Change in the value function due to learning after taking action a^* :

$$v(ba^{*}) - v(b) = \sum_{b'} p(b' \mid b, a^{*}) \sum_{a} \pi(a \mid b') q(b', a) - \sum_{a} \pi(a \mid b) q(b, a)$$

$$= \sum_{a} \left(\sum_{b'} p(b' \mid b, a^{*}) \pi(a \mid b') q(b', a) - \pi(a \mid b) q(b, a) \right)$$

$$= \sum_{a} \left(\sum_{b'} p(b' \mid b, a^{*}) \pi(a \mid b') q(b', a) - \pi(a \mid b) q(b, a) \right)$$

$$+ \pi(a \mid b) \sum_{b'} p(b' \mid b, a^{*}) q(b', a) - \pi(a \mid b) \sum_{b'} p(b' \mid b, a^{*}) q(b', a) \right)$$

$$= \sum_{a} \left(\sum_{b'} p(b' \mid b, a^{*}) \left(\pi(a \mid b') - \pi(a \mid b) \right) q(b', a) \right)$$

$$+ \pi(a \mid b) \sum_{b'} p(b' \mid b, a^{*}) \left(q(b', a) - q(b, a) \right) \right)$$

$$= \sum_{b'} p(b' \mid b, a^{*}) \sum_{a} \left(\left(\pi(a \mid b') - \pi(a \mid b) \right) q(b', a) \right)$$

$$+ \pi(a \mid b) \left(q(b', a) - q(b, a) \right) \right)$$

Expanding q(b', a) - q(b, a):

$$q(b',a) - q(b,a) = \sum_{b''} p(b'' \mid b',a) \left[r(b',a) + \gamma v(b'') \right]$$

$$- \sum_{b'} p(g' \mid b,a) \left[r(b,a) + \gamma v(g') \right]$$

$$= r(b',a) + \gamma \sum_{b''} p(b'' \mid b',a) v(b'')$$

$$- r(b,a) + \gamma \sum_{g'} p(g' \mid b,a) v(g')$$

$$= r(b',a) - r(b,a) + \gamma \left[\sum_{b''} p(b'' \mid b',a) v(b'') - \sum_{g'} p(g' \mid b,a) v(g') \right]$$
Difference in the expected immediate return

Difference in the expected Difference in the expected immediate return

So overall the EVB decomposes as:

$$v(ba^{*}) - v(b) = \mathbb{E}_{b' \sim p(b'|b,a^{*})} \Big[\sum_{a} (\pi(a \mid b') - \pi(a \mid b)) q(b',a)$$

$$+ \mathbb{E}_{a \sim \pi(a|b)} [r(b',a) - r(b,a)]$$

$$+ \mathbb{E}_{a \sim \pi(a|b)} \Big[\gamma \sum_{b''} p(b'' \mid b',a) v(b'') - \gamma \sum_{a'} p(g' \mid b,a) v(g') \Big] \Big]$$
(3)