the D course could either be E♭ or D♯, the second fret of the course E and the third is F. The distance between E and F is a diatonic semitone which makes the distance between the second and third fret a diatonic semitone. Because of this one requirement in tuning between two notes on a single course, the distance of a diatonic semitone between these two frets will apply to all other courses as well. Given these requirements, Dowland's chromatic sixth-comma fret in table 2.7 seems impossible, because this would create something flatter than F. Gerle's diatonic fret, on the other hand, makes more sense in this context.

The other consideration players can make when choosing semitone size is that some chromatic notes are more common than others. For example, it is more common to find an F♯ in 17th-century music than it is a G♭. Returning to our previous example of the D course, the fourth fret makes better sense as a chromatic semitone, giving us an F♯, instead of a diatonic one which would have produced a G♭. If we use this same logic and move back to the first fret of that course, we could choose an E♭ over a D♯ by using the reasoning that we are probably more likely to encounter an E♭ than a D♯, although this is not always the case.

For the sake of argument, let us say that we have settled on the semitone choices we have discussed above: diatonic first and third frets, followed by a chromatic forth fret. The result of these semitone sizes are summarized in figure 3.1, and as we can see, this has an interesting impact on the rest of the pitches on the other courses of the same frets. Since we have chosen E♭ over D♯ for the first fret, this results in a B♭ for the third course, which is a good choice; however, the fourth and fifth courses are G♭ and D♭, instead of the more likely F♯ and C♯. This also creates a problem because the G♭ found on the first fret of the fourth course will not match the F♯ on the fourth fret of the second course. Similarly, the D♭ on the fifth course will not match the C♯ on the third.

The alternative solution is to have a chromatic semitone for the first fret instead of a diatonic one, seen in figure 3.2, but this only makes things worse. It allows for an F♯, C♯ and G♯ on the lower courses, matching the C♯ and F♯ of the fourth fret, but the D♯ and A♯ on the upper courses proves to be a problem. A♯ is an especially

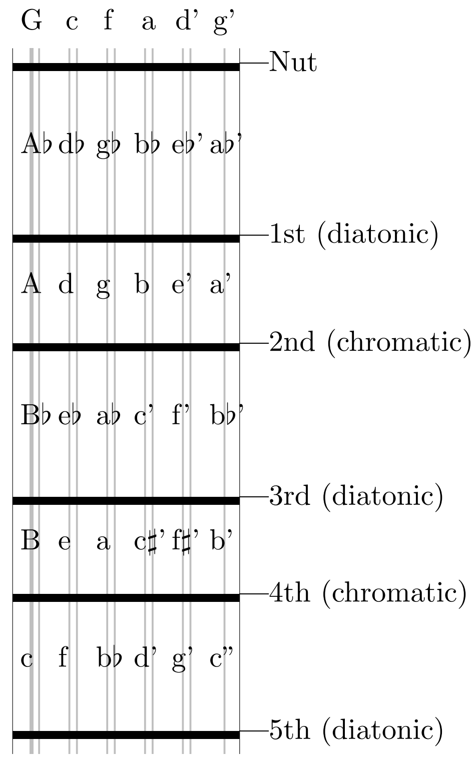

Figure 3.1: Standard Quarter-comma Fretting

unlikely chromatic note and it does not match the B♭ found on the third fret of the sixth course.

There are two main reasons why the alternative solution of a chromatic first fret is undesirable. First, it seems that most of the historical sources agree that the first fret was some kind of diatonic semitone. Referring back to table 2.7, Dowland, Gerle, Ganassi and Bermudo all agreed that the first fret was diatonic in nature. Although theirs was closer to sixth-comma than quarter-comma, it indicates a preference for the diatonic semitone, or notes that have the ♭ accidental as opposed to a ♯ accidental.

The second reason that a chromatic first fret is unlikely is because it would create false unisons between the B♭ and the A♯ , as well as the E♭ on the fifth and the D♯ on the second. These unisons are quite common in lute tablature, and comprise some of the most common chord shapes used in continuo playing. A chromatic first fret would

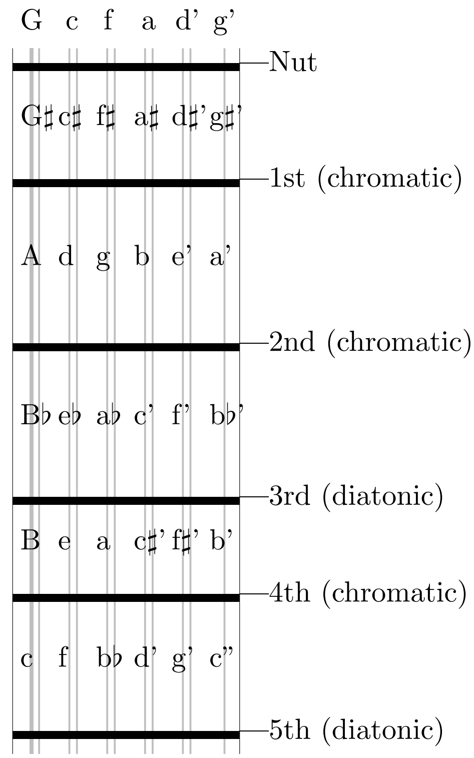

Figure 3.2: Alternate Quarter-comma Fretting for the Lute

render those chords unplayable without additional adjustments, making it unlikely that a chromatic semitone would be used in either a solo or ensemble context.

3.2 The Theorbo

As lutes were used more and more in ensembles towards the end of the sixteenth century, a new type of lute was invented specifically intended to play in ensembles. The theorbo, as it was called, was a much larger instrument and had additional bass strings that extended beyond the length of the neck. Although all kinds of lutes, including the theorbo, were generally fretted the same way, the tuning of the theorbo presented other alternative fretting solutions that were not available on the the lute. The theorbo's nominal pitch was almost always A, instead of the G as with other lutes of the time. Technically, any lute or theorbo can be tunned to any key, and it was not uncommon to find lutes pitched to F, G, A and even D. This applied to lutes in a consort where each existed in a variety of sizes. The pitch could also result from the instrument's mensur length. The top string would be tuned as high as possible without breaking and where that point was determined the overall pitch of the instrument.

When taken in an ensemble context, the pitch of a lute or theorbo had to be standardized so that it could play with other instruments. In English consort music as well as most all lute song publications in England, the standard lute pitched was G. In Italy, however, the theorbo was usually pitched to A. No matter the pitch of a lute, courses were tuned so that the preceding course was always lower than the one following it. (see figure 3.3) This was not the case for the theorbo and it had first and second courses that were an octave lower. (see figure 3.4)

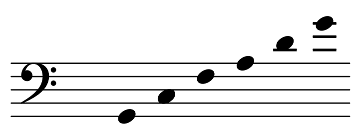

Figure 3.3: Standard lute tuning in G

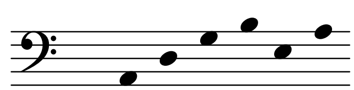

Figure 3.4: Theorbo tuned in A with re-entrant first and second courses

Theorbos were designed for accompaniment and needed to provide more volume than other lutes of the day; therefore, the body size was much larger and the strings were longer. Because of the increased mensur length, it was not possible to preserve the low to high arrangement of courses as they were on the lute. Players found that as they tried to tune the upper strings to their normal lute pitches, the strings would break and it was not possible to fashion a gut string thin enough to hold the pitch at that length. To solve the problem, they tuned the strings to the same pitch but at an octave lower and thus preserving the same interval relationships between strings as they were on the lute This made chord shapes identical between instruments and only altered the voicings of the chords. Since the theorbo was primarily a continuo instrument, the change in voicing did not present a problem. In fact, it became more of an advantage. The re-entrant tuning kept the overall tessitura of the instrument lower and away from the accompanied singer or instrumentalist.

All theorbos had eight additional bass strings that descended diatonically in pitch from the A on the sixth course. Therefore, the seventh, eighth and ninth courses would be G, F and E, continuing on to an octave G on the fourteenth course. The disposition of these lower courses varied somewhat from instrument to instrument. Praetorius discussed two kinds of theorbos, one which he called a Roman style theorbo that had six courses on the fretboard and all eight of the lower courses on the extended neck of the instrument.[2] The other type, which he called the Paduan-style theorbo, had eight courses on the fretboard and the rest of the bass courses were on the extended neck.

