Optimising Replay for Partially Observable Domains

Georgy, lab meeting 09/03/22

talk outline

- planning
- DYNA, prioritised sweeping
- explore-exploit
- normative theory of hippocampal replay
- replay in Bayesian bandits
- replay in BAMDPs
- limitations and future directions

planning

- generally speaking, planning refers to the process of computing a policy – i.e., coming up with an action to execute
- in value-based planning, this is achieved by estimating values associated with the available actions
- planning typically requires a model of the environment
- several aproaches exist: dynamic programming (DP), sample/simulation-based

planning $\rightarrow$ dynamic programming

- Bellman optimality equation

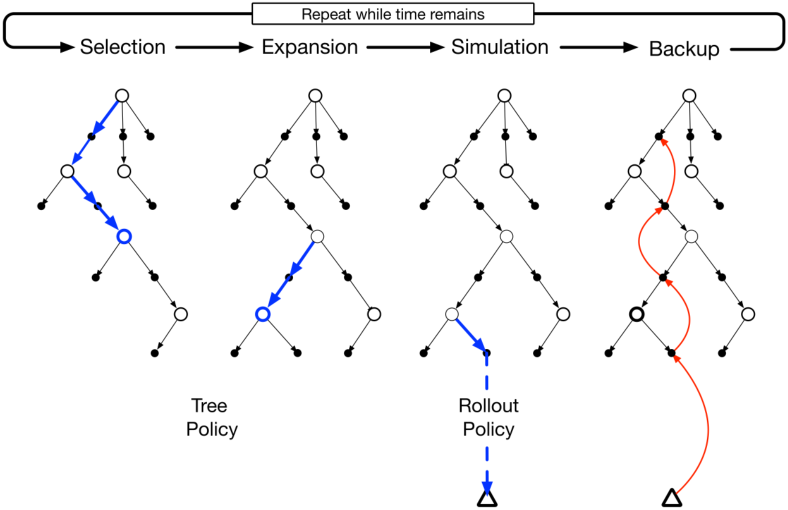
$$V^*(s) = \max_a \sum_{s'} \mathcal{P}^a_{ss'}[\mathcal{R}^a_{ss'} + \gamma V^*(s')]$$

- value iteration is one example value-based planning algorithm

$$V_k(s) \leftarrow \max_a \sum_{s'} \mathcal{P}^a_{ss'}[\mathcal{R}^a_{ss'} + \gamma V_{k-1}(s')]$$

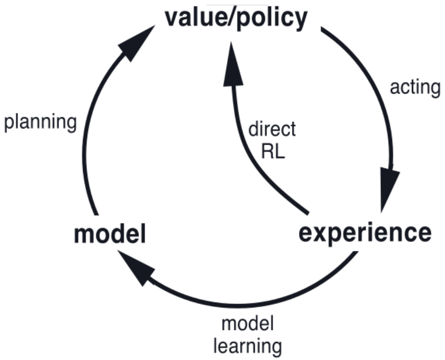
- assumes a known model of the environment $\mathcal{P}^a_{ss'}$
- performs synchronous sweeps over all states; computationally expensive; time-consuming

planning → MCTS



- MCTS is a simulation-based planning algorithm
- Tree and rollout policies
- Difficult to balance exploration and exploitation

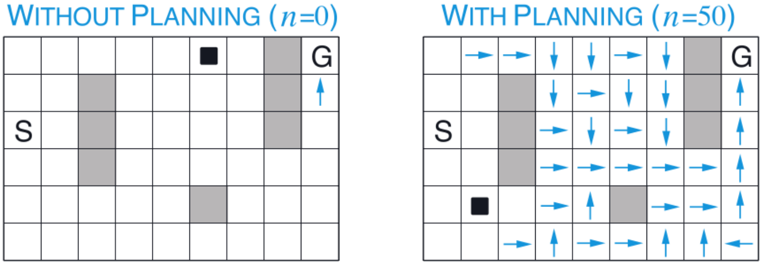
DYNA



Sutton (1990)

- DYNA is an integrated architecture
- combines a *reactive* MF policy and a *deliberate* MB system
- MB system is used offline to provide additional training to MF values

DYNA → example



- agent discovers online prediction erros (e.g., a goal)
- model inversion to additionally train MF values

$$Q^{MF}(s,a) \leftarrow Q^{MF}(s,a) + \alpha[R^{MB}(s') + \gamma \max_{a'} Q^{MB}(s',a') - Q^{MF}(s,a)]$$

- asynchronous DP

prioritised sweeping

- asynchronous DP updates can be optimised
- online discovery of a prediction error results in high offline prediction errors for the immediately preceeding states
- the idea of prioritised sweeping (Moore et al., 1993) is to execute the individual updates according to a priority queue, for instance:

$$p(s,a) = |Q^{MF}(s,a) + \alpha[R(s') + \gamma \max_{a'} Q^{MF}(s',a') - Q^{MF}(s,a)]|$$

prioritised sweeping $\rightarrow$ example

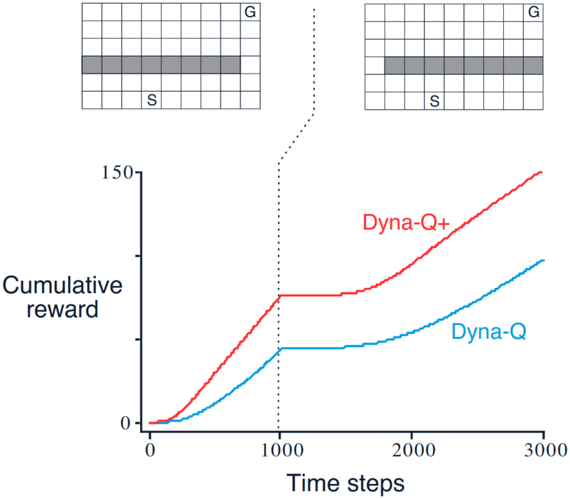
explore-exploit

- Optimal behaviour necessitates optimal balance of exploration and exploitation
- Multiple heuristics have been devised to encourage exploration
- One prominent heuristic is called an *exploration bonus*
- Sutton (1991) proposed to add an exploration bonus to the values of state-action pairs which have not been visited recently:

$$Q^{MB}(s,a) \leftarrow Q^{MB}(s,a) + \kappa\sqrt{\epsilon_{(s,a)}}$$

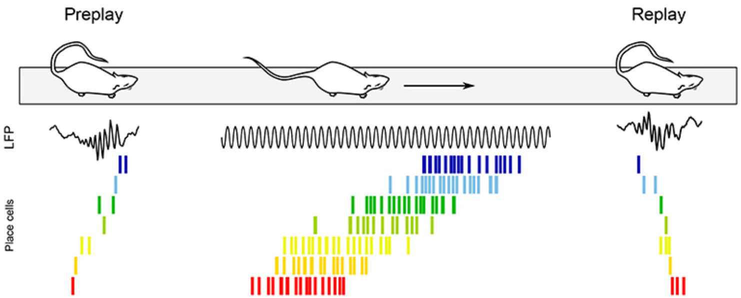
  where $\epsilon_{(s,a)}$ grows with the number of time steps elapsed since the state-action pair (s,a) was last tried, and κ controls the rate of exploration

explore-exploit $\rightarrow$ DYNA-Q+



- note that this exploration bonus is myopic
- propagates from distal locations due to the Q-learning rule
- however, it is blind towards what could be the consequences of exploration

hippocampal replay



Drieu et al. (2019)

