

Introduction to Functional Programming with Haskell - RCOS Presentation

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What characterizes Functional Programming?

- ▶ Abstraction of common patterns via higher order functions
- ▶ A mathematical use of the term "function" (as a pure mapping from input to output), as opposed to the concept of procedures (e.g. in C)

What is Haskell?

A general-purpose programming language that:

- ▶ Encourages a functional style of programming
- ▶ Enables very short, readable code
- ▶ Has static typechecking, with type inference
- ▶ Compiles to native code, in the same efficiency class as Java/C# (can be made as performant as Assembly/C/C++, with some effort)

Hello world

► `main = putStrLn "Hello, world!"`

► `main = do
 putStrLn "Enter a string: "
 str <- getLine
 putStrLn ("Echo: " ++ str)`

Examples with numbers

- ▶

```
factorial n = product [1..n]
```
- ▶

```
dotProduct xs ys = sum $ zipWith (*) xs ys
dotProduct' xs ys = sum (zipWith (*) xs ys) -- equivalent definition
magnitude xs = sqrt $ dotProduct xs xs
```
- ▶

```
divides n i = (n `mod` i) == 0
isPrime n = not $ any (divides n) [1..n]
isPrime' n = not (any (divides n) [1..n]) -- equivalent definition
```

Examples with numbers (continued)

- ▶

```
fibonaccis = 1 : 1 : zipWith (+) fibonaccis (tail fibonaccis)
```
- ▶

```
approxDerivative epsilon f x = (f (x+epsilon) - f x) / epsilon
```
- ▶

```
stars n = replicate (floor n) '*'  
showLines = putStr . unlines  
prepend = zipWith ((++) . show)  
plot f xs = showLines $ prepend xs (map (stars . f) xs)
```

Execution of examples

```
ghci> :load haskell_lecture_numberslide.hs
[1 of 1] Compiling Main                ( haskell_lecture_numberslide.hs, interpreted )
Ok, modules loaded: Main.
ghci> :browse
factorial :: (Enum a, Num a) => a -> a
dotProduct :: Num a => [a] -> [a] -> a
magnitude :: Floating a => [a] -> a
divides :: Integral a => a -> a -> Bool
isPrime :: Integral a => a -> Bool
fibonacci :: Num a => [a]
approxDerivative :: Fractional a => a -> (a -> a) -> a -> a
stars :: RealFrac a => a -> [Char]
showLines :: [String] -> IO ()
prepend :: Show a => [a] -> [[Char]] -> [[Char]]
plot :: (RealFrac b, Show a) => (a -> b) -> [a] -> IO ()
ghci> map factorial [1..10]
[1,2,6,24,120,720,5040,40320,362880,3628800]
ghci> magnitude [1,1]
1.4142135623730951
ghci> take 10 $ filter isPrime [2..]
[2,3,5,7,11,13,17,19,23,29]
ghci> take 10 fibonacci
[1,1,2,3,5,8,13,21,34,55]
ghci> plot (^2) [0..7]
0.0
1.0*
2.0****
3.0*****
4.0*****
5.0*****
6.0*****
7.0*****
ghci> plot (approxDerivative 0.01 (^2)) [0..7]
0.0
1.0**
2.0****
3.0*****
4.0*****
5.0*****
6.0*****
7.0*****
ghci> █
```

Functions used in previous examples

```
($) :: (a -> b) -> a -> b
```

```
f $ x = f x
```

```
(.) :: (b -> c) -> (a -> b) -> (a -> c)
```

```
(f . g) x = f (g x)
```

```
not :: Bool -> Bool
```

```
any, all :: (a -> Bool) -> [a] -> Bool
```

```
zip :: [a] -> [b] -> [(a, b)]
```

```
zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]
```

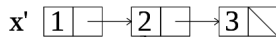
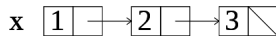
```
sum, product :: Num a => [a] -> a
```

```
replicate :: Int -> a -> [a]
```

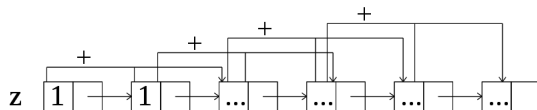
```
putStr, putStrLn :: String -> IO ()
```


Linked lists - Boxes and Pointers!

```
x = [1,2,3]
x' = 1:2:3:[]
y = 1 : 2 : y
z = 1 : 1 : zipWith (+) z (tail z)
```



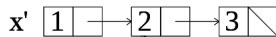
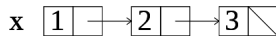
tail x'



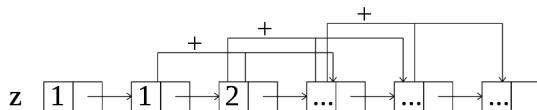
tail z

Linked lists - Boxes and Pointers!