Because of the variation Praetorius describes, players today will often have the seventh as well as the eighth course on their fretboards before the additional strings

2. Praetorius, *Syntagma Musicum II: De Organographia Parts I and II*, 59.

on the extended neck. See figure 3.5 below. The advantage to having these additional

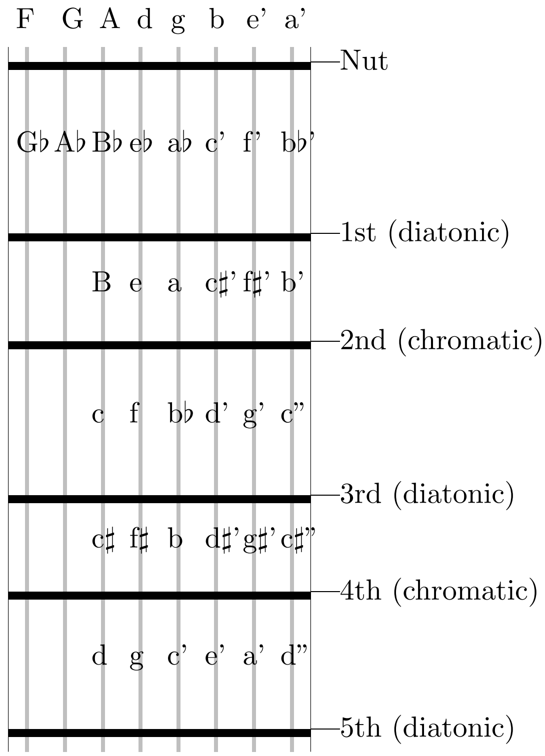

Figure 3.5: Theorbo with extend courses

courses on the neck is that a player is able to fret additional chromatic notes with the left hand. On the longer strings that are attached to the extension, this is not possible and any chromatic changes in the pitches of those stings must be done using the tuning pegs prior to playing.

While it might have been possible to tune a lute's first course to a chromatic semitone, this was impossible on the theorbo. For example, the first fret had to be diatonic because of the open E♮ and B♮ on the second and third courses so that the pitches on those courses of the first fret would be F♮ and C♮. If a chromatic semitone was used, an E♯ and B♯ would result, making this type of semitone unusable. Similarly, the presence of a B♭ at the third fret of the fourth course dictates that the next fret must be chromatic to create a B♮ at the fourth fret of the same course. Even the sixth fret is determined to be diatonic because of the E♮ to F♮ that occurs on the third course.

Essentially, the frets of a theorbo tuned to meantone temperament were "fixed" in their positions because the location of the diatonic semitones between B and C, and E and F determined which fret was either chromatic or diatonic. If we also take into consideration the same issue of octaves that affect the position of frets on the lute in G, it makes the most sense for a theorbo to have its initial five frets arranged the same way as a lute and alternate between diatonic and chromatic semitones, beginning with a diatonic semitone at the first fret. Since this limits the type of semitones that are available, we have to find alternative methods of playing semitones if we are to be successful in a quarter-comma meantone temperament.

3.2.1 Solutions utilizing re-entrant tuning

The theorbo has the advantage of using both chromatic and the diatonic semitones within the same octave. Because of the re-entrant nature of the theorbo's tuning, notes that are an octave apart on the lute are on separate courses; however, on a theorbo, these notes are unisons. This offers a possible solution to some of the problems of semitone size because a particular pitch could be a diatonic semitone at one fret while being a chromatic one at a different fret, and still be in the same octave. Referring to figure 3.5, the A♭ on the first fret of the fourth course and the G♯ found on the fourth fret of the second course are in the same octave, whereas on a lute in standard G tuning they are an octave apart. Other notes were still an octave apart, such as the E♭ on the fifth course of the first fret and the D♯ on the third course of the fourth fret. However, in continuo playing octave displacement did not matter and players were able to freely substitute different octaves as needed. Therefor, all the player has to do is choose the appropriate fingering for the left hand to obtain either the chromatic or diatonic semitone.

For chords that require chromatic semitones, such as a G♯ or a D♯, the player can use pitches found on the fourth fret. Some of the more common left-hand chord patterns that use this fret include the E major triad and different kinds of chords with a sixth above the bass. For other chords that require diatonic semitones, such as

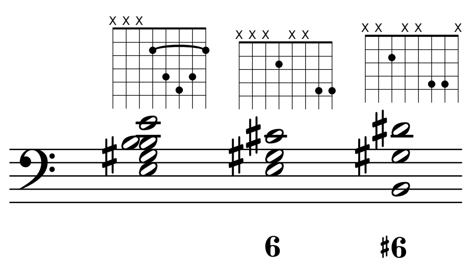

Figure 3.6: Chords using chromatic semitones on the fourth fret

the A♭ or E♭, the player may use pitches on the first fret. This includes A♭ major, F minor and C minor. Although instances of an A♭ major triad are rare, all the needed

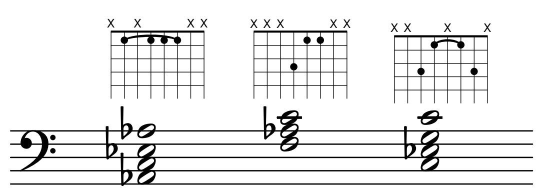

Figure 3.7: Chords using diatonic semitones on the first fret

pitches are on the first fret and the F minor and C minor triads are both possible using a limited number of voices.

While re-entrant tuning makes it possible to play chords with different kinds of semitones, sometimes the left-hand chord fingerings that result are not the easiest to execute, nor are they as idiomatic to the instrument as other more commonly used fingerings. More ideal chords for the theorbo are easier to execute, have more potential voices, and favor open strings whenever possible. The fingerings for F minor and C minor listed in figure 3.7 are not commonly found in existing theorbo tablatures of the time, nor are they used very often among modern players. The more common fingerings for these chords use the fourth fret. Additionally, the E major chord in figure 3.6 uses the chromatic semitone on the fourth fret, but ignores the open E and B on the third and second courses. More common chord fingerings below in figure 3.8 indicate that a player would more likely use a fully-voiced F minor or C minor chord with a barre at the third fret than those listed previously. An E major chord that makes use of the open B and E strings sounda much more resonant and is easier to play for the left-hand. The obvious problem with these more idiomatic chord shapes

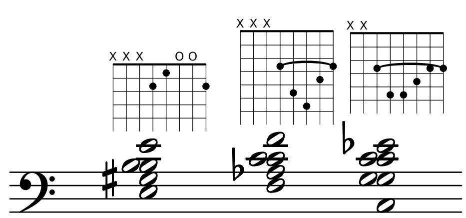

Figure 3.8: Common theorbo chord shapes

is that if they are used on a theorbo in meantone temperament, their semitones are opposite of what they should be. The E major chord shown above would have a diatonic A♭ instead of the chromatic G♯ and the thirds of the F minor and C minor chords would be chromatic in nature instead of diatonic.

3.2.2 Tastini

The advantages that re-entrant tuning might offer has the side-effect of rendering our more idiomatic chord shapes unusable in a meantone temperament. If we want to be able to use these shapes, yet still be able to play in meantone temperament, we need the ability to locally apply different semitones within a fret instead of being forced to have all the pitches at one fret of a certain size. In other words, we need to mix both chromatic and diatonic semitone sizes within the same fret. For example, consider the pitches of the extended bass courses. The first chromatic note on the seventh and eighth courses is determined by the quality of the first fret. Since the first fret on a theorbo is a diatonic semitone, this would make the pitches on this fret for these two lower courses A♭ and G♭, respectively. It is far more likely that a G♯ and an F♯ are needed, but shifting the entire first fret to a chromatic semitone would alter the rest of the notes and result in a B♯ on the third course instead of a C.

To correct this problem and apply a chromatic semitone localized only to one or two courses, lute players during this time employed the use of *tastini*. The diminutive form of *tasto*, the Italian word for fret, these “little frets” were small pieces of wood that were glued on to the fretboard to create a chromatic semitone on one or two courses while the remainder of the courses on the fret were diatonic. Courses beyond the sixth on a theorbo were used for bass support and any that were on the fretboard, such as the seventh and eighth course, were only stopped at the first fret. This made tastini an ideal choice since it only affected the first fret. Players now had the ability to use an F♯ and G♯ while keeping the rest of the pitches on the first fret at their original diatonic position. See figure 3.9.