- reinstatement of behaviourally-relevant neural activity during periods of quiet wakefullness and sleep (offline periods)
- the order of the replayed experiences is highly specific
- can proceed in forward and reverse directions
- forward replay seems to be predictive of the subsequent animal choices; reverse replay is highly sensitive to reward

hippocampal replay $\rightarrow$ normative theory

- Mattar & Daw (2018) realised that hippocampal replay might be a candidate mechanism for offline generative planning – acting in accordance with the DYNA system by supplying MB information to the animal's MF policy
- each replay experience, according to Mattar & Daw, corresponds to an update of an MF value for a state-action pair
- each replay update therefore changes the animal's policy at the state where that update is executed
- the fact that the order in which replay experiences proceed is highly specific suggests some sort of prioritisation (prioritised sweeping)

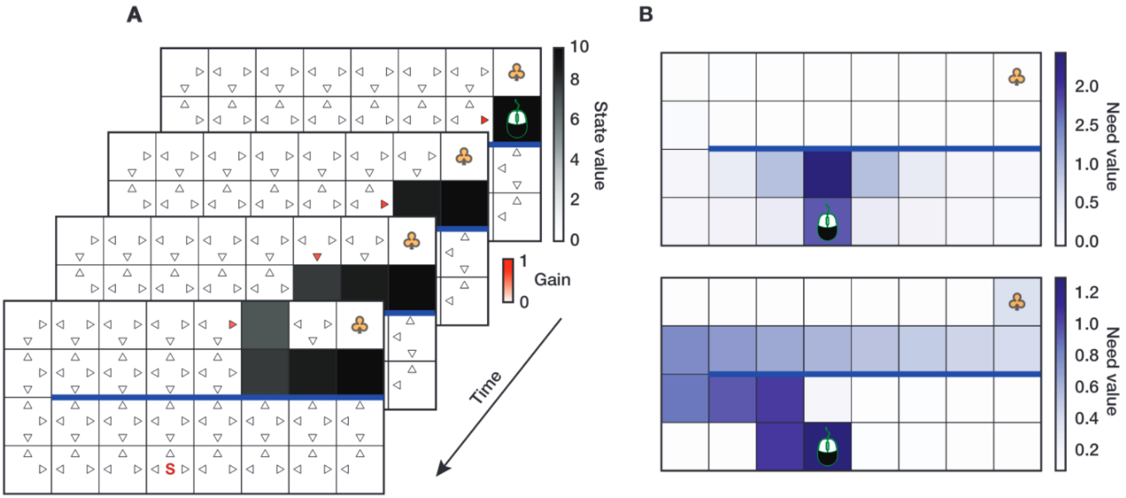
hippocampal replay $\rightarrow$ normative theory

- by the repeated unrolling of $v_{\pi_{\text{new}}}(s) - v_{\pi_{\text{old}}}(s)$, where s is the animal's current location, M&D showed that the value of computation (i.e., replay update) performed at state s_k for action a_k can be decomposed into two terms

$$\text{EVB}(s_k, a_k) = \sum_{s' \in \mathcal{S}} \underbrace{\sum_{i=0}^{\infty} \gamma^i P(s \rightarrow s', i, \pi_{\text{old}})}_{\text{Need}} \times \underbrace{\sum_a [\pi_{\text{new}}(a \mid s') - \pi_{\text{old}}(a \mid s')] q_{\pi_{\text{new}}}(s', a)}_{\text{Gain}}$$

- Need is the expected discounted future occupancy of state s' under the animal's policy prior to the update, π_{old}
- Gain quantifies the local policy improvement at the state where the potential replay update is considered

hippocampal replay → normative theory



hippocampal replay $\rightarrow$ normative theory

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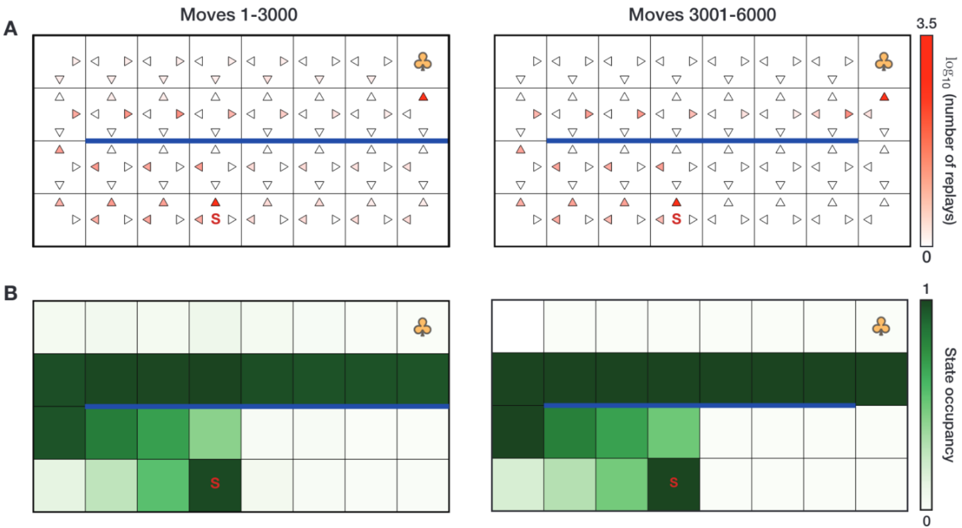
Assumptions of the M&D model:

- policy updates are local, and thus M&D get rid of the first sum over $\mathcal{S}$
-
  - Note this means that the benefit of policy change at a distal state is only considered at the animal's current state
- Gain is expressed in terms of the *true* Q-values implied by the new policy, $q_{\pi_{\text{new}}}$
- the transition model P is known

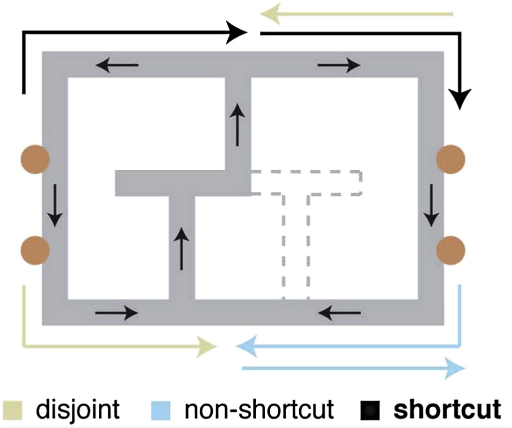
hippocampal replay → exploration?



