

# EV Decomposition

Change in the value function due to learning after taking action  $a^*$ :

$$\begin{aligned}
 v(ba^*) - v(b) &= \sum_{b'} p(b' | b, a^*) (v(b') - v(b)) \\
 &= \sum_{b'} p(b' | b, a^*) \left( \sum_a \pi(a | b') q(b', a) - \sum_a \pi(a | b) q(b, a) \right) \\
 &= \sum_{b'} p(b' | b, a^*) \sum_a \left( (\pi(a | b') - \pi(a | b)) q(b', a) \right. \\
 &\quad \left. + \pi(a | b) (q(b', a) - q(b, a)) \right)
 \end{aligned} \tag{1}$$

Expanding  $q(b', a) - q(b, a)$ :

$$\begin{aligned}
 q(b', a) - q(b, a) &= \sum_{b''} p(b'' | b', a) [r(b', a) + \gamma v(b'')] \\
 &\quad - \sum_{b''} p(b'' | b, a) [r(b, a) + \gamma v(b'')] \\
 &= r(b', a) + \gamma \sum_{b''} p(b'' | b', a) v(b'') \\
 &\quad - r(b, a) + \gamma \sum_{b''} p(b'' | b, a) v(b'') \\
 &= \underbrace{r(b', a) - r(b, a)}_{\text{Difference in the expected immediate return}} + \underbrace{\gamma \left[ \sum_{b''} p(b'' | b', a) v(b'') - \sum_{b''} p(b'' | b, a) v(b'') \right]}_{\text{Difference in the expected future return}}
 \end{aligned} \tag{2}$$

So overall the EVB decomposes as:

$$\begin{aligned}
 v(ba^*) - v(b) &= \mathbb{E}_{b' \sim p(b' | b, a^*)} \left[ \sum_a (\pi(a | b') - \pi(a | b)) q(b', a) \right. \\
 &\quad \left. + \mathbb{E}_{a \sim \pi(a | b)} [r(b', a) - r(b, a)] \right. \\
 &\quad \left. + \mathbb{E}_{a \sim \pi(a | b)} \left[ \gamma \sum_{b''} p(b'' | b', a) v(b'') - \gamma \sum_{b''} p(b'' | b, a) v(b'') \right] \right]
 \end{aligned} \tag{3}$$

# Simulations

Prioritisation pattern with horizon  $h = 3$ .