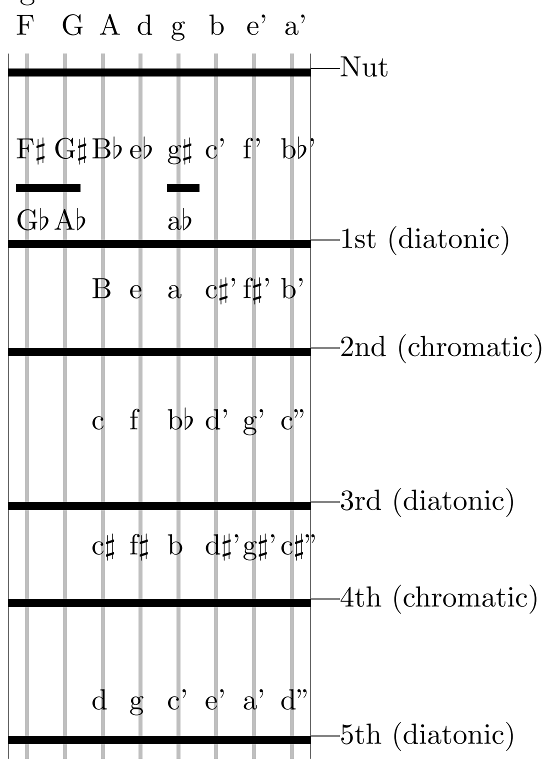

Figure 3.9: Theorbo with added tastini

Modern-day players have employed tastini on other frets as well, which when applied can help solve the previous problems of idiomatic chord shapes in meantone frettings. Referring again to figure 3.9, an additional tastini on the fourth course can provide us with a G♯ which would enable us to play the more common E major chord shape described in figure 3.8 using the open strings on courses two and three. The problem of C minor and F minor chords, however, still remains. We could switch the entire fourth fret to a diatonic semitone, thereby giving us the needed pitches, but we would loose the ability to play some of our sixth chords that were described in figure 3.6. One could argue that additional tastini on the fourth fret could correct this problem, but such a solution might become unwieldy. Also, the solution then becomes sacrificing several chromatic semitones, in this case C♯, F♯ and G♯, for the sake of two diatonic ones: E♭ and A♭.

While modern players have embraced the use of tastini, there are no surviving instruments with their tastini intact. Yet, it is obvious they were in use because of the different historical accounts that describe them. The earliest of these comes from Vicenzo Galilei's *Fronimo*, where he did not have very good things to say about them. Galilei's description of tastini suggests that players were using them in different places on the instrument and not just the first fret. Galilei was writing at a time shortly before the theorbo came into use, so he described their use on the lute, such as a tastino on the just beneath the second fret of the fifth course, in order to make thirds less sharp.[3] Although in today's uses, most players do not use a tastino in this location.

Galilei's main disagreement over the use of tastini was that it made adjustments to one fret only in a certain pitch context, for example when you might want an F♯ instead of a G♭, but that one adjustment does not work in other pitch contexts or match the same pitch of the instrument in a different location on the fingerboard. He also maintained that the lute was tuned in equal semitones and a well-placed fretting system was sufficient to play all the pitches necessary. Tastini ruined that sort of system because in his mind, if the lute was tuned in equal semitones, there would be no need for a tastino because the fret would function correctly as either a chromatic

3. Vincenzo Galilei, *Fronimo*, Musicological Studies and Documents, volume 39, Translated and edited by Carol MacClintock (Rome: American Institute of Musicology, 1985), 165.

or diatonic semitone.

Whether we follow Galilei's advice or not, his attitude towards tastini is the most important indicator that tastini were in use. Some players must have used them at this time otherwise, Galilei would not have mentioned it. However, we must note that the manner in which Galilei describes their usage appears to have no direct application to correcting the problems found in quarter-comma meantone. After all, since Galilei felt the lute was tuned equally, meantone was not an issue. All of this point to the fact that if we are to use tastini on our theorbo or lute, it would be in a way Galilei had not envisioned, so we left to create our own solutions.

Bermudo has a brief account of tastini in the context of the vihuela. His description of their use is in a more positive light and fits very closely with our modern-day usage of them:

> For faults that may arise, take the advice given before of looking for the notes on other frets, or with the pressure of the finger when stopping the note, of by placing another fret in front of the principle fret, which, when placed for this purpose, should be thicker than the first fret so that it does not rub against the string. This [extra] fret can be placed by dividing the distance from the third fret to the bridge into eight parts, and wherever the compass reaches [downward from the third fret] will be the first fret, which will form *fa*.[4]

Recalling table 2.7, Bermudo's first fret is a diatonic semitone or what he calls *fa*. In order to obtain the chromatic fret, or mi, Bermudo proposes an additional fret that is front of the first fret, or place between the nut and the first fret. He says the fret should be slightly thicker, which makes sense because if it were not, then the *fa* fret would prevent it from working properly. Although these are not true tastini

4. Espinosa, "Juan Bermudo 'On Playing the Vihuela' ('De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555)," 115-116.

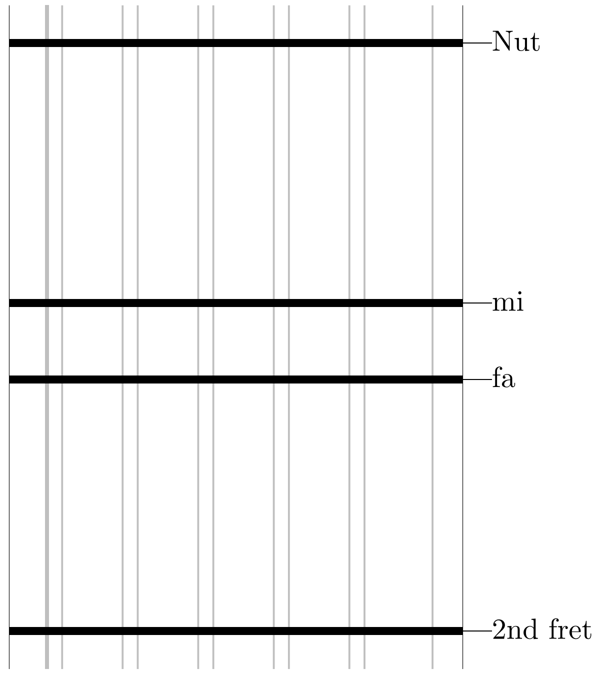

Figure 3.10: Bermudo's *mi* and *fa* first frets

because his fret spans the entire fingerboard instead of just the affected course, it is the strongest indication we know of for any kind of additional frets creating both the chromatic and diatonic semitones.

A later reference to tastini from the seventeenth-century, comes from Jean Denis who was a harpsichord builder during the first half of the the century. He refers to "staggered" frets on the lute which could be made of ivory. The reference appears in Lindley's book, and the context in which Denis was discussing tastini was because someone had tuned a harpsichord to equal temperament. Denis criticizes this approach stating that someone should perfect the lute and viola da gamba so that they may accomodate unequal semitones instead of "ruining a good and perfect tuning in order to accomodate imperfect instruments.".[5]

Another reference comes from Christopher Simpson's *A Compendium of Practi-*

5. Lindley, *Lutes, viols and temperaments*, 47.

cal Music and appears to describe tastini as they are commonly used today on the theorbo:

> I do not deny but that the slitting of the keys in harpsichords and organs, as also the placing of a middle fret near the top or nut of the viol or theorbo where the space is wide, may be useful in some cases for the sweetening of such dissonances as may happen in those places; but I do not conceive that the enharmonic scale is therein concerned, seeing those dissonances are sometimes more, sometimes less, and seldom that any of them do hit precisely the quarter of a note.[6]

The first part of Simpson's description matches Bermudo's additional fret exactly, as he describes an additional fret between the first fret and the nut. He does not state whether or not this middle fret spans the entire fretboard or not. The second part of his statement refers to the difference between the diatonic and chromatic semitone. Simpson calls these "quarter notes" like we might today refer to quarter tones, or half of a semitone. He seems to think that from a practical standpoint, these semitones are seldom precisely what they are supposed to be, either diatonic or chromatic.Simpsons' opinions aside, he does provide us with evidence that tastini or additional frets were historically used in the same places on the fretboard of a lute as players today might use them. Beyond tastini, there are yet more possibilities to overcome some of the issues with quarter-comma meantone temperament and frets.