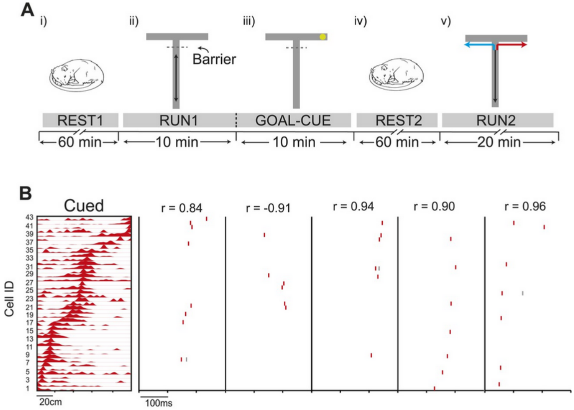
hippocampal replay $\rightarrow$ exploration?

- turns out the model of M&D doesn't explore well
- weird because the original intention of DYNA was to encourage exploration
- Need acts as a regulariser – once a policy is learnt it biases the selection of replay updates towards those states which the current policy already expects to visit (pure exploitation!)
- Gain (in fact, Need as well) doesn't account for the potential information which can be learnt and utilised in the future

hippocampal replay → exploration?



Gupta et al. (2010)



Ólafsdóttir et al. (2015)

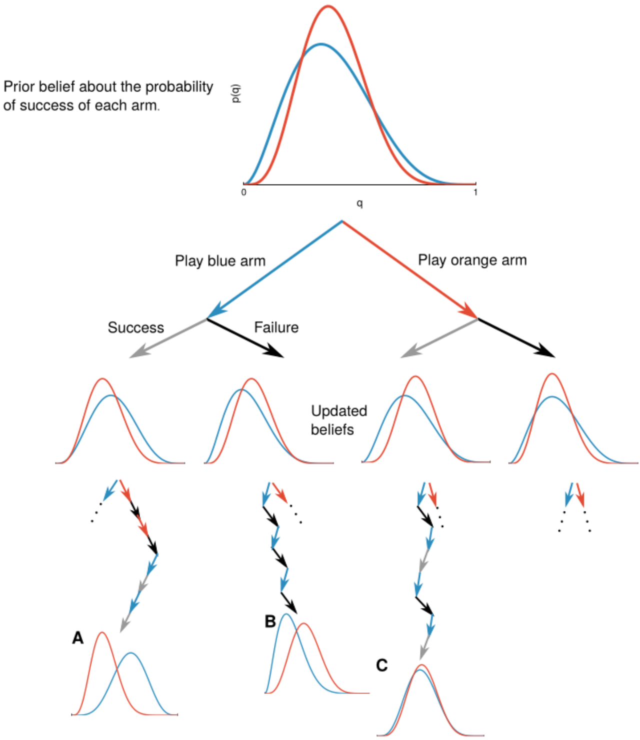
explore-exploit revisited

- in Bayesian RL, one often assumes some prior belief b over the unknown parameters
- in Bayes-adaptive MDPs [BAMDPs; Duff (1995)], the prior is over the unknown transition model parameters:

$$P(s' \mid s, a) = \int_{\Theta} P(s' \mid s, a, \theta) b(\theta) d\theta$$

- once integrated out, the model becomes known; moreover, it incorporates the agent's epistemic uncertainty – i.e., the 'known unknowns'
- the resulting policies are known as Bayes-adaptive policies, since they optimally trade-off exploration and exploitation

- planning in bandits can be visualised as belief trees



Guez (2015)

- optimal solution is known as the Gittins indices (Gittins, 1979), which correspond to the DP solution in the belief space

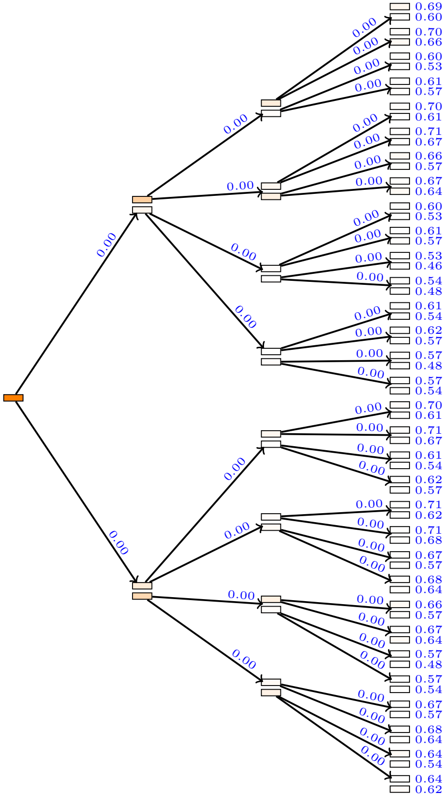
replay in Bayesian bandits

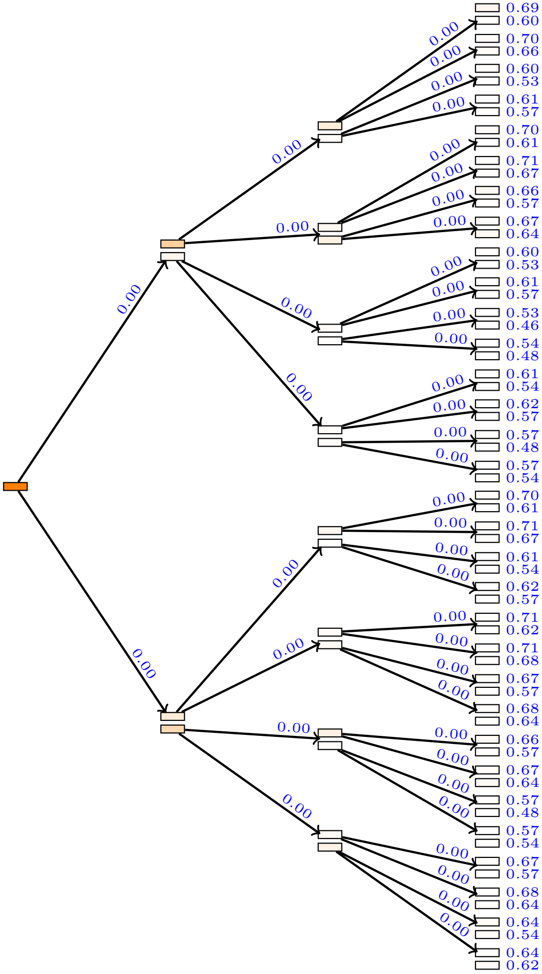
- we can do the same decomposition as in M&D but for belief states:

$$\text{EVB}(b_k, a_k) = \gamma^h P(b_\rho \to b_k, h, \pi_{\text{old}}) \times \sum_a [\pi_{\text{new}}(a \mid b_k) - \pi_{\text{old}}(a \mid b_k)] q_{\pi_{\text{new}}}(a, b_k)$$

