Final Review

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1 Law of Large Numbers

Definition. X_n *converges in probability* to X if for any $\varepsilon > 0$,

$$\lim_{n\to\infty} \mathbb{P}(|X_n - X| \geq \varepsilon) = 0$$

denoted by $X_n \xrightarrow{p} X$.

Definition. X_n *converges almost surely* to X if

$$\mathbb{P}(\lim_{n\to\infty} X_n = X) = 1.$$

denoted by $X_n \xrightarrow{a.s.} X$.

Theorem. Suppose $\{X_n\}$ are i.i.d random variables with $X_n \xrightarrow{d} X, \mathbb{E}[X] < \infty$, then

$$\lim_{n\to\infty} \frac{X_1 + X_2 + \cdots + X_n}{n} \xrightarrow{p \ \& \ a.s.} \mathbb{E}[X].$$