3.2.3 Other Solutions

Aside from tastini, there were assorted other methods in which lutenists were able to coax meantone temperaments from their fretting. One of these involved placing frets

6. Christopher Simpson, *A Compendium of Practical Music in Five Parts*, Reprinted from the Second Edition of 1667, Edited and with an introduction by Phillip J. Lord (Oxford: Basil Blackford, 1970), 51.

at angles so that a fret could be diatonic on one side of the fingerboard and chromatic on the other. For the theorbo, this could be used as a substitute method or tastini. It is possible to use an angled first fret, for example, to achieve the chromatic semitones necessary on the seventh and eighth courses. Instead of placing tastini at the left side of the fingerboard, the first fret could be slanted so that it moved at an angle reaching towards the chromatic side of the semitone as it moved towards the lower courses.

The problem with this is that pitches in the middle of the fret are somewhere between chromatic and diatonic. Juan Bermudo discusses the practice of angled frets on the vihuela and comes to the same conclusions:

> [...] some players hope to fix the abovementioned faults by putting the frets where the said faults occur at an angle, taking them out of line. This is not a solution but a cover-up [...] Take a fret where there is a fault (where it is *mi* for strings but needs to be *fa* for others) and you will find that, by slanting the fret, it does not hit any string in the right place.[7]

While we might might be able to achieve a chromatic semitone at the eighth course, each successive course would be slightly sharper until reaching the top course. Only the first and last courses would be truly either chromatic or diatonic, the courses in the middle would be something in between and not in a specific temperament.

Despite what Bermudo and other writers of the time have said about angled frets, we can find use for them. Angled frets work best when they are strategically located and used for frets that might have only one or two useful pitches on them. In a "standard" quarter-comma meantone fretting system, as shown in figure A.2 of the apendix, the sixth fret is diatonic and duplicates the E♭ and A♭ found at the first fret.

7. Espinosa, "Juan Bermudo 'On Playing the Vihuela' ('De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555)," 112-113.

A common left-hand fingering for the first inversion triad or a chord with a *6* above the bass, uses the bass on the fifth and sixth courses. These include the first-inversion D major triad with the F♯ as well as the first-inversion A major triad with the C♯, both found on the fourth fret. However, we lack the G♯ or D♯ for either E major or B major tonalities. If we move our sixth fret, which is commonly diatonic, so that it is placed at an angle, it is possible to get a very close approximation of a chromatic semitone for these pitches (see figure 3.11). The slanted sixth fret allows us to play the needed triads with G♯ and D♯ in the bass, disadvantages are minimized because the sixth fret is not commonly used in other left-hand chord shapes. Alternatively,

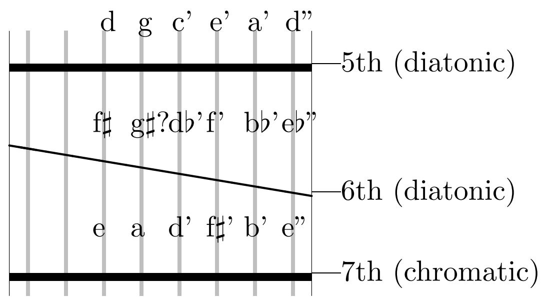

Figure 3.11: Theorbo with angled sixth fret

we could employ a tastini underneath these two courses and avoid the slanted fret altogether, but such a solution would be at the player's discretion.

Other solutions for achieving a successful quarter-comma meantone temperament do not involve adjusting frets at all, but involve positioning the left hand so that individual courses are pulled in one direction or another to raise their pitch slightly and compensate for frets that are using an incorrect semitone. Praetorius describes such a method in his *Syntagma Musicum*, referring to the frets viols and lutes as "intermediate" and between chromatic and diatonic semitones:

> Thus the semitones cannot be either major nor minor, but are, perforce,

> “intermediate” if anything. For I reckon that each fret [...] contains four-and-a-half commas, whereas the major semitone contains five and the minor semitone only four. Since the error is only half a comma either way, the ear hardly notices it with these instruments [...] Major and minor semitones are both produced by the same fret, both sound in tune, [...] especially since by particular applications of the finger to the string, over the fret, it is possible to have some control over the pitch of the note produced.[8]

First off, it is clear that Praetorius is describing meantone temperament because he refers to chromatic (minor) and diatoni (major) semitones as having a different number of commas. However, Praetorius is actually referring to sixth-comma meantone temperament which divides its wholetones into nine commas, split four to five, versus a quarter-comma meantone which contains five commas per wholetone and is divided two to three.

The *Syntagma* was published in 1619, so we might assume that sixth-comma meantone had begun to replace quarter-comma in some musical circles, but it is still a meantone temperament. More importantly, Praetorius describes fretted instruments as having equal semitones, divided exactly in the middle between chromatic and diatonic, but essentially played unequally. According to him, the player has the ability to change the quality of the semitones so that they might be close to chromatic or diatonic as the music requires.

Additional evidence of using finger pressure to correct pitch problems is found in Bermudo's treatise. If we recall the section quoted above where Bermudo describes using additional frets for chromatic and diatonic semitones, he first advises the player to either locate the note somewhere else on the fingerboard or use finger pressure to

8. Praetorius, *Syntagma Musicum II: De Organographia Parts I and II*, 68.

alter it. He refers to this method several times in his treatise, indicating it might have been a preferential method before resorting to additional frets.[9]

Bending pitches using variable pressure in the left-hand, such as Praetorius and Bermudo describe, is possible but generally not easily done. Another common way for a lutenist to bend pitches is to pull the course with the finger to one side or the other and alter the pitch that way. This technique is very common in twentieth-century classical guitar literature where extreme fluctuations of pitch are exploited for various compositional reasons. Although there is no such evidence of its use in historical lute tablatures or other musical sources for lute or theorbo, it is possible to entertain the idea that players during that time might have found a way to utilize it in some form or another to adjust to meantone ensemble playing.

3.3 Meantone Fretting in Tablature Sources

Thus far, we have discussed the methods in which lute players can adapt to the problems of quarter-comma meantone fretting systems in ensemble situations. The methods mostly require the player to place certain notes on certain frets. In ensemble music, where basso continuo is used, the player has the ability to do this because the music is in staff notation and no left-hand pitches are dictated anywhere. In fact, it is customary for players to move bass notes into different octaves when necessary and even use reduction methods that omit repetitive notes. In essence, the player may re-compose sections of his or her part to fit the instrument's compass and make it

9. Espinosa, "Juan Bermudo 'On Playing the Vihuela' ('De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555)," 106.

sound as idiomatic as possible.

In lute tablature sources, the placement of notes in the left-hand is dictated exactly, so the player may not move notes to different locations on the fretboard and thereby compensate for any potentially incorrect semitones in a meantone temperament. If we explore some examples from the repertoire, we find quarter-comma meantone temperament does not fit the solo literature for the lute in the later sixteen and early seventeenth century. Furthermore, ensemble music for lute and voice, where the lute is an accompaniment instrument but written using tablature and not basso continuo, indicates further that a different kind of temperament such as sixth-comma meantone would work better in this context.

The lute song repertory is a unique form of ensemble music where the lute part is written in tablature specifying where the pitches are to be played. The ensemble consists of the lute, one or more voices, and sometimes a bowed bass. Because the lute part is not written in basso continuo, players may not take the same liberties with pitch placement that they can when playing continuo.