- where b_ρ is the prior belief at the root, and h is the horizon of belief b_k
- note that Need here is not cumulative; this is because each belief state can be visited at most once due to continual learning

- each rectangle is a belief state
- colour intensity corresponds to Need, $\gamma^h P(b_\rho \rightarrow b_k, h, \pi_{\text{old}})$
- each arrow is an action
- blue numbers are Q-values
- in this tree example, the root belief is $(\alpha_0 = 5, \beta_0 = 1, \alpha_1 = 2, \beta_1 = 4)$
- the tree policy is a softmax with $\beta = 4$





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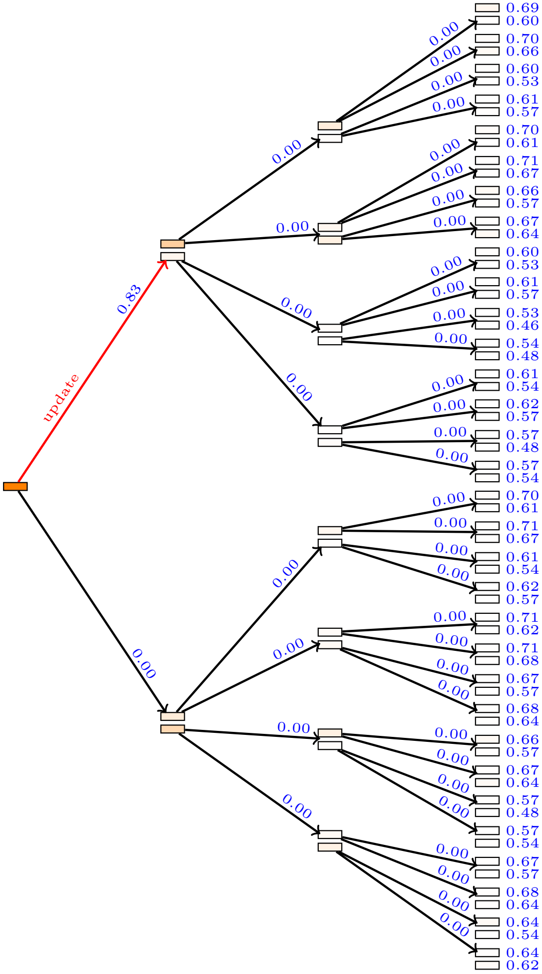
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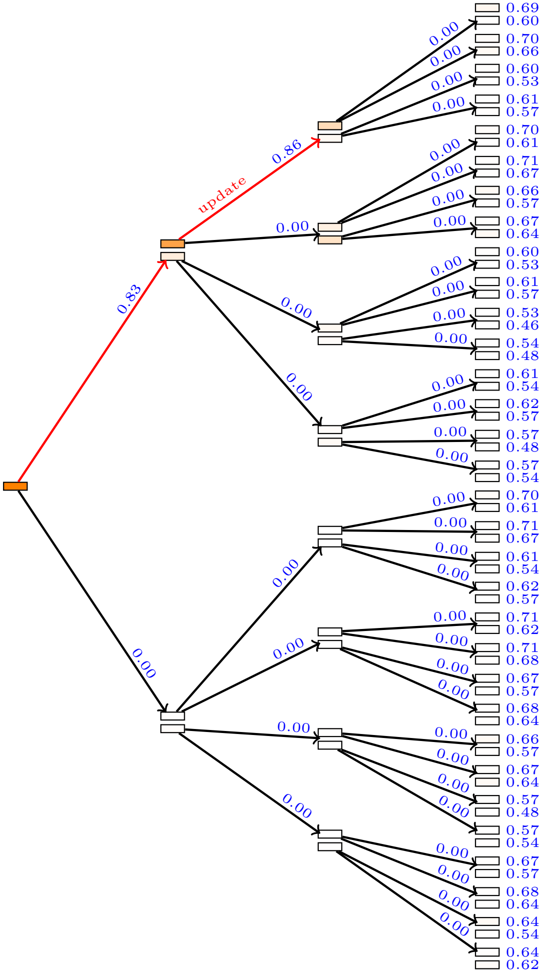

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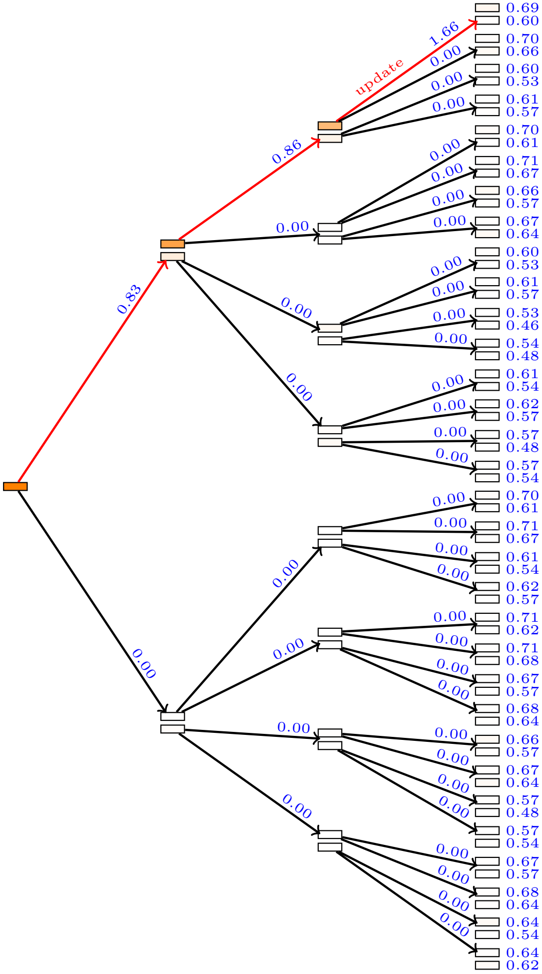

