

Change in the value function due to learning after taking action  $a^*$ :

$$v(ba^*) - v(b) = \sum_{b'} p(b' | b, a^*) \sum_a \pi(a | b') q(b', a) - \sum_a \pi(a | b) q(b, a) \quad (1)$$

$$\begin{aligned} &= \sum_a \left( \sum_{b'} p(b' | b, a^*) \pi(a | b') q(b', a) - \pi(a | b) q(b, a) \right) \\ &= \sum_a \left( \sum_{b'} p(b' | b, a^*) \pi(a | b') q(b', a) - \pi(a | b) q(b, a) \right) \\ &\quad + \pi(a | b) \sum_{b'} p(b' | b, a^*) q(b', a) - \pi(a | b) \sum_{b'} p(b' | b, a^*) q(b', a) \\ &= \sum_a \left( \sum_{b'} p(b' | b, a^*) (\pi(a | b') - \pi(a | b)) q(b', a) \right) \\ &\quad + \pi(a | b) \sum_{b'} p(b' | b, a^*) (q(b', a) - q(b, a)) \\ &= \sum_{b'} p(b' | b, a^*) \sum_a ((\pi(a | b') - \pi(a | b)) q(b', a) \\ &\quad + \pi(a | b) (q(b', a) - q(b, a))) \end{aligned} \quad (2)$$

$$\begin{aligned} q(b', a) - q(b, a) &= \sum_{b''} p(b'' | b', a) [r(b', a) + \gamma v(b'')] \\ &\quad - \sum_{b'} p(b' | b, a) [r(b, a) + \gamma v(b')] \\ &= r(b', a) + \gamma \sum_{b''} p(b'' | b', a) v(b'') \\ &\quad - r(b, a) + \gamma \sum_{b'} p(b' | b, a) v(b') \\ &= \underbrace{r(b', a) - r(b, a)}_{\text{Difference in the expected immediate return}} + \gamma \underbrace{\left[ \sum_{b''} p(b'' | b', a) v(b'') - \sum_{b'} p(b' | b, a) v(b') \right]}_{\text{Difference in the expected future return}} \end{aligned} \quad (3)$$