John Dowland published the first book of music written for this ensemble in 1597. Called simply, *The First Booke of Songs or Ayres*, his accompaniments were very thorough and equal in caliber to his solo works for lute. The keys for the songs are typical for the lute, such as G minor and major, for a Renaissance lute in standard tuning. Near the end of Dowland's first book of Ayres, is a well-known song entitled "Come heavy sleep" which opens in G minor but has a very striking key change to B major about mid-way through the song at mm 9. Such a change of tonality befits the subject matter of the song, but if we look closely at Downland's placement of the pitches on the lute, they are at odds with a quarter-comma fretting system. In figure 3.12, there are repeat instances of F♯ and D♯ in the first measure of our

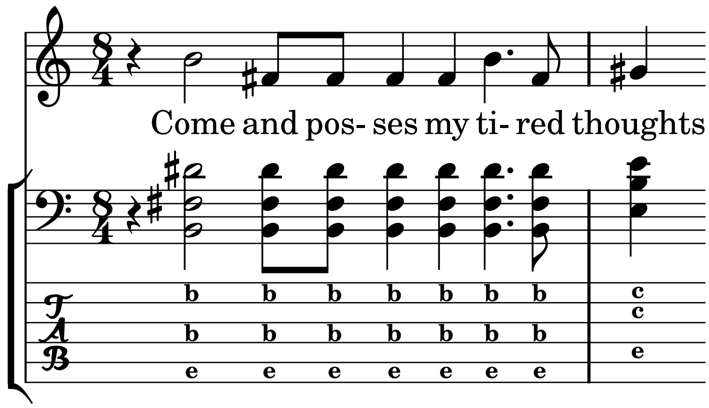

Figure 3.12: "Come heavy sleep" from *The First Booke of Songs or Ayres* (1597), mm. 9

example, which are represented in the tablature part by the character *b*. English lute music was written in the French system of tablature, so the frets are indicated with letters. The tablature character *a* being the open string and the character *b* is the first fret. If we were trying to follow the meantone fretting system I outlined earlier, these notes would be G♭ and E♭ and would sound quite strident against the B. Recalling Dowland's own choice for the first fret, as shown in table 2.7, the quality of semitone is diatonic, but is in sixth-comma and not quarter-comma. A sixth-comma fret would certainly be more palatable in this case and is perhaps why Dowland himself was advocating for a sixth-comma diatonic semitone instead of a quarter-comma one.

Other examples from Dowland's works highlight the central problem with employing meantone fretting systems on lutes. Because of how fretted instruments are tuned, there are cases when both the diatonic and chromatic semitone are required at the same fret. For example, in his song *I saw my lady weepe*, there are both G minor chords and D major chords, which require a B♭ and the F♯. However, if look at an excerpt from the song below, we can see that these notes are both placed on the same fret. The notes and their corresponding tablature characters are highlighted in

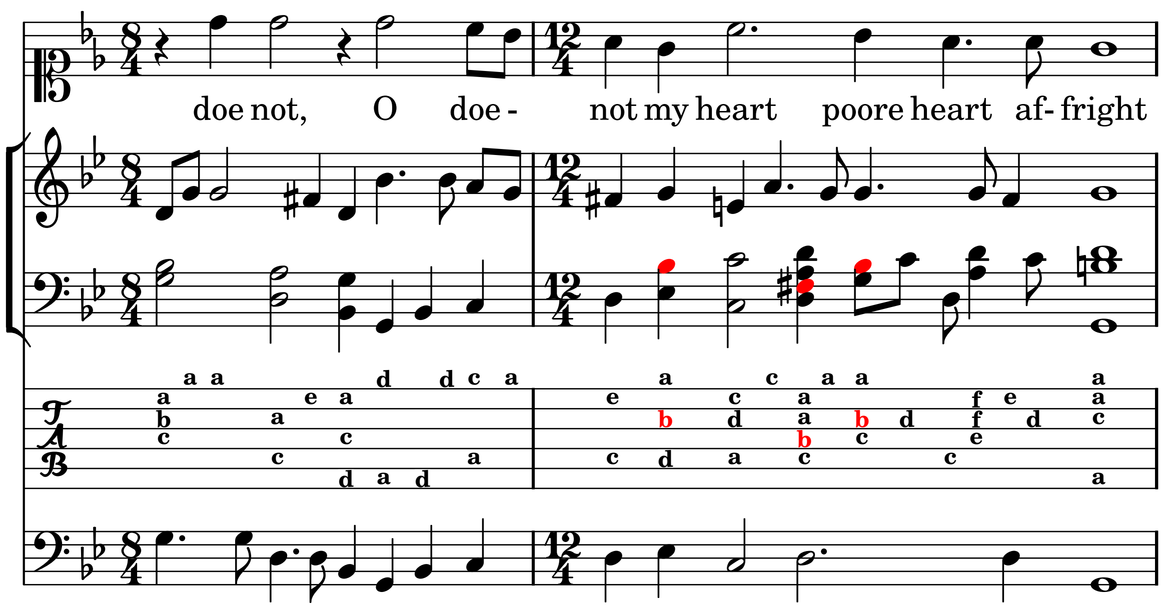

Figure 3.13: "I saw my lady weepe" from *The Second Booke of Songs or Ayres* (1600), mm. 9

red. Both notes occur on the first fret, where there is the tablature character *b*. If we were to use a meantone fretting system, the B♭ would be true but the F♯ would be a G♭ instead, unless we were using a tastino to correct the problem. This kind of issue results when frets determine the quality of the semitone regardless of what the particular pitch might be. Keyboard and other instruments are exempt from this kind of issue because their semitones can be tuned independent of one another. A keyboard or organ can very easily have F♯ and B♭ existing at the same time.

Referring back to Dowland's own fretting instructions from the first chapter, he did recommend a sixth-comma diatonic semitone at the first fret which would ease the problem of having the F♯ and B♭ on the same fret. Although the problem of having a G♭ instead of an F♯ would remain, the difference would not sound quite as pronounced as a quarter-comma difference. Since other composers and lute players were subject to the same tuning constraints, it is understandable that some might have opted for an equal semitone approach by splitting the difference between the positions or opting for positions that simply satisfied their ears. Yet, the ensemble problem still would have remained and applying an equal semitone solution, a sixth-comma meantone solution, or a completely original system of fretting would mean that the lute would be attempting to play in tune with an ensemble that was using a different temperament.

The same issues that affect fretting systems for Renaissance lute also affect the theorbo as well. Although they are less common, there are tablature accompaniments for the theorbo and just as we studied lute tablatures for clues regarding temperament choices, we can also examine theorbo accompaniment tablatures for the same information. An example of an early seventeenth-century accompaniment comes from Girolamo Kapsberger, a theorbo player and composer who was active in Rome. In addition to publishing several books of music for solo theorbo, he published four books of villanelle written for for voices, bowed bass, guitar and theorbo. The vocal parts and bass part are written in staff notation, while the guitar and theorbo parts are written in their own specialized notation. In the case of the guitar, alphabetic notation is used, where a series of different letters indicate which chord to play. The theorbo part is in Italian-style tablature, where numbers are used in place of letters to indicate fret placement and the order of strings is actually inverted from French-style tablature so the top string of the theorbo is bottom line of the tablature staff. For sake of consistency, I have transcribed Kapsberger's tablature part into French tablature so we can make comparisons between the examples with lute and theorbo.

The song "All' ombra", from his first book of villanelle, contains a cadence in A major towards the end of the piece. The G♯ and C that are used are highlighted in red. Similar to our previous example from Dowland, these two notes are found on

Figure 3.14: "All' ombra" from *Di Villanelle, bk. 1* (16??), mm. 21-22

the same fret, indicated with the tablature character b highlighted in red. If we were employing a meantone fretting on our theorbo, the G♯ would instead be an A♭. In this example, the sound of the theorbo in such a temperament was either accepted or the instrument used a different temperament, such as sixth-comma or another kind of temperament that used equal semitones, in order to make different between the G♯ a bit more palatable.

If Kapsberger's theorbo was definitely in quarter-comma meantone, a tastino would have been the only solution to avoid the A♭ issue on his first fret. An alternative solution that I proposed earlier would be to change the left-hand fingering of the E major chord so that the G♯ on the fourth fret is used resulting in a chromatic semitone and not a diatonic one. However, Kapsberger's tablature clearly indicates that the G♯ on the first fret is to be used.

While these examples represent only a fraction of the type of problems that lute players faced when tuning to meantone temperaments, the issue of chromatic versus diatonic frets was so pervasive in the literature that it was impossible to ignore. Given the historical evidence, lute players were apt to try various different techniques to address the problem, some of which were evidently considered controversial. When we are playing in meantone temperaments today, we must be willing to do the same and apply our owns solutions, controversial or otherwise. In the concluding chapter of this study, I will summarize all of the solutions presented here and how they might be used in the context of various kinds of ensembles in which the lute and theorbo played a part during the late sixteenth and early seventeenth centuries.

Chapter 4

Summary of Solutions

Lute players today who want to perform music from the sixteenth and seventeenth centuries must address the issue of using historically appropriate temperaments on their instruments. Because of the nature of fretted instruments, using temperaments with unequal semitones requires a different approach than is used with keyboards or other instruments. In some ways, lutes have greater flexibility with temperaments than keyboards do, but this flexibility results in a multitude of choices and options that players must consider when fretting their instruments. The choices can also reflect opinions of personal choice and go beyond the importance of historical accuracy.