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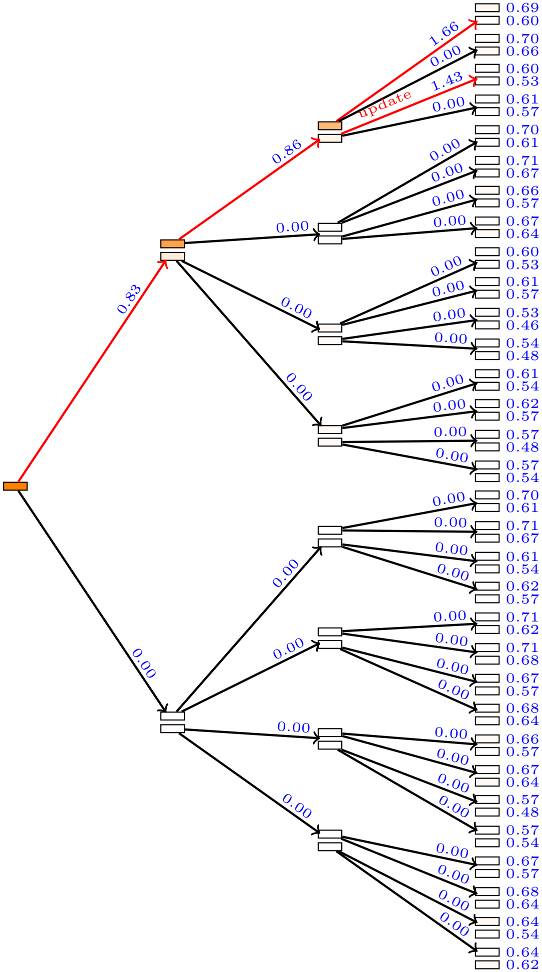
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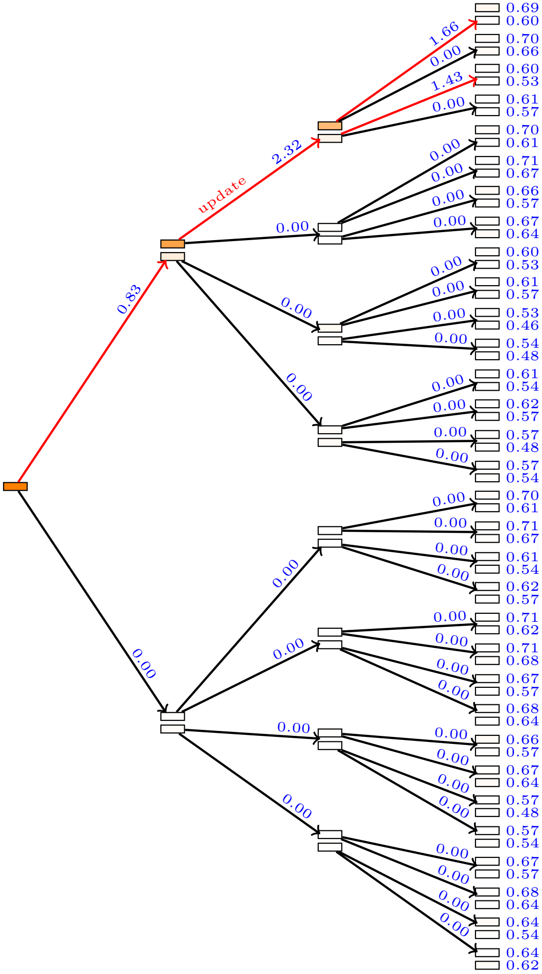

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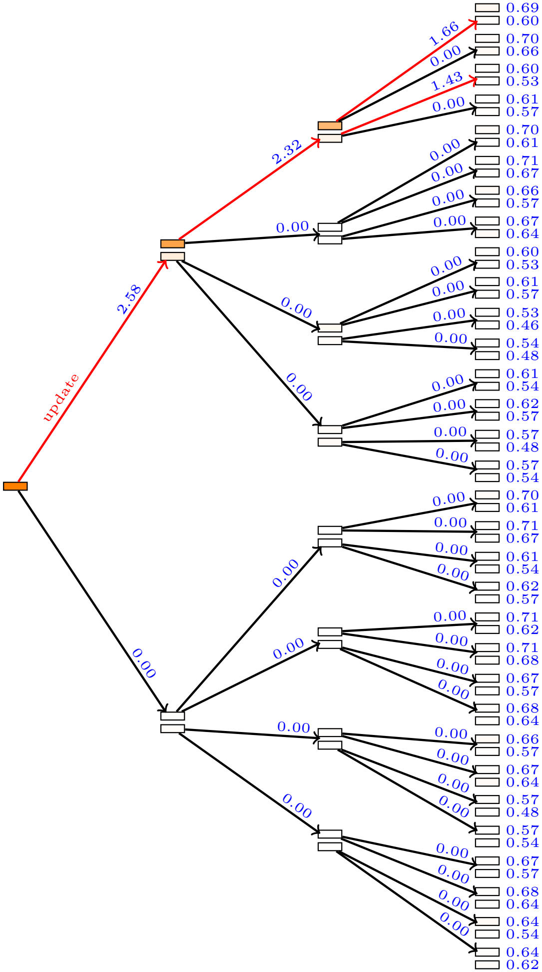
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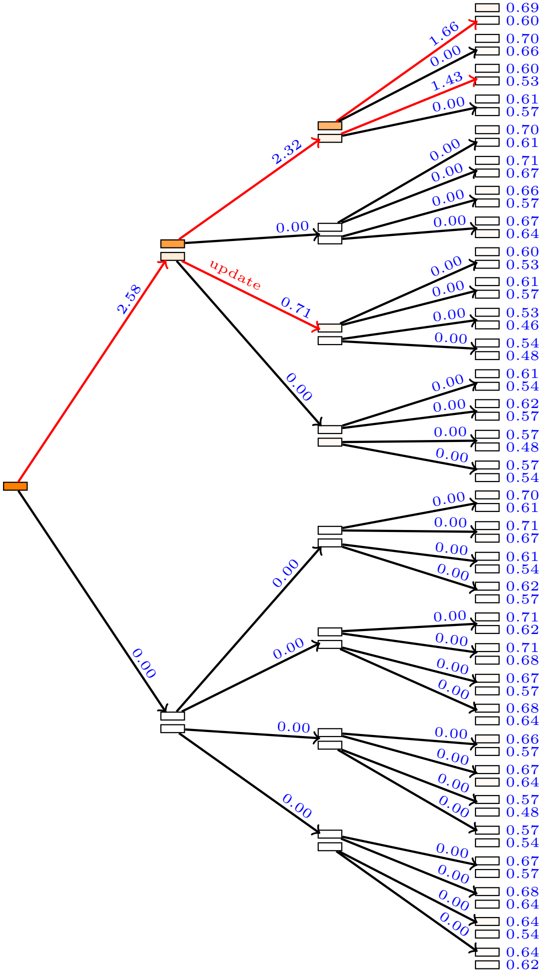