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```



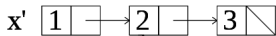
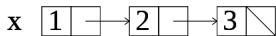
tail x'



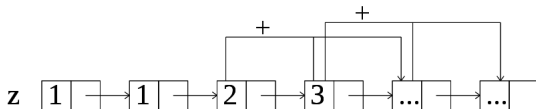
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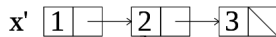
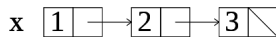
tail x'



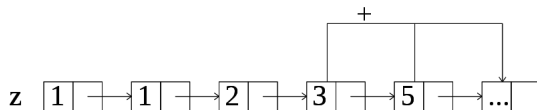
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Linked lists - Boxes and Pointers!

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```



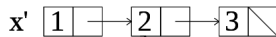
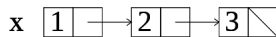
tail x'



tail z

Linked lists - Boxes and Pointers!

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x' = 1:2:3:[]
y = 1 : 2 : y
z = 1 : 1 : zipWith (+) z (tail z)
```



tail x'



tail z

Map and Reduce

```
map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x:xs) = (f x):(map f xs)

map' f = foldr ((:) . f) [] -- equivalent definition

foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f acc [] = acc
foldr f acc (x:xs) = f x (foldr f acc xs)
```

map applies its function to every element of a list.

foldr uses the provided function to reduce/aggregate the list into some value, starting from a seed.

Folds are very general, and can be used to implement many types of loops.

Definition of foldl in Python:

```
def foldl(f, init, xs):
    acc = init
    for x in xs:
        acc = f(acc, init)
    return acc
```

Reduce (continued)

More definitions of things in terms of `foldr`:

```
map f = foldr ((:) . f) []  
  
sum = foldr (+) 0  
  
product = foldr (*) 1  
  
concat = foldr (++) []  
  
all p = foldr ((&&) . p) True  
  
any p = foldr ((||) . p) False  
  
average = uncurry (/) . foldr (\x (s, t) -> (s+x, t+1)) (0, 0)
```

Typeclasses

With this definition:

```
instance Num a => Num [a] where
    (+) = zipWith (+)
    (*) = zipWith (*)
    (-) = zipWith (-)
    negate = map negate
    abs = map abs
    signum = map signum
    fromInteger = cycle . return . fromInteger
```

Expressions like these become valid:

```
1 + [1,2,3]
2 * [2..10]
-[6,2,8]
abs [-5..5]
```


Currying, Sections, and Infix

The following are all equivalent (via currying):

```
f g x y = zipWith g x y
f g x = zipWith g x
f g = zipWith g
f = zipWith
f g x = \y -> zipWith g x y
f g = \x y -> zipWith g x y
f = \g x y -> zipWith g x y
```

These are equivalent triples (via sections, and infix):

```
g x = x + 1
g = (+1)
g x = (+) x 1
```

```
h x = x 'mod' 2
h = ('mod' 2)
h x = mod x 2
```

```
i x = 2 'mod' x
i = (2 'mod')
i x = mod 2 x
```

Note that $h \neq i$, since mod is not commutative.

TIMTOWTDI - There's More Than One Way To Do It

Functions introduced:

```
ord :: Char -> Int
chr :: Int -> Char

interact :: (String -> String) -> IO ()

forever :: Monad m => m a -> m b
```

```
import Control.Monad
import Data.Char

caesarCipher shift = map (chr . ('mod' 256) . (+ shift) . ord)

main1 = forever $ do
    plaintext <- getLine
    let ciphertext = caesarCipher 3 plaintext
    putStrLn (caesarCipher 3 plaintext)

main2 = forever (getLine >>= (putStrLn . caesarCipher 3))

main3 = interact (caesarCipher 3)
```

Datatype declarations

```
data TrafficLight = RedLight | YellowLight | GreenLight
```

```
import qualified Data.Foldable as F

data List a = Node a (List a) | Empty

instance Foldable List where
    foldr f acc Empty = acc
    foldr f acc (Node x xs) = f x (F.foldr f acc xs)
```

```
import qualified Data.Map as M

data JSVal =
    JSUndefined
  | JSNull
  | JSBoolean Bool
  | JSString String
  | JSNumber Double
  | JSObject (M.Map String JSVal)

jsAdd (JSNumber x) (JSNumber y) = x + y
jsAdd (JSString x) (JSString y) = x ++ y
jsAdd _ _ = error "Runtime type error in jsAdd"
```

Binary Search Trees

```
{-# LANGUAGE NoMonomorphismRestriction #-}
import Data.Foldable as F

data BinaryTree a = Node a (BinaryTree a) (BinaryTree a) | Empty
    deriving (Eq, Show, Ord)

instance Foldable BinaryTree where
    foldr f acc Empty = acc
    foldr f acc (Node x l r) = F.foldr f (f x (F.foldr f acc r)) 1
```