To realize temperaments with unequal semitones on the lute, there are two basic approaches, both of which are supported by the historical evidence. The first is the "fixed semitone" method where each fret is one type of semitone: chromatic, diatonic, equal or something unique. The second approach treats the lute as an enharmonic instrument, similar to harpsichords with split keys. In this method, two kinds of semitones, such as the diatonic and chromatic semitone, are available at certain locations on the fretboard using split or slanted frets, or with the aid of tastini. The choice of method is entirely at the discretion of the player, and either technique can work regardless of the context in which it is used.

In this concluding chapter, I will elaborate on the flexibility of these approaches and describe scenarios in which either can be employed successfully. The nature of lute temperaments as this time indicates that they were not universally applied in the same way. While the modern musical world relies on established standards of pitch as well as temperament, such standards did not exist at this time in musical history. Pitch could vary from city to city, and even from ensemble to ensemble. This same variation applied to temperaments as well, not only between different ensembles, but between different instruments in the same ensemble. This is perhaps why the lute held such an important place because it could conform to the different requirements of temperament more easily than keyboards; however, this is not to say that specialized lute temperaments were only for ensemble music.

4.1 Frets with fixed semitones

As lute players, we have the option to "fix it and forget it" or in other words, set our frets to a certain position and leave them. This requires a customized kind of fretting system in which we accept the limitations of the temperament and either set our frets in a way that makes the entire fretboard available to us or pick and choose which semitones we want to play on which fret. The historical sources presented in chapter two showed a predominant use of these kinds of customized temperaments consisting of different kinds of semitones used throughout the octave. They are neither completely one kind of meantone, such as quarter-comma or sixth-comma, nor are they consistently Pythagorean or equal in nature.

Evidence clearly indicates that fretting systems approximating equal temperament had their place and purpose on the lute, while most other instruments preferred to use a non-equal temperament. So why is it that fretted instruments held this exception? The reason seems to be that it was just easier for a player to divide the octave into twelve approximately equal semitones than it was for a keyboard. A keyboardist would temper intervals aurally, counting beats between notes, while a lute player can visualize the fret distances and create quasi-equal semitones by simply making a good visual approximation between two existing frets and placing the fret somewhere in the middle.

The visual manner in which a player could approximate equal semitones could also take an iterative approach as well, where the fret is adjusted several times while playing to find the right spot in which the semitones sound the best. Whether by visual placement or trial and error, the semitones that resulted from such methods were not true Equal Temperament, as today's modern standard, but where an "equal-ish" temperament. These kinds of customized temperaments would be irregular in nature, with semitones of potentially varying size throughout the octave thus still technically remaining slightly unequal by nature. Depending on the semitone, however, they would be much closer to modern-day equal temperament than some of the existing meantone temperaments of the time.

The use of temperaments that could approximate equal semitones was a distinguishing feature of the lute and would have appealed to amateurs. The treatises from which we take our fretting instructions were often written for amateur musicians and intended for their education. A simple fretting solution that yields an equal-ish temperament would simplify matters for someone who was still learning how to play the instrument. Once experienced enough, however, the player could start moving frets around to his or her liking creating their own temperament that could selectively use different sized semitones. Bermudo describes this when he refers to musicians who move their frets according to their ears, but he instead wants to make the vihuela "more prefect" with equal semitones for inexperienced players.[1]

Equal-ish temperaments and other customized schemes would also lend themselves to solo lute music and repertoire for small ensembles such as the lute and voice. The example of the Dowland song *Come heavy sleep* in the previous chapter, would benefit from a temperament that did not tune its fifths quite as flat. For example, by using less-tempered fifths, the D♯ and F♯ would not sound quite as strident as they would if were in quarter-comma meantone. Some of the custom fretting solutions discussed in the previous chapter could work in this regard, with fifths wider that quarter-comma, however yet another kind of solution is to use a different type of meantone.

Different varieties of meantone were presented in chapter one and in chapter two we saw that many of the historical fretting sources preferred sixth-comma meantone for certain frets. Sixth-comma figured quite prominently in these for the first, third and sixth frets (see table 2.7). A similar temperament was also typical for other

1. Espinosa, "Juan Bermudo 'On Playing the Vihuela' ('De tañer vihuela) from Declaración de instrumentos musicales (Osuna, 1555)," 78.

fretted instruments such as cittern, which was commonly tunned using a temperament somewhere in between quarter-comma meantone and equal temperament.[2] Sixth-comma meantone is essentially the midway point between quarter-comma meantone and equal temperament because its thirds are tunned slightly wider than pure, but are not as wide as they are in equal temperament. Other meantone temperaments whose thirds were wider than pure and therefore similar to a sixth-comma and other meantone varieties, work well in early fifteenth-century repertoires such as the music of vihuela composers Alonzo Mudarra and Luis Milan.[3] Lastly, we can refer back to Praetorius, in his *Syntagma Musicum*, describing frets with $4\frac{1}{2}$ commas.[4] Although he is describing a semitone divided equally, it is still sixth-comma in nature because the wholetone has 9 parts, indicating that the frets were initially set in that kind of temperament.

Sixth-comma meantone is an excellent choice for fretted instruments because it does not restrict the instrument as much as quarter-comma does, and generally does not require many modifications to make it successful. At the same time, it makes a good compromise between equal semitones and the pure thirds of quarter-comma meantone, allowing the lute to effectively play all the semitones in the octave without much difficulty. Different irregular temperaments that share many of the characteristics of sixth-comma meantone were used well into the seventeenth and eighteenth centuries, such as Vallotti and Werkmeister. These temperaments lend themselves

2. Peter Forrester, "Wood and wire: cittern building," *Lute News* 75 (2005): 12.

3. William Bernard Hearn, *Performing the Music of Alonso Mudarra: An Investigation into Performance Practive in the Music of the 'Vihuelistas'* (Doctor of Musical Arts, University of Arizona, 1995), 56.

4. Praetorius, *Syntagma Musicum II: De Organographia Parts I and II*, 68.

quite well to the lute and I can speak both personally and anecdotally that many lutenists prefer to use either a regular sixth-comma meantone or its irregular varieties as their default temperament for most music.

Custom fretting solutions and other temperaments such as sixth comma meantone are ideal for the lute in either solo music or small ensembles. For example, in lute song repertoire, ensembles with additional fretted instruments, or instruments that do not have a fixed semitone size such as a violin or wind instrument, customized temperaments do not present a problem because the instruments can conform to the same temperament as the lute or other fretted instruments. Fixed-fret solutions, however, become problematic when using keyboards that are not tuned in sixth-comma, or if the members of the ensemble wish to choose quarter-comma meantone over other temperaments.

It is possible to play a lute in quarter-comma meantone using a fixed, one semitone per fret, system; however, the player, must restrict where certain the semitones are placed on the fretboard. He or she would choose the semitones needed for a particular piece, and perform them in their designated location on the fretboard. For a theorbo in A, for example, we can fret our lute using the standard quarter-comma fretting pattern described in chapter three (see figure A.2 in the appendix for the complete fretting). In this pattern, C♯ and F♯ are found on the fourth fret, and if we slant our sixth fret as described in figure 3.11, we would also have the G♯ as well. Depending on the piece of music, that might be sufficient for our needs.

If the needed semitones vary from piece to piece, or between sections of a larger work such as an opera, players could change the position of the frets if they have enough time during which they are not playing. Additional solutions could involve slanting other frets, either before or during performance, or choosing to tie a double fret so that it can be split and one side moved higher or lower than the other. Using varied pressure with the left-hand to alter the pitch is also another possibility, but this has a limited effect as it will also reduce the resonance of the stopped note, muffling it slightly. Beyond these types of surface fixes, we must turn to extended methods if we want to have more than one kind of semitone at a fret.

4.2 Enharmonic fretting

If we truly need two different kinds of semitones available to us, and a fixed-fret approach does not work, then we must turn to using either tastini or additional frets tied to the fingerboard. This offers the best possible solution and treats the lute as an enharmonic instrument, allowing both the chromatic and diatonic semitones to be played. The drawback is it increases the complexity of the instrument substantially, and requires added technical skills on the part of the player. Players who work in ensembles that use temperaments such as quarter-comma will opt for enharmonic fretting if they wish to have a complete working solution. However, anyone is free to use an enharmonic fretting in solo literature as well.