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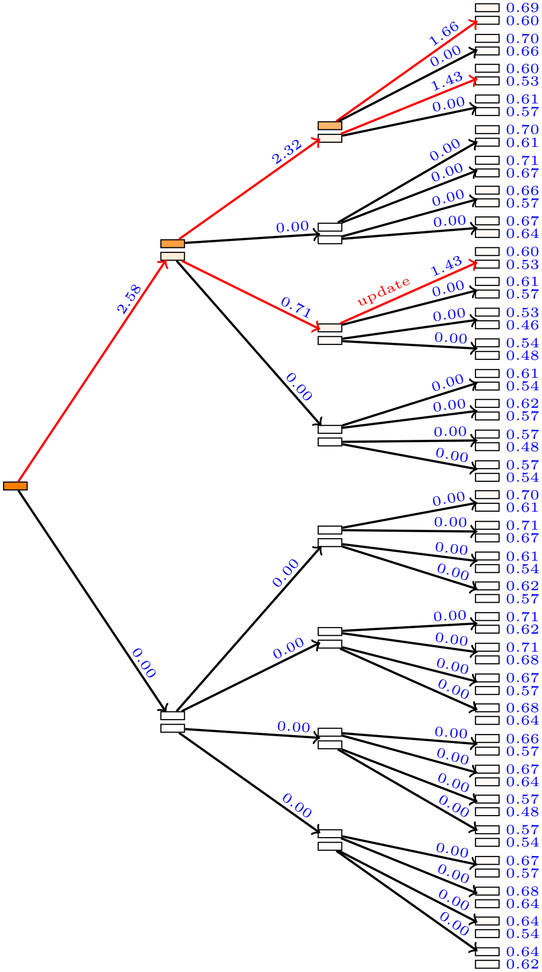

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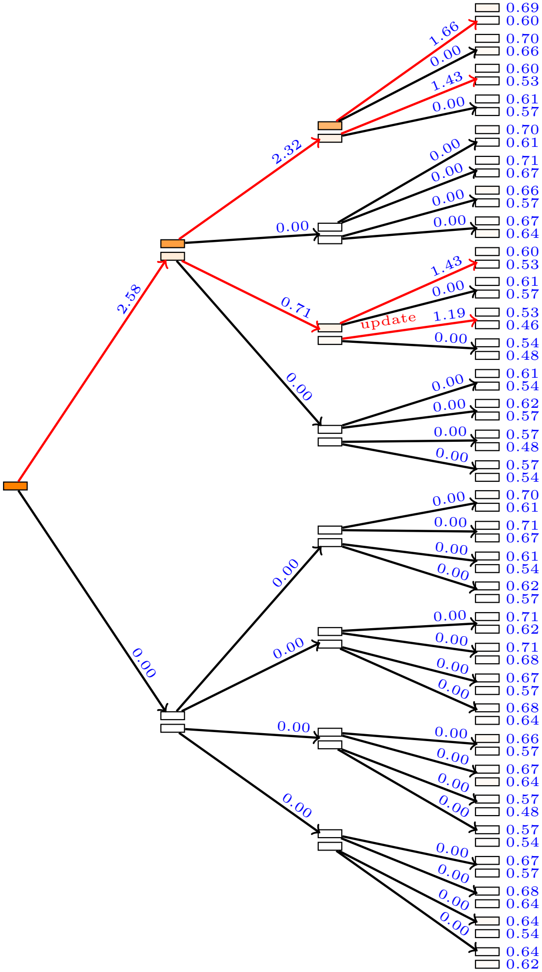
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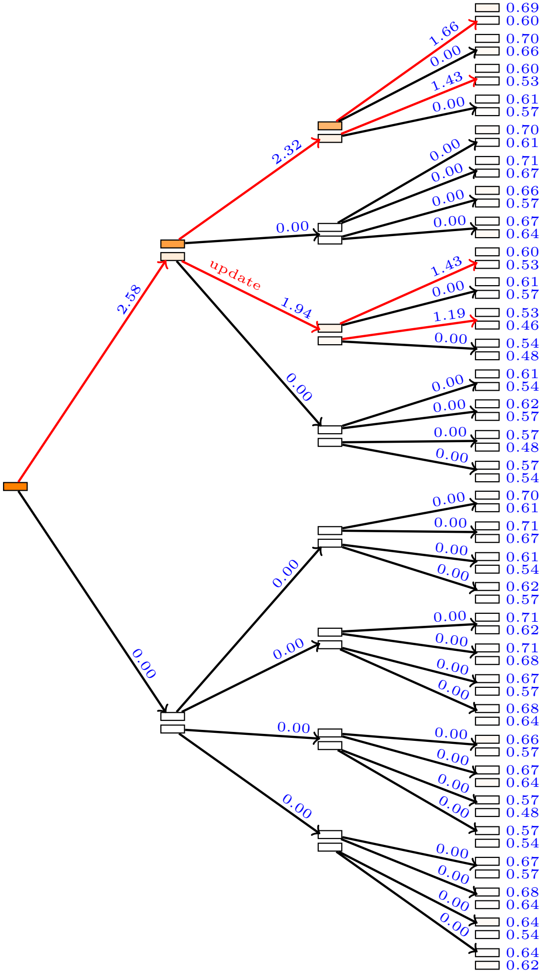