```
#include <memory>
using std::shared_ptr;
using std::make_shared;

template <class A> struct BinaryTree {
    A elem;
    shared_ptr<BinaryTree<A>> left, right;
};

template <class A> shared_ptr<BinaryTree<A>> makeTree(A x,
    shared_ptr<BinaryTree<A>> l, shared_ptr<BinaryTree<A>> r) {
    return make_shared({elem: x, left: l, right: r});
}

template <class A> shared_ptr<BinaryTree<A>> emptyTree() {
    return shared_ptr<BinaryTree<A>>(nullptr);
}
```

Binary Search Trees (continued)

```
treeInsert x Empty = Node x Empty Empty
treeInsert x (Node y l r) = case compare x y of
    LT -> Node y (treeInsert x l) r
    GT -> Node y l (treeInsert x r)
    EQ -> Node y l r
```

```
template <class A> shared_ptr<BinaryTree<A>>
treeInsert(A x, shared_ptr<BinaryTree<A>> tree) {
    if(tree.get() == nullptr) {
        return makeTree(x, emptyTree<A>(), emptyTree<A>());
    } else if(x < tree->elem) {
        return makeTree(tree->elem, treeInsert(x, tree->left), tree->right);
    } else if(tree->elem < x) {
        return makeTree(tree->elem, tree->left, treeInsert(x, tree->right));
    } else { // equal
        return tree;
    }
}
```

Binary Search Trees (continued)

```
treeFind x Empty = Empty
treeFind x (Node y l r) = case compare x y of
    LT -> treeFind x l
    GT -> treeFind x r
    EQ -> Node y l r
```

```
template <class A> shared_ptr<BinaryTree<A>>
treeFind(A x, shared_ptr<BinaryTree<A>> tree) {
    if(tree.get() == nullptr) {
        return tree;
    } else if(x < tree->elem) {
        return treeFind(x, tree->left);
    } else if(tree->elem < x) {
        return treeFind(x, tree->right);
    } else { // equal
        return tree;
    }
}
```

Binary Search Trees (continued)

```
treeRemove x Empty = Empty
treeRemove x (Node y l r) = case compare x y of
  LT -> Node y (treeRemove x l) r
  GT -> Node y l (treeRemove x r)
  EQ -> case (l, r) of
    (Empty, Empty) -> Empty
    (_, Empty) -> l
    (Empty, _) -> r
    (_, _) -> let lMax = F.maximum l in
      Node lMax (treeRemove lMax l) r
```

Binary Search Trees (continued)

```
template <class A> shared_ptr<BinaryTree<A>>
treeRemove(A x, shared_ptr<BinaryTree<A>> tree) {
    if(tree.get() == nullptr) {
        return tree;
    } else if(x < tree->elem) {
        return treeRemove(x, tree->left);
    } else if(tree->elem < x) {
        return treeRemove(x, tree->right);
    } else { // equal
        if((tree->left.get() == nullptr) &&
            (tree->right.get() == nullptr)) {
            return emptyTree<A>();
        } else if(tree->right.get() == nullptr) {
            return tree->left;
        } else if(tree->left.get() == nullptr) {
            return tree->right;
        } else {
            // requires implementation of tree iterators, which was not done
            // for this presentation due to space constraints
            A lMax = std::max(begin(tree->left), end(tree->left));
            return makeTree(lMax, treeRemove(lMax, tree->left), tree->right);
        }
    }
}
```


Quicksort

```
{-# LANGUAGE NoMonomorphismRestriction #-}
import Control.Monad
import Control.Monad.ST
import Data.STRef
import qualified Data.Vector as V
import qualified Data.Vector.Mutable as VM

import Test.QuickCheck

naiveQuicksort [] = []
naiveQuicksort lst = naiveQuicksort less ++ eq ++ naiveQuicksort more where
    pivot = lst !! (length lst `div` 2)
    [less, eq, more] = map (\op -> filter ('op' pivot) lst) [(<), (==), (>)]

partitionByST cmp vec pivot lo hi = aux lo lo hi where
    aux i j n | j <= n = do
        a_j <- VM.read vec j
        case cmp a_j pivot of
            LT -> VM.swap vec i j >> aux (i+1) (j+1) n
            GT -> VM.swap vec j n >> aux i j (n-1)
            EQ -> aux i (j+1) n
    aux i j n = return (i, j)
```

Quicksort (continued)

```
quicksortByST cmp vec = aux 0 (VM.length vec - 1) where
  aux lo hi = when (lo < hi) $ do
    let pivotIdx = lo + ((hi-lo) 'div' 2)
    pivot <- VM.read vec pivotIdx
    (leftMid, rightMid) <- partitionByST cmp vec pivot lo hi
    aux lo leftMid
    aux rightMid hi

quicksortBy cmp vec = V.modify (quicksortByST cmp) vec

quicksort = quicksortBy compare

quicksortList = V.toList . quicksort . V.fromList

runTests = quickCheck naiveSort_matches_STSort where
  naiveSort_matches_STSort :: [Int] -> Bool
  naiveSort_matches_STSort x = naiveQuicksort x == quicksortList x
```

Questions?

Thanks

- ▶ RCOS
- ▶ Professor Goldschmidt
- ▶ Professor Moorthy
- ▶ Sean O'Sullivan