Generally speaking, one tastino will make a chromatic semitone for a single course, while the remaining courses are diatonic. A common example of this are the tastini found between the nut and first fret, as we saw in figure 3.9. Usually, the left-hand finger is placed in the area beyond the main fret or tastino, which equates to one semitone type per course. Some players cultivate the skill of playing both the diatonic and chromatic notes on the course where the tastino is found. In that case, the diatonic semitone is played with the left-hand finger placed in the area of the fretboard between the tastino and main fret. For the chromatic fret, the finger is placed just behind the tastino like the other courses.

Tastini can be affixed to the fretboard in various ways. Historically, ivory was used, but today, a toothpick and a piece of double-sided tape will work just fine. The main disadvantage to tastini, once a player is accustomed to using them, is they may slip or break. More permanent solutions are possible, such as using glue and a more durable wood, but this forces the instrument into a particular temperament. If professional needs dictate tuning in a variety of different temperaments, permanent tastini may not be the ideal solution, in which case the "tape and toothpick" approach would be the only option.

For players who wish to extend a lute's capabilities even further, as well as avoid some of the problems associated with tastini, we can have the chromatic and diatonic semitones available on all courses. Recalling Bermudo's example of an additional *mi* fret in figure 3.10, if an extra fret spans the entire width of the fretboard, we have the option of playing either semitone on every course. Bermudo seems to suggest that players then were comfortable doing this with their left hand. The scale of the instrument would have a direct effect on the the difficulty of such a technique. For example, on smaller lutes with a mensur of 60 centimeters or less, the distance between a diatonic and chromatic semitone in quarter-comma meantone at the first fret–where the distances are greatest–is just over a centimeter. Depending on the size of one's fingers, it might be challenging to finger the diatonic semitone in this space. On instruments with a longer mensur length, such as the theorbo where distances of 85 centimeters or larger are not uncommon, this distance increases to almost two centimeters.

Instruments with longer mensur lengths have a greater distance between their semitones and therefore could hold a small advantage in situations where different semitones are required. The larger distance would make it easier for an experienced player to choose the diatonic semitone over the chromatic one, and vice-versa. The thickness of the fret would also be a factor because the chromatic fret would need to be slightly larger than the diatonic one in order to prevent the courses from vibrating against a fret that was too high. A thicker fret would also decrease the available fretboard space between pitches, making larger instruments such as the theorbo an ideal choice for enharmonic fretting solutions. Ensemble situations are often the main cause for temperament problems with fretted instruments. Theorbos and archlutes, which also have longer mensur lengths, can enjoy preferred status not only because of their increased volume due to their body size and string length, but also because they may navigate issues of semitone size better than some of their smaller counterparts.

4.3 Conclusion

Regardless of the instrument's size, tuning, or repertoire, lute players have many different options when it comes to choosing a temperament. One can review all the historical evidence, and propose as many different fretting solutions as one is able, but the principle will always remain the same. The choice of fretting will always rest with the player. The sources are quite clear that there were a multitude of different options available and certain options aroused heated arguments between musicians. The same is true today as well. One can find very spirited discussions of tuning and fretting on various online forums dedicated to topics on tuning and lute playing. Similar arguments can be found in print too, and that will never change.[5]

As lute playing continues to evolve through the 21st century, we will continue to rehash these arguments and hopefully continue to reach the same conclusions. The lute can and did play in a variety of temperaments, both in solo and in ensemble contexts, but the efficacy of its performance shall always rest with the skill of the player. No one would ever question the choice of temperament from exquisite performance. Therefore, it should make no difference what temperament we use, so long as it best serves the music, giving it the warmth and brilliance it justly deserves.

5. David Dolata, "Tunings and Temperaments: Theory in the Service of Practice," *LSA Quarterly* (February 1995): 26–29.

Appendix A

Complete Fretting Diagrams

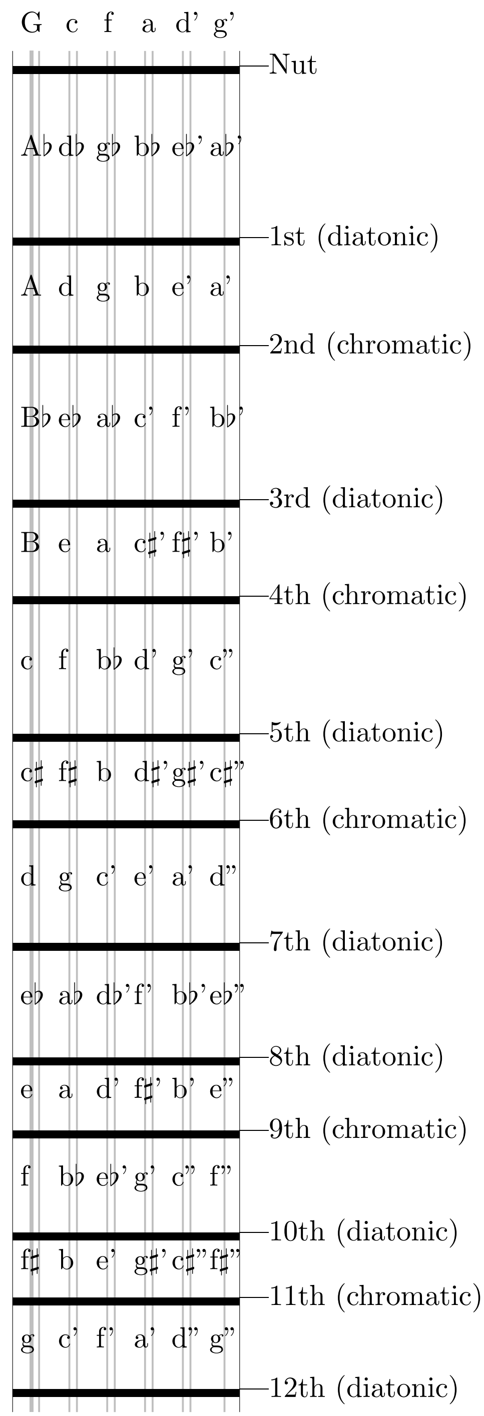

Figure A.1: Complete quarter-comma fretting for lute in G

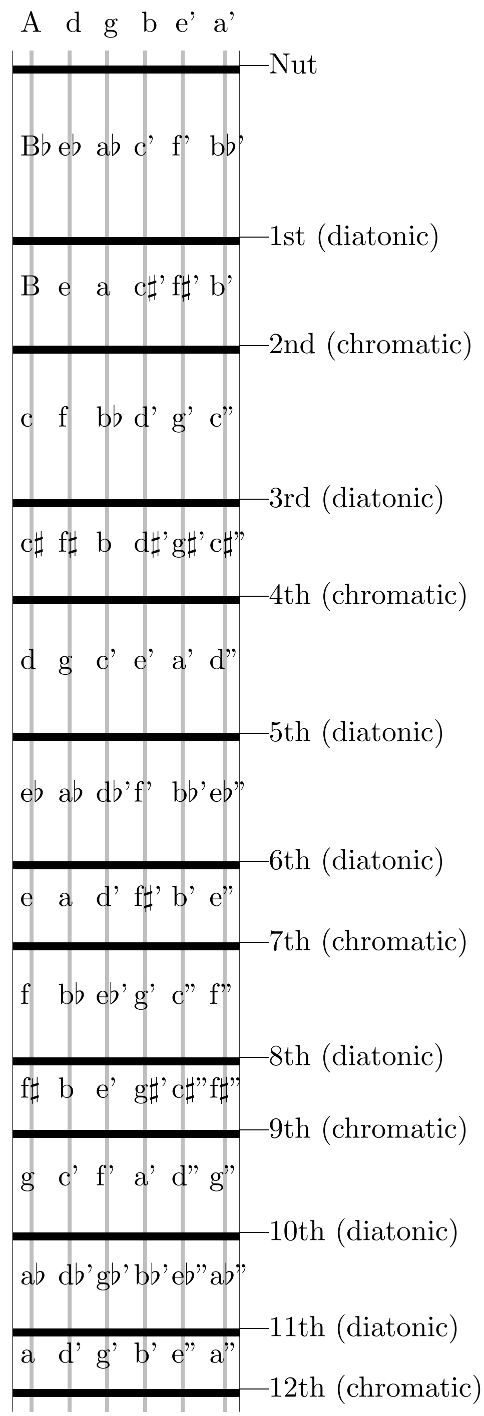

Figure A.2: Complete quarter-comma fretting for theorbo (extended courses not shown)

Appendix B

Calculations

Here are the mathematical details used calculate whole-number ratios for each fret. Formulas are grouped according to source and the order of frets according to how they appear in each source. Each instruction method focuses on where to place the fret on the neck of the instrument, but in order to determine the ratio we must find the vibrating length. Therefore, each passage will have two calculations associated with it. One denoted F which represents the location of the fret and another denoted V which represents the vibrating length. In all cases, the mensur length of the string will be represented by the constant m.