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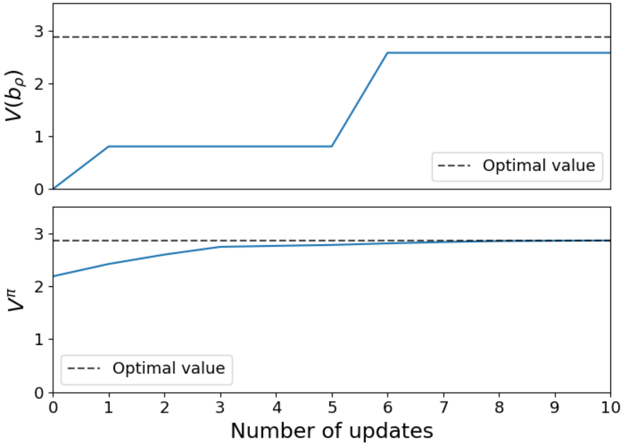
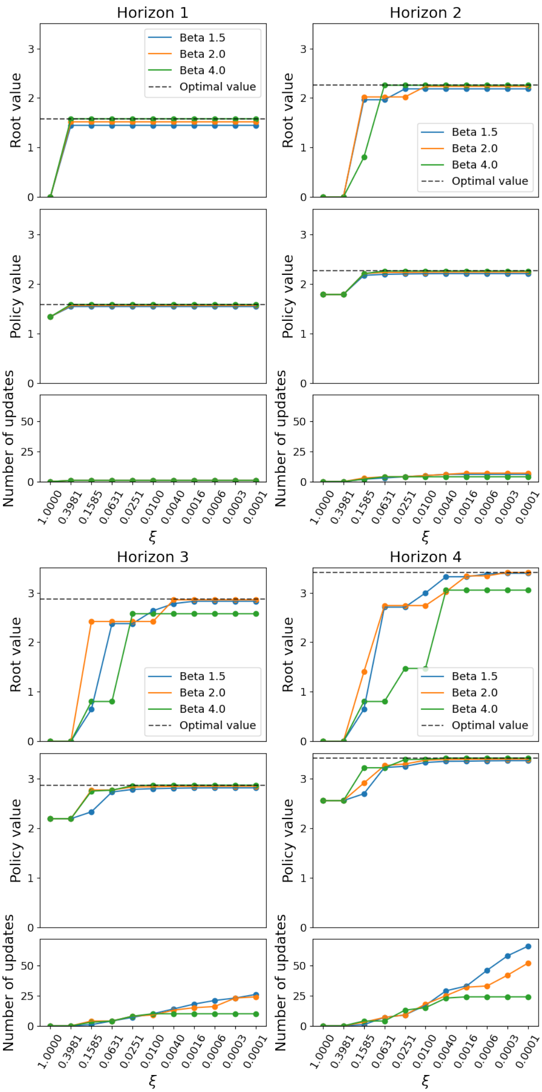
replay in Bayesian bandits

- the effects of each consequitive update on the root value, $V(b_\rho)$, as well as the value of the new (updated) policy evaluated in the tree, V^π

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Beta 1.5
Beta 2.0
Beta 4.0
Optimal value
Root value
Policy value
Number of updates
1.0000
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0.1585
0.0631
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0.0100
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0.0016
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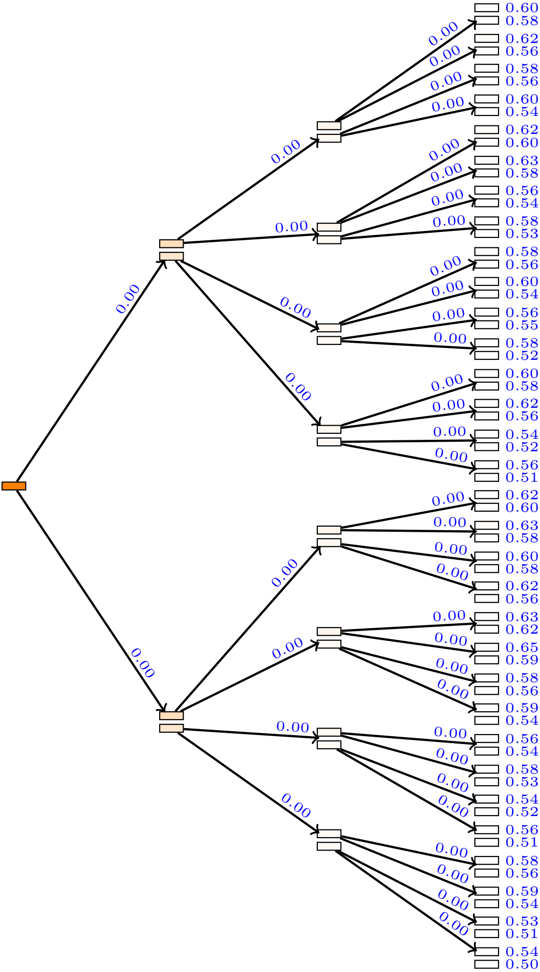
replay in Bayesian bandits

- one more example
- this time for the prior $(\alpha_0 = 14, \beta_0 = 10, \alpha_1 = 4, \beta_1 = 3)$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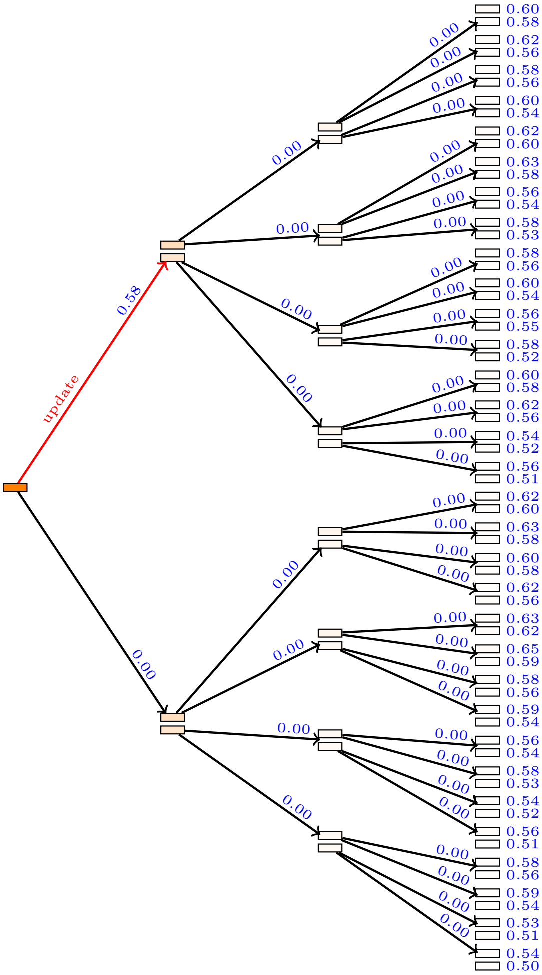

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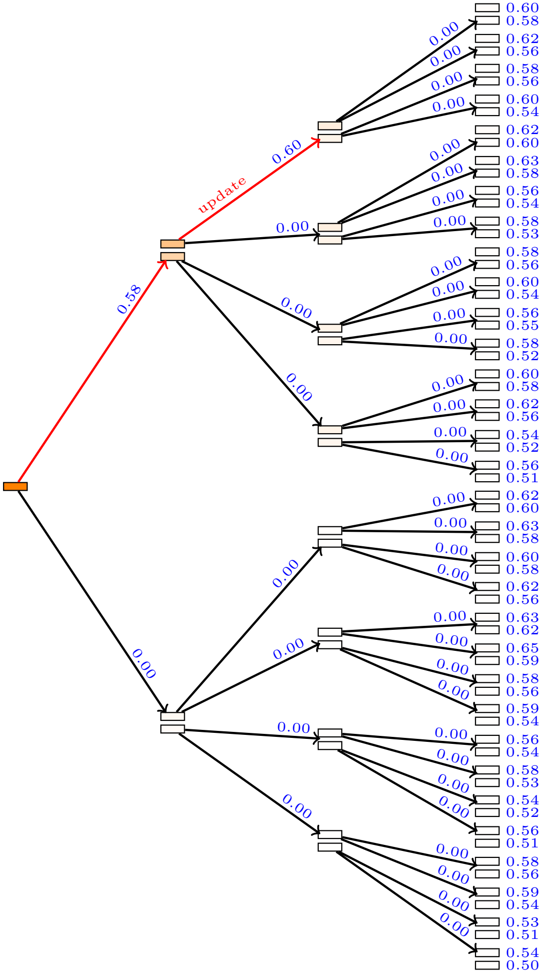

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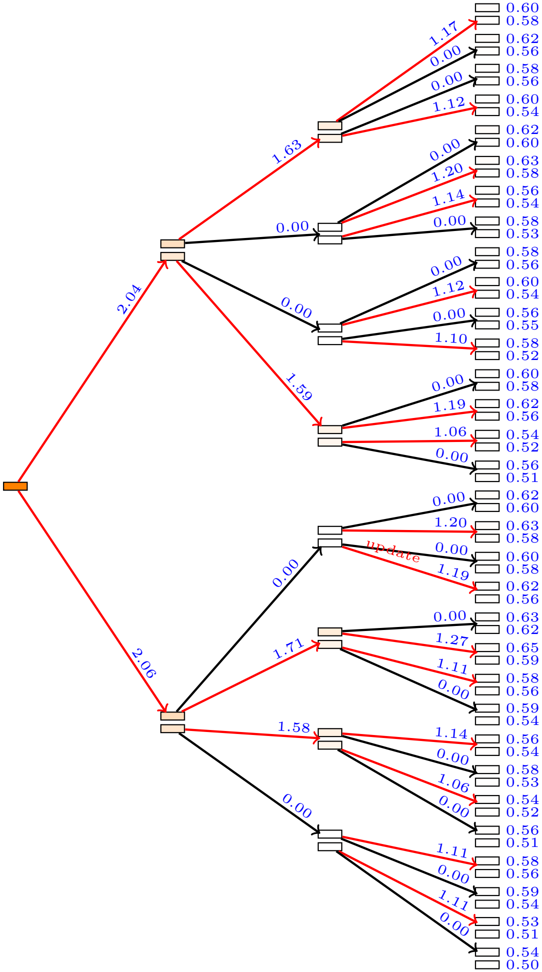

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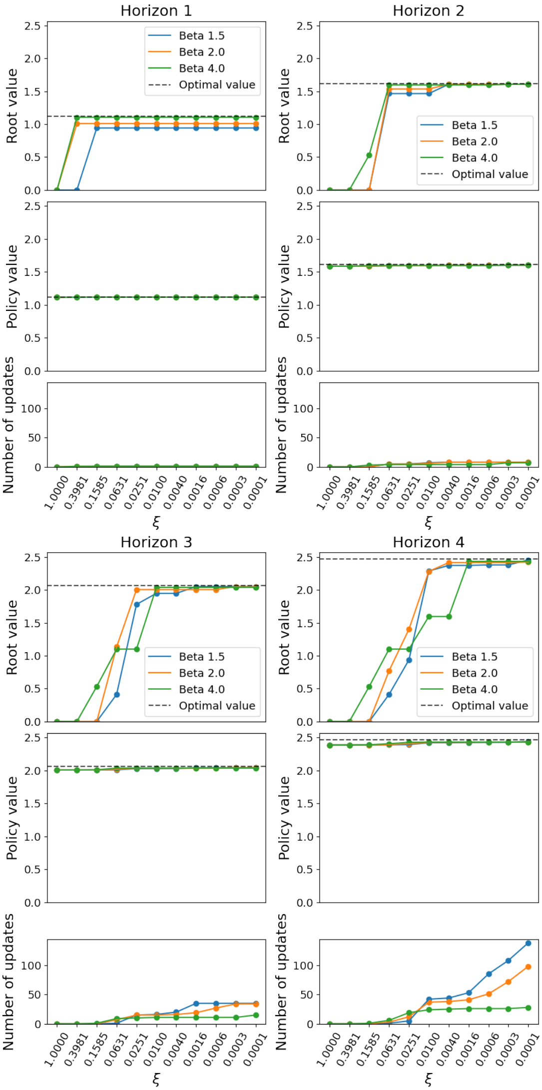
Horizon 1
Horizon 2
Horizon 3
Horizon 4
Root value
Policy value
Number of updates
Beta 1.5
Beta 2.0
Beta 4.0
Optimal value
1.0000
0.3981
0.1585
0.0631
0.0251
0.0100
0.0040
0.0016
0.0006
0.0003
0.0001
ξ

replay in Bayesian bandits

- planning in belief space can be optimised
- free generalisation for subsequent beliefs
- note the use of a softmax policy
- we are ultimately interested in exploration in DYNA-like systems which have an MF policy; MB system needs to convince the MF policy that something is worth exploring
- otherwise, the Need term $\gamma^h P(b_\rho \to b_k, h, \pi_{\text{old}})$ would be zero for those beliefs which the current MF policy doesn't expect to visit
- similar issues in MCTS – e.g., UCB

replay in BAMPDs

- in BAMDPs, we transition through belief states as well as physical states. Jointly, these are referred to as information states $z = \langle s, b \rangle$
- the prioritisation in BAMDPs thus takes the following form:

$$\text{EVB}(z_k, a_k) = \sum_{z' \in \mathcal{Z}} \sum_{i=0}^{\infty} \gamma^i P(z \to z', i, \pi_{\text{old}}) \times \sum_a [\pi_{\text{new}}(a \mid z') - \pi_{\text{old}}(a \mid z')] q_{\pi_{\text{new}}}(z', a)$$

- note that each $z = \langle s, b \rangle$ can still be visited at most once
- we know, however, that although the belief changes continuosly, the agent should still expect to visit the same physical state over and over again
- moreover, the summation over $\mathcal{Z}$ allows us to account for generalisation – that is, how updates at single information states (revealing a piece of information) affect policy at other beliefs (not quite there yet)