The calculation of fret placement (F) is determined according to the given passage from the instructions, while the vibrating length (V) is calculated as the difference between the fret distance and mensur length.

$$V_x = m - F_x$$

As calculations proceed through a given source, the source often refers back to frets that have already been placed. For this, each fret calculation is referred to by name, such as F_3 which would refer to the fret placement calculation for the third fret.

B.1 Hans Gerle

Fret 12

$$F_{12} = \frac{m}{2}$$
$$V_{12} = m - F12 = m - \frac{m}{2} = \frac{2m}{2} - \frac{1m}{2} = \frac{1m}{2} \rightarrow 2:1$$

Fret 7

$$F_7 = 2 * (\frac{F_{12}}{3}) = 2 * (\frac{\frac{m}{2}}{3}) = \frac{\frac{2m}{2}}{3} = \frac{m}{3}$$
$$V_7 = m - \frac{m}{3} = \frac{2m}{3} \rightarrow 3:2$$

Fret 1

$$F_1 = 2 * (\frac{F_7}{11}) = 2 * (\frac{\frac{m}{3}}{11}) = \frac{\frac{2m}{3}}{11} = \frac{2m}{33}$$
$$V_1 = m - \frac{2m}{33} = \frac{31m}{33} \rightarrow 33:31$$

Fret 2

$$\begin{aligned} F_2 &= \frac{F_7}{3} = \frac{\frac{m}{3}}{3} = \frac{m}{9} \\ V_2 &= m - F_2 = m - \frac{m}{9} = \frac{8m}{9} \rightarrow 9:8 \end{aligned}$$

Fret 5

$$\begin{aligned} F_5 &= \frac{F_{12}}{2} = \frac{\frac{m}{2}}{2} = \frac{m}{4} \\ V_5 &= m - F_5 = m - \frac{m}{4} = \frac{3m}{4} \rightarrow 4:3 \end{aligned}$$

Fret 6

$$\begin{aligned} F_6 &= \frac{F_5 + F_7}{2} = \frac{\frac{m}{4} + \frac{m}{3}}{2} = \frac{\frac{7m}{12}}{2} = \frac{7m}{24} \\ V_6 &= m - F_6 = m - \frac{7m}{24} = \frac{17m}{24} \rightarrow 24:17 \end{aligned}$$

Fret 3

$$F_3 = (3+5) * (\frac{F_1}{3}) = 8 * (\frac{\frac{2m}{33}}{3} = 8 * \frac{2m}{99} = \frac{16m}{99}$$

$$V_3 = m - F_3 = m - \frac{16m}{99} = \frac{83m}{99} \rightarrow 99:83$$

Fret 4

$$
\begin{aligned}
F_4 &= \frac{F_3 + F_5}{2} = \frac{\frac{16m}{99} + \frac{m}{9}}{2} = \frac{\frac{64m}{396} + \frac{99m}{396}}{2} = \frac{\frac{163m}{396}}{2} = \frac{163m}{792} \\
V_4 &= m - F_4 = m - \frac{163m}{792} = \frac{629m}{792} \rightarrow 792 : 629
\end{aligned}
$$

B.2 John Dowland

All frets are identical to Gerle's ratios except:

Fret 3

$$
\begin{aligned}
F_{3Dowland} &= (3 + 4 + \frac{1}{2}) * (\frac{F_{1Dowland}}{3}) \\
&= (7 + \frac{1}{2}) * (\frac{\frac{2m}{33}}{3}) = (7 + \frac{1}{2}) * (\frac{2m}{99}) \\
&= \frac{14m}{99} + \frac{2m}{198} = \frac{28m}{198} + \frac{2m}{198} = \frac{30m}{198} \\
V_{3Dowland} &= m - F_{3Dowland} = m - \frac{30m}{198} = \frac{168m}{198} \rightarrow 198 : 168
\end{aligned}
$$

Fret 4

$$
\begin{aligned}
F_{4Dowland} &= \frac{F_{2Dowland} + F_{5Dowland}}{2} \\
&= \frac{\frac{30m}{198} + \frac{m}{4}}{2} = \frac{\frac{120m}{792} + \frac{198m}{792}}{2} = \frac{\frac{318m}{792}}{2} = \frac{318m}{1584} \\
V_{4Dowland} &= m - F_{4Dowland} = m - \frac{318m}{1584} = \frac{1266m}{1584} \rightarrow 1584 : 1266
\end{aligned}
$$

Frets 8, 9 and 10

$$
\begin{aligned}
F_{8Dowland} &= \frac{m - F_{1Dowland}}{3} = \frac{m - \frac{2m}{33}}{3} = \frac{\frac{31m}{33}}{3} = \frac{31m}{99} \\
F_{9Dowland} &= \frac{m - F_{2Dowland}}{3} = \frac{m - \frac{m}{9}}{3} = \frac{\frac{8m}{9}}{3} = \frac{8m}{27} \\
F_{10Dowland} &= \frac{m - F_{3Dowland}}{3} = \frac{m - \frac{30m}{198}}{3} = \frac{\frac{168m}{198}}{3} = \frac{168m}{594} \\
V_{8Dowland} &= m - F_{8Dowland} = m - \frac{31m}{99} = \frac{68m}{99} \rightarrow 99 : 68 \\
V_{9Dowland} &= m - F_{9Dowland} = m - \frac{8m}{27} = \frac{19m}{27} \rightarrow 27 : 19 \\
V_{10Dowland} &= m - F_{10Dowland} = m - \frac{168m}{594} = \frac{426m}{594} \rightarrow 594 : 426
\end{aligned}
$$

B.3 Silvestro Ganassi

Fret 1

$$
\begin{aligned}
F_{1Ganassi} &= \frac{F_2}{2} = \frac{\frac{m}{9}}{2} = \frac{m}{18} \\
V_{1Ganassi} &= m - F_1 = m - \frac{m}{18} = \frac{17}{18} \rightarrow 18 : 17
\end{aligned}
$$

Fret 3

$$
\begin{aligned}
F_{3Ganassi} &= F_{1Ganassi} + F_{2Ganassi} = \frac{m}{18} + \frac{m}{9} = \frac{m}{18} + \frac{2m}{18} = \frac{3m}{18} = \frac{m}{6} \\
V_{3Ganassi} &= m - F_{3Ganassi} = m - \frac{m}{6} = \frac{5m}{6} \rightarrow 6 : 5
\end{aligned}
$$

Fret 4

$$
\begin{aligned}
F_{4Ganassi} &= \frac{F_{3Ganassi} + F_{5Ganassi}}{2} = \frac{\frac{m}{6} + \frac{m}{4}}{2} = \frac{\frac{4m}{24} + \frac{6m}{24}}{2} = \frac{\frac{10m}{24}}{2} = \frac{10m}{48} \\
V_{4Ganassi} &= m - F_{4Ganassi} = m - \frac{10m}{48} = \frac{38m}{48} \rightarrow 48 : 38
\end{aligned}
$$

Fret 8

$$
\begin{aligned}
F_{8Ganassi} &= F_{7Ganassi} + (F_{6Ganassi} - F_{5Ganassi}) \\
&= \frac{m}{3} + \frac{7m}{24} - \frac{m}{4} = \frac{m}{3} + \frac{7m}{24} - \frac{6m}{24} = \frac{m}{3} + \frac{m}{24} \\
&= \frac{8m}{24} + \frac{m}{24} = \frac{9m}{24} = \frac{3m}{8} \\
V_{8Ganassi} &= m - F_{8Ganassi} = m - \frac{3m}{8} = \frac{5m}{8} \rightarrow 8 : 5
\end{aligned}
